```
> restart:
  with(DETools):
  #1
  ecdif:=diff(x(t),t)-k*x(t)=0;
  cond:=x(0)=100;
```

$$ecdif := \frac{d}{dt} x(t) - k\,x(t) = 0$$

$$cond := x(0) = 100 \tag{1}$$

```
> #a
  sol:=dsolve({ecdif,cond},x(t));
```

$$sol := x(t) = 100\,e^{kt} \tag{2}$$

```
> #b
  sol_unapplied:=unapply(rhs(sol),t,k);
```

$$sol\_unapplied := (t,k) \rightarrow 100\,e^{kt} \tag{3}$$

```
> k:=solve(sol_unapplied(50,k)=200);
```

$$k := \frac{1}{50}\,\ln(2) \tag{4}$$

```
> evalf(k);
```

$$0.01386294361 \tag{5}$$

```
> restart:
  with(DETools):
  #2
  ecdif:=2*diff(y(x),x,x)+8*y(x)=2*(sin(x)+cos(x));
```

$$ecdif := 2\left(\frac{d^2}{dx^2}\,y(x)\right) + 8\,y(x) = 2\,\sin(x) + 2\,\cos(x) \tag{6}$$

```
> #a
  dsolve(ecdif,y(x));
```

$$y(x) = \sin(2\,x)\,\_C2 + \cos(2\,x)\,\_C1 + \frac{1}{3}\,\sin(x) + \frac{1}{3}\,\cos(x) \tag{7}$$

```
> #b
  cond:=y(0)=Pi/2,D(y)(0)=Pi/2;
```

$$cond := y(0) = \frac{1}{2}\,\pi, \mathrm{D}(y)(0) = \frac{1}{2}\,\pi \tag{8}$$
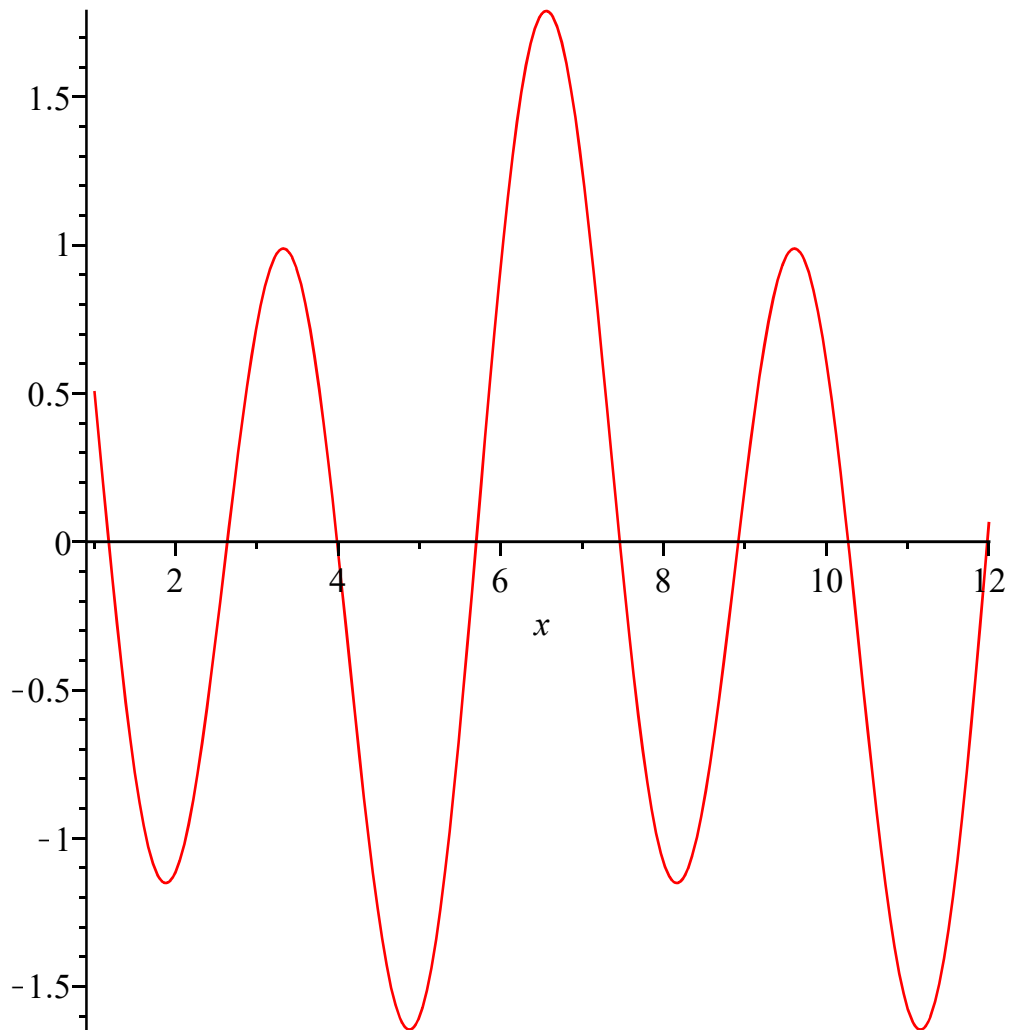
```
> sol:=dsolve({ecdif,cond},{y(x)});
```

