S-matrix approach to the $\rho - \omega$ interference

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Amplitude of the ρ/ω interferece is constructed in K-matrix formalism. Parameters of the scattering matrix are fixed using scattering phase shift and known partial width of the ρ, and ω mesons.

I. DECAY AMPLITUDE FOR $\chi_{c1}(3872) \to J/\psi\pi^+\pi^-$

The process is described by a transition amplitude $M_{\lambda_0,\lambda_1}(m_{\pi^+\pi^-})$, with λ_0 and λ_1 being the helicity of χ_{c1} and J/ψ. The $\pi^+\pi^-$ system needs to be in P-wave with $J^{PC} = 1^{--}$ to give positive C parity to χ_{c1}. The decay amplitude reads:

$$M_{\lambda_0,\lambda_1} = h_{\lambda_0+\lambda_1,\lambda_1}\, A_{\pi\pi}\, d^1_{\lambda_0+\lambda_1,0}(\theta_{\pi\pi}) \tag{1}$$

where $A_{\pi\pi}$ is a the $\pi\pi$ production amplitude, h is the helicity coupling. The later simplifies in the LS basis since the parity and charge conjugation constraints can be enforced.

$$h_{\lambda_0+\lambda_1,\lambda_1} = \sum_{S,L} \sqrt{\frac{2L+1}{3}}\, \langle 1, \lambda_0+\lambda_1; 1, -\lambda_1 | S, \lambda_0\rangle\, \langle L0; S, \lambda_0 | 1, \lambda_0\rangle\, h_{LS}\, q^L, \tag{2}$$

$$\tag{3}$$

where we put explicit threshold factor q^L with with $q = \lambda^{1/2}(m^2_{\chi_{c1}}, m^2_{J/\psi}, m^2_{\pi\pi})$. [1] Due to the parity conservation only even waves are present.

Angular dependence vanishes once the decay in integrated over the scattering angle $\theta_{\pi\pi}$. We also neglect D-wave in the $\chi_{c1} \to J\psi\rho$ decay.

$$I_{\pi\pi} \equiv \frac{\mathrm{d}N}{\mathrm{d}m_{\pi\pi}} = pq \int \frac{\mathrm{d}\cos\theta}{2} \sum_{\lambda_0,\lambda_1} |M_{\lambda_0,\lambda_1}|^2 \tag{4}$$

$$= N\, pq\, |A_{\pi\pi}|^2. \tag{5}$$

where $p = \sqrt{s/4 - m_\pi^2}$ is a pion break-up momentum, N is the overall normalization constant.

II. TWO-CHANNELS SCATTERING

We consider coupling between two channels:

- $J^{PC} = 1^{--}$: $\pi\pi$ P-wave, and
- $J^{PC} = 1^{--}$: $\rho\pi$ P-wave.

First we factor out kinematic singularity related to the P-wave

$$A_{\pi^+\pi^-} = \hat{A}_{\pi^+\pi^-} p, \tag{6}$$

The scattering and production amplitudes are expressed through the K-matrix:

$$\hat{T} = [1 - iK\rho]^{-1}\, T \tag{7}$$

$$\hat{A} = [1 - iK\rho]^{-1}\, N. \tag{8}$$

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[1] We use the triangle function $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$.

