Contents

1 Basic

1.1 vimrc

```
set nocp nu rnu cul ai ci cin si sta
set sc si ts=4 sw=4 sts=4 bs=2 et
set hls sm is ic scs bg=dark
set ru stal=2 ls=2 so=5 wrap lbr
filetype plugin indent on
syntax enable
colo delek
no ; :
no <C-l> :nohl<CR>
au filetype c,cpp ino <F9> <ESC>:w<CR>:!~/r.sh '%'<CR>
au filetype c,cpp  no <F9> <ESC>:w<CR>:!~/r.sh '%'<CR>
let leader = '\'
function! Tg()
    s,^\(\s*\)\?,\1// ,e
    s,^\(\s*\)\(// \?\)\{2},\1,e
endfunc
au filetype c,cpp no <leader><leader> :call Tg()<CR>
```

1.2 readchar

```cpp
#include <unistd.h>
const int S = 65536;
inline char RC() {
  static char buf[S], *p = buf, *q = buf;
  return p == q and (q =
      (p = buf) + read(0, buf, S)) == buf ? -1 : *p++;
}
inline int RI() {
  static char c; int a;
  while (((c = RC
      ()) < '0' or c > '9') and c != '-' and c != -1);
  if (c == '-') { a = 0;
    while ((c = RC()
        ) >= '0' and c <= '9') a *= 10, a -= c ^ '0'; }
  else { a = c ^ '0';
    while ((c = RC()
        ) >= '0' and c <= '9') a *= 10, a += c ^ '0'; }
  return a;
}
```

1.3 Black Magic

```cpp
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> // rb_tree
#include <ext/rope> // rope
#include
    <tr2/dynamic_bitset> // tr2::dynamic_bitset<> a(n)
using namespace __gnu_pbds;
using namespace __gnu_cxx; // rope
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
  heap h1, h2; // max heap
  h1.push(1), h1.push(3), h2.push(2), h2.push(4);
  h1.join(h2); // h1 = {1, 2, 3, 4}, h2 = {};
  tree<int, null_type, less<int>, rb_tree_tag
    , tree_order_statistics_node_update> st;
  tree<int, int, less<int>, rb_tree_tag
    , tree_order_statistics_node_update> mp;
  for (int x : {0, 3, 20, 50}) st.insert(x);
  assert(st.
      order_of_key(3) == 1 && st.order_of_key(4) == 2);
  assert(*st.find_by_order
      (2) == 20 && *st.lower_bound(4) == 20);
  rope<char> *root[10]; // nsqrt(n)
  root[0] = new rope<char>();
  root[1] = new rope<char>(*root[0]);
  // root[1]->insert(pos, 'a');
  // root[1]->at(pos); 0-base
  // root[1]->erase(pos, size);
}
// __int128_t,__float128_t
// for (int i = bs._Find_first
    (); i < bs.size(); i = bs._Find_next(i));
```

2 Graph

2.1 BCC Vertex*

```cpp
vector<int> G[N]; // 1-base
vector<int> nG[N * 2], bcc[N];
int low[N], dfn[N], Time;
int bcc_id[N], bcc_cnt; // 1-base
bool is_cut[N]; // whether is av
bool cir[N * 2];
int st[N], top;

void dfs(int u, int pa = -1) {
  int child = 0;
  low[u] = dfn[u] = ++Time;
  st[top++] = u;
  for (int v : G[u])
    if (!dfn[v]) {
      dfs(v, u), ++child;
      low[u] = min(low[u], low[v]);
      if (dfn[u] <= low[v]) {
        is_cut[u] = 1;
        bcc[++bcc_cnt].clear();
        int t;
        do {
          bcc_id[t = st[--top]] = bcc_cnt;
          bcc[bcc_cnt].eb(t);
        } while (t != v);
```

```
        bcc_id[u] = bcc_cnt;
        bcc[bcc_cnt].eb(u);
      }
    } else if (dfn[v] < dfn[u] && v != pa)
      low[u] = min(low[u], dfn[v]);
  if (pa == -1 && child < 2) is_cut[u] = 0;
}

void bcc_init(int n) { // TODO: init {nG, cir}[1..2n]
  Time = bcc_cnt = top = 0;
  for (int i = 1; i <= n; ++i)
    G[i].clear(), dfn[i] = bcc_id[i] = is_cut[i] = 0;
}

void bcc_solve(int n) {
  for (int i = 1; i <= n; ++i)
    if (!dfn[i]) dfs(i);
  // block-cut tree
  for (int i = 1; i <= n; ++i)
    if (is_cut[i])
      bcc_id[i] = ++bcc_cnt, cir[bcc_cnt] = 1;
  for (int i = 1; i <= bcc_cnt && !cir[i]; ++i)
    for (int j : bcc[i])
      if (is_cut[j])
        nG[i].eb(bcc_id[j]), nG[bcc_id[j]].eb(i);
}
```

2.2 Bridge*

```
int low[N], dfn[N], Time; // 1-base
vector<pii> G[N], edge;
vector<bool> is_bridge;

void init(int n) {
  Time = 0;
  for (int i = 1; i <= n; ++i)
    G[i].clear(), low[i] = dfn[i] = 0;
}

void add_edge(int a, int b) {
  G[a].eb(pii(b, SZ(edge))), G[b].eb(pii(a, SZ(edge)));
  edge.eb(pii(a, b));
}

void dfs(int u, int f) {
  dfn[u] = low[u] = ++Time;
  for (auto i : G[u])
    if (!dfn[i.X])
      dfs(i.X, i.Y), low[u] = min(low[u], low[i.X]);
    else if (i.Y != f) low[u] = min(low[u], dfn[i.X]);
  if (low[u] == dfn[u] && f != -1) is_bridge[f] = 1;
}

void solve(int n) {
  is_bridge.resize(SZ(edge));
  for (int i = 1; i <= n; ++i)
    if (!dfn[i]) dfs(i, -1);
}
```

2.3 2SAT (SCC)*

```
struct SAT { // 0-base
  int low[N], dfn[N], bln[N], n, Time, nScc;
  bool instack[N], istrue[N];
  stack<int> st;
  vector<int> G[N], SCC[N];
  void init(int _n) {
    n = _n; // assert(n * 2 <= N);
    for (int i = 0; i < n + n; ++i) G[i].clear();
  }
  void add_edge(int a, int b) { G[a].eb(b); }
  int rv(int a) {
    if (a >= n) return a - n;
    return a + n;
  }
  void add_clause(int a, int b) {
    add_edge(rv(a), b), add_edge(rv(b), a);
  }
  void dfs(int u) {
    dfn[u] = low[u] = ++Time;
    instack[u] = 1, st.push(u);
    for (int i : G[u])
      if (!dfn[i])
        dfs(i), low[u] = min(low[i], low[u]);
      else if (instack[i] && dfn[i] < dfn[u])
        low[u] = min(low[u], dfn[i]);
    if (low[u] == dfn[u]) {
      int tmp;
      do {
        tmp = st.top(), st.pop();
        instack[tmp] = 0, bln[tmp] = nScc;
      } while (tmp != u);
      ++nScc;
    }
  }
  bool solve() {
    Time = nScc = 0;
    for (int i = 0; i < n + n; ++i)
      SCC[i].clear(), low[i] = dfn[i] = bln[i] = 0;
    for (int i = 0; i < n + n; ++i)
      if (!dfn[i]) dfs(i);
    for (int i = 0; i < n + n; ++i) SCC[bln[i]].eb(i);
    for (int i = 0; i < n; ++i) {
      if (bln[i] == bln[i + n]) return false;
      istrue[i] = bln[i] < bln[i + n];
      istrue[i + n] = !istrue[i];
    }
    return true;
  }
};
```

2.4 MinimumMeanCycle*

```
int road[N][N]; // input here
struct MinimumMeanCycle {
  int dp[N + 5][N], n;
  pii solve() {
    int a = -1, b = -1, L = n + 1;
    for (int i = 2; i <= L; ++i)
      for (int k = 0; k < n; ++k)
        for (int j = 0; j < n; ++j)
          dp[i][j] =
            min(dp[i - 1][k] + road[k][j], dp[i][j]);
    for (int i = 0; i < n; ++i) {
      if (dp[L][i] >= INF) continue;
      int ta = 0, tb = 1;
      for (int j = 1; j < n; ++j)
        if (dp[j][i] < INF &&
          ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
          ta = dp[L][i] - dp[j][i], tb = L - j;
      if (ta == 0) continue;
      if (a == -1 || a * tb > ta * b) a = ta, b = tb;
    }
    if (a != -1) {
      int g = __gcd(a, b);
      return pii(a / g, b / g);
    }
    return pii(-1LL, -1LL);
  }
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
      for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
  }
};
```

2.5 Virtual Tree*

```
vector<int> vG[N];
int top, st[N];

void insert(int u) {
  if (top == -1) return st[++top] = u, void();
  int p = LCA(st[top], u);
  if (p == st[top]) return st[++top] = u, void();
  while (top >= 1 && dep[st[top - 1]] >= dep[p])
    vG[st[top - 1]].eb(st[top]), --top;
  if (st[top] != p)
    vG[p].eb(st[top]), --top, st[++top] = p;
  st[++top] = u;
}

void reset(int u) {
  for (int i : vG[u]) reset(i);
  vG[u].clear();
}

void solve(vector<int> &v) {
  top = -1;
  sort(ALL(v),
    [&](int a, int b) { return dfn[a] < dfn[b]; });
  for (int i : v) insert(i);
```

```cpp
  while (top > 0) vG[st[top - 1]].eb(st[top]), --top;
  // do something
  reset(v[0]);
}
```

2.6 Maximum Clique Dyn*

```cpp
struct MaxClique { // fast when N <= 100
  bitset<N> G[N], cs[N];
  int ans, sol[N], q, cur[N], d[N], n;
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) G[i].reset();
  }
  void add_edge(int u, int v) {
    G[u][v] = G[v][u] = 1;
  }
  void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
    if (l < 4) {
      for (int i : r) d[i] = (G[i] & mask).count();
      sort(ALL(r)
          , [&](int x, int y) { return d[x] > d[y]; });
    }
    vector<int> c(SZ(r));
    int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
    cs[1].reset(), cs[2].reset();
    for (int p : r) {
      int k = 1;
      while ((cs[k] & G[p]).any()) ++k;
      if (k > rgt) cs[++rgt + 1].reset();
      cs[k][p] = 1;
      if (k < lft) r[tp++] = p;
    }
    for (int k = lft; k <= rgt; ++k)
      for (int p = cs[k]._Find_first
          (); p < N; p = cs[k]._Find_next(p))
        r[tp] = p, c[tp] = k, ++tp;
    dfs(r, c, l + 1, mask);
  }
  void dfs(vector<
      int> &r, vector<int> &c, int l, bitset<N> mask) {
    while (!r.empty()) {
      int p = r.back();
      r.pb(), mask[p] = 0;
      if (q + c.back() <= ans) return;
      cur[q++] = p;
      vector<int> nr;
      for (int i : r) if (G[p][i]) nr.eb(i);
      if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
      else if (q > ans) ans = q, copy_n(cur, q, sol);
      c.pb(), --q;
    }
  }
  int solve() {
    vector<int> r(n);
    ans = q = 0, iota(ALL(r), 0);
    pre_dfs(r, 0, bitset<N>(string(n, '1')));
    return ans;
  }
};
```

2.7 Minimum Steiner Tree*

```cpp
struct SteinerTree { // 0-base
  int n, dst[N][N], dp[1 << T][N], tdst[N];
  int vcst[N]; // the cost of vertexs
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) {
      fill_n(dst[i], n, INF);
      dst[i][i] = vcst[i] = 0;
    }
  }
  void chmin(int &x, int val) {
    x = min(x, val);
  }
  void add_edge(int ui, int vi, int wi) {
    chmin(dst[ui][vi], wi);
  }
  void shortest_path() {
    for (int k = 0; k < n; ++k)
      for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
          chmin(dst[i][j], dst[i][k] + dst[k][j]);
  }
  int solve(const vector<int>& ter) {
    shortest_path();
    int t = SZ(ter), full = (1 << t) - 1;
    for (int i = 0; i <= full; ++i)
      fill_n(dp[i], n, INF);
    copy_n(vcst, n, dp[0]);
    for (int msk = 1; msk <= full; ++msk) {
      if (!(msk & (msk - 1))) {
        int who = __lg(msk);
        for (int i = 0; i < n; ++i)
          dp[msk
              ][i] = vcst[ter[who]] + dst[ter[who]][i];
      }
      for (int i = 0; i < n; ++i)
        for (int sub = (
            msk - 1) & msk; sub; sub = (sub - 1) & msk)
          chmin(dp[msk][i],
              dp[sub][i] + dp[msk ^ sub][i] - vcst[i]);
      for (int i = 0; i < n; ++i) {
        tdst[i] = INF;
        for (int j = 0; j < n; ++j)
          chmin(tdst[i], dp[msk][j] + dst[j][i]);
      }
      copy_n(tdst, n, dp[msk]);
    }
    return *min_element(dp[full], dp[full] + n);
  }
}; // O(V 3^T + V^2 2^T)
```

2.8 Dominator Tree*

```cpp
struct dominator_tree { // 1-base
  vector<int> G[N], rG[N];
  int n, pa[N], dfn[N], id[N], Time;
  int semi[N], idom[N], best[N];
  vector<int> tree[N]; // dominator_tree
  void init(int _n) {
    n = _n;
    for (int i = 1; i <= n; ++i)
      G[i].clear(), rG[i].clear();
  }
  void add_edge(int u, int v) {
    G[u].eb(v), rG[v].eb(u);
  }
  void dfs(int u) {
    id[dfn[u] = ++Time] = u;
    for (auto v : G[u])
      if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
  }
  int find(int y, int x) {
    if (y <= x) return y;
    int tmp = find(pa[y], x);
    if (semi[best[y]] > semi[best[pa[y]]])
      best[y] = best[pa[y]];
    return pa[y] = tmp;
  }
  void tarjan(int root) {
    Time = 0;
    for (int i = 1; i <= n; ++i) {
      dfn[i] = idom[i] = 0;
      tree[i].clear();
      best[i] = semi[i] = i;
    }
    dfs(root);
    for (int i = Time; i > 1; --i) {
      int u = id[i];
      for (auto v : rG[u])
        if (v = dfn[v]) {
          find(v, i);
          semi[i] = min(semi[i], semi[best[v]]);
        }
      tree[semi[i]].eb(i);
      for (auto v : tree[pa[i]]) {
        find(v, pa[i]);
        idom[v] =
          semi[best[v]] == pa[i] ? pa[i] : best[v];
      }
      tree[pa[i]].clear();
    }
    for (int i = 2; i <= Time; ++i) {
      if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
      tree[id[idom[i]]].eb(id[i]);
    }
  }
};
```

2.9 Minimum Arborescence*

```
/* TODO
DSU: disjoint set
- DSU(n), .boss(x), .Union(x, y)
min_heap<
    T, Info>: min heap for type {T, Info} with lazy tag
- .push({w, i}),
    .top(), .join(heap), .pop(), .empty(), .add_lazy(v)
*/
struct E { int s, t; int w; }; // 0-base
vector<int> dmst(const vector<E> &e, int n, int root) {
  vector<min_heap<int, int>> h(n * 2);
  for (int i = 0; i < SZ(e); ++i)
    h[e[i].t].push({e[i].w, i});
  DSU dsu(n * 2);
  vector<int> v(n * 2, -1), pa(n * 2, -1), r(n * 2);
  v[root] = n + 1;
  int pc = n;
  for (int i = 0; i < n; ++i) if (v[i] == -1) {
    for (int p = i; v[p]
        == -1 || v[p] == i; p = dsu.boss(e[r[p]].s)) {
      if (v[p] == i) {
        int q = p; p = pc++;
        do {
          h[q].add_lazy(-h[q].top().X);
          pa[q] = p, dsu.Union(p, q), h[p].join(h[q]);
        } while ((q = dsu.boss(e[r[q]].s)) != p);
      }
      v[p] = i;
      while (!h[p].
          empty() && dsu.boss(e[h[p].top().Y].s) == p)
        h[p].pop();
      if (h[p].empty()) return {}; // no solution
      r[p] = h[p].top().Y;
    }
  }
  vector<int> ans;
  for (int i = pc
      - 1; i >= 0; i--) if (i != root && v[i] != n) {
    for (int f = e[r[i]].t; ~f && v[f] != n; f = pa[f])
      v[f] = n;
    ans.eb(r[i]);
  }
  return ans; // default minimize, returns edgeid array
} // O(Ef(E)), f(E) from min_heap
```

2.10 Vizing's theorem*

```
namespace vizing { // returns
     edge coloring in adjacent matrix G. 1 - based
const int N = 105;
int C[N][N], G[N][N], X[N], vst[N], n;
void init(int _n) { n = _n;
  for (int i = 0; i <= n; ++i)
    for (int j = 0; j <= n; ++j)
      C[i][j] = G[i][j] = 0;
}
void solve(vector<pii> &E) {
  auto update = [&](int u)
  { for (X[u] = 1; C[u][X[u]]; ++X[u]); };
  auto color = [&](int u, int v, int c) {
    int p = G[u][v];
    G[u][v] = G[v][u] = c;
    C[u][c] = v, C[v][c] = u;
    C[u][p] = C[v][p] = 0;
    if (p) X[u] = X[v] = p;
    else update(u), update(v);
    return p;
  };
  auto flip = [&](int u, int c1, int c2) {
    int p = C[u][c1];
    swap(C[u][c1], C[u][c2]);
    if (p) G[u][p] = G[p][u] = c2;
    if (!C[u][c1]) X[u] = c1;
    if (!C[u][c2]) X[u] = c2;
    return p;
  };
  fill_n(X + 1, n, 1);
  for (int t = 0; t < SZ(E); ++t) {
    int u = E[t
        ].X, v0 = E[t].Y, v = v0, c0 = X[u], c = c0, d;
    vector<pii> L;
    fill_n(vst + 1, n, 0);
    while (!G[u][v0]) {
      L.emplace_back(v, d = X[v]);
      if (!C[v][c]) for (int a = SZ(
          L) - 1; a >= 0; --a) c = color(u, L[a].X, c);
      else if (!C[u][d]) for (int a = SZ(L
          ) - 1; a >= 0; --a) color(u, L[a].X, L[a].Y);
      else if (vst[d]) break;
      else vst[d] = 1, v = C[u][d];
    }
    if (!G[u][v0]) {
      for (; v; v = flip(v, c, d), swap(c, d));
      if (int a; C[u][c0]) {
        for (
            a = SZ(L) - 2; a >= 0 && L[a].Y != c; --a);
        for (; a >= 0; --a) color(u, L[a].X, L[a].Y);
      }
      else --t;
    }
  }
}
} // namespace vizing
```

