# 运行说明

## 推导本征能量方程

由于V为偶函数, 不失一般性可假设波函数解为奇函数或偶函数, 波函数解形式如下:

$$\text{偶数能级} u(x) = \begin{cases} Be^{kx}, x < -a \\ Acosqx, -a < x < a \\ Be^{-kx}, x > a \end{cases}$$

$$\text{奇数能级} u(x) = \begin{cases} De^{kx}, x < -a \\ Csinqx, -a < x < a \\ -De^{-kx}, x > a \end{cases}$$

无量纲化$\kappa = ka$, $\xi = \frac{x}{a}$, $\theta = qa$. 由于奇偶性, 仅需考虑势阱一侧波函数的连续性.

$$\begin{aligned}
&\text{偶数能级} \\
&u(\xi) = \begin{cases} Be^{\kappa\xi}, \xi < -1 \\ Acos\theta\xi, -1 < \xi < 1 \\ Be^{-\kappa\xi}, \xi > 1 \end{cases} \\
&u'(\xi) = \begin{cases} B\kappa e^{\kappa\xi}, \xi < -1 \\ -A\theta sin\theta\xi, -1 < \xi < 1 \\ -B\kappa e^{-\kappa\xi}, \xi > 1 \end{cases} \\
&u(-1_-) = u(-1_+)\text{得}Be^{-\kappa} = Acos\theta \\
&u'(-1_-) = u'(1_+)\text{得}B\kappa e^{-k} = A\theta sin(\theta) \\
&\text{故}\kappa = \theta tan(\theta), \text{即}\sqrt{\frac{|E|}{V_0 - |E|}} = tan\frac{\sqrt{2m(V_0 - |E|)}a}{\hbar} \\
&\text{奇数能级} \\
&u(\xi) = \begin{cases} De^{\kappa\xi}, \xi < -1 \\ Csin\theta\xi, -1 < \xi < 1 \\ -De^{-\kappa\xi}, \xi > 1 \end{cases} \\
&u'(\xi) = \begin{cases} D\kappa e^{\kappa\xi}, \xi < -1 \\ C\theta cos\theta\xi, -1 < \xi < 1 \\ D\kappa e^{-\kappa\xi}, \xi > 1 \end{cases} \\
&u(-1_-) = u(-1_+)\text{得}De^{-\kappa} = -Csin\theta \\
&u'(-1_-) = u'(-1_+)\text{得}D\kappa e^{-\kappa} = C\theta cos(\theta) \\
&\text{故} -\theta cot(\theta) = \kappa, \text{即} -\sqrt{\frac{|E|}{V_0 - |E|}} = cot\frac{\sqrt{2m(V_0 - |E|)}a}{\hbar}
\end{aligned}$$

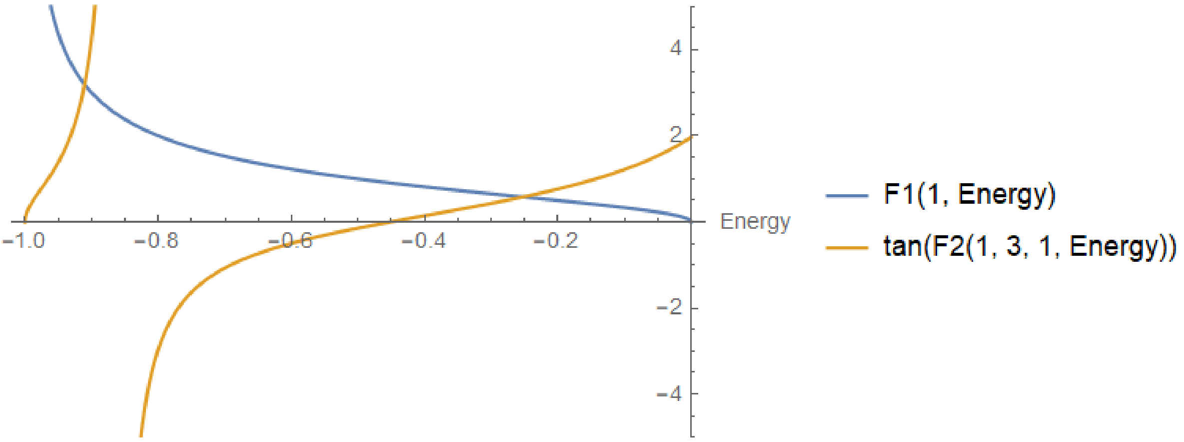
代入参数, 使用Mathematica对偶数和奇数能级的本征能量方程作图. 由于已选用自然单位制, 可略去$\hbar$. 由于粒子处于束缚态, 有E<0, 故|E|=-E, 两个方程化为:

