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Width Of Convection Zones In Stars

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ABSTRACT
Growth of convection zones in stars ..

Key words: turbulence – convection zone – stellar evolution

1 INTRODUCTION

One of the issues in stellar evolution is location of convection boundaries, hence size of star's convection zones at any time during its life. Unlike modelling approach based on Swarzschild/Ledoux criterion with overshooting (see eg. Pedersen et al. (2018)) or model with parametrization of entrainment rate using bulk Richardson number approach (Meakin & Arnett 2006), to cope with location of convection boundaries, we propose to extend hydrodynamic stellar structure equations with additional physics represented by evolution equations for Reynolds stress, temperature flux and temperature variance, and get size of convection zones naturally from their solution (Biferale et al. 2011).

Size of the convection zones, or entrainment rates are emergent properties of convection zone, where underlying processes can be understood by RANS equations. When we solve those equations with correct models for unknown correlation, we get the size of the convection zone and entrainemnt rates right.

While deriving models or entrainment theory, keep in mind its domain of applicability (densities, temperatures, Reynolds number, Peclet nubmer etc.)

RANS equation provides insight to underlying physics of turbulent flow. But fluxes or any other correlations in them are very difficult to measure even in a simple lab experiment (reference the experiment you heard about in Newcastle, Igor's references too). Hence, it'll be difficult to verify them anytime soon. In stars this is close to impossible in near future, requiring a probe to be send inside a star.

$\mathcal{T}_R, \mathcal{T}_{\sigma_T}, \mathcal{T}_{f_T}$: turbulent entrainment (correct size of convection zone arises naturally from solution (Biferale et al. 2011))

$$\mathcal{T}_R = -\nabla_r(2f_k^r + 2f_p) + 2W_b - 2\widetilde{R}_{rr}\partial_r\widetilde{u}_r + \mathcal{C}_R^t + 2\mathcal{G}_k^r - \overline{\rho}\widetilde{u}_r\partial_r\widetilde{u''_r u''_r} + \varepsilon \quad (1)$$

$$\mathcal{T}_{\sigma_T} = -\nabla_r f_{\sigma_T} - \mathcal{C}_\sigma^o - 2f_T\partial_r\overline{T} - \mathcal{C}_{\sigma_T}^w + \mathcal{C}_{\sigma_T}^e + \mathcal{H}_{\sigma_T}^T + \mathcal{A}_{\sigma_T}^{\mathrm{nuc}} - \widetilde{u}_r\partial_r\sigma_T \quad (2)$$

$$\mathcal{T}_{f_T} = -\nabla_r f_T^r - f_T\partial_r\overline{u}_r - \overline{u'_r u''_r}\partial_r\overline{T} - \mathcal{P}_{f_T} - \mathcal{C}_{f_T}^o - \mathcal{C}_{f_T}^w - \mathcal{C}_{f_T}^t + \mathcal{C}_{f_T}^s + \mathcal{A}_{f_T}^{\mathrm{nuc}} + \mathcal{H}_{f_T}^T - \widetilde{u}_r\partial_r f_T + \mathcal{G}_T \quad (3)$$

