Full Turbulent Velocity Field Hypothesis

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ABSTRACT
Full turbulence field hypothesis ...

Key words: turbulence – velocity field

1 INTRODUCTION

Full turbulence velocity field hypothesis (Eq.1, Fig.4) is a system of equations, whose solution should provide us with full velocity vector of turbulence in hydrostatic equilibrium. The solutions itself, however, remains elusive.

It is based on dilatation flux relations found in RANS evolution equation for acoustic flux (Sect.2, Fig.2), various identities (Sect.3, Fig.3) and assumption of hydrostatic equilibrium ($\partial_r P = -\rho g_r$).

$$
\begin{aligned}
\overline{u'_r \nabla_r u'_r} + \overline{u'_r \nabla_\theta u'_\theta} + \overline{u'_r \nabla_\phi u'_\phi} &\sim \overline{\rho}\ \overline{u'_r u'_r}\ \overline{g}_r/\Gamma_1\ \overline{P} \\
\overline{u'_\theta \nabla_r u'_r} + \overline{u'_\theta \nabla_\theta u'_\theta} + \overline{u'_\theta \nabla_\phi u'_\phi} &\sim \overline{\rho}\ \overline{u'_\theta u'_r}\ \overline{g}_r/\Gamma_1\ \overline{P} \\
\overline{u'_\phi \nabla_r u'_r} + \overline{u'_\phi \nabla_\theta u'_\theta} + \overline{u'_\phi \nabla_\phi u'_\phi} &\sim \overline{\rho}\ \overline{u'_\phi u'_r}\ \overline{g}_r/\Gamma_1\ \overline{P} \qquad (1)
\end{aligned}
$$

Validation of these equations was performed on data from 3D simulation of turbulence in a box based on code comparison project setup and 3D simulation of oxygen burning shell in a massive star in spherical geometry (Fig.1)

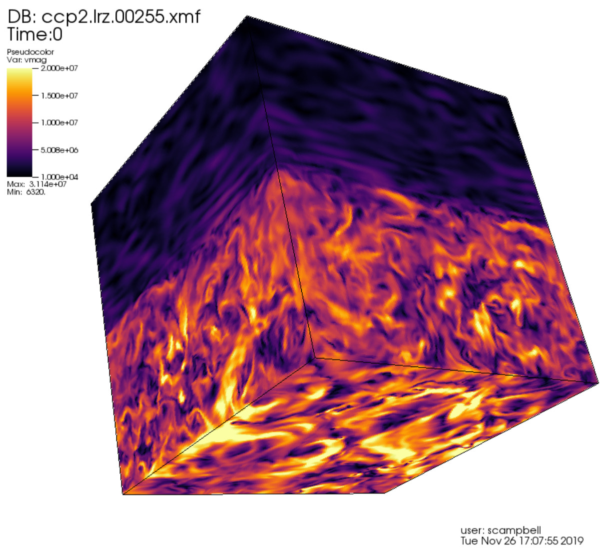
2 ACOUSTIC FLUX EQUATIONS

Terms in acoustic flux equations (Mocák et al. 2014) based on our 3D models (Fig.1) can be found in Figure 2.

Based on this analysis, we find the following:

$$
\begin{aligned}
&\overline{u'_r u''_r \partial_r \overline{P}} \sim +\Gamma_1 \overline{u'_r P d} \qquad -\overline{\rho} \overline{u'_r u''_r} \widetilde{g}_r \sim -\Gamma_1 \overline{P}\ \overline{u'_r d''} \qquad (2) \\
&\overline{u'_\theta u''_r \partial_r \overline{P}} \sim +\Gamma_1 \overline{u'_\theta P d} \qquad -\overline{\rho} \overline{u'_\theta u''_r} \widetilde{g}_r \sim -\Gamma_1 \overline{P}\ \overline{u'_\theta d''} \qquad (3) \\
&\overline{u'_\phi u''_r \partial_r \overline{P}} \sim +\Gamma_1 \overline{u'_\phi P d} \qquad -\overline{\rho} \overline{u'_\phi u''_r} \widetilde{g}_r \sim -\Gamma_1 \overline{P}\ \overline{u'_\phi d''} \qquad (4)
\end{aligned}
$$

For the identities on the right, we use equation for hydrostatic equilibrium $\partial_r \overline{P} = -\overline{\rho}\widetilde{g}$ and identities derived/proved in Sect.3 and Fig.3.



