On Evolution Of Turbulent Fluxes in Stars

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ABSTRACT
Turbulent fluxes in stars ..

Key words: turbulence – turbulent fluxes –

1 INTRODUCTION

Based on RANS analysis of flux evolution equations (Mocák et al. 2014), Fig.2 we find that the density, pressure, temperature and energy gradients (possibly also others) in turbulent regions are controlled by Reynolds stresses and dilatation flux (Sect.7). The results and plots presented here were deduced from 3D simulation of stable-unstable layers in Cartesian geometry (Fig.1) based on code comparison project setup started in Exeter 2019.

1.1 Turbulent Flux

Turbulent flux is a correlation of any thermodynamic field fluctuations with velocity fluctuations. In order to calculate evolution equations for correlations of two arbitrary fluctuations, we can derive the following general formula.

$$\overline{\rho}\widetilde{D}_t \widetilde{c''d''} = +\overline{c''\rho D_t d} - \overline{\rho}\widetilde{c''u''_n}\partial_n\widetilde{d} + \\ + \overline{d''\rho D_t c} - \overline{\rho}\widetilde{d''u''_n}\partial_n\widetilde{c} - \overline{\partial_n \rho c'' d'' u''_n} \quad (1)$$

$$\widetilde{D}_t\overline{c'd'} = +\overline{c'D_t d} - \overline{c'u''_n}\partial_n\overline{d} + \overline{d'D_t c} - \quad (2)$$

$$\overline{d'u''_n}\partial_n\overline{c} - \partial_n\overline{u''_n c'd'} + \overline{c'd'\partial_n u''_n} \quad (3)$$

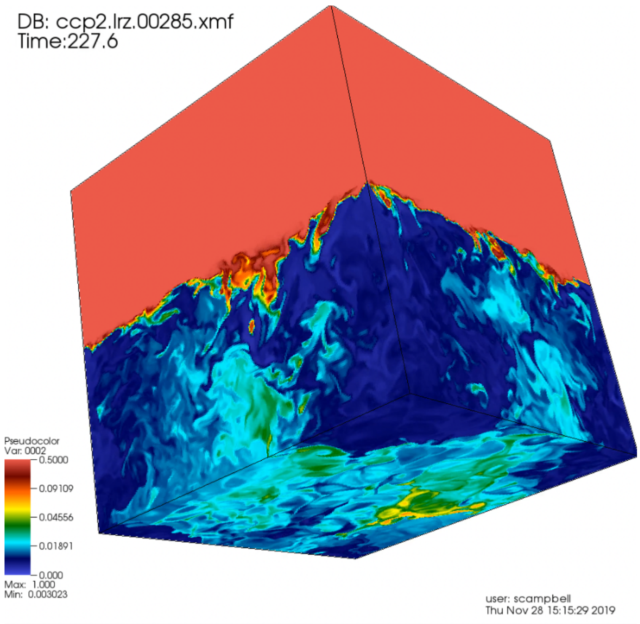

Figure 1. 3D two-layer hydrodynamic simulation with convectively unstable region below a stable one based on code-comparison project started in Exeter 2019

2 TURBULENT MASS FLUX EQUATION

3 INTERNAL ENERGY FLUX EQUATION

4 ACOUSTIC FLUX EQUATION

5 TEMPERATURE FLUX EQUATION

6 ENTHALPY FLUX EQUATION

7 DILATATION FLUX AND REYNOLDS STRESS

The dilatation flux $\overline{u'_x d'}$ can be explained as (hot) expanding ($d' > 0$) gas going upwards ($u'_r > 0$) and (cold) compressing ($d' < 0$) gas going down ($u'_r < 0$), hence you get dilatation flux positive everywhere in convective region (Fig.4).

$$d = \nabla \cdot \mathbf{u} = -\partial_t \ln \rho \quad \text{no stratification} \quad \text{(Pope 2000)} \quad (4)$$

Based on the RANS analysis of flux evolution equations (Fig.2), the density, pressure, temperature, energy gradients in turbulent regions (let's call them $\partial_x\overline{Q}$) are controlled by Reynolds stresses $\widetilde{R}_{xx}$ and dilatation flux $\overline{u'_x d''}$ (Eq.5).

