# On-the-Job Search, Human Capital Formation, and Lifecycle Wages

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#### **Abstract**

I build an equilibrium lifecycle model of wages that combines human capital accumulation with on-the-job search in a frictional labor market. In the model, heterogeneous workers endogenously invest in human capital accumulation and search effort while firms post jobs. I discipline the model using microdata from the SIPP. Using the calibrated model, I show that (1) on-the-job search is the driving force behind lifecycle wage growth, (2) heterogeneous human capital accumulation is the driving force behind lifecycle wage dispersion, and (3) there is significant heterogeneity in how workers approach increasing their wages over the life cycle. Then, I use the model as a laboratory to study the effects of tax and transfer progressivity. An increase in progressivity decreases wages, primarily due to reduced on-the-job search effort. Furthermore, interactions between human capital, search, and job posting amplify the decrease in wages. Surprisingly, increasing progressivity has little effect on wage dispersion because the effects from the human capital and search channels offset each other.

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# 1 Introduction

Contemporary macroeconomists tend to focus on two general theories for wage growth: human capital theory and job ladder theory. In human capital theory (Ben-Porath, 1967), workers increase their wages by accumulating human capital.. If the labor market is frictionless, then the worker's wage equals his marginal product of labor. So, as a worker accumulates human capital, he becomes more productive, and his wage increases.

In job ladder theory, workers increase their wages by making job-to-job transitions, often to more productive firms. When a worker moves to a more productive firm, he and the firm negotiate over a new, higher wage. The theory relies on search frictions; if workers could start in the most productive firm, there would be no wage growth.

In this paper, I develop an equilibrium model where both channels interact endogenously. Going forward, I refer to the human capital part as the "human capital channel" and the search-and-matching, job ladder part as the "search channel." The model combines endogenous human capital accumulation, endogenous on-the-job search, endogenous job posting, and a life cycle. Workers choose how much effort to invest in human capital accumulation and job search effort, and firms choose how many job vacancies to post in equilibrium. Both types of investment increase wages, but to varying degrees, and workers choose an optimal mix of investments according to their fixed learning ability, their current state, and the state of the labor market. To my knowledge, a quantitative model with all of these elements has not been studied.

What is the benefit of endogenizing all of these elements? Current empirical research (cited below) shows that both the human capital and search channels are necessary to account for wage levels, lifecycle wage growth, and wage inequality. It follows that both channels may respond to changes in policy. Therefore, it is useful to understand how the channels interact for explaining the current state of the world as well as for predicting the effects of policy.

In this paper, I use the model to evaluate tax and transfer progressivity. Progressive labor taxation is a natural choice for counterfactual policy experiments in my model. It

is well known that tax progressivity differs vastly across developed countries (Guvenen et al., 2014). Also, it is well known that the labor market operates differently across countries with respect to lifecycle wage growth, job-to-job transitions, and unemployment (Engbom, 2022). To what extent are the differences across labor markets attributable to tax policy? This is an important question to which my model can contribute.

In the model, workers are heterogeneous at the beginning of life in fixed learning ability and initial human capital. Firms are heterogeneous in productivity. Human capital and firm productivity are complements in production. When employed by a firm, a worker earns a wage which is the product of three components: the worker's human capital, the firm's productivity, and an endogenous bargaining component (similar to a rental rate of human capital). The bargaining component arises from a surplus sharing rule as in Cahuc et al. (2006).

Workers are risk averse and maximize lifetime utility of consumption. To do so, workers choose how much effort to invest in human capital accumulation and searching for a new job. When a worker invests in human capital accumulation, he increases his stock of human capital as in Ben-Porath (1967); when a worker invests in search effort, he increases the probability of meeting an outside firm. Each activity is costly in the sense that the worker endures convex disutility. However, both activities are beneficial because they lead to wage growth and increased future consumption.

In addition, workers also face negative risk from exogenous unemployment shocks. Low-wage workers are more likely to experience unemployment, and human capital depreciates while a worker is unemployed.

Firms enter the model by posting job vacancies. Meeting probabilities are determined by an aggregate matching function which depends on the number of open job vacancies and aggregate search effort. In equilibrium, the number of open vacancies satisfies a free entry condition which states that firms are indifferent toward posting the marginal vacancy. From the firm's perspective, the optimal number of job postings depends on the distribution of human capital and search effort in the labor market.

Workers face a clear tradeoff: if a worker invests in human capital accumulation or job search effort today, he may increase his wage tomorrow. His optimal mix of human capital investment and search effort depends on his learning ability, age, and current levels of human capital, firm productivity, and the bargaining term. Therefore, workers in different situations will (a) choose different levels of investment and (b) choose different mixes of investment.

For example, consider how fixed learning ability affects investment decisions.<sup>1</sup> Analytically, I show that a worker with high learning ability will invest more in human capital accumulation and less in job search. The opposite is true for workers with low learning ability. In this sense, workers *substitute* between investment in human capital and search. Compared to a pure human capital model, this is good news for low-ability workers; they can invest heavily in job search instead of struggling to accumulate human capital.

I calibrate the model using microdata from the Survey of Income and Program Participation (SIPP) for 1990-2019. The SIPP is a large panel data set which tracks respondents for several years at a time. It is particularly useful for this paper because wages and jobs are reported monthly. Therefore, I calibrate the model to match monthly wage growth and job changes.

The calibrated model successfully replicates lifecycle profiles for mean wages, wage dispersion, and job-to-job transitions. Additionally, the model matches monthly wage growth rates for workers who stay at the same job or switch jobs, as well as transition rates to and from unemployment.

Since I experiment with tax progressivity, I also carefully calibrate a government sector. I estimate an average tax function over wages that replicates income taxes paid and means-tested transfer payments received for an average household in the US. Some workers pay negative taxes; these workers receive more in transfers than they pay in taxes. In counterfactual exercises, I increase the progressivity of the system. Greater progressivity implies that low-wage workers receive more transfers while high-wage workers pay

<sup>&</sup>lt;sup>1</sup>In the quantitative calibrated model, heterogeneity in learning ability turns out to be a key variable in accounting for worker heterogeneity.

more taxes.

Before I conduct policy experiments, I derive insights from the calibrated benchmark model. On average, the search channel is the most important driver of lifecycle wage growth; about 70% of lifecycle wage growth comes from workers climbing the job ladder, with the remaining coming from an increase in human capital.

However, there is significant heterogeneity between workers, and heterogeneity is amplified by the interaction of the two endogenous channels. Heterogeneity in fixed learning ability generates differential returns to human capital investment, and workers with greater learning ability will invest more in human capital accumulation, especially at the beginning of life. Across learning abilities, I observe vast differences in human capital accumulation, but roughly equal job ladder outcomes. The result is that workers with high learning ability accumulate wages faster than those with low learning ability, and the difference is driven by human capital. Thus, over the lifecycle, the increase in wage dispersion is driven by an increase in the dispersion of human capital. Taken together, we can generalize my results as suggesting that wage growth comes from the job ladder while wage dispersion comes from human capital.

I then turn to counterfactual tax policy experiments. Greater tax progressivity implies that, when a worker increases his wage, he will take home a smaller part of the wage increase. Thus, workers are disincentivized from growing their wages, which puts downward pressure on both human capital investment and search effort. There is also an equilibrium effect; because workers have less human capital and exert less search effort, firms are discouraged from posting jobs, leading to fewer open vacancies in equilibrium.