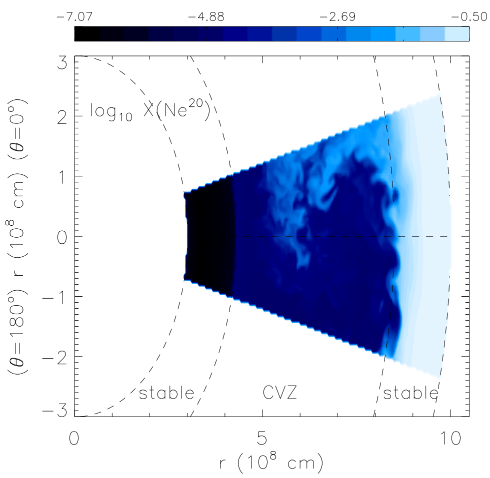


Figure 1. Visualization of 3D turbulence in a box based on code comparision setup and 3D oxygen burning shell in spherical geometry.

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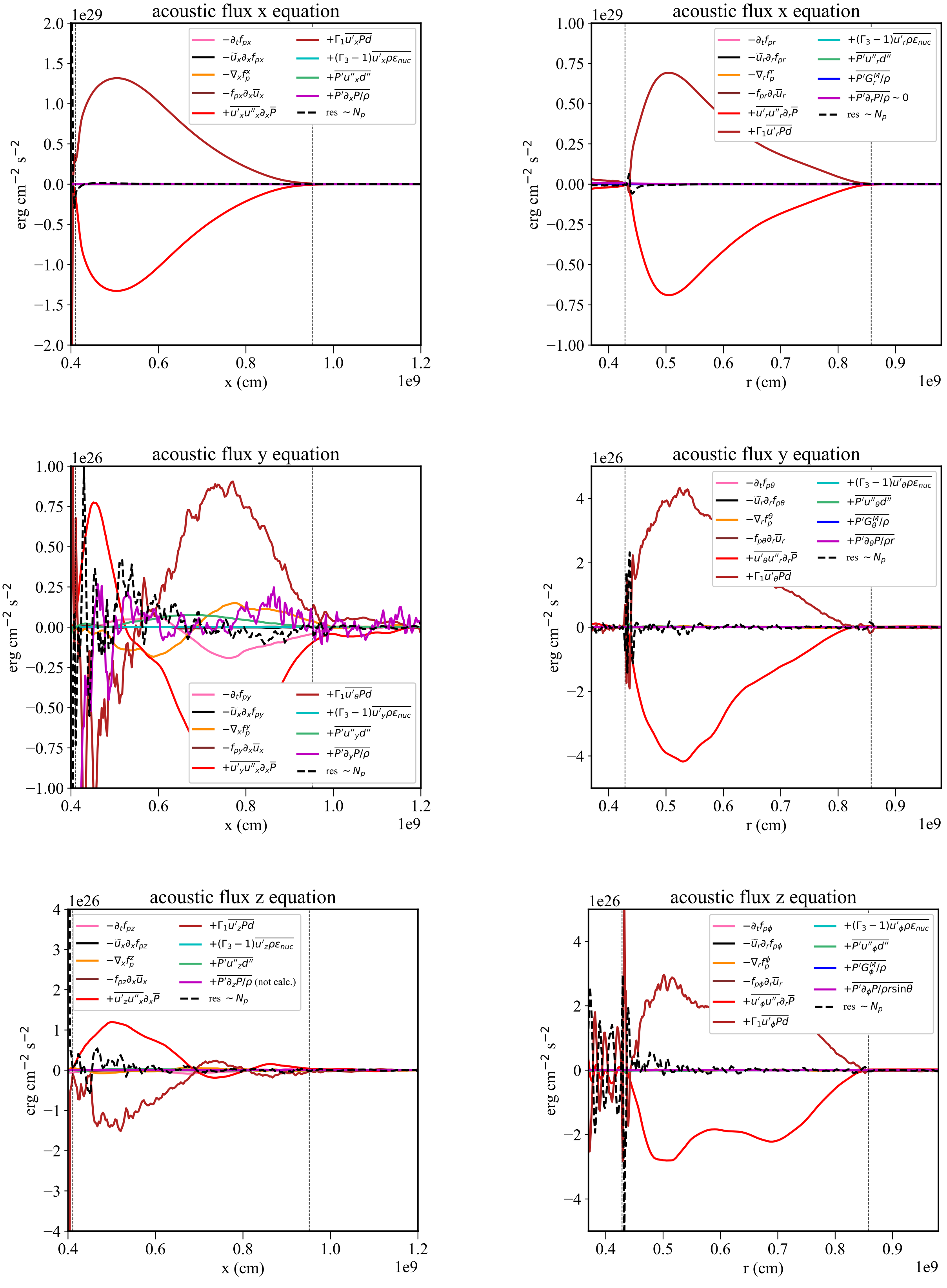

