S-matrix approach to the $\rho - \omega$ interference

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I. CONSTRUCTION OF THE SCATTERING MATRIX

Here we explore an idea to introduce $\rho - \omega$ interference by a small coupling between two channels:

- $J^{PC} = 1^{--}$: $\pi\pi$ P-wave
- $J^{PC} = 1^{--}$: $\rho\pi$ P-wave.

The production amplitude A is calculated from the scattering matrix as follows.

$$A_{\pi\pi} = \hat{A}_{\pi\pi} B_1^{1/2}(p), \tag{1}$$

where $p = \sqrt{s/4 - m_\pi^2}$ is a pion break-up momentum, $B_1(p)$ is a threshold factor (+ barrier factor, e.g. the Blatt-Weisskopf function, $B_1(p) = p^2/(1 + R^2 p^2)$, $R = 5/\mathrm{GeV}$).

$$T = [1 - iK\rho]^{-1}\, T \tag{2}$$

$$A = [1 - iK\rho]^{-1}\, N. \tag{3}$$

where ρ is a diagonal matrix, $\rho = \mathrm{diag}(\rho_1, \rho_2)$ with $\rho_1 = \sqrt{1 - 4m_\pi^2/s}\; B_1(p)$, and $\rho_2 = 1$.

The K-matrix contains a pole at every channel and a small non-diagonal coupling between two channels.

$$K = \frac{1}{m_1^2 - s}\begin{pmatrix} g_1^2 & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{m_2^2 - s}\begin{pmatrix} h^2 & hg_2 \\ hg_2 & g_2^2 \end{pmatrix} \tag{4}$$

The coefficient h^2 is proportional $\Gamma_{\omega\to 2\pi}$, i.e. extremely small compare to g_2^2.

The denominator of the scattering amplitude is proportional to the $\det(\mathbb{I} - i\rho K)$, that is a product of the ρ and ω inverse propagators up to the terms proportional to g_2^2.

$$\begin{aligned} D &= (m_1^2 - s)(m_2^2 - s)^2 \det(\mathbb{I} - i\rho K) \\ &= (m_1^2 - s - ig_1^2\rho_1)(m_2^2 - s - ig_2^2\rho_2)(m_2^2 - s) + O(h^2). \end{aligned} \tag{5}$$

Using the Q-vector construction of the production vector,

$$N = K\,[\alpha_1, \alpha_2]^T, \tag{6}$$

we obtain an expression for the amplitude:

$$\hat{A}_{\pi\pi} = \alpha_1 A^\rho + \alpha_2 A^{\rho/\omega} \tag{7}$$

where the amplitudes A^ρ and $A^{\rho/\omega}$ are given by the expressions:

$$\begin{aligned} A^\rho &= \frac{\left(g_1^2\left(m_2^2 - s\right) + h^2\left(m_1^2 - s\right)\right)\left(m_2^2 - s - ig_2^2\rho_2\right) + ih^2 g_2^2 \rho_2 (m_1^2 - s)}{D} \\ &= \frac{g_1^2}{m_1^2 - s - ig_1^2\rho_1} + O(h^2) \\ A^{\rho/\omega} &= \frac{hg_2\left(m_1^2 - s\right)\left(m_2^2 - s\right)}{D} = \frac{hg_2(m_1^2 - s)}{(m_1^2 - s - ig_1^2\rho_1)(m_2^2 - s - ig_2^2\rho_2)} + O(h^2). \end{aligned} \tag{8}$$

where we indicated a limit with $h \to 0$.

The pole in the numerator of the $A^{\rho/\omega}$ produces a zero at the value of the bare ρ mass. Since the bare mass does not have physical meaning and can be shifted arbitrary, the zero does not need to be enforced there. It is removed with the production coefficients:

$$\alpha_1 = \mathrm{Pol}_m(s), \qquad \alpha_2 = \frac{\mathrm{Pol}_n(s)}{m_1^2 - s}, \tag{9}$$

with $\mathrm{Pol}_i(s)$ being a real polynomial of the order i. One should be able to obtain a decent fit with $m = 1$, $n = 0$. The higher order polynomials should be tried for systematic studies.