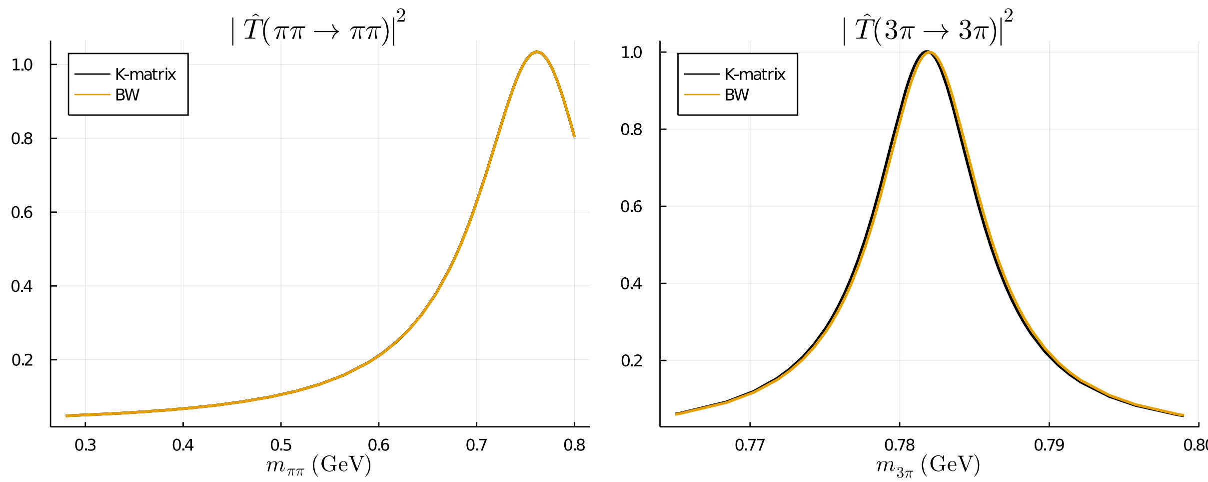

FIG. 1. Diagonal terms of the K matrix ampltude in comparison with the BW amplitde.

where ρ is a diagonal matrix of the phase space elements with the square of the break-up momentum:

$$\rho_1(s) = \frac{2p}{\sqrt{s}}\, B_1(p) \tag{9}$$

$$\rho_2(s) = \frac{1}{s}\int_{4m_\pi^2}^{(\sqrt{s}-m_\pi)^2} \frac{\mathrm{d}\sigma}{2\pi\sigma}\, \frac{\lambda^{1/2}(s,\sigma,m_\pi^2)\lambda^{1/2}(\sigma,m_\pi^2,m_\pi^2)}{(m_\rho^2-\sigma)^2+(m_\rho\Gamma_\rho)^2} B_1(p(\sigma))B_1(k(\sigma)), \tag{10}$$

with $s \equiv m_{\pi\pi}$, $k(\sigma) = \lambda^{1/2}(s,\sigma,m_\pi^2)/(2\sqrt{s})$, and $p(\sigma) = \lambda^{1/2}(\sigma,m_\pi^2,m_\pi^2)/(2\sqrt{\sigma})$. The second expression represent a convolution of the two-body phase, $\rho^+\pi^-$ P-wave with the lineshape of the ρ meson, $\rho \to \pi\pi$ P-wave. We have neglected symmetrization of pions in the decay $\omega \to \pi^+\pi^-\pi^0$.

The K-matrix contains a pole at every channel and a small non-diagonal coupling between two channels.

$$K = \frac{1}{m_1^2 - s}\begin{pmatrix} g_1^2 & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{m_2^2 - s}\begin{pmatrix} h_1^2 & h_1 h_2 \\ h_1 h_2 & h_2^2 \end{pmatrix} \tag{11}$$

The parameters of the K matrix are completely fixed by the widths of the resonances and branching fractions:

$$\begin{aligned} g_1^2 &= m_\rho \Gamma_\rho / \rho_1(m_\rho^2), \\ h_1^2 &= m_\omega \Gamma_\omega \, \mathrm{Br}(\omega \to \pi^+\pi^-)/\rho_1(m_\omega^2), \\ h_2^2 &= m_\omega \Gamma_\omega \, \mathrm{Br}(\omega \to 3\pi)/\rho_2(m_\omega^2). \end{aligned} \tag{12}$$

We note that coefficient h_1^2 is extremely small compare to g_2^2.

A general expression (Q and P vectors) for the production vector follow

$$N = K\,[\alpha_1, \alpha_2]^T + [f_1, f_2]^T, \tag{13}$$

where α_i and f_i are constants. the case $F = 0$ corresponds to the Q-vector approach.

$$\hat{A}_{\pi^+\pi^-} = \alpha_1 \hat{T}_{1,1} + \alpha_2 \hat{T}_{1,2}. \tag{14}$$