I perform a simple tax policy experiment where I increase the progressivity of the tax and transfer system from the US benchmark to that of a country like Denmark, Finland, Germany, the Netherlands, or Sweden. In total, when I increase tax progressivity, the wage level in the economy decreases by about 4%. Approximately 2/3 of the decrease is due to the job search channel, inclusive of the job posting effect, and the remaining 1/3 is due to the human capital channel. For the average worker, job-to-job transitions

are the primary method for generating wage growth and thus the most relevant margin for adjustment. Furthermore, interactions between human capital, search, and job posting amplify the decrease in wages, implying that a model that does not endogenize each channel will understate the decrease in the wage level. In particular, 15% of the decrease is due to a decrease in vacancy posting, implying that equilibrium effects are also significant.

Surprisingly, increasing tax progressivity has no effect on the variance of wages because the effects of the human capital and search channels offset one another. A human capital model predicts that an increase in tax progressivity will decrease the variance of wages; since an increase in progressivity has more "bite" at the top of wage distribution, workers with high wages are relatively more disincentivized from accumulating human capital. So, wages at the top of the distribution decrease more than wages at the bottom, and the gap between high wages and low wages decreases.

In contrast, in a job ladder model, an increase in tax progressivity increases wage dispersion. In my model (and as is typical in job ladder models), workers meet firms from a fixed productivity distribution. Over the lifecycle, workers progressively move to firms with more productivity, and workers eventually bunch up near the top of the distribution. With more tax progressivity, workers exert less search effort and make fewer job-to-job transitions. So, there is less bunching at high-productivity firms, and the the variance of wages is more spread. Simply put, since there are fewer job-to-job transitions, workers are more likely to remain an unproductive firms that would otherwise not be able to retain workers. Taken together, the result casts some doubt on the notion that increasing tax progressivity can decrease wage inequality.

Finally, I compare the results of my model with models where I turn off certain channels. In response to an increase in tax-and-transfer progressivity, I find that a pure human capital model (without the search channel) will understate the decrease in wage levels and overstate the decrease in lifecycle wage growth. The opposite is true for a pure job ladder model (without the human capital channel).

The remainder of the paper proceeds as follows. First, I contextualize my paper within the literature in Section 2. Section 3 presents the model. I analytically investigate the model mechanisms in Section 4. My calibration strategy is described in Section 5. I analyze the benchmark model in Section 6 before analyzing counterfactual taxation experiments in Section 7. Section 8 concludes.

# 2 Related Literature

To my knowledge, my model is the first to combine a life cycle, endogenous human capital accumulation, endogenous search effort, and endogenous job posting. There have been papers which combine subsets of these ingredients. My model is most similar to the model in Engbom (2022), which features endogenous human capital accumulation and job posting. In addition to the framework in Engbom (2022), my model includes two key features: endogenous search effort, and heterogeneity in learning ability and initial human capital. And while Engbom (2022) investigates wedges to the creation of new jobs, I investigate tax policy. I show that endogenous search effort is the more relevant margin for many workers and that a model without endogenous search effort will understate the effects of tax progressivity on wages.

Bowlus and Liu (2013) analyzes a model with endogenous human capital accumulation and endogenous search effort, but does not do policy analysis. I add firms which post jobs, which also requires that I adopt a wage bargaining scheme. The inclusion of the firm side allows for more careful counterfactual policy experiments, and I show endogenous job posting amplifies the effects of tax progressivity. Rauh and Santos (2022) build a search-and-matching model with human capital accumulation, endogenous job posting, and a carefully-calibrated government sector, and use the model to investigate transfer payments. However, human capital accumulation is exogenous and there is no on-the-job search.<sup>2</sup>

<sup>&</sup>lt;sup>2</sup>On the other hand, the model in Rauh and Santos (2022) includes incomplete asset markets, which is a potentially important feature.

I rely on well-established methods for modeling human capital accumulation and job search. This is intentional; it allows me to easily isolate the effects of their interaction. With regards to human capital accumulation, my model builds on Ben-Porath (1967) and Huggett et al. (2011). For modeling on-the-job search, the model borrows from Burdett and Mortensen (1998)<sup>3</sup> and Bagger et al. (2014). I model endogenous search effort and job posting as in Pissarides (2000) and Mortensen and Pissarides (1994). Finally, the wage bargaining protocol comes from Cahuc et al. (2006) and Bagger et al. (2014).

Quantitative human capital models, such as those in Huggett et al. (2011) and Badel et al. (2020), rely on idiosyncratic shocks to match wage dispersion. Such idiosyncratic shocks are thought to represent unemployment spells or job-to-job transitions. In a sense, my model provides an explicit which underlies those idiosyncratic shocks.

My paper also draws from a literature which decomposes wage growth and inequality between the human capital and search channels (Ozkan et al., 2023; Gregory, 2021; Bagger et al., 2014; Pavan, 2011; Carrillo-Tudela, 2012; Veramendi, 2012; Omer, 2004; Schönberg, 2007; Dustmann and Meghir, 2005).<sup>4</sup> In particular, my model resembles Ozkan et al. (2023) and Gregory (2021).<sup>5</sup> However, in my model, human capital accumulation, job finding rates, and labor market tightness are endogenously determined and, thus, variant to policy.

Since workers and firms can alter their behavior in response to policy changes, my model is a more appropriate setting for counterfactual experiments.

In my counterfactual experiments, I contribute to two strands of literature which study the effects of progressive taxes and transfers on lifecycle wage growth and inequality. The first strand of literature investigates labor taxation in models with endogenous on-the-job search and job posting but fixed human capital (Bagger et al., 2021, 2019; Sleet and Yazıcı, 2017; Kreiner et al., 2015; Gentry and Hubbard, 2004).<sup>6</sup> The second strand

<sup>&</sup>lt;sup>3</sup>See also Mortensen (2003).

<sup>&</sup>lt;sup>4</sup>Acabbi et al. (2023) further the literature by investigating the how human capital and job search interact over the business cycle.

<sup>&</sup>lt;sup>5</sup>Ozkan et al. (2023) studies sophisticated worker heterogeneity while Gregory (2021) studies firms as heterogeneously promoting human capital.

<sup>&</sup>lt;sup>6</sup>A related literature shows that some types of labor market regulation and redistribution may be efficient

investigates labor taxation in human capital models without labor market frictions (Badel et al., 2020; Guvenen et al., 2014; Blandin, 2018; Kapička, 2015, 2006).<sup>7</sup> I show that these channels interact in meaningful ways in response to a change in tax policy and that a model without both will misstate the effects of tax progressivity.

## 3 Model

My focus is the stationary equilibrium. I define the recursive stationary equilibrium in Appendix C.

## 3.1 Life Cycle

For computational efficiency, the model is in continuous time. I model overlapping generations using a stochastic lifecycle.<sup>8</sup> There are I+1 stages of life, I working stages and a retirement stage. For  $i \in \{1, 2, ..., I\}$ , workers transition from stage i to i+1 with probability  $\zeta$ . Retired workers die with probability  $\overline{\zeta}$  after which they are replaced by newborns in the first working stage. To fix ideas, when I calibrate the model, I set I=4 and calibrate the working part of life to ages 23 to 65. So, each stage of working life is approximately a decade.

# 3.2 Wages

Workers are heterogeneous in human capital h and firms are heterogeneous in productivity p. A match between a worker with human capital h and a firm with productivity p produces hp of the numeraire consumption good. Note that human capital and firm productivity are complements in production. Of the total production of the match, work-

in a frictional labor market (Cubas and Silos, 2020; Lise et al., 2016).

<sup>&</sup>lt;sup>7</sup>In these papers, workers make human capital decisions each period. One can also study progressive taxation in a model with an endogenous human capital/education choice which only occurs at the beginning of the life cycle (Heathcote et al., 2020, 2017; Esfahani, 2020; Krueger and Ludwig, 2016).

