

# Accuracy of Mathematical Functions in Julia

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Workshop on Approximate Computing in Numerical Linear Algebra

Sorbonne University, Paris, France

9 October, 2025



# IEEE 754: correct rounding

## correct rounding (IEEE 754-2019 Clause 2.1)

*“[...] method of converting an infinitely precise result to a floating-point number, as determined by the applicable rounding direction.”*

Correct rounding assures implementations have bit-wise equivalent outputs for each input.

IEEE 754-2019 Clause 9.2 “Additional mathematical operations”:

*A conforming operation shall return results correctly rounded for the applicable rounding direction for all operands in its domain.*

## Rounding

Rounding itself is easy—it is the approximation of a function that causes difficulties.

# IEEE 754: required operations—correct rounding applies

- Arith:  $+$ ,  $-$ ,  $\times$ ,  $\div$ ,  $\sqrt{x}$ ,  $ab + c$  (FMA)
- Round to integer
- Convert to/from int
- Convert formats
- Convert FP to decimal char. sequenc.

## Building blocks

Expressions computed by correctly rounded operations are not correctly rounded—mathematical functions cannot be computed naively in the working prec.

# IEEE 754: mathematical functions (recommended)

$\underline{e^x}$ ,  $\underline{e^x - 1}$  (expm1),  $\underline{2^x}$ ,  $\underline{2^x - 1}$  (exp2m1),  $\underline{10^x}$ ,  $\underline{10^x - 1}$  (exp10m1),  
 $\underline{\log_e(x)}$ ,  $\underline{\log_2(x)}$ ,  $\underline{\log_{10}(x)}$ ,  $\underline{\log_e(1+x)}$  (logp1),  $\underline{\log_2(1+x)}$  (log2p1),  
 $\underline{\log_{10}(1+x)}$  (log10p1),  
 $\underline{\sqrt{x^2 + y^2}}$  (hypot),  $\underline{1/\sqrt{x}}$ ,  $\underline{(1+x)^n}$  (compound),  $\underline{x^{1/n}}$  (rootn),  $\underline{x^n}$ ,  
 $\underline{x^y}$ ,  
 $\underline{\sin(x)}$ ,  $\underline{\cos(x)}$ ,  $\underline{\tan(x)}$ ,  $\underline{\sin(x\pi)}$  (sinpi),  $\underline{\cos(x\pi)}$  (cospi),  $\underline{\tan(x\pi)}$   
(tanpi),  
 $\underline{\text{asin}(x)}$ ,  $\underline{\text{acos}(x)}$ ,  $\underline{\text{atan}(x)}$ ,  $\underline{\sinh(x)}$ ,  $\underline{\cosh(x)}$ ,  $\underline{\tanh(x)}$ ,  $\underline{\text{asinh}(x)}$ ,  
 $\underline{\text{acosh}(x)}$ ,  $\underline{\text{atanh}(x)}$ .

# Correct rounding of math functions: the challenge?

## Rounding breakpoints

Inputs at which the rounding function changes values.

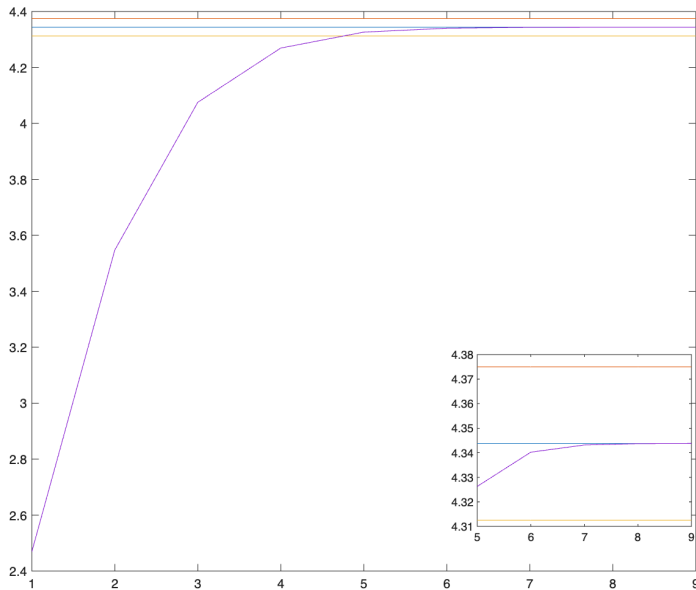
## Table Maker's Dilemma

How accurate does the approximation need to be so that we are on the right side of the *rounding breakpoints*?