2.11 Minimum Clique Cover*

```
struct Clique_Cover { // 0-base, O(n2^n)
  int co[1 << N], n, E[N];
  int dp[1 << N];
  void init(int _n) {
    n = _n, fill_n(dp, 1 << n, 0);
    fill_n(E, n, 0), fill_n(co, 1 << n, 0);
  }
  void add_edge(int u, int v) {
    E[u] |= 1 << v, E[v] |= 1 << u;
  }
  int solve() {
    for (int i = 0; i < n; ++i)
      co[1 << i] = E[i] | (1 << i);
    co[0] = (1 << n) - 1;
    dp[0] = (n & 1) * 2 - 1;
    for (int i = 1; i < (1 << n); ++i) {
      int t = i & -i;
      dp[i] = -dp[i ^ t];
      co[i] = co[i ^ t] & co[t];
    }
    for (int i = 0; i < (1 << n); ++i)
      co[i] = (co[i] & i) == i;
    fwt(co, 1 << n, 1);
    for (int ans = 1; ans < n; ++ans) {
      int sum = 0; // probabilistic
      for (int i = 0; i < (1 << n); ++i)
        sum += (dp[i] *= co[i]);
      if (sum) return ans;
    }
    return n;
  }
};
```

2.12 NumberofMaximalClique*

```
struct BronKerbosch { // 1-base
  int n, a[N], g[N][N];
  int S, all[N][N], some[N][N], none[N][N];
  void init(int _n) {
    n = _n;
    for (int i = 1; i <= n; ++i)
      for (int j = 1; j <= n; ++j) g[i][j] = 0;
  }
  void add_edge(int u, int v) {
    g[u][v] = g[v][u] = 1;
  }
  void dfs(int d, int an, int sn, int nn) {
    if (S > 1000) return; // pruning
    if (sn == 0 && nn == 0) ++S;
    int u = some[d][0];
    for (int i = 0; i < sn; ++i) {
      int v = some[d][i];
      if (g[u][v]) continue;
      int tsn = 0, tnn = 0;
      copy_n(all[d], an, all[d + 1]);
      all[d + 1][an] = v;
      for (int j = 0; j < sn; ++j)
        if (g[v][some[d][j]])
          some[d + 1][tsn++] = some[d][j];
      for (int j = 0; j < nn; ++j)
        if (g[v][none[d][j]])
          none[d + 1][tnn++] = none[d][j];
      dfs(d + 1, an + 1, tsn, tnn);
      some[d][i] = 0, none[d][nn++] = v;
```

```
    }
  }
  int solve() {
    iota(some[0], some[0] + n, 1);
    S = 0, dfs(0, 0, n, 0);
    return S;
  }
};
```

3 Data Structure

3.1 BIT kth*

```
vector<int> bit(N+1); // N = 2 ^ k
int query_kth(int k) {
    int res = 0;
    for (int i = N >> 1; i >= 1; i >>= 1)
        if (bit[res + i] < k)
            k -= bit[res += i];
    return res + 1;
}
```

3.2 Interval Container*

```
/* Add and
     remove intervals from a set of disjoint intervals.
 * Will merge the added interval with
      any overlapping intervals in the set when adding.
 * Intervals are [inclusive, exclusive). */
set<pii>::
    iterator addInterval(set<pii>& is, int L, int R) {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->X <= R) {
    R = max(R, it->Y);
    before = it = is.erase(it);
  }
  if (it != is.begin() && (--it)->Y >= L) {
    L = min(L, it->X);
    R = max(R, it->Y);
    is.erase(it);
  }
  return is.insert(before, pii(L, R));
}
void removeInterval(set<pii>& is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->Y;
  if (it->X == L) is.erase(it);
  else (int&)it->Y = L;
  if (R != r2) is.emplace(R, r2);
}
```

3.3 Centroid Decomposition*

```
struct Cent_Dec { // 1-base
  vector<pii> G[N];
  pii info[N]; // store info. of itself
  pii upinfo[N]; // store info. of climbing up
  int n, pa[N], layer[N], sz[N], done[N];
  int dis[__lg(N) + 1][N];
  void init(int _n) {
    n = _n, layer[0] = -1;
    fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
    for (int i = 1; i <= n; ++i) G[i].clear();
  }
  void add_edge(int a, int b, int w) {
    G[a].eb(pii(b, w)), G[b].eb(pii(a, w));
  }
  void get_cent(
    int u, int f, int &mx, int &c, int num) {
    int mxsz = 0;
    sz[u] = 1;
    for (pii e : G[u])
      if (!done[e.X] && e.X != f) {
        get_cent(e.X, u, mx, c, num);
        sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
      }
    if (mx > max(mxsz, num - sz[u]))
      mx = max(mxsz, num - sz[u]), c = u;
  }
  void dfs(int u, int f, int d, int org) {
    // if required, add self info or climbing info
    dis[layer[org]][u] = d;
    for (pii e : G[u])
      if (!done[e.X] && e.X != f)
        dfs(e.X, u, d + e.Y, org);
  }
  int cut(int u, int f, int num) {
    int mx = 1e9, c = 0, lc;
    get_cent(u, f, mx, c, num);
    done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
    for (pii e : G[c])
      if (!done[e.X]) {
        if (sz[e.X] > sz[c])
          lc = cut(e.X, c, num - sz[c]);
        else lc = cut(e.X, c, sz[e.X]);
        upinfo[lc] = pii(), dfs(e.X, c, e.Y, c);
      }
    return done[c] = 0, c;
  }
  void build() { cut(1, 0, n); }
  void modify(int u) {
    for (int a = u, ly = layer[a]; a;
         a = pa[a], --ly) {
      info[a].X += dis[ly][u], ++info[a].Y;
      if (pa[a])
        upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
    }
  }
  int query(int u) {
    int rt = 0;
    for (int a = u, ly = layer[a]; a;
         a = pa[a], --ly) {
      rt += info[a].X + info[a].Y * dis[ly][u];
      if (pa[a])
        rt -=
          upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
  }
};
```

3.4 LiChaoST*

```
struct L {
  int m, k, id;
  L() : id(-1) {}
  L(int a, int b, int c) : m(a), k(b), id(c) {}
  int at(int x) { return m * x + k; }
};
class LiChao { // maintain max
private:
  int n; vector<L> nodes;
  void insert(int l, int r, int rt, L ln) {
    int m = (l + r) >> 1;
    if (nodes[rt].id == -1)
      return nodes[rt] = ln, void();
    bool atLeft = nodes[rt].at(l) < ln.at(l);
    if (nodes[rt].at(m) < ln.at(m))
      atLeft ^= 1, swap(nodes[rt], ln);
    if (r - l == 1) return;
    if (atLeft) insert(l, m, rt << 1, ln);
    else insert(m, r, rt << 1 | 1, ln);
  }
  int query(int l, int r, int rt, int x) {
    int m = (l + r) >> 1; int ret = -INF;
    if (nodes[rt].id != -1) ret = nodes[rt].at(x);
    if (r - l == 1) return ret;
    if (x
       < m) return max(ret, query(l, m, rt << 1, x));
    return max(ret, query(m, r, rt << 1 | 1, x));
  }
public:
  LiChao(int n_) : n(n_), nodes(n * 4) {}
  void insert(L ln) { insert(0, n, 1, ln); }
  int query(int x) { return query(0, n, 1, x); }
};
```

3.5 Link cut tree*

```
struct Splay { // xor-sum
  static Splay nil;
  Splay *ch[2], *f;
  int val, sum, rev, size;
  Splay (int
       _val = 0) : val(_val), sum(_val), rev(0), size(1)
  { f = ch[0] = ch[1] = &nil; }
  bool isr()
  { return f->ch[0] != this && f->ch[1] != this; }
  int dir()
  { return f->ch[0] == this ? 0 : 1; }
```

```
  void setCh(Splay *c, int d) {
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  }
  void give_tag(int r) {
    if (r) swap(ch[0], ch[1]), rev ^= 1;
  }
  void push() {
    if (ch[0] != &nil) ch[0]->give_tag(rev);
    if (ch[1] != &nil) ch[1]->give_tag(rev);
    rev = 0;
  }
  void pull() {
    // take care of the nil!
    size = ch[0]->size + ch[1]->size + 1;
    sum = ch[0]->sum ^ ch[1]->sum ^ val;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
  }
} Splay::nil;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
  Splay *p = x->f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
  p->setCh(x->ch[!d], d);
  x->setCh(p, !d);
  p->pull(), x->pull();
}
void splay(Splay *x) {
  vector<Splay*> splayVec;
  for (Splay *q = x;; q = q->f) {
    splayVec.eb(q);
    if (q->isr()) break;
  }
  reverse(ALL(splayVec));
  for (auto it : splayVec) it->push();
  while (!x->isr()) {
    if (x->f->isr()) rotate(x);
    else if (x->dir() == x->f->dir())
      rotate(x->f), rotate(x);
    else rotate(x), rotate(x);
  }
}
Splay* access(Splay *x) {
  Splay *q = nil;
  for (; x != nil; x = x->f)
    splay(x), x->setCh(q, 1), q = x;
  return q;
}
void root_path(Splay *x) { access(x), splay(x); }
void chroot(Splay *x) {
  root_path(x), x->give_tag(1);
  x->push(), x->pull();
}
void split(Splay *x, Splay *y) {
  chroot(x), root_path(y);
}
void link(Splay *x, Splay *y) {
  root_path(x), chroot(y);
  x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
  split(x, y);
  if (y->size != 5) return;
  y->push();
  y->ch[0] = y->ch[0]->f = nil;
}
Splay* get_root(Splay *x) {
  for (root_path(x); x->ch[0] != nil; x = x->ch[0])
    x->push();
  splay(x);
  return x;
}
bool conn(Splay *x, Splay *y) {
  return get_root(x) == get_root(y);
}
Splay* lca(Splay *x, Splay *y) {
  access(x), root_path(y);
  if (y->f == nil) return y;
  return y->f;
}
void change(Splay *x, int val) {
  splay(x), x->val = val, x->pull();
}
int query(Splay *x, Splay *y) {
  split(x, y);
  return y->sum;
}
```

3.6 KDTree

```
namespace kdt {
int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn],
  yl[maxn], yr[maxn];
point p[maxn];
int build(int l, int r, int dep = 0) {
  if (l == r) return -1;
  function<bool(const point &, const point &)> f =
    [dep](const point &a, const point &b) {
      if (dep & 1) return a.x < b.x;
      else return a.y < b.y;
    };
  int m = (l + r) >> 1;
  nth_element(p + l, p + m, p + r, f);
  xl[m] = xr[m] = p[m].x;
  yl[m] = yr[m] = p[m].y;
  lc[m] = build(l, m, dep + 1);
  if (~lc[m]) {
    xl[m] = min(xl[m], xl[lc[m]]);
    xr[m] = max(xr[m], xr[lc[m]]);
    yl[m] = min(yl[m], yl[lc[m]]);
    yr[m] = max(yr[m], yr[lc[m]]);
  }
  rc[m] = build(m + 1, r, dep + 1);
  if (~rc[m]) {
    xl[m] = min(xl[m], xl[rc[m]]);
    xr[m] = max(xr[m], xr[rc[m]]);
    yl[m] = min(yl[m], yl[rc[m]]);
    yr[m] = max(yr[m], yr[rc[m]]);
  }
  return m;
}
bool bound(const point &q, int o, long long d) {
  double ds = sqrt(d + 1.0);
  if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
    q.y < yl[o] - ds || q.y > yr[o] + ds)
    return false;
  return true;
}
long long dist(const point &a, const point &b) {
  return (a.x - b.x) * 1ll * (a.x - b.x) +
    (a.y - b.y) * 1ll * (a.y - b.y);
}
void dfs(
  const point &q, long long &d, int o, int dep = 0) {
  if (!bound(q, o, d)) return;
  long long cd = dist(p[o], q);
  if (cd != 0) d = min(d, cd);
  if ((dep & 1) && q.x < p[o].x ||
    !(dep & 1) && q.y < p[o].y) {
    if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    if (~rc[o]) dfs(q, d, rc[o], dep + 1);
  } else {
    if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    if (~lc[o]) dfs(q, d, lc[o], dep + 1);
  }
}
void init(const vector<point> &v) {
  for (int i = 0; i < v.size(); ++i) p[i] = v[i];
  root = build(0, v.size());
}
long long nearest(const point &q) {
  long long res = 1e18;
  dfs(q, res, root);
  return res;
}
} // namespace kdt
```

4 Flow/Matching

4.1 Dinic

```
template <typename Cap = int>
struct Dinic { // 0-base
  struct Edge { int to, rev; Cap cap, fl; };
  const Cap INF = numeric_limits<Cap>::max() >> 1;
  vector<vector<Edge>> G;
  vector<int> dis, cur;
  int s, t, n;
```

```
  Cap dfs(int u, Cap cap) {
    if (u == t or !cap) return cap;
    for (int &i = cur[u]; i < SZ(G[u]); ++i) {
      Edge &e = G[u][i];
      if (dis[e.to] == dis[u] + 1 and e.fl != e.cap) {
        Cap df = dfs(e.to, min(e.cap - e.fl, cap));
        if (df) {
          e.fl += df;
          G[e.to][e.rev].fl -= df;
          return df;
        }
      }
    }
    dis[u] = -1;
    return 0;
  }
  bool bfs() {
    fill(ALL(dis), -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
      int tmp = q.front(); q.pop();
      for (Edge &e : G[tmp])
        if (!~dis[e.to] and e.fl != e.cap)
          dis[e.to] = dis[tmp] + 1, q.push(e.to);
    }
    return dis[t] != -1;
  }
  Cap maxflow(int _s, int _t) {
    s = _s, t = _t;
    Cap flow = 0, df;
    while (bfs()) {
      fill(ALL(cur), 0);
      while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
  }
  void init(int _n) {
    n = _n;
    G.assign(n, vector<Edge>());
    dis.resize(n), cur.resize(n);
  }
  void reset() {
    for (int i = 0; i < n; ++i)
      for (Edge &e : G[i]) e.fl = 0;
  }
  void add_edge(int u, int v, int cap) {
    G[u].eb(v, SZ(G[v]), cap, 0);
    G[v].eb(u, SZ(G[u]) - 1, 0, 0);
  }
};
```

4.2 Bipartite Matching

```
struct Bipartite_Matching { // 0-base
  int l, r;
  vector<int> mp, mq, dis, cur;
  vector<vector<int>> G;
  bool dfs(int u) {
    for (int &i = cur[u]; i < SZ(G[u]); ++i) {
      int e = G[u][i];
      if (mq[e] == l
          or (dis[mq[e]] == dis[u] + 1 and dfs(mq[e])))
        return mp[mq[e] = u] = e, 1;
    }
    return dis[u] = -1, 0;
  }
  bool bfs() {
    queue<int> q;
    fill(ALL(dis), -1);
    for (int i = 0; i < l; ++i)
      if (!~mp[i]) dis[i] = 0, q.push(i);
    while (!q.empty()) {
      int u = q.front(); q.pop();
      for (int e : G[u]) if (!~dis[mq[e]]) {
        dis[mq[e]] = dis[u] + 1, q.push(mq[e]);
      }
    }
    return dis[l] != -1;
  }
  int matching() {
    int res = 0;
    fill(ALL(mp), -1), fill(ALL(mq), l);
    while (bfs()) {
      fill(ALL(cur), 0);
      for (int i = 0; i < l; ++i)
        res += (!~mp[i] and dfs(i));
    }
    return res; // (i, mp[i] != -1)
  }
  void add_edge(int s, int t) { G[s].eb(t); }
  void init(int _l, int _r) {
    l = _l, r = _r;
    mp.resize(l), mq.resize(r);
    dis.resize(l+1), cur.resize(l);
    G.assign(l+1, vector<int>());
  }
};
```

4.3 Kuhn Munkres*

```
struct KM { // 0-base, maximum matching
  int w[N][N], hl[N], hr[N], slk[N];
  int fl[N], fr[N], pre[N], qu[N], ql, qr, n;
  bool vl[N], vr[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
      fill_n(w[i], n, -INF);
  }
  void add_edge(int a, int b, int wei) {
    w[a][b] = wei;
  }
  bool Check(int x) {
    if (vl[x] = 1, ~fl[x])
      return vr[qu[qr++] = fl[x]] = 1;
    while (~x) swap(x, fr[fl[x] = pre[x]]);
    return 0;
  }
  void bfs(int s) {
    fill_n(slk
        , n, INF), fill_n(vl, n, 0), fill_n(vr, n, 0);
    ql = qr = 0, qu[qr++] = s, vr[s] = 1;
    for (int d; ; ) {
      while (ql < qr)
        for (int x = 0, y = qu[ql++]; x < n; ++x)
          if (!vl[x] && slk
              [x] >= (d = hl[x] + hr[y] - w[x][y])) {
            if (pre[x] = y, d) slk[x] = d;
            else if (!Check(x)) return;
          }
      d = INF;
      for (int x = 0; x < n; ++x)
        if (!vl[x] && d > slk[x]) d = slk[x];
      for (int x = 0; x < n; ++x) {
        if (vl[x]) hl[x] += d;
        else slk[x] -= d;
        if (vr[x]) hr[x] -= d;
      }
      for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !Check(x)) return;
    }
  }
  int solve() {
    fill_n(fl
        , n, -1), fill_n(fr, n, -1), fill_n(hr, n, 0);
    for (int i = 0; i < n; ++i)
      hl[i] = *max_element(w[i], w[i] + n);
    for (int i = 0; i < n; ++i) bfs(i);
    int res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
  }
};
```

4.4 MincostMaxflow*

```
struct MinCostMaxFlow { // 0-base
  struct Edge {
    int from, to, cap, flow, cost, rev;
  } *past[N];
  vector<Edge> G[N];
  int inq[N], n, s, t;
  int dis[N], up[N], pot[N];
  bool BellmanFord() {
    fill_n(dis, n, INF), fill_n(inq, n, 0);
    queue<int> q;
    auto relax = [&](int u, int d, int cap, Edge *e) {
      if (cap > 0 && dis[u] > d) {
        dis[u] = d, up[u] = cap, past[u] = e;
        if (!inq[u]) inq[u] = 1, q.push(u);
      }
```

```
    };
    relax(s, 0, INF, 0);
    while (!q.empty()) {
      int u = q.front();
      q.pop(), inq[u] = 0;
      for (auto &e : G[u]) {
        int d2 = dis[u] + e.cost + pot[u] - pot[e.to];
        relax
            (e.to, d2, min(up[u], e.cap - e.flow), &e);
      }
    }
    return dis[t] != INF;
  }
  void solve(int _s,
      int _t, int &flow, int &cost, bool neg = true) {
    s = _s, t = _t, flow = 0, cost = 0;
    if (neg) BellmanFord(), copy_n(dis, n, pot);
    for (; BellmanFord(); copy_n(dis, n, pot)) {
      for (int
          i = 0; i < n; ++i) dis[i] += pot[i] - pot[s];
      flow += up[t], cost += up[t] * dis[t];
      for (int i = t; past[i]; i = past[i]->from) {
        auto &e = *past[i];
        e.flow += up[t], G[e.to][e.rev].flow -= up[t];
      }
    }
  }
  void init(int _n) {
    n = _n, fill_n(pot, n, 0);
    for (int i = 0; i < n; ++i) G[i].clear();
  }
  void add_edge(int a, int b, int cap, int cost) {
    G[a].eb(Edge{a, b, cap, 0, cost, SZ(G[b])});
    G[b].eb(Edge{b, a, 0, 0, -cost, SZ(G[a]) - 1});
  }
};
```

