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Transport Of Density In Stars

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ABSTRACT
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Key words: turbulence – convection zone – stellar evolution

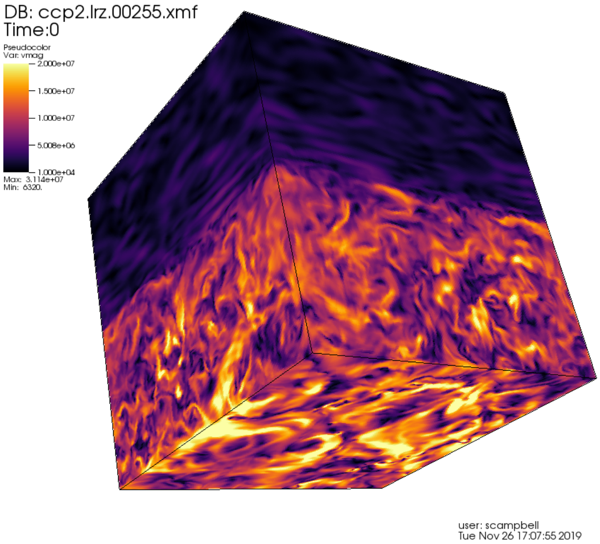

Figure 1. Visualization of turbulence in a 3D box under study.

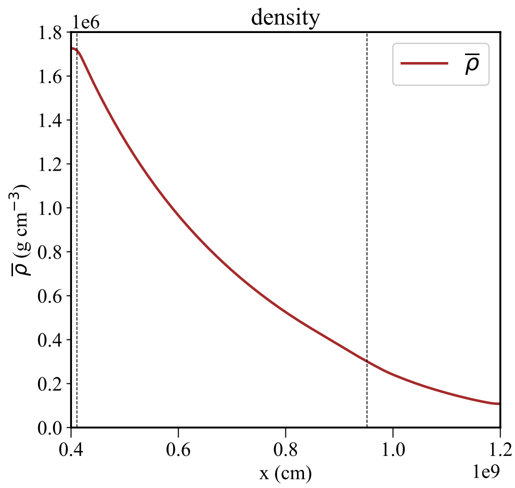

Figure 2. Density radial profile of the 3D model.

1 INTRODUCTION

What does RANS transport equation for density (Mocák et al. 2014) tell us about convection zones in stars?

Time-dependency $\partial_t\overline{\rho}$, compressibility $\overline{\rho}\nabla_x\widetilde{u}_x$ and non-local physics $\nabla_x f_\rho$ play important role in convection zone.

The following relation between the dilatation terms and non-local physics can be deduced from Fig.3:

$$-\overline{\rho}\nabla_x\widetilde{u}_x \sim -\nabla_x f_\rho + f_\rho/\overline{\rho}\partial_r\overline{\rho} - \overline{\rho}\nabla_x\overline{u}_x \quad (1)$$

Linearized continuity equation leads to incomplete picture of density physics in stars (see Sect.2).

2 LINEARIZED CONTINUITY EQUATION

$$\partial_t\rho + \nabla\cdot\rho\mathbf{u} = 0 \quad (2)$$

Let us assume, that $\rho(\mathbf{r},t) = \rho_0(\mathbf{r}) + \rho'(\mathbf{r},t)$ where $\rho_0(\mathbf{r})$ is time-independent background density state around which we'll linearize the continuity equation. We get:

$$\partial_t\left[\rho_0(\mathbf{r}) + \rho'(\mathbf{r},t)\right] + \nabla\cdot\left[\rho_0(\mathbf{r}) + \rho'(\mathbf{r},t)\right]\mathbf{u}(\mathbf{r},t) = 0 \quad (3)$$

$$\cancelto{0}{\partial_t\rho_0(\mathbf{r})} + \partial_t\rho'(\mathbf{r},t) + \nabla\cdot\left[\rho_0(\mathbf{r})\mathbf{u}(\mathbf{r},t)\right] + \nabla\cdot\left[\rho'(\mathbf{r},t)\mathbf{u}(\mathbf{r},t)\right] = 0 \quad (4)$$

And because we also assume that $\rho' << \rho_0$, we have that $\nabla\cdot[\rho_0(\mathbf{r})\mathbf{u}(\mathbf{r},t)] >> \nabla\cdot[\rho'(\mathbf{r},t)\mathbf{u}(\mathbf{r},t)]$ and get:

$$\partial_t\rho' + \nabla\cdot(\rho_0\mathbf{u}) \sim 0 \quad (5)$$

Furthermore, we know that $\mathbf{u} \sim \mathbf{u}'$ and therefore we can write:

