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Hydrodynamic Stellar Structure Equations

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ABSTRACT
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Key words: turbulence – mixing – nuclear burning – stellar evolution

1 INTRODUCTION

The true nature of turbulent regions in stars is hydrodynamic (citations). And because stars evolve all the time, their evolution is intrinsically time-dependent (citations). Besides, stars are gas, and gas is compressible, solids are incompressible (citations). All the emerging evidence based hydrodynamic experiments, observations and simulations forces us to reconsider our basic assumptions used for solutions to observable problems in stellar evolution (citations, what problems there are currently).

Besides models for magnetic fields and rotation, a model for stellar turbulence is one of such assumptions. With the supercomputers available today, we finally start to be able to simulate such turbulent layers in stars with decent resolution (Reynolds numbers $> 10^3$) and sufficient extend in time (over several convective turnover timescales) in order to get robust statistics of various thermodynamic fields (mean density, pressure, temperature, fluxes, variances etc.). With such data, the complexity of underlying physics of turbulent convective flow in stars (reactive or not) can be accurately understood by the analysis of the RANS equations (Mocák et al. 2014). Their complex shape represents the unforgiving reality of turbulence, where its understanding for a stratified and compressibles structure of stars is a complicated matter.

We use the RANS analysis to derive hydrodynamic stellar structure equations (Eqs.4,5,6,7,8, Fig.2,3,4), which are main focus of this paper. We also derive alternative sets of stellar structure equations based on dilatation flux, Reynolds stresses and assumption of hydrostatic equilibrium (Eqs.9-13). Validation of the equations is based on RANS analysis of 3D hydrodynamic simulation of oxygen burning shell (Fig.2,3,4) in a massive supernova progenitor star (citation).

The paper is organized as follows. Initial model and 3D simulations are described first. Hydrodynamic stellar structure equations for conservation of mass, momentum and energy are presented next followed by descriptions of temperature and composition equations as well. Summary of results and conclusions completes the paper at the end.

check paper by Tao Chai, 2018, Apj NUMERICAL ANALYSIS OF NONLOCAL CONVECTION | COMPARISON WITH THREE-DIMENSIONAL NUMERICAL SIMULATIONS OF EFFICIENT TURBULENT CONVECTION (Cai 2018)

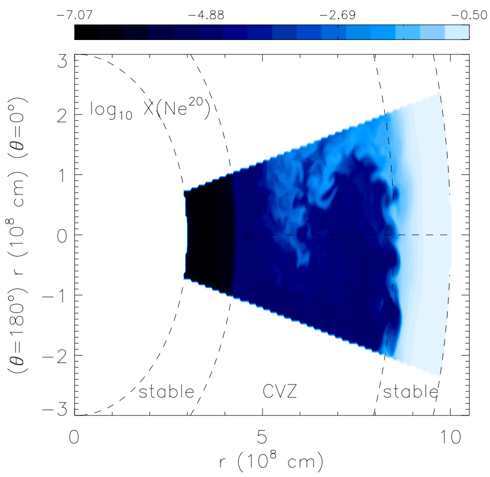

Figure 1. Oxygen burning shell - old graphics, will be replaced

2 INITIAL MODEL AND 3D SIMULATIONS

3 CONSERVATION OF MASS

4 CONSERVATION OF MOMENTUM

5 CONSERVATION OF ENERGY

the luminosity equations

6 TEMPERATURE EQUATION

7 ALTERNATIVE HYDRODYNAMIC STELLAR STRUCTURE EQUATIONS

Based on the RANS analysis of flux evolution equations, the density, pressure, temperature, energy gradients in turbulent

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regions are controlled by reynolds stresses and dilatation flux.

$$\widetilde{R}_{rr}\partial_r\overline{Q} \sim -\overline{\rho}\ \overline{Q}\ \overline{u'_r d''} \quad (1)$$

$$\partial_r\overline{P} \sim -\ \overline{\rho}\ \overline{g}_r \quad (2)$$

$$\overline{u'_r d''} \sim \frac{\widetilde{R}_{rr}}{\Gamma_1}\frac{\overline{\rho}}{\overline{P}} \quad (3)$$

hydrodynamic stellar structure equations (various versions)