Example in bfloat16 (8-bit precision):

- 1 Take Maclaurin series  $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$
- 2  $e^{1.46875} = \boxed{100.01011} 00000000011 \dots$  (due to Lefèvre & Muller)
- 3 Correct rounding (to zero):  
$$\text{RZ}\left(\boxed{100.01011} 00000000011 \dots\right) = 100.01011$$
- 4 For  $k = 8$  and  $\boxed{100.01010} 1111111110010 \dots$  (would yield incorrect rounding)

# Correct rounding of math functions: $e^{1.46875}$ in bfloat16



## Correctly Rounded Evaluation of a Function: Why, How, and at What Cost?

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The goal of this article is to give a survey on the various computational and mathematical issues and progress related to the problem of providing efficient correctly rounded elementary functions in floating-point arithmetic. We also aim at convincing the reader that a future standard for floating-point arithmetic should require the availability of a correctly rounded version of a well-chosen core set of elementary functions. We discuss the interest and feasibility of this requirement.

CCS Concepts: • **Mathematics of computing** → **Mathematical software**; **Numerical analysis**; • **Computer systems organization** → **Reliability**;

Additional Key Words and Phrases: computer arithmetic, floating-point arithmetic, elementary functions, standardization, correct rounding, table maker's dilemma

### ACM Reference Format:

Nicolas Brisebarre, Guillaume Hanrot, Jean-Michel Muller, and Paul Zimmermann. 2025. Correctly Rounded Evaluation of a Function: Why, How, and at What Cost?. *ACM Comput. Surv.* 58, 1, Article 27 (September 2025), 34 pages. <https://doi.org/10.1145/3747840>

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# Floating-point formats in Julia

Type	$e_{\min}$	$e_{\max}$	$p$	$s_{\min}$	$f_{\min}$	$f_{\max}$	Finite numbers
binary16	-14	15	11	$2^{-24}$	$2^{-14}$	65504	$2^{16} - 2^{11} = 63488$
binary32	-126	127	24	$2^{-149}$	$2^{-126}$	$\sim 3.4 \times 10^{38}$	$2^{32} - 2^{24} = 4278190080$
binary64	-1022	1023	53	$2^{-1074}$	$2^{-1022}$	$\sim 1.79 \times 10^{308}$	$2^{64} - 2^{53} \approx 1.8 \times 10^{19}$



# ULP errors

Denote a floating-point format with  $\mathbb{F}\langle e_{\min}, e_{\max}, p \rangle$  and take  $x \in \mathbb{R}$ , then

$$\text{ulp}(x) = \begin{cases} 2^{\max(e_{\min}, \lfloor \log_2 |x| \rfloor) - p + 1} & \text{if } x \neq 0, \\ 2^{e_{\min} - p + 1} & \text{otherwise.} \end{cases}$$

## Worst-case ulp errors

We report the *worst-case errors* or the *known worst-case errors* in units of ULP. For example, 2.5ULPs of error tells us that the approximation of a function  $f$  is not farther away from  $f$  than  $2.5\text{ulp}(f(x))$  for any  $x$  in the input domain of the approximation of  $f$ , where  $f(x)$  is a reference solution.

## Error measure

$E(x) := \frac{|\hat{f}(x) - f(x)|}{\text{ulp}(\text{RZ}(f(x)))}$  (Use RZ to avoid hitting next power of two where ulp doubles).  $f(x)$  computed to  $p + 20$  bits using MPFR.

