Programming Language: MatLab

1st Semester 2015

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Content

- From coupled oscillator to triple oscillator
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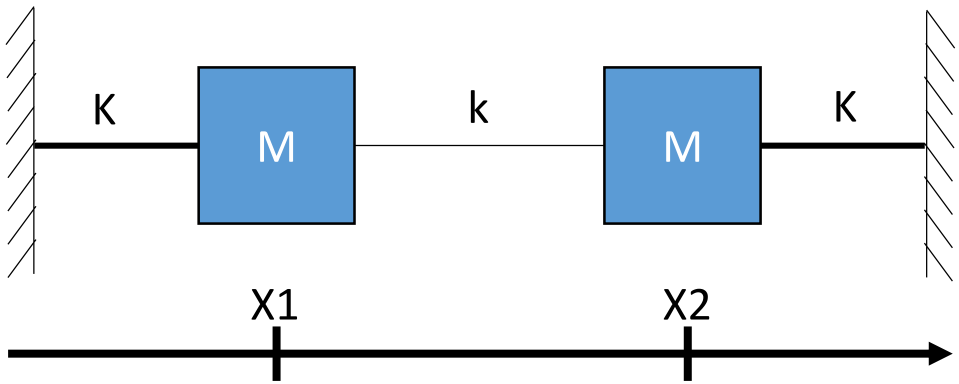
Review

Two blocks with mass M are connected to the wall by spring with spring constant K, and they are coupled with a soft spring with k<K. Their equilibrium positions are X1 and X2, respectively. R1 and R2 are the displacement deviated from their equilibrium positions.



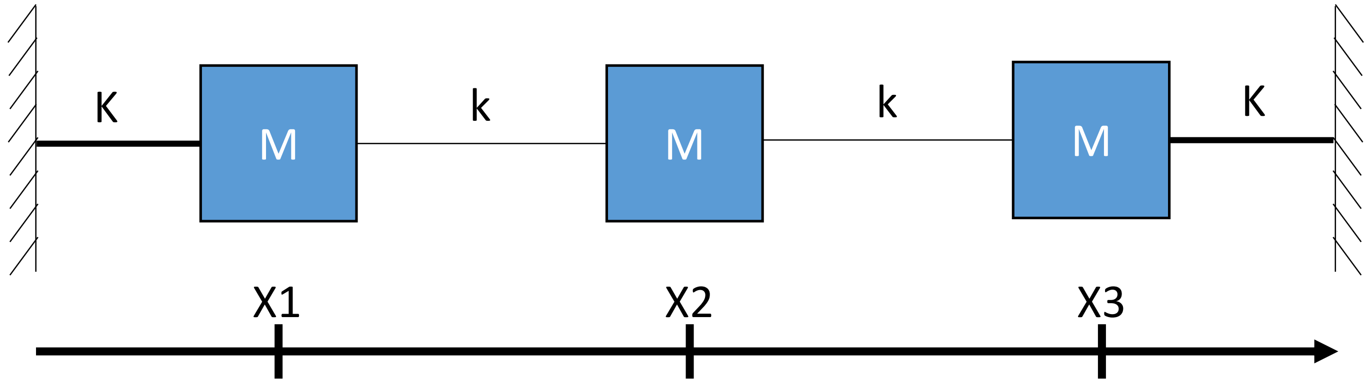
The force exert on each block

$$M\ddot{R}_1 = -KR_1 - k(R_1 - R_2)$$
$$M\ddot{R}_2 = -KR_2 - k(R_2 - R_1)$$

$$\Rightarrow$$

$$M\ddot{R}_1 + (K+k)R_1 - kR_2 = 0$$
$$M\ddot{R}_2 - kR_1 + (K+k)R_2 = 0$$

Triple coupled oscillator



Xi: equilibrium position of block i
Ri: displacement deviated from equilibrium of block i
k: coupling constant between two blocks
K: coupling constant to the wall
M: the mass of the block