4.5 Maximum Simple Graph Matching

```
struct Matching { // 0-base
  queue<int> q; int n;
  vector<int> fa, s, vis, pre, match;
  vector<vector<int>> G;
  int Find(int u)
  { return u == fa[u] ? u : fa[u] = Find(fa[u]); }
  int LCA(int x, int y) {
    static int tk = 0; tk++; x = Find(x); y = Find(y);
    for (;; swap(x, y)) if (x != n) {
      if (vis[x] == tk) return x;
      vis[x] = tk;
      x = Find(pre[match[x]]);
    }
  }
  void Blossom(int x, int y, int l) {
    for (; Find(x) != l; x = pre[y]) {
      pre[x] = y, y = match[x];
      if (s[y] == 1) q.ee(y), s[y] = 0;
      for (int z: {x, y}) if (fa[z] == z) fa[z] = l;
    }
  }
  bool Bfs(int r) {
    iota(ALL(fa), 0); fill(ALL(s), -1);
    q = queue<int>(); q.ee(r); s[r] = 0;
    for (; !q.empty(); q.pop()) {
      for (int x = q.front(); int u : G[x])
        if (s[u] == -1) {
          if (pre[u] = x, s[u] = 1, match[u] == n) {
            for (int a = u, b = x, last;
                b != n; a = last, b = pre[a])
              last =
                  match[b], match[b] = a, match[a] = b;
            return true;
          }
          q.ee(match[u]); s[match[u]] = 0;
        } else if (!s[u] && Find(u) != Find(x)) {
          int l = LCA(u, x);
          Blossom(x, u, l); Blossom(u, x, l);
        }
    }
    return false;
  }
  Matching(int _n) : n(_n), fa(n+1), s(n+1)
      , vis(n+1), pre(n+1, n), match(n+1, n), G(n) {}
  void add_edge(int u, int v)
  { G[u].eb(v), G[v].eb(u); }
  int solve() {
    int ans = 0;
    for (int x = 0; x < n; ++x)
      if (match[x] == n) ans += Bfs(x);
    return ans;
  } // match[x] == n means not matched
};
```

4.6 Maximum Weight Matching*

```
#define REP(i, l, r) for (int i=(l); i<=(r); ++i)
struct WeightGraph { // 1-based
  struct edge { int u, v, w; }; int n, nx;
  vector<int> lab; vector<vector<edge>> g;
  vector<int> slk, match, st, pa, S, vis;
  vector<vector<int>> flo, flo_from; queue<int> q;
  WeightGraph(int n_) : n(n_), nx(n * 2), lab(nx + 1),
    g(nx + 1, vector<edge>(nx + 1)), slk(nx + 1),
    flo(nx + 1), flo_from(nx + 1, vector(n + 1, 0)) {
    match = st = pa = S = vis = slk;
    REP(u, 1, n) REP(v, 1, n) g[u][v] = {u, v, 0};
  }
  int E(edge e)
  { return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2; }
  void update_slk(int u, int x, int &s)
  { if (!s || E(g[u][x]) < E(g[s][x])) s = u; }
  void set_slk(int x) {
    slk[x] = 0;
    REP(u, 1, n)
      if (g[
          u][x].w > 0 and st[u] != x and S[st[u]] == 0)
        update_slk(u, x, slk[x]);
  }
  void q_push(int x) {
    if (x <= n) q.push(x);
    else for (int y : flo[x]) q_push(y);
  }
  void set_st(int x, int b) {
    st[x] = b;
    if (x > n) for (int y : flo[x]) set_st(y, b);
  }
  vector<int> split_flo(auto &f, int xr) {
    auto it = find(ALL(f), xr);
    if (auto pr = it - begin(f); pr % 2 == 1)
      reverse(1 + ALL(f)), it = end(f) - pr;
    auto res = vector(begin(f), it);
    return f.erase(begin(f), it), res;
  }
  void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    int xr = flo_from[u][g[u][v].u];
    auto &f = flo[u], z = split_flo(f, xr);
    REP(i, 0, SZ(z) - 1) set_match(z[i], z[i ^ 1]);
    set_match(xr, v); f.insert(end(f), ALL(z));
  }
  void augment(int u, int v) {
    for (;;) {
      int xnv = st[match[u]]; set_match(u, v);
      if (!xnv) return;
      set_match(v = xnv, u = st[pa[xnv]]);
    }
  }
  int lca(int u, int v) {
    static int t = 0; ++t;
    for (++t; u || v; swap(u, v)) if (u) {
      if (vis[u] == t) return u;
      vis[u] = t, u = st[match[u]];
      if (u) u = st[pa[u]];
    }
    return 0;
  }
  void add_blossom(int u, int o, int v) {
    int b = find(n + 1 + ALL(st), 0) - begin(st);
    lab[b] = 0, S[b] = 0, match[b] = match[o];
    vector<int> f = {o};
    for (int t : {u, v}) {
      reverse(1 + ALL(f));
      for (int x = t, y; x != o; x = st[pa[y]])
        f.eb(x), f.eb(y = st[match[x]]), q_push(y);
    }
    flo[b] = f; set_st(b, b);
    REP(x, 1, nx) g[b][x].w = g[x][b].w = 0;
    fill(ALL(flo_from[b]), 0);
    for (int xs : flo[b]) {
      REP(x, 1, nx)
        if (g[b][x].w == 0 || E(g[xs][x]) < E(g[b][x]))
```

```
      g[b][x] = g[xs][x], g[x][b] = g[x][xs];
    REP(x, 1, n)
      if (flo_from[xs][x]) flo_from[b][x] = xs;
  }
  set_slk(b);
}
void expand_blossom(int b) {
  for (int x : flo[b]) set_st(x, x);
  int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
  for (int x : split_flo(flo[b], xr)) {
    if (xs == -1) { xs = x; continue; }
    pa[xs] = g[x][xs].u, S[xs] = 1, S[x] = 0;
    slk[xs] = 0, set_slk(x), q_push(x), xs = -1;
  }
  for (int x : flo[b])
    if (x == xr) S[x] = 1, pa[x] = pa[b];
    else S[x] = -1, set_slk(x);
  st[b] = 0;
}
bool on_found_edge(const edge &e) {
  if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
    int nu = st[match[v]]; pa[v] = e.u; S[v] = 1;
    slk[v] = slk[nu] = S[nu] = 0; q_push(nu);
  } else if (S[v] == 0) {
    if (int o = lca(u, v)) add_blossom(u, o, v);
    else return augment(u, v), augment(v, u), true;
  }
  return false;
}
bool matching() {
  fill(ALL(S), -1), fill(ALL(slk), 0);
  q = queue<int>();
  REP(x, 1, nx) if (st[x] == x and !match[x])
    pa[x] = S[x] = 0, q_push(x);
  if (q.empty()) return false;
  for (;;) {
    while (SZ(q)) {
      int u = q.front(); q.pop();
      if (S[st[u]] == 1) continue;
      REP(v, 1, n)
        if (g[u][v].w > 0 and st[u] != st[v]) {
          if (E(g[u][v]) != 0)
            update_slk(u, st[v], slk[st[v]]);
          else if
               (on_found_edge(g[u][v])) return true;
        }
    }
    int d = INF;
    REP(b, n + 1, nx) if (st[b] == b and S[b] == 1)
      d = min(d, lab[b] / 2);
    REP(x, 1, nx)
      if (int
          s = slk[x]; st[x] == x and s and S[x] <= 0)
        d = min(d, E(g[s][x]) / (S[x] + 2));
    REP(u, 1, n)
      if (S[st[u]] == 1) lab[u] += d;
      else if (S[st[u]] == 0) {
        if (lab[u] <= d) return false;
        lab[u] -= d;
      }
    REP(b, n + 1, nx) if (st[b] == b and S[b] >= 0)
      lab[b] += d * (2 - 4 * S[b]);
    REP(x, 1, nx)
      if (int s = slk[x]; st[x] == x and
          s and st[s] != x and E(g[s][x]) == 0)
        if (on_found_edge(g[s][x])) return true;
    REP(b, n + 1, nx)
      if (st[b] == b and S[b] == 1 and lab[b] == 0)
        expand_blossom(b);
  }
  return false;
}
pii solve() {
  fill(ALL(match), 0);
  REP(u, 0, n) st[u] = u, flo[u].clear();
  int w_max = 0;
  REP(u, 1, n) REP(v, 1, n) {
    flo_from[u][v] = (u == v ? u : 0);
    w_max = max(w_max, g[u][v].w);
  }
  fill(ALL(lab), w_max);
  int n_matches = 0, tot_weight = 0;
  while (matching()) ++n_matches;
  REP(u, 1, n) if (match[u] and match[u] < u)
    tot_weight += g[u][match[u]].w;
  return make_pair(tot_weight, n_matches);
}
void add_edge(int u, int v, int w)
{ g[u][v].w = g[v][u].w = w; }
};
```

4.7 SW-mincut

```
struct SW{ // global min cut, O(V^3)
  #define REP for (int i = 0; i < n; ++i)
  static const int MXN = 514, INF = 2147483647;
  int vst[MXN], edge[MXN][MXN], wei[MXN];
  void init(int n) {
    REP fill_n(edge[i], n, 0);
  }
  void addEdge(int u, int v, int w){
    edge[u][v] += w; edge[v][u] += w;
  }
  int search(int &s, int &t, int n){
    fill_n(vst, n, 0), fill_n(wei, n, 0);
    s = t = -1;
    int mx, cur;
    for (int j = 0; j < n; ++j) {
      mx = -1, cur = 0;
      REP if (wei[i] > mx) cur = i, mx = wei[i];
      vst[cur] = 1, wei[cur] = -1;
      s = t; t = cur;
      REP if (!vst[i]) wei[i] += edge[cur][i];
    }
    return mx;
  }
  int solve(int n) {
    int res = INF;
    for (int x, y; n > 1; n--){
      res = min(res, search(x, y, n));
      REP edge[i][x] = (edge[x][i] += edge[y][i]);
      REP {
        edge[y][i] = edge[n - 1][i];
        edge[i][y] = edge[i][n - 1];
      } // edge[y][y] = 0;
    }
    return res;
  }
} sw;
```

4.8 BoundedFlow*(Dinic*)

```
struct BoundedFlow { // 0-base
  struct edge {
    int to, cap, flow, rev;
  };
  vector<edge> G[N];
  int n, s, t, dis[N], cur[N], cnt[N];
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n + 2; ++i)
      G[i].clear(), cnt[i] = 0;
  }
  void add_edge(int u, int v, int lcap, int rcap) {
    cnt[u] -= lcap, cnt[v] += lcap;
    G[u].eb(edge{v, rcap, lcap, SZ(G[v])});
    G[v].eb(edge{u, 0, 0, SZ(G[u]) - 1});
  }
  void add_edge(int u, int v, int cap) {
    G[u].eb(edge{v, cap, 0, SZ(G[v])});
    G[v].eb(edge{u, 0, 0, SZ(G[u]) - 1});
  }
  int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < SZ(G[u]); ++i) {
      edge &e = G[u][i];
      if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
        int df = dfs(e.to, min(e.cap - e.flow, cap));
        if (df) {
          e.flow += df, G[e.to][e.rev].flow -= df;
          return df;
        }
      }
    }
    dis[u] = -1;
    return 0;
  }
  bool bfs() {
    fill_n(dis, n + 3, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
```

```
      int u = q.front();
      q.pop();
      for (edge &e : G[u])
        if (!~dis[e.to] && e.flow != e.cap)
          q.push(e.to), dis[e.to] = dis[u] + 1;
    }
    return dis[t] != -1;
  }
  int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
      fill_n(cur, n + 3, 0);
      while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
  }
  bool solve() {
    int sum = 0;
    for (int i = 0; i < n; ++i)
      if (cnt[i] > 0)
        add_edge(n + 1, i, cnt[i]), sum += cnt[i];
      else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
    if (sum != maxflow(n + 1, n + 2)) sum = -1;
    for (int i = 0; i < n; ++i)
      if (cnt[i] > 0)
        G[n + 1].pb(), G[i].pb();
      else if (cnt[i] < 0)
        G[i].pb(), G[n + 2].pb();
    return sum != -1;
  }
  int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pb(), G[_s].pb(), x;
  }
};
```

4.9 Gomory Hu tree*

```
MaxFlow Dinic;
int g[maxn];
void GomoryHu(int n) { // 0-base
  fill_n(g, n, 0);
  for (int i = 1; i < n; ++i) {
    Dinic.reset();
    add_edge(i, g[i], Dinic.maxflow(i, g[i]));
    for (int j = i + 1; j <= n; ++j)
      if (g[j] == g[i] && ~Dinic.dis[j])
        g[j] = i;
  }
}
```

4.10 Minimum Cost Circulation*

```
struct MinCostCirculation { // 0-base
  struct Edge {
    int from, to, cap, fcap, flow, cost, rev;
  } *past[N];
  vector<Edge> G[N];
  int dis[N], inq[N], n;
  void BellmanFord(int s) {
    fill_n(dis, n, INF), fill_n(inq, n, 0);
    queue<int> q;
    auto relax = [&](int u, int d, Edge *e) {
      if (dis[u] > d) {
        dis[u] = d, past[u] = e;
        if (!inq[u]) inq[u] = 1, q.push(u);
      }
    };
    relax(s, 0, 0);
    while (!q.empty()) {
      int u = q.front();
      q.pop(), inq[u] = 0;
      for (auto &e : G[u])
        if (e.cap > e.flow)
          relax(e.to, dis[u] + e.cost, &e);
    }
  }
  void try_edge(Edge &cur) {
    if (cur.cap > cur.flow) return ++cur.cap, void();
    BellmanFord(cur.to);
    if (dis[cur.from] + cur.cost < 0) {
      ++cur.flow, --G[cur.to][cur.rev].flow;
      for (int
             i = cur.from; past[i]; i = past[i]->from) {
        auto &e = *past[i];
        ++e.flow, --G[e.to][e.rev].flow;
      }
    }
    ++cur.cap;
  }
  void solve(int mxlg) {
    for (int b = mxlg; b >= 0; --b) {
      for (int i = 0; i < n; ++i)
        for (auto &e : G[i])
          e.cap *= 2, e.flow *= 2;
      for (int i = 0; i < n; ++i)
        for (auto &e : G[i])
          if (e.fcap >> b & 1)
            try_edge(e);
    }
  }
  void init(int _n) { n = _n;
    for (int i = 0; i < n; ++i) G[i].clear();
  }
  void add_edge(int a, int b, int cap, int cost) {
    G[a].eb(Edge
        {a, b, 0, cap, 0, cost, SZ(G[b]) + (a == b)});
    G[b].eb(Edge{b, a, 0, 0, 0, -cost, SZ(G[a]) - 1});
  }
} mcmf; // O(VE * ElogC)
```

4.11 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
  1. Construct super source S and sink T.
  2. For each edge (x,y,l,u), connect $x \to y$ with capacity $u-l$.
  3. For each vertex v, denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$.
     - To maximize, connect $t \to s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T. If $f \neq \sum_{v\in V, in(v)>0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
     - To minimize, let f be the maximum flow from S to T. Connect $t \to s$ with capacity ∞ and let the flow from S to T be f'. If $f+f' \neq \sum_{v\in V, in(v)>0} in(v)$, there's no solution. Otherwise, f' is the answer.
  5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X,Y)
  1. Redirect every edge: $y \to x$ if $(x,y) \in M$, $x \to y$ otherwise.
  2. DFS from unmatched vertices in X.
  3. $x \in X$ is chosen iff x is unvisited.
  4. $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
  1. Consruct super source S and sink T
  2. For each edge (x,y,c), connect $x \to y$ with $(cost,cap) = (c,1)$ if $c > 0$, otherwise connect $y \to x$ with $(cost,cap) = (-c,1)$
  3. For each edge with $c < 0$, sum these cost as K, then increase $d(y)$ by 1, decrease $d(x)$ by 1
  4. For each vertex v with $d(v) > 0$, connect $S \to v$ with $(cost,cap) = (0,d(v))$
  5. For each vertex v with $d(v) < 0$, connect $v \to T$ with $(cost,cap) = (0,-d(v))$
  6. Flow from S to T, the answer is the cost of the flow $C+K$
- Maximum density induced subgraph
  1. Binary search on answer, suppose we're checking answer T
  2. Construct a max flow model, let K be the sum of all weights
  3. Connect source $s \to v$, $v \in G$ with capacity K
  4. For each edge (u,v,w) in G, connect $u \to v$ and $v \to u$ with capacity w
  5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - (\sum_{e\in E(v)} w(e)) - 2w(v)$
  6. T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
  1. For each $v \in V$ create a copy v', and connect $u' \to v'$ with weight $w(u,v)$.
  2. Connect $v \to v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v.
  3. Find the minimum weight perfect matching on G'.
- Project selection problem
  1. If $p_v > 0$, create edge (s,v) with capacity p_v; otherwise, create edge (v,t) with capacity $-p_v$.
  2. Create edge (u,v) with capacity w with w being the cost of choosing u without choosing v.
  3. The mincut is equivalent to the maximum profit of a subset of projects.
- Dual of minimum cost maximum flow
  1. Capacity c_{uv}, Flow f_{uv}, Cost w_{uv}, Required Flow difference for vertex b_u.

