Automatic Differentiation

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Introduction

- Automatic Differentiation is a set of techniques to calculate the derivative of a specified function.
  - Capitalizes on the elementary capabilities that computers have (multiplication, subtract, add, divide)
  - Uses the chain run

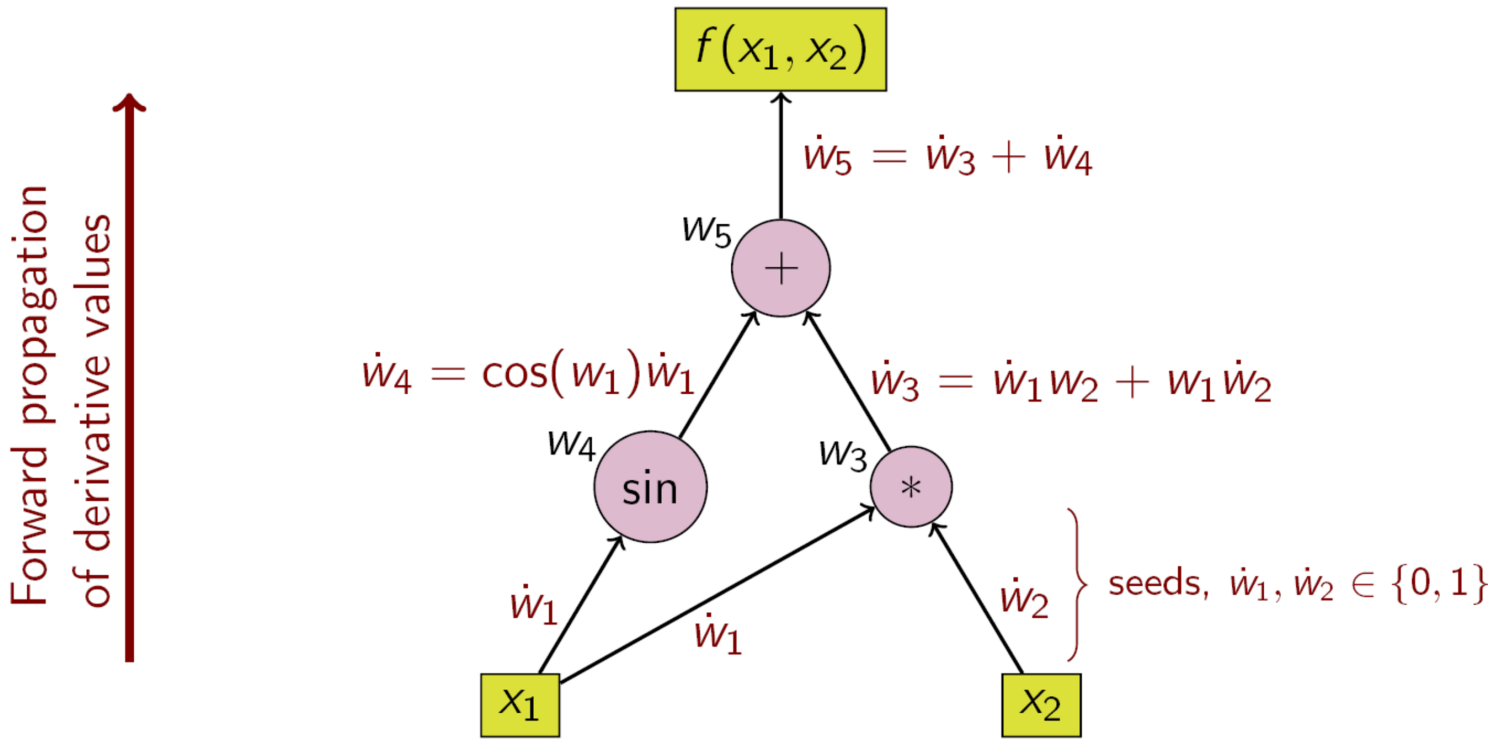
$$(f \circ g)' = (f' \circ g) \cdot g' \quad \text{or} \quad (f(g(x)))' = f'(g(x)) \cdot g'(x)$$

- Automatic Differentiation is not Symbolic Differentiation and it is not Numerical Differentiation
  - Symbolic Differentiation - software developed for the manipulation of mathematical expressions or objects
  - Numerical Differentiation - numerical analysis and algorithms for approximating functions or derivatives of functions

- Automatic Differentiation solves problems for calculating partial derivatives with respect to many inputs quickly
  - Can calculate higher order derivatives with less error than Symbolic/Numerical computation methods.

