

# Programming Language: MatLab

1<sup>st</sup> Semester 2015

Chong-Wai

W15, 23<sup>rd</sup> Dec

# Content

- Format and requirements for the final report
- Topic for final report
- A review of Euler method
- Runge–Kutta method
- Comparison of Euler method and Runge-Kutta method

# Format and requirements for the final report

What should be included in your final project?

- Title, Name & student ID, Group number, Date (1<sup>st</sup> page)
- Abstract (2<sup>nd</sup> page)
- Background
- Method/Algorithm
- Result(s)
- Conclusion

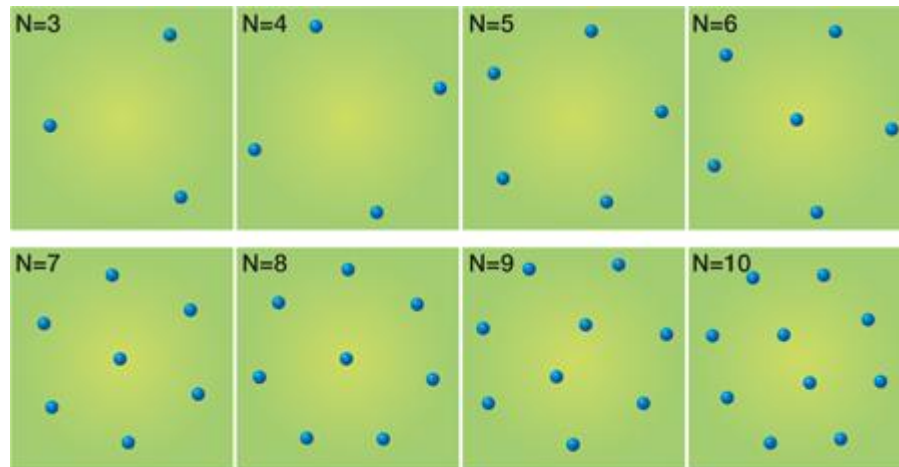
# Format and requirements for the final report

## Requirements:

- Select a topic you are interested, or you can prepare a topic you want
- Please submit your file **before 23:59, 10<sup>th</sup> Jan**
- Practice your presentation in advance
- Do not exceed **6 min** (over 7 min you will have a penalty in your score)
- Each member is required to present at least 2 min
- Each group share one file

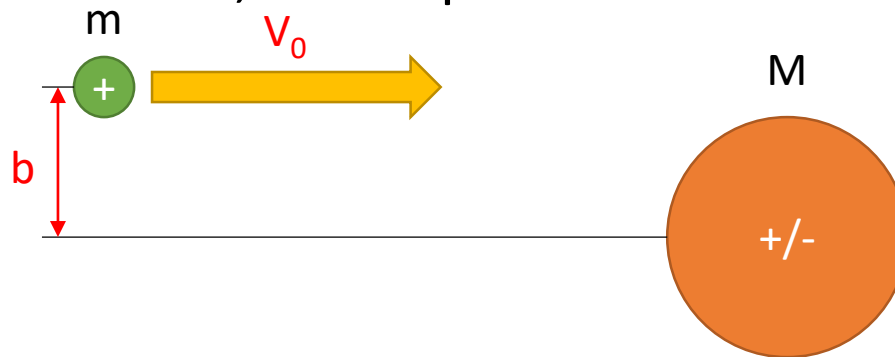
# Topic 1

- Consider  $N$  ( $2 \sim 40$ ) charged particles are confined in a 2D parabolic potential
- The 2D potential is given as:  $U(X,Y) = k(X^2+Y^2)$
- Capture the picture of their equilibrium positions
- Hints: Wigner crystal



## Topic 2

- Consider there are two charged particle with mass “ $m$ ” and “ $M$ ”, respectively.
- $m$  has a initial speed  $V_0$  move along X direction (as the figure below), and  $M \gg m$ , the displacement of  $M$  is negligible