2. If all w_{uv} are integers, then optimal solution can happen when all p_u are integers.

$$\min \sum_{uv} w_{uv} f_{uv}$$
$$-f_{uv} \ge -c_{uv} \Leftrightarrow \quad \min \sum_{u} b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv})$$
$$\sum_{v} f_{vu} - \sum_{v} f_{uv} = -b_u \qquad p_u \ge 0$$

4.12 MCMF HLPP

```
// Push-Relabel
     implementation of the cost-scaling algorithm
// Runs in O( <max_flow
   > * log(V * max_edge_cost)) = O( V^3 * log(V * C))
// Operates on integers
template<typename flow_t = int, typename cost_t = int>
struct mcSFlow{
  struct Edge{
    cost_t c;
    flow_t f;
    int to, rev;
    Edge(int _to, cost_t _c, flow_t _f
      , int _rev):c(_c), f(_f), to(_to), rev(_rev){}
  };
  const
      cost_t INFCOST = numeric_limits<cost_t>::max()/2;
  const
      cost_t INFFLOW = numeric_limits<flow_t>::max()/2;
  cost_t epsilon;
  int N, S, T;
  vector<vector<Edge> > G;
  vector<unsigned int> isEnqueued, state;
  mcSFlow(int _N, int _S,
       int _T):epsilon(0), N(_N), S(_S), T(_T), G(_N){}
  void add_edge(int a, int b, cost_t cost, flow_t cap){
    if(a==b){assert(cost>=0); return;}
    cost*=N;// to preserve integer-values
    epsilon = max(epsilon, abs(cost));
    assert(a>=0&&a<N&&b>=0&&b<N);
    G[a].emplace_back(b, cost, cap, G[b].size());
    G[b].emplace_back(a, -cost, 0, G[a].size()-1);
  }
  flow_t calc_max_flow(){ // Dinic max-flow
    vector<flow_t> dist(N), state(N);
    vector<Edge*> path(N);
    auto cmp = [](Edge*a, Edge*b){return a->f < b->f;};
    flow_t addFlow, retflow=0;;
    do{
      fill(dist.begin(), dist.end(), -1);
      dist[S]=0;
      auto head = state.begin(), tail = state.begin();
      for(*tail++ = S;head!=tail;++head){
        for(Edge const&e:G[*head]){
          if(e.f && dist[e.to]==-1){
            dist[e.to] = dist[*head]+1;
            *tail++=e.to;
          }
        }
      }
      addFlow = 0;
      fill(state.begin(), state.end(), 0);
      auto top = path.begin();
      Edge dummy(S, 0, INFFLOW, -1);
      *top++ = &dummy;
      while(top != path.begin()){
        int n = (*prev(top))->to;
        if(n==T){
          auto next_top
               = min_element(path.begin(), top, cmp);
          flow_t flow = (*next_top)->f;
          while(--top!=path.begin()){
            Edge &e=**top, &f=G[e.to][e.rev];
            e.f-=flow;
            f.f+=flow;
          }
          addFlow=1;
          retflow+=flow;
          top = next_top;
          continue;
        }
        for(int &i=state[n], i_max
             = G[n].size(), need = dist[n]+1;;++i){
          if(i==i_max){
            dist[n]=-1;
            --top;
            break;
          }
          if(dist[G[n][i].to] == need && G[n][i].f){
            *top++ = &G[n][i];
            break;
          }
        }
      }
    }while(addFlow);
    return retflow;
  }
  vector<flow_t> excess;
  vector<cost_t> h;
  void push(Edge &e, flow_t amt){
    //cerr << "push: "
         << G[e.to][e.rev].to << " -> " << e.to << " ("
         << e.f << "/" << e.c << ") : " << amt << "\n";
    if(e.f < amt) amt=e.f;
    e.f-=amt;
    excess[e.to]+=amt;
    G[e.to][e.rev].f+=amt;
    excess[G[e.to][e.rev].to]-=amt;
  }
  void relabel(int vertex){
    cost_t newHeight = -INFCOST;
    for(unsigned int i=0;i<G[vertex].size();++i){
      Edge const&e = G[vertex][i];
      if(e.f && newHeight < h[e.to]-e.c){
        newHeight = h[e.to] - e.c;
        state[vertex] = i;
      }
    }
    h[vertex] = newHeight - epsilon;
  }
  const int scale=2;
  pair<flow_t, cost_t> minCostFlow(){
    cost_t retCost = 0;
    for(int i=0;i<N;++i){
      for(Edge &e:G[i]){
        retCost += e.c*(e.f);
      }
    }
    //find feasible flow
    flow_t retFlow = calc_max_flow();
    excess.resize(N);h.resize(N);
    queue<int> q;
    isEnqueued.assign(N, 0); state.assign(N,0);
    for(;epsilon;epsilon>>=scale){
      //refine
      fill(state.begin(), state.end(), 0);
      for(int i=0;i<N;++i)
        for(auto &e:G[i])
          if(h[i
             + e.c - h[e.to] < 0 && e.f) push(e, e.f);
      for(int i=0;i<N;++i){
        if(excess[i]>0){
          q.push(i);
          isEnqueued[i]=1;
        }
      }
      while(!q.empty()){
        int cur=q.front();q.pop();
        isEnqueued[cur]=0;
        // discharge
        while(excess[cur]>0){
          if(state[cur] == G[cur].size()){
            relabel(cur);
          }
          for(unsigned int &i=state[
              cur], max_i = G[cur].size();i<max_i;++i){
            Edge &e=G[cur][i];
            if(h[cur] + e.c - h[e.to] < 0){
              push(e, excess[cur]);
              if(excess
                  [e.to]>0 && isEnqueued[e.to]==0){
                q.push(e.to);
                isEnqueued[e.to]=1;
              }
              if(excess[cur]==0) break;
            }
          }
        }
      }
      if(epsilon>1 && epsilon>>scale==0){
        epsilon = 1<<scale;
      }
    }
```

```
    for(int i=0;i<N;++i){
      for(Edge &e:G[i]){
        retCost -= e.c*(e.f);
      }
    }
    //cerr << " -> " << retFlow << " / "
         << retCost << " bzw. " << retCost/2/N << "\n";
    return make_pair(retFlow, retCost/2/N);
  }
  flow_t getFlow(Edge const &e){
    return G[e.to][e.rev].f;
  }
};
```

5 String

5.1 KMP

```
int F[maxn];
vector<int> match(string A, string B) {
  vector<int> ans;
  F[0] = -1, F[1] = 0;
  for (int i = 1, j = 0; i < SZ(B); F[++i] = ++j) {
    if (B[i] == B[j]) F[i] = F[j]; // optimize
    while (j != -1 && B[i] != B[j]) j = F[j];
  }
  for (int i = 0, j = 0; i < SZ(A); ++i) {
    while (j != -1 && A[i] != B[j]) j = F[j];
    if (++j == SZ(B)) ans.eb(i + 1 - j), j = F[j];
  }
  return ans;
}
```

5.2 Z-value*

```
int z[maxn];
void make_z(const string &s) {
  int l = 0, r = 0;
  for (int i = 1; i < SZ(s); ++i) {
    for (z[i] = max(0, min(r - i + 1, z[i - l]));
         i + z[i] < SZ(s) && s[i + z[i]] == s[z[i]];
         ++z[i])
      ;
    if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
  }
}
```

5.3 Manacher*

```
int z[maxn]; // 0-base
/* center i: radius z[i * 2 + 1] / 2
   center i, i + 1: radius z[i * 2 + 2] / 2
   both aba, abba have radius 2 */
void Manacher(string tmp) {
  string s = "%";
  int l = 0, r = 0;
  for (char c : tmp) s.eb(c), s.eb('%');
  for (int i = 0; i < SZ(s); ++i) {
    z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
    while (i - z[i] >= 0 && i + z[i] < SZ(s)
           && s[i + z[i]] == s[i - z[i]]) ++z[i];
    if (z[i] + i > r) r = z[i] + i, l = i;
  }
}
```

5.4 SAIS*

```
namespace sfx {
bool _t[N * 2];
int SA[N * 2], H[N], RA[N];
int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2];
// zero based, string content MUST > 0
// SA[i]: SA[i]-th
    suffix is the i-th lexigraphically smallest suffix.
// H[i]: longest
    common prefix of suffix SA[i] and suffix SA[i - 1].
void pre(int *sa, int *c, int n, int z)
{ fill_n(sa, n, 0), copy_n(c, z, x); }
void induce
    (int *sa, int *c, int *s, bool *t, int n, int z) {
  copy_n(c, z - 1, x + 1);
  for (int i = 0; i < n; ++i)
    if (sa[i] && !t[sa[i] - 1])
      sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
  copy_n(c, z, x);
  for (int i = n - 1; i >= 0; --i)
    if (sa[i] && t[sa[i] - 1])
      sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
}
void sais(int *s, int *sa
  , int *p, int *q, bool *t, int *c, int n, int z) {
  bool uniq = t[n - 1] = true;
  int nn = 0,
      nmxz = -1, *nsa = sa + n, *ns = s + n, last = -1;
  fill_n(c, z, 0);
  for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
  partial_sum(c, c + z, c);
  if (uniq) {
    for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
    return;
  }
  for (int i = n - 2; i >= 0; --i)
    t[i] = (
        s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
  pre(sa, c, n, z);
  for (int i = 1; i <= n - 1; ++i)
    if (t[i] && !t[i - 1])
      sa[--x[s[i]]] = p[q[i] = nn++] = i;
  induce(sa, c, s, t, n, z);
  for (int i = 0; i < n; ++i)
    if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
      bool neq = last < 0 || !equal
          (s + sa[i], s + p[q[sa[i]] + 1], s + last);
      ns[q[last = sa[i]]] = nmxz += neq;
    }
  sais(ns,
       nsa, p + nn, q + n, t + n, c + z, nn, nmxz + 1);
  pre(sa, c, n, z);
  for (int i = nn - 1; i >= 0; --i)
    sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
  induce(sa, c, s, t, n, z);
}
void mkhei(int n) {
  for (int i = 0, j = 0; i < n; ++i) {
    if (RA[i])
      for (; _s[i + j] == _s[SA[RA[i] - 1] + j]; ++j);
    H[RA[i]] = j, j = max(0, j - 1);
  }
}
void build(int *s, int n) {
  copy_n(s, n, _s), _s[n] = 0;
  sais(_s, SA, _p, _q, _t, _c, n + 1, 256);
  copy_n(SA + 1, n, SA);
  for (int i = 0; i < n; ++i) RA[SA[i]] = i;
  mkhei(n);
}}
```

5.5 Aho-Corasick Automatan

```
struct AC_Automatan {
  int nx[len][sigma], fl[len], cnt[len], ord[len], top;
  int rnx[len][sigma]; // node actually be reached
  int newnode() {
    fill_n(nx[top], sigma, -1);
    return top++;
  }
  void init() { top = 1, newnode(); }
  int input(string &s) {
    int X = 1;
    for (char c : s) {
      if (!~nx[X][c - 'A']) nx[X][c - 'A'] = newnode();
      X = nx[X][c - 'A'];
    }
    return X; // return the end node of string
  }
  void make_fl() {
    queue<int> q;
    q.push(1), fl[1] = 0;
    for (int t = 0; !q.empty(); ) {
      int R = q.front();
      q.pop(), ord[t++] = R;
      for (int i = 0; i < sigma; ++i)
        if (~nx[R][i]) {
          int X = rnx[R][i] = nx[R][i], Z = fl[R];
          for (; Z && !~nx[Z][i]; ) Z = fl[Z];
          fl[X] = Z ? nx[Z][i] : 1, q.push(X);
        }
        else rnx[R][i] = R > 1 ? rnx[fl[R]][i] : 1;
    }
  }
  void solve() {
    for (int i = top - 2; i > 0; --i)
```

```
    cnt[fl[ord[i]]] += cnt[ord[i]];
  }
} ac;
```

5.6 De Bruijn sequence*

```
constexpr int maxc = 10, maxn = 1e5 + 10;
struct DBSeq {
  int C, N, K, L, buf[maxc * maxn]; // K <= C^N
  void dfs(int *out, int t, int p, int &ptr) {
    if (ptr >= L) return;
    if (t > N) {
      if (N % p) return;
      for (int i = 1; i <= p && ptr < L; ++i)
        out[ptr++] = buf[i];
    } else {
      buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
      for (int j = buf[t - p] + 1; j < C; ++j)
        buf[t] = j, dfs(out, t + 1, t, ptr);
    }
  }
  void solve(int _c, int _n, int _k, int *out) {
    int p = 0;
    C = _c, N = _n, K = _k, L = N + K - 1;
    dfs(out, 1, 1, p);
    if (p < L) fill(out + p, out + L, 0);
  }
} dbs;
```

5.7 Extended SAM*

```
struct exSAM {
  int len[N * 2], link[N * 2]; // maxlength, suflink
  int next[N * 2][CNUM], tot; // [0, tot), root = 0
  int lenSorted[N * 2]; // topo. order
  int cnt[N * 2]; // occurence
  int newnode() {
    fill_n(next[tot], CNUM, 0);
    len[tot] = cnt[tot] = link[tot] = 0;
    return tot++;
  }
  void init() { tot = 0, newnode(), link[0] = -1; }
  int insertSAM(int last, int c) {
    int cur = next[last][c];
    len[cur] = len[last] + 1;
    int p = link[last];
    while (p != -1 && !next[p][c])
      next[p][c] = cur, p = link[p];
    if (p == -1) return link[cur] = 0, cur;
    int q = next[p][c];
    if (len
        [p] + 1 == len[q]) return link[cur] = q, cur;
    int clone = newnode();
    for (int i = 0; i < CNUM; ++i)
      next[
          clone][i] = len[next[q][i]] ? next[q][i] : 0;
    len[clone] = len[p] + 1;
    while (p != -1 && next[p][c] == q)
      next[p][c] = clone, p = link[p];
    link[link[cur] = clone] = link[q];
    link[q] = clone;
    return cur;
  }
  void insert(const string &s) {
    int cur = 0;
    for (auto ch : s) {
      int &nxt = next[cur][int(ch - 'a')];
      if (!nxt) nxt = newnode();
      cnt[cur = nxt] += 1;
    }
  }
  void build() {
    queue<int> q;
    q.push(0);
    while (!q.empty()) {
      int cur = q.front();
      q.pop();
      for (int i = 0; i < CNUM; ++i)
        if (next[cur][i])
          q.push(insertSAM(cur, i));
    }
    vector<int> lc(tot);
    for (int i = 1; i < tot; ++i) ++lc[len[i]];
    partial_sum(ALL(lc), lc.begin());
    for (int i
         = 1; i < tot; ++i) lenSorted[--lc[len[i]]] = i;
  }
  void solve() {
    for (int i = tot - 2; i >= 0; --i)
      cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
  }
};
```

5.8 PalTree*

```
struct palindromic_tree {
  struct node {
    int next[26], fail, len;
    int cnt, num; // cnt: appear times, num: number of
                  // pal. suf.
    node(int l = 0) : fail(0), len(l), cnt(0), num(0) {
      for (int i = 0; i < 26; ++i) next[i] = 0;
    }
  };
  vector<node> St;
  vector<char> s;
  int last, n;
  palindromic_tree() : St(2), last(1), n(0) {
    St[0].fail = 1, St[1].len = -1, s.eb(-1);
  }
  inline void clear() {
    St.clear(), s.clear(), last = 1, n = 0;
    St.eb(0), St.eb(-1);
    St[0].fail = 1, s.eb(-1);
  }
  inline int get_fail(int x) {
    while (s[n - St[x].len - 1] != s[n])
      x = St[x].fail;
    return x;
  }
  inline void add(int c) {
    s.eb(c -= 'a'), ++n;
    int cur = get_fail(last);
    if (!St[cur].next[c]) {
      int now = SZ(St);
      St.eb(St[cur].len + 2);
      St[now].fail =
        St[get_fail(St[cur].fail)].next[c];
      St[cur].next[c] = now;
      St[now].num = St[St[now].fail].num + 1;
    }
    last = St[cur].next[c], ++St[last].cnt;
  }
  inline void count() { // counting cnt
    auto i = St.rbegin();
    for (; i != St.rend(); ++i) {
      St[i->fail].cnt += i->cnt;
    }
  }
  inline int size() { // The number of diff. pal.
    return SZ(St) - 2;
  }
};
```

6 Math

6.1 Modular Struct*

```
template <typename T> struct M {
  static T MOD; // change to constexpr if already known
  T v;
  M(T x = 0) {
    v = (-MOD <= x and x < MOD) ? x : x % MOD;
    if (v < 0) v += MOD;
  }
  explicit operator T() const { return v; }
  bool
      operator==(const M &b) const { return v == b.v; }
  bool
      operator!=(const M &b) const { return v != b.v; }
  M operator-() { return M(-v); }
  M operator+(M b) { return M(v + b.v); }
  M operator-(M b) { return M(v - b.v); }
  M operator
      *(M b) { return M((__int128)v * b.v % MOD); }
  // change
       implementation to extgcd if MOD is not prime
  M operator/(M b) { return *this * b.inv(); }
  M pow(M b) {
    M r(1);
    for (M a = *this; b; b >>= 1, a *= a)
      if (b & 1) r *= a;
```

```
    return r;
  }
  M inv(M b) { return b.pow(MOD - 2); }
  M operator+=(const M &b) {
    if ((v += b.v) >= MOD) v -= MOD;
    return *this;
  }
  M operator-=(const M &b) {
    if ((v -= b.v) < 0) v += MOD;
    return *this;
  }
  friend M &operator*=(M &a, M b) { return a = a * b; }
  friend M &operator/=(M &a, M b) { return a = a / b; }
};
using Mod = M<int>;
template <> int Mod::MOD = 1'000'000'007;
int &MOD = Mod::MOD;
```

6.2 ax+by=gcd(only exgcd *)

```
pii exgcd(int a, int b) {
  if (b == 0) return pii(1, 0);
  int p = a / b;
  pii q = exgcd(b, a % b);
  return pii(q.Y, q.X - q.Y * p);
}
/* ax+by=res, let x be minimum non-negative
g, p = gcd(a, b), exgcd(a, b) * res / g
if p.X < 0: t = (abs(p.X) + b / g - 1) / (b / g)
else: t = -(p.X / (b / g))
p += (b / g, -a / g) * t */
```

6.3 Floor and Ceil

```
int floor(int a, int b)
{ return a / b - (a % b && (a < 0) ^ (b < 0)); }
int ceil(int a, int b)
{ return a / b + (a % b && (a < 0) ^ (b > 0)); }
```

6.4 Floor Enumeration

```
// enumerating x = floor(n / i), [l, r]
for (int l = 1, r; l <= n; l = r + 1) {
  int x = n / l;
  r = n / x;
}
```

6.5 Mod Min

```
// min{k | l <= ((ak) mod m) <= r}, no solution -> -1
int mod_min(int a, int m, int l, int r) {
  if (a == 0) return l ? -1 : 0;
  if (int k = (l + a - 1) / a; k * a <= r)
    return k;
  int b = m / a, c = m % a;
  if (int y = mod_min(c, a, a - r % a, a - l % a))
    return (l + y * c + a - 1) / a + y * b;
  return -1;
}
```

