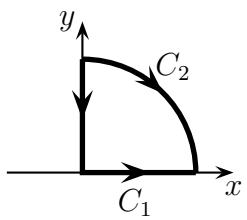
PHYS 320: Methods of Theoretical Physics I name: _____
Exam I

1. (*5 pts*) Without taking derivatives, find the first two nonzero terms in the Taylor approximation for the function $f(x) = \sqrt{3 - x + e^x}$ for small $x \ll 1$.

2. (*10 pts*) Consider the vector field $\vec{f} = (xy^2 + \sin y)\hat{\imath} + (yz^2 + \sin z)\hat{\jmath} + (zx^2 + \sin x)\hat{k}$.
   (a) Identify the regions of space that are a source, a sink, and neither for $\vec{f}$.
   (b) Consider a sphere of radius R centered on the origin. Using the outward normal, what is the surface integral of $\vec{f}$ over the surface of the sphere?

3. (*10 pts*) Consider the vector field $\vec{f} = xy^2\hat{\imath} + yz^2\hat{\jmath} + zx^2\hat{k}$.
   (a) Calculate the path integral for paths C_1 and C_2 on the right. Both paths lie in the x-y plane, start at $x = 0, y = R$ and end at $x = R, y = 0$. C_2 follows a circle of radius R.
   (b) Does your answer imply $\vec{f}$ is conservative or nonconservative or is it inconclusive? Explain



4. (*10 pts*) Suppose the altitude of the terrain is

$$f(x, y) = 2xy - x + 1 \ .$$

   (a) Standing at the origin (x=y=0), you take a tiny step toward the point $x = y = 1$. What is the slope at the origin along this path.
   (b) There is one critical point. Where is it? What is the height of the hill at this point?
   (c) Is the critical point a maximum, minimum, or saddle point?