$$\begin{aligned}
\partial_r\overline{m} &= +4\pi r^2\overline{\rho} + (4\pi r^3/3\widetilde{u}_r)\left[-\nabla_r f_\rho + (f_\rho/\overline{\rho})\partial_r\overline{\rho} - \overline{\rho}\overline{d} - \partial_t\overline{\rho}\right] && (4)\\
\partial_r\overline{P} &= +\overline{\rho}\overline{g} - \overline{\rho}\partial_t\widetilde{u}_r - \nabla_r\widetilde{R}_{rr} - \overline{G}^M_r - \overline{\rho}\widetilde{u}_r\partial_r\widetilde{u}_r && (5)\\
\partial_r\widetilde{L} &= +4\pi r^2\overline{\rho}\widetilde{\epsilon}_{nuc} + 4\pi r^2\left[-\nabla_r(f_i + f_{th} + f_K + f_p) - \overline{Pd} - \widetilde{R}_{ir}\partial_r\widetilde{u}_i + W_b + \overline{\rho}\widetilde{D}_t\widetilde{u}_i\widetilde{u}_i/2 - \overline{\rho}\partial_t\widetilde{\epsilon}_t\right] + \widetilde{\epsilon}_t\partial_r 4\pi r^2\overline{\rho}\widetilde{u}_r && (6)\\
\partial_r\overline{T} &= +(1/\overline{u}_r)\left[-\nabla_r f_T + (1-\Gamma_3)\overline{T}\ \overline{d} + (2-\Gamma_3)\overline{T'd'} + \epsilon_{nuc}/c_v + \nabla\cdot f_{th}/(\rho c_v) - \partial_t T\right] && (7)\\
\partial_t\widetilde{X}_i &= +\widetilde{\dot{X}}_i^{nuc} - (1/\overline{\rho})\nabla_r f_i - \widetilde{u}_r\partial_r\widetilde{X}_i && (8)
\end{aligned}$$

$$\begin{aligned}
\partial_r\overline{m} &= -\overline{\rho}\ \overline{m}\ \ \overline{u'_r d''}/\ \widetilde{R}_{rr} + 4\pi r^2\overline{\rho} && (9)\\
\partial_r\overline{P} &= -\Gamma_1\ \overline{\rho}\ \overline{P}\ \overline{u'_r d''}/\ \widetilde{R}_{rr} && (10)\\
\partial_r\widetilde{L} &= +\widetilde{\epsilon}_t\partial_r 4\pi r^2\overline{\rho}\widetilde{u}_r - 4\pi r^2\overline{\rho}\ \widetilde{u}_r\ \overline{P}\ \overline{u'_r d''}/\ \widetilde{R}_{rr} && (11)\\
\partial_r\overline{T} &= -(\Gamma_3 - 1)\ \overline{\rho}\ \overline{T}\ \overline{u'_r d''}/\ \widetilde{R}_{rr} && (12)\\
\partial_t\widetilde{X}_i &= +\widetilde{\dot{X}}_i^{nuc} - (1/\overline{\rho})\nabla_r f_i - \widetilde{u}_r\partial_r\widetilde{X}_i && (13)
\end{aligned}$$

$$\begin{aligned}
\partial_r\overline{m} &= -\overline{\rho}\ \overline{m}\ \overline{g}_r/\Gamma_1\overline{P} + 4\pi r^2\overline{\rho} && (14)\\
\partial_r\overline{P} &= -\overline{\rho}\ \overline{g}_r && (15)\\
\partial_r\widetilde{L} &= -4\pi r^2\widetilde{u}_r\ \overline{\rho}\ \overline{g}_r/\Gamma_1 + \widetilde{\epsilon}_t\partial_r 4\pi r^2\overline{\rho}\widetilde{u}_r && (16)\\
\partial_r\overline{T} &= -(\Gamma_3 - 1)\ \overline{\rho}\ \overline{T}\ \overline{g}_r/\Gamma_1\overline{P} && (17)\\
\partial_t\widetilde{X}_i &= +\widetilde{\dot{X}}_i^{nuc} - (1/\overline{\rho})\nabla_r f_i - \widetilde{u}_r\partial_r\widetilde{X}_i && (18)
\end{aligned}$$

