Exam Signal and Image Processing 2014

- During this exam you may use a computer
- The use of the internet/mailing/instant messaging is strictly prohibited
- Ensure that you hand in the matlab .m files of the questions that you answered using a computer
- You have to hand in this sheet with exam questions after the test
- Good luck!

1. (15 points) Huffman encoding and imaging
   (a) Given the following string: "goederenwagon". Convert this string to a vector by mapping each letter to its index in the alphabet, that is "a" is 1 and "z" is 26. Next use Huffmann encoding to encode this vector.
   (b) Compute how much bits would be needed to transmit the string with and without Huffmann encoding and compute the compression ratio.
   (c) Given the image of my tape dispenser in "imageSIP2014Cropped.jpg"; perform edge detection on the intensity channel in the HSI representation using Haar wavelets. Show the edges in an image.

2. (10 points) Fourier series and complex exponential functions

   Compute *analytically* the Fourier series coefficients c_k of the following 2π periodic function that is given for $t \in [-\pi, \pi)$ as:
   $$f(t) = t^2 - \pi$$

3. (25 points) Filtering linear systems, and Fourier analysis

   The file `SignalExercise4.mat` contains a discrete-time sequence representing an audio signal which has been sampled with the sampling frequency Fs =44100Hz. The signal is corrupted with high-frequency noise concentrated in the frequency region above approximately 5000 Hz.

   Given an FIR causal filter $H(z)$ characterised by the impulse response `B` stored in `filterExercise4.mat`:
   (a) Determine if this filter is suitable to remove the high frequency noise corrupting the audio signal. Please, justify your answer.
   (b) Apply this filter to the audio signal by using the function *filter*. Display the amplitude spectrum of the original and the filtered signal and comment the performance of the filter in removing the high-frequency noise.
   (c) Display the original and the filtered signal in the same plot. What do you notice at the beginning of the filtered signal? Why this happens?
   (d) Show that the filtered signal is still corrupted by power line interference at 50Hz.
   (e) Design a notch filter with two zeros and two poles to filter out the powerline interference and plot its frequency response.
   (f) Filter the filtered signal by means of the notch filter implemented at the previous point. Display the input and the output signal to the notch filter in the same plot. Make some considerations on the result of the filtering.

4. (20 points) Principal component analysis (PCA)

   Load the file `SignalExercise5.mat` into Matlab. Matrix `X` contains 13 observations for 3 variables.
   (a) Generate a 3-D scatter plot of the tri-variate variable `X` by exploiting the Matlab function *plot3*.

(b) The PCA model for the tri-variate variable `X` is `X=AZ`, where `Z` is the matrix of principal components and `A` is the transfer matrix. Use a singular value decomposition approach to find out `Z` and `A`.

(c) Which one of the original variables is more reflected in the first principal component? Please, justify your answer.

(d) Reconstruct `X` by exploiting only the first n principal components which account for 95% of the variance of `X`. Then, generate a 3-D scatter plot of the original tri-variate variable `X` and its approximation.

5. (15 points) Orthogonal filters

(a) Someone is constructing an orthogonal wavelet structure with four filter coefficients. It is given that $h_2 = 0.35\sqrt{2}$ and that the filter has unit energy. Find the other low-pass filter coefficients.

(b) You are engaged in a discussion with a colleague that disagrees with your viewpoint that a certain signal is in fact a polynomial. How can you employ Daubechies wavelets to prove him wrong? Explain!

6. (15 points) Wavelet shrinkage
Load the ECG signal stored in "data2014Question6.mat". Denoise the signal by wavelet shrinkage with soft thresholding. Use a 4 level decomposition of the Daubechies 4 wavelet (4 vanishing moments). Use Visushrink as described in the book and discussed in the lectures. Give a plot of the original and the two denoised signal superimposed.