 $<sup>^8</sup>$ One can think of this structure as perpetual youth with multiple stages.

ers earn a piece-rate  $r \in [0,1]$ . Therefore, before taxes and transfers, the worker earns the wage hpr and the firm earns profit hp(1-r). r is endogenously negotiated between workers and firms as described below.

## 3.3 Utility

Workers discount the future at discount rate  $\rho$ . They enjoy utility from consumption where consumption is hand-to-mouth. In other words, consumption equals the worker's after-tax-and-transfer wage. (In other words, workers are hand-to-mouth). I assume that utility of consumption is logarithmic,  $\ln(c)$ . Given the wage hpr and the average tax rate function T(hpr) (explained below), c = [1 - T(hpr)]hpr.

Workers are employed or unemployed. Employed workers choose how much effort to invest in human capital accumulation, *l* (for "learning"), and/or job search, *s*. Unemployed workers can search but cannot accumulate human capital. Both types of investment are costly in that workers experience disutility from effort.

Disutility of effort is a convex function. For the employed, disutility of effort is  $\phi(l+s)^{1+\gamma_E}$  with  $\phi>0$  and  $\gamma_E>0$ , and for the unemployed, disutility of effort is  $\phi s^{1+\gamma_U}$  with  $\gamma_U>0$ . So, unemployed workers are subject to the same disutility function with a different curvature parameter and no possibility for human capital investment.<sup>9</sup>

# 3.4 Human Capital Accumulation

Workers accumulate human capital as in Ben-Porath (1967). Starting from birth, workers are heterogeneous in fixed learning ability, a > 0. Human capital evolves according to the law of motion

$$\frac{dh}{dt} = a(lh)^{\omega} - \delta h \tag{1}$$

<sup>&</sup>lt;sup>9</sup>The fact that employed and unemployed workers have a different curvatures of disutility reflects that unemployed workers value their time differently than employed workers. Later, I estimate  $\gamma_U > \gamma_E$ , which means that unemployed workers can exert relatively more search effort before their disutility of effort gets prohibitively steep. Therefore, unemployed workers will tend to exert more search effort and get more job offers. This modeling choice ultimately allows the model to replicate higher job finding rates for unemployed workers than employed workers.

where  $\omega \in (0,1)$  governs the level of decreasing returns to human capital accumulation and  $\delta \in [0,1]$  is the human capital depreciation rate. Thus, a is the worker's efficiency of human capital accumulation. All else equal, workers with a higher a have greater returns to human capital investment.

#### 3.5 Labor Market Frictions

All workers, employed and unemployed, meet open vacancies at rate  $sm(\theta)$  where s is search effort as described above,  $m(\theta)$  is a meeting function, and  $\theta$  is the labor market tightness ratio. Therefore, when a worker exerts search effort, he increases his probability of meeting an outside firm.  $\theta$  is defined as the number of vacancies  $\theta$  per unit of aggregate search effort  $\theta$ ,  $\theta = v/s$ . Search is random in the sense that workers and firms randomly meet in a single market and neither party can direct their search toward certain types of firms or workers.

Upon meeting an open vacancy, the worker observes the firm's productivity, p, drawn from a distribution F(p) over  $[\underline{p}, \overline{p}]$ . The worker and firm then bargain over the wage. Since h and p are fixed at this point, wage bargaining is over the piece rate r. They will form a match if the worker is better off with the new firm relative to his outside option.

Firms post and maintain jobs at cost  $\kappa$ . From the firm's perspective, an open job vacancy meets a worker with probability  $m_f(\theta)$ . Matches are subject to exogenous job destruction shocks which are a decreasing function of wage,  $\Lambda(hpr)$ . I assume that the job destruction rate depends on the wage for two reasons: (1) it is a clear feature in the data, and (2) the increased risk of unemployment for low-wage workers is key for accounting for increasing wage dispersion over the life cycle (Ozkan et al., 2023; Jarosch, 2023).

<sup>&</sup>lt;sup>10</sup>Mukoyama et al. (2018) empirically show that greater search effort (in the form of time spent searching for a job) is significantly and positively correlated with the probability of finding a job.

 $<sup>^{11}</sup>S$  is mathematically defined in Appendix A.1 Equation (23), but it does not need to be solved in computing the model; the relevant object is  $\theta$ .

## 3.6 Government

I model both income taxes and means-tested transfers in a single average tax equation with the functional form from Bénabou (2002). At wage *hpr*, workers pay the average tax rate

$$T(hpr) = 1 - \tau_0 (hpr)^{-\tau_1}$$
 (2)

which subsumes both income taxes paid and means-tested transfers received. It is possible that workers with low wages pay negative taxes. If so, it implies that transfer payments exceed taxes paid. Going forward, I refer to Equation (2) as a tax function for simplicity, though it should be understood that the function is carefully calibrated to combine progressive taxation and means-tested transfers.

The two parameters of the tax function are easily interpretable:  $\tau_1$  determines the progressivity of the tax and transfer system while  $\tau_0$  determines its level. Later in the paper, I will experiment with adjusting  $\tau_1$ , and I let  $\tau_0$  adjust so that the government budget constraint holds in equilibrium.

Unemployed workers earn unemployment benefits B(w) where w is the most recent wage the worker earned before becoming unemployed. Unemployment benefits expire with probability  $\chi$ , after which the worker gets a transfer payment  $T_0$ . I denote whether unemployed workers qualify for unemployment benefits with the indicator  $\iota \in \{0,1\}$ . Unemployed workers pay taxes on their unemployment benefits while unemployment workers without unemployment benefits do not pay taxes.

Finally, retired workers receive a flat social security payment SS.

# 3.7 Wage Bargaining

I use the wage bargaining protocol of Cahuc et al. (2006) as applied in Bagger et al. (2014). The protocol determines the endogenous values of r as well as the worker's reservation strategies for accepting a new job. Every resulting object is a function which depends on the characteristics of the worker and firm(s). This section develops the functions. Behind

every function is a surplus sharing rule where, if the worker has the opportunity to go to a new firm, he extracts the total surplus from his outside option plus a fraction  $\eta \in [0,1]$  of the additional worker surplus at the winning firm.

Let  $E_{ai}(h, p, r)$  be the value of employment for a worker with learning ability a, in stage of life i, with human capital h, and currently working for a firm with productivity p at the piece rate r. I define  $E_{ai}(h, p, r)$  mathematically in Section 3.8. Currently, the worker earns the wage hpr. The greatest wage that the worker can earn while working at the current firm is hp, the wage when r = 1.

If r = 1, then the firm makes zero profit. And in equilibrium, the value of the firm's outside option is zero. So, if r = 1, the firm is indifferent toward continuing the match.

There are two scenarios where wage bargaining arises: when an employed worker meets an outside firm, and when an unemployed worker meets a firm. Consider first the scenario of a meeting between an employed worker with value  $E_{ai}(h, p, r)$  and an outside firm with productivity p'. The incumbent firm and the outside firm will commence Bertrand competition for the worker, making alternating bids on the worker's wage. As the firms offer progressively higher wages, they reach a point where the firm with lower productivity cannot pay the worker a higher wage without earning negative profit. For the firm with lower productivity, that wage is given by r = 1. At that point, the firm with higher productivity can offer a marginally larger wage and win the worker.