$$\widetilde{R}_{xx}\partial_x\overline{Q} \sim -\overline{\rho}\ \overline{Q}\ \overline{u'_x d''} \quad (5)$$

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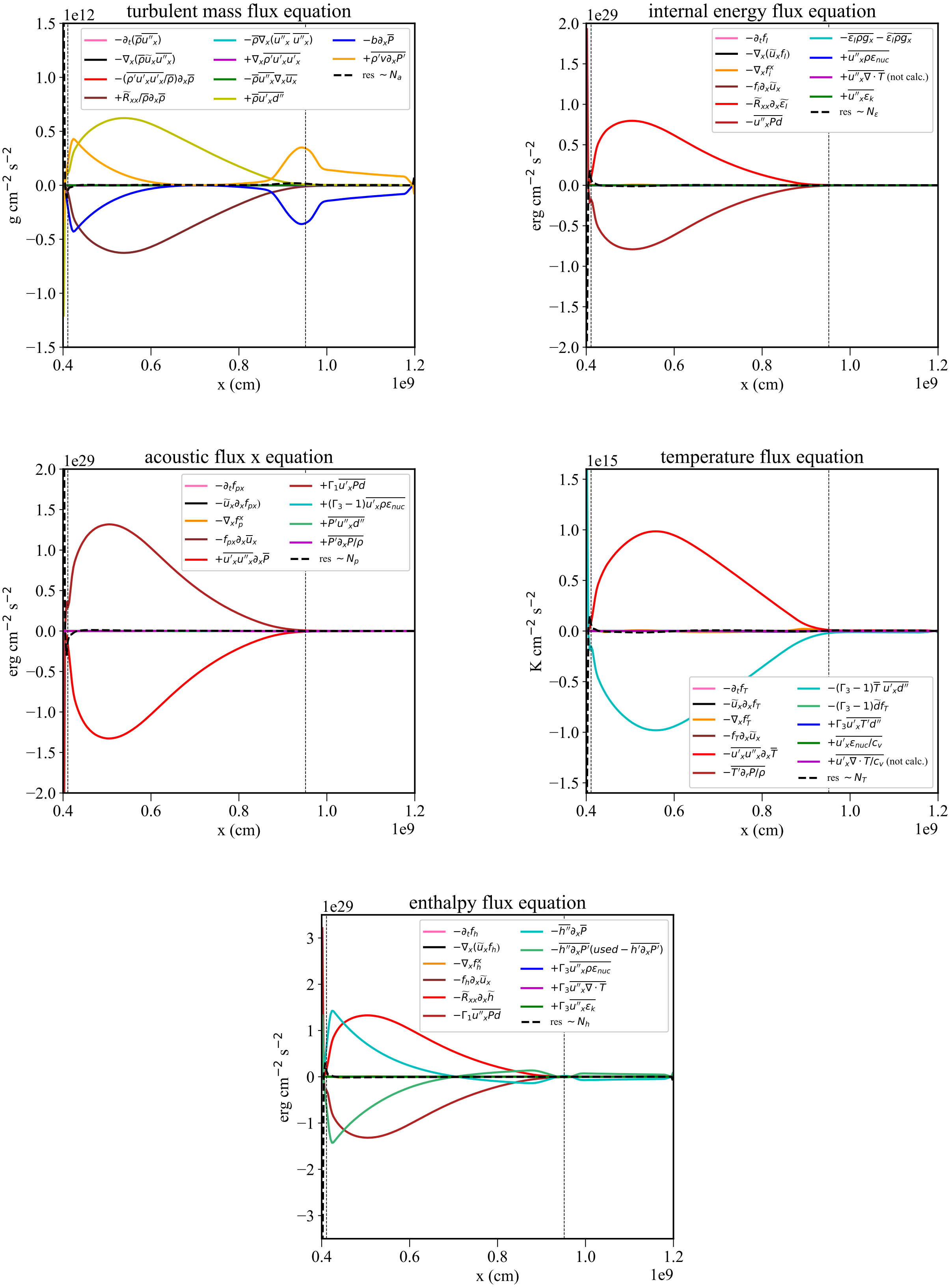

Figure 2. Top-Left: Turbulent mass flux equation Top-Right: Internal energy flux equation Middle-Right: Acoustic flux equation Middle-Left: Temperature flux equation Bottom: Enthalpy flux equation All profiles based on 3D two-layer (stable-unstable configuration) simulated with code-comparison setup specs.

