S-matrix approach to the $\rho - \omega$ interference

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1 Construction of the matrix element

Here we explore an idea to introduce $\rho - \omega$ interference by a small coupling between two channels:

- $J^{PC} = 1^{--}$: $\pi\pi$ P-wave
- $J^{PC} = 1^{--}$: $\rho\pi$ P-wave.

The production amplitude A is calculated from the scattering matrix as follows.

$$A_{\pi\pi} = \hat{A}_{\pi\pi} B_1^{1/2}(p), \tag{1}$$

where $p = \sqrt{s/4 - m_\pi^2}$ is a pion break-up momentum, $B_1(p)$ is a threshold factor (+ barrier factor, e.g. the Blatt-Weisskopf function, $B_1(p) = p^2/(1 + R^2 p^2)$, $R = 5/\mathrm{GeV}$).

$$A = [1 - iK\rho]^{-1} N. \tag{2}$$

ρ is a diagonal matrix with $\rho_1 = \sqrt{1 - 4m_\pi^2/s} B_1(p)$, and $\rho_2 = 1$.

The K-matrix contains a pole at every channel and a small non-diagonal coupling between two channels.

$$K = \frac{1}{m_1^2 - s}\begin{pmatrix} g_1^2 & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{m_2^2 - s}\begin{pmatrix} 0 & k \\ k & g_2^2 \end{pmatrix} \tag{3}$$

The coefficient k can be calculated using $\Gamma_{\omega\to 2\pi}$. It is extremely small, so, we drop in the amplitude denominator to simplify the expression.

$$\begin{aligned} D &= (m_1^2 - s)(m_2^2 - s)\,\det(\mathbb{I} - i\rho K) \\ &= (m_1^2 - s - ig_1^2\rho_1)(m_2^2 - s - ig_2^2\rho_2) + k^2\rho_1\rho_2(m_1^2 - s)/(m_2^2 - s) \\ &\approx (m_1^2 - s - ig^2\rho_1)(m_2^2 - s - ig_2^2\rho_2). \end{aligned} \tag{4}$$

An investigation of the dropped term in vicinity of the ω mass where the dropped term is enhanced requires a separate investigation.

1.1 Parametrization of the production amplitude

We consider two common constructions for the vector N:

1.1.1 Q-vector amplitude

$$N = K\,[\alpha_1, \alpha_2]^T. \tag{5}$$

Then, the full amplitude reads:

$$\hat{A}_{\pi\pi} = \frac{g_1^2}{m_1^2 - s - ig_1^2\rho_1}\left[\alpha_1 + \alpha_2\frac{k(m_1^2 - s)}{m_2^2 - s - ig_2^2\rho_2}\right]. \tag{6}$$

We find that the interference term has a zero at $s = m_1^2$. One should not, however, take it literarily since the bare masses are not physical can be shifted to any value by replacing the phase space factor by the dispersive representation (the fixed real can be shifted arbitrary).

1.1.2 P-vector amplitude

P-vector production gives more flexible parametrization

$$N_i = \sum_R \left(\frac{\alpha_i^R}{m_R^2 - s} + f_i\right). \tag{7}$$

With an assumption that direct decay of X to $J/\psi\,3\pi$ is negligible, i.e. $f_2 = 0$, we get:

$$\begin{aligned}\hat{A}_{\pi\pi} &= \frac{1}{m_1^2 - s - ig_1^2\rho_1}\left(\alpha_1^\rho + \frac{k\alpha_2^\omega(i\rho_2)(m_1^2 - s)}{(m_2^2 - s - ig_2^2\rho_2)(m_2^2 - s)}\right) + \frac{f_1(m_1^2 - s)}{m_1^2 - s - ig_1^2\rho_1} \\ &= \frac{1}{m_1^2 - s - ig_1^2\rho_1}\left(\alpha_1^{\rho\prime} + \frac{k\alpha_2^\omega(m_1^2 - s)}{m_2^2 - s - ig_2^2\rho_2}\right) + \frac{f_1(m_1^2 - s)}{m_1^2 - s - ig_1^2\rho_1}\end{aligned} \tag{8}$$

1.2 The final reasonable form

We find that unitality-guided amplitude contains two type of terms: ρ-term and $\rho\times\omega$-term with, in principle, arbitrary numerator functions. The pragmatic approach would be to leave freedom adjust ρ-meson lineshape at the full range of spectrum and allow for local modification in vicinity of the ω mass.

$$\hat{A}_{\pi\pi} = \frac{c^\rho + c^{\pi\pi}(m_1^2 - s)}{m_1^2 - s - ig_1^2\rho_1} + \frac{c^\rho}{(m_2^2 - s - ig_2^2\rho_2)(m_1^2 - s - ig_1^2\rho_1)} \tag{9}$$

$$\text{OR,} = \frac{a + bs}{m_1^2 - s - ig_1^2\rho_1}\left(1 + \frac{c}{m_2^2 - s - ig_2^2\rho_2}\right). \tag{10}$$