$$\begin{aligned}
\partial_t\overline{m} &= +3\overline{\rho}\widetilde{u}_r/r - \nabla_r f_\rho + (f_\rho/\overline{\rho})\partial_r\overline{\rho} - \overline{\rho}\overline{d} && (19)\\
\overline{\rho}\partial_t\widetilde{u}_r &= +\overline{\rho}\overline{g} - \partial_r\overline{P} - \nabla_r\widetilde{R}_{rr} - \overline{G}^M_r - \overline{\rho}\widetilde{u}_r\partial_r\widetilde{u}_r && (20)\\
\overline{\rho}\partial_t\widetilde{\epsilon}_t &= +4\pi r^2\left[-\nabla_r(f_i + f_{th} + f_K + f_p) + \overline{\rho}\widetilde{\epsilon}_{nuc} - \overline{Pd} - \widetilde{R}_{ir}\partial_r\widetilde{u}_i + W_b + \overline{\rho}\widetilde{D}_t\widetilde{u}_i\widetilde{u}_i/2 - \partial_r\widetilde{L}\right] + \widetilde{\epsilon}_t\partial_r 4\pi r^2\overline{\rho}\widetilde{u}_r && (21)\\
\partial_t\overline{T} &= -\nabla_r f_T + (1-\Gamma_3)\overline{T}\ \overline{d} + (2-\Gamma_3)\overline{T'd'} + \epsilon_{nuc}/c_v + \nabla\cdot f_{th}/(\rho c_v) - \overline{u}_r\partial_r\overline{T} && (22)\\
\overline{\rho}\partial_t\widetilde{X}_i &= +\overline{\rho}\widetilde{\dot{X}}_i^{nuc} - \nabla_r f_i - \overline{\rho}\widetilde{u}_r\partial_r\widetilde{X}_i && (23)
\end{aligned}$$

$$M = \int \rho dV = \int \frac{m}{V} dV = m(r)\ln V(r) \quad ??(check) \quad (24)$$

REFERENCES

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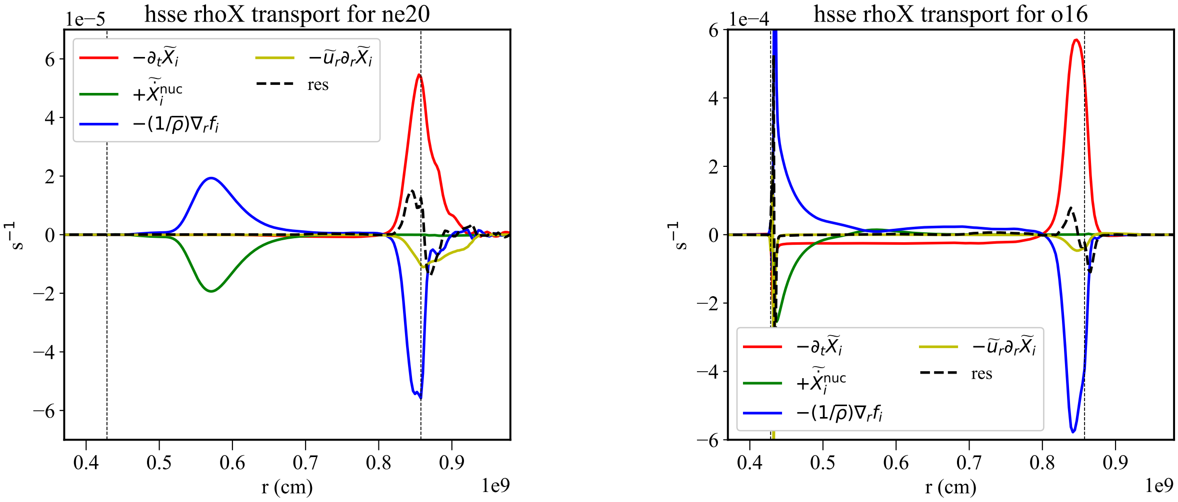

Figure 2. Left: Hydrodynamic stellar structure transport equations for Ne^{20} and Right: for O^{16}.