II. VALUES FOR THE COUPLINGS

The parameters of the K matrix in Eq. (4) are completely fixed by the widths of the resonances and branching fractions:

$$\begin{aligned} g_1^2 &= m_\rho \Gamma_\rho / \rho_1(m_\rho^2), \\ g_2^2 &= m_\omega \Gamma_\omega \, \mathrm{Br}(\omega \to 3\pi)/\rho_2(m_\omega^2), \\ h^2 &= m_\omega \Gamma_\omega \, \mathrm{Br}(\omega \to \pi\pi)/\rho_2(m_\omega^2). \end{aligned}$$

III. FURTHER IMPROVEMENTS

One can improve analytic structure of the amplitude by replacing the phase-space factors by their dispersive representation:

$$i\rho_i \to (\mathrm{CM}_i - \mathrm{Re}\,\mathrm{CM}_i(m_i^2)), \quad \mathrm{CM}_i = \frac{s}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s'\, \rho_i(s')}{s'(s' - s - i\epsilon)} \tag{10}$$

Instread of using a constant for the expression for the $\rho\pi$ P-wave, the quasi-two-body phase space can be calculated:

$$\rho_2 \to \frac{1}{2\pi s} \int_{4m_\pi^2}^{(\sqrt{s}-m_\pi)^2} \frac{\mathrm{d}\sigma}{2\pi\sigma} \, \frac{\lambda^{1/2}(s,\sigma,m_\pi^2)\lambda^{1/2}(\sigma,m_\pi^2,m_\pi^2)}{(m_\rho^2 - \sigma)^2 + (m_\rho \Gamma_\rho)^2} B_1(k) B_1(q), \tag{11}$$

with $k = \sqrt{\sigma/4 - m_\pi^2}$, $k = \lambda^{1/2}(s,\sigma,m_\pi^2)/(2\sqrt{s})$, and $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$.

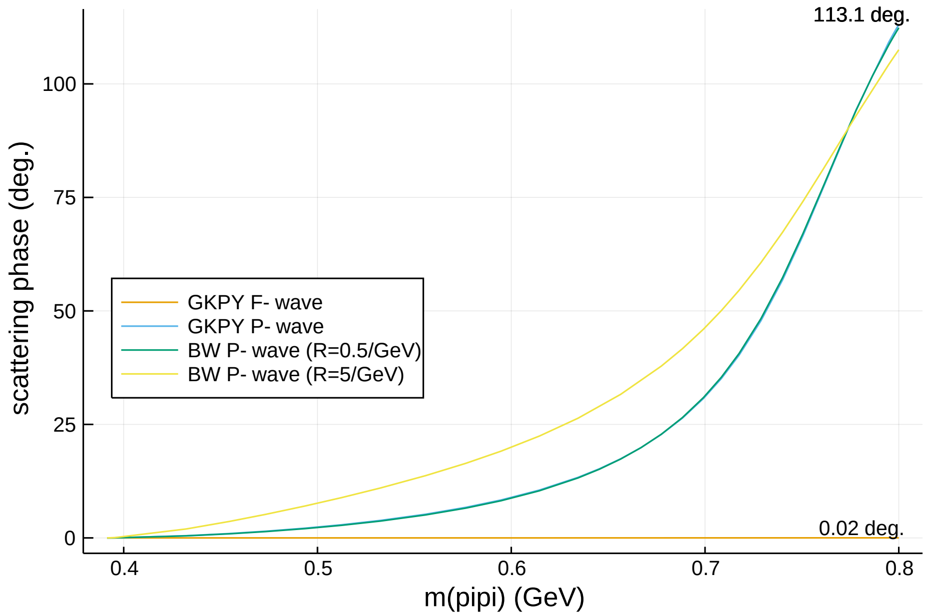

FIG. 1. The $\pi\pi$ P-wave and F-wave scattering phases from the phenomenological analysis of Ref. [1] and the single-pole amplitude with CM function. Note that the green line is right on top of the blue line with a little deviation at the limit of the phase space.

Appendix A: Contribution of the higher resonances

In this section we address contribution of $\rho(1450)$ and F-wave and argue that for the region of $m_{\pi\pi}$ below 0.8 GeV these contributions are irrelevant. $\pi\pi$ P-wave is essentially elastic below 1 GeV, therefore the scattering/production amplitudes are proportional to the sine of the scattering phase δ_1. These scattering phases are well established, e.g. in analysis of the Madrid group [1]. Fig. 1 shows the phase of the F-wave as well as the phase of P-wave in several models. The F-wave reaches just 0.02 deg. at $m_{\pi\pi}^{(\max)} = 0.8\,\text{GeV}$. compare to 113.1 deg. of P-wave. It gives three order of magnitude suppression of the F-wave amplitude if the same production strength is used for both waves.

