Fall
2025

46711
Stability and control in electric power systems

Assignment 2

Small-Signal Stability

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Learning Objectives of the Assignment

As you work your way through the assignment, you should bear in mind the learning objectives for this assignment, which are listed below:

- You should understand what a dynamic mode is and how it is characterized (by an equation)
- You should understand the overall objective of a power system stabilizer (which are the characteristics of the electrical torque component generated because of the PSS)
- You should know which are the different types of electromechanical modes (local vs. global oscillations)
- You should understand which information the mode shape of a given electromechanical mode provides. Furthermore, you should know how to obtain the mode shapes (which matrix contains the mode shapes).
- You should understand what participation factors and the participation matrix can be used for.
- You should understand how a power system stabilizer can be applied and tuned to improve the damping of electromechanical modes

When having completed this assignment, you should capable of answering each of the above, if the learning objectives of this assignments have been achieved. It is a good idea to address these points in your report (for example in your conclusion) to illustrate that you have met the assignment's learning objectives.

Introduction

In this assignment, a small signal analysis will be carried out on two different systems. The first system to be analyzed is a single-machine infinite bus system where it is investigated which effect the different forms of generator excitation control have on the system stability. The second system to be analyzed consists of four machines that aim to investigate different modes of electromechanical oscillations as well as the effect that a power system stabilizer has on the damping of these oscillations.

The mathematical model of the synchronous machines that is used throughout this assignment is of 6th order. In the machine model, the direct axis and quadrature axis transient and sub-transient voltages are used as state variables. The states used for the description of the machine behavior are listed below:

δ	=	The machine rotor angle $[rad]$
ω	=	The rotor speed in $[pu]$
$e_q\prime$	=	q-axis transient voltage $[pu]$
$e_d\prime$	=	d-axis transient voltage $[pu]$
$e_q\prime\prime$	=	q-axis subtransient voltage $[pu]$
$e_d\prime\prime$	=	d-axis subtransient voltage $[pu]$

The second part is relying on the machine flux linkages. The difference being, that the formulation is extended by including an additional damper winding in the q-axis. The state variable of the 6th order machine model based on flux linkages is indicated below. Generally, these models only differentiate in the choice of state variables, while they exhibit the same dynamic behaviour.

δ	=	The machine rotor angle $[rad]$
ω	=	The rotor speed in $[pu]$
ψ_f	=	field winding flux linkage $[pu]$
ψ_{kd1}	=	d-axis damper winding flux linkage $[pu]$
ψ_{kq1}	=	first q-axis damper winding flux linkage $[pu]$
ψ_{kq2}	=	second q-axis damper winding flux linkage $[pu]$

These states are involved in the mathematical model of the two systems that will be analyzed in this assignment. The system dynamics can be described in terms of non-linear differential equations:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) \tag{1}$$

When a small signal analysis is to be performed, the focus is on the response of the system to small disturbances for a given operating point of the system. If the initial operating point of the system is described by the initial state vector $\mathbf{x_0}$, the non-linear system equations can be linearized around $\mathbf{x_0}$ which results in the following linearized differential equations:

$$\Delta\dot{\mathbf{x}}(t) = \mathbf{A}\,\Delta\mathbf{x}(t) \tag{2}$$

An analysis of the linearized system matrix $\mathbf{A}$ can provide meaningful information about the small signal stability of the system. In this assignment, the linearized system matrix $\mathbf{A}$ for a given operating point will be provided for each of the different system configurations. Your task is to analyze these matrices in order to assess the small signal stability for the different system configurations.

1 Small signal analysis of a single machine infinite bus system

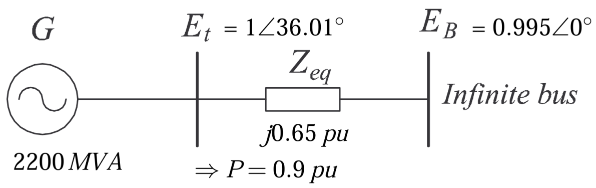

Figure 1: A single machine infinite bus system that is used in the first part of this assignment. The system will be analyzed for the operating conditions shown in the Figure where the power flowing from the machine into the infinite bus is 0.9 pu

Figure 1 shows a diagram of the system that will be analyzed during the first part of the assignment. The system consist of a single 2200MVA machine that is connected through an impedance to an infinite bus. The infinite bus represents a large and strong power system and therefore the voltage on the infinite bus (E_B) is considered to be constant and the system frequency is considered to be fixed during system disturbances.