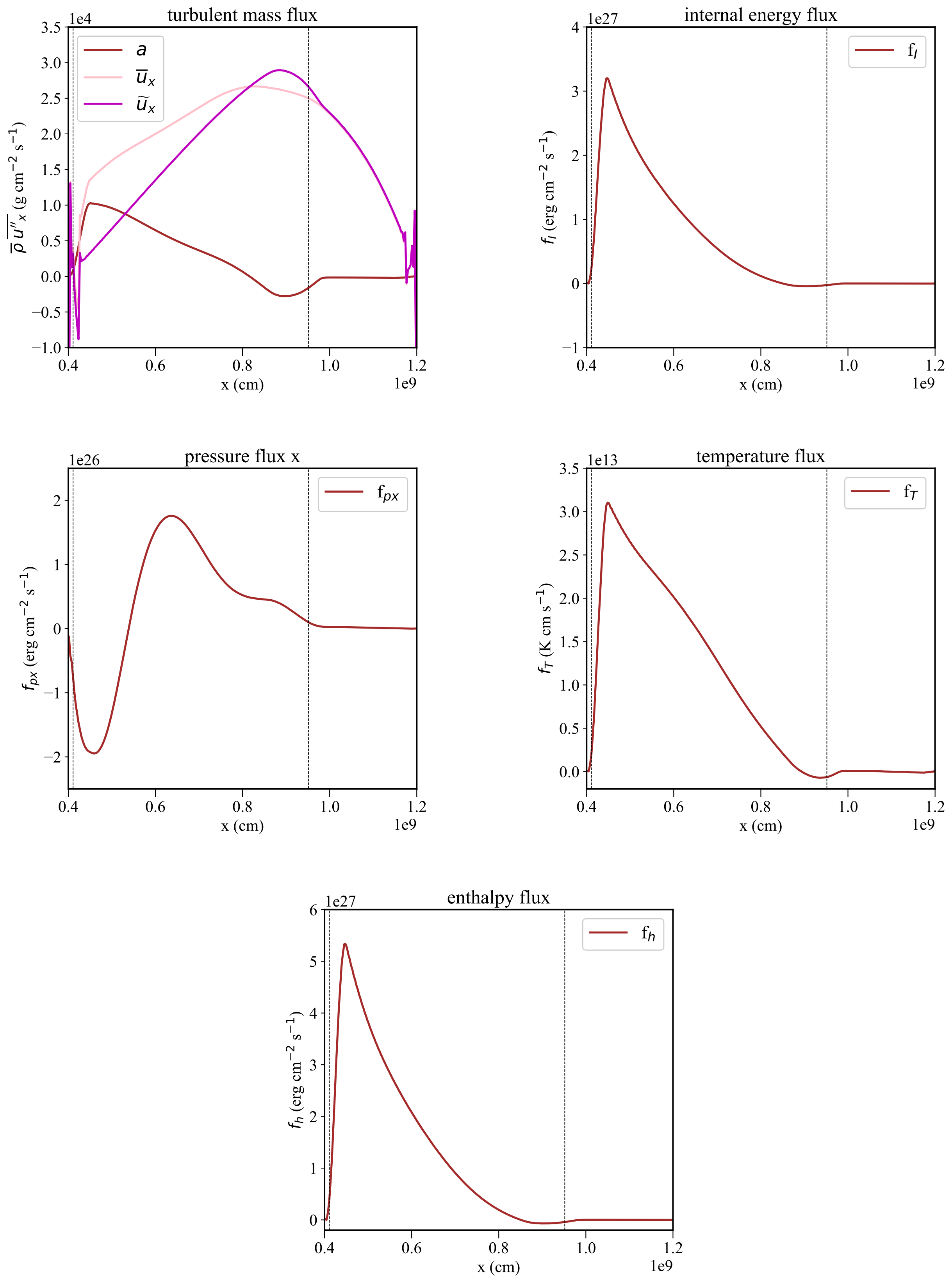

Figure 3. Top-Left: Turbulent mass flux Top-Right: Internal energy flux Middle-Right: Acoustic flux Middle-Left: Temperature flux Bottom: Enthalpy flux. All profiles based on 3D two-layer (stable-unstable configuration) simulated with code-comparison setup specs.