6.6 Gaussian integer gcd

```
cpx gaussian_gcd(cpx a, cpx b) {
#define rnd
    (a, b) ((a >= 0 ? a * 2 + b : a * 2 - b) / (b * 2))
  int c = a.real() * b.real() + a.imag() * b.imag();
  int d = a.imag() * b.real() - a.real() * b.imag();
  int r = b.real() * b.real() + b.imag() * b.imag();
  if (c % r == 0 && d % r == 0) return b;
  return gaussian_gcd
      (b, a - cpx(rnd(c, r), rnd(d, r)) * b);
}
```

6.7 floor sum*

```
int floor_sum(int n, int m, int a, int b) {
    int ans = 0;
    if (a >= m)
        ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m)
        ans += n * (b / m), b %= m;
    int y_max
         = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
}// sum^{
    n-1}_0 floor((a * i + b) / m) in log(n + m + a + b)
```

6.8 Miller Rabin*

```
// n < 4,759,123,141       3 : 2, 7, 61
// n < 1,122,004,669,633   4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383   6 : primes <= 13
// n < 2^64                7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(int a, int n) {
  if ((a = a % n) == 0) return 1;
  if (n % 2 == 0) return n == 2;
  int tmp = (n - 1) / ((n - 1) & (1 - n));
  int t = __lg(((n - 1) & (1 - n))), x = 1;
  for (; tmp; tmp >>= 1, a = mul(a, a, n))
    if (tmp & 1) x = mul(x, a, n);
  if (x == 1 || x == n - 1) return 1;
  while (--t)
    if ((x = mul(x, x, n)) == n - 1) return 1;
  return 0;
}
```

6.9 Simultaneous Equations

```
struct matrix { //m variables, n equations
  int n, m;
  fraction M[maxn][maxn + 1], sol[maxn];
  int solve() { //-1: inconsistent, >= 0: rank
    for (int i = 0; i < n; ++i) {
      int piv = 0;
      while (piv < m && !M[i][piv].n) ++piv;
      if (piv == m) continue;
      for (int j = 0; j < n; ++j) {
        if (i == j) continue;
        fraction tmp = -M[j][piv] / M[i][piv];
        for (int k = 0; k <=
             m; ++k) M[j][k] = tmp * M[i][k] + M[j][k];
      }
    }
    int rank = 0;
    for (int i = 0; i < n; ++i) {
      int piv = 0;
      while (piv < m && !M[i][piv].n) ++piv;
      if (piv == m && M[i][m].n) return -1;
      else if (piv
           < m) ++rank, sol[piv] = M[i][m] / M[i][piv];
    }
    return rank;
  }
};
```

6.10 Pollard Rho*

```
map<int, int> cnt;
void PollardRho(int n) {
  if (n == 1) return;
  if (prime(n)) return ++cnt[n], void();
  if (n % 2
      == 0) return PollardRho(n / 2), ++cnt[2], void();
  int x = 2, y = 2, d = 1, p = 1;
  #define f(x, n, p) ((mul(x, x, n) + p) % n)
  while (true) {
    if (d != n && d != 1) {
      PollardRho(n / d);
      PollardRho(d);
      return;
    }
    if (d == n) ++p;
    x = f(x, n, p), y = f(f(y, n, p), n, p);
    d = gcd(abs(x - y), n);
  }
}
```

6.11 chineseRemainder

```
int solve(int x1, int m1, int x2, int m2) {
  int g = gcd(m1, m2);
  if ((x2 - x1) % g) return -1; // no sol
  m1 /= g; m2 /= g;
  pii p = exgcd(m1, m2);
  int lcm = m1 * m2 * g;
  int res = p.first * (x2 - x1) * m1 + x1;
  // be careful with overflow
  return (res % lcm + lcm) % lcm;
}
```

6.12 Factorial without prime factor*

```
// O(p^k + log^2 n), pk = p^k
int prod[maxp];
int fac_no_p(int n, int p, int pk) {
  prod[0] = 1;
  for (int i = 1; i <= pk; ++i)
    if (i % p) prod[i] = prod[i - 1] * i % pk;
    else prod[i] = prod[i - 1];
  int rt = 1;
  for (; n; n /= p) {
    rt = rt * mpow(prod[pk], n / pk, pk) % pk;
    rt = rt * prod[n % pk] % pk;
  }
  return rt;
} // (n! without factor p) % p^k
```

6.13 QuadraticResidue*

```
int Jacobi(int a, int m) {
  int s = 1;
  for (; m > 1; ) {
    a %= m;
    if (a == 0) return 0;
    const int r = __builtin_ctz(a);
    if ((r & 1) && ((m + 2) & 4)) s = -s;
    a >>= r;
    if (a & m & 2) s = -s;
    swap(a, m);
  }
  return s;
}

int QuadraticResidue(int a, int p) {
  if (p == 2) return a & 1;
  const int jc = Jacobi(a, p);
  if (jc == 0) return 0;
  if (jc == -1) return -1;
  int b, d;
  for (; ; ) {
    b = rand() % p;
    d = (1LL * b * b + p - a) % p;
    if (Jacobi(d, p) == -1) break;
  }
  int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
  for (int e = (1LL + p) >> 1; e; e >>= 1) {
    if (e & 1) {
      tmp = (1LL *
          g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
      g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
      g0 = tmp;
    }
    tmp = (1LL
        * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
    f1 = (2LL * f0 * f1) % p;
    f0 = tmp;
  }
  return g0;
}
```

6.14 PiCount*

```
int PrimeCount(int n) { // n ~ 10^13 => < 2s
  if (n <= 1) return 0;
  int v = sqrt(n), s = (v + 1) / 2, pc = 0;
  vector<int> smalls(v + 1), skip(v + 1), roughs(s);
  vector<int> larges(s);
  for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
  for (int i = 0; i < s; ++i) {
    roughs[i] = 2 * i + 1;
    larges[i] = (n / (2 * i + 1) + 1) / 2;
  }
  for (int p = 3; p <= v; ++p) {
    if (smalls[p] > smalls[p - 1]) {
      int q = p * p;
      ++pc;
      if (1LL * q * q > n) break;
      skip[p] = 1;
      for (int i = q; i <= v; i += 2 * p) skip[i] = 1;
      int ns = 0;
      for (int k = 0; k < s; ++k) {
        int i = roughs[k];
        if (skip[i]) continue;
        int d = 1LL * i * p;
        larges[ns] = larges[k] - (d <= v ? larges
            [smalls[d] - pc] : smalls[n / d]) + pc;
        roughs[ns++] = i;
      }
      s = ns;
      for (int j = v / p; j >= p; --j) {
        int c =
            smalls[j] - pc, e = min(j * p + p, v + 1);
        for (int i = j * p; i < e; ++i) smalls[i] -= c;
      }
    }
  }
  for (int k = 1; k < s; ++k) {
    const int m = n / roughs[k];
    int t = larges[k] - (pc + k - 1);
    for (int l = 1; l < k; ++l) {
      int p = roughs[l];
      if (1LL * p * p > m) break;
      t -= smalls[m / p] - (pc + l - 1);
    }
    larges[0] -= t;
  }
  return larges[0];
}
```

6.15 Discrete Log*

```
int DiscreteLog(int s, int x, int y, int m) {
  constexpr int kStep = 32000;
  unordered_map<int, int> p;
  int b = 1;
  for (int i = 0; i < kStep; ++i) {
    p[y] = i;
    y = 1LL * y * x % m;
    b = 1LL * b * x % m;
  }
  for (int i = 0; i < m + 10; i += kStep) {
    s = 1LL * s * b % m;
    if (p.find(s) != p.end()) return i + kStep - p[s];
  }
  return -1;
}
int DiscreteLog(int x, int y, int m) {
  if (m == 1) return 0;
  int s = 1;
  for (int i = 0; i < 100; ++i) {
    if (s == y) return i;
    s = 1LL * s * x % m;
  }
  if (s == y) return 100;
  int p = 100 + DiscreteLog(s, x, y, m);
  if (fpow(x, p, m) != y) return -1;
  return p;
}
```

6.16 Berlekamp Massey

```
template <typename T>
vector<T> BerlekampMassey(const vector<T> &output) {
  vector<T> d(SZ(output) + 1), me, he;
  for (int f = 0, i = 1; i <= SZ(output); ++i) {
    for (int j = 0; j < SZ(me); ++j)
      d[i] += output[i - j - 2] * me[j];
    if ((d[i] -= output[i - 1]) == 0) continue;
    if (me.empty()) {
      me.resize(f = i);
      continue;
    }
    vector<T> o(i - f - 1);
    T k = -d[i] / d[f]; o.eb(-k);
    for (T x : he) o.eb(x * k);
    o.resize(max(SZ(o), SZ(me)));
    for (int j = 0; j < SZ(me); ++j) o[j] += me[j];
    if (i - f + SZ(he) >= SZ(me)) he = me, f = i;
    me = o;
  }
  return me;
}
```

6.17 Characteristic Polynomial

```
template <class Tp>
     void hessenberg_reduction(vector<vector<Tp>> &M) {
  // assert(M.size() == M[0].size());
  const int N = M.size();
  for (int r = 0; r < N - 2; r++) {
    int piv = -1;
    for (int h = r + 1; h < N; ++h) {
```

```
      if (M[h][r] != 0) {
        piv = h;
        break;
      }
    }
    if (piv < 0) continue;
    for (int i
           = 0; i < N; i++) swap(M[r + 1][i], M[piv][i]);
    for (int i
           = 0; i < N; i++) swap(M[i][r + 1], M[i][piv]);
    const auto rinv = Tp(1) / M[r + 1][r];
    for (int i = r + 2; i < N; i++) {
      const auto n = M[i][r] * rinv;
      for (int j
             = 0; j < N; j++) M[i][j] -= M[r + 1][j] * n;
      for (int j
             = 0; j < N; j++) M[j][r + 1] += M[j][i] * n;
    }
  }
}
template <class Tp> vector
    <Tp> characteristic_poly(vector<vector<Tp>> M) {
  hessenberg_reduction(M);
  const int N = M.size();
  vector<vector<Tp>> p(N + 1);
  p[0] = {1};
  for (int i = 0; i < N; i++) {
    p[i + 1].assign(i + 2, 0);
    for (int j =
           0; j < i + 1; j++) p[i + 1][j + 1] += p[i][j];
    for (int j = 0; j
          < i + 1; j++) p[i + 1][j] -= p[i][j] * M[i][i];
    Tp betas = 1;
    for (int j = i - 1; j >= 0; j--) {
      betas *= M[j + 1][j];
      Tp hb = -M[j][i] * betas;
      for (int k = 0;
             k < j + 1; k++) p[i + 1][k] += hb * p[j][k];
    }
  }
  return p[N];
}
```

6.18 Primes

```
/* 12721 13331 14341 75577 123457 222557
     556679 999983 1097774749 1076767633 100102021
    999997771 1001010013 1000512343 987654361 999991231
     999888733 98789101 987777733 999991921 1010101333
     1010102101 1000000000039 1000000000000037
     2305843009213693951 4611686018427387847
     9223372036854775783 18446744073709551557 */
```

6.19 Theorem

- Cramer's rule

$$\begin{array}{l} ax+by=e \\ cx+dy=f \end{array} \Rightarrow \begin{array}{l} x=\dfrac{ed-bf}{ad-bc} \\ y=\dfrac{af-ec}{ad-bc} \end{array}$$

- Vandermonde's Identity

$$C(n+m,k)=\sum_{i=0}^{k}C(n,i)C(m,k-i)$$

- Kirchhoff's Theorem
  Denote L be a $n\times n$ matrix as the Laplacian matrix of graph G, where $L_{ii}=d(i)$, $L_{ij}=-c$ where c is the number of edge (i,j) in G.
  - The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
  - The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.
- Tutte's Matrix
  Let D be a $n\times n$ matrix, where $d_{ij}=x_{ij}$ (x_{ij} is chosen uniformly at random) if $i<j$ and $(i,j)\in E$, otherwise $d_{ij}=-d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.
- Cayley's Formula
  - Given a degree sequence $d_1,d_2,\ldots,d_n$ for each *labeled* vertices, there are $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees.
  - Let $T_{n,k}$ be the number of *labeled* forests on n vertices with k components, such that vertex $1,2,\ldots,k$ belong to different components. Then $T_{n,k}=kn^{n-k-1}$.
- Erdős–Gallai theorem
  A sequence of nonnegative integers $d_1\ge\cdots\ge d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1+\cdots+d_n$ is even and $\sum_{i=1}^{k}d_i\le k(k-1)+\sum_{i=k+1}^{n}\min(d_i,k)$ holds for every $1\le k\le n$.
- Gale–Ryser theorem
  A pair of sequences of nonnegative integers $a_1\ge\cdots\ge a_n$ and $b_1,\ldots,b_n$ is bigraphic if and only if $\sum_{i=1}^{n}a_i=\sum_{i=1}^{n}b_i$ and $\sum_{i=1}^{k}a_i\le\sum_{i=1}^{n}\min(b_i,k)$ holds for every $1\le k\le n$.
- Fulkerson–Chen–Anstee theorem
  A sequence $(a_1,\ b_1),\ \ldots,\ (a_n,\ b_n)$ of nonnegative integer pairs with $a_1\ \ge\ \cdots\ \ge\ a_n$ is digraphic if and only if $\sum_{i=1}^{n}a_i=\sum_{i=1}^{n}b_i$ and $\sum_{i=1}^{k}a_i\le\sum_{i=1}^{k}\min(b_i,k-1)+\sum_{i=k+1}^{n}\min(b_i,k)$ holds for every $1\le k\le n$.
- Pick's theorem
  For simple polygon, when points are all integer, we have $A=\#\{\text{lattice points in the interior}\}+\frac{\#\{\text{lattice points on the boundary}\}}{2}-1$.
- Möbius inversion formula
  - $f(n)=\sum_{d|n}g(d)\Leftrightarrow g(n)=\sum_{d|n}\mu(d)f(\frac{n}{d})$
  - $f(n)=\sum_{n|d}g(d)\Leftrightarrow g(n)=\sum_{n|d}\mu(\frac{d}{n})f(d)$
- Spherical cap
  - A portion of a sphere cut off by a plane.
  - r: sphere radius, a: radius of the base of the cap, h: height of the cap, θ: $\arcsin(a/r)$.
  - Volume $=\pi h^2(3r-h)/3=\pi h(3a^2+h^2)/6=\pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$.
  - Area $=2\pi rh=\pi(a^2+h^2)=2\pi r^2(1-\cos\theta)$.
- Lagrange multiplier
  - Optimize $f(x_1,\ldots,x_n)$ when k constraints $g_i(x_1,\ldots,x_n)=0$.
  - Lagrangian function $\mathcal{L}(x_1,\ \ldots\ ,x_n,\ \lambda_1,\ \ldots\ ,\lambda_k)\ =\ f(x_1,\ \ldots\ ,x_n)\ -\ \sum_{i=1}^{k}\lambda_i g_i(x_1,\ldots,x_n)$.
  - The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.
- Nearest points of two skew lines
  - Line 1 $:\boldsymbol{v}_1=\boldsymbol{p}_1+t_1\boldsymbol{d}_1$
  - Line 2 $:\boldsymbol{v}_2=\boldsymbol{p}_2+t_2\boldsymbol{d}_2$
  - $\boldsymbol{n}=\boldsymbol{d}_1\times\boldsymbol{d}_2$
  - $\boldsymbol{n}_1=\boldsymbol{d}_1\times\boldsymbol{n}$
  - $\boldsymbol{n}_2=\boldsymbol{d}_2\times\boldsymbol{n}$
  - $\boldsymbol{c}_1=\boldsymbol{p}_1+\frac{(\boldsymbol{p}_2-\boldsymbol{p}_1)\cdot\boldsymbol{n}_2}{\boldsymbol{d}_1\cdot\boldsymbol{n}_2}\boldsymbol{d}_1$
  - $\boldsymbol{c}_2=\boldsymbol{p}_2+\frac{(\boldsymbol{p}_1-\boldsymbol{p}_2)\cdot\boldsymbol{n}_1}{\boldsymbol{d}_2\cdot\boldsymbol{n}_1}\boldsymbol{d}_2$

6.20 Estimation

- Estimation
  - The number of divisors of n is at most around 100 for $n<5e4$, 500 for $n<1e7$, 2000 for $n<1e10$, 200000 for $n<1e19$.
  - The number of ways of writing n as a sum of positive integers, disregarding the order of the summands. $1,1,2,3,5,7,11,15,22,30$ for $n=0\sim9$, 627 for $n=20$, $\sim2e5$ for $n=50$, $\sim2e8$ for $n=100$.
  - Total number of partitions of n distinct elements: $B(n)=1,1,2,5,15,52,203,877,4140,21147,115975,678570,4213597,27644437,190899322,\ldots$.

