RESEARCH PROJECT PROPOSAL

Name: Miliano Mikol
Major: Computer Science
Minor: Pure Mathematics
Adviser: Dr. Joshua Hallam, Mathematics
Title: Tight Forests and the Chromatic Polynomial

STATEMENT OF THE PROBLEM

The chromatic polynomial of a graph is an important combinatorial polynomial. It tracks several important properties of the graph that it encodes. The coefficients of the chromatic polynomial have a well-known combinatorial interpretation. However, for some graphs, it is difficult to use the same interpretation. As a result, it is useful to think about other interpretations. In this project, we wish to classify which graphs have such an interpretation in terms of graphs known as "tight forests". We will also look at other properties of tight forests.

BACKGROUND AND CONTEXT

In the paper, *Increasing Spanning Forests in Graphs and Simplicial Complexes* [1], the authors developed some the theory for tight forests. They gave a labeling that relates tight forests to the chromatic polynomial. However, what remains an open question is which graphs would have such labelings. Solving this problem would be very helpful in the field of combinatorics. I am personally interested in this research project for two reasons. First, this research project is part of my exploration into finding my "niche" for graduate school. The second reason is that I am fascinated by mathematics and its applications in computer science. One of my favorite math classes explored problems related to graph theory (a subsection of mathematics with a superfluous amount of applications). After talking with Dr. Hallam about potential projects, this one struck me as interesting and exciting.

HYPOTHESIS AND/OR RESEARCH QUESTIONS

Due to the nature of this project, forming a hypothesis is not necessarily applicable. This is a fairly unexplored area of combinatorics. Thus, we cannot predict or guess the structures of graphs that would have labelings such that they are tight forests and relate to the chromatic polynomial. Nonetheless, the research question that we are hoping to answer is: "Which graphs have labelings such that the tight forest interpretation of the chromatic polynomial applies?".

RESEARCH DESIGN AND METHODS OF DATA GATHERING

Sage is a computer system with features for a variety of areas in mathematics, including combinatorics and graph theory. Sage would be used to automate and scale a variety of tests. With the collected data, we would proceed with analysis and mathematical proofs of new findings.

JUSTIFICATION/IMPORTANCE OF THE PROJECT

Discoveries from this project would contribute substantially to the combinatorics community through the advancement of learning. The significance of such unexplored findings is a matter of study itself and is to be seen.

TIMELINE OF PROJECT ACTIVITIES

Week 1: background/literature review; project proposal; Overleaf setup; Github setup; programming dependencies setup; learning Sage programming basics.

Week 2: research; powerpoint preparation but no presentation due to travel; learning Sage programming basics.

Week 3: research; powerpoint preparation (for week 4).

Week 4: research; powerpoint presentation; powerpoint preparation (for week 5).

Week 5: research; presentation presentation.

Week 6: research/wrap-up with an evaluation of current progress and goals for future work during the academic year; powerpoint preparation/presentation; finish abstract; finish poster; finish updated research proposal; finish summary of research experience.

July 2: submit all required documents (abstract, poster, updated research proposal, and summary of research experience)

LITERATURE CITED

[1] Hallam, Joshua & Martin, Jeremy & Sagan, Bruce. (2016). *Increasing spanning forests in graphs and simplicial complexes.* European Journal of Combinatorics. 76. 10.1016/j.ejc.2018.09.011.

[2] Sagan, Bruce. (2020). *Combinatorics: The Art of Counting.*