Table 1. Definitions:

ρ density	g_r radial gravitational acceleration
$m = \rho V = \rho \frac{4}{3}\pi r^3$ mass	$M = \int \rho(r) dV = \int \rho(r) 4\pi r^2 dr$ integrated mass
T temperature	$\mathcal{S} = \rho \epsilon_{\rm nuc}(q)$ nuclear energy production (cooling function)
P pressure	$\tau_{ij} = 2\mu S_{ij}$ viscous stress tensor (μ kinematic viscosity)
u_r, u_θ, u_ϕ velocity components	$S_{ij} = (1/2)(\partial_i u_j + \partial_j u_i)$ strain rate
$\mathbf{u} = u(u_r, u_\theta, u_\phi)$ velocity	$\widetilde{R}_{ij} = \overline{\rho}\widetilde{u''_i u''_j}$ Reynolds stress tensor
$f_i = \overline{\rho}\widetilde{X''_i u''_r}$	

(25)

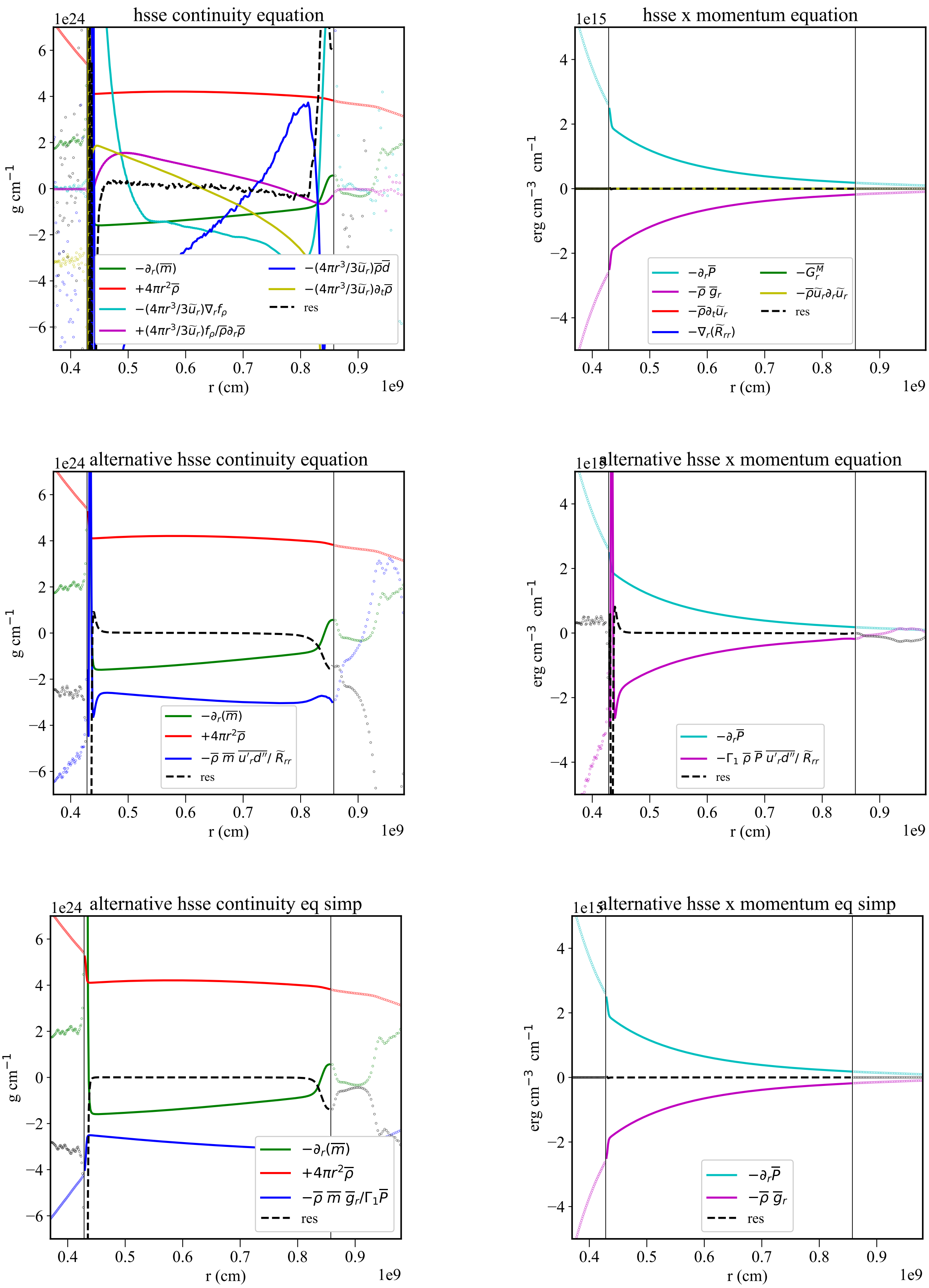

Figure 3. Left: Hydrodynamic stellar structure continuity equations Right: Hydrodynamic stellar structure momentum equations.

