Assignment 1
CSC376, Fall 2024

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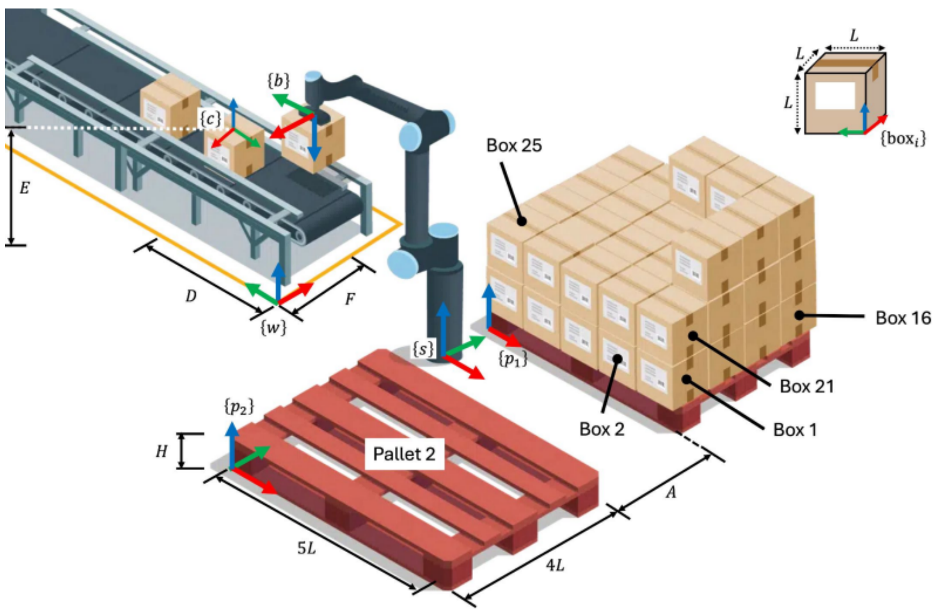

Figure 1: A palletizing robot receives boxes from a conveyor belt and places them on pallets for shipping.

(Q1) Solution:

1. Write down the transformation matrix T_{wc} given the dimensions in Figure 1.

   The rotation from w to c is one by π about the z axis, so, using the guide from lecture we get our rotation matrix to be:

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   And for p:

$$\begin{bmatrix} F \\ D \\ E \end{bmatrix}$$

   So, for T_{wc}, we have:

$$\begin{bmatrix} -1 & 0 & 0 & F \\ 0 & -1 & 0 & D \\ 0 & 0 & 1 & E \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Write down the transformation matrix $T_{p_1p_2}$ given the dimensions in Figure 1.
   There is no change in rotation between p_1 and p_2 so our R will be the default. For p, we have a translation along the y axis of magnitude $A + 4L$ in the negative y direction. So, our p:
   $$\begin{bmatrix} 0 \\ -A - 4L \\ 0 \end{bmatrix}$$
   So, for $T_{p_1p_2}$, we have:
   $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -A - 4L \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Determine the transformation matrix T_{bc} as a product of known transformation matrices.

   In the prompt, we are given T_{ws} and T_{sb}. Additionally, we calculated T_{wc} in 1.1. These three matrices are sufficient to calculate T_{bc} using formulas from lecture, as seen below:
   $$T_{bc} = (T_{ws}T_{sb})^{-1}T_{wc} = (T_{wb})^{-1}T_{wc} = T_{bw}T_{wc} = T_{bc}$$

4. Determine the transformation matrix $T_{p_1box_1}$.

   From the diagram of a box given in the upper right corner of figure 1, we can see that from p_1 to box_1 there is a $\frac{\pi}{2}$ rotation about the z axis which gives the following R:
   $$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
   Additionally, there is movement in the z and x directions; respectively by H and $5L$ which gives our p:
   $$\begin{bmatrix} 5L \\ 0 \\ H \end{bmatrix}$$
   So, $T_{p_1box_1}$ is:
   $$\begin{bmatrix} 0 & -1 & 0 & 5L \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. Determine the transformation matrix $T_{p_1box_{16}}$.

   We have the same R as in 1.4 and the same p except for the addition of a translation by $3L$ in the y direction. So, altogether we have:
   $$\begin{bmatrix} 0 & -1 & 0 & 5L \\ 1 & 0 & 0 & 3L \\ 0 & 0 & 1 & H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6. Determine the location of Box 50 on Pallet 1 with respect to the box arrival location c on the conveyor belt, i.e. the transformation matrix $T_{cbox_{50}}$.

   First, we calculate $T_{p_1box_{50}}$. We once again have the same R but our p is instead:
   $$\begin{bmatrix} L \\ L \\ 2L + H \end{bmatrix}$$

Which for $T_{p_1 box_{50}}$ gives us:

$$\begin{bmatrix} 0 & -1 & 0 & L \\ 1 & 0 & 0 & L \\ 0 & 0 & 1 & 2L+H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, $T_{cbox_{50}}$ can be acquired via known matrix multiplication:

$$T_{cbox_{50}} = (T_{wc})^{-1} T_{wp_1} T_{p_1 box_{50}} = T_{cw} T_{wp_1} T_{p_1 box_{50}} = T_{cbox_{50}}$$

7. Determine the location of Box 1 on Pallet 2 with respect to the robot's space frame s, i.e. the transformation matrix T_{sbox_1}.