Exlicitely,

$$\hat{A}_{\pi^+\pi^-} = \frac{\alpha_1\left(ih_1^2h_2^2\rho_2\left(m_1^2-s\right) - \left(g_1^2\left(m_2^2-s\right)+h_1^2\left(m_1^2-s\right)\right)\left(ih_2^2\rho_2 - m_2^2 + s\right)\right) + \alpha_2 h_1 h_2 \left(m_1^2-s\right)\left(m_2^2-s\right)}{h_1^2h_2^2\rho_1\rho_2(m_1^2-s) + \left(i\rho_1\left(g_1^2\left(m_2^2-s\right)+h_1^2\left(m_1^2-s\right)\right) - \left(m_1^2-s\right)\left(m_2^2-s\right)\right)\left(ih_2^2\rho_2 - m_2^2+s\right)} \tag{15}$$

When setting h_1^2 to zero, the amplitude becomes:

$$\hat{A}_{\pi^+\pi^-}\Big|_{h^2\to 0} = \frac{\alpha_1 g_1^2}{m_1^2 - s - ig_1^2\rho_1} + \frac{\alpha_2 h_1 h_2 (m_1^2 - s)}{(m_1^2 - s - ig_1^2\rho_1)(m_2^2 - s - ih_2^2\rho_2)}. \tag{16}$$

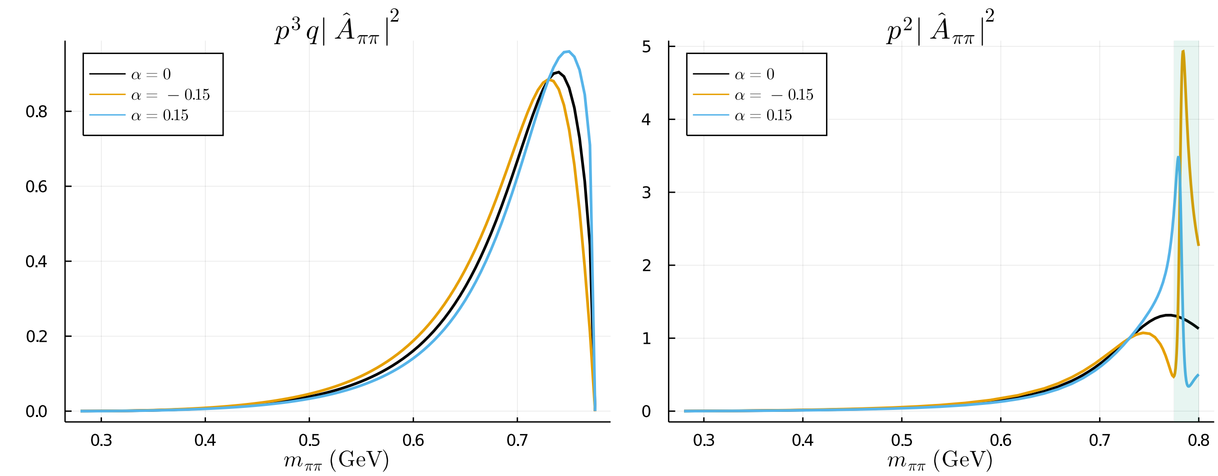

FIG. 2. Two-pion production amplitude. The phase-space factor is removed on the light plot to highlight the region of large ρ/ω mixing.

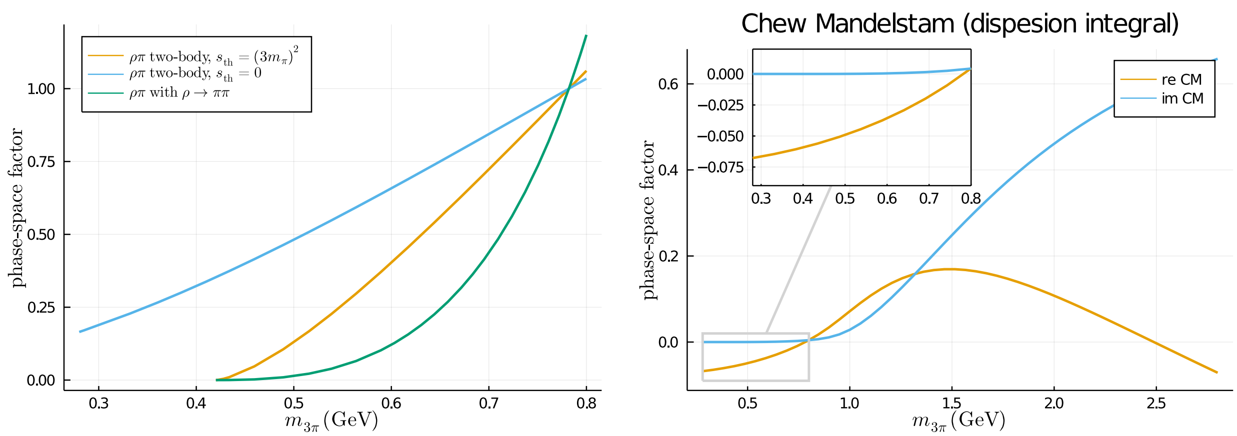

FIG. 3.