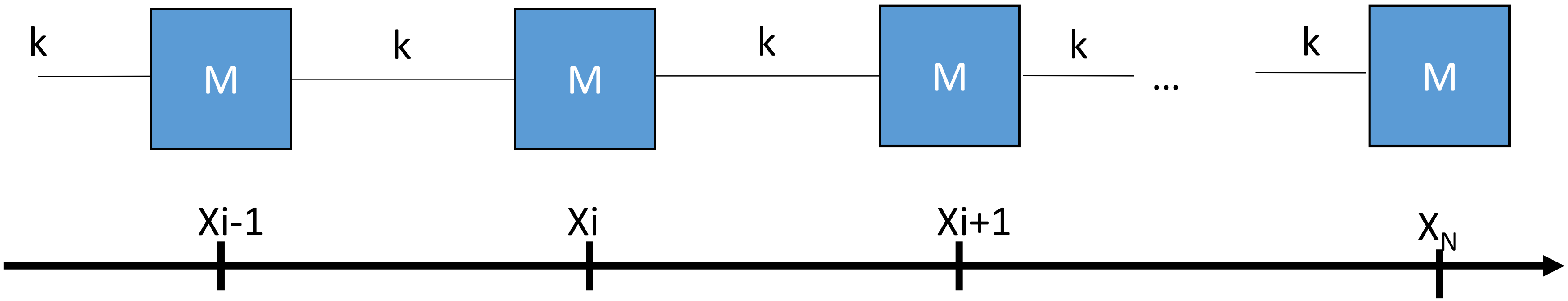
The force exert on each block

$$\mathrm{M}\ddot{\mathrm{R}}_1 = -KR_1 - k(R_1 - R_2)$$

$$\mathrm{M}\ddot{\mathrm{R}}_2 = F_{32} - F_{21} = [k(R_3 - R_2)] - [k(R_2 - R_1)]$$

$$\mathrm{M}\ddot{\mathrm{R}}_3 = -KR_3 - k(R_3 - R_2)$$

Many coupled oscillators



Xi: equilibrium position of block i
Ri: displacement deviated from equilibrium of block i
k: coupling constant between two blocks
K: coupling constant to the wall
M: the mass of the block

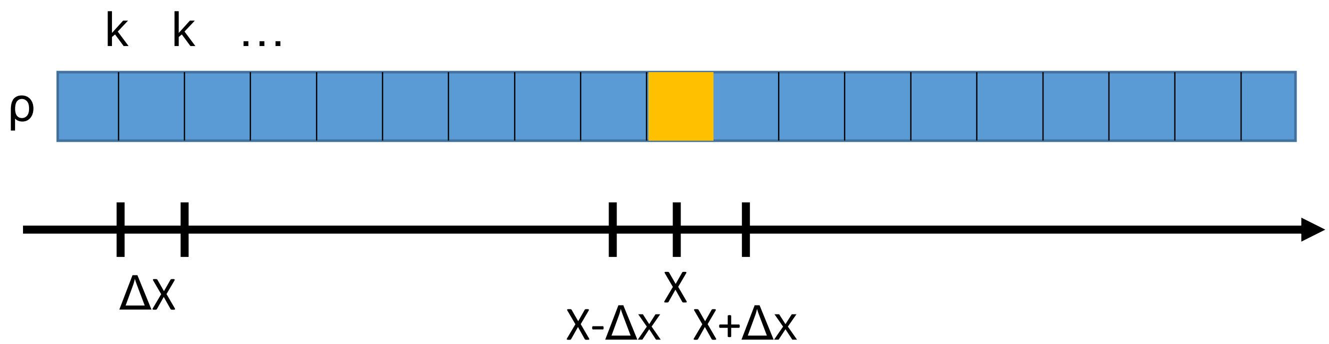
The force exert on block i

$$M\ddot{R}_i = F_{i+1} - F_i$$
$$= [k(R_{i+1} - R_i)] - [k(R_i - R_{i-1})]$$

$N \longrightarrow \infty$

$\Delta X \equiv X_{i+1} - X_i \longrightarrow 0$

Coupled oscillator to continuous system



R(x): displacement deviated from equilibrium of segment i

k: coupling constant between two segments

ρ: the mass density per unit length

The net force exert on segment i

$$M\ddot{\mathrm{R}}(\mathrm{x};\mathrm{t}) = F(x+\Delta x/2) - F(x-\Delta x/2)$$