Within this scenario, there are three possible outcomes. First, suppose that the outside firm has greater productivity that the incumbent firm, p' > p. In this case, the worker will be poached and will make a job-to-job transition to the outside firm. The poaching firm will pay the worker a piece rate  $R_{ai}^P(h, p, p')$  which solves

$$E_{ai}\left(h, p', R_{ai}^{P}(h, p, p')\right) = E_{ai}(h, p, 1) + \eta \left[E_{ai}(h, p', 1) - E_{ai}(h, p, 1)\right]. \tag{3}$$

After getting poached by the firm with productivity p', the worker earns a piece rate  $r = R_{ai}^P(h, p, p')$  such that the value of employment equals the maximum surplus possible from the incumbent firm,  $E_{ai}(h, p, 1)$ , plus a fraction  $\eta$  of the additional worker surplus

from the match.<sup>12</sup>

Now, suppose that the outside firm has lower productivity than the incumbent firm, p' < p. Regardless of the value of p', the incumbent firm will retain the worker because it can pay the worker a greater wage while remaining profitable. However, if the outside firm has a high enough p', it is possible that the outside firm could pay the worker a greater wage than it earns now. This is the second possible outcome. In this case, the outside firm triggers a renegotiation between the worker and the incumbent firm. Similarly, the firms will make alternating bids over the worker until the firm with lower productivity (in this case, the outside firm) earns zero profit. The worker will stay at the current firm but get wage increase; the worker will earn a piece rate  $R_{ai}^R(h, p, p')$  which solves

$$E_{ai}(h, p, R_{ai}^{R}(h, p, p')) = E_{ai}(h, p, 1) + \eta \left[E_{ai}(h, p, 1) - E_{ai}(h, p', 1)\right]. \tag{4}$$

Now, the worker's outside option is the maximum surplus from the unsuccessful outside firm. The worker earns a piece rate  $r = R_{ai}^R(h, p, p')$  such that value of employment is the maximum that the outside firm could offer,  $E_{ai}(h, p', 1)$ , plus a fraction of the additional worker surplus.

The third and final outcome is that the outside firm has a lower productivity than the incumbent firm, p' < p, but the outside firm cannot pay the worker a greater wage even if it offers the worker the maximum r = 1. In this case, the worker ignores the outside firm and stays at the current firm at the same piece rate.

 $q_{ai}(h, p, r)$  defines the minimum p' such that, if the worker meets an outside firm

 $<sup>^{12}</sup>$ A slight clarification is in order. The worker earns a fraction  $\eta$  of the additional *potential worker* surplus from the match, not the additional *total* surplus from the match. This is slightly different from the original scheme in Cahuc et al. (2006) which uses a version of the Rubinstein (1982) infinite-horizon alternating-offers bargaining game. In Cahuc et al. (2006), workers and firms have linear utility over the wage. Thus,  $E_{ai}(h,p,1)$  is equivalent to the total surplus of the match, regardless of how the total surplus is shared. So, Equation (3) states that the worker earns a fraction  $\eta$  of the additional total surplus of the match. The same is true in Engbom (2022). However, as Bagger et al. (2014) points out, when workers have curvature in utility and firms have linear utility, the total amount of surplus from the match is not independent of r and therefore not fixed, and the present scheme may not be a Nash equilibrium. My case is further complicated by the fact that workers also have disutility over effort. I elect to follow Bagger et al. (2014) and impose this wage structure even with curvature in utility and disutility over effort. For an approach which uses total surplus but allows for curvature in utility, see Lise et al. (2016) or Gregory (2021).

with  $p' \in [q_{ai}(h, p, r), p]$ , the meeting will trigger a renegotiation with the current firm.  $q_{ai}(h, p, r)$  solves

$$E_{ai}(h, p, r) = E_{ai}(h, q_{ai}(h, p, r), 1) + \eta \left[ E_{ai}(h, p, 1) - E_{ai}(h, q_{ai}(h, p, r), 1) \right].$$
 (5)

To summarize the scenario where an employed worker meets an outside firm: If p' > p, the worker is poached, and the worker earns the piece rate  $R_{ai}^P(h, p, p')$ . If  $q_{ai}(h, p, r) < p' < p$ , the worker stays at their current firm but leverages the outside offer into a greater piece rate  $R_{ai}^R(h, p, p')$ . And if  $p' < q_{ai}(h, p, r)$ , the outside firm cannot compete with the current firm and the worker stays with the same firm at the same piece rate.

Next, consider the scenario of a meeting between an unemployed worker and a firm with productivity p'. In this case, the firm is not competing against another firm, but rather is competing against the worker's outside option of remaining unemployed. Let  $U_{ai}(h, w, \iota)$  be the value of unemployment for a worker with learning ability a, in stage of life i, with human capital h, who previously earned the pre-tax wage w, and is eligible for unemployment benefits with indicator  $\iota$ . Suppose that the outside firm has a high enough p' such that it can make an offer which will entice the worker to leave unemployment and form a match. The worker will earn the piece rate  $R_{ai}^U(h, w, \iota, p')$  which solves

$$E_{ai}(h, p', R_{ai}^{U}(h, w, \iota, p')) = U_{ai}(h, w, \iota) + \eta \left[ E_{ai}(h, p', 1) - U_{ai}(h, w, \iota) \right].$$
 (6)

The piece rate is set such that the worker gets the value of his outside option,  $U_{ai}(h, w, \iota)$ , plus a fraction  $\eta$  of the additional worker surplus from the match.

It is possible that a firm cannot make the unemployed worker better off, even if the firm pays the worker r = 1. Let  $z_{ai}(h, w, \iota)$  be the lowest value of p' which will entice the

worker to leave unemployment.  $z_{ai}(h, w, \iota)$  solves<sup>13</sup>

$$U_{ai}(h, w, \iota) = E_{ai}(h, z_{ai}(h, w, \iota), 1).$$
 (7)

## 3.8 Hamilton-Jacobi-Bellman Equations for Workers

The value of employment for a worker with learning ability a, in stage of life i, with human capital h, and currently working for a firm with productivity p at the piece rate r solves

$$\rho E_{ai}(h, p, r) = \max_{l,s} u(c) - d_{E}(l+s) + (a(lh)^{\omega} - \delta h) \frac{\partial E_{ai}(h, p, r)}{\partial h} 
+ \Lambda(hpr) \left[ U_{ai}(h, hpr, 1) - E_{ai}(h, p, r) \right] + \zeta \left[ E_{a,i+1}(h, p, r) - E_{ai}(h, p, r) \right] 
+ sm(\theta) \left( \int_{q_{ai}(h, p, r)}^{p} \left[ E_{ai} \left( h, p, R_{ai}^{R}(h, p, p') \right) - E_{ai}(h, p, r) \right] dF(p') \right) 
+ \int_{p}^{\overline{p}} \left[ E_{ai} \left( h, p', R_{ai}^{P}(h, p, p') \right) - E_{ai}(h, p, r) \right] dF(p') \right)$$
(8)

subject to

$$c = [1 - T(hpr)]hpr. (9)$$

An employed worker chooses how much effort to invest in human capital accumulation l and search s. The worker consumes the value of his after-tax-and-transfer wage and experiences the disutility associated with human capital accumulation and search effort. He accumulates human capital as in Equation (1). With probability  $\Lambda(hpr)$ , the match is destroyed and the worker becomes unemployed with unemployment benefits. With probability  $\zeta$ , the worker ages to the next stage of life. And with probability  $sm(\theta)$ , the

$$U_{ai}(h,w,\iota) = U_{ai}(h,w,\iota) + \eta \left[ E_{ai}\left(h,z_{ai}(h,w,\iota),1\right) - U_{ai}(h,w,\iota) \right],$$

which simplifies to Equation (7).

