运行说明

代码思路

有限元素法

1.数学推导

$$
\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0
$$

$$
\varphi(x,0)=\varphi(x,1)=0,\varphi(0,y)=\varphi(1,y)=1
$$

对于泊松方程$\nabla^2\varphi=\frac{-\rho}{\epsilon}$和第一类边界条件$\varphi|_L=\varphi_0$

泛函极值问题：

$$
\delta I(\varphi)=0
$$

$$
I(\phi)=\int_{D(L)}[\frac{\epsilon}{2}(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2})-\rho\varphi]dxdy
$$

$$
\varphi|_L=\varphi_0
$$

对于此题可取$\rho=0,\epsilon=2,$则

$$
I(\phi)=\int_{D(L)}[\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}]dxdy
$$

整个区域$I(\phi)=\Sigma_e\int_e[\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}]dxdy=\Sigma I_e(\varphi)$

对于每一个元素定义顶点函数值矩阵和三角形的型函数,可求得泛函矩阵形式

$$
I_e(\varphi)=\frac{1}{2}(\Phi)_e^T(K)_e(\phi)_e
$$

对元素泛函求和并变分可得待求线性方程组$(K)(\Phi)=0$

2.对元素的划分

按三角形划分。考察第一类边界条件，编号时前n_0个为内部节点，n_0+1到n个位边界节点。
代入边界条件$\varphi_{n_0+i}=\varphi_0$后，方程改写为$(k_{11})(\Phi_1)=-(K_{12})(\Phi_2)$。
Φ_2即为边界点对应值，只需利用K求解内点值（超松弛迭代）

3.超松弛迭代求解线性方程组

原理同上一次作业。

实现细节

在主函数中设置边界，分割宽度等参数，计算边界点个数及坐标。定义函数，传入坐标等值，进行有限元素分割, 计算矩阵K和Ph_2。返回元素编号和矩阵。定义函数，传入编号和矩阵，使用超松弛迭代计算各元素对应值，根据编号和x,y坐标对应关系计算最终结果，返回该矩阵。定义函数，传入边界坐标分割和最终结果，进行作图。

优化松弛参数：改变ω值进行循环，不断缩小范围至迭代次数最小。
使用该范围内的ω计算结果并作图。

运行结果

取松弛参数[1.0,1.9]，间距0.05：

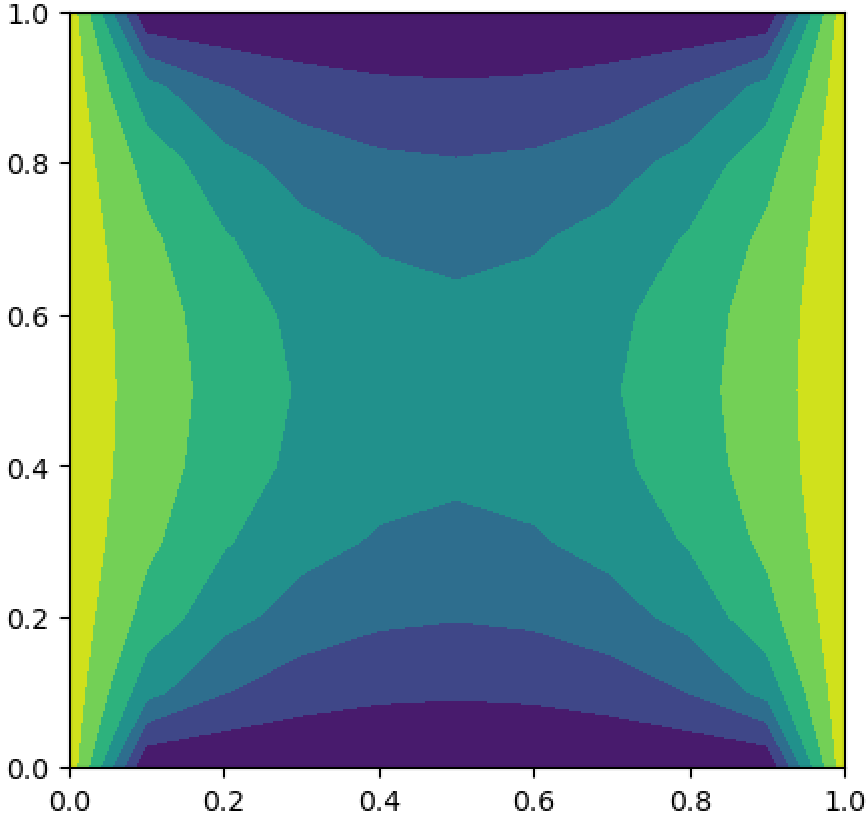
```
omega = 1.000000, Converges in 114 steps
omega = 1.050000, Converges in 104 steps
omega = 1.100000, Converges in 94 steps
omega = 1.150000, Converges in 86 steps
omega = 1.200000, Converges in 77 steps
omega = 1.250000, Converges in 70 steps
omega = 1.300000, Converges in 62 steps
omega = 1.350000, Converges in 55 steps
omega = 1.400000, Converges in 48 steps
omega = 1.450000, Converges in 42 steps
omega = 1.500000, Converges in 34 steps
omega = 1.550000, Converges in 28 steps
omega = 1.600000, Converges in 31 steps
omega = 1.650000, Converges in 40 steps
omega = 1.700000, Converges in 42 steps
omega = 1.750000, Converges in 52 steps
omega = 1.800000, Converges in 64 steps
omega = 1.850000, Converges in 87 steps
omega = 1.900000, Converges in 133 steps
```

取松弛参数[1.50，1.60]，间距0.005：

```
omega = 1.500000, Converges in 34 steps
omega = 1.505000, Converges in 33 steps
omega = 1.510000, Converges in 32 steps
omega = 1.515000, Converges in 31 steps
omega = 1.520000, Converges in 30 steps
omega = 1.525000, Converges in 29 steps
omega = 1.530000, Converges in 28 steps
omega = 1.535000, Converges in 27 steps
omega = 1.540000, Converges in 27 steps
omega = 1.545000, Converges in 27 steps
omega = 1.550000, Converges in 28 steps
omega = 1.555000, Converges in 27 steps
omega = 1.560000, Converges in 28 steps
omega = 1.565000, Converges in 29 steps
omega = 1.570000, Converges in 29 steps
omega = 1.575000, Converges in 29 steps
omega = 1.580000, Converges in 29 steps
omega = 1.585000, Converges in 30 steps
omega = 1.590000, Converges in 30 steps
omega = 1.595000, Converges in 31 steps
omega = 1.600000, Converges in 31 steps
```

由此可知最小收敛步数27，不妨取omega = 1.54

作图：


1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0