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You must sign this form before taking the exam. You will not receive any credit if your signature (handwritten or digital) is not on this paper.

Name: Michael Mendez

Signature: Michael Mendez

Note: 14 questions on both sides, maximum 100 points.

1. [5 points] Let  $f$  be an image and  $g$  be a Gaussian filter. When we compute  $x$  image gradient, why do we want to apply Gaussian filter first, i.e.,  $\frac{\partial}{\partial x}(f * g)$ ?
  - We want to apply the Gaussian filter first in order to skip a step in the whole process, if we don't apply the Gaussian filter we will have an extra step with the kernel.
  
2. [5 points] Let  $f$  be an image and  $g$  be a Gaussian filter. When we compute  $x$  gradient, why can we first compute  $\frac{\partial}{\partial x}g$  and then convolve an image  $f$  with  $\frac{\partial}{\partial x}g$ ? What are the advantages?
  - We can compute the gradient first because since it is a convolution, this makes it so it has the property of associativity and it allows us to switch which process is first, since both lead to the same answer. Like the answer above, this allows us to skip a step and save us time.
  
3. [5 points] Let  $f$  be an image and  $g$  be a Gaussian filter. When we find the zero crossing on the  $x$  image gradient, why can we convolve an image  $f$  with  $\frac{\partial^2}{\partial x^2}g$  directly? What are the advantages?
  - After finding the zero crossing on the  $x$  image gradient, the result is the maximum points in the image. This in terms is already the result of the first and second derivative of the image. This is advantages because it allows us to see the results of the first and second derivative without actually calculating them and just finding the zero crossing.

4. [5 points] Canny edge detector. Which of the following statement is true? Explain your answers for full credits.

- a. Non-maximum suppression is used to select a pixel that is close to the true edge
- b. The edges found by a Canny edge detector are determined by the Gaussian kernel scale
- c. In hysteresis process, we start with low thresholds and then high thresholds
- d. a, b and c are correct
- e. a and b are correct

- e is true because a. when the pixel is not a maximum it detects the pixels around it and estimates which direction the edge is going in order to find the true edge. b. A gaussian kernel scale is needed in order to find an edge by Canny edge detector.

5. [10 points] For Harris point detector, the second moment matrix at a pixel  $p$  is computed by

$$M = \begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix} \text{ where } I_x, I_y \text{ are } x \text{ and } y \text{ image gradients. Let the first and second eigenvalues}$$

of  $M$  be  $\lambda_1, \lambda_2, \lambda_1 \geq \lambda_2$ . Explain why the eigenvalues can tell us the whether we find an edge, corner, or flat region at pixel  $p$ ?

- If the eigenvalues  $\lambda_1$  and  $\lambda_2$  are a large number it is indicated by Harris point detector that it is an corner, if eigen values  $\lambda_1$  and  $\lambda_2$  are both small and approximately the same value then by the same principle it is a flatregion . Also, if  $\lambda_1$  is a small number and  $I_1$  is a very large number then by the same principle then we will have a edge. This is all true because Harris point detector uses the shifting of points in order to determine if the findings is either an edge, corner, or flatregion.

6. [10 points] For Harris point detector, the corner response  $R = \det(M) - 0.04 \operatorname{tr}(M)^2$  where  $\det$

and  $\operatorname{tr}$  are the determinant and trace of a matrix. For a point where  $M = \begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix} =$

$\begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.9 \end{bmatrix}$  where  $I_x, I_y$  are  $x$  and  $y$  image gradients, is this point on an edge? a corner? or a flat region?

- The point is on a corner, this is because we have all the information to solve the equation and since the threshold is 0 and our answer is greater than zero, we get a corner. When solving for the equation we are left with  $\det(M) = 0.68$ ,  $0.04 * \operatorname{tr}(M)^2 = 0.1156$ . When plugging in these values in to the given equation, we are left with  $R = 0.5644$ . Note: there is a typo in the equation given, in lab 7 the trace is squared and the given it is not, so I added the square on the trace resulting in these numbers.

