Exam Signal and Image Processing 2015

- During this exam you may use a computer
- The use of the internet/mailing/instant messaging is strictly prohibited
- Ensure that you hand in the Matlab .m files of the questions that you answered using a computer
- You have to hand in this sheet with exam questions after the test
- Good luck!

1. (15 points) Huffman encoding and imaging

   (a) Given the following string Huffman encoded string:
   "111101110101000011011010100001100001100001100001101101100111111010001110111011111101111100"
   and the following encoding table

e	00
g	111100
h	111101
i	111111
l	1110
n	01
o	110
s	111110
t	10

   Decrypt the original string. (feel free to use a computer to help you if you think that this will expedite things)

   (b) Reconstruct the Huffman encoding tree that has been used and verify whether it is correct or not. State this explicitly in your answer.

   (c) Given the image `imageSIP2015.jpg`; perform edge detection using Haar wavelets to reveal the contours of the people in the picture. Plot in one figure the original picture, the decomposition and the edges.

2. (10 points) Fourier series and complex exponential functions

   Compute *analytically* the Fourier series coefficients c_k of the following 2π periodic function that is given for $t \in [-\pi, \pi)$ as:

$$f(t) = \begin{cases} \pi & -\pi \leq t < 0 \\ 2t & 0 \leq t < \pi \end{cases}$$

3. (25 points) Filtering linear systems, and Fourier analysis

   The file `SignalExercise3SIPReg2015.mat` contains a discrete-time sequence y representing a signal which has been sampled with a sampling frequency Fs = 11025 Hz.

   Given an IIR band-pass filter $H(z)$ characterized by the following transfer function $H(z)$:

$$H(z) = \frac{0.0201 - 0.0402z^{-2} + 0.0201z^{-4}}{1 - 2.1192z^{-1} + 2.6952z^{-2} - 1.6924z^{-3} + 0.6414z^{-4}}$$

   (a) Plot the *frequency response* of $H(z)$. To accomplish that, look at the *help* of the function `freqz` to see how to correctly generate the vectors `B` and `A` (which contain the coefficients of the numerator and denominator of $H(z)$, respectively).

   (b) Determine if this filter is suitable to preserve mainly the frequency content in the range 1400 Hz and 1900 Hz. Motivate your answer.

(c) Apply this filter to the signal y by using the function *filter*. Display the original and filtered signals and their amplitude spectra and comment on the performance of the filter.

(d) When looking at the amplitude spectrum of the filtered signal, you should notice that the filtered signal is still corrupted by a strong sinusoidal interference at 1600 Hz. Design a notch filter with two zeros and two poles to filter out the 1600 Hz interference and plot its frequency response.

(e) Filter the filtered signal by means of the notch filter implemented at the previous point. Display the input and the output signal to the notch filter in the same plot. Make some considerations on the result of the filtering.

4. (20 points) Principal component analysis (PCA)

   Load the file `dataExercise4SIPReg2015.mat` into Matlab. Matrix `data` contains 501 observations of 5 random variables.

   (a) Generate a PCA model of the multivariate dataset `data` by means of a singular value decomposition (SVD) approach, to investigate the variance of the dataset.

   (b) Which one of the original variables the first principal component is more reflected on? Please, justify your answer.

   (c) Reconstruct `data` by exploiting only the first n principal components which account for 60% of the variance of `data`.

   (d) Display the second variable of `data` and its 60% approximation in the same plot.

5. (15 points) Orthogonal filters

   (a) In the file `filtercoefficients_Question_5_2015.mat` there is a set of filter coefficients: `c` and `d`. Investigate (and show) whether this set of filter coefficients constitutes a Daubechies wavelet filter bank and if not list what properties of the Daubechies wavelet this filter bank does have and which properties it doesn't share. Ensure that the filters are scaled as you expect them to be and otherwise rescale them.

   (b) Consider the following proposition: "If one samples the wavelet function at dyadic points, one actually gets the overall filter from signal, through the cascade of n filter banks to the high-pass output at scale n." Two statements on this proposition are made:

      1. "Since the scaling function at dyadic points is discovered by iterating the low-pass filter n times and the wavelet function is discovered by iterating the low-pass filter $n-1$ times, followed by iterating the high-pass filter once, this is true"
      2. "Since this would mean that sampling the continuous wavelet transform at scale n at the appropriate dyadic points would give you the high-pass output of the discrete wavelet transform at scale n, this cannot be true."

      explain whether statement 1 or statement 2 is true.

6. (15 points) Wavelet shrinkage
   Load the ECG signal stored in `dataQuestion6_2015`. Denoise the signal by wavelet shrinkage with soft thresholding. Use a 6 level decomposition of the Daubechies 3 wavelet (3 vanishing moments). Use Visushrink as described in the book and discussed in the lectures. Give a plot of the original and the two denoised signal superimposed.