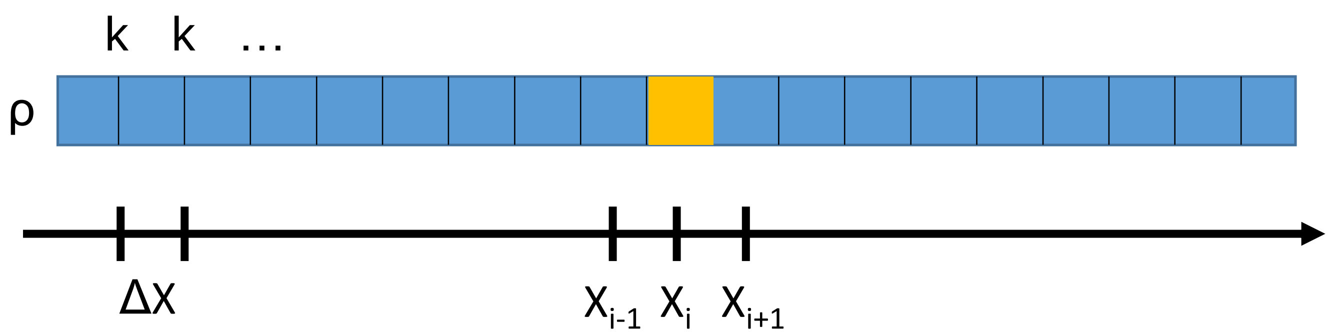
$$k[R(x+\Delta x) - R(x)] \qquad k[R(x) - R(x-\Delta x)]$$

$$\Delta x \frac{k[R(x+\Delta x) - R(x)]}{\Delta x} \qquad \Delta x \frac{k[R(x) - R(x-\Delta x)]}{\Delta x}$$

$\boxed{\Delta X \to 0}$

$$\lim_{\Delta x \to 0} \frac{k[R(x+\Delta x) - R(x)]}{\Delta x} = \lim_{\Delta x \to 0} k \left.\frac{\Delta R(x)}{\Delta x}\right|_{x+\Delta x/2} = k \frac{\partial}{\partial x} R(x;t)\Big|_{x+\Delta x/2}$$

Coupled oscillator to continuous system



R(x): displacement deviated from equilibrium of segment i

k: coupling constant between two segments

ρ: the mass density per unit length

The net force exert on segment i

$$M\ddot{\mathrm{R}}(\mathrm{x};\mathrm{t}) = F(x+\Delta x/2) - F(x-\Delta x/2)$$