$$sol := y(x) = \sin(2\,x)\left(\frac{1}{4}\,\pi - \frac{1}{6}\right) + \cos(2\,x)\left(\frac{1}{2}\,\pi - \frac{1}{3}\right) + \frac{1}{3}\,\sin(x) + \frac{1}{3}\,\cos(x) \tag{9}$$

```
> sol_plottable:=unapply(rhs(sol),x);
  with(plots):
  plot(sol_plottable(x),x=1..12);
```

$$sol\_plottable := x \rightarrow \sin(2\,x)\left(\frac{1}{4}\,\pi - \frac{1}{6}\right) + \cos(2\,x)\left(\frac{1}{2}\,\pi - \frac{1}{3}\right) + \frac{1}{3}\,\sin(x) + \frac{1}{3}\,\cos(x)$$

```
> restart:
  with(DETools):
  #3
  f:=x->(x+1)*x*(1-x);
  ecdif:=diff(x(t),t)=f(x(t));
```

$$f := x \rightarrow (x+1)\, x\, (1-x)$$

$$ecdif := \frac{\mathrm{d}}{\mathrm{d}t}\, x(t) = (x(t)+1)\, x(t)\, (1-x(t))$$

(10)

```
> #a
  pe:=solve(f(x)=0);
```

$$pe := 0,\, 1,\, -1$$

(11)

```
> D(f)(0);#instabil
```

$$1$$

(12)

```
> D(f)(1);#local as stabil
```

$$-2$$

(13)

```
> D(f)(-1);#local as stabil
```

$$-2$$

(14)

```
> #b
  DEplot(ecdif,x(t),t=0..2,[[x(0)=-2],[x(0)=-1/2],[x(0)=0],[x(0)=
  1/2],[x(0)=1],[x(0)=3/2],[x(0)=2]]);
```
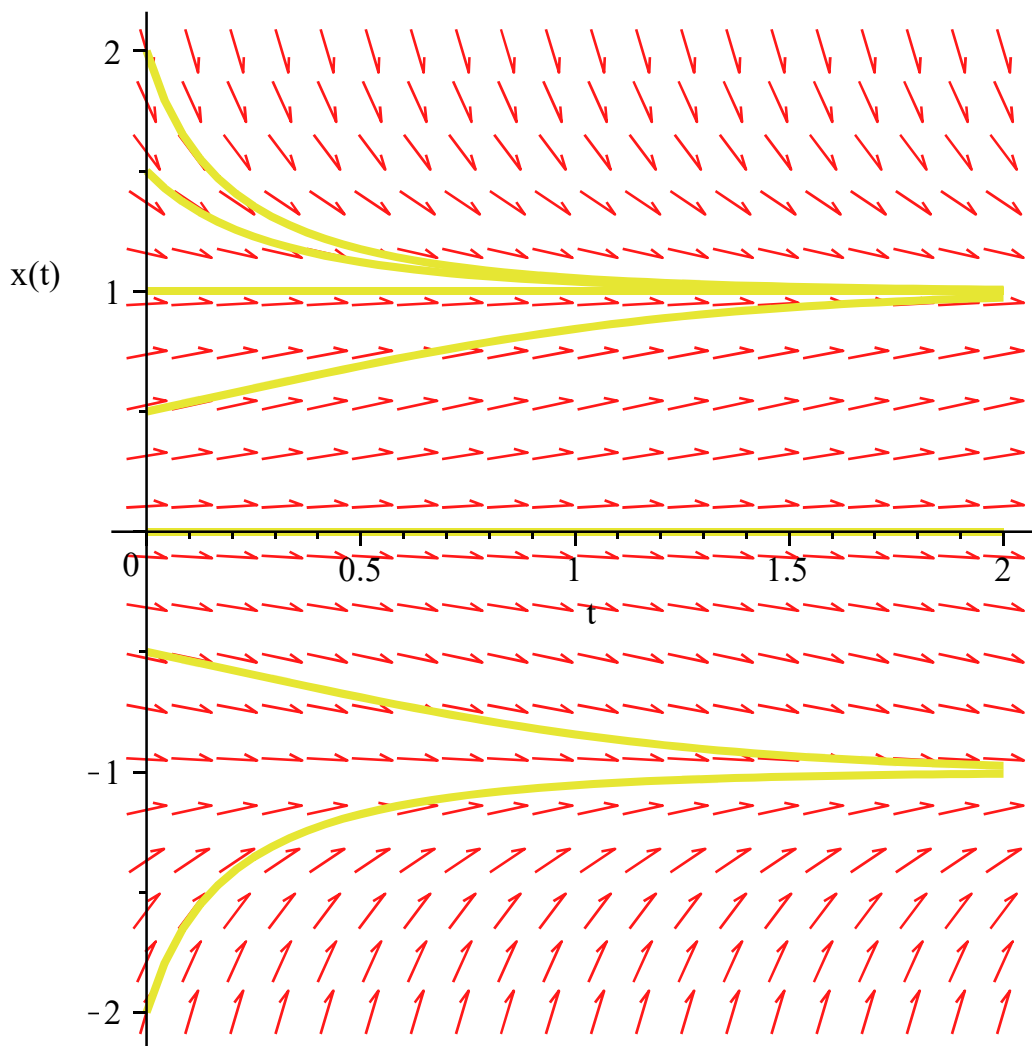
```
> restart:
  with(DETools):
