THEORY OF ρ-ω INTERFERENCE IN $\pi^+\pi^-$ PRODUCTION *

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The Coleman-Glashow model for ρ - ω mass mixing permits unambiguous analysis of recent $\pi^+\pi^-$ production data. We confirm production phase predictions of a Regge model with exchange degenerate trajectories. We explain the anomalous rho width observed in colliding beam reactions.

Experimental attempts to positively identify ρ^0 - ω interference have for several years resembled a search for a needle in a haystack. The recent observation by Goldhaber et al. [1] ‡ of a sizeable dip at the ω mass in the dipion mass distribution in $\pi^+ p \to \pi^+\pi^-\Delta^{++}$ is rather dramatic evidence for the existence and magnitude of the effect. In this letter we combine various models for rho and omega production mechanisms with the mass mixing theory of $\omega \to \pi^+\pi^-$ decay [2-5]. Our predictions for hadronic and photoproduction experiments and for $e^+e^- \to \pi^+\pi^-$ agree well with present data and are summarized in table 1. The experimental sign and magnitude of the $\omega \to \pi^+\pi^-$ amplitude agree, within a factor of two, with the symmetry breaking theory of Coleman and Glashow [3].

The mass matrix formalism to describe the 2π decays of the ρ^0 and ω has been described by many authors [2,4]. Any 2π production amplitude may be written

$$S(\pi^+\pi^-) = [A(\rho) \quad A(\omega)]\ \boldsymbol{P}(m) \begin{bmatrix} T(\rho^0 \to \pi^+\pi^-) \\ T(\omega \to \pi^+\pi^-) \end{bmatrix} \quad (1)$$

where the matrix propagator is given by

$$[\boldsymbol{P}(m)]^{-1} = m\mathbf{1} - \boldsymbol{M}\ ,$$
$$\boldsymbol{M} = \begin{bmatrix} m_\rho - \frac{1}{2}\mathrm{i}\Gamma_\rho & -\delta \\ -\delta & m_\omega - \frac{1}{2}\mathrm{i}\Gamma_\omega \end{bmatrix}\ ; \quad (2)$$

$A(\rho)$ and $A(\omega)$ are the respective vector meson production amplitudes; the T's are decay amplitudes in the absence of mass mixing, and m is the invariant mass of the dipion system.

A priori, $\delta = -\langle \rho^0 | M | \omega \rangle$ where M is the electromagnetic mass operator, is a free parameter. However, Coleman and Glashow [3] have related such matrix elements to the medium strong, SU(3) breaking, mass splitting of the well-known baryon and meson states; they predict

$$\delta = -\langle \rho^0 | M | \omega \rangle \approx \frac{(m_{\mathrm{K}^*}^2 - m_\rho^2)}{\sqrt{3} m_\rho} \frac{\langle \pi' \rangle}{\langle \eta' \rangle} \approx 2.5\ \mathrm{MeV}$$

for the value $\langle \pi' \rangle / \langle \eta' \rangle = 0.018$ determined from baryon mass splittings. The fact that δ if large, must be real is simply Watson's theorem that if the $\pi\pi$ intermediate state dominates then $\omega \to 2\pi$ must have the same phase as $\pi\pi$ elastic scattering

Neglecting the intrinsic decay amplitude $T(\omega \to \pi^+\pi^-)$, we have for the production amplitude [5]

$$S(\pi^+\pi^-) = \frac{A(\rho)\, T(\rho \to \pi^+\pi^-)}{m_\rho - m - \frac{1}{2}\mathrm{i}\Gamma_\rho} \times \times \left\{ 1 + \frac{A(\omega)}{A(\rho)} \frac{1}{m_\omega - m - \frac{1}{2}\mathrm{i}\Gamma_\omega} \right\} . \quad (3)$$

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‡ Such effects were predicted long ago, and in some detail, by Bernstein and Feinberg [2], using a mass matrix approach similar to ours. The present work differs from theirs in our attention to the production mechanism and our ability (thanks to a recent experiment) to compute the mixing parameter.

