Practical Assignment 4 – Time-frequency representation and Haar wavelets

Haar wavelet in signals

During the lecture the Haar wavelet transform was discussed. In this exercise we will practice a bit more. Remember that the Haar wavelet transform can be implemented with a filter bank with a low-pass filter:

$$h_0 = \left\{ \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\}$$

and a high-pass filter:

$$h_1 = \left\{ -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\}$$

After filtering with these two filters in parallel down sampling takes place, yielding a vector with scaling coefficients a and a vector with wavelet coefficients b, both with length n for an input signal of length 2n.

Question 4.1

Complete the prototype function `dwlt.m` *such that it calculates the Haar wavelet transform of a signal x when passing the proper filter coefficients to c and d. Ensure that you avoid any border problems that may arise due to the "filter" function. Test your function on the signal x = (1, 4, 2, 3, 1, 0, 0, 2), and verify the conservation of energy property of the wavelet transform.*

Question 4.2

Write a function `idwlt.m` *that calculates the inverse Haar wavelet transform of your approximation and detail coefficients.*

Question 4.3

The multiresolution structure for the discrete wavelet transform has been discussed during the lecture. Finish the function `wldecom.m` *such that for each cascade in the multiresolution approach the detail coefficients* b_k *are appended in front of the vector C and finally the coarsest approximation coefficients are appended too, yielding* $C=[a_k, b_k, b_{k-1}, \ldots, b_1]$*, where the index runs from fine to coarse.*

Test your function on the same signal as before.

Haar wavelet in images

Question 4.4

Load the ronald.mat file. Display the image with imagesc and colormap gray.

On this image we want to perform a 2D-Haar wavelet transform. A 1-level wavelet transform of an image s can be calculated using two simple steps:

Perform a single level of the regular wavelet transform on each row of the image yielding two images t_a *and* t_d *which can be concatenated to give a single image t that has the same dimensions as s.*

Repeat the previous step on the columns of t.

This produces a single step of a 2D wavelet transform of s which can be symbolized as:

$$s \rightarrow \begin{pmatrix} b_1 & v_1 \\ h_1 & d_1 \end{pmatrix}$$

The sub image a1 will be a coarse approximation of s since it is formed by low-pass filtering in both directions. h_1 *will pick up the horizontal features since it is low-pas filtered along the rows and high-pass filtered along the columns. The vertical features will be picked up by* v_1*. Perform a 1-level Haar wavelet transform on Ronald.*

Question 4.5

Perform a 3-level Haar wavelet transform on Ronald.

Haar wavelets and edge detection

When applying the Haar high-pass filter to a signal, you are actually differencing two consecutive signal values, which is the discrete-time equivalent to differentiation. This is quite similar to how the Sobel edge detector works (p.211) which considers the Gradient of an image.

The Haar low-pass filter is a smoothing filter, hence the wavelet coefficients at coarser scales are the difference of smoothed signal values, i.e., the gradient of an image preprocessed by a smoothing mask as is done for the Canny edge detector (p.211).

Question 4.6

By taking `foo.png` *picture and applying one scale of the Haar wavelet 2D transform you obtain a decomposition partitioned in a blur image, horizontal details, vertical details and diagonal differences. The blur image is purely the result of smoothing and downsampling, the other ones involve the discrete-time equivalent of derivatives hence combined form something that resembles the gradient.*

Decompose foo.png, set the blur image to zero and reconstruct the filtered image. What is it that you have obtained?

This method is a bit too sensitive because it will pick up any dark/light changes in the picture. To require that only transitions that are strong enough are picked up, one can use a thresholding technique.

Question 4.7

Incorporate the automatic thresholding technique from page 214 to detect the edges in the picture.

Question 4.8

Perform 3 scales of the 2D Haar wavelet transform on the image, set the blur image to the zero matrix and reconstruct. What is the difference with the result from 4.6?

Short-term Fourier transforms

Given a dataset `data2_1.mat` *with ECG data of a driver, sampled at 15.5 Hz.*

Question 4.9

Compute the short-time FFT using a Hamming window in order to plot the spectrogram of the data. Ensure that you have ample of time resolution. Obviously if you read this remark this isn't always the case by default.

Question 4.10

Can you find out what the dominant frequencies are and at what time instances there is a problem with baseline wander?

Question 4.11

Can you, using this data and its spectrogram, discover which frequencies are involved with the QRS complex?