Programming Language: MatLab

1st Semester 2015

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W11, 23rd Nov

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Vectorization

Try to arrange your operations to *vector form* instead of *for-loop*

Comparing the difference between two methods below:

NonVec

```
clear all;
tic
A=0:0.000001:10;
B=0:0.000001:10;
Z=zeros(size(A));
for i = 1:10000001
    Z(i) = sin(0.5*A(i))*exp(B(i)^2);
end
sum(Z)
toc
```

Recording the starting time

Recording the end time

```
Command Window
>> nonVec

ans =

  -1.3042e+48

Elapsed time is 1.132052 seconds.
>>
```

Vec

```
clear all;
tic
A=0:0.000001:10;
B=0:0.000001:10;
Z=zeros(size(A));
Z = sin(0.5*A).*exp(B.^2);
sum(Z)
toc
```

```
Command Window
>> Vec

ans =

  -1.3042e+48

Elapsed time is 0.555933 seconds.
>>
```

Pre-allocation array

Although it is easier to constructor the program by resizing array, it costs many unnecessary actions for array relocation.
Try to allocate (define) the array before the calculation.

```
% Without Pre-allocation
N = 10000;
tic
x=1;
for i = 2:N
    x(i) = 2*i;
end
toc

% Pre-allocation
tic
y=zeros(1,N);
for i = 1:N
    y(i) = 2*i;
end
toc
```

Command Window

```
>> Preallocate
Elapsed time is 0.005087 seconds.
Elapsed time is 0.000071 seconds.
>>
```

Practice:
Try to compare process times for difference N from N = 10^{α}, for α= 2,3,4,5 and 6

* Drawing the figure in log scale

Column first in for-loop

Row first

```
clear;
N = 5000;
X = magic(N);
Y = zeros(N,N);
tic
for i = 1:N
    for j = 1:N
        Y(i,j) = X(i,j);
    end
end
toc
```

Column first

```
clear;
N = 5000;
X = magic(N);
Y = zeros(N,N);
tic
for j = 1:N
    for i = 1:N
        Y(i,j) = X(i,j);
    end
end
toc
```

Command Window

```
>> Row_first
Elapsed time is 1.458254 seconds.
>> Col_first
Elapsed time is 0.458113 seconds.
>>
```

Miscellaneous

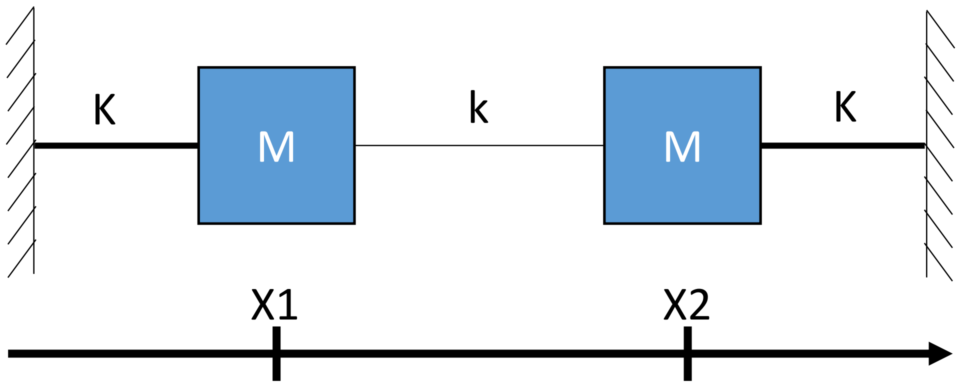
- Minimize file Read/Write within the loop

  File read/write is very expensive. Try to load/store the data into variable(s) at once (before the loop), and then those variables can be operated inside the loop.

- Minimize dynamically changing the variable class
- Minimize the number and size of variables
- Minimize dynamically changing the folder
- Keeping the code readability

Coupled oscillator

Two blocks with mass M are connected to the wall by spring with spring constant K, and they are coupled with a soft spring with k<K. Their equilibrium positions are X1 and X2, respectively. R1 and R2 are the displacement deviated from their equilibrium positions.