An analysis of the system in Figure 1 will be performed for the following configurations:

1. When the generator is manually excited
2. When the generator excitation is controlled by an automatic voltage regulator (AVR)
3. When the generator excitation is controlled by an AVR and a power system stabilizer (PSS)

1.1 Manual excitation

In this part of the assignment, the system in Figure 1 will be analysed when the generator is manually excited. The only states involved in the system description are the states of the synchronous machine and and the state vector $\Delta\mathbf{x}$ has the following form:

$$\Delta\mathbf{x} = \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta e_q\prime \\ \Delta e_d\prime \\ \Delta e_q\prime\prime \\ \Delta e_d\prime\prime \end{bmatrix} \tag{3}$$

The file `system_q1a.mat` was uploaded together with this assignment text on DTU-Learn. The file contains the 6×6 system matrix `A_q1a` and a 6×1 vector `names_q1a` containing the names of the state variables of the system. There is also included an additional 6×1 name vector `latex_names_q1a` which contains the state variables names in LaTeX format.

Load the file `system_q1a.mat` into Python and answer the following:

Q.1.1.1 Find the eigenvalues of the system matrix and determine the damping, frequency of the oscillating modes.

1.2 Effects of Automatic Voltage Regulator (AVR)

In this part of the assignment an AVR is used to control the excitation of the machine. In order to describe the dynamic behavior of the AVR, three additional states are needed into the state vector $\Delta\mathbf{x}$. Rows 1-6 in the new state vector $\Delta\mathbf{x}$ are the same as in Q.1.1.1. The additional states are stored in rows 7-9 in the state vector. The additional states are listed below:

$$\Delta\mathbf{x}_{[7:9]} = \begin{bmatrix} \Delta v_{m,exc,G_1} \\ \Delta v_{r3,exc,G_1} \\ \Delta v_{f,exc,G_1} \end{bmatrix} \tag{4}$$

Where $\Delta v_{m,exc,G_1}$ represents the measured value of the terminal voltage, $\Delta v_{r3,exc,G_1}$ represents an internal state in the AVR and $\Delta v_{f,exc,G_1}$ is the output from the AVR (the excitation voltage).

The file `system_q1b.mat` contains the 9×9 system matrix `A_q1b` along with the 9×1 name vectors `names_q1b` and `latex_names_q1b`. Load the file `system_q1b.mat` into Python and answer the following:

Q.1.2.1 Find the eigenvalues of the system matrix. Determine as well the damping and the frequency of the oscillating modes. Identify which of the oscillating modes causes electromechanical oscillations. (Hint: there are two oscillating modes now. Use the participation matrix to identify the mode of electromechanical oscillations.)

Q.1.2.2 How does an AVR affect the damping and synchronizing torque components, when the generator output is high as in this case?

1.3 Effects of AVR together with a Power System Stabilizer (PSS)

In order to improve the damping of the electromechanical oscillations in the system, a PSS is added to the system. The aim of a PSS is to create an additional electrical torque component ΔT_e which is in phase with the speed deviations $\Delta\omega$ (if the machine accelerates, the electrical loading of the machine is increased and vice versa). In order to describe the dynamics of the PSS, four dynamic state variables are required. This results in a dynamic system of 13th order (a 13×13 system matrix) where the four additional states are as follows:

$$\Delta \mathbf{x}_{[10:13]} = \begin{bmatrix} \Delta v_{1,pss,G_1} \\ \Delta v_{2,pss,G_1} \\ \Delta v_{3,pss,G_1} \\ \Delta v_{ss,pss,G_1} \end{bmatrix} \tag{5}$$

where $\Delta v_{1,pss,G_1}$, $\Delta v_{2,pss,G_1}$ and $\Delta v_{3,pss,G_1}$ are internal states in the PSS and $\Delta v_{ss,pss,G_1}$ is the output signal from the PSS.