# Determining input ranges of functions

## Reducing input space of functions

Given some  $\mathbb{F}\langle e_{\min}, e_{\max}, p \rangle$ , find  $[a, b]$ ,  $a, b \in \mathbb{F}\langle e_{\min}, e_{\max}, p \rangle$  for each function  $f$  such that

- the function is defined
- does not overflow ( $|f(x)| \leq f_{\max}$ ) if correctly rounded to any of the standard rounding modes
- does not converge to a constant output value, such as zero when raising  $e$  to large negative power

## Hardcoded ranges

Solving simple equations by substituting  $f_{\max}$  or a convergence value for each function/format, we found the ranges. These are hardcoded exactly in the source code.

## Example: function input ranges for binary32

Function	Input domain $I_f$
acos	$[-1.0, 1.0]$
acosh	$[1.0, f_{\max}]$
asin	$[-1.0, 1.0]$
asinh	$[-f_{\max}, f_{\max}]$
atan	$[-f_{\max}, f_{\max}]$
atanh	$[\text{nextfloat}(-1.0), \text{prevfloat}(1.0)]$
cbrt	$[-f_{\max}, f_{\max}]$
cos	$[-f_{\max}, f_{\max}]$
cosh	$[-89.415985107421875, 89.415985107421875]$
exp	$[-103.27892303466796875, 88.72283172607421875]$
exp10	$[-44.853466033935546875, 38.531841278076171875]$
exp2	$[-149, 127.99999237060546875]$
log	$[s_{\min}, f_{\max}]$
log10	$[s_{\min}, f_{\max}]$
log1p	$[\text{nextfloat}(-1), f_{\max}]$
log2	$[s_{\min}, f_{\max}]$
sin	$[-f_{\max}, f_{\max}]$
sinh	$[-89.415985107421875, 89.415985107421875]$
sqrt	$[0.0, f_{\max}]$
tan	$[-f_{\max}, f_{\max}]$
tanh	$[-9.01091289520263671875, 9.01091289520263671875]$
cospi	$[-f_{\max}, f_{\max}]$
sinpi	$[-f_{\max}, f_{\max}]$
tanpi	$[-f_{\max}, f_{\max}]$

