Symmetric Polynomials Solutions

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Problem 6

Consider the polynomial with roots $r+s$, $s+t$, and $r+t$. We will find its coefficients and show that it is the desired polynomial. Using Vieta's, we can see that

$$A = -2(r+s+t) = -14.$$

We can also see that

$$B = (r+s)(s+t) + (s+t)(r+t) + (r+s)(r+t).$$

Expanding and simplifying with Vieta's, we get $B = 52$.

The C term is slightly more involved, but we can use a combination of Newton sums and grouping of terms to get $C = -23$.

All these terms are rational, so overall, our answer is $A+B+C = -14+52-23 = 15$.