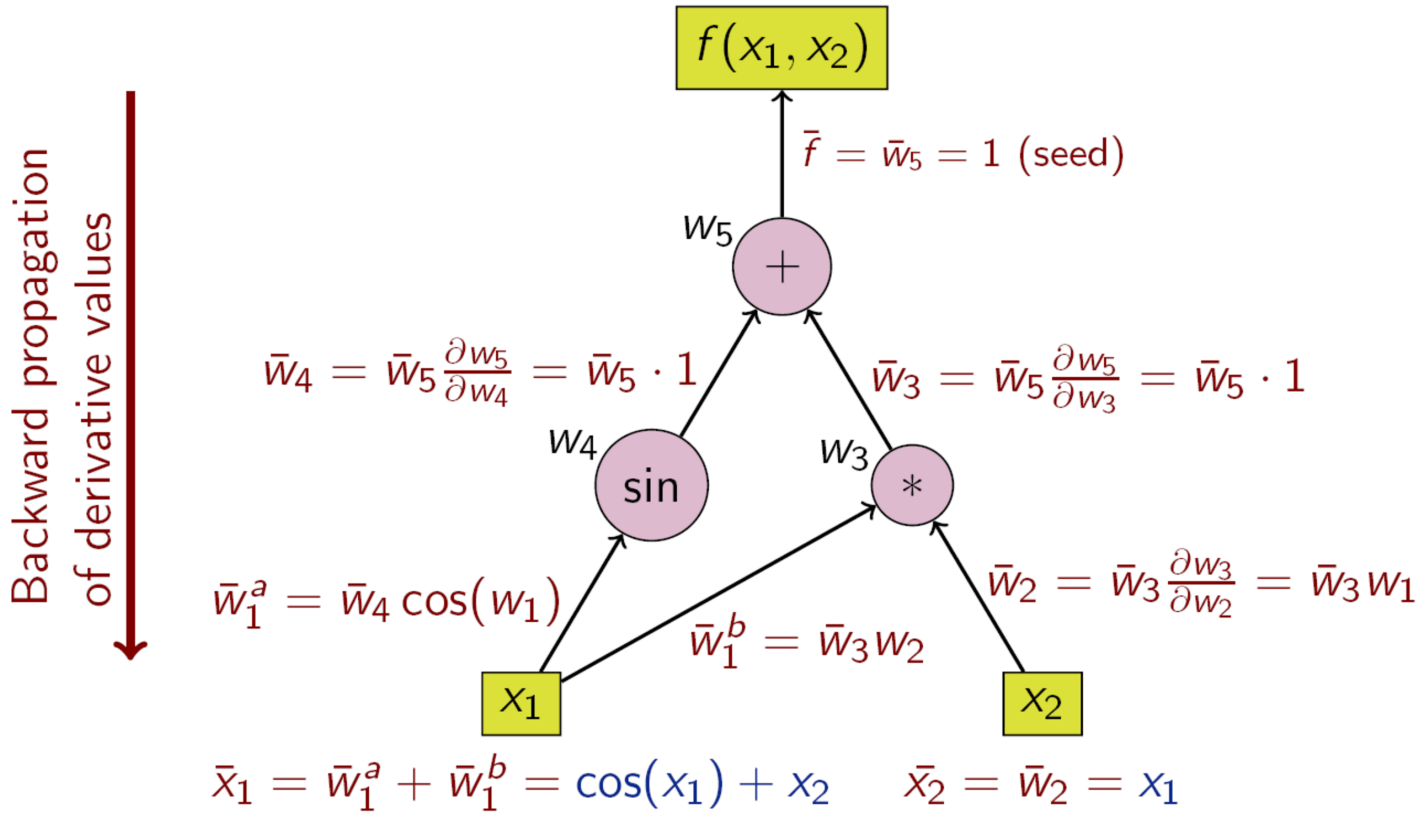
Forward Accumulation

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial w_1}\frac{\partial w_1}{\partial x} = \frac{\partial y}{\partial w_1}\left(\frac{\partial w_1}{\partial w_2}\frac{\partial w_2}{\partial x}\right) = \frac{\partial y}{\partial w_1}\left(\frac{\partial w_1}{\partial w_2}\left(\frac{\partial w_2}{\partial w_3}\frac{\partial w_3}{\partial x}\right)\right) = \ldots$$



Backward Accumulation

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial w_1}\frac{\partial w_1}{\partial x} = \left(\frac{\partial y}{\partial w_2}\frac{\partial w_2}{\partial w_1}\right)\frac{\partial w_1}{\partial x} = \left(\left(\frac{\partial y}{\partial w_3}\frac{\partial w_3}{\partial w_2}\right)\frac{\partial w_2}{\partial w_1}\right)\frac{\partial w_1}{\partial x} = \dots$$



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