   This can be acquired via known matrix multiplication:

$$T_{sbox_1} = (T_{ws})^{-1} T_{wp_1} T_{p_1 box_1} = T_{sw} T_{wp_1} T_{p_1 box_1} = T_{sbox_1}$$

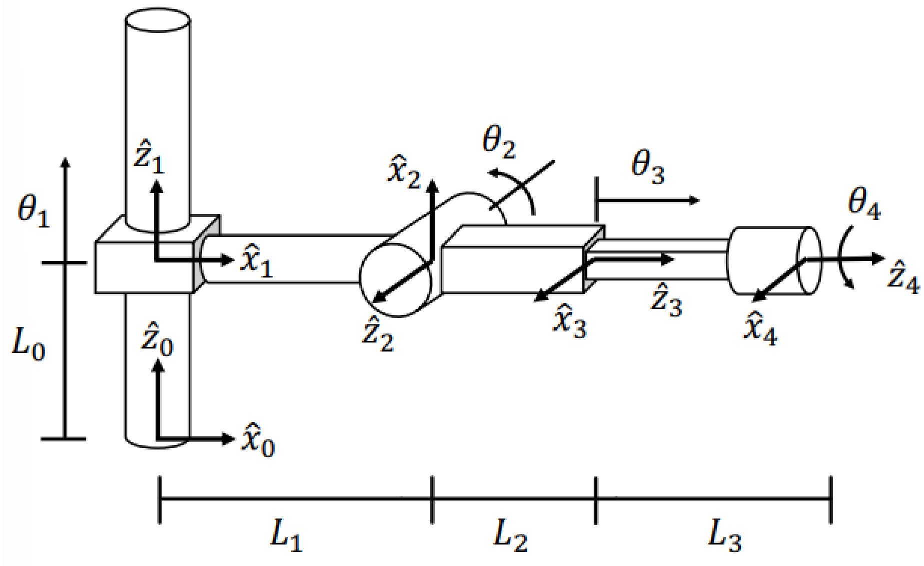

Figure 2: PRPR robot in its zero configuration with assigned link frames.

8. **Solution**:

Row 1: There is no change in the orientation between frame 0 and frame 1 so α_0 and ϕ_0 are both 0. There is no movement in the $\vec{x}_0$ direction so our a_0 is also 0. Lastly, there is a L_0 translation in the positive $\vec{z}_1$ direction from $\vec{x}_0$ to $\vec{x}_1$ but we also have a prismatic joint so d_1 is $L_0 + \theta_1$

Row 2: From $\vec{z}_1$ to $\vec{z}_2$ there is a $\frac{\pi}{2}$ rotation about $\vec{x}_1$ so, α_1 is $\frac{\pi}{2}$. From $\vec{x}_1$ to $\vec{x}_2$ there is also a $\frac{\pi}{2}$ rotation about $\vec{z}_2$ while in zero position. The second joint is revolute with θ_2, so, ϕ_2 is $\theta_2 + \frac{\pi}{2}$. From $\vec{z}_1$ to $\vec{z}_2$ there is a L_1 movement in the $\vec{x}_1$ direction so, a_1 is L_1. Lastly, There is no movement from $\vec{x}_1$ to $\vec{x}_2$ in the $\vec{z}_2$ direction so d_2 is 0.

Row 3: From $\vec{z}_2$ to $\vec{z}_3$ there is a $\frac{\pi}{2}$ rotation about $\vec{x}_2$ so, α_2 is $\frac{\pi}{2}$. From $\vec{x}_2$ to $\vec{x}_3$ there is also a $\frac{\pi}{2}$ rotation about $\vec{z}_3$ so ϕ_3 is $\frac{\pi}{2}$. From $\vec{z}_2$ to $\vec{z}_3$ there is no movement in the $\vec{x}_2$ direction, so a_2 is 0. Lastly, there is a L_2 translation in the positive $\vec{z}_3$ direction from $\vec{x}_2$ to $\vec{x}_3$ but we also have a prismatic joint so d_3 is $L_2 + \theta_3$

Row 4: From frame 3 to frame 4 there is no change in orientation so, α_3 and ϕ_4 are 0. From $\vec{z}_3$ to $\vec{z}_4$ there is no movement in the $\vec{x}_3$ direction so, a_3 is 0. Lastly, There is L_3 movement from $\vec{x}_3$ to $\vec{x}_4$ in the $\vec{z}_4$ direction so d_4 is L_3.

i	α_{i-1}	a_{i-1}	d_i	ϕ_i
1	0	0	$L_0 + \theta_1$	0
2	$\frac{\pi}{2}$	L_1	0	$\theta_2 + \frac{\pi}{2}$
3	$\frac{\pi}{2}$	0	$L_2 + \theta_3$	$\frac{\pi}{2}$
4	0	0	L_3	θ_4

Table 1: DH Parameters