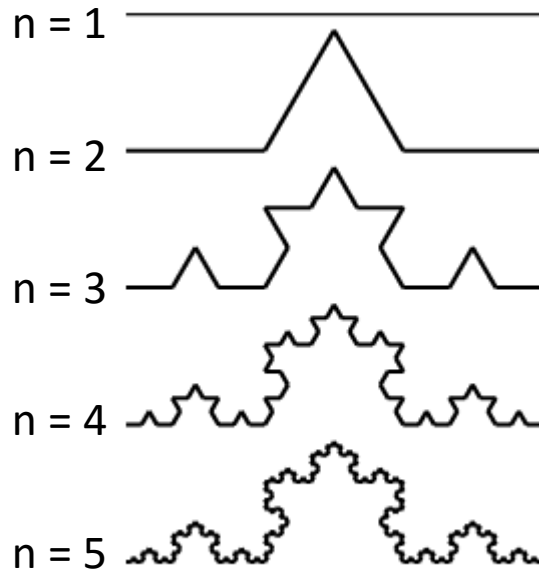
- There is another factor  $b$  (impact parameter), which describe the initial y position
- Try to find out the characteristics of different  $b$  and  $V_0$  for  $M$  with positive and negative charge.
- Hint: Scattering

# Topic 3

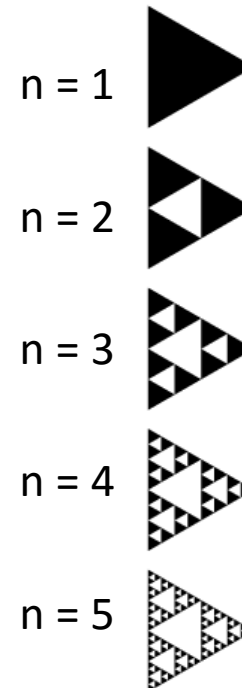
- Consider a ultra fine geometric sets (fractal) as below:
- Plotting the sets to for  $n = 1 \sim 10$
- Try to calculate the length of set 1 vs  $n$ , and area of set 2 vs  $n$

- Hint: Fractal

**Set 1**



**Set 2**



# Topic 4

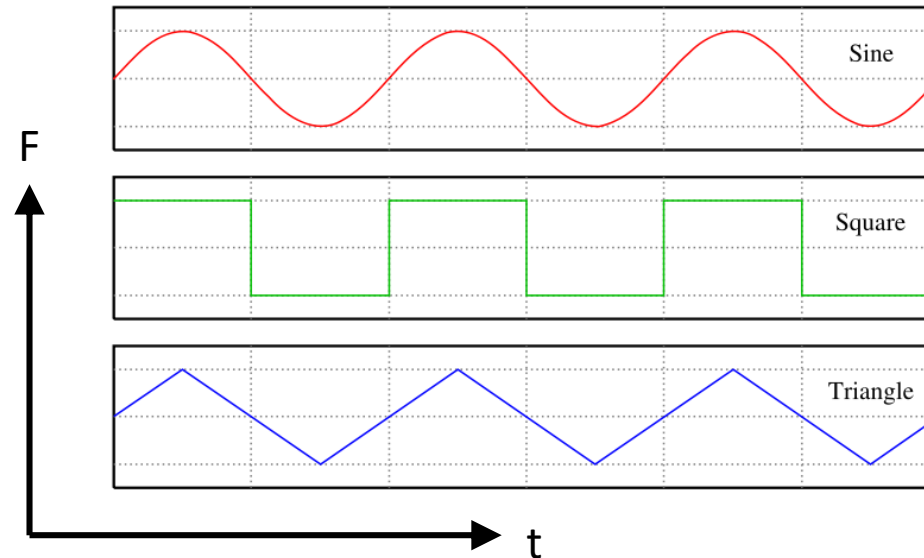
- Try to simulate the motion of **Solar System** (8 Planets) by using gravitation force
- Find out the data (Mass, speed, phase, and position)
- Plotting their trajectories and prepare a movie





# Topic 5

- Forced oscillator
- Try to simulate the motion of a damped (under damp) oscillator with external forces with different frequency and wave forms
  - i) sine
  - ii) square
  - iii) triangular



# Topic 6

- If you throw 15 dices, the possible point are ranged from 15 to 90
- Try to compute the relative probability (number of configuration) of each point  
 $P_{15}:P_{16}:P_{17}: \dots :P_{90},$
- Plotting the histogram
- Hint :  $P_{15}:P_{16} = 1:6$