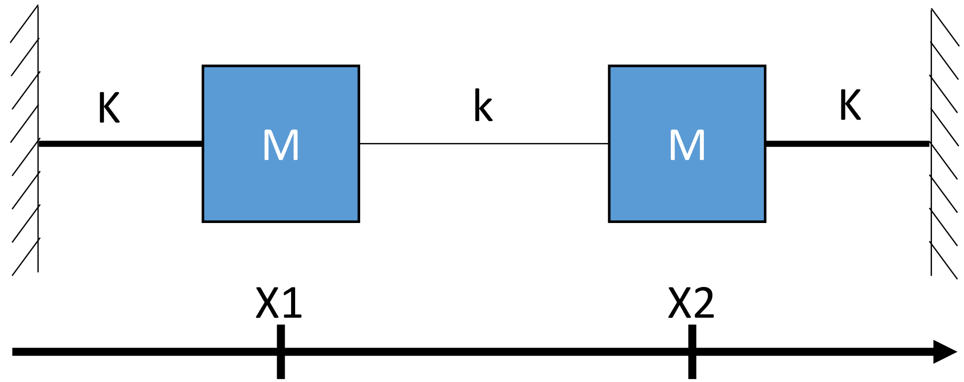
The force exert on each block

$$M\ddot{R}_1 = -KR_1 - k(R_1 - R_2) \qquad M\ddot{R}_1 + (K+k)R_1 - kR_2 = 0$$
$$M\ddot{R}_2 = -KR_2 - k(R_2 - R_1) \qquad M\ddot{R}_2 - kR_1 + (K+k)R_2 = 0$$

Coupled oscillator

The force exert on each block

$$M\ddot{R}_1 + (K+k)R_1 - kR_2 = 0$$
$$M\ddot{R}_2 - kR_1 + (K+k)R_2 = 0$$



According to the solution of SHO

$$R_i(t) = A_i \exp(i\omega t)$$

If R_1 and $R_2 \neq 0$

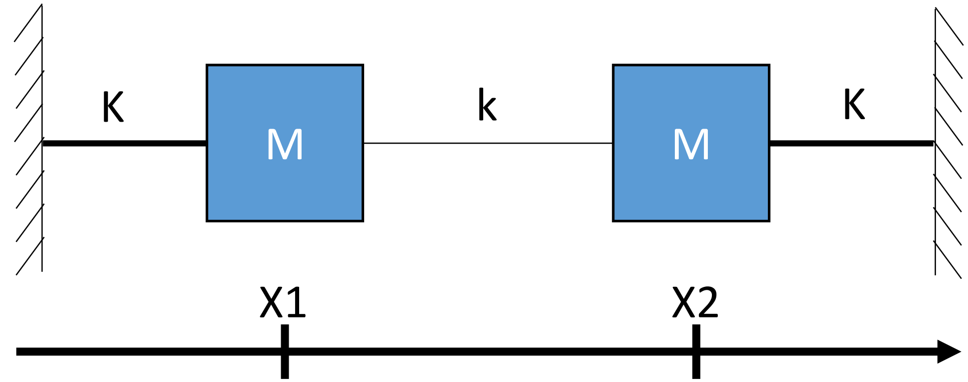
$$\begin{pmatrix} -M\omega^2 + K + k & -k \\ -k & -M\omega^2 + K + k \end{pmatrix} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = 0 \Rightarrow \begin{vmatrix} -M\omega^2 + K + k & -k \\ -k & -M\omega^2 + K + k \end{vmatrix} = 0$$

$$\Rightarrow \left(-M\omega^2 + K + k\right)^2 - k^2 = 0 \Rightarrow \omega = \pm\sqrt{\frac{K + k \pm k}{M}}$$