Figure 2. Left: Pressure flux equation derived from 3D cartesian box turbulence Right: The same as on left but derived from 3D oxygen burning shell in spherical geometry.

3 VARIOUS IDENTITIES

$$+\overline{u'_r Pd} = -\overline{P' u'_r}\widetilde{d} - \overline{P}\ \overline{u'_r d''} - \overline{P' u'_r d''} \quad (5)$$
$$+\overline{u'_\theta Pd} = -\overline{P' u'_\theta}\widetilde{d} - \overline{P}\ \overline{u'_\theta d''} - \overline{P' u'_\theta d''} \quad (6)$$
$$+\overline{u'_\phi Pd} = -\overline{P' u'_\phi}\widetilde{d} - \overline{P}\ \overline{u'_\phi d''} - \overline{P' u'_\phi d''} \quad (7)$$

$$+\overline{u'_r Pd} \sim -\overline{P}\ \overline{u'_r d''} \quad (8)$$
$$+\overline{u'_\theta Pd} \sim -\overline{P}\ \overline{u'_\theta d''} \quad (9)$$
$$+\overline{u'_\phi Pd} \sim -\overline{P}\ \overline{u'_\phi d''} \quad (10)$$

It looks like Reynolds and Favrian decomposition of velocity field leads to very similar Reynolds stresses and dilatation fluxes:

$$\overline{u'_r u''_r} \sim \overline{u'_r u'_r} \quad (11)$$
$$\overline{u'_\theta u''_r} \sim \overline{u'_\theta u'_r} \quad (12)$$
$$\overline{u'_\phi u''_r} \sim \overline{u'_\phi u'_r} \quad (13)$$

$$\overline{u'_r d''} \sim \overline{u'_r d'} \quad (14)$$
$$\overline{u'_\theta d''} \sim \overline{u'_\theta d'} \quad (15)$$
$$\overline{u'_\phi d''} \sim \overline{u'_\phi d'} \quad (16)$$

4 DILATATION FLUX RELATIONS

These are inferred from mean field acoustic flux equations in r, θ, ϕ.

$$\overline{u'_r d'} \sim \frac{\overline{\rho}\ \overline{R}_{rr}\ \overline{g}_r}{\Gamma_1\ \overline{P}} \quad (17)$$
$$\overline{u'_\theta d'} \sim \frac{\overline{\rho}\ \overline{R}_{\theta r}\ \overline{g}_r}{\Gamma_1\ \overline{P}} \quad (18)$$
$$\overline{u'_\phi d'} \sim \frac{\overline{\rho}\ \overline{R}_{\phi r}\ \overline{g}_r}{\Gamma_1\ \overline{P}} \quad (19)$$

