1 ransX to PROMPI implementation

1.1 Composition transport equation

$$
\begin{aligned}
\overline{\rho}\widetilde{D}_t\widetilde{X}_i &= -\nabla_r f_i + \overline{\rho}\widetilde{\dot{X}}_i^{\rm nuc} \\
\overline{\rho}\partial_t\widetilde{X}_i + \overline{\rho}\widetilde{u}_r\partial_r\widetilde{X}_i &= -\nabla_r\overline{\rho}\widetilde{X_i''u_r''} + \overline{\rho}\widetilde{\dot{X}}_i^{\rm nuc} \\
\overline{\rho}\partial_t\widetilde{X}_i + \overline{\rho}\widetilde{u}_r\partial_r\widetilde{X}_i &= -\nabla_r\overline{\rho}(\widetilde{X_iu_r} - \widetilde{X}_i\widetilde{u}_r) + \overline{\rho}\widetilde{\dot{X}}_i^{\rm nuc} \\
\overline{\rho}\partial_t\overline{\rho X_i}/\overline{\rho} + \overline{\rho u_r}\partial_r\overline{\rho X_i}/\overline{\rho} &= -\nabla_r\left(\overline{\rho X_i u_r} - \overline{\rho X_i}\,\overline{\rho u_r}/\overline{\rho}\right) + \overline{\rho\dot{X}_i^{\rm nuc}} \\
dd\ \partial_t\ ddxi/dd + ddux\ \partial_r\ ddxi/dd &= -\nabla_r\left(ddxiux - ddxi*ddux/dd\right) + ddxidot
\end{aligned} \tag{1}
$$

1.2 Composition variance equation

$$
\begin{aligned}
\overline{\rho}\widetilde{D}_t\sigma_i &= -\nabla_r f_i^r - 2f_i\partial_r\widetilde{X}_i + 2\overline{X_i''\rho\dot{X}_i^{\rm nuc}} \\
\overline{\rho}\widetilde{D}_t\widetilde{X_i''X_i''} &= -\nabla_r(\overline{\rho X_i''X_i''u_r''}) - 2\overline{\rho}\widetilde{X_i''u_r''}\partial_r\widetilde{X}_i + 2\overline{X_i''\rho\dot{X}_i^{\rm nuc}} \\
\overline{\rho}\partial_t(\widetilde{X_iX_i} - \widetilde{X}_i\widetilde{X}_i) + \overline{\rho}\widetilde{u}_r\partial_r(\widetilde{X_iX_i} - \widetilde{X}_i\widetilde{X}_i) &= -\nabla_r(\overline{\rho X_iX_iu_r} - 2\widetilde{X}_i\overline{\rho X_iu_r} - \widetilde{u}_r\overline{\rho X_iX_i} + 2\widetilde{X}_i\widetilde{X}_i\overline{\rho u_r}) \\
&\quad - 2\overline{\rho}(\widetilde{X_iu_r} - \widetilde{X}_i\widetilde{u}_r)\partial_r\widetilde{X}_i + (\overline{X_i\rho\dot{X}_i} - \widetilde{X}_i\overline{\rho\dot{X}_i}) \\
dd\ \partial_t\ (ddxisq/dd - ddxi*ddxi/dd*dd) & \\
+\ ddux\ \partial_r\ (ddxisq/dd - ddxi*ddxi/dd*dd) &= -\nabla_r(ddxisqux - 2*ddxi/dd*ddxiux - ddux/dd*ddxisq + 2*ddxi*ddxi*ddux/dd*dd) \\
&\quad -\ 2*dd\ (ddxiux/dd - ddxi*ddux/dd*dd)*\partial_r\ ddxi/dd \\
&\quad + 2*(ddxixidot - ddxi/dd*ddxidot)
\end{aligned} \tag{2}
$$