<sup>&</sup>lt;sup>13</sup>The algebra is as follows: if  $p' = z_{ai}(h, w, \iota)$ , then unemployed worker is indifferent between remaining unemployed and working for the firm at the highest possible wage. So,

worker meets an outside firm with productivity p' drawn from the distribution F(p'). If  $p' \in (q_{ai}(h, p, r), p]$ , the worker leverages the outside offer and renegotiates a higher wage with the incumbent firm. If  $p' \in (p, \overline{p}]$ , the worker is poached.

The value of unemployment for a worker with learning ability a, in stage of life i, with human capital h, who previously earned the pre-tax wage w, and earns unemployment benefits with indicator  $\iota$  solves

$$\rho U_{ai}(h, w, \iota) = \max_{s} u(c) + d_{U}(s) - \delta h \frac{\partial U_{ai}(h, w, \iota)}{\partial h} + \zeta \left[ U_{a,i+1}(h, w, \iota) - U_{ai}(h, w, \iota) \right] + \chi \left[ U_{ai}(h, w, \iota) - U_{ai}(h, w, 0) \right] + sm(\theta) \int_{z_{ai}(h, w, \iota)}^{\overline{p}} \left[ E_{ai} \left( h, p', R_{ai}^{U}(h, w, p') \right) - U_{ai}(h, w, \iota) \right] dF(p')$$
(10)

subject to

$$c = \iota \left[ 1 - T(B(w)) \right] B(w) + (1 - \iota) T_0 \tag{11}$$

An unemployed worker chooses how much effort to invest in search, enjoys utility from consumption, endures disutility from search effort, and experience human capital depreciation. The worker's consumption level depends of if he still qualifies for unemployment benefits. With probability  $\zeta$ , the worker ages to the next stage of life, and with probability  $\chi$ , his unemployment benefits expire (if they have not already). With probability  $sm(\theta)$ , the worker meets an outside firm with productivity p', and if  $p \in (z_{ai}(h, w, \iota), \overline{p}]$ , the worker will accept a job from the firm.

Finally, the value of retirement,  $\overline{E}$ , solves

$$\left(\rho + \overline{\zeta}\right)\overline{E} = u(SS). \tag{12}$$

Employed and unemployed workers transition to retirement according to

$$E_{a,I+1}(h,p,r) = \overline{E}. {13}$$

and

$$U_{a,I+1}(h,w,\iota) = \overline{E}. \tag{14}$$

## 3.9 Firms

Firms are modeled as one worker-one job matches. Firms have linear utility over after-tax profit. There is a flat tax on profits  $\tau_b$ , so after-tax profits are  $(1 - \tau_b)hp(1 - r)$ .

I assume free entry in the labor market. In equilibrium, firms post a quantity of vacancies such that firms are indifferent toward to the marginal job posting. The optimal level of job vacancies will depend on the distribution of workers in the economy. All else equal, if all workers in the economy increase their human capital, then, because h and p are complements, filled jobs become more profitable, the benefit of job posting increases. So, firms will post more jobs. Similarly, if workers increase search effort, then firms have a greater probability of converting an open vacancy to a filled job, the benefit of job posting increases, and firms will post more jobs.

Let  $l_{ai}(h, p, r)$ ,  $s_{Eai}(h, p, r)$ , and  $s_{Uai}(h, w, \iota)$  denote the policy functions for employed workers' human capital investment, employed workers' search effort, and unemployed workers' search effort, respectively. For a firm with productivity p, the value of a filled job with a worker of learning ability a, in stage of life i, with human capital h, and with the worker earning the piece rate r solves

$$\rho J_{ai}(h, p, r) = (1 - \tau_b) h p (1 - r) + (a(l_{ai}(h, p, r)h)^{\omega} - \delta h) \frac{\partial J_{ai}(h, p, r)}{\partial h} + \zeta \left[ J_{a,i+1}(h, p, r) - J_{ai}(h, p, r) \right] + s_{Eai}(h, p, r) m(\theta) \int_{q_{ai}(h, p, r)}^{p} \left[ J_{ai}\left(h, p, R_{ai}^{R}(h, p, p')\right) - J_{ai}(h, p, r) \right] dF(p') + \left[ s_{Eai}(h, p, r) m(\theta) \left( F(\overline{p}) - F(p) \right) + \Lambda(hpr) \right] (-J_{ai}(h, p, r))$$
(15)

Except for the last line, Equation (15) closely resembles Equation (8).

If the match is destroyed, the firm is left with zero profit. There are three ways the match can be destroyed: (1) the worker is poached by a firm with higher productivity,

which occurs if the worker meets an outside firm with greater productivity at probability  $s_{Eai}(h, p, r)m(\theta)$  ( $F(\overline{p}) - F(p)$ ); (2) the job is exogenously destroyed at rate  $\Lambda(hpr)$ ; or (3) the worker retires. The latter is made explicit by

$$J_{a,I+1}(h,p,r) = 0. (16)$$

Let  $\Psi_E(h, p, r|a, i)$ ,  $\Psi_U(h, w|a, i, \iota)$ , and  $\Psi_R(a)$  denote the distributions of employed, unemployed, and retired workers, respectively. These distributions are defined such that

$$1 = \sum_{a,i} \int_{\Psi_E} d\Psi_E(h,p,r|a,i) + \sum_{a,i,\iota} \int_{\Psi_U} d\Psi_U(h,w|a,i,\iota) + \sum_a \Psi_R(a).$$

The free entry condition is

$$\kappa = m_{f}(\theta) \left[ \sum_{a,i} \int_{\Psi_{E}} s_{Eai}(h,p,r) \int_{p}^{\overline{p}} J_{ai} \left( h, p', R_{ai}^{P}(h,p,p') \right) dF(p') d\Psi_{E}(h,p,r|a,i) \right. \\
\left. + \sum_{a,i,\iota} \int_{\Psi_{U}} s_{Uai}(h,w) \int_{z_{ai}(h,w)}^{\overline{p}} J_{ai} \left( h, p', R_{ai}^{U}(h,w,p') \right) dF(p') d\Psi_{U}(h,w|a,i,\iota) \right].$$
(17)

The left had side of Equation (17),  $\kappa$ , is the cost of posting and maintaining a vacancy. The right hand side is the expected benefit of posting a vacancy. It consists of two terms, both multiplied by the  $m_f(\theta)$  (the probability that an open job vacancy meets a worker). The first term is the probability and expected value of poaching an employed worker; the second is the probability and expected value of hiring an unemployed worker. In equilibrium, the cost of posting a vacancy equals the benefit.

# 3.10 Government Budget Constraint

When I perform counterfactual experiments with tax policy, I discipline the model such that a government budget constraint must hold in equilibrium. Mathematically, the gov-

ernment budget constraint is

$$\sum_{a,i} \int_{\Psi_{E}} T(hpr)hpr \, d\Psi_{E}(h,p,r|a,i) + \sum_{a,i} \int_{\Psi_{U}} T(bw)bw \, d\Psi_{U}(h,w|a,i,\iota = 1) 
+ \sum_{a,i} \int_{\Psi_{E}} \tau_{b}(1-r)hp \, d\Psi_{E}(h,p,r|a,i) = \sum_{a,i} \int_{\Psi_{U}} bw \, d\Psi_{U}(h,w|a,i,\iota = 1) 
+ \sum_{a,i} \int_{\Psi_{U}} T_{0} \, d\Psi_{U}(h,w|a,i,\iota = 0) + \sum_{a} \Psi_{R}(a)SS + \overline{g}.$$
(18)

The left hand side consists of tax revenue from employed workers, unemployed workers, and firms. Strictly speaking, since those at the low end of the wage distribution pay negative taxes (receive means-tested transfers), there are also government outlays in the left hand side of (18). The right hand side consists of other outlays in the form of unemployment benefits, transfers to workers without unemployment benefits, social security payments, and public consumption,  $\overline{g}$ . I calculate  $\overline{g}$  in my benchmark economy such that the government budget constraint holds, and I assume that the government must spend  $\overline{g}$  in counterfactual experiments.