Table 1
Predictions of the interference in various reactions

Reaction	Exchange	Phase (ω/ρ)	Prediction
$\pi^+p \to \pi^+\pi^-\Delta^{++}$	π, B	+i	dip
$\pi^+n \to \pi^+\pi^-\Delta^+$	π, B	+i	dip
$\to \pi^+\pi^-p$	π, B	+i	dip
$\pi^-p \to \pi^+\pi^-n$	π, B	-i	peak
$\to \pi^+\pi^-\Delta^0$	π, B	-i	peak
$\pi^-n \to \pi^+\pi^-\Delta^-$	π, B	-i	peak
$K^-p \to \pi^+\pi^-(\Lambda, \Sigma^0)$	$K, K^*(1320)$	+1	constructive below ω, destructive above
$K^-n \to \pi^+\pi^-\Sigma^-$	or	+1	
$\overline{K^0p} \to \pi^+\pi^-\Sigma^+$	$K^*(890), K^*(1420)$	-1	destructive below ω, constructive above
$\overline{K^0n} \to \pi^+\pi^-(\Lambda, \Sigma^0)$		-1	
$e^+e^- \to \pi^+\pi^-$	γ	+1	constructive below ω, destructive above
$\gamma A \to \pi^+\pi^-A$	(π), P	+1	

We now turn to the problem of estimating the phase of $A(\omega)/A(\rho)$. As we shall indicate shortly, a simple Regge model with some experimental support implies

$$A(\omega)/A(\rho) = \pm i \text{ (real, positive number)} \quad (4)$$

for the reactions $\pi^{\pm}N \to (\rho^0, \omega)N^{(*)}$. This sign is theoretically determined to be plus for incident π^+ and minus for incident π^- Consequently we expect a dip (rise), of maximum size at the ω mass, coming from the factor

$$(1 \mp (2\delta/\Gamma_\omega)|A(\omega)/A(\rho)|)^2 ,$$

for incident $\pi^{\pm}$. Such a dip and peak have been seen in two different high statistics experiments [1, 6] ‡:

$$\pi^+p \to \pi^+\pi^-\Delta^{++} \quad \text{at 3.7 GeV}/c;\ \text{dip} \quad \text{(I)}$$

$$\pi^-p \to \pi^+\pi^-n \quad \text{at 3 to 5 GeV}/c;\ \text{peak}\ . \quad \text{(II)}$$

The present best estimate of $|A(\omega)/A(\rho)|$ for reaction (I) gives $\delta \approx 4$ MeV, in fair agreement with our crude estimate. These data confirm the sign predictions of both exchange degeneracy [see eq. (4)] and the Coleman-Glashow mass-mixing theory.

A reaction with relatively real production amplitudes is $e^+e^- \to (\rho^0, \omega)$, where universal vector meson-photon coupling suggests

$$A(\omega)/A(\rho) \approx +\tfrac{1}{3}\ .$$

We therefore deduce a modulating function in the ρ^0-peak,

$$\left| 1 + \frac{\frac{1}{3}\delta}{m_\omega - m - \frac{1}{2}i\Gamma_\omega} \right|^2 \quad (5)$$

which gives a shoulder or kink on the high side of the dipion ρ peak. We understand [9] † that such a shape is seen in the preliminary data from Orsay. We note in passing that the processes $\pi N \to l^+l^-N^{(*)}$ are also amenable to detailed analysis [11] in our picture.

Our claim for the relative phases of the ρ^0 and ω production amplitudes is justified by a simple exchange degeneracy [12] argument. Since

‡ These results are preliminary, pending understanding of possible biases. Also see, for example ref. 7. The compilation of Pišut and Roos [8] does not exhibit strong features at the omega mass. However, Roos' sample contains both π^+ and π^- data for which the predicted dip and bump will lead to a cancellation of the net effect. Again, the quoted resolution of 30 MeV is insufficiently good to see any real structure.