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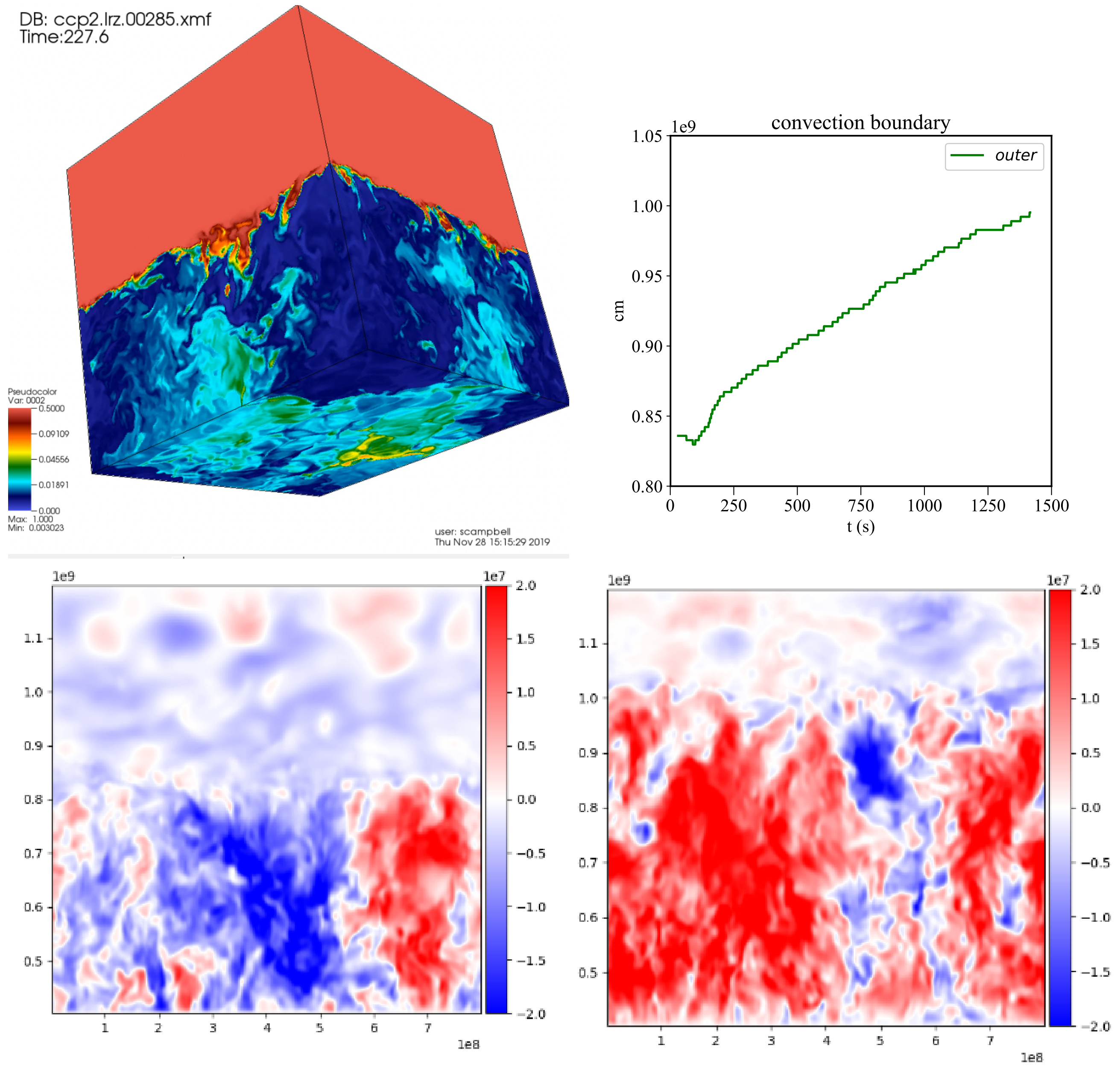

Figure 1. Top-Left: Visualization of turbulence in a 3D cartesian box (resolution 256x256x256) based on code comparison project setup Top-Right: Temporal evolution of outer convection boundary position. Bottom: Vertical slices of radial velocity field in the 3D simulation at times 200 seconds (left) and 1500 seconds (right).

Table 1. Definitions:

ρ density	g_r radial gravitational acceleration
T temperature	$\mathcal{S} = \rho\epsilon_{\rm nuc}(q)$ nuclear energy production (cooling function)
P pressure	$\tau_{ij} = 2\mu S_{ij}$ viscous stress tensor (μ kinematic viscosity)
u_r, u_θ, u_ϕ velocity components	$S_{ij} = (1/2)(\partial_i u_j + \partial_j u_i)$ strain rate
$\mathbf{u} = u(u_r, u_\theta, u_\phi)$ velocity	$\widetilde{R}_{ij} = \overline{\rho}\widetilde{u''_i u''_j}$ Reynolds stress tensor
$m = \rho V = \rho \frac{4}{3}\pi r^3$ mass	$M = \int \rho(r) dV = \int \rho(r) 4\pi r^2 dr$ integrated mass