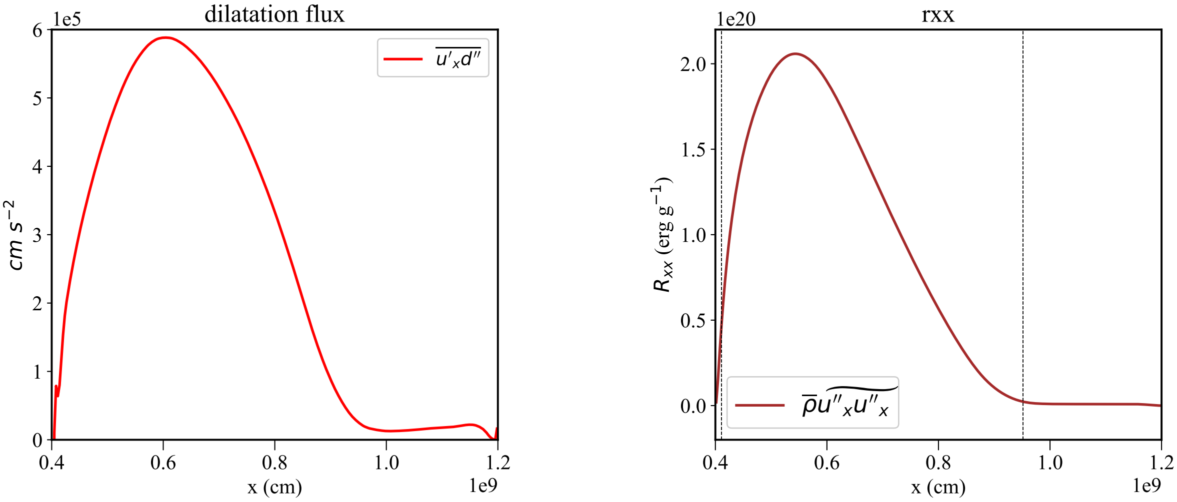

Figure 4. Left: Dilatation flux and Right: Reynolds stress. Profile based on 3D two-layer (stable-unstable configuration) simulated with code-comparison setup specs.

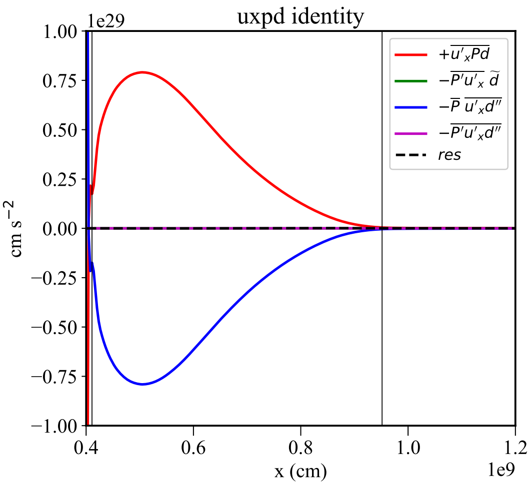

Figure 5. Dilatation flux relation.

7.1 Pressure Gradient

From pressure flux equation, we find that:

$$+\overline{u'_x u''_x}\partial_x \overline{P} \sim -\Gamma_1 \overline{u'_x P d} \tag{6}$$

We also find that (Fig.5):

$$\overline{u'_x P d} \sim \overline{P}\ \overline{u'_r d''} \tag{7}$$

From Eqs.6,7 and multiplying by $\overline{\rho}$ we get:

$$\partial_r \overline{P} \sim \ -\Gamma_1 \ \overline{\rho}\ \overline{P}\ \overline{u'_r d''}/\ \widetilde{R}_{rr} \tag{8}$$

And assuming hydrostatic equilibrium

$$\partial_x \overline{P} \sim -\ \overline{\rho}\ \overline{g}_x \tag{9}$$

we get:

$$\overline{u'_x d''} \sim +\frac{\widetilde{R}_{xx}}{\Gamma_1}\frac{\overline{\rho}}{\overline{P}} \tag{10}$$

8 SUMMARY

REFERENCES

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