# Prev. work (Gladman, Innocente, Mather, Zimmermann)

## Binary32 (exhaustive)

library version	GNU libc 2.42	IML 2025.2.1	AMD 5.1	Newlib 4.5.0	OpenLibm 0.8.7	Musl 1.2.5	Apple 15.3.2	LLVM 20.1.8	MSVC 2022	FreeBSD 14.3	ArmPL 25.07	CUDA 12.8	ROCm 6.2.4
acos	<b>0.500</b>	0.501	0.897	0.899	0.918	0.918	0.634	<b>0.500</b>	0.669	0.895	1.32	1.34	1.47
acosh	<b>0.500</b>	0.501	0.504	2.01	2.01	2.01	0.502	<b>0.500</b>	2.89	2.01	2.79	2.18	0.564
asin	<b>0.500</b>	0.501	0.781	0.926	0.743	0.743	0.634	<b>0.500</b>	0.861	0.672	2.41	1.36	2.54
asinh	<b>0.500</b>	0.527	0.518	1.78	1.78	1.78	0.515	<b>0.500</b>	1.99	1.78	3.57	1.78	0.573
atan	<b>0.500</b>	0.541	0.501	0.853	0.853	0.853	0.722	<b>0.500</b>	0.501	0.853	2.88	1.21	2.10
atanh	<b>0.500</b>	0.507	0.547	1.73	1.73	1.73	0.511	<b>0.500</b>	2.35	1.73	3.09	3.16	0.574
cbrt	<b>0.500</b>	0.520	0.548	3.56	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	1.83	<b>0.500</b>	1.53	1.17	1.14
cos	0.561	0.548	0.729	2.91	0.501	0.501	0.846	<b>0.500</b>	0.530	0.501	0.561	1.52	1.61
cosh	<b>0.500</b>	0.506	1.03	2.51	1.36	1.03	0.579	<b>0.500</b>	<b>0.500</b>	1.36	1.89	2.34	0.567
erf	<b>0.500</b>	0.780	0.531	0.968	0.943	0.968	0.501	<b>0.500</b>	3.99	0.890	1.93	1.04	1.51
erfc	<b>0.500</b>	0.934		63.9	3.17	3.13	0.750		6.66	3.18	1.64	4.49	3.33
exp	0.502	0.506	0.501	0.911	0.911	0.502	0.514	<b>0.500</b>	0.501	0.911	0.502	1.94	1.00
exp10	0.502	0.507	0.501	1.00		3.88	0.514	<b>0.500</b>				2.07	1.00
exp2	0.502	0.519	0.501	1.00	0.501	0.502	0.514	<b>0.500</b>	2.14	0.501	0.502	2.39	0.871
expm1	<b>0.500</b>	0.544	0.508	0.813	0.813	0.813	0.687	<b>0.500</b>	3.02	0.813	1.51	1.45	1.45
j0	9.00	<b>0.678</b>		6.18e6	3.66e6	3.66e6				3.66e6		3.78e10	7.60e7
j1	9.00	<b>1.69</b>		1.68e7	2.25e6	2.25e6				2.25e6		7.48e9	7.53e7
lgamma	<b>0.500</b>	0.510		7.50e6	7.50e6	7.50e6	0.501		2.92e5	7.50e6		1.35e7	7.50e6
log	0.818	0.509	0.577	0.888	0.888	0.818	0.511	<b>0.500</b>	0.562	0.888	0.818	0.865	1.89
log10	<b>0.500</b>	0.508	1.40	2.10	0.832	0.832	0.502	<b>0.500</b>	0.626	0.832	0.82	2.09	1.71
log1p	<b>0.500</b>	0.525	0.501	1.30	0.839	0.835	0.513	<b>0.500</b>	1.44	0.839	2.02	0.887	0.579
log2	0.752	0.507	0.766	1.65	0.865	0.752	0.502	<b>0.500</b>	2.04	0.865	0.752	0.919	1.00
sin	0.561	0.546	0.530	1.37	0.501	0.501	0.846	<b>0.500</b>	0.530	0.501	0.561	1.50	1.61
sinh	<b>0.500</b>	0.538	<b>0.500</b>	2.51	1.83	1.83	0.579	<b>0.500</b>	0.501	1.83	2.26	2.94	0.922
sqrt	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>		<b>0.500</b>	<b>0.500</b>
tan	<b>0.500</b>	0.520	0.509	3.48	0.800	0.800	0.746	<b>0.500</b>	0.502	0.800	3.30	3.10	2.33
tanh	<b>0.500</b>	0.514	<b>0.500</b>	2.19	2.19	2.19	0.817	<b>0.500</b>	1.27	2.19	2.59	1.82	1.41