† The same features are seen in the DESY photoproduction data [10], and in the preliminary photoproduction data of the SLAC--LRL--U. C. Berkeley--Tufts Collaboration. We thank K. Moffeit for bringing these data to our attention. M. Gourdin (preprint, 1969) has calculated the rho-omega interference in e^+e^- scattering in terms of a vacuum polarization diagram. Although this gives the right phase for δ it drastically underestimates the $\omega \to \pi^+\pi^-$ rate.

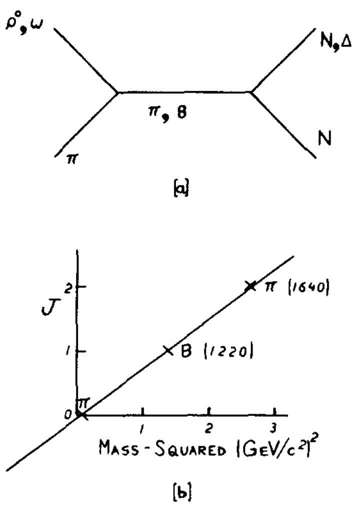

Fig. 1. a) Regge exchange diagram for the production of ρ^0 or ω in πN collisions.
b) Chew-Frautschi plot of the postulated π - B trajectory. We assume the $\pi_A(1640)$ to be the first recurrence of the pion. This agreement has yet to be established. It may well be that the π trajectory has a larger slope, and that the exact exchange degeneracy is broken.

the production reactions primarily populate the density matrix element ρ_{00}, we expect $\rho^0(\omega)$ production to be dominated by π(B) exchange near the forward direction ‡. The mechanism is indicated in fig. 1a). The meson spectrum suggests [fig. 1b] that the π and B may lie on the same trajectory. To apply exchange degeneracy, we consider the SU(3) related reaction,

$$K^+p \to K^{*0}\Delta^{++} ,$$

which also proceeds by π and B exchange. From the absence of s-channel resonances, we infer that the imaginary parts of the π and B amplitudes [the phases of which are $1 \pm \exp(-i\pi\alpha)$, respectively] must cancel. Consequently the π and B must lie on the same trajectory, and must have the same residue functions. The SU(3) rotations ‡‡ to the reaction of interest then give the result ‡‡‡, $A(\omega)/A(\rho) = -i \tan(\frac{1}{2}\pi\alpha(t))$.

We would like to note that the familiar role played by the pion and A_1 in current algebra has led to some confusion [14] about their roles in

‡ This is confirmed by detailed fits [13].

‡‡ This just involves standard SU(3) Clebsch-Gordan coefficients plus the assumption that $B \to \pi\phi = 0$. Such a relation is expected from the quark model and supported experimentally by the small cross section for ϕ production in πN collisions.