Comparison of the P-wave phase of the standard Breit-Wigner amplitude ($R = 5/\text{GeV}$) to the one extracted from phenomenological analysis [1] shows a large difference (compare yellow curve and the blue curve). However, the difference almost vanishes once the size parameter R is tuned to 0.5/GeV and the Chew-Mandelstam function, Eq. (10) is used for the energy-dependent width. The further difference between the Madrid phase (orange line) shows and the adjusted curve shows potential contributions of the other poles, i.e. $\rho'(1450)$.

Appendix B: Adam's model

Adam suggested writing amplitude in a form:

$$\begin{aligned} T_{11} &= \dots \\ T_{12} &= \dots \\ T_{13} &= \dots \\ T_{14} &= \dots \end{aligned}$$

One find the matrix representation of these equations:

$$T = G + G\,\Sigma\,T, \tag{B1}$$

where

$$G = \begin{pmatrix} 0 & 0 & g_{13} & g_{14} \\ 0 & 0 & 0 & g_{24} \\ g_{13} & 0 & 0 & 0 \\ g_{14} & g_{24} & 0 & 0 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} \rho_1 & 0 & 0 & 0 \\ 0 & \rho_2 & 0 & 0 \\ 0 & 0 & \frac{1}{m_1^2-s} & 0 \\ 0 & 0 & 0 & \frac{1}{m_2^2-s}. \end{pmatrix}. \tag{B2}$$

Solving Eq. (B1) for T, we find that it is exactly equivalent to Eq. (2), with the following correspondence in Eq. (4),

$$g_{13} \Rightarrow g_1, \qquad g_{14} \Rightarrow h, \qquad g_{24} \Rightarrow g_2. \tag{B3}$$

Appendix C: Other check with different production amplitude

P-vector production gives more flexible parametrization

$$N_i = \sum_R \left(\frac{\alpha_i^R}{m_R^2 - s} + f_i \right). \tag{C1}$$

With an assumption that direct decay of X to $J/\psi\, 3\pi$ is negligible, i.e. $f_2 = 0$, we get:

$$\begin{aligned} \hat{A}_{\pi\pi} &= \frac{1}{m_1^2 - s - i g_1^2 \rho_1} \left(\alpha_1^\rho + \frac{k \alpha_2^\omega (i\rho_2)(m_1^2 - s)}{(m_2^2 - s - i g_2^2 \rho_2)(m_2^2 - s)} \right) + \frac{f_1 (m_1^2 - s)}{m_1^2 - s - i g_1^2 \rho_1} \\ &= \frac{1}{m_1^2 - s - i g_1^2 \rho_1} \left(\alpha_1^{\rho'} + \frac{k \alpha_2^\omega (m_1^2 - s)}{m_2^2 - s - i g_2^2 \rho_2} \right) + \frac{f_1 (m_1^2 - s)}{m_1^2 - s - i g_1^2 \rho_1} \end{aligned} \tag{C2}$$

1. The final reasonable forms

We find that unitality-guided amplitude contains two type of terms: ρ-term and $\rho \times \omega$-term with, in principle, arbitrary numerator functions. The pragmatic approach would be to leave freedom adjust ρ-meson lineshape at the full range of spectrum and allow for local modification in vicinity of the ω mass.

$$\hat{A}_{\pi\pi} = \frac{c^\rho + c^{\pi\pi}(m_1^2 - s)}{m_1^2 - s - i g_1^2 \rho_1} + \frac{c^\rho}{(m_2^2 - s - i g_2^2 \rho_2)(m_1^2 - s - i g_1^2 \rho_1)} \tag{C3}$$

$$\text{OR,} = \frac{a + bs}{m_1^2 - s - i g_1^2 \rho_1} \left(1 + \frac{c}{m_2^2 - s - i g_2^2 \rho_2} \right). \tag{C4}$$

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[1] R. Garcia-Martin, R. Kaminski, J. Pelaez, J. Ruiz de Elvira, and F. Yndurain, Phys. Rev. D **83**, 074004 (2011), arXiv:1102.2183 [hep-ph].