# Topic 7 and X

- Using your creativity to prepare a GUI App/game
- Open for any interesting topic you want

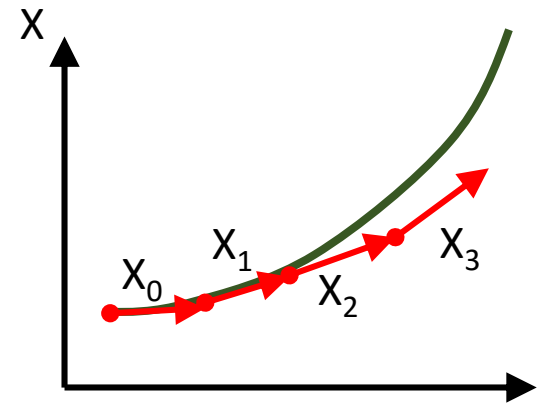
# A review of Euler method

For a equation of motion:

$$\dot{X} = f(X;t)$$

## Euler method

$$X_{n+1} = X_n + f(X;t)\Delta t$$



## Algorithm:

$$X_1 = X_0 + f(X_0;t)\Delta t$$

$$X_2 = X_1 + f(X_1;t)\Delta t$$

$$X_3 = X_2 + f(X_2;t)\Delta t$$

$\vdots$

# Runge–Kutta method

For a equation of motion:

$$\dot{X} = f(X;t)$$

Runge-Kutta method (RK4):

$$X_{n+1} = X_n + \frac{\Delta t}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

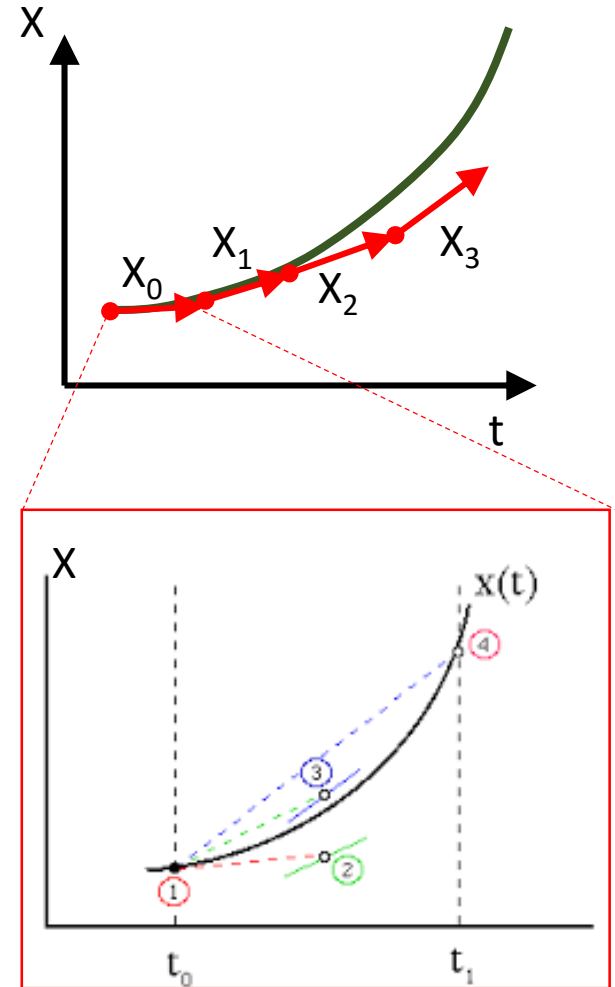
where

$$k_1 = f(X_n; t)$$

$$k_2 = f\left(X_n + \frac{\Delta t}{2} k_1; t + \frac{\Delta t}{2}\right)$$

$$k_3 = f\left(X_n + \frac{\Delta t}{2} k_2; t + \frac{\Delta t}{2}\right)$$

$$k_4 = f(X_n + k_3 \Delta t; t + \Delta t)$$



# Comparison

Example: Simple Harmonic Oscillator 
$$\begin{cases} \dot{X} = V \\ \dot{V} = -(k/m)X = -cX \end{cases}$$