# Prev. work (Gladman, Innocente, Mather, Zimmermann)

## Binary64 (known worst cases)

library version	GNU libc 2.42	IML 2025.2.1	AMD 5.1	Newlib 4.5.0	OpenLibm 0.8.7	Musl 1.2.5	Apple 15.3.2	LLVM 20.1.8	MSVC 2022	FreeBSD 14.3	ArmPL 25.07	CUDA 12.8	ROCm 6.2.4
acos	<b>0.523</b>	0.531	1.36	0.930	0.930	0.930	1.06		0.934	0.930	1.52	1.53	0.772
acosh	2.25	<b>0.509</b>	1.32	2.25	2.25	2.25	2.25		3.22	2.25	2.66	2.52	0.662
asin	<b>0.516</b>	0.531	1.06	0.981	0.981	0.981	0.709		1.05	0.981	2.69	1.99	0.710
asinh	1.92	<b>0.507</b>	1.65	1.92	1.92	1.92	1.58		2.05	1.92	2.04	2.57	0.661
atan	<b>0.523</b>	0.528	0.863	0.861	0.861	0.861	0.876		0.863	0.861	2.24	1.77	1.73
atanh	1.78	<b>0.507</b>	1.04	1.81	1.81	1.80	2.01		2.50	1.81	3.00	2.50	0.664
cbrt	3.67	0.523	0.503	0.670	0.668	0.668	0.729	<b>0.500</b>	1.86	0.668	1.79	0.501	0.501
cos	0.516	0.518	0.919	0.887	0.834	0.834	0.948	<b>0.500</b>	0.897	0.834		1.52	0.798
cosh	1.93	<b>0.516</b>	1.85	2.67	1.47	1.04	0.523		1.91	1.47	1.93	1.40	0.563
erf	1.43	<b>0.773</b>	1.00	1.02	1.02	1.02	6.41		4.62	1.02	2.29	1.50	1.12
erfc	5.19	<b>0.827</b>		4.08	4.08	3.72	10.7		8.46	4.08	1.71	4.51	4.08
exp	0.511	0.530	1.01	0.949	0.949	0.511	0.521	<b>0.500</b>	1.50	0.949	0.511	0.928	0.929
exp10	0.513	0.538	1.02	0.896		4.14	0.521	<b>0.500</b>			0.510	1.11	1.11
exp2	0.511	0.535	1.05	0.896	0.751	0.511	0.521	<b>0.500</b>	2.23	0.751	0.509	0.948	0.947
expm1	0.913	0.512	0.670	0.909	0.909	0.909	0.706	<b>0.500</b>	3.06	0.909	2.18	1.18	1.91
j0	4.51e14	<b>0.600</b>		9.01e15	4.51e14	4.51e14	3.83e14		1.88e26	4.51e14		3.08e20	1.25e13
j1	4.47e14	<b>0.615</b>		9.01e15	1.10e15	1.10e15	1.10e15		3.85e26	1.10e15		1.73e21	4.80e13
lgamma	11.1	<b>0.515</b>		4.45e15	4.45e15	4.45e15	2.33e16		5.10e13	4.45e15		5.11e15	4.45e15
log	0.520	0.518	0.611	0.946	0.946	0.520	0.508	<b>0.500</b>	0.577	0.946	0.520	0.564	0.663
log10	1.62	0.532	1.09	2.08	0.814	0.814	0.514	<b>0.500</b>	0.633	0.814	1.62	1.43	0.785
log1p	0.899	0.521	0.636	0.896	0.896	0.900	0.667	<b>0.500</b>	1.44	0.896	1.74	1.50	1.00
log2	0.548	0.509	1.72	2.06	0.921	0.555	0.515	<b>0.500</b>	0.812	0.921	0.554	1.31	0.734
sin	0.516	0.518	0.895	0.888	0.831	0.831	0.944	<b>0.500</b>	0.799	0.831		1.52	0.800
sinh	1.93	<b>0.521</b>	1.49	2.67	1.88	1.88	0.539		1.51	1.88	2.59	1.51	0.868
sqrt	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>	<b>0.500</b>		<b>0.500</b>	<b>0.500</b>
tan	0.619	0.550	1.38	1.02	1.02	1.02	3.53	<b>0.500</b>	1.32	1.02		2.09	1.30
tanh	2.21	<b>0.556</b>	1.40	2.22	2.22	2.22	0.613		1.44	2.22	2.76	1.48	0.866

# Covering input ranges of functions

## How many inputs to test

We can cover all inputs for all functions easily for binary16 and binary32 (univariate). For the cases where it is not feasible, we need a *search strategy*.

## Determining $N$ the number of tests (per thread)

Given an input domain in  $\mathbb{F}\langle e_{\min}, e_{\max}, p \rangle$  of a function  $f$  for a particular floating-point format,  $I_f$ , form  $I \subset I_f$  according to one of the strategies below. Take  $N$  members of  $I_f$  to compile  $I = \{x_1, \dots, x_N\}$ .

- Seconds:  $N = \lfloor \frac{10^9}{t} \rfloor$ .
- Minutes:  $N = \lfloor \frac{60 \times 10^9}{t} \rfloor$ .
- Hours:  $N = \lfloor \frac{3600 \times 10^9}{t} \rfloor$ .
- Days:  $N = \lfloor \frac{24 \times 3600 \times 10^9}{t} \rfloor$ .
- Exhaustive:  $I = I_f$ ,  $N = \text{length}_{\mathbb{F}\langle e_{\min}, e_{\max}, p \rangle}(I_f)$ .