7. [5 points] Hough transform. Given one points  $(x, y) = (2, 6)$  in the image plane, write down the corresponding line in the Hough parameter space (describe a line in terms of  $m$  and  $b$ , your answer should be  $m = \underline{\hspace{1cm}}$  ).

- Hough transform is given by the general term  $y = mx + b$ . Since we only have a point the Hough parameter space is given for a point  $(x_0, y_0)$  as  $b = -x_0 * m + y_0$ . Solving in terms of  $m$  we are left with the equation  $m = -(b - y_0 / x_0)$ . Plugging in the points we are given, the Hough parameter space is a line in terms of  $m$  and  $b$  as:  $m = -(b - 6) / 2$ .

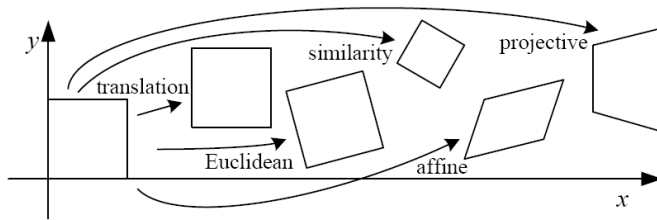
8. [5 points] Which of the following statements regarding line fitting is true?

- a. Line fitting with least squares minimization gives a closed form solution.
  - b. Line fitting with least squares minimization is not sensitive to outliers.
  - c. Hough transform can be efficiently applied to model fitting with a large number of parameters.
  - d. Model fitting with RANSAC does not the same answer every time.
  - e. a and d are correct
- e is true because first, line fitting does give a closed form solution. This is due to the fact that in order for least square minimization to have a better outcome, it needs a small dataset. This is due to the fact that if it is a large dataset  $X'X$  might not have an inverse making the problem unsolvable. Also, RANSAC doesn't have the same answer every time because it will always choose random samples through each iteration which will always cause it to have a different answer.

9. [5 points] Which of the following statements is true when we use RANSAC to fit data points with an objective function?

- a. Applicable to an objective function with more parameters than the Hough transform
  - b. Optimization parameters are easier to choose than Hough transform
  - c. Computational time grows quickly with fraction of outliers
  - d. Not good for getting multiple fits
  - e. a, b, and d are correct
- All options are true because a. since it chooses each parameter at random, it doesn't need to have a certain amount of parameters in order for it to have an objective function that works with the model. b. since it will always choose parameters in random you can always optimize any parameter with ease to make the result better. c. With more parameters the computational time grows quickly with the fraction of outliers. This is because it has to compute the set of inliers to the model from the whole data set, and it has to repeat this step until the model with the most inliers are found over all samples. d. this is not good for getting multiple fits, because the computational time grows quickly it will take too much time to get multiple fits.

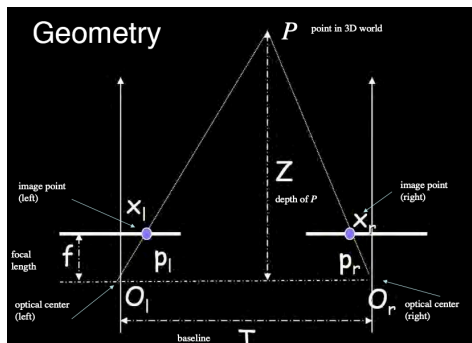
10. [5 points] Which of the following statements regarding 2D transformation are correct?



- a. Euclidean transformation has 3 parameters
  - b. Similarity transformation has 4 parameters
  - c. Affine transformation has 6 parameters
  - d. a, b, and c are true
  - e. a and c are true
- d is true because those are just the values that the transformations have, and it was given to us.