6.21 Euclidean Algorithms

- $m=\lfloor\frac{an+b}{c}\rfloor$
- Time complexity: $O(\log n)$

$$f(a,b,c,n)=\sum_{i=0}^{n}\lfloor\frac{ai+b}{c}\rfloor$$

$$=\begin{cases}\lfloor\frac{a}{c}\rfloor\cdot\frac{n(n+1)}{2}+\lfloor\frac{b}{c}\rfloor\cdot(n+1) & \\ +f(a\bmod c,b\bmod c,c,n), & a\ge c\lor b\ge c\\ 0, & n<0\lor a=0\\ nm-f(c,c-b-1,a,m-1), & \text{otherwise}\end{cases}$$

$$g(a,b,c,n)=\sum_{i=0}^{n}i\lfloor\frac{ai+b}{c}\rfloor$$

$$=\begin{cases}\lfloor\frac{a}{c}\rfloor\cdot\frac{n(n+1)(2n+1)}{6}+\lfloor\frac{b}{c}\rfloor\cdot\frac{n(n+1)}{2} & \\ +g(a\bmod c,b\bmod c,c,n), & a\ge c\lor b\ge c\\ 0, & n<0\lor a=0\\ \frac{1}{2}\cdot(n(n+1)m-f(c,c-b-1,a,m-1) & \\ -h(c,c-b-1,a,m-1)), & \text{otherwise}\end{cases}$$

$$h(a,b,c,n)=\sum_{i=0}^{n}\lfloor\frac{ai+b}{c}\rfloor^2$$

$$=\begin{cases}\lfloor\frac{a}{c}\rfloor^2\cdot\frac{n(n+1)(2n+1)}{6}+\lfloor\frac{b}{c}\rfloor^2\cdot(n+1) \\ +\lfloor\frac{a}{c}\rfloor\cdot\lfloor\frac{b}{c}\rfloor\cdot n(n+1) \\ +h(a \bmod c,b \bmod c,c,n) \\ +2\lfloor\frac{a}{c}\rfloor\cdot g(a \bmod c,b \bmod c,c,n) \\ +2\lfloor\frac{b}{c}\rfloor\cdot f(a \bmod c,b \bmod c,c,n), & a\geq c\vee b\geq c \\ 0, & n<0\vee a=0 \\ nm(m+1)-2g(c,c-b-1,a,m-1) \\ -2f(c,c-b-1,a,m-1)-f(a,b,c,n), & \text{otherwise}\end{cases}$$

6.22 General Purpose Numbers

- Bernoulli numbers
  $B_0-1, B_1^{\pm}=\pm\frac{1}{2}, B_2=\frac{1}{6}, B_3=0$
  $\sum_{j=0}^{m}\binom{m+1}{j}B_j=0$, EGF is $B(x)=\frac{x}{e^x-1}=\sum_{n=0}^{\infty}B_n\frac{x^n}{n!}$.
  $S_m(n)=\sum_{k=1}^{n}k^m=\frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k^+n^{m+1-k}$
- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.
  $S(n,k)=S(n-1,k-1)+kS(n-1,k), S(n,1)=S(n,n)=1$
  $S(n,k)=\frac{1}{k!}\sum_{i=0}^{k}(-1)^{k-i}\binom{k}{i}i^n$
  $x^n=\sum_{i=0}^{n}S(n,i)(x)_i$
- Pentagonal number theorem
  $\prod_{n=1}^{\infty}(1-x^n)=1+\sum_{k=1}^{\infty}(-1)^k\left(x^{k(3k+1)/2}+x^{k(3k-1)/2}\right)$
- Catalan numbers
  $C_n^{(k)}=\frac{1}{(k-1)n+1}\binom{kn}{n}$
  $C^{(k)}(x)=1+x[C^{(k)}(x)]^k$
- Eulerian numbers
  Number of permutations $\pi\in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j)>\pi(j+1)$, $k+1$ j:s s.t. $\pi(j)\geq j$, k j:s s.t. $\pi(j)>j$.
  $E(n,k)=(n-k)E(n-1,k-1)+(k+1)E(n-1,k)$
  $E(n,0)=E(n,n-1)=1$
  $E(n,k)=\sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$

6.23 Tips for Generating Functions

- Ordinary Generating Function $A(x)=\sum_{i\geq 0}a_ix^i$
  - $A(rx)\Rightarrow r^na_n$
  - $A(x)+B(x)\Rightarrow a_n+b_n$
  - $A(x)B(x)\Rightarrow\sum_{i=0}^{n}a_ib_{n-i}$
  - $A(x)^k\Rightarrow\sum_{i_1+i_2+\cdots+i_k=n}a_{i_1}a_{i_2}\ldots a_{i_k}$
  - $xA(x)'\Rightarrow na_n$
  - $\frac{A(x)}{1-x}\Rightarrow\sum_{i=0}^{n}a_i$
- Exponential Generating Function $A(x)=\sum_{i\geq 0}\frac{a_i}{i!}x_i$
  - $A(x)+B(x)\Rightarrow a_n+b_n$
  - $A^{(k)}(x)\Rightarrow a_{n+k}$
  - $A(x)B(x)\Rightarrow\sum_{i=0}^{n}\binom{n}{i}a_ib_{n-i}$
  - $A(x)^k\Rightarrow\sum_{i_1+i_2+\cdots+i_k=n}\binom{n}{i_1,i_2,\ldots,i_k}a_{i_1}a_{i_2}\ldots a_{i_k}$
  - $xA(x)\Rightarrow na_n$
- Special Generating Function
  - $(1+x)^n=\sum_{i\geq 0}\binom{n}{i}x^i$
  - $\frac{1}{(1-x)^n}=\sum_{i\geq 0}\binom{i}{n-1}x^i$

7 Polynomial

7.1 Fast Fourier Transform

```
template<int maxn>
struct FFT {
  using val_t = complex<double>;
  const double PI = acos(-1);
  val_t w[maxn];
  FFT() {
    for (int i = 0; i < maxn; ++i) {
      double arg = 2 * PI * i / maxn;
      w[i] = val_t(cos(arg), sin(arg));
    }
  }
  void bitrev(val_t *a, int n); // see NTT
  void trans
      (val_t *a, int n, bool inv = false); // see NTT;
  // remember to replace LL with val_t
};
```

7.2 Number Theory Transform*

```
//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
template<int P, int RT> //maxn must be 2^k
struct NTT {
  int maxn;
  vector<int> w;
  NTT(int n) {
    maxn = 2 << __lg(n+1);
    w.resize(maxn);
    int dw = mpow(RT, (P - 1) / maxn);
    w[0] = 1;
    for (int
        i = 1; i < maxn; ++i) w[i] = w[i - 1] * dw % P;
  }
  void bitrev(vector<int> &a) {
    int i = 0;
    for (int j = 1; j < maxn - 1; ++j) {
      for (int k = maxn >> 1; (i ^= k) < k; k >>= 1);
      if (j < i) swap(a[i], a[j]);
    }
  }
  void operator()(vector
      <int> &a, bool inv = false) { //0 <= a[i] < P
    bitrev(a);
    for (int L = 2; L <= maxn; L <<= 1) {
      int dx = maxn / L, dl = L >> 1;
      for (int i = 0; i < maxn; i += L) {
        for (int
            j = i, x = 0; j < i + dl; ++j, x += dx) {
          int tmp = a[j + dl] * w[x] % P;
          if ((a[j
              + dl] = a[j] - tmp) < 0) a[j + dl] += P;
          if ((a[j] += tmp) >= P) a[j] -= P;
        }
      }
    }
    if (inv) {
      reverse(1 + begin(a), end(a));
      int invn = fpow(maxn, P-2); // do fpow
      for (int
          i = 0; i < maxn; ++i) a[i] = a[i] * invn % P;
    }
  }
};
```

7.3 Fast Walsh Transform*

```
/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
  for (int L = 2; L <= n; L <<= 1)
    for (int i = 0; i < n; i += L)
      for (int j = i; j < i + (L >> 1); ++j)
        a[j + (L >> 1)] += a[j] * op;
}
const int N = 21;
int f[
    N][1 << N], g[N][1 << N], h[N][1 << N], ct[1 << N];
void
    subset_convolution(int *a, int *b, int *c, int L) {
  // c_k = \sum_{i | j = k, i & j = 0} a_i * b_j
  int n = 1 << L;
  for (int i = 1; i < n; ++i)
    ct[i] = ct[i & (i - 1)] + 1;
  for (int i = 0; i < n; ++i)
    f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
  for (int i = 0; i <= L; ++i)
    fwt(f[i], n, 1), fwt(g[i], n, 1);
  for (int i = 0; i <= L; ++i)
    for (int j = 0; j <= i; ++j)
      for (int x = 0; x < n; ++x)
        h[i][x] += f[j][x] * g[i - j][x];
  for (int i = 0; i <= L; ++i)
    fwt(h[i], n, -1);
  for (int i = 0; i < n; ++i)
    c[i] = h[ct[i]][i];
}
```

7.4 Polynomial Operation

```
#define
    fi(s, n) for (int i = (int)(s); i < (int)(n); ++i)
template<int maxn, int P, int RT> // maxn = 2^k
struct Poly : vector<int> { // coefficients in [0, P)
  using vector<int>::vector;
  static NTT<maxn, P, RT> ntt;
  int n() const { return (int)size(); } // n() >= 1
  Poly(const Poly &p, int m) : vector<int>(m) {
    copy_n(p.data(), min(p.n(), m), data());
  }
  Poly& irev()
       { return reverse(data(), data() + n()), *this; }
  Poly& isz(int m) { return resize(m), *this; }
  Poly& iadd(const Poly &rhs) { // n() == rhs.n()
    fi(0, n()) if
         (((*this)[i] += rhs[i]) >= P) (*this)[i] -= P;
    return *this;
  }
  Poly& imul(int k) {
    fi(0, n()) (*this)[i] = (*this)[i] * k % P;
    return *this;
  }
  Poly Mul(const Poly &rhs) const {
    int m = 1;
    while (m < n() + rhs.n() - 1) m <<= 1;
    Poly X(*this, m), Y(rhs, m);
    ntt(X.data(), m), ntt(Y.data(), m);
    fi(0, m) X[i] = X[i] * Y[i] % P;
    ntt(X.data(), m, true);
    return X.isz(n() + rhs.n() - 1);
  }
  Poly Inv() const { // (*this)[0] != 0, 1e5/95ms
    if (n() == 1) return {ntt.minv((*this)[0])};
    int m = 1;
    while (m < n() * 2) m <<= 1;
    Poly Xi = Poly(*this, (n() + 1) / 2).Inv().isz(m);
    Poly Y(*this, m);
    ntt(Xi.data(), m), ntt(Y.data(), m);
    fi(0, m) {
      Xi[i] *= (2 - Xi[i] * Y[i]) % P;
      if ((Xi[i] %= P) < 0) Xi[i] += P;
    }
    ntt(Xi.data(), m, true);
    return Xi.isz(n());
  }
  Poly Sqrt()
       const { // Jacobi((*this)[0], P) = 1, 1e5/235ms
    if (n()
        == 1) return {QuadraticResidue((*this)[0], P)};
    Poly
        X = Poly(*this, (n() + 1) / 2).Sqrt().isz(n());
    return
         X.iadd(Mul(X.Inv()).isz(n())).imul(P / 2 + 1);
  }
  pair<Poly, Poly> DivMod
      (const Poly &rhs) const { // (rhs.)back() != 0
    if (n() < rhs.n()) return {{0}, *this};
    const int m = n() - rhs.n() + 1;
    Poly X(rhs); X.irev().isz(m);
    Poly Y(*this); Y.irev().isz(m);
    Poly Q = Y.Mul(X.Inv()).isz(m).irev();
    X = rhs.Mul(Q), Y = *this;
    fi(0, n()) if ((Y[i] -= X[i]) < 0) Y[i] += P;
    return {Q, Y.isz(max(1, rhs.n() - 1))};
  }
  Poly Dx() const {
    Poly ret(n() - 1);
    fi(0,
         ret.n()) ret[i] = (i + 1) * (*this)[i + 1] % P;
    return ret.isz(max(1, ret.n()));
  }
  Poly Sx() const {
    Poly ret(n() + 1);
    fi(0, n())
         ret[i + 1] = ntt.minv(i + 1) * (*this)[i] % P;
    return ret;
  }
  Poly _tmul(int nn, const Poly &rhs) const {
    Poly Y = Mul(rhs).isz(n() + nn - 1);
    return Poly(Y.data() + n() - 1, Y.data() + Y.n());
  }
  vector<int> _eval(const
       vector<int> &x, const vector<Poly> &up) const {
    const int m = (int)x.size();
    if (!m) return {};
    vector<Poly> down(m * 2);
    // down[1] = DivMod(up[1]).second;
    // fi(2, m *
      2) down[i] = down[i / 2].DivMod(up[i]).second;
    down[1] = Poly(up[1])
         .irev().isz(n()).Inv().irev()._tmul(m, *this);
    fi(2, m * 2) down[i]
        = up[i ^ 1]._tmul(up[i].n() - 1, down[i / 2]);
    vector<int> y(m);
    fi(0, m) y[i] = down[m + i][0];
    return y;
  }
  static vector<Poly> _tree1(const vector<int> &x) {
    const int m = (int)x.size();
    vector<Poly> up(m * 2);
    fi(0, m) up[m + i] = {(x[i] ? P - x[i] : 0), 1};
    for (int i = m - 1; i
         > 0; --i) up[i] = up[i * 2].Mul(up[i * 2 + 1]);
    return up;
  }
  vector<int
      > Eval(const vector<int> &x) const { // 1e5, 1s
    auto up = _tree1(x); return _eval(x, up);
  }
  static Poly Interpolate(const vector
      <int> &x, const vector<int> &y) { // 1e5, 1.4s
    const int m = (int)x.size();
    vector<Poly> up = _tree1(x), down(m * 2);
    vector<int> z = up[1].Dx()._eval(x, up);
    fi(0, m) z[i] = y[i] * ntt.minv(z[i]) % P;
    fi(0, m) down[m + i] = {z[i]};
    for (int i = m -
        1; i > 0; --i) down[i] = down[i * 2].Mul(up[i
        * 2 + 1]).iadd(down[i * 2 + 1].Mul(up[i * 2]));
    return down[1];
  }
  Poly Ln() const { // (*this)[0] == 1, 1e5/170ms
    return Dx().Mul(Inv()).Sx().isz(n());
  }
  Poly Exp() const { // (*this)[0] == 0, 1e5/360ms
    if (n() == 1) return {1};
    Poly X = Poly(*this, (n() + 1) / 2).Exp().isz(n());
    Poly Y = X.Ln(); Y[0] = P - 1;
    fi(0, n())
        if ((Y[i] = (*this)[i] - Y[i]) < 0) Y[i] += P;
    return X.Mul(Y).isz(n());
  }
  // M := P(P - 1). If k >= M, k := k % M + M.
  Poly Pow(int k) const {
    int nz = 0;
    while (nz < n() && !(*this)[nz]) ++nz;
    if (nz * min(k, (int)n()) >= n()) return Poly(n());
    if (!k) return Poly(Poly {1}, n());
    Poly X(data() + nz, data() + nz + n() - nz * k);
    const int c = ntt.mpow(X[0], k % (P - 1));
    return X.Ln().imul
        (k % P).Exp().imul(c).irev().isz(n()).irev();
  }
  static int LinearRecursion
      (const vector<int> &a, const vector
      <int> &coef, int n) { // a_n = \sum c_j a_(n-j)
    const int k = (int)a.size();
    assert((int)coef.size() == k + 1);
    Poly C(k + 1), W(Poly {1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = coef[i] ? P - coef[i] : 0;
    C[k] = 1;
    while (n) {
      if (n % 2) W = W.Mul(M).DivMod(C).second;
      n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    int ret = 0;
    fi(0, k) ret = (ret + W[i] * a[i]) % P;
    return ret;
  }
};
#undef fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};
```

7.5 Value Polynomial

```
struct Poly {
  mint base; // f(x) = poly[x - base]
  vector<mint> poly;
  Poly(mint b = 0, mint x = 0): base(b), poly(1, x) {}
  mint get_val(const mint &x) {
    if (x >= base && x < base + SZ(poly))
```

```
      return poly[x - base];
    mint rt = 0;
    vector<mint> lmul(SZ(poly), 1), rmul(SZ(poly), 1);
    for (int i = 1; i < SZ(poly); ++i)
      lmul[i] = lmul[i - 1] * (x - (base + i - 1));
    for (int i = SZ(poly) - 2; i >= 0; --i)
      rmul[i] = rmul[i + 1] * (x - (base + i + 1));
    for (int i = 0; i < SZ(poly); ++i)
      rt += poly[i] * ifac[i] * inegfac
          [SZ(poly) - 1 - i] * lmul[i] * rmul[i];
    return rt;
  }
  void raise() { // g(x) = sigma{base:x} f(x)
    if (SZ(poly) == 1 && poly[0] == 0)
      return;
    mint nw = get_val(base + SZ(poly));
    poly.eb(nw);
    for (int i = 1; i < SZ(poly); ++i)
      poly[i] += poly[i - 1];
  }
};
```

7.6 Newton's Method

Given $F(x)$ where

$$F(x)=\sum_{i=0}^{\infty}\alpha_i(x-\beta)^i$$

for β being some constant. Polynomial P such that $F(P)=0$ can be found iteratively. Denote by Q_k the polynomial such that $F(Q_k)=0 \pmod{x^{2^k}}$, then

$$Q_{k+1}=Q_k-\frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

8 Geometry

8.1 Default Code

```
typedef pair<double, double> pdd;
typedef pair<pdd, pdd> Line;
struct Cir{ pdd O; double R; };
const double eps = 1e-8;
pdd operator+(pdd a, pdd b)
{ return pdd(a.X + b.X, a.Y + b.Y); }
pdd operator-(pdd a, pdd b)
{ return pdd(a.X - b.X, a.Y - b.Y); }
pdd operator*(pdd a, double b)
{ return pdd(a.X * b, a.Y * b); }
pdd operator/(pdd a, double b)
{ return pdd(a.X / b, a.Y / b); }
double dot(pdd a, pdd b)
{ return a.X * b.X + a.Y * b.Y; }
double cross(pdd a, pdd b)
{ return a.X * b.Y - a.Y * b.X; }
double abs2(pdd a)
{ return dot(a, a); }
double abs(pdd a)
{ return sqrt(dot(a, a)); }
int sign(double a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1; }
int ori(pdd a, pdd b, pdd c)
{ return sign(cross(b - a, c - a)); }
bool collinearity(pdd p1, pdd p2, pdd p3)
{ return sign(cross(p1 - p3, p2 - p3)) == 0; }
bool btw(pdd p1, pdd p2, pdd p3) {
  if (!collinearity(p1, p2, p3)) return 0;
  return sign(dot(p1 - p3, p2 - p3)) <= 0;
}
bool seg_intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
  int a123 = ori(p1, p2, p3);
  int a124 = ori(p1, p2, p4);
  int a341 = ori(p3, p4, p1);
  int a342 = ori(p3, p4, p2);
  if (a123 == 0 && a124 == 0)
    return btw(p1, p2, p3) || btw(p1, p2, p4) ||
      btw(p3, p4, p1) || btw(p3, p4, p2);
  return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
  double a123 = cross(p2 - p1, p3 - p1);
  double a124 = cross(p2 - p1, p4 - p1);
  return (p4
      * a123 - p3 * a124) / (a123 - a124); // C^3 / C^2
}
pdd perp(pdd p1)
{ return pdd(-p1.Y, p1.X); }
pdd projection(pdd p1, pdd p2, pdd p3)
{ return p1 + (
    p2 - p1) * dot(p3 - p1, p2 - p1) / abs2(p2 - p1); }
pdd reflection(pdd p1, pdd p2, pdd p3)
{ return p3 + perp(p2 - p1
    ) * cross(p3 - p1, p2 - p1) / abs2(p2 - p1) * 2; }
pdd linearTransformation
    (pdd p0, pdd p1, pdd q0, pdd q1, pdd r) {
  pdd dp = p1 - p0
      , dq = q1 - q0, num(cross(dp, dq), dot(dp, dq));
  return q0 + pdd(
      cross(r - p0, num), dot(r - p0, num)) / abs2(dp);
} // from line p0--p1 to q0--q1, apply to r
```

8.2 PointSegDist*

```
double PointSegDist(pdd q0, pdd q1, pdd p) {
  if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
  if (sign(dot(q1 - q0,
      p - q0)) >= 0 && sign(dot(q0 - q1, p - q1)) >= 0)
    return fabs(cross(q1 - q0, p - q0) / abs(q0 - q1));
  return min(abs(p - q0), abs(p - q1));
}
```