$$\sqrt{\frac{-E}{V_0 + E}} = tan\frac{\sqrt{2m(V_0 + E)}a}{\hbar}$$

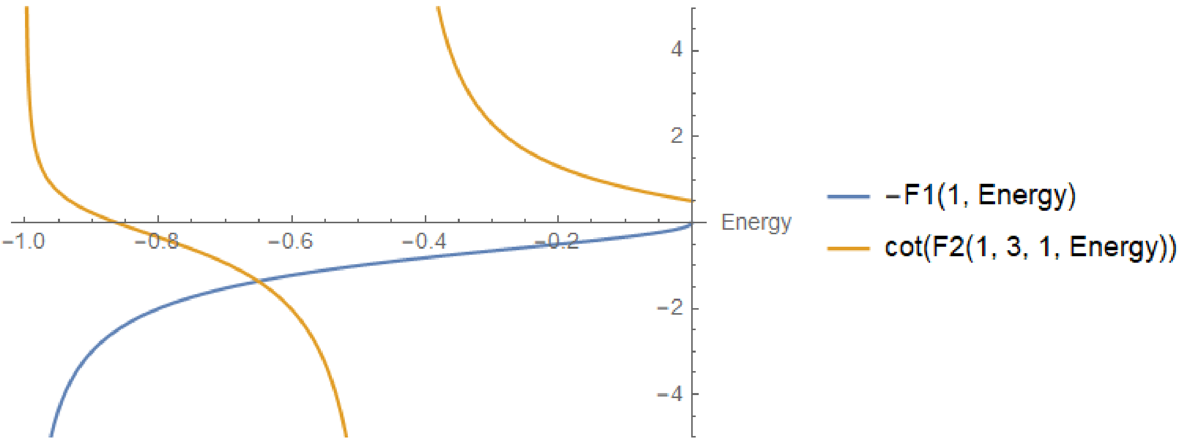
$$-\sqrt{\frac{-E}{V_0 + E}} = cot\frac{\sqrt{2m(V_0 + E)}a}{\hbar}$$

作图得:

偶数能级:




奇数能级:




故该系统本征能量共有三个解. 使用数值解法求出三个本征能量为:

$$\text{偶数能级}: E1 = -0.911GeV, E3 = -0.253GeV$$
$$\text{奇数能级}: E2 = -0.650GeV$$

代入方程并利用归一化条件求解系数, 得波函数表达式并作图.

$$\text{对于偶数能级}, A = \frac{Be^{-\kappa}}{cos\theta}$$

$$\text{定义}u_{even}(\xi) = \begin{cases} e^{\kappa\xi}, \xi < -1 \\ \frac{e^{-\kappa}}{cos\theta}cos(\theta\xi), -1 < \xi < 1 \\ e^{-\kappa\xi}, \xi > 1 \end{cases}$$

$$\text{归一化}: I_{even} = \int_{-\infty}^{\infty} u_{even}(\xi)d\xi \qquad \text{则}B = \sqrt{1/I_{even}}$$

$$\text{对于奇数能级}, C = \frac{-De^{-\kappa}}{sin\theta}$$

$$\text{定义}u_{odd}(\xi) = \begin{cases} e^{\kappa\xi}, \xi < -1 \\ \frac{-e^{-\kappa}}{sin\theta}sin(\theta\xi), -1 < \xi < 1 \\ -e^{-\kappa\xi}, \xi > 1 \end{cases}$$

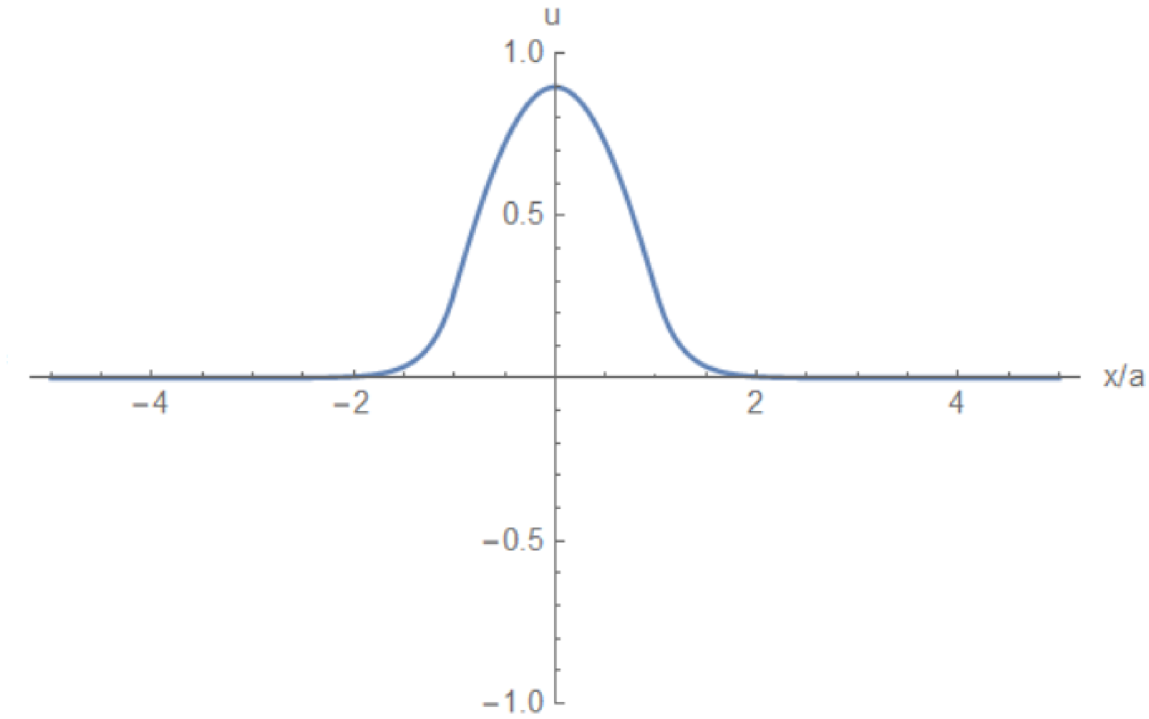
$$\text{归一化}: I_{odd} = \int_{-\infty}^{\infty} u_{odd}(\xi)d\xi$$