One finds an artificial zero at the second term originating from the K-matrix construction. The zero is located at the value of the bare ρ mass. Since the bare mass does not have physical meaning and can be shifted arbitrary, the zero does not need to be enforced there. We remove it with the production coefficients:

$$\alpha_1 = \mathrm{Pol}_m(s), \qquad \alpha_2 = \frac{\mathrm{Pol}_n(s)}{m_1^2 - s}, \tag{17}$$

with $\mathrm{Pol}_i(s)$ being a real polynomial of the order i. One should be able to obtain a decent fit with $m = 1$, $n = 0$. The higher order polynomials should be tried for systematic studies.

III. A COMMENT ON $\omega \to \rho\pi$ PHASE SPACE

We compated several expressions that approximate $\rho\pi$ P-wave phase space factor in Fig. 3. Due to the small width of ω the parametrization does not influence the $\pi\pi$ amplitude.

IV. MATCHING THE AMPLITUDE TO KNOWN $\pi^+\pi^-$ PHASE SHIFTS

In this section we address contribution of $\rho(1450)$ and F-wave and argue that for the region of $m_{\pi^+\pi^-}$ below 0.8 GeV these contributions are irrelevant. $\pi^+\pi^-$ P-wave is essentially elastic below 1 GeV, therefore the scattering/production

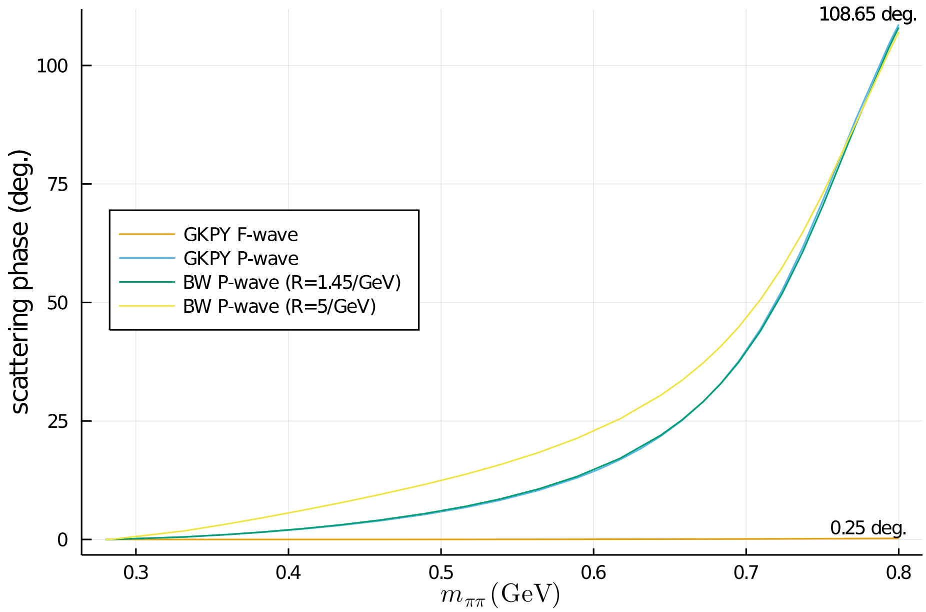

FIG. 4. The $\pi^+\pi^-$ P-wave and F-wave scattering phases from the phenomenological analysis of Ref. [1] and the single-pole amplitude with CM function. Note that the green line is right on top of the blue line with a little deviation at the limit of the phase space.