8.3 Heart

```
pdd circenter
    (pdd p0, pdd p1, pdd p2) { // radius = abs(center)
  p1 = p1 - p0, p2 = p2 - p0;
  double x1 = p1.X, y1 = p1.Y, x2 = p2.X, y2 = p2.Y;
  double m = 2. * (x1 * y2 - y1 * x2);
  center.X = (x1 * x1
       * y2 - x2 * x2 * y1 + y1 * y2 * (y1 - y2)) / m;
  center.Y = (x1 * x2
       * (x2 - x1) - y1 * y1 * x2 + x1 * y2 * y2) / m;
  return center + p0;
}
pdd incenter
    (pdd p1, pdd p2, pdd p3) { // radius = area / s * 2
  double a =
      abs(p2 - p3), b = abs(p1 - p3), c = abs(p1 - p2);
  double s = a + b + c;
  return (a * p1 + b * p2 + c * p3) / s;
}
pdd masscenter(pdd p1, pdd p2, pdd p3)
{ return (p1 + p2 + p3) / 3; }
pdd orthcenter(pdd p1, pdd p2, pdd p3)
{ return masscenter
    (p1, p2, p3) * 3 - circenter(p1, p2, p3) * 2; }
```

8.4 point in circle

```
// return q'
    s relation with circumcircle of tri(p[0],p[1],p[2])
bool in_cc(const array<pii, 3> &p, pii q) {
  __int128 det = 0;
  for (int i = 0; i < 3; ++i)
    det += __int128(abs2(p[i]) - abs2(q)) *
        cross(p[(i + 1) % 3] - q, p[(i + 2) % 3] - q);
  return det > 0; // in: >0, on: =0, out: <0
}
```

8.5 Convex hull*

```
void hull(vector<pii> &dots) { // n=1 => ans = {}
  sort(dots.begin(), dots.end());
  vector<pii> ans(1, dots[0]);
  for (int ct = 0; ct < 2; ++ct, reverse(ALL(dots)))
    for (int i = 1,
        t = SZ(ans); i < SZ(dots); ans.eb(dots[i++]))
      while (SZ(ans) > t && ori
          (ans[SZ(ans) - 2], ans.back(), dots[i]) <= 0)
        ans.pb();
  ans.pb(), ans.swap(dots);
}
```

8.6 PointInConvex*

```
bool PointInConvex
    (const vector<pii> &C, pii p, bool strict = true) {
  int a = 1, b = SZ(C) - 1, r = !strict;
  if (SZ(C) == 0) return false;
  if (SZ(C) < 3) return r && btw(C[0], C.back(), p);
  if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
  if (ori
      (C[0], C[a], p) >= r || ori(C[0], C[b], p) <= -r)
```

```
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (ori(C[0], C[c], p) > 0 ? b : a) = c;
  }
  return ori(C[a], C[b], p) < r;
}
```

8.7 TangentPointToHull*

```
/* The point should be strictly out of hull
  return arbitrary point on the tangent line */
pii get_tangent(vector<pii> &C, pii p) {
  auto gao = [&](int s) {
    return cyc_tsearch(SZ(C), [&](int x, int y)
    { return ori(p, C[x], C[y]) == s; });
  };
  return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0
```

8.8 Intersection of line and convex

```
int TangentDir(vector<pii> &C, pii dir) {
  return cyc_tsearch(SZ(C), [&](int a, int b) {
    return cross(dir, C[a]) > cross(dir, C[b]);
  });
}
#define cmpL(i) sign(cross(C[i] - a, b - a))
pii lineHull(pii a, pii b, vector<pii> &C) {
  int A = TangentDir(C, a - b);
  int B = TangentDir(C, b - a);
  int n = SZ(C);
  if (cmpL(A) < 0 || cmpL(B) > 0)
    return pii(-1, -1); // no collision
  auto gao = [&](int l, int r) {
    for (int t = l; (l + 1) % n != r; ) {
      int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(t) ? l : r) = m;
    }
    return (l + !cmpL(r)) % n;
  };
  pii res = pii(gao(B, A), gao(A, B)); // (i, j)
  if (res.X == res.Y) // touching the corner i
    return pii(res.X, -1);
  if (!
      cmpL(res.X) && !cmpL(res.Y)) // along side i, i+1
    switch ((res.X - res.Y + n + 1) % n) {
      case 0: return pii(res.X, res.X);
      case 2: return pii(res.Y, res.Y);
    }
  /* crossing sides (i, i+1) and (j, j+1)
  crossing corner i is treated as side (i, i+1)
  returned
       in the same order as the line hits the convex */
  return res;
} // convex cut: (r, l]
```

8.9 minMaxEnclosingRectangle*

```
const double INF = 1e18, qi = acos(-1) / 2 * 3;
pdd solve(vector<pii> &dots) {
#define diff(u, v) (dots[u] - dots[v])
#define vec(v) (dots[v] - dots[i])
  hull(dots);
  double Max = 0, Min = INF, deg;
  int n = SZ(dots);
  dots.eb(dots[0]);
  for (int i = 0, u = 1, r = 1, l = 1; i < n; ++i) {
    pii nw = vec(i + 1);
    while (cross(nw, vec(u + 1)) > cross(nw, vec(u)))
      u = (u + 1) % n;
    while (dot(nw, vec(r + 1)) > dot(nw, vec(r)))
      r = (r + 1) % n;
    if (!i) l = (r + 1) % n;
    while (dot(nw, vec(l + 1)) < dot(nw, vec(l)))
      l = (l + 1) % n;
    Min = min(Min, (double)(dot(nw, vec(r)) - dot
        (nw, vec(l))) * cross(nw, vec(u)) / abs2(nw));
    deg = acos(dot(diff(r
        , l), vec(u)) / abs(diff(r, l)) / abs(vec(u)));
    deg = (qi - deg) / 2;
    Max = max(Max, abs(diff
        (r, l)) * abs(vec(u)) * sin(deg) * sin(deg));
  }
  return pdd(Min, Max);
}
```

8.10 VectorInPoly*

```
// ori(a
    , b, c) >= 0, valid: "strict" angle from a-b to a-c
bool btwangle(pii a, pii b, pii c, pii p, int strict) {
  return
      ori(a, b, p) >= strict && ori(a, p, c) >= strict;
}
// whether vector
    {cur, p} in counter-clockwise order prv, cur, nxt
bool inside
    (pii prv, pii cur, pii nxt, pii p, int strict) {
  if (ori(cur, nxt, prv) >= 0)
    return btwangle(cur, nxt, prv, p, strict);
  return !btwangle(cur, prv, nxt, p, !strict);
}
```

8.11 PolyUnion*

```
double rat(pii a, pii b) {
  return sign
      (b.X) ? (double)a.X / b.X : (double)a.Y / b.Y;
} // all poly. should be ccw
double polyUnion(vector<vector<pii>> &poly) {
  double res = 0;
  for (auto &p : poly)
    for (int a = 0; a < SZ(p); ++a) {
      pii A = p[a], B = p[(a + 1) % SZ(p)];
      vector
          <pair<double, int>> segs = {{0, 0}, {1, 0}};
      for (auto &q : poly) {
        if (&p == &q) continue;
        for (int b = 0; b < SZ(q); ++b) {
          pii C = q[b], D = q[(b + 1) % SZ(q)];
          int sc = ori(A, B, C), sd = ori(A, B, D);
          if (sc != sd && min(sc, sd) < 0) {
            double sa = cross(D
                - C, A - C), sb = cross(D - C, B - C);
            segs.emplace_back
                (sa / (sa - sb), sign(sc - sd));
          }
          if (!sc && !sd &&
              &q < &p && sign(dot(B - A, D - C)) > 0) {
            segs.emplace_back(rat(C - A, B - A), 1);
            segs.emplace_back(rat(D - A, B - A), -1);
          }
        }
      }
      sort(ALL(segs));
      for (auto &s : segs) s.X = clamp(s.X, 0.0, 1.0);
      double sum = 0;
      int cnt = segs[0].second;
      for (int j = 1; j < SZ(segs); ++j) {
        if (!cnt) sum += segs[j].X - segs[j - 1].X;
        cnt += segs[j].Y;
      }
      res += cross(A, B) * sum;
    }
  return res / 2;
}
```

8.12 PolyCut

```
vector<pdd> cut(vector<pdd> poly, pdd s, pdd e) {
  vector<pdd> res;
  for (int i = 0; i < SZ(poly); ++i) {
    pdd cur
        = poly[i], prv = i ? poly[i - 1] : poly.back();
    bool side = ori(s, e, cur) < 0;
    if (side != (ori(s, e, prv) < 0))
      res.eb(intersect(s, e, cur, prv));
    if (side)
      res.eb(cur);
  }
  return res;
}
```

8.13 Trapezoidalization

```
template<class T>
struct SweepLine {
  struct cmp {
    cmp(const SweepLine &_swp): swp(_swp) {}
    bool operator()(int a, int b) const {
      if (abs(swp.get_y(a) - swp.get_y(b)) <= swp.eps)
        return swp.slope_cmp(a, b);
```

```
    return swp.get_y(a) + swp.eps < swp.get_y(b);
  }
  const SweepLine &swp;
} _cmp;
T curTime, eps, curQ;
vector<Line> base;
multiset<int, cmp> sweep;
multiset<pair<T, int>> event;
vector<typename multiset<int, cmp>::iterator> its;
vector
    <typename multiset<pair<T, int>>::iterator> eits;
bool slope_cmp(int a, int b) const {
  assert(a != -1);
  if (b == -1) return 0;
  return sign(cross(base
      [a].Y - base[a].X, base[b].Y - base[b].X)) < 0;
}
T get_y(int idx) const {
  if (idx == -1) return curQ;
  Line l = base[idx];
  if (l.X.X == l.Y.X) return l.Y.Y;
  return ((curTime - l.X.X) * l.Y.Y
      + (l.Y.X - curTime) * l.X.Y) / (l.Y.X - l.X.X);
}
void insert(int idx) {
  its[idx] = sweep.insert(idx);
  if (its[idx] != sweep.begin())
    update_event(*prev(its[idx]));
  update_event(idx);
  event.emplace(base[idx].Y.X, idx + 2 * SZ(base));
}
void erase(int idx) {
  assert(eits[idx] == event.end());
  auto p = sweep.erase(its[idx]);
  its[idx] = sweep.end();
  if (p != sweep.begin())
    update_event(*prev(p));
}
void update_event(int idx) {
  if (eits[idx] != event.end())
    event.erase(eits[idx]);
  eits[idx] = event.end();
  auto nxt = next(its[idx]);
  if (nxt ==
       sweep.end() || !slope_cmp(idx, *nxt)) return;
  auto t = intersect(base[idx].
      X, base[idx].Y, base[*nxt].X, base[*nxt].Y).X;
  if (t + eps < curTime || t
       >= min(base[idx].Y.X, base[*nxt].Y.X)) return;
  eits[idx] = event.emplace(t, idx + SZ(base));
}
void swp(int idx) {
  assert(eits[idx] != event.end());
  eits[idx] = event.end();
  int nxt = *next(its[idx]);
  swap((int&)*its[idx], (int&)*its[nxt]);
  swap(its[idx], its[nxt]);
  if (its[nxt] != sweep.begin())
    update_event(*prev(its[nxt]));
  update_event(idx);
}
// only expected to call the functions below
SweepLine(T t, T e, vector
    <Line> vec): _cmp(*this), curTime(t), eps(e)
    , curQ(), base(vec), sweep(_cmp), event(), its(SZ
    (vec), sweep.end()), eits(SZ(vec), event.end()) {
  for (int i = 0; i < SZ(base); ++i) {
    auto &[p, q] = base[i];
    if (p > q) swap(p, q);
    if (p.X <= curTime && curTime <= q.X)
      insert(i);
    else if (curTime < p.X)
      event.emplace(p.X, i);
  }
}
void setTime(T t, bool ers = false) {
  assert(t >= curTime);
  while (!event.empty() && event.begin()->X <= t) {
    auto [et, idx] = *event.begin();
    int s = idx / SZ(base);
    idx %= SZ(base);
    if (abs(et - t) <= eps && s == 2 && !ers) break;
    curTime = et;
    event.erase(event.begin());
    if (s == 2) erase(idx);
    else if (s == 1) swp(idx);
    else insert(idx);
  }
  curTime = t;
}
T nextEvent() {
  if (event.empty()) return INF;
  return event.begin()->X;
}
int lower_bound(T y) {
  curQ = y;
  auto p = sweep.lower_bound(-1);
  if (p == sweep.end()) return -1;
  return *p;
}
};
```

8.14 Polar Angle Sort*

```
int cmp(pii a, pii b, bool same = true) {
#define is_neg(k) (
    sign(k.Y) < 0 || (sign(k.Y) == 0 && sign(k.X) < 0))
  int A = is_neg(a), B = is_neg(b);
  if (A != B)
    return A < B;
  if (sign(cross(a, b)) == 0)
    return same ? abs2(a) < abs2(b) : -1;
  return sign(cross(a, b)) > 0;
}
```

8.15 Half plane intersection*

```
pii area_pair(Line a, Line b)
{ return pii(cross(a.Y
    - a.X, b.X - a.X), cross(a.Y - a.X, b.Y - a.X)); }
bool isin(Line l0, Line l1, Line l2) {
  // Check inter(l1, l2) strictly in l0
  auto [a02X, a02Y] = area_pair(l0, l2);
  auto [a12X, a12Y] = area_pair(l1, l2);
  if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
  return i128(a02Y) * a12X - i128(a02X) * a12Y > 0;
}
/* Having solution, check size > 2 */
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> arr) {
  sort(ALL(arr), [&](Line a, Line b) -> int {
    if (cmp(a.Y - a.X, b.Y - b.X, 0) != -1)
      return cmp(a.Y - a.X, b.Y - b.X, 0);
    return ori(a.X, a.Y, b.Y) < 0;
  });
  deque<Line> dq(1, arr[0]);
  auto pop_back = [&](int t, Line p) {
    while (SZ(dq) >= t
        && !isin(p, end(dq)[-2], end(dq)[-1])) dq.pb();
  };
  auto pop_front = [&](int t, Line p) {
    while (
        SZ(dq) >= t && !isin(p, dq[0], dq[1])) dq.pf();
  };
  for (auto p : arr)
    if (cmp(
        dq.back().Y - dq.back().X, p.Y - p.X, 0) != -1)
      pop_back(2, p), pop_front(2, p), dq.eb(p);
  pop_back(3, dq[0]), pop_front(3, dq.back());
  return vector<Line>(ALL(dq));
}
```

8.16 RotatingSweepLine

```
void rotatingSweepLine(vector<pii> &ps) {
  int n = SZ(ps), m = 0;
  vector<int> id(n), pos(n);
  vector<pii> line(n * (n - 1));
  for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j)
      if (i != j) line[m++] = pii(i, j);
  sort(ALL(line), [&](pii a, pii b) {
    return cmp(ps[a.Y] - ps[a.X], ps[b.Y] - ps[b.X]);
  }); // cmp(): polar angle compare
  iota(ALL(id), 0);
  sort(ALL(id), [&](int a, int b) {
    if (ps[a].Y != ps[b].Y) return ps[a].Y < ps[b].Y;
    return ps[a] < ps[b];
  }); // initial order, since (1, 0) is the smallest
  for (int i = 0; i < n; ++i) pos[id[i]] = i;
  for (int i = 0; i < m; ++i) {
    auto l = line[i];
```

```
    // do something
    tie(pos[l.X], pos[l.Y], id[pos[l.X]], id[pos[l.Y
        ]]) = make_tuple(pos[l.Y], pos[l.X], l.Y, l.X);
  }
}
```

8.17 Minimum Enclosing Circle*

```
pdd Minimum_Enclosing_Circle
    (vector<pdd> dots, double &r) {
  pdd cent;
  random_shuffle(ALL(dots));
  cent = dots[0], r = 0;
  for (int i = 1; i < SZ(dots); ++i)
    if (abs(dots[i] - cent) > r) {
      cent = dots[i], r = 0;
      for (int j = 0; j < i; ++j)
        if (abs(dots[j] - cent) > r) {
          cent = (dots[i] + dots[j]) / 2;
          r = abs(dots[i] - cent);
          for(int k = 0; k < j; ++k)
            if(abs(dots[k] - cent) > r)
              cent = excenter
                  (dots[i], dots[j], dots[k], r);
        }
    }
  return cent;
}
```

8.18 Intersection of two circles*

```
bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
  pdd o1 = a.O, o2 = b.O;
  double r1 =
       a.R, r2 = b.R, d2 = abs2(o1 - o2), d = sqrt(d2);
  if(d < max
      (r1, r2) - min(r1, r2) || d > r1 + r2) return 0;
  pdd u = (o1 + o2) * 0.5
       + (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
  double A = sqrt((r1 + r2 + d) *
       (r1 - r2 + d) * (r1 + r2 - d) * (-r1 + r2 + d));
  pdd v
       = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2);
  p1 = u + v, p2 = u - v;
  return 1;
}
```

8.19 Intersection of polygon and circle*

```
// Divides into multiple triangle, and sum up
const double PI=acos(-1);
double _area(pdd pa, pdd eb, double r){
  if(abs(pa)<abs(eb)) swap(pa, eb);
  if(abs(eb)<eps) return 0;
  double S, h, theta;
  double a=abs(eb),b=abs(pa),c=abs(eb-pa);
  double cosB = dot(eb,eb-pa) / a / c, B = acos(cosB);
  double cosC = dot(pa,eb) / a / b, C = acos(cosC);
  if(a > r){
    S = (C/2)*r*r;
    h = a*b*sin(C)/c;
    if (h < r && B
        < PI/2) S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
  }
  else if(b > r){
    theta = PI - B - asin(sin(B)/r*a);
    S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
  }
  else S = .5*sin(C)*a*b;
  return S;
}
double area_poly_circle(const
    vector<pdd> poly,const pdd &O,const double r){
  double S=0;
  for(int i=0;i<SZ(poly);++i)
    S+=_area(poly[i]-O,poly[(i+1)%SZ(poly
        )]-O,r)*ori(O,poly[i],poly[(i+1)%SZ(poly)]);
  return fabs(S);
}
```

8.20 Intersection of line and circle*

```
vector<pdd> circleLine(pdd c, double r, pdd a, pdd b) {
  pdd p
       = a + (b - a) * dot(c - a, b - a) / abs2(b - a);
  double s = cross
      (b - a, c - a), h2 = r * r - s * s / abs2(b - a);
  if (h2 < 0) return {};
  if (h2 == 0) return {p};
  pdd h = (b - a) / abs(b - a) * sqrt(h2);
  return {p - h, p + h};
}
```