5 FULL TURBULENCE VELOCITY FIELD HYPOTHESIS

$$\overline{u'_r \nabla_r u'_r} + \overline{u'_r \nabla_\theta u'_\theta} + \overline{u'_r \nabla_\phi u'_\phi} \sim \overline{\rho}\ \overline{u'_r u'_r}\ \overline{g}_r/\Gamma_1\ \overline{P} \quad (20)$$
$$\overline{u'_\theta \nabla_r u'_r} + \overline{u'_\theta \nabla_\theta u'_\theta} + \overline{u'_\theta \nabla_\phi u'_\phi} \sim \overline{\rho}\ \overline{u'_\theta u'_r}\ \overline{g}_r/\Gamma_1\ \overline{P} \quad (21)$$
$$\overline{u'_\phi \nabla_r u'_r} + \overline{u'_\phi \nabla_\theta u'_\theta} + \overline{u'_\phi \nabla_\phi u'_\phi} \sim \overline{\rho}\ \overline{u'_\phi u'_r}\ \overline{g}_r/\Gamma_1\ \overline{P} \quad (22)$$

These equations will give us full turbulence velocity field in convection zone in hydrostatic equilibrium. The hypothesis is formulated using Reynolds fluctuations but its formulation using Favrian fluctuations is in this case the same.

6 SIMPLIFIED TURBULENCE VELOCITY FIELD HYPOTHESIS

$$\overline{u'_r \nabla_r u'_r} + \overline{u'_r \nabla_\theta u'_\theta} + \overline{u'_r \nabla_\phi u'_\phi} \sim \overline{\rho}\ \overline{u'_r u'_r}\ \overline{g}_r/\Gamma_1\ \overline{P} \quad (23)$$
$$\overline{u'_\theta \nabla_\phi u'_\phi} \sim \overline{\rho}\ \overline{u'_\theta u'_r}\ \overline{g}_r/\Gamma_1\ \overline{P} \quad (24)$$
$$\overline{u'_\phi \nabla_\phi u'_\phi} \sim \overline{\rho}\ \overline{u'_\phi u'_r}\ \overline{g}_r/\Gamma_1\ \overline{P} \quad (25)$$

$$\nabla_r u'_r = \partial_r u'_r$$
$$\nabla_\theta u'_\theta = \left(\partial_\theta u'_\theta + u'_r\right)/r$$
$$\nabla_\phi u'_\phi = \left(\partial_\phi u'_\phi + u'_r \sin\theta + u'_\theta \cos\theta\right)/r\sin\theta$$
$$d' = \nabla \cdot u' = \nabla_r u'_r + \nabla_\theta u'_\theta + \nabla_\phi u'_\phi \quad \text{dilatation: trace of covariant derivative}$$

$$\Gamma_1 = \partial\ ln\ P/\partial\ ln\ \rho|_s$$
ρ density

$$R_{rr} = u'_r u'_r$$
$$R_{\theta r} = u'_\theta u'_r$$
$$R_{\phi r} = u'_\phi u'_r$$

Definitions:

REFERENCES

Mocák M., Meakin C., Viallet M., Arnett D., 2014, arXiv e-prints, p. arXiv:1401.5176