1.3 Composition flux equation

$$
\begin{aligned}
\overline{\rho}\widetilde{D}_t(f_i/\overline{\rho}) = & -\nabla_r f_i^r - f_i \partial_r \widetilde{u}_r - \widetilde{R}_{rr}\partial_r \widetilde{X}_i - \overline{X_i''}\partial_r \overline{P} - \overline{X_i''\partial_r P'} + \overline{u_r''\rho \dot{X}_i^{\mathrm{nuc}}} + \mathcal{G}_i \qquad (3) \\
\overline{\rho}\partial_t \widetilde{X_i''u_r''} + \overline{\rho}\widetilde{u}_r \partial_r \widetilde{X_i''u_r''} = & -\nabla_r \overline{\rho}\widetilde{X_i''u_r''u_r''} - \overline{\rho}\widetilde{X_i''u_r''}\partial_r \widetilde{u}_r - \overline{\rho}\widetilde{u_r''u_r''}\partial_r \widetilde{X}_i - \overline{X_i''}\partial_r \overline{P} - \overline{X_i''\partial_r P'} + \overline{u_r''\rho \dot{X}_i^{\mathrm{nuc}}} + \overline{G_r^i} - \overline{X_i'' G_r^M} \\
\overline{\rho}\partial_t \widetilde{X_i''u_r''} + \overline{\rho}\widetilde{u}_r \partial_r \widetilde{X_i''u_r''} = & -\nabla_r \overline{\rho}\widetilde{X_i''u_r''u_r''} - \overline{\rho}\widetilde{X_i''u_r''}\partial_r \widetilde{u}_r - \overline{\rho}\widetilde{u_r''u_r''}\partial_r \widetilde{X}_i - \overline{X_i''}\partial_r \overline{P} - \overline{X_i''\partial_r P'} + \overline{u_r''\rho \dot{X}_i^{\mathrm{nuc}}} \\
& - \overline{\rho X_i'' u_\theta'' u_\theta''/r} - \overline{\rho X_i'' u_\phi'' u_\phi''/r} + \overline{\rho X_i'' u_\theta u_\theta/r} + \overline{\rho X_i'' u_\phi u_\phi/r} \\
\overline{\rho}\partial_t(\widetilde{X_i u_r} - \widetilde{X}_i\widetilde{u}_r) + \overline{\rho}\widetilde{u}_r\partial_r(\widetilde{X_i u_r} - \widetilde{X}_i\widetilde{u}_r) = & -\nabla_r(\overline{\rho X_i u_r u_r} - \widetilde{X}_i\overline{\rho u_r u_r} - 2\widetilde{u}_r\overline{\rho X_i u_r} + 2\overline{\rho}\widetilde{X}_i\widetilde{u}_r\widetilde{u}_r) \\
& - \overline{\rho}(\widetilde{X_i u_r} - \widetilde{X}_i\widetilde{u}_r)\partial_r\widetilde{u}_r - \overline{\rho}(\widetilde{u_r u_r} - \widetilde{u}_r\widetilde{u}_r)\partial_r\widetilde{X}_i \\
& - (\overline{X_i}\partial_r\overline{P} - \widetilde{X}_i\partial_r\overline{P}) - (\overline{X_i\partial_r P} - \overline{X_i}\partial_r\overline{P}) + (\overline{u_r\rho\dot{X}_i^{\mathrm{nuc}}} - \widetilde{u}_r\overline{\rho\dot{X}_i^{\mathrm{nuc}}}) \\
& - (\overline{\rho X_i u_\theta u_\theta} - \widetilde{X}_i\overline{\rho u_\theta u_\theta} - 2\widetilde{u}_\theta\overline{\rho X_i u_\theta} + 2\overline{\rho}\widetilde{X}_i\widetilde{u}_\theta\widetilde{u}_\theta)/r \\
& - (\overline{\rho X_i u_\phi u_\phi} - \widetilde{X}_i\overline{\rho u_\phi u_\phi} - 2\widetilde{u}_\phi\overline{\rho X_i u_\phi} + 2\overline{\rho}\widetilde{X}_\phi\widetilde{u}_\phi\widetilde{u}_\phi)/r \\
& + (\overline{\rho X_i u_\theta u_\theta} - \widetilde{X}_i\overline{\rho u_\theta u_\theta})/r \\
& + (\overline{\rho X_i u_\phi u_\phi} - \widetilde{X}_i\overline{\rho u_\phi u_\phi})/r
\end{aligned}
$$

