

运行说明

代码思路

区域为 $(x, y) \in [0, \pi] \times [0, \pi]$, 已 $h = \frac{\pi}{90}$ 进行分割, 则

$$n = 0, 1, \dots, 90, \text{有} x_i = \frac{i\pi}{90}, y_i = \frac{j\pi}{90},$$

$$\begin{aligned} \phi_{i,j}^{(k+1)} &= \phi_{i,j}^{(k)} + \frac{\omega}{4}(\phi_{i+1,j}^{(k)} + \phi_{i-1,j}^{(k)} + \phi_{i,j+1}^{(k)} + \phi_{i,j-1}^{(k)} - h^2 q_{i,j} - 4\phi_{i,j}^{(k)}) \\ &= \phi_{i,j}^{(k)} + \frac{\omega}{4}(\phi_{i+1,j}^{(k)} + \phi_{i-1,j}^{(k)} + \phi_{i,j+1}^{(k)} + \phi_{i,j-1}^{(k)} - 4\phi_{i,j}^{(k)}) \end{aligned}$$

对边界点赋值并保持不变。

$$\text{即} \phi_{0,j} = \phi_{90,j} = \phi_{i,0} = 0$$

$$\phi_{i,90} = \sin(\frac{i\pi}{90})$$

内点取初值为零进行迭代。

$$\text{范数判断标准} ep = \frac{||\Delta^{(k)}||}{||\phi^{(k)}||}, \Delta_{i,j} = \phi_{i,j}^{(k)} - \phi_{i,j}^{(k-1)}$$

取 $ep < 10^{-6}$ 停止迭代。

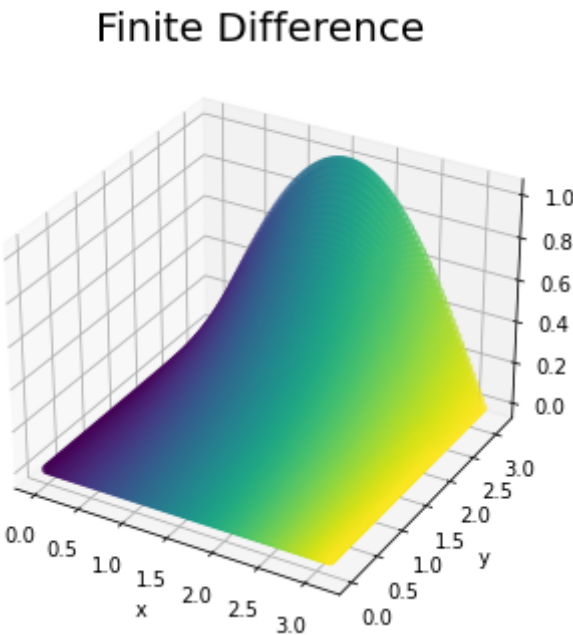
实现细节

见代码注释。对差分法所得解, 精确解, 并计算二者误差 (用无穷范数度量)

$$error = \frac{||\Delta||}{||\phi^{exact}||}, \Delta = \phi_{i,j}^{final} - \phi_{i,j}^{exact}$$

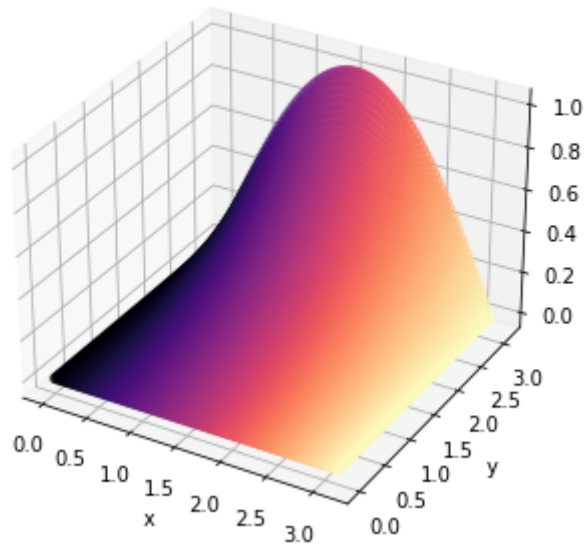
运行结果

有限差分法作图:



精确解作图

Exact Solution



范数误差

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error: 5.2057364068631924e-05
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二者结果相同