Rows 1-9 in the state vector are the same as in previous question. The file `system_q1c.mat` contains the 13×13 system matrix `A_q1c` along with the 13×1 name vectors `names_q1c` and `latex_names_q1c`.

Load the file `system_q1c.mat` into Python and answer the following:

Q.1.3.1 Find the eigenvalues of the system matrix and determine the damping and the frequency of the oscillating modes.

Q.1.3.2 Plot the time response of the rotor angle $\Delta\delta$, for all of the three cases (the system with manual excitation, the system with AVR and the system with AVR and PSS) when the system is perturbed at $t = 0$ by setting $\Delta\delta(0) = 5^\circ = 0.087266\,rad$. The time interval for the plots should be $0 - 5s$.

2 Small signal analysis of a two-area system

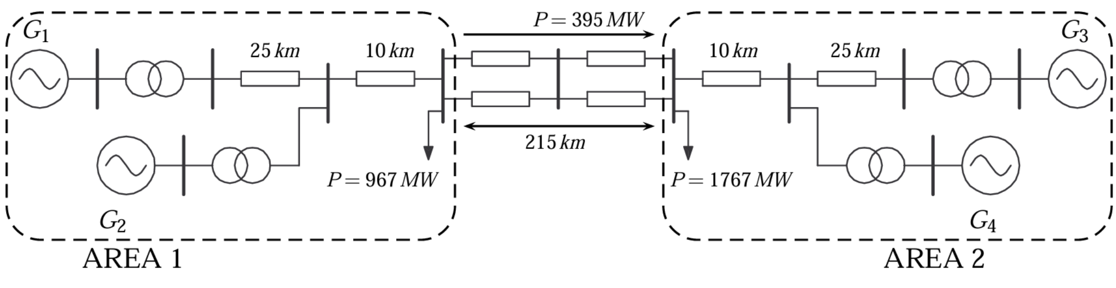

Figure 2: The simple two area system used in the second part of the assignment.

Figure 2 shows a diagram of the system that will be analyzed in the second part of the assignment. The system consist of four $900\,MVA$ generators (G_1, G_2, G_3 and G_4) where G_1 and G_2 are located in the same area (area 1) while G_3 and G_4 are located in area 2. The two areas are connected by two parallel 215 *km* long transmission lines. Your task is to analyze the small signal stability of the system which is highly loaded and with power transfer from area 1 to area 2 of $395\,MW$.

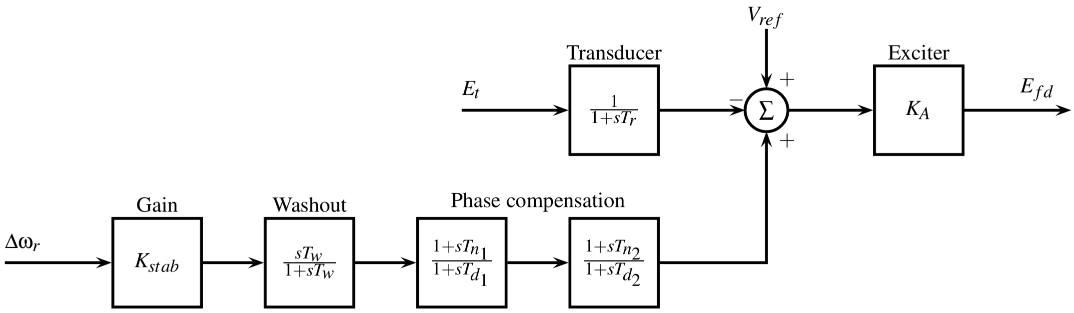

Figure 3: Excitation system including power system stabilizer

The general structure of the excitation system used within the second part of the assignment is shown in figure 3. In the initial state all the generators are equipped with an AVR but no power system stabilizers are installed. Hence, only the top half of the control system structure in figure 3 is active. Your next task will then be to identify locations where power system stabilizers have the largest impact and further identify suitable parameters for the gain and time constants of the stabilizer transfer function indicated in the bottom half of figure 3. The overall purpose is to ensure that all electromechanical system modes are adequately damped ($\zeta > 10\%$).