$$
\begin{aligned}
dd\ \partial_t(ddxiux/dd - ddxi*ddux/dd*dd) + ddux\ \partial_r(ddxiux/dd - ddxi*ddux/dd*dd) = \\
-\nabla_r(ddxiuxux - ddxi/dd*dduxux - 2*ddux/dd*ddxiux + 2*ddxi*ddux*ddux/dd*dd) \\
-(ddxiux - ddxi*ddux/dd)*\partial_r ddux/dd - (dduxux - ddux*ddux/dd)*\partial_r ddxi/dd \\
-(xi\ \partial_r\ pp - ddxi/dd\ \partial_r\ pp) - (xigradxpp - xi\ \partial_r\ pp) + (ddxidotux - ddux/dd*ddxidot) \\
-(ddxiuyuy - ddxi/dd*dduyuy - 2*dduy/dd*ddxiuy + 2*ddxi*dduy*dduy/dd*dd)/r \\
-(ddxiuzuz - ddxi/dd*dduzuz - 2*dduz/dd*ddxiuz + 2*ddxi*dduz*dduz/dd*dd)/r \\
+(ddxiuyuy - ddxi/dd*dduyuy)/r \\
+(ddxiuzuz - ddxi/dd*dduzuz)/r
\end{aligned}
$$

1.4 MLT velocity

$$
\begin{aligned}
u_{MLT} \equiv (u'_{rms}) &= \frac{F_c}{\alpha_E c_P (T'_{rms})} = \frac{\overline{\rho}\widetilde{h'' u''_r}}{\alpha_E \widetilde{c_P} (\widetilde{TT} - \widetilde{T}\widetilde{T})^{1/2}} \sim \frac{\overline{\rho}\overline{h' u'_r}}{\alpha_E \overline{c_P} (\overline{TT} - \overline{T}\ \overline{T})^{1/2}} ? \qquad (4) \\
u_{MLT} \equiv (u'_{rms}) &= \frac{\overline{\rho}(\widetilde{hu_r} - \widetilde{h}\widetilde{u_r})}{\alpha_E \widetilde{c_P} (\widetilde{TT} - \widetilde{T}\widetilde{T})^{1/2}} \sim \frac{\overline{\rho}(\overline{hu_r} - \overline{h}\overline{u_r})}{\alpha_E \overline{c_P} (\overline{TT} - \overline{T}\ \overline{T})^{1/2}} \\
u_{MLT} \equiv (u'_{rms}) &= \frac{ddhhux - ddhh * ddux/dd}{\alpha_E * ddcp/dd\ (ddttsq/dd - ddtt * ddtt/dd * dd)^{1/2}} \sim \frac{dd * hhux - dd * hh * ux}{\alpha_E * cp\ (ttsq - tt * tt)^{1/2}}
\end{aligned}
$$

1.5 Usefull identities

$$\overline{a''} = \overline{a - \widetilde{a}} = \overline{a} - \widetilde{b} \qquad (5)$$

$$\widetilde{a''b''} = \widetilde{(a - \widetilde{a}) * (b - \widetilde{b})} = \widetilde{ab} - \widetilde{a}\widetilde{b} \qquad (6)$$

$$\widetilde{a''b''c''} = \widetilde{(a - \widetilde{a}) * (b - \widetilde{b}) * (c - \widetilde{c})} = \widetilde{abc} - \widetilde{a}\widetilde{bc} - \widetilde{b}\widetilde{ac} - \widetilde{c}\widetilde{ab} + 2\widetilde{a}\widetilde{b}\widetilde{c} \qquad (7)$$

$$\overline{a''bc} = \overline{(a - \widetilde{a})bc} = \overline{abc} - \widetilde{a}\overline{bc} \qquad (8)$$

$$\overline{a''\partial_r b'} = \overline{(a - \widetilde{a})\partial_r b'} = \overline{a\partial_r b'} - \cancelto{0}{\widetilde{a}\partial_r \overline{b'}} = \overline{a\partial_r b} - \overline{a}\partial_r \overline{b} \qquad (9)$$