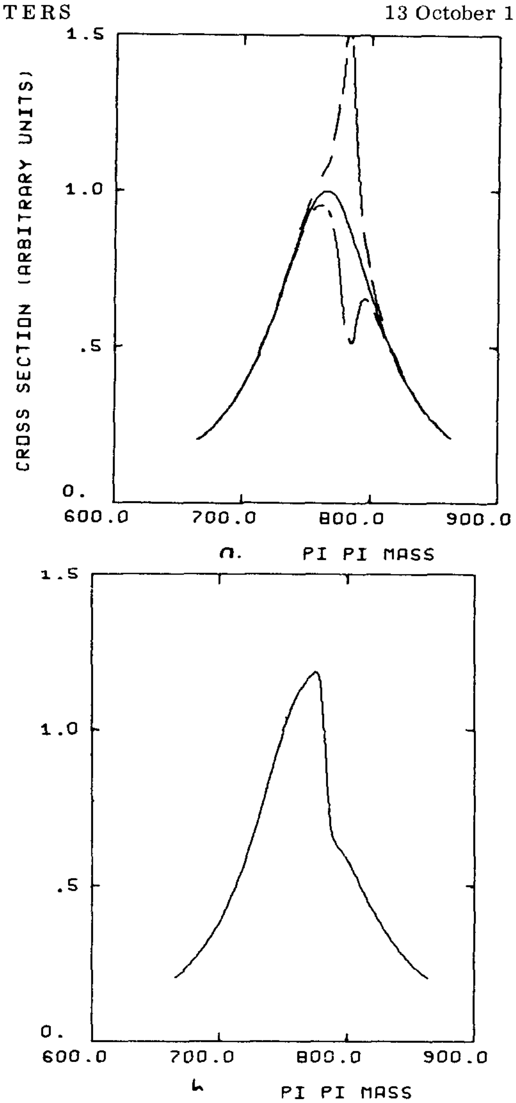

Fig. 2. a) Typical predictions for $\pi N \to \pi^+\pi^- N^{(*)}$: $\delta\,|A(\omega)/A(\rho)| = -2, 0, +2$ for constructive, no, and destructive interference.
b) Results for $e^+e^- \to \pi^+\pi^-$: $\delta = +4$. Our calculations should be compared with $|F_\pi(m_{\pi\pi})|^2$, after the kinematic dependence from the photon propagator has been removed.

‡‡‡ We realize that the simple π - B Regge pole model does not fit the data quantitatively. (See the contortions performed in ref. 13.) In particular the predicted size of the ratio $|A(\omega)/A(\rho)|^2 = \tan^2(\frac{1}{2}\pi\alpha)$ underestimates the size of omega production (using $\alpha = \alpha_\pi(t)$). However, it is easy to see that the effective α is very sensitive to cuts while the sign prediction is rather stable. A quantitative study of absorptive cut effects is in progress. Further, our predictions hold only in the intermediate energy range (as in ref. 1, for example) where the B exchange contribution dominates omega production.

Regge theory. From factorization and exchange degeneracy for the reactions $NN \to NN$, $NN \to N\Delta$, and $KN \to K^*N$, one may in fact prove that it is π and B and *not* π and A_1 that are exchange degenerate. The essential point is that π and A_1 couple to different $N\bar{N}$ spin states.

From the prediction for reaction (I), we obtain by isospin rotations five other results for pion-nucleon reactions. All of our results are summarized in table 1. Quantitative predictions may be obtained from eqs. (3) and (5), and either the theoretical or experimental estimates of the mixing parameter δ. In fig. 2 we show typical predictions for $\pi N \to \pi\pi N^{(*)}$ and for $e^+e^- \to \pi^+\pi^-$. We have modulated a pure Breit-Wigner for simplicity. In our example, the unadorned Breit-Wigner has $\Gamma_\rho = 100$. The distribution for the $e^+e^- \to \pi^+\pi^-$ reaction displays i) a rho peak shifted toward the omega mass and ii) a full-width at half-maximum of 80 MeV.

Thus we predict that colliding beam experiments should see an anomalously small rho width. The same shape is predicted for diffractive photoproduction of rho and omega [9].

For reactions with kaons incident, we can also apply exchange degeneracy to obtain the relative production phase †, $A(\omega)/A(\rho) = \pm 1$.

The high energy kaon data are consistent with the prediction of equal omega and rho production amplitudes. However, it is not sufficiently precise to resolve the rho-omega interference in the two pion decay mode and so test the relative phase of the amplitudes. The low energy data [16] contain substantial background ($\approx 75\%$) under the rho, and a clean test is impossible.

Finally it should be emphasized that one needs good resolution in addition to high statistics to see the ρ-ω interference phenomena. It is perhaps plausible that a resolution of about Γ_ω (≈ 10 MeV) is necessary, and detailed calculations confirm this [17]. In the theoretical dream of an experiment with infinite statistics we could hope to use ρ-ω interference to investigate the t-dependence of the phase of $A(\omega)/A(\rho)$. Similarly one can examine its value in different amplitudes by looking at various moments of the decaying 2π system.