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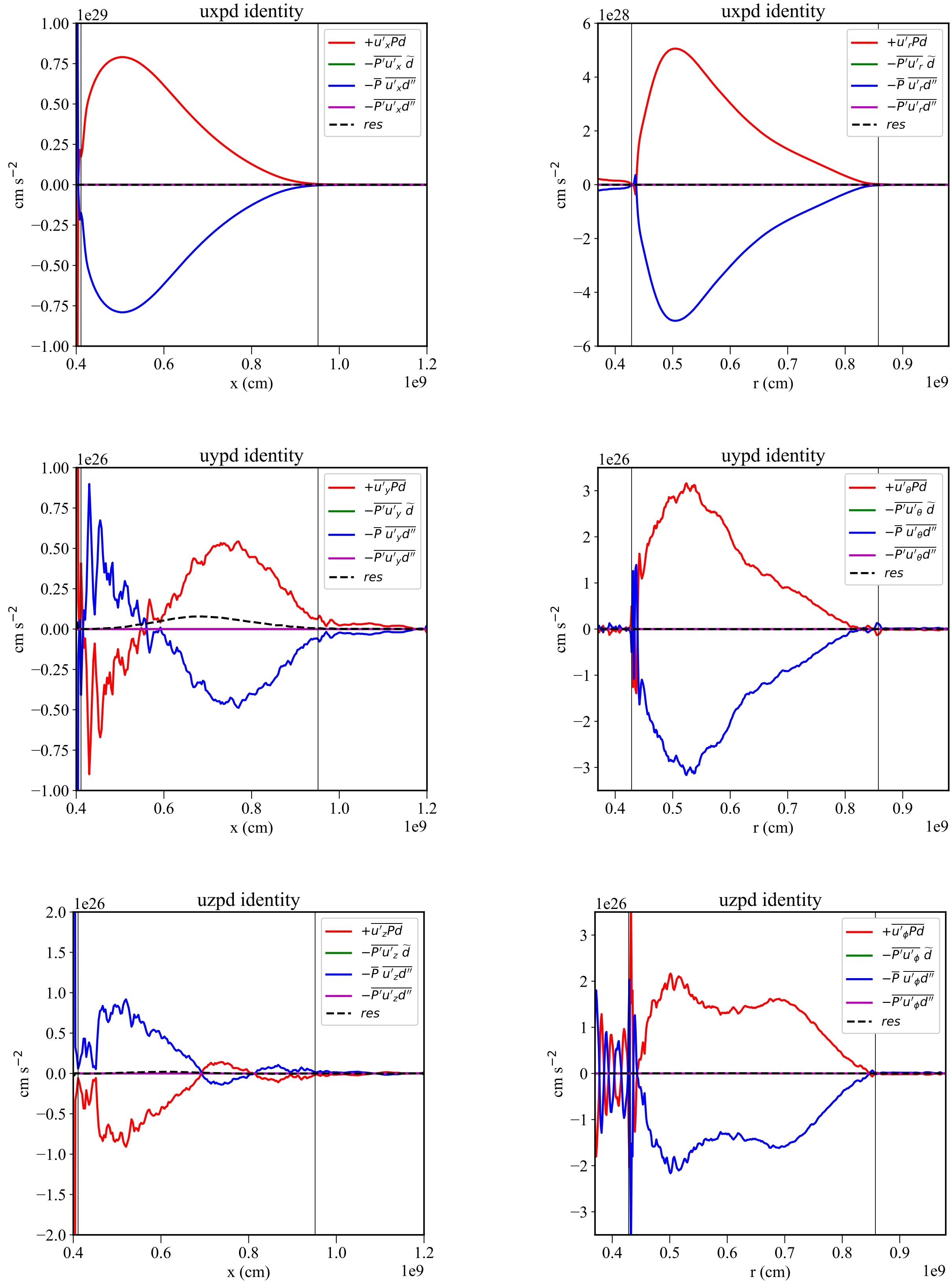

Figure 3. Left: Dilatation flux identities derived from 3D cartesian box turbulence Right: The same as on left but derived from 3D oxygen burning shell in spherical geometry.