  #4
  f1:=(x,y)->y^2-8*x;
  f2:=(x,y)->x^2-y;
```

$$f1 := (x, y) \rightarrow y^2 - 8\,x$$
$$f2 := (x, y) \rightarrow x^2 - y \tag{15}$$

```
> ecdif1:=diff(x(t),t)=f1(x(t),y(t));
  ecdif2:=diff(y(t),t)=f2(x(t),y(t));
```

$$ecdif1 := \frac{\mathrm{d}}{\mathrm{d}t}\,x(t) = y(t)^2 - 8\,x(t)$$

$$ecdif2 := \frac{\mathrm{d}}{\mathrm{d}t}\,y(t) = x(t)^2 - y(t) \tag{16}$$

```
> #a
  pe:=solve({f1(x,y)=0,f2(x,y)=0},{x,y});
```

$$pe := \{x = 0, y = 0\}, \{x = 2, y = 4\}, \{x = 2\,RootOf(\_Z^2 + \_Z + 1), y = -4 - 4\,RootOf(\_Z^2 + \_Z \tag{17}$$
$$+ 1)\}$$

```
> #b
  with(linalg):
```

```
J:=jacobian([f1(x,y),f2(x,y)],[x,y]);
```

$$J := \begin{bmatrix} -8 & 2\,y \\ 2\,x & -1 \end{bmatrix}$$

(18)

```
> A1:=subs(pe[1,1],pe[1,2],eval(J));
```

$$A1 := \begin{bmatrix} -8 & 0 \\ 0 & -1 \end{bmatrix}$$

(19)

```
> eigenvals(A1);
```

$$-8, -1$$

(20)

```
> A2:=subs(pe[2,1],pe[2,2],eval(J));
```

$$A2 := \begin{bmatrix} -8 & 8 \\ 4 & -1 \end{bmatrix}$$

(21)

```
> eigenvals(A2);
```

$$-\frac{9}{2} + \frac{1}{2}\,\sqrt{177}, \ -\frac{9}{2} - \frac{1}{2}\,\sqrt{177}$$

(22)