While our results do not embody any revolutionary theoretical concepts, they do illustrate a pleasing union of some simple and appealing ideas. First we have the mass mixing theory of the $\omega \to \pi^+\pi^-$ amplitude itself. This kind of model was previously familiar from the superweak theory [18] of the *CP*-violating $K_2^0 \to \pi^+\pi^-$. The hypothesis of octet dominance for the electromagnetic mass splitting relates the mass mixing parameter (and thus the $\omega \to \pi^+\pi^-$ amplitude) to the $I = 1$ electromagnetic mass differences. Secondly, we find exchange degenerate Regge trajectories. This notion has received much attention lately in both πN and KN scattering [19], and appears, by virtue of our results, to be valid in vector meson production reactions as well. Thirdly, the relative sign prediction of the universal (ρ^0, ω)-photon couplings [see eq. (5)] is tested in $e^+e^- \to \pi^+\pi^-$.

† The exchange degeneracy prediction is not uniquely specified for these kaon reactions. One must also assume that both members of the exchange degenerate pair have the same F/D ratio at the baryon-antibaryon vertex. This can be established by using factorization plus exchange degeneracy in baryon-baryon reaction [15].

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References

1. G. Goldhaber, W. R. Butler, D. G. Coyne, B. H. Hall, J. N. MacNaughton and G. H. Trilling, UCRL-18894, Proc. Conf. on $\pi\pi$ and $K\pi$ interactions, Argonne National Laboratory, May, 1969, to be published.
2. J. Bernstein and G. Feinberg, Nuovo Cimento 25 (1962) 1343.
3. S. Coleman and S. Glashow, Phys. Rev. Letters 6 (1961) 423; Phys. Rev. 134 (1964) B671;
S. L. Glashow, Phys. Rev. Letters 7 (1961) 469;
S. Coleman and H. J. Schnitzer, Phys. Rev. 136 (1964) B223;
S. Coleman, S. L. Glashow, H. J. Schnitzer, R. Socolow, Proc. XII Int. Conf. on High energy physics, Dubna (1964), Vol. I, p. 785;
R. Socolow, Phys. Rev. 137 (1965) B1221.
4. S. Coleman and H. J. Schnitzer, Phys. Rev. 134 (1965) B863;
John Harte and R. G. Sachs, Phys. Rev. 135 (1965) B459.
5. Our unpublished preprint, UCRL-19241, presented at the 1969 Boulder Conference, contains a more detailed description of the calculation of the $\omega \to \pi^+\pi^-$ amplitude.
6. T. N. Rangaswamy, A. R. Clark, Bruce Cork, T. Elioff, L. T. Kerth, W. A. Wenzel (LRL, Berkeley) reported at Conf. on $\pi\pi$ and $K\pi$ Interactions, Argonne National Laboratory, 1969.
7. S. Marateck et al., Phys. Rev. Letters 21 (1969) 1613.

8. J. Pišut and M. Roos, Nucl. Phys. B6 (1968) 325.
9. J. E. Augustin et al., Lund Conference, 1969.
10. E. Lohrman, Proc. 1967 International Symposium on Electron and Photon Interactions at High Energies.
11. J. T. Donohue, University of Illinois Thesis, 1967 (unpublished);
L. Durand and Y. T. Chiu, Phys. Rev. Letters 14 (1965) 1039.
These papers do not consider the important issue of the production amplitudes.
12. R. Arnold, Phys. Rev. Letters 14 (1965) 657.
13. G. C. Fox and L. Sertorio, Phys. Rev. 176 (1968) 1739.
14. E. S. Abers and V. L. Teplitz, Phys. Rev. Letters 22 (1969) 909. Their conflict with positivity obtained for the assumption of $\pi - A_1$ exchange degeneracy does not arise for a $\pi - B$ exchange degeneracy.
15. R. H. Capps, Purdue preprint, 1969, Exchange degeneracy and SU(3).
16. S. M. Flatté et al., Phys. Rev. 145 (1050) 1966.
17. A detailed discussion of the effects of resolution will be presented in a paper in preparation.
18. L. Wolfenstein, Phys. Rev. Letters 13 (1964) 562.
19. C. Schmid, Lettere al Nuovo Cimento 1 (1969) 165.

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