Coupled oscillator

The oscillation frequency

$$\omega_A = \pm\sqrt{\frac{\mathrm{K}+2k}{\mathrm{M}}} \qquad \omega_B = \pm\sqrt{\frac{\mathrm{K}}{\mathrm{M}}}$$



The general solution is

$$\mathrm{R}_1(t) = A^+ e^{i\sqrt{\frac{K+2k}{M}}t} + A^- e^{-i\sqrt{\frac{K+2k}{M}}t} + B^+ e^{i\sqrt{\frac{K}{M}}t} + B^- e^{-i\sqrt{\frac{K}{M}}t}$$

$$\mathrm{R}_2(t) = a^+ e^{i\sqrt{\frac{K+2k}{M}}t} + a^- e^{-i\sqrt{\frac{K+2k}{M}}t} + b^+ e^{i\sqrt{\frac{K}{M}}t} + b^- e^{-i\sqrt{\frac{K}{M}}t}$$

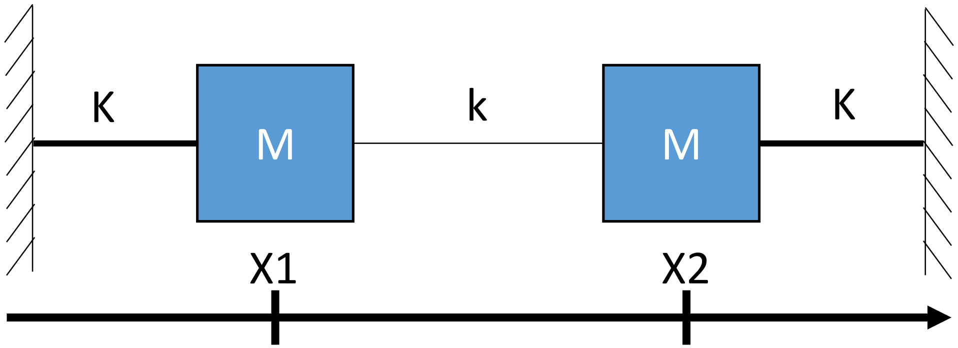
Since $\begin{pmatrix} -\mathrm{M}\omega^2 + K + k & -k \\ -k & -\mathrm{M}\omega^2 + K + k \end{pmatrix}\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = 0$ $\quad \begin{matrix} \omega = \omega_\mathrm{A} \\ \omega = \omega_\mathrm{B} \end{matrix} \Rightarrow \begin{matrix} \mathrm{A} = -a \\ B = b \end{matrix}$

$$\mathrm{R}_1(t) = A^+ e^{i\sqrt{\frac{K+2k}{M}}t} + A^- e^{-i\sqrt{\frac{K+2k}{M}}t} + B^+ e^{i\sqrt{\frac{K}{M}}t} + B^- e^{-i\sqrt{\frac{K}{M}}t}$$

$$\mathrm{R}_2(t) = -A^+ e^{i\sqrt{\frac{K+2k}{M}}t} - A^- e^{-i\sqrt{\frac{K+2k}{M}}t} + B^+ e^{i\sqrt{\frac{K}{M}}t} + B^- e^{-i\sqrt{\frac{K}{M}}t}$$

Homework

Consider there are two blocks with mass M, which are connected to the wall by spring with spring constant K, and they are coupled with a soft spring $k<K$. Their equilibrium positions are X1 and X2, respectively. R1 and R2 are the displacement deviated from their equilibrium positions.



a) Plotting the figure for R1 and R2 vs t , For M = 2.5 Kg; K = 1 N/m; R1 = 2 m, R2 =V1=V2= 0 at t = 0., plotting. Also plotting the figure of energy for each block.

b) Try to use the new coordinate $\eta 1 \equiv (R1+R2)/2$ and $\eta 2 \equiv (R1-R2)/2$, to plot $\eta 1(t)$ and $\eta 2(t)$.

c) If R1 = -R2 = 3 m at t = 0, try to compare R1(t), R2(t), $\eta 1(t)$ and $\eta 2(t)$