# 4 Mechanisms in Simplified Model

Before proceeding to quantitative exercises, I analytically inspect the economics of the worker's problem. To do so, I use slightly simplified discrete-time two-period model. Suppose workers live for two periods and discount the second period at discount factor  $\beta$ . They experience utility from consumption both periods, but can only invest in human capital accumulation or job search in the first period. Workers earn a constant piece rate r=1 and there is no human capital depreciation. With probability  $sm(\theta)$ , workers move from firm productivity p to  $p+\Delta_p$ . I write the worker's problem in Appendix B.

For a worker in the first period of life with the first order conditions state that the

optimal choices for human capital effort *l* and search effort *s* solve

$$\phi(1+\gamma)(l+s)^{\gamma} = \frac{\beta(1-\tau_1)\omega}{(lh)^{1-\omega}/a+l} = \frac{\beta(1-\tau_1)}{p/(m(\theta)\Delta_p)+s}.$$
 (19)

The left hand side of this equation is the marginal cost of investing in human capital or search effort,<sup>14</sup> the middle part is the marginal benefit of human capital accumulation, and the right hand side is the marginal benefit of search effort. When the worker is behaving optimally, all three are equal. Note that tax progressivity,  $\tau_1$ , enters directly in the marginal benefit of human capital accumulation and search effort.<sup>15</sup>

Consider two otherwise identical workers where one has greater learning ability than the other,  $a_1 > a_0$ . The marginal benefit of human capital investment, the middle part of Equation (19), is greater for the worker with high a. In order for Equation (19) to hold, the worker must increase l and decrease the middle part. However, by increasing l, the worker increases the marginal cost of effort on the left hand side. Thus, for Equation (19) to hold, the worker must also decrease s. In all cases, the worker will not adjust l or s without adjusting the other.

It can be shown that the optimal decision rules for the two agents satisfy  $l_1 > l_0$ ,  $s_1 < s_0$ , and  $l_1 + s_1 > l_0 + s_0$ . In words, the agent with more learning ability will invest more in human capital accumulation, less in search, and more overall. So, the worker with greater learning ability will substitute away from search toward human capital, but the substitution is imperfect (the decrease in l exceeds the decrease in s.) Within the period, it follows that human capital and job search investment are akin to imperfect substitutes.

Compared to a pure human capital model, endogenous search effort has an equalizing force in my model. Instead of struggling to accumulate human capital, workers with low learning ability can direct their efforts toward on-the-job search.

<sup>&</sup>lt;sup>14</sup>Because disutility is simply a function of l + s, the marginal cost of investing in either activity will always be equivalent.

<sup>&</sup>lt;sup>15</sup>Note that the tax level,  $\tau_0$ , has no effect on worker decisions in the simplified model. This is a result of log utility of consumption. Similar to how the tax level does not affect a labor supply decision with log utility, if the tax level changes, the income and substitution effects perfectly offset each other (Boskin, 1975).

<sup>&</sup>lt;sup>16</sup>The mathematical proof comes from the fact that all other possibilities lead to a contradiction.

In a dynamic sense, there is also some complementarity at work in Equation (19). Consider two otherwise identical workers where one worker works for a more productive firm,  $p_1 > p_0$ . It can be shown that  $l_1 > l_0$ ,  $s_1 < s_0$ , and  $l_1 + s_1 < l_0 + s_0$ . In words, the worker at the more productive firm will invest more in human capital accumulation, less in search, and less overall. Recall that h and p are complements in production. So, at a more productive firm, the benefit of increasing human capital is greater. Thus, the worker at the high-productivity firm is incentivized to invest in h such that the level of h "catches up" with the level of p. With symmetry, the same logic holds for differences in h. Therefore, contrary to the substitution story above, there is reason to believe that workers with high learning ability — and, therefore, high human capital — may work at more productive firms.

Finally, consider otherwise identical economies except that tax progressivity,  $\tau_1$ , is greater in one economy. If  $\tau_1$  is greater, then the marginal benefits of human capital and search investment both decrease. The result is that both l and s will be lower in the economy with higher tax progressivity. More progressive taxes discourage wage growth in all of its forms because, either way, when the worker's wage increases, the benefit of the increased wage is smaller.

## 5 Calibration

#### 5.1 Data

To parameterize the model, I rely on microdata from the Survey of Income and Program Participation (SIPP). The SIPP is a panel data set with interviews every four months.<sup>17</sup> Within interviews, respondents report on what occurred in the months between interviews. I use every SIPP panel between the years 1990 and 2019, 12 panels in total.<sup>18</sup>

Two features of the SIPP make it convenient for my setting. First, respondents report

<sup>&</sup>lt;sup>17</sup>In 2018, the SIPP transitioned from interviewing respondents every four months to interviewing respondents every year with an event history design. I use three panels with this design.

<sup>&</sup>lt;sup>18</sup>I use the 1990, 1991, 1992, 1993, 1996, 2001, 2004, 2008, 2014, 2018, 2019, and 2020 panels.

their earnings and hours every month, which allows me to observe wage growth across months. Second, the SIPP tracks worker-job matches, which allows me to observe job switches. I convert monthly transfer rates (unemployment to employment, employment to unemployment, and job switching) to continuous-time arrival rates and correct for time aggregation bias using the methods developed in Mukoyama (2014).

I restrict the data to males between the ages of 23 and 65 who are never out of the labor force, I convert earnings data to hourly wages, and drop the self-employed. Observations with wages which are below the federal minimum wage are dropped. For every statistic, I calculate the weighted mean within the panel using panel weights, then take the weighted mean across panels where each panel is weighted by the number of months is covers. I drop the first two and last two months of each panel. Though the SIPP has weekly labor force indicators, I aggregate to a monthly frequency and use the labor force indicator for the second week of the month to mirror the CPS.<sup>19</sup>

## 5.2 Functional Forms and Distributions

I use a Cobb-Douglas matching function. From the perspective of a worker, the probability of meeting an open vacancy per unit of search effort is  $m(\theta) = \xi \theta^{1-\alpha}$  with  $\xi > 0$  and  $\alpha \in (0,1)$ . For a firm, the probability of meeting a worker is  $m_f(\theta) = \xi \theta^{-\alpha}$ .

I assume that the distribution of firm productivity, F(p), is Pareto with level parameter  $\mu_p$  and tail parameter  $1/\lambda_p$ . The distributions for learning ability and initial human capital follow Badel et al. (2020). Learning ability a is drawn from a Pareto lognormal distribution,  $a \sim PLN(\mu_a, \sigma_a, 1/\lambda_a)$ , where  $\mu_a$  is the level parameter,  $\sigma_a$  is the dispersion parameter, and  $1/\lambda_a$  is the tail parameter.<sup>20</sup> The distribution of initial human capital,  $h_0$ , is a linear function of a,

$$\ln(h_0) = \beta_0 + \beta_1 \ln(a) + \ln(\varepsilon)$$

<sup>&</sup>lt;sup>19</sup>Unemployment-to-employment and employment-to-unemployment rates in the SIPP are consistently lower than the familiar values in the CPS. See the discussion in Footnote 16 in Menzio et al. (2016).