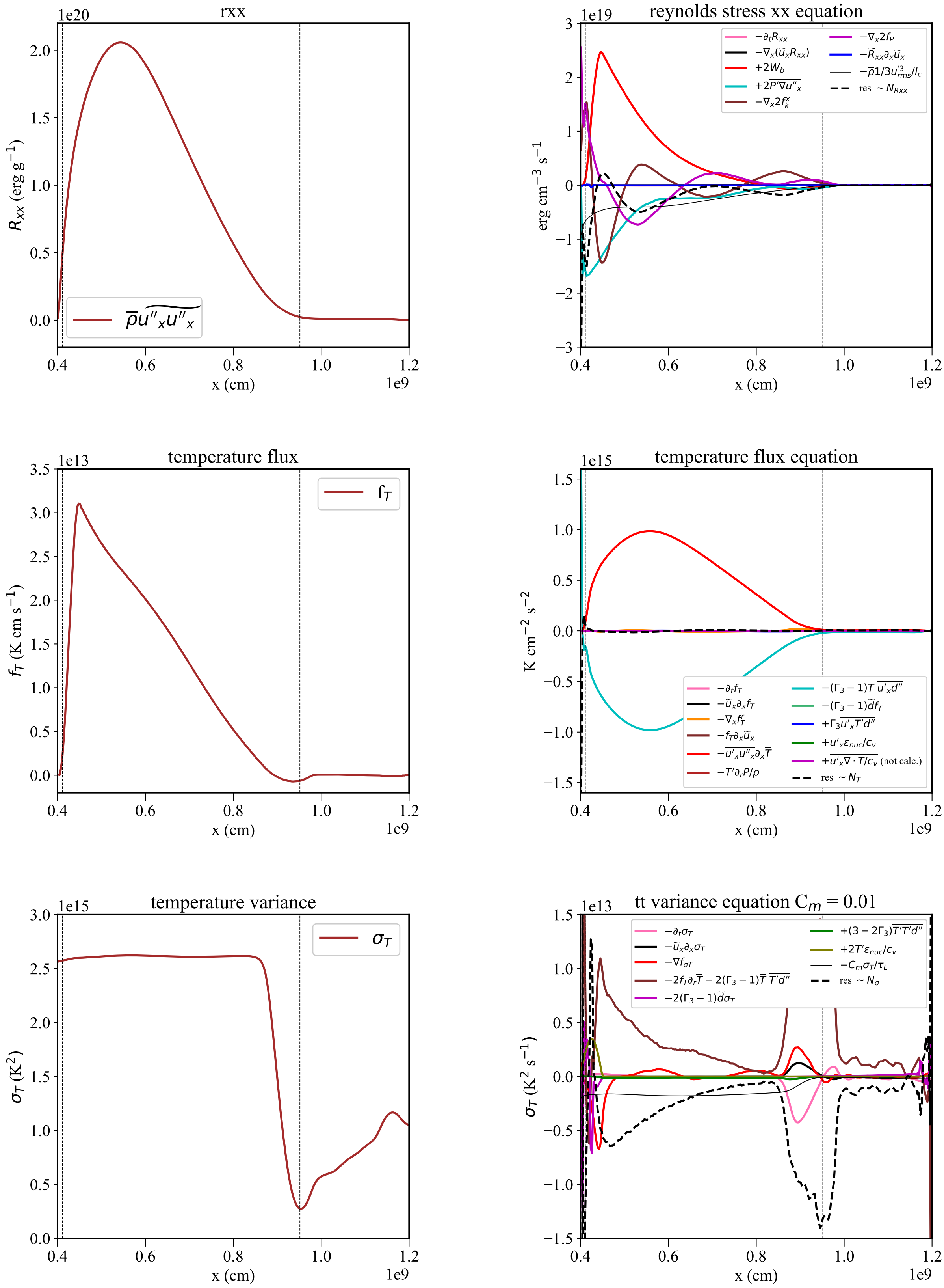

Figure 2. Top: Profiles of Reynolds stress and its evolution equation. Middle: Profiles of temperature flux and its evolution equation. Bottom: Profiles of temperature variance and its evolution equation.

Table 2. Definitions:

$R_{rr} = \overline{\rho}\widetilde{u''_r u''_r}$	$d = \nabla \cdot \mathbf{u}$	$\mathcal{H}_\sigma^T = \overline{(2/\rho c_v) T' \nabla \cdot F_{th}}$
$\mathcal{T}_R = \overline{\rho}\partial_t \widetilde{u''_r u''_r}$	$f_{\sigma_T} = \overline{(T'T'u''_r)}$	$\mathcal{C}_{f_T}^o = (\Gamma_3 - 1)(\overline{T}\ \overline{u'_r d''})$
$\mathcal{T}_{f_T} = \partial_t \overline{T'u'_r}$	$f_T = \overline{T'u''_r}$	$\mathcal{C}_{f_T}^w = (\Gamma_3 - 1)(\widetilde{d}\ \overline{u'_r T'})$
$\mathcal{T}_{\sigma_T} = \partial_t \overline{T'T'}$	$\sigma_T = \overline{T'T'}$	$\mathcal{C}_{f_T}^t = (\Gamma_3 - 1)(\overline{u'_r T' d''})$
$\mathcal{T}_T = \partial_t \overline{T}$	$\mathcal{C}_\sigma^o = 2(\Gamma_3 - 1)\overline{T}\ \overline{T'd''}$	$\mathcal{C}_{f_T}^s = \overline{T'u'_r d''}$
$\mathcal{S} = 4\pi r^2$	$\mathcal{C}_\sigma^w = 2(\Gamma_3 - 1)\widetilde{d}\sigma_T$	$\mathcal{C}_{f_T}^T = \overline{u'_r \nabla \cdot F_{th}/\rho c_v}$
$\mathcal{P}_{f_T} = \overline{T'\partial_r P/\rho}$	$\mathcal{C}_\sigma^e = (3 - 2\Gamma_3)\overline{T'T'd''}$	$\mathcal{A}_{f_T}^{\mathrm{nuc}} = \overline{2T'\epsilon_{\mathrm{nuc}}/c_v}$
	$\mathcal{H}_T^T = (1/\rho c_v)\ \nabla \cdot F_{th}$	$\mathcal{A}_{\sigma_T}^{\mathrm{nuc}} = \overline{u'_r \epsilon_{\mathrm{nuc}}/c_v}$