2.1 System without power system stabilizers

For the purpose of this assignment a simple AVR consisting of a transducer and a gain as indicated in figure 3 is used. The resulting system matrix $\mathbf{A}$ is therefore a 28×28 matrix since each of the machines is modeled by six state variables and 4 additional states are introduced due to the AVR. The components of the 28×1 state vector $\Delta\mathbf{x}$ are listed below:

$$\Delta\mathbf{x}_{[1:7]} = \begin{bmatrix} \Delta\delta_{G_1} \\ \Delta\omega_{G_1} \\ \Delta\psi_{f,G_1} \\ \Delta\psi_{kd1,G_1} \\ \Delta\psi_{kq1,G_1} \\ \Delta\psi_{kq2,G_1} \\ \Delta v_{m,exc,G_1} \end{bmatrix}, \quad \Delta\mathbf{x}_{[8:14]} = \begin{bmatrix} \Delta\delta_{G_2} \\ \Delta\omega_{G_2} \\ \Delta\psi_{f,G_2} \\ \Delta\psi_{kd1,G_2} \\ \Delta\psi_{kq1,G_2} \\ \Delta\psi_{kq2,G_2} \\ \Delta v_{m,exc,G_2} \end{bmatrix}, \quad \Delta\mathbf{x}_{[15:21]} = \begin{bmatrix} \Delta\delta_{G_3} \\ \Delta\omega_{G_3} \\ \Delta\psi_{f,G_3} \\ \Delta\psi_{kd1,G_3} \\ \Delta\psi_{kq1,G_3} \\ \Delta\psi_{kq2,G_3} \\ \Delta v_{m,exc,G_3} \end{bmatrix}, \quad \Delta\mathbf{x}_{[22:28]} = \begin{bmatrix} \Delta\delta_{G_4} \\ \Delta\omega_{G_4} \\ \Delta\psi_{f,G_4} \\ \Delta\psi_{kd1,G_4} \\ \Delta\psi_{kq1,G_4} \\ \Delta\psi_{kq2,G_4} \\ \Delta v_{m,exc,G_4} \end{bmatrix}$$

The file `system_q2.mat` contains the 28×28 system matrix (`A`) and two 28×1 vectors (`names_q2a` and `latex_names_q2a`) containing the names of the involved states. The remaining files will be used for the second part of Q2.

Load the file `system_q2.mat` into Python and answer the following:

Q.2.1.1 Construct the participation matrix $\mathbf{P}$ for the system. The table should be stored in appendix of your report.

(Hint: In order to make the documentation of the participation matrix easier, LaTeX users can use the Python function `latex_P_matrix()` (located under file sharing on DTU-Learn). Excel users can use the code of `latex_P_matrix()` as an inspiration to write a code that exports the participation matrix into Excel).

Q.2.1.2 Construct a table that shows the eigenvalues of the system, the frequency and the damping of the oscillating modes and a description of the dominating states in the mode. The table should be similar to the one at page 815 in the textbook.

Q.2.1.3 Identify the modes of electromechanical oscillation and determine whether one of these modes has a critically low damping ($\zeta \leq 0.1$). Furthermore, identify the zero-modes and explain why they appear in your results.

Q.2.1.4 Plot the shape of the oscillating electromechanical modes and describe the mode characteristics inter-area mode or local mode?). Explain how the plots should be interpreted. Hint: in order to plot the modes, the function `plot_phasor()` can be used. The function can be found in the file-sharing folder on Learn.)

Q.2.1.5 Plot the linearized time response of all four rotor angles when only the inter area mode is excited. How is the mode shape related to the time response?

2.2 Power system stabilizer design and performance

In this part of the assignment you are asked to design power system stabilizers to improve the small signal stability of the initial system. The aim is to achieve a minimum damping ratio of 10% for all electromechanical mode in the system.