Euler method:

$$\begin{aligned} X(i+1) &= X(i) + V(i) * \Delta t \\ V(i+1) &= V(i) - c * X(i) * \Delta t \end{aligned}$$

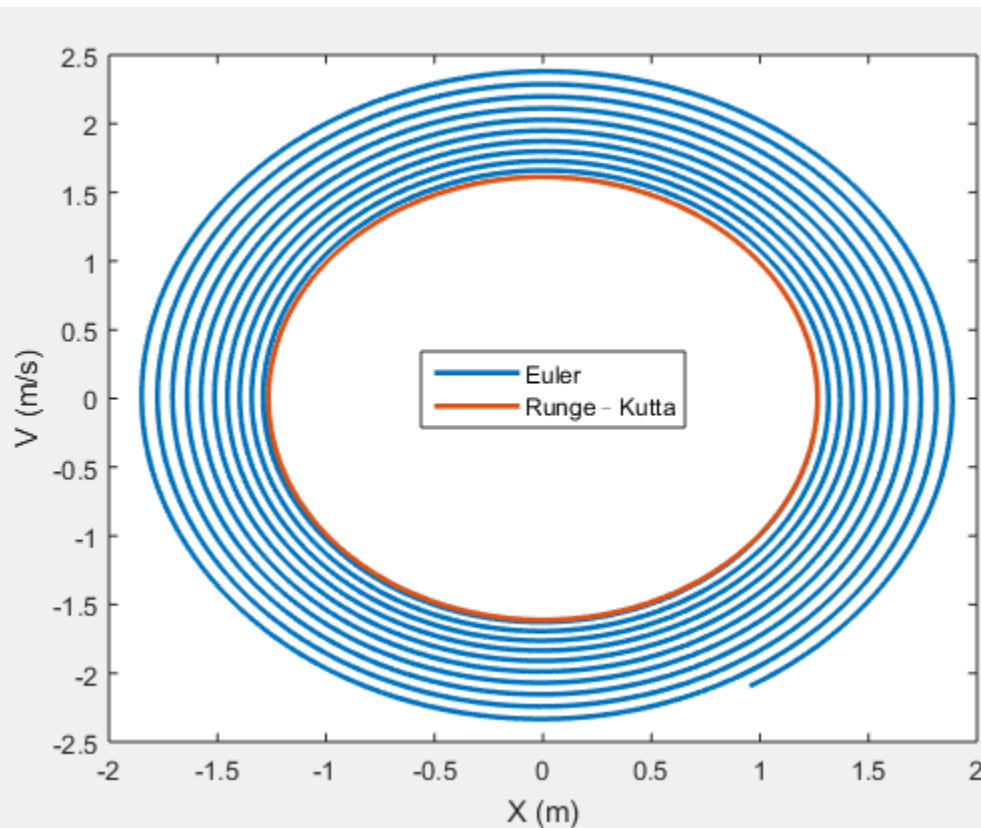
Runge-Kutta method (RK4):

$$\begin{aligned} K_{x1} &= V(i) \\ K_{v1} &= -c * X(i) \\ K_{x2} &= X(i) + K_{x1} * \Delta t / 2 \\ K_{v2} &= V(i) + K_{v1} * \Delta t / 2 \\ K_{x3} &= X(i) + K_{x2} * \Delta t / 2 \\ K_{v3} &= V(i) + K_{v2} * \Delta t / 2 \\ K_{x4} &= X(i) + K_{x3} * \Delta t \\ K_{v4} &= V(i) + K_{v3} * \Delta t \end{aligned}$$

$$\begin{aligned} X(i+1) &= X(i) + (K_{x1} + 2 K_{x2} + 2 K_{x3} + K_{x4}) * \Delta t / 6 \\ V(i+1) &= V(i) + (K_{v1} + 2 K_{v2} + 2 K_{v3} + K_{v4}) * \Delta t / 6 \end{aligned}$$