amplitudes are proportional to the sine of the scattering phase δ_1. These scattering phases are well established, e.g. in analysis of the Madrid group [1]. Fig. 4 shows the phase of the F-wave as well as the phase of P-wave in several models. The F-wave reaches just 0.25 deg. at $m_{\pi^+\pi^-}^{(\max)} = 0.8\,\mathrm{GeV}$. compare to 108.65 deg. of P-wave. It gives three order of magnitude suppression of the F-wave amplitude if the same production strength is used for both waves. The phase shift of the P-wave for the standard Breit-Wigner amplitude to the one extracted from phenomenological analysis [1] shows a large difference (compare yellow curve and the blue curve) The difference almost vanishes once the size parameter R is tuned to 1.4/GeV. The value of this parameter in the range from 1.3 to 1.5 gives the phase shift bearely distringuishable from the Madrid $\pi\pi$ phase shift.

Appendix A: Alternative formulations

The amplitude can be written in the other form:

$$T = G\,\Sigma\,G\,\Sigma\,G\,\Sigma\,G\cdots = (1 - G\Sigma)^{-1}G, \tag{A1}$$

where Σ and G are 4×4 matrices

$$G = \begin{pmatrix} 0 & 0 & g_{13} & g_{14} \\ 0 & 0 & 0 & g_{24} \\ g_{13} & 0 & 0 & 0 \\ g_{14} & g_{24} & 0 & 0 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} i\rho_1 & 0 & 0 & 0 \\ 0 & i\rho_2 & 0 & 0 \\ 0 & 0 & \frac{1}{m_1^2 - s} & 0 \\ 0 & 0 & 0 & \frac{1}{m_2^2 - s}. \end{pmatrix}, \tag{A2}$$

i.e. G is a matrix of couplings, Σ is either particle propagator, or a particle loop. The coupling matrix makes sure that the particle propagator, and a particle loop are always interchange (only element at the off-diagonal 2×2 blocks are allowed).

The production amplitude has the form:

$$\hat{A}_{\pi^+\pi^-} = \sum_{i=1}^{4} T_{1,i}\alpha_i \tag{A3}$$

The submatrix $T_{(1,2),(1,2)}$ in Eq. (A1) it is exactly equal to Eq. (7), with the following correspondence in Eq. (11),

$$g_{13} \Rightarrow g_1, \qquad g_{14} \Rightarrow h_1, \qquad g_{24} \Rightarrow h_2. \tag{A4}$$

Hence, the $T_{1,1}\alpha_1 + T_{1,2}\alpha_2$ is exactly equal to the sum in Eq.(14).

To identify the terms $T_{1,3}$ and $T_{1,4}$ in Eq. (A3) further, we show explicit expressions for $T_{1,i}$ in the limit $g14^2 \to 0$:

$$\hat{T}_{1,1} = \frac{g_{13}^2}{m_\rho^2 - s - ig_{13}^2\rho_1} \tag{A5}$$

$$\hat{T}_{1,2} = \frac{g_{14}g_{24}(m_\rho^2 - s)}{(m_\rho^2 - s - ig_{13}^2\rho_1)(m_\omega^2 - s - ig_{24}^2\rho_2)} \tag{A6}$$

$$\hat{T}_{1,3} = \frac{g_{13}^2(m_\rho^2 - s)}{m_\rho^2 - s - ig_{13}^2\rho_1} \tag{A7}$$

$$\hat{T}_{1,4} = \frac{g_{14}(m_\rho^2 - s)(m_\omega^2 - s)}{(m_\rho^2 - s - ig_{13}^2\rho_1)(m_\omega^2 - s - ig_{24}^2\rho_2)}. \tag{A8}$$

Both terms, $\hat{T}_{1,3}$ and $\hat{T}_{1,3}$ have artificial zeros $(m_\rho - s)$ or/and $(m_\rho - s)$ in the numerators. If we cancel this zero by a pole in the production vector as we did in Eq. 17, these terms becomes the same as $T_{1,1}$ and $T_{1,2}$. If only $(m_1^2 - s)$ term suppressed in $T_{1,2}$ and $T_{1,4}$, adding $T_{1,3}\alpha_3$ and $T_{1,4}\alpha_4$ is equivalent of using Pol_1 in Eq. 17.

[1] R. Garcia-Martin, R. Kaminski, J. Pelaez, J. Ruiz de Elvira, and F. Yndurain, Phys. Rev. D **83**, 074004 (2011), arXiv:1102.2183 [hep-ph].