Q.2.2.1 Use participation factors and residues to identify suitable locations for installing power system stabilizers. Keep in mind that there might be both, local and interarea modes with insufficient damping. This needs to be considered when choosing appropriate locations. Identify three locations that you will use further on. Hint: use `Identify_PSS_locations_help` code provided on Learn

Q.2.2.2 Use the `Python-control` library to obtain the phase response of the system for the frequency range of the electromechanical modes. Hint: use the code `PSS_design_help.py` provided on Learn.

Q.2.2.3 Identify suitable values for the phase compensation $T_{n_1}, T_{d_1}, T_{n_2}, T_{d_2}$ that maximize the efficiency of the PSS across the relevant frequency range. The gain of the stabilizer is fixed to $K_{stab} = 20$ and the time constant of the washout filter T_w should be in the range $10s \leq T_w \leq 30s$. As a starting point use the settings provided in the textbook:

$$K_{stab} = 20.0, \quad T_w = 10.0, \quad T_{n_1} = 0.05, \quad T_{d_1} = 0.02, \quad T_{n_2} = 3.0, \quad T_{d_2} = 5.4$$

Q.2.2.4 Test the performance of your solution by: (Use `Add_PSS_help.py` for this part)

1. Adding only a single stabilizer to the system. Repeat this process for each of your chosen locations and compare the system performance. Discuss the differences in performance in terms of which modes are affected and how their damping is affected depending on the location.
2. Adding two of your stabilizers to the system. Are these two stabilizers enough to damp all electromechanical system modes?
3. Comparing the small signal stability of the system using your stabilizers with the performance of stabilizers in the same locations using the initial values provided in Q.2.2.3.

A function adding power system stabilizers to the A matrix will be provided as part of the help files.

Q.2.2.5 Plot the time response of all four rotor angles for your final solution when only the inter area mode is excited. Discuss the effect of the power system stabilizer when comparing this plot to the one from [Q.2.1.5].

Guidelines for Writing of the Report

When writing the report for the assignment, the points listed below should be followed:

- The report should not exceed 15 pages (exclusive of the front page and appendices).
- The front page of the report should contain your name and student number.
- Your Python code used for the solution of this assignment has to be included in the appendix.
- The report shall contain the answer to each of the questions. The answers shall be clearly separated from each other and come in the same order as the questions in the assignment. It is not necessary to repeat the text from the assignment in your report.
- When answering the questions, both the results as well as an explanation of how you came up with your results should be included in the answer.
- Always when asked to provide plots of something (time responses, curves etc.) you must provide your interpretation of the figure.
- Make sure that the assignment's learning objectives are reflected in your report.
- The report should be ended by a conclusion explaining what you have learned by working on the assignment.
- When your reports are evaluated, points are awarded for the general setup of the report. We are looking for a professional look of the report, where among others the following is considered:
  - Readability (appropriate font sizes, margins etc.) and consistency in styles applied (same look for body text, headings, captions etc. throughout the report).
  - Presentation of equations should be of an appropriate quality (do not copy/paste equations as screenshot pictures from the lecture handouts).
  - Plots and figures should be of good quality (no fuzzy looking screenshots in the report). You can always ask the teachers how you can export figures of good quality in Python.
  - Figures and tables must have a label and caption (e.g., Figure 1: plot of active power with respect to ....).
  - If you are citing external material, you need to include a properly set-up reference list (you can use any citation style you prefer).
  - Does the report contain a conclusion section?
- You will also be awarded points for the general formulation of the report where focus is on:
  - The flow in the text when explaining how you came up with your answers and correct use of relevant terminology.
  - Interpretation and reflection of results when appropriate.
  - Grammar and spelling.

Guidelines for the Upload of the Report and Python Code

The report submission is carried out on DTU Learn, where the following shall be uploaded:

- PDF version of your report, with your Python code in the appendix
- Your Python code used for solving the assignment (the *.py files). The code shall include your comments where you explain the meaning behind the code