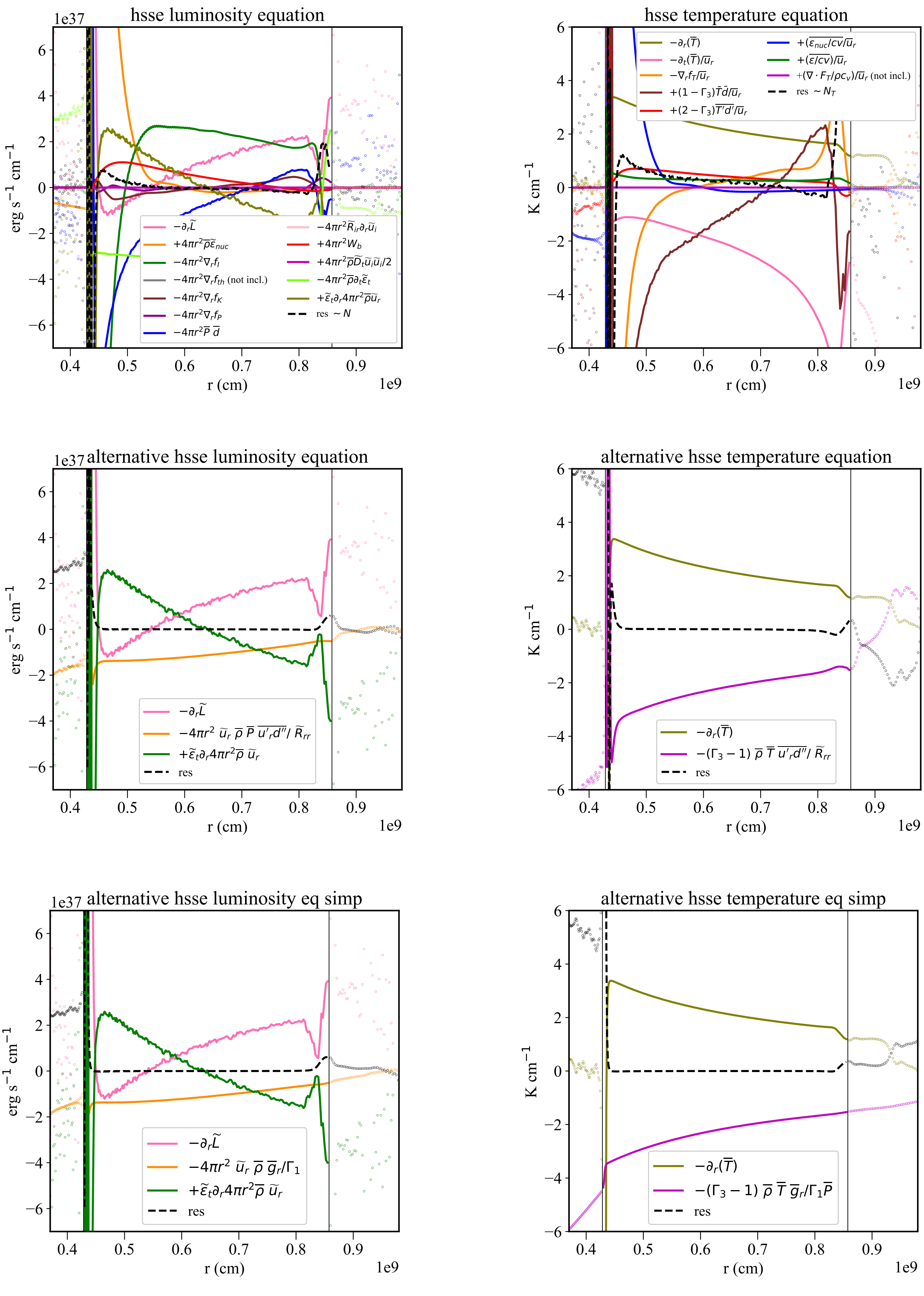

Figure 4. Left: Hydrodynamic stellar structure luminosity equations Right: Hydrodynamic stellar structure temperature equations.