# Testbench configuration: JSON interface

```
{  
  "test-binary16RN-exhaustive-nofastmath" : {  
    "format" : "binary16",  
    "rounding" : "RN",  
    "fastmath" : 0,  
    "search" : "exhaustive"  
  },  
  "test-binary32RN-exhaustive-nofastmath" : {  
    "format" : "binary32",  
    "rounding" : "RN",  
    "fastmath" : 0,  
    "search" : "exhaustive"  
  },  
  "test-binary64RN-hours-nofastmath" : {  
    "format" : "binary64",  
    "rounding" : "RN",  
    "fastmath" : 0,  
    "search" : "hours"  
  }  
}
```

Run: `julia -t <T> MathBenchmark.jl`

# Results: system specification

University of Leeds AIRE machine.

Two AMD EPYC 9634 84-core CPUs running at 2.2GHz, overall providing 168 cores. We have used Julia 1.11.6 (at the time of writing, latest is 1.11.7 (Sep. 2025)).

2 days of runtime, mainly testing binary64.



# Results: binary16 (exhaustive)

Function	ULPs	Input	Output	MPFR	Tests
exp2	0.50001	0.0007042885	1	1.0004882943	39,424
atan	0.50003	-0.0283966064	-0.0283813477	-0.0283889774	63,487
sqrt	0.49994	0.0002440214	0.0156173706	0.0156211848	31,744
sinh	0.49998	-7.34375	-773	-773.2499918938	37,833
tan	0.50002	-0.0357666016	-0.0357666016	-0.0357818608	63,487
asinh	0.50007	-258.5	-6.25	-6.2480466142	63,487
asin	0.49993	-0.1364746094	-0.1368408203	-0.1369018472	30,721
cospi	0.49994	-0.0070343018	0.9995117188	0.9997558291	63,487
exp10	0.50024	-3.763671875	0.0001723766	0.000172317	35,850
acosh	0.50005	1.22265625	0.6552734375	0.6555176014	16,384
log10	0.50006	2,976	3.47265625	3.4736329261	31,743
sinpi	0.49989	-0.0000471473	-0.0001480579	-0.0001481175	63,487
exp	0.50003	0.0226898193	1.0234375	1.0229491908	38,324
cosh	0.50004	-0.03125	1	1.0004883213	37,833
cbirt	0.49998	-10,888	-22.171875	-22.1640627384	63,487
log	0.50004	0.0053405762	-5.234375	-5.2324217334	31,743
atanh	0.49986	-0.0357666016	-0.0357971191	-0.0357818647	30,719
log1p	0.50001	-0.0058479309	-0.0058670044	-0.005865097	47,103
log2	0.49991	0.2834472656	-1.8193359375	-1.8188477429	31,743
tanpi	0.50006	-0.0001212358	-0.0003809929	-0.0003808737	63,487
acos	0.50002	-0.556640625	2.16015625	2.1611328535	30,721
tanh	0.49994	-0.0283966064	-0.0283813477	-0.0283889762	35,075
cos	0.50001	-0.0584716797	0.998046875	0.9982910184	63,487
sin	0.50004	-300	1	0.9997558398	63,487