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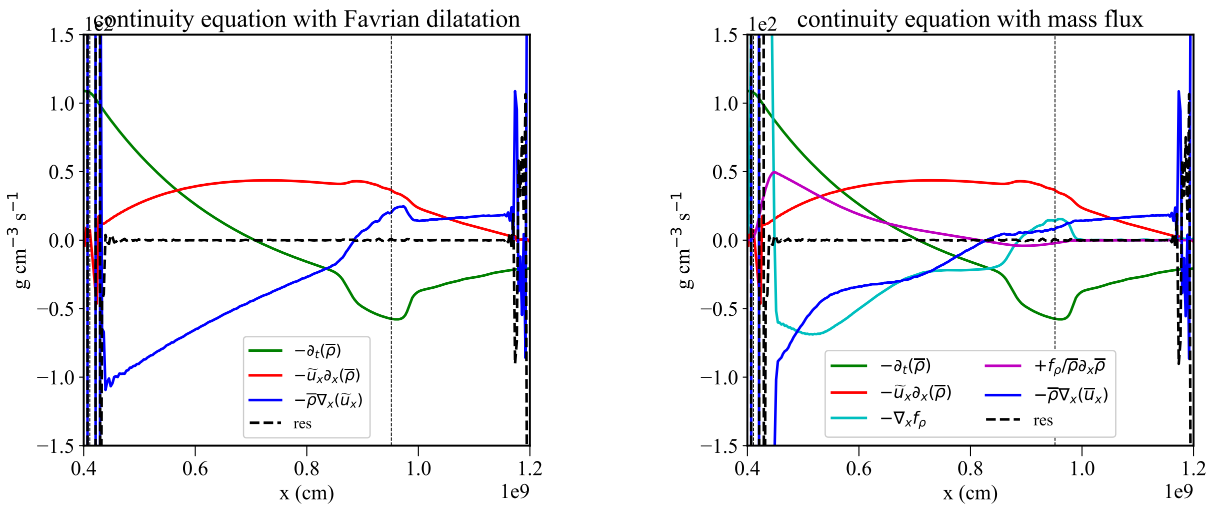

Figure 3. Transport density equation with Left: Favrian dilatation $\nabla_r \widetilde{u}_x$ and Right: turbulent mass flux f_ρ

$$\partial_t \rho' + \nabla \cdot (\rho_0 \mathbf{u}') \sim 0 \tag{6}$$

This is equation 23 from (Viallet et al. 2013).

3 MEAN LINEARIZED CONTINUITY EQUATION

$$\partial_t \rho' + \nabla \cdot (\rho_0 \mathbf{u}) \sim 0 \tag{7}$$

After space-time averaging of this linearized continuity equation, we get:

$$\overline{\partial_t \rho'} + \overline{\nabla \cdot (\rho_0 \mathbf{u})} \sim 0 \tag{8}$$

$$\cancelto{0}{\overline{\partial_t \rho'}} + \overline{\nabla \cdot (\rho_0 \mathbf{u})} \sim 0 \tag{9}$$

$$\nabla_r \overline{\rho_0 u_r} \sim 0 \tag{10}$$

$$\tag{11}$$

But because $\rho_0 \equiv \overline{\rho}$, we can write:

$$\nabla_r \overline{\rho}\ \overline{u}_r \sim 0 \tag{12}$$

$$\overline{\rho} \nabla_r \overline{u}_r + \overline{u}_r \partial_r \overline{\rho} \sim 0 \tag{13}$$

$$\overline{\rho} \nabla_r \overline{u}_r + (\overline{u''}_r + \widetilde{u}_r) \partial_r \overline{\rho} \sim 0 \tag{14}$$

$$\overline{\rho} \nabla_r \overline{u}_r + \overline{u''}_r \partial_r \overline{\rho} + \widetilde{u}_r \partial_r \overline{\rho} \sim 0 \tag{15}$$

$$\overline{\rho} \nabla_r \overline{u}_r + \overline{\rho' u'_r}/\overline{\rho}\ \partial_r \overline{\rho} + \widetilde{u}_r \partial_r \overline{\rho} \sim 0 \tag{16}$$

$$\overline{\rho} \nabla_r \overline{u}_r - f_\rho/\overline{\rho}\ \partial_r \overline{\rho} + \widetilde{u}_r \partial_r \overline{\rho} \sim 0 \tag{17}$$

But from my latest analysis of the full (non-linearzed) continuity equation, it turns out that:

$$\overline{\rho} \nabla_r \overline{u}_r - f_\rho/\overline{\rho}\ \partial_r \overline{\rho} + \widetilde{u}_r \partial_r \overline{\rho} \neq 0 \tag{18}$$

Or, when multiply this by -1:

$$-\overline{\rho} \nabla_r \overline{u}_r + f_\rho/\overline{\rho}\ \partial_r \overline{\rho} - \widetilde{u}_r \partial_r \overline{\rho} \neq 0 \tag{19}$$

Linearization relies on time-independent background state around which you can linearize, which turns out not to be our case and $\partial_t \rho_0$ or in other words $\partial_t \overline{\rho} \neq 0$ (green curve). Also it gets rid of the transport of turbulent density field $\nabla_r f_\rho$ (cyan curve) (Fig.3):

REFERENCES

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