$$\text{则}D = \sqrt{1/I_{odd}k}$$

使用$\xi = x/a$无量纲化, 有:

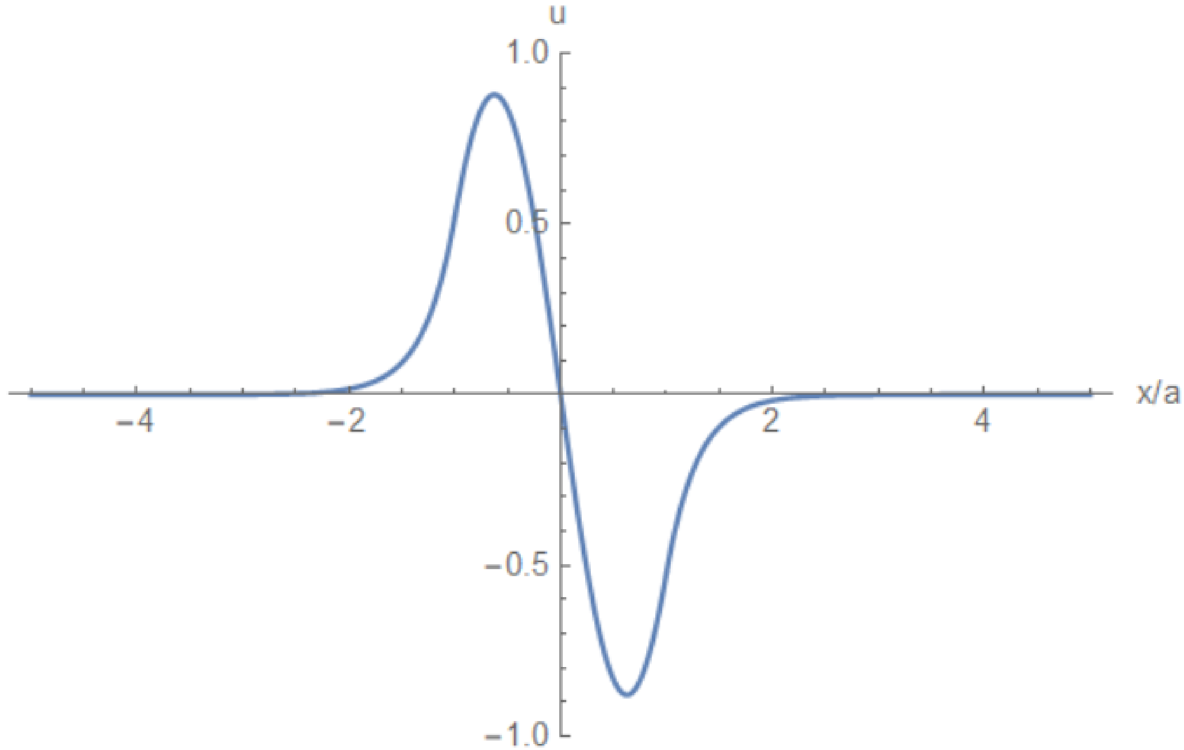
$E1=-0.911GeV$时, $u_1(x)=$

$$\begin{cases} 0.267521\times 2.71828^{4.0489+4.0489\xi} & \xi<-1. \\ 0.895509\,\mathrm{Cos}[1.26743\,\xi] & -1.<\xi<1. \\ 0.267521\times 2.71828^{4.0489-4.0489\xi} & \xi>1. \\ 0. & \mathrm{True} \end{cases}$$




$E2=-0.650GeV$时, $u_2(x)=$

$$\begin{cases} 2.71828^{3.42134\xi} & \xi<-1. \\ -0.0552444\,\mathrm{Sin}[2.50887\,\xi] & -1.<\xi<1. \\ -1.\times 2.71828^{-3.42134\xi} & \xi>1. \\ 0. & \mathrm{True} \end{cases}$$




$E3 = -0.253GeV$时,$u_3(x) =$

$$\begin{cases} 0.520191 \times 2.71828^{3.42134+3.42134\,\xi} & \xi < -1. \\ -0.879673\,\mathrm{Sin}[2.50887\,\xi] & -1. < \xi < 1. \\ -0.520191 \times 2.71828^{3.42134-3.42134\,\xi} & \xi > 1. \\ 0. & \mathrm{True} \end{cases}$$