Comparing metaheuristic approaches to solving the quadratic assignment problem

COMP4240 project presentation

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Outline

Background and motivation

Introduced by Koopmans and Beckmann (1957)

'Assignment Problems and the Location of Economic Activities'

The quadratic assignment problem (QAP)

Assign manufacturing plants to locations in a way that maximises total revenue.

Account for the following complicating factors:

- The revenue of each plant is dependent on its location;
- Pairs of plants must transport a given number of commodity bundles between them per unit time;
- Transportation cost is proportional to distance.

Given the matrices:

- $[r_{ki}]$: revenue of plant k at location i
- $[a_{kl}]$: required commodity *flow* between plants k and l
- $[b_{ij}]$: cost of transport per unit flow between locations i and j

Find a permutation $\pi^* \in S_n$ that maximises the total revenue:

$$\pi^* = \max_{\pi} \left(\sum_k r_{k\pi(k)} - \sum_k \sum_l a_{kl} b_{\pi(k)\pi(l)} \right)$$

where $\pi(k) = i$ indicates that plant k is to be placed at location i.

Given the matrices:

- $[a_{kl}]$: required commodity *flow* between plants k and l
- $[b_{ij}]$: cost of transport per unit flow between locations i and j

Find a permutation $\pi^* \in S_n$ that minimises the transportation cost:

$$\pi^* = \min_{\pi} \sum_{k} \sum_{l} a_{kl} b_{\pi(k)\pi(l)}$$

where $\pi(k) = i$ indicates that plant k is to be placed at location i.

Difficulty

- NP-Hard.
- Contains the Travelling Salesman Problem as a special case.

"one of the most difficult problems in the NP-hard class"
(Loiola et al., 2007)

Applications of the QAP

The facilities layout problem (FLP):

- Optimally locate manufacturing plants to maximise revenue.
- Used by Koopmans and Beckmann to motivate the QAP.
- The most common application of the QAP (Loiola et al., 2007).

Loiola et al. (2007) surveys 365 papers published between 1957 and 2007.

Recent FLP applications

Samanta et al. (2015): Layout optimisation of a bus body manufacturing plant.

Feng and Su (2015): Layout of departments in a hospital.

Alguliyev et al. (2015): Unsupervised document summarisation.

Azab (2015): Machine features of a product in an optimal sequence in order to minimise handling time, given a set of precedence constraints between features.

The QAP in the literature

Memetic algorithm (Neri et al., 2012)

A metaheuristic combining a population-based approach with a local improvement method.

(Misevicius, 2012): Iterated tabu-search (ITS) - alternates *controlled chained mutations* with a variant of tabu-search. Includes several rules to deter ‘stagnant behaviour’.

(Harris et al., 2015): A memetic algorithm. Employs a ternary tree to structure the population. No mutation operator. Uses tabu-search and uniform crossover.

(Benlic and Hao, 2015): BMA - Breakout local search (BLS) with uniform crossover and chained mutation;

(Helal et al., 2015): TBH-PSO - Hierarchical Particle Swarm Optimisation (HPSO) with Tabu Local Search;

QAPLIB (Burkard et al., 1997)

A widely used library of QAP instances intended as test instances for the research community.

QAPLIB results in the literature:

Publication	Gap (%)	Time Limit	Time (s)
(Misevicius, 2012)	0.5	5 min	
(Harris et al., 2015)	0.049	5 min	3.93
(Benlic and Hao, 2015)	0.016	30 min	150
(Helal et al., 2015)	0.0919		44

Comparison of metaheuristics

This work compares four metaheuristics:

- Simulated Annealing (Kirkpatrick et al., 1983; Černỳ, 1985);
- Iterated tabu search (Misevicius, 2012);
- BMA (Benlic and Hao, 2015);
- A simple evolutionary algorithm.

Iterated tabu search (ITS) (Misevicius, 2012)

Alternates two steps:

1. *Controlled chained mutation*
    - Performs a *chained mutation*
    - Chooses the most *disruptive* mutation from a set
    - Controls mutation size and disruptiveness
2. *Improved robust tabu-search* Tabu search with extra rules to deter ‘stagnant behaviour’:
    - periodically performs steepest descent search
    - periodically ignores the tabu-list
    - halves all tabu-counts when a new local optimum is reached

BMA (Benlic and Hao, 2015)

BMA has the following features:

Local search: Breakout local search (BLS) (Benlic and Hao, 2013);

Crossover: ‘The’ uniform crossover (UX) operator;

Mutation: Chained sequence mutation.

‘Breakout local search for the quadratic assignment problem’ (Benlic and Hao, 2013).

Each iteration:

1. Perform steepest descent search using a 2-swap neighbourhood.
2. Perform a number of perturbation moves:
    - Either random moves or tabu search moves;
    - Perturbation type chosen based on last improving iteration;
    - Number of moves increases with visits to the same local optimum;

An evolutionary algorithm without a local improvement method:

- Maintains a population of N solutions;
- Each iteration:
  1. Remove all but the K best solutions from the population.
  2. Generate $N - K$ new individuals using either crossover or mutation, chosen randomly:
     - Chained sequence mutation
     - Uniform crossover

Experiment

The performance of the algorithms was compared on the full set of QAPLIB problems (Burkard et al., 1997).

- Ran each algorithm on all 135 of the 136 problem instances (the trivial instance `esc16f` was excluded).
- Time limit of 5 s per run.
- Record best solution, time that best solution was found, and actual time taken.
- Test significance of performance difference using Wilcoxon signed rank tests.

Preliminary results

- The algorithms were written in Rust and compiled with rustc 1.3.0 and cargo 0.5.0
- Experiments were run on an early 2013 Macbook Pro 15” with a 2.7 GHz Intel Core i7 processor and 16 GB of RAM
- The neighbourhood evaluation acceleration due to Frieze et al. (1989) was not used for ITS and BMA

Experiment results averaged over all QAPLIB problems:

Algorithm	Deviation (%)	BKV Time (s)	% BKV Solutions
SA	7.40	2.70	22.22
ITS	6.88	0.29	18.52
BMA	0.63	0.52	67.41
EA	10.54	0.60	9.63

Wilcoxon signed rank tests on the % deviation from the BKV:

Algorithms		p-value
BMA	ITS	<0.0001
BMA	EA	<0.0001
BMA	SA	<0.0001
ITS	EA	<0.0001
ITS	SA	0.9333
EA	SA	>0.9999

Conclusion

Conclusion

This work compared four metaheuristics based on recent approaches from the literature to solving the QAP.

- BMA significantly outperformed the other algorithms
- The evolutionary method without a tabu-list or local-improvement method was ineffective
- Simulated annealing performed surprisingly well / ITS may be have implemented incorrectly

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Questions?