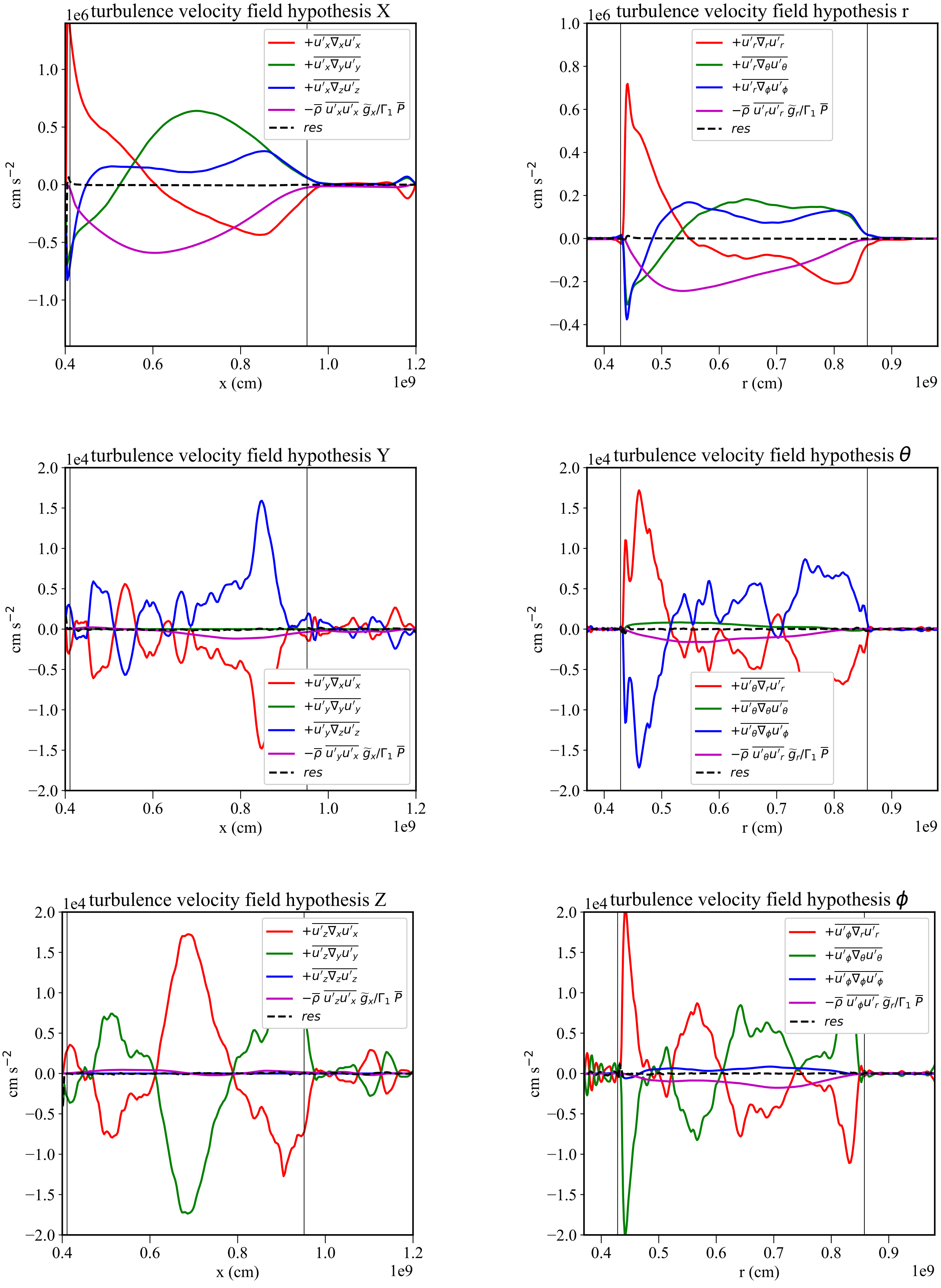

Figure 4. Left: Full turbulence field hypothesis fields derived from 3D cartesian box turbulence Right: The same as on left but derived from 3D oxygen burning shell in spherical geometry.