<sup>&</sup>lt;sup>20</sup>There is more than one type of Pareto lognormal distribution (Hajargasht and Griffiths, 2013). I use the type which consists of a lognormal distribution with a Pareto right tail.

with  $\varepsilon \sim LN(0, \sigma_{\varepsilon})^{2}$ . Appendix D.2 documents how I discretize the distribution of a.

To help the model fit the data, I make one final adjustment and allow for "godfather shocks." With probability  $\psi$ , an employed worker experiences a godfather shock, which means that he meets an outside firm and must accept a job at the new firm.<sup>22</sup> Though ad-hoc, godfather shocks are useful for generating one feature in the data: of workers who switch jobs, 30% earn a lower wage in their new job compared to their old job. In my model, it is impossible for a worker to make a job-to-job transition without a wage increase.<sup>23</sup> Quantitatively, I estimate a small value for  $\psi$ , but it is very helpful in matching this fact. All equations which are affected by godfather shocks are updated in Appendix A.2.

#### 5.3 Taxes and Transfers

My goal in calibrating taxes and transfers is to accurately represent the US system while maintaining a simple structure. I therefore estimate Equation (2) such that it fits both the progressive income tax and means-tested transfer systems in the US. My strategy consists of borrowing an estimated average tax function and an estimated means-tested transfer function from the literature, then re-estimating Equation (2) on the combination of both functions. Each function is over wage *y* normalized such that the mean wage is one.

For taxes, I use an estimate of federal income taxes from Guner et al. (2014) for all households (married and unmarried) which takes the earned income tax credit (EITC) into account.<sup>24</sup> To this function, I add a flat state and local income tax rate of 5%.<sup>25</sup> Using these estimates, the average tax rate is

$$Tax(y) = -0.294 + 0.382y^{0.164} + 0.05.$$

 $<sup>^{21}</sup>$ Badel et al. (2020), in the appendix, show that this implies that  $h_0$  is also distributed Pareto lognormally.  $^{22}$ The worker is made "an offer he can't refuse."

<sup>&</sup>lt;sup>23</sup>See Dorn (2018) and Tjaden and Wellschmied (2014) for investigations into job-to-job transitions with a wage decrease, and see Moscarini and Postel-Vinay (2018) for an example of godfather shocks in use.

<sup>&</sup>lt;sup>24</sup>I use the power function specification in Table A5 in the appendix.

<sup>&</sup>lt;sup>25</sup>I take the value of 5% from (Guner et al., 2022). The notion of a flat state and local income tax rate is supported by Fleck et al. (2021).

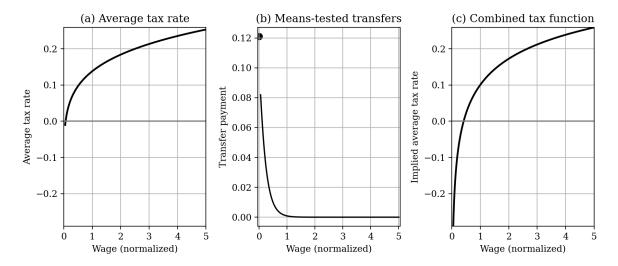


Figure 1: Combined tax function and its ingredients

Panel (a): average tax rate function as estimated in Guner et al. (2014). Panel (b): means-tested transfer function as estimated in Guner et al. (2023). Panel (c): combined average tax function with combines panels (a) and (b) and re-estimates an average tax function with form of Equation (2). Wage is normalized such that the mean wage equals one.

The average tax function is plotted in Panel (a) of Figure 1.

For transfers, I use an estimated means-tested transfer function for all households from (Guner et al., 2023). The means-tested transfer function takes the following programs into account: WIC (the Special Supplemental Nutrition Program for Women, Infants, and Children), SSI (Supplemental Security Income, for those with disabilities), SNAP (Supplemental Nutrition Assistance Program, formerly known as Food Stamps), TANF (Temporary Assistance for Needy Families), and housing. For a discussion of these programs, see Guner et al. (2023). The estimated transfer function is

$$Tr(y) = \begin{cases} e^{-2.122}e^{-4.954y}y^{0.044}, & y > 0\\ 0.121, & y = 0 \end{cases}$$
 (20)

and is plotted in Panel (b) of Figure 1. I use the top line in my estimation of a tax-and-transfer function; I use 0.121 to parameterize transfers for the unemployed without unemployment benefits,  $T_0$ .

When I combine taxes and transfers and estimate the parameters of Equation (2), I get  $\tau_0 = 0.899$  and  $\tau_1 = 0.120^{.26}$  The new tax function is plotted in Panel (c) of Figure 1. Comparing Panel (c) and Panel (a), it is clear that including transfers significantly increases progressivity.

## 5.4 External Parameters

External parameter choices are summarized in Table 1. The model is period is monthly. Working life is ages 23 to 65, 42 years total, and I set the number of working stages of life to I = 4. So, each stage is 42/4 = 10.5 years, or 126 months, and the rate of transitioning from one stage of life to the next is  $\zeta = 1/126$ . Retirement is 10 years, so  $\overline{\zeta} = 1/(10 \times 12)$ .

Unemployment benefits pay a fraction b of the previous wage up to a maximum,  $\bar{b}$ . So,  $B(w) = \min \left\{ bw, \bar{b} \right\}$ . In accordance with typical unemployment benefits the US, I set b = 0.5 and  $\bar{b} = 0.5$ . The expiration rate of unemployment benefits is  $\chi = 1/6$  in keeping with the typical rule that unemployment benefits can be collected for a maximum of six months. I normalize SS = .5. The flat tax rate on firm profit is  $\tau_b = 0.243$ , the estimated average tax on business income in Cooper et al. (2016).

As for other parameters, I set  $\alpha=0.5$  (Petrongolo and Pissarides, 2001) and  $\eta=0.4$  (Bagger et al., 2014).<sup>28</sup>  $\kappa$  is chosen such that equilibrium  $\theta$  is normalized to one. I set  $\rho=0.0033$  in accordance with a 4% risk-free annual real interest rate, and I set  $\delta=0.00116$  to match the decline in wages at the end of life.<sup>29</sup>  $\beta_0$  is normalized such that the lowest possible  $h_0$  is the bottom point on the human capital grid.

Finally, I estimate the job loss function  $\Lambda(hpr)$  directly from the SIPP.  $\Lambda(hpr)$  is presented graphically in Appendix D.1.

<sup>&</sup>lt;sup>26</sup>These estimates are similar to Holter et al. (2019).

<sup>&</sup>lt;sup>27</sup>The choice of *SS* is immaterial to my results. If social security payments are equal across workers and workers have no choice but to transition to retirement eventually, retirement does not affect choices during working life.

<sup>&</sup>lt;sup>28</sup>Engbom (2022) estimates a similar value for  $\eta$ .

<sup>&</sup>lt;sup>29</sup> Assume that workers do not invest in human capital in the last stage of life, i=I. Without job switching or unemployment, wages are multiplied by  $1-\delta$  every month. If there are x months in the last stage of life, then  $\frac{w_I}{w_{I-1}}=(1-\delta)^x$ . Since each working stage is 126 months on average, and given that  $w_I/w_{I-1}=0.864$  in the data, we have  $\delta=0.00116$ .