# Comparison

Example: Simple Harmonic Oscillator  $\begin{cases} \dot{X} = V \\ \dot{V} = -(k/m)X = -cX \end{cases}$



```
M = 2.0;
K = 3.25;
Dur = 50.0;
dt = 0.01;
dt_RK = dt*4; % 4X time step for RK4

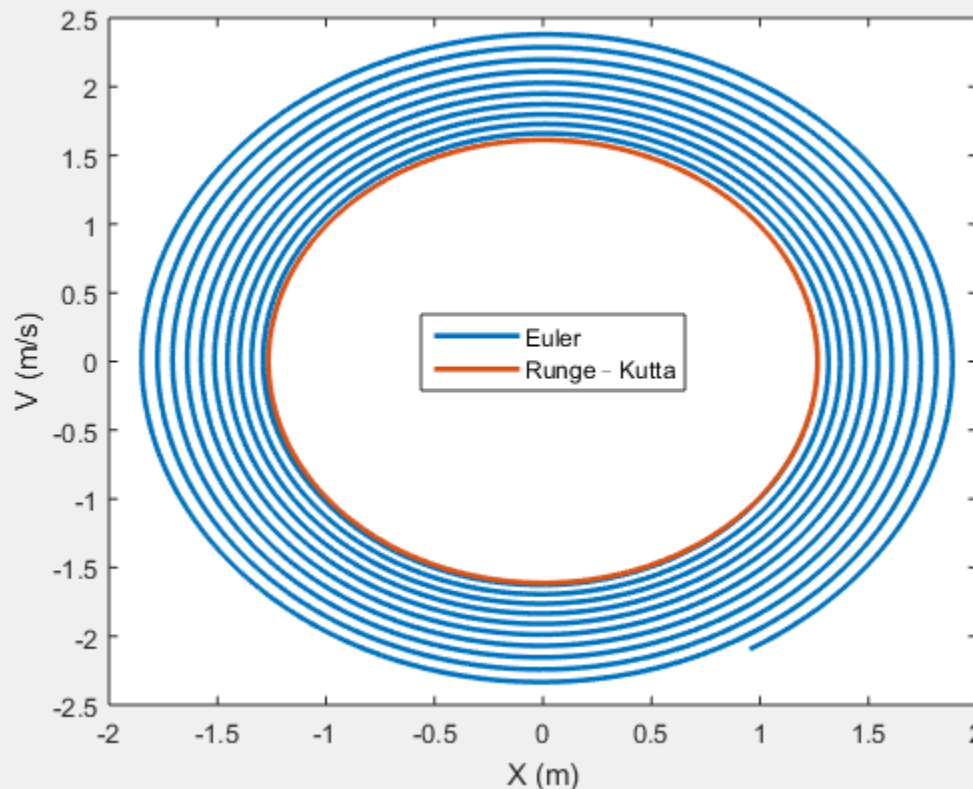
N_step = ceil(Dur/dt);
N_step_RK = ceil(Dur/dt_RK);

X_E = zeros(N_step,1);
V_E = zeros(N_step,1);
X_RK = zeros(N_step_RK,1);
V_RK = zeros(N_step_RK,1);
KX = zeros(4,1);
KV = zeros(4,1);

X_E(1) = 1.25;
V_E(1) = -0.25;
X_RK(1) = 1.25;
V_RK(1) = -0.25;
```

# Comparison

Example: Simple Harmonic Oscillator  $\begin{cases} \dot{X} = V \\ \dot{V} = -(k/m)X = -cX \end{cases}$



```
for i = 2:N_step % Euler method
    V_E(i) = V_E(i-1) - dt*K*X_E(i-1)/M;
    X_E(i) = X_E(i-1) + dt*V_E(i-1);
    t(i) = (i-1)*dt;
end

for i = 2:N_step_RK % RK4
    KX(1) = V_RK(i-1);
    KV(1) = -K/M*X_RK(i-1);
    for j = 2:3
        KX(j) = V_RK(i-1) + KV(j-1)*dt_RK/2;
        KV(j) = -K/M*(X_RK(i-1)+KX(j-1)*dt_RK/2);
    end
    KX(4) = V_RK(i-1) + KV(3)*dt_RK;
    KV(4) = -K/M*(X_RK(i-1)+KX(3)*dt_RK);

    X_RK(i) = X_RK(i-1) +
dt_RK*(KX(1)+2*KX(2)+2*KX(3)+KX(4))/6;

    V_RK(i) = V_RK(i-1) +
dt_RK*(KV(1)+2*KV(2)+2*KV(3)+KV(4))/6;
end
```