Exam Signal and Image Processing 2/2/2012

- During this exam you may use a computer
- The use of the internet/mailing/instant messaging is strictly prohibited
- Ensure that you hand in the matlab .m files of the questions that you answered using a computer
- You have to hand in this sheet with exam questions after the test
- Good luck!

1. Huffman encoding and imaging
   (a) Given the following quantified signal $s = \{3, 5, 0, 0, 3, 5, 2, 0, 0, 3, 0, 5, 2, 3, 9, 0, 1\}$. Use Huffman encoding to encode it.
   (b) Load the image of my culinary enterprize `quiche.jpg`, and perform histogram equalization on the luminance channel. (You might need to convert it to another color system first)
   (c) Use a wavelet approach on the luminance channel to ensure that the structure of the table cloth is clearly visible. If needed, you may crop the image to a convenient size.

2. Fourier series and complex exponential functions
   Compute the Fourier series coefficients of the following 2π periodic function that is given for $t \in [-\pi, \pi)$ as:
   $$f(t) = 2t.$$

3. Filtering linear systems, and Fourier analysis
   Load the signal stored in `dataQ3.mat` into Matlab. This signal is corrupted by power line interference $f_p = 50$Hz and by some higher frequency noise. A physician wants to built a suitable notch filter in order to filter out both noises, but he does not know the frequency of the higher noise. Knowing that the pair of conjugate zeros to filter out the power line interference at 50Hz is $z_1 = 0.3090 + i0.9511$ and $z_2 = 0.3090 - i0.9511$:
   (a) Find out the sampling frequency f_s of the system.
   (b) Find out the other frequency f_h (not being 50Hz) that is to filter out.
   (c) Find out the pair of conjugate zeros z_3 and z_4 so as to filter the highest frequency out of the signal by appropriately placing the zeros in the z-domain.
   (d) Design a notch filter so as to filter both the power line interference f_p and the highest frequency f_h out of the signal, by exploiting z_1, z_2, z_3, z_4.
   (e) Filter the original signal by exploiting the notch filter designed at the previous point. Give a plot of the original and filtered signals superposed. What do you notice?

4. Principal component analysis (PCA)
   Load the tri-variate signal stored in `dataQ4.mat` into Matlab. Each raw of matrix `X` represents one of the three variables, and each column represent an observation.
   (a) Generate a 3-D scatter plot of the tri-variate signal by exploiting the Matlab function `plot3`.
   (b) The PCA model for the tri-variate signal X is $X = AZ$, where Z is the matrix of principal components and A is the transfer matrix. Exploit function `eig` (which given a matrix M provides the eigenvalue decomposition of M) to find out Z and A.
   (c) Estimate a linear approximation of the original tri-variate signal by exploiting the projection of the first principal component only on the original 3-D subspace. Then, generate a 3-D scatter plot of the original tri-variate signal and its linear approximation.

5. Orthogonal wavelets

(a) Given the following set of low-pass filter coefficients:

$$c = \{\frac{3}{5}, p, \frac{-1}{10}, q\}$$

Find the coefficients p and q that lead to an orthogonal filter structure. Note that this doesn't have to be a Daubechies wavelet and that you have some freedom in how to choose the scaling convention.

(b) Calculate the Daubechies 6 (i.e. with three vanishing moments) low-pass filter coefficients yourself using the appropriate conditions.

6. Wavelet shrinkage
Load the signal stored in `dataQ7.mat`. Denoise the signal by wavelet shrinkage with hard thresholding. Use a 4 level decomposition of the Daubechies 1 wavelet. Use Visushrink as described in the book and told in the lectures. Give a plot of the original and the denoised signal superposed.