8.21 Tangent line of two circles

```
vector<Line
    > go( const Cir& c1 , const Cir& c2 , int sign1 ){
  // sign1 = 1 for outer tang, -1 for inter tang
  vector<Line> ret;
  double d_sq = abs2(c1.O - c2.O);
  if (sign(d_sq) == 0) return ret;
  double d = sqrt(d_sq);
  pdd v = (c2.O - c1.O) / d;
  double c = (c1.R - sign1 * c2.R) / d;
  if (c * c > 1) return ret;
  double h = sqrt(max(0.0, 1.0 - c * c));
  for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
    pdd n = pdd(v.X * c - sign2 * h * v.Y,
      v.Y * c + sign2 * h * v.X);
    pdd p1 = c1.O + n * c1.R;
    pdd p2 = c2.O + n * (c2.R * sign1);
    if (sign(p1.X - p2.X) == 0 and
        sign(p1.Y - p2.Y) == 0)
      p2 = p1 + perp(c2.O - c1.O);
    ret.eb(Line(p1, p2));
  }
  return ret;
}
```

8.22 CircleCover*

```
const int N = 1021;
struct CircleCover {
  int C;
  Cir c[N];
  bool g[N][N], overlap[N][N];
  // Area[i] : area covered by at least i circles
  double Area[ N ];
  void init(int _C){ C = _C;}
  struct Teve {
    pdd p; double ang; int add;
    Teve() {}
    Teve(pdd _a
        , double _b, int _c):p(_a), ang(_b), add(_c){}
    bool operator<(const Teve &a)const
    {return ang < a.ang;}
  }eve[N * 2];
  // strict: x = 0, otherwise x = -1
  bool disjuct(Cir &a, Cir &b, int x)
  {return sign(abs(a.O - b.O) - a.R - b.R) > x;}
  bool contain(Cir &a, Cir &b, int x)
  {return sign(a.R - b.R - abs(a.O - b.O)) > x;}
  bool contain(int i, int j) {
    /* c[j] is non-strictly in c[i]. */
    return (sign
        (c[i].R - c[j].R) > 0 || (sign(c[i].R - c[j].
        R) == 0 && i < j)) && contain(c[i], c[j], -1);
  }
  void solve(){
    fill_n(Area, C + 2, 0);
    for(int i = 0; i < C; ++i)
      for(int j = 0; j < C; ++j)
        overlap[i][j] = contain(i, j);
    for(int i = 0; i < C; ++i)
      for(int j = 0; j < C; ++j)
        g[i][j] = !(overlap[i][j] || overlap[j][i] ||
            disjuct(c[i], c[j], -1));
    for(int i = 0; i < C; ++i){
      int E = 0, cnt = 1;
      for(int j = 0; j < C; ++j)
        if(j != i && overlap[j][i])
          ++cnt;
      for(int j = 0; j < C; ++j)
        if(i != j && g[i][j]) {
          pdd aa, bb;
          CCinter(c[i], c[j], aa, bb);
          double A =
               atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
          double B =
               atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
          eve[E++] = Teve
              (bb, B, 1), eve[E++] = Teve(aa, A, -1);
          if(B > A) ++cnt;
```

```
      }
      if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
      else{
        sort(eve, eve + E);
        eve[E] = eve[0];
        for(int j = 0; j < E; ++j){
          cnt += eve[j].add;
          Area[cnt
              ] += cross(eve[j].p, eve[j + 1].p) * .5;
          double theta = eve[j + 1].ang - eve[j].ang;
          if (theta < 0) theta += 2. * pi;
          Area[cnt] += (theta
                 - sin(theta)) * c[i].R * c[i].R * .5;
        }
      }
    }
  }
};
```

8.23 Minkowski Sum*

```
vector<pii> Minkowski
    (vector<pii> A, vector<pii> B) { // |A|,|B| >= 3
  hull(A), hull(B);
  vector<pii> C(1, A[0] + B[0]), s1, s2;
  for (int i = 0; i < SZ(A); ++i)
    s1.eb(A[(i + 1) % SZ(A)] - A[i]);
  for (int i = 0; i < SZ(B); i++)
    s2.eb(B[(i + 1) % SZ(B)] - B[i]);
  for (int i = 0, j = 0; i < SZ(A) || j < SZ(B);)
    if (j >= SZ
        (B) || (i < SZ(A) && cross(s1[i], s2[j]) >= 0))
      C.eb(B[j % SZ(B)] + A[i++]);
    else
      C.eb(A[i % SZ(A)] + B[j++]);
  return hull(C), C;
}
```

9 Else

9.1 Cyclic Ternary Search*

```
/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
  if (n == 1) return 0;
  int l = 0, r = n; bool rv = pred(1, 0);
  while (r - l > 1) {
    int m = (l + r) / 2;
    if (pred(0, m) ? rv: pred(m, (m + 1) % n)) r = m;
    else l = m;
  }
  return pred(l, r % n) ? l : r % n;
}
```

9.2 Mo's Algorithm(With modification)

```
/*
Mo's Algorithm With modification
Block: N^{2/3}, Complexity: N^{5/3}
*/
struct Query {
  int L, R, LBid, RBid, T;
  Query(int l, int r, int t):
    L(l), R(r), LBid(l / blk), RBid(r / blk), T(t) {}
  bool operator<(const Query &q) const {
    if (LBid != q.LBid) return LBid < q.LBid;
    if (RBid != q.RBid) return RBid < q.RBid;
    return T < b.T;
  }
};
void solve(vector<Query> query) {
  sort(ALL(query));
  int L=0, R=0, T=-1;
  for (auto q : query) {
    while (T < q.T) addTime(L, R, ++T); // TODO
    while (T > q.T) subTime(L, R, T--); // TODO
    while (R < q.R) add(arr[++R]); // TODO
    while (L > q.L) add(arr[--L]); // TODO
    while (R > q.R) sub(arr[R--]); // TODO
    while (L < q.L) sub(arr[L++]); // TODO
    // answer query
  }
}
```

9.3 Mo's Algorithm On Tree

```
/*
Mo's Algorithm On Tree
Preprocess:
1) LCA
2) dfs with in[u] = dft++, out[u] = dft++
3) ord[in[u]] = ord[out[u]] = u
4) bitset<maxn> inset
*/
struct Query {
  int L, R, LBid, lca;
  Query(int u, int v) {
    int c = LCA(u, v);
    if (c == u || c == v)
      q.lca = -1, q.L = out[c ^ u ^ v], q.R = out[c];
    else if (out[u] < in[v])
      q.lca = c, q.L = out[u], q.R = in[v];
    else
      q.lca = c, q.L = out[v], q.R = in[u];
    q.Lid = q.L / blk;
  }
  bool operator<(const Query &q) const {
    if (LBid != q.LBid) return LBid < q.LBid;
    return R < q.R;
  }
};
void flip(int x) {
    if (inset[x]) sub(arr[x]); // TODO
    else add(arr[x]); // TODO
    inset[x] = ~inset[x];
}
void solve(vector<Query> query) {
  sort(ALL(query));
  int L = 0, R = 0;
  for (auto q : query) {
    while (R < q.R) flip(ord[++R]);
    while (L > q.L) flip(ord[--L]);
    while (R > q.R) flip(ord[R--]);
    while (L < q.L) flip(ord[L++]);
    if (~q.lca) add(arr[q.lca]);
    // answer query
    if (~q.lca) sub(arr[q.lca]);
  }
}
```

9.4 Additional Mo's Algorithm Trick

- Mo's Algorithm With Addition Only
  - Sort querys same as the normal Mo's algorithm.
  - For each query $[l,r]$:
  - If $l/blk = r/blk$, brute-force.
  - If $l/blk \neq curL/blk$, initialize $curL := (l/blk+1) \cdot blk, curR := curL-1$
  - If $r > curR$, increase $curR$
  - decrease $curL$ to fit l, and then undo after answering
- Mo's Algorithm With Offline Second Time
  - Require: Changing answer $\equiv$ adding $f([l,r],r+1)$.
  - Require: $f([l,r],r+1) = f([1,r],r+1) - f([1,l),r+1)$.
  - Part1: Answer all $f([1,r],r+1)$ first.
  - Part2: Store $curR \to R$ for $curL$ (reduce the space to $O(N)$), and then answer them by the second offline algorithm.
  - Note: You must do the above symmetrically for the left boundaries.

9.5 Hilbert Curve

```
int hilbert(int n, int x, int y) {
  int res = 0;
  for (int s = n / 2; s; s >>= 1) {
    int rx = (x & s) > 0;
    int ry = (y & s) > 0;
    res += s * 1ll * s * ((3 * rx) ^ ry);
    if (ry == 0) {
      if (rx == 1) x = s - 1 - x, y = s - 1 - y;
      swap(x, y);
    }
  }
  return res;
} // n = 2^k
```

9.6 DynamicConvexTrick*

```
// only works for integer coordinates!! maintain max
struct Line {
  mutable int a, b, p;
  bool operator
      <(const Line &rhs) const { return a < rhs.a; }
  bool operator<(int x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
  static const int kInf = 1e18;
  int Div(int a,
      int b) { return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) { x->p = kInf; return 0; }
    if (x
        ->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
    else x->p = Div(y->b - x->b, x->a - y->a);
    return x->p >= y->p;
  }
  void addline(int a, int b) {
    auto z = insert({a, b, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin
        () && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin
        () && (--x)->p >= y->p) isect(x, erase(y));
  }
  int query(int x) {
    auto l = *lower_bound(x);
    return l.a * x + l.b;
  }
};
```

9.7 All LCS*

```
void all_lcs(string s, string t) { // 0-base
  vector<int> h(SZ(t));
  iota(ALL(h), 0);
  for (int a = 0; a < SZ(s); ++a) {
    int v = -1;
    for (int c = 0; c < SZ(t); ++c)
      if (s[a] == t[c] || h[c] < v)
        swap(h[c], v);
    // LCS(s[0, a], t[b, c]) =
    // c - b + 1 - sum([h[i] >= b] | i <= c)
    // h[i] might become -1 !!
  }
}
```

9.8 DLX*

```
#define TRAV(i, link, start)
     for (int i = link[start]; i != start; i = link[i])
template<
    bool E> // E: Exact, NN: num of 1s, RR: num of rows
struct DLX {
  int lt[NN], rg[NN], up[NN], dn[NN
      ], rw[NN], cl[NN], bt[NN], s[NN], head, sz, ans;
  int rows, columns;
  bool vis[NN];
  bitset<RR> sol, cur; // not sure
  void remove(int c) {
    if (E) lt[rg[c]] = lt[c], rg[lt[c]] = rg[c];
    TRAV(i, dn, c) {
      if (E) {
        TRAV(j, rg, i)
          up[dn[j]]
               = up[j], dn[up[j]] = dn[j], --s[cl[j]];
      } else {
        lt[rg[i]] = lt[i], rg[lt[i]] = rg[i];
      }
    }
  }
  void restore(int c) {
    TRAV(i, up, c) {
      if (E) {
        TRAV(j, lt, i)
          ++s[cl[j]], up[dn[j]] = j, dn[up[j]] = j;
      } else {
        lt[rg[i]] = rg[lt[i]] = i;
      }
    }
    if (E) lt[rg[c]] = c, rg[lt[c]] = c;
  }
  void init(int c) {
    rows = 0, columns = c;
    for (int i = 0; i < c; ++i) {
      up[i] = dn[i] = bt[i] = i;
      lt[i] = i == 0 ? c : i - 1;
      rg[i] = i == c - 1 ? c : i + 1;
      s[i] = 0;
    }
    rg[c] = 0, lt[c] = c - 1;
    up[c] = dn[c] = -1;
    head = c, sz = c + 1;
  }
  void insert(const vector<int> &col) {
    if (col.empty()) return;
    int f = sz;
    for (int i = 0; i < (int)col.size(); ++i) {
      int c = col[i], v = sz++;
      dn[bt[c]] = v;
      up[v] = bt[c], bt[c] = v;
      rg[v] = (i + 1 == (int)col.size() ? f : v + 1);
      rw[v] = rows, cl[v] = c;
      ++s[c];
      if (i > 0) lt[v] = v - 1;
    }
    ++rows, lt[f] = sz - 1;
  }
  int h() {
    int ret = 0;
    fill_n(vis, sz, false);
    TRAV(x, rg, head) {
      if (vis[x]) continue;
      vis[x] = true, ++ret;
      TRAV(i, dn, x) TRAV(j, rg, i) vis[cl[j]] = true;
    }
    return ret;
  }
  void dfs(int dep) {
    if (dep + (E ? 0 : h()) >= ans) return;
    if (rg[head
        ] == head) return sol = cur, ans = dep, void();
    if (dn[rg[head]] == rg[head]) return;
    int w = rg[head];
    TRAV(x, rg, head) if (s[x] < s[w]) w = x;
    if (E) remove(w);
    TRAV(i, dn, w) {
      if (!E) remove(i);
      TRAV(j, rg, i) remove(E ? cl[j] : j);
      cur.set(rw[i]), dfs(dep + 1), cur.reset(rw[i]);
      TRAV(j, lt, i) restore(E ? cl[j] : j);
      if (!E) restore(i);
    }
    if (E) restore(w);
  }
  int solve() {
    for (int i = 0; i < columns; ++i)
      dn[bt[i]] = i, up[i] = bt[i];
    ans = 1e9, sol.reset(), dfs(0);
    return ans;
  }
};
```

9.9 Matroid Intersection

Start from $S=\emptyset$. In each iteration, let

- $Y_1=\{x\notin S \mid S\cup\{x\}\in I_1\}$
- $Y_2=\{x\notin S \mid S\cup\{x\}\in I_2\}$

If there exists $x\in Y_1\cap Y_2$, insert x into S. Otherwise for each $x\in S, y\notin S$, create edges

- $x\to y$ if $S-\{x\}\cup\{y\}\in I_1$.
- $y\to x$ if $S-\{x\}\cup\{y\}\in I_2$.

Find a *shortest* path (with BFS) starting from a vertex in Y_1 and ending at a vertex in Y_2 which doesn't pass through any other vertices in Y_2, and alternate the path. The size of S will be incremented by 1 in each iteration. For the weighted case, assign weight $w(x)$ to vertex x if $x\in S$ and $-w(x)$ if $x\notin S$. Find the path with the minimum number of edges among all minimum length paths and alternate it.

9.10 AdaptiveSimpson*

```
template<typename Func, typename d = double>
struct Simpson {
  using pdd = pair<d, d>;
  Func f;
  pdd mix(pdd l, pdd r, optional<d> fm = {}) {
    d h = (r.X - l.X) / 2, v = fm.value_or(f(l.X + h));
    return {v, h / 3 * (l.Y + 4 * v + r.Y)};
  }
  d eval(pdd l, pdd r, d fm, d eps) {
```

```
    pdd m((l.X + r.X) / 2, fm);
    d s = mix(l, r, fm).second;
    auto [flm, sl] = mix(l, m);
    auto [fmr, sr] = mix(m, r);
    d delta = sl + sr - s;
    if (abs(delta
        ) <= 15 * eps) return sl + sr + delta / 15;
    return eval(l, m, flm, eps / 2) +
      eval(m, r, fmr, eps / 2);
  }
  d eval(d l, d r, d eps) {
    return eval
        ({l, f(l)}, {r, f(r)}, f((l + r) / 2), eps);
  }
  d eval2(d l, d r, d eps, int k = 997) {
    d h = (r - l) / k, s = 0;
    for (int i = 0; i < k; ++i, l += h)
      s += eval(l, l + h, eps / k);
    return s;
  }
};
template<typename Func>
Simpson<Func> make_simpson(Func f) { return {f}; }
```

9.11 Simulated Annealing

```
double factor = 100000;
const int base = 1e9; // remember to run ~ 10 times
for (int it = 1; it <= 1000000; ++it) {
    // ans:
        answer, nw: current value, rnd(): mt19937 rnd()
    if (exp(-(nw - ans
        ) / factor) >= (double)(rnd() % base) / base)
        ans = nw;
    factor *= 0.99995;
}
```

9.12 Tree Hash*

```
ull seed;
ull shift(ull x) {
  x ^= x << 13;
  x ^= x >> 7;
  x ^= x << 17;
  return x;
}
ull dfs(int u, int f) {
  ull sum = seed;
  for (int i : G[u])
    if (i != f)
      sum += shift(dfs(i, u));
  return sum;
}
```

9.13 Binary Search On Fraction

```
struct Q {
  int p, q;
  Q go(Q b, int d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p,q <= N
Q frac_bs(int N) {
  Q lo{0, 1}, hi{1, 0};
  if (pred(lo)) return lo;
  assert(pred(hi));
  bool dir = 1, L = 1, H = 1;
  for (; L || H; dir = !dir) {
    int len = 0, step = 1;
    for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
      if (Q mid = hi.go(lo, len + step);
          mid.p > N || mid.q > N || dir ^ pred(mid))
        t++;
      else len += step;
    swap(lo, hi = hi.go(lo, len));
    (dir ? L : H) = !!len;
  }
  return dir ? hi : lo;
}
```

9.14 Min Plus Convolution*

```
// a is convex a[i+1]-a[i] <= a[i+2]-a[i+1]
vector<int> min_plus_convolution
    (vector<int> &a, vector<int> &b) {
  int n = SZ(a), m = SZ(b);
  vector<int> c(n + m - 1, INF);
  auto dc = [&](auto Y, int l, int r, int jl, int jr) {
    if (l > r) return;
    int mid = (l + r) / 2, from = -1, &best = c[mid];
    for (int j = jl; j <= jr; ++j)
      if (int i = mid - j; i >= 0 && i < n)
        if (best > a[i] + b[j])
          best = a[i] + b[j], from = j;
    Y(Y, l,
        mid - 1, jl, from), Y(Y, mid + 1, r, from, jr);
  };
  return dc(dc, 0, n - 1 + m - 1, 0, m - 1), c;
}
```

9.15 Bitset LCS

```
cin >> n >> m;
for (int i = 1, x; i <= n; ++i)
  cin >> x, p[x].set(i);
for (int i = 1, x; i <= m; i++) {
  cin >> x, (g = f) |= p[x];
  f.shiftLeftByOne(), f.set(0);
  ((f = g - f) ^= g) &= g;
}
cout << f.count() << '\n';
```

10 Python

10.1 Misc

```
from decimal import *
setcontext(Context(prec
    =MAX_PREC, Emax=MAX_EMAX, rounding=ROUND_FLOOR))
print(Decimal(input()) * Decimal(input()))
from fractions import Fraction
Fraction
    ('3.14159').limit_denominator(10).numerator # 22
```

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