11. [10 points] Given a pair of stereo images. Show every step on how to compute depth  $Z =$

$$f \frac{T}{x_r - x_l}$$



Using Similar triangles the steps are:

$$T + X_i - X_r / Z - f = T / Z \text{ (Cross Multiply)}$$

$$Z(T + X_i - X_r) = T(Z - f) \text{ (Distribute out Z and T)}$$

$$ZT + ZX_i - ZX_r = ZT - Tf \text{ (Subtract ZT from both sides)}$$

$$ZX_i - ZX_r = -Tf \text{ (Take out like terms Z)}$$

$$Z(X_i - X_r) = -Tf \text{ (Divide to get Z alone)}$$

$$Z = -Tf / (X_i - X_r) \text{ (distribute the negative)}$$

$$Z = Tf / X_r - X_t$$

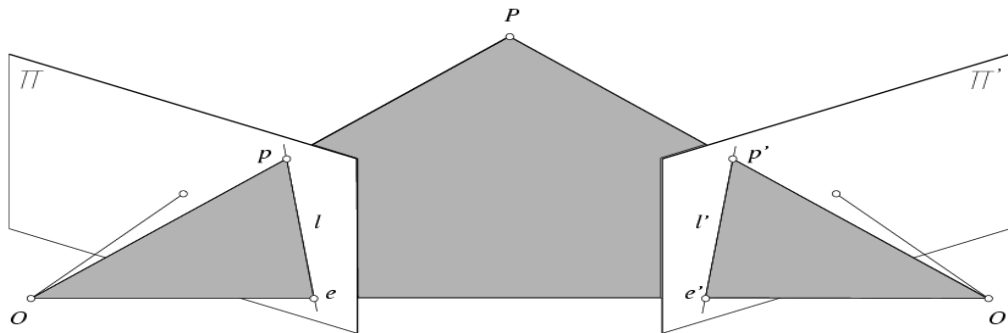
12. [10 points] Which of the following statements are true? Explain your answers.

- a. Given a pair of images from a calibrated stereo camera, for each pixel in one image, we can use the essential matrix to compute the corresponding epipolar line in the other image
  - b. Give a pair of images from an uncalibrated stereo camera, for each pixel in one image, we can use the essential matrix to compute the corresponding epipolar line in the other image.
  - c. Epipolar lines are always horizontal lines on an image
  - d. When we use larger window for search correspondence, we can capture more details
  - e. a and c are correct
- a is true because because since it is a calibrated stereo camera, we can use normalize imaging in order to compute the essential matrix directly and from there we can use the essential matrix to compute the corresponding epipolar line in the other image.

13. [10 points] Given a pair of left and right images for a calibrated stereo camera, which of the following statements for the calibrated stereo camera are true? Explain your answers.

- a. Given one point in one image, the corresponding point in the second image of a stereo pair is on a line passing through its epipole.
  - b. We can use the essential matrix to map a point in the left image to a line in the right image.
  - c. Depth is inversely proportional to disparity
  - d. a and c are correct
  - e. a, b and c are correct
- e is true because a. This is true because of the answer above. We can use an essential matrix that is computed directly with normalize imaging in order to use that essential matrix to find the corresponding epipolar line and find the given point in the corresponding second image. b. This is also true given the example in answer 12 and part a. also proves this to be correct. c. is correct because we can use similar triangles to derive depth based on disparity, which will lead to it being inversely proportional.

14. [10 points] Epipolar geometry. Given a point  $P$  in the world coordinate with two mapped points  $p$  and  $p'$  on two image planes with two optical centers  $O$  and  $O'$ . Derive the following equations.



$$\vec{Op} \cdot [\vec{OO'} \times \vec{O'p'}] = 0$$

Explain every step (what does the cross product of two vectors do and what does the inner product of two vectors do?) to earn full credit.

$$\begin{aligned} &Op \\ &OO' \\ &O'p' \\ &OO' \times O'p' \\ &Op \cdot [OO' \times O'p'] = 0 \end{aligned}$$