# Results: binary32 (exhaustive)

Function	ULPs	Input	Output	Tests
exp2	0.87476	-126.42215	$8.77285 \cdot 10^{-39}$	2,249,523,202
atan	0.85211	-0.6922	-0.60547	4,278,190,081
sqrt	0.5	$8.8959 \cdot 10^{-39}$	$9.43181 \cdot 10^{-20}$	2,139,095,040
sinh	2.41409	-89.37019	$-3.25051 \cdot 10^{38}$	2,238,032,378
tan	0.79986	$-6.46722 \cdot 10^{10}$	-7.99478	4,278,190,081
asinh	1.52777	-0.12024	-0.11996	4,278,190,081
asin	0.72989	-0.50073	-0.52445	2,130,706,435
cospi	0.50007	-0.42245	0.24123	4,278,190,080
exp10	1.04779	4.08913	12,278.02344	2,219,674,257
acosh	1.99449	1.00775	0.1244	1,073,741,826
log10	0.58669	1.06589	0.02771	2,139,095,041
sinpi	0.50007	-0.08006	-0.24886	4,278,190,080
exp	0.89347	26.68997	$3.90217 \cdot 10^{11}$	2,239,758,569
cosh	2.41409	-89.37019	$3.25051 \cdot 10^{38}$	2,238,032,379
cbirt	0.5	$-3.33621 \cdot 10^{38}$	$-6.93561 \cdot 10^{12}$	4,278,190,081
log	0.55072	1.06556	0.0635	2,139,095,041
atanh	1.94114	$-7.79411 \cdot 10^{-3}$	$-7.79427 \cdot 10^{-3}$	2,130,706,433
log1p	0.56355	0.082	0.07881	3,204,448,257
log2	0.57073	1.08145	0.11297	2,139,095,041
tanpi	0.50008	-0.24998	-0.99986	4,278,190,080
acos	0.91792	-0.50042	2.09488	2,130,706,435
tanh	1.36637	-0.12551	-0.12486	2,183,158,121
cos	0.5009	$-1.20222 \cdot 10^{32}$	0.89931	4,278,190,081
sin	0.5009	$-1.02122 \cdot 10^{35}$	-0.97786	4,278,190,081

# Results: binary64 (one hour per function, 168 cores)

Function	ULPs	Input	Output	Tests
exp2	0.75875	$-1,022.03188$	$2.17645 \cdot 10^{-308}$	13,775,094,148
atan	0.85251	0.69962	0.61047	18,443,002,540
sqrt	0.5	$2.91022 \cdot 10^{-309}$	$5.39465 \cdot 10^{-155}$	23,709,682,418
sinh	1.92221	$-710.47184$	$-1.79048 \cdot 10^{308}$	23,702,916,724
tan	1.04147	$3.02146 \cdot 10^{88}$	$-0.79934$	18,693,623,932
asinh	1.55278	0.52074	0.49969	15,313,993,634
asin	0.88156	$-0.5023$	$-0.52625$	28,884,011,020
cospi	0.90973	$-1.2756 \cdot 10^7$	0.72012	27,455,885,860
exp10	0.75859	$-307.70809$	$1.95843 \cdot 10^{-308}$	24,004,809,868
acosh	2.01525	1.00699	0.11819	21,778,279,324
log10	0.62611	1.06618	0.02783	17,515,452,364
sinpi	0.90815	$-8.78737 \cdot 10^6$	0.71028	23,496,667,156
exp	0.75758	$-708.6358$	$1.75138 \cdot 10^{-308}$	32,446,565,428
cosh	1.92221	$-710.47184$	$1.79048 \cdot 10^{308}$	34,457,676,796
cbrt	0.66707	$-1.50513 \cdot 10^{-160}$	$-5.31934 \cdot 10^{-54}$	35,003,831,524
log	0.55516	1.08195	0.07877	22,568,962,084
atanh	1.98804	$1.9499 \cdot 10^{-3}$	$1.9499 \cdot 10^{-3}$	31,533,156,148
log1p	0.56563	0.07422	0.0716	24,852,795,412
log2	0.59812	1.08182	0.11346	18,707,470,156
tanpi	2.4518	$-1.55527 \cdot 10^7$	$-0.98508$	26,232,941,620
acos	0.9007	$-0.52243$	2.1205	26,590,058,380
tanh	1.5873	0.50202	0.4637	35,662,764,484
cos	0.82527	$-1.1852 \cdot 10^{280}$	0.72064	26,920,755,796
sin	0.82936	$6.86426 \cdot 10^{30}$	0.71468	24,285,228,748


# Conclusions

- We have developed an easy-to-run testsuite for Julia.
- We find that most functions are not correctly rounded, incl. binary16.
- Increases awareness of incorrectly rounded math. func.
- Future work: bivariate functions, different search strategy for binary64, bigger proportion of input space.

## Preprint available

M. Mikaitis and T. Rizyal. *Accuracy of Mathematical Functions in Julia*.

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Slides at <http://mmikaitis.github.io/talks>

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