$$\boxed{k' = k\Delta x}$$

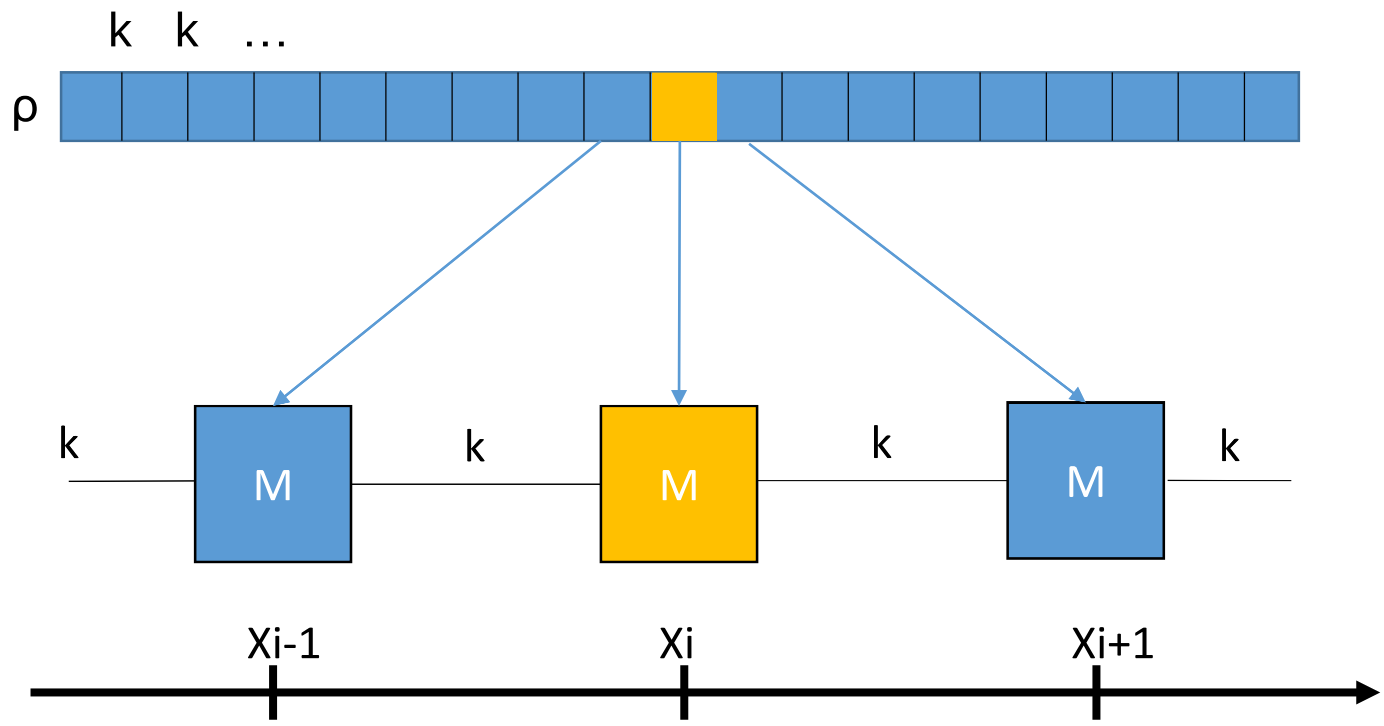
$$\frac{M\ddot{\mathrm{R}}(\mathrm{x};\mathrm{t})}{\Delta x} = \frac{\rho\Delta x\ddot{\mathrm{R}}(\mathrm{x};\mathrm{t})}{\Delta x} = \frac{F(x+\Delta x/2) - F(x-\Delta x/2)}{\Delta x}$$

$$\boxed{k'\frac{\partial}{\partial x}R(x;t)\Big|_x}$$

$$\boxed{\Delta \mathrm{X} \to 0} \quad \rho\ddot{\mathrm{R}}(\mathrm{x}) = \lim_{\Delta x \to 0}\frac{\Delta F}{\Delta x} \Rightarrow \rho\ddot{\mathrm{R}}(\mathrm{x}) = \frac{\partial F(x)}{\partial x} \Rightarrow \boxed{\frac{\partial^2}{\partial t^2}R(x;t) = \frac{k'}{\rho}\frac{\partial^2}{\partial x^2}R(x;t)}$$

How to model a continuous system

Cutting the continuous system to discrete system with small size

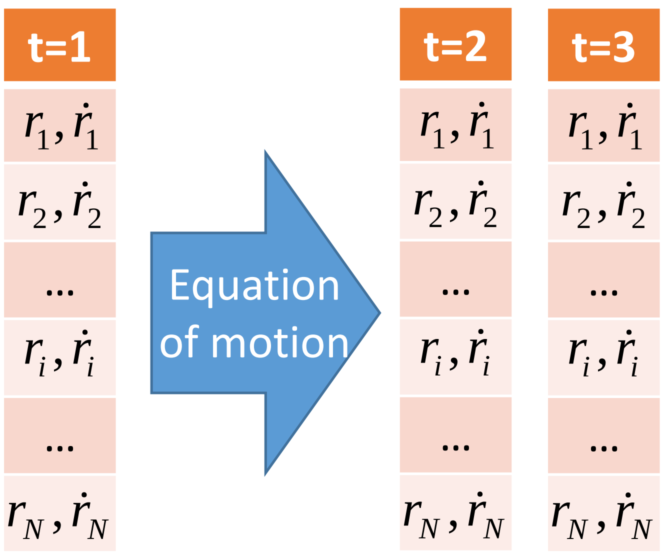


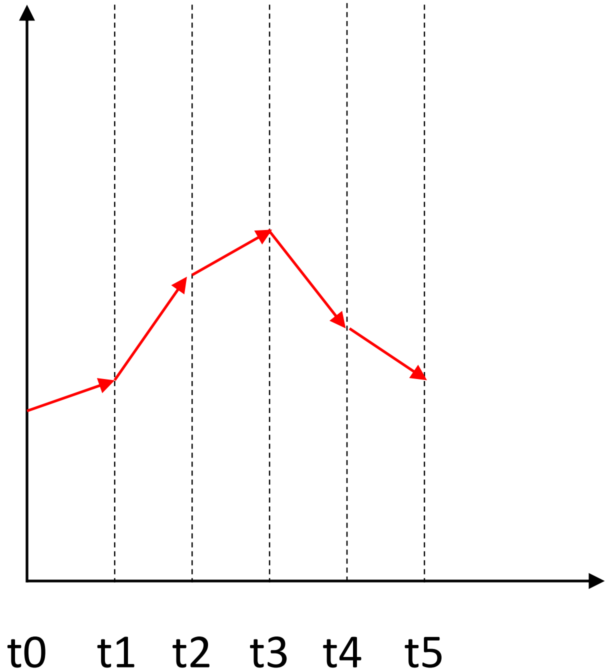
Converting the differential equation to discrete subtraction

$$\left.\frac{df(x)}{dx}\right|_x \Rightarrow \frac{f(x+\Delta x/2)-f(x-\Delta x/2)]}{\Delta x} \qquad \left.\frac{d^2 f(x)}{dx^2}\right|_x \Rightarrow \frac{f(x+\Delta x)-2f(x)+f(x-\Delta x)]}{\Delta x^2}$$

How to model a continuous system

Using extrapolation to estimate the evolution





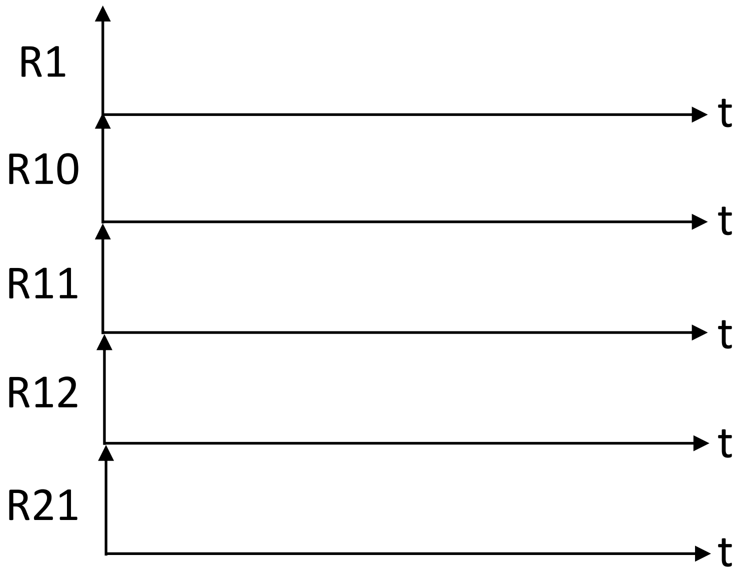
Homework

a) Try to analysis the unit of "k'/ρ" in the equation below

$$\frac{\partial^2}{\partial t^2} R(x;t) = \frac{k'}{\rho} \frac{\partial^2}{\partial x^2} R(x;t)$$

b) For a system with many coupled oscillators (example in page 5) but with N = 21; M_i = 2.5 Kg (except i = 11); K = 1 N/m; R1 = 2 m, R2 =V1=V2= 0 at t = 0, if M_{11} = 5.0, two ends are free

Try to plot a figure shown below



c) For M_{11} = 1.0,

Try to plot the same figure again

(Don't forget to attach you code)