Table 1: Externally calibrated parameters

| Parameter          | Meaning  | Value                                    | Explanation/source   |
|--------------------|--|--|--|
| Lifecycle          |  |  |  |
| I                  | Stages of life                                   | 4  | By choice  |
| ζ                  | Transition pobability from one stage to the next | $\left(\frac{42\times12}{I}\right)^{-1}$ | Working for 42 years on average (ages 23-65)                         |
| $\overline{\zeta}$ | Probability of death for the retired             | $(10 \times 12)^{-1}$                    | Retired for 10 years on average                                      |
| Policy             |  |  |  |
| $	au_0$            | Tax+transfer progressivity                       | 0.899                                    | Guner et al. (2014, 2023)  |
| $	au_1$            | Tax+transfer level                               | 0.120                                    | Guner et al. (2014, 2023)  |
| $T_0$              | Transfer for worker without UI                   | 0.121                                    | Guner et al. (2023)  |
| $	au_b$            | Business tax rate                                | 0.243                                    | Cooper et al. (2016)   |
| b                  | Unemployment benefit replacement rate            | 0.5                                      | Standard in US   |
| $\overline{b}$     | Unemployment benefit maximum                     | 0.5                                      | Standard in US   |
| χ                  | Unemployment benefit expiration rate             | 1/6                                      | Standard US maximum of 6 months                                      |
| SS                 | Social security payment                          | 0.5                                      | Normalization  |
| g                  | Public consumption                               | 0.084                                    | Equalizes government budget constraint in benchmark equilibrium      |
| Search             |  |  |  |
| α                  | Meeting function elasticity                      | 0.5                                      | Petrongolo and Pissarides (2001)                                     |
| $\eta$             | Worker's bargaining power                        | 0.4                                      | Bagger et al. (2014)   |
| К                  | Job posting cost                                 | 0.124                                    | Normalizes benchmark equilibrium $\theta$ to 1                       |
| Other              |  |  |  |
| ρ                  | Discount rate                                    | 0.00330                                  | 4 percent annual interest rate                                       |
| δ                  | Human capital depreciation rate                  | 0.00116                                  | Matches decline of wages in last stage of life                       |
| $eta_0$            | Initial human capital intercept                  | 0.305                                    | Normalized such the lowest $h_0$ is the bottom point on the $h$ grid |

Parameters set outside the model.

## 5.5 Targeted Moments

I am left with 13 parameters to calibrate internally. Internal parameters are estimated jointly using simulated method of moments such that the simulated model hits 16 moments from the SIPP.<sup>30</sup>

I target the lifeycle profiles of the mean log wage, variance of log wages, and the job switching rate. For each profile, I target the starting point, the ending point, and the midpoint. I also target five moments which are not associated with the life cycle: the average unemployment to employment rate, cross-sectional log wage skewness, monthly wage growth for those who stay in the same job, monthly wage growth for those who switch jobs, and monthly wage growth for those who switch jobs and increase their wage. Finally, I target two normalizations: I normalize the mean wage to 1 (in order to be consistent with the job destruction function and tax function), and I normalize mean  $l + s_E$  to equal  $0.1.^{31}$ 

## 5.6 Identification

Before proceeding to my calibration results, I provide an informal discussion about identification. Since I use SMM, all parameters are jointly determined, and most parameters affect more than one moment. Nevertheless, I will describe how each parameter relates to moments in the data with the goal of arguing that my internal parameters are well-identified. The 13 internal parameters are listed and described in Table 3.

First, there are some clear one-to-one relationships between moments and parameters.  $\gamma_U$  pins down the unemployment-to-employment rate. And the level parameter for the firm productivity distribution,  $\mu_p$ , is immaterial except for establishing the mean wage in the economy. So,  $\mu_p$  is used to normalize mean log wages to 1.

<sup>&</sup>lt;sup>30</sup>I solve the SMM minimization problem using MIDACO, a general-purpose ant colony optimization algorithm (Schlüter et al., 2009).

 $<sup>^{31}</sup>$ The final normalization is not necessary, but it is convenient because it keeps other parameter levels contained. Since I have level parameters on the disutility of effort ( $\phi$ ), the return to human capital investment (a), and the return to job search effort ( $\xi$ ), l and s can be any level and these three parameters will adjust to get an equally tight fit.

The two moments associated with wage growth for job switchers are determined by two parameters,  $\lambda_p$  and  $\psi$ .  $\lambda_p$  determines the size of wage jumps that workers experience while changing jobs. Conditional on  $\lambda_p$ ,  $\psi$  identifies the difference between wage growth for those switching jobs with a wage increase and those switching jobs with a wage decrease. In other words, the frequency of godfather shocks is pinned down by the relative wage growth from switching jobs when the wage decreases.

Next, the level parameter of the learning ability distribution,  $\mu_a$ , pins down the mean monthly wage increase for job stayers. This leaves  $\omega$  as the parameter most closely related to the concave shape of wages over the lifecycle.<sup>32</sup>

Following the logic of Huggett et al. (2011) (and given the parameters and moment accounted for to this point), I associate the dispersion parameters of the learning ability and initial human capital distribution  $(\sigma_a, \lambda_a, \beta_1, \sigma_{\varepsilon})$  with the variance of wages, skewness of wages, and lifecycle profile of the variance of wages. Given that it determines the skewness of a,  $\lambda_a$  is mostly closely related to the skewness of wages. With  $\lambda_p$  accounted for,  $\sigma_a$  is identified by the overall level of variance of wages. The lifecycle profile of variance is thus identified by  $\beta_1$  and  $\sigma_{\varepsilon}$ .

We are left with three parameters for three moments: the parameters are  $\phi$ ,  $\gamma_E$ , and  $\xi$ , and the moments are the level of the job switching rate, the lifecycle profile of the job switching rate, and mean human capital investment plus mean search effort for the employed. Since  $\phi$  and  $\gamma_E$  parameterize the function for total disutility,  $\phi(l+s)^{1+\gamma_E}$ , they are strongly related to the level of total wage investment, but immaterial to the mix of human capital investment and search investment. Therefore,  $\phi$  and  $\gamma_E$  are closely related with the normalization of  $s_E + l$ . In contrast,  $\xi$  determines the relative benefit of search investment compared to human capital investment.

 $<sup>^{32}</sup>$ One difficulty with identification is that both the human capital and search channels are generate curvature in lifecycle wages. Because I use the Ben-Porath (1967) human capital accumulation function, there are decreasing returns to human capital investment give by  $\omega$ , which generates concavity in the lifecycle human capital profile. Similarly, since the distribution of firm productivity F(p) is fixed, we would expect to see greater wage increases at the beginning of life (as workers pick the "low hanging fruit" of firms), but wage growth will become slower as workers move up the job ladder. This curvature is determined by  $\lambda_p$ . However,  $\lambda_p$  is also closely related to monthly wage growth for job switchers. Thus, we can say that the residual curvature in lifecycle wages is identified by  $\omega$ .

Generally, it is important that I match monthly wage growth for job stayers versus job switchers. The difference allows me to differentiate the month-to-month wage growth which comes from the human capital channel versus the search channel. One may object that in the wage bargaining protocol of Cahuc et al. (2006) and Bagger et al. (2014), workers who stay at the same firm can experience wage growth from human capital accumulation or from renegotiation, the latter of which is part of the search channel. Thus, it is not the case that the wage growth of job stayers only reflects human capital accumulation. However, we know that the wage growth for job switchers is due to search. So, for job stayers, the residual wage growth which is unidentified from the search channel comes from human capital growth.

#### 5.7 Calibration Results

The fit between the model and lifecycle moments is illustrated in Figure 5.7, and the fit between the model and non-lifecycle moments is presented in Table 2. The parameters which deliver this fit are listed in Table 3.

A handful of my parameter estimates deserve a brief discussion. First, my estimate of the curvature parameter in the human capital production function,  $\omega \approx 0.5$ , is in line with micro estimates (Browning et al., 1999). Second, the disutility function has significantly more curvature for the unemployed . This means that unemployed workers can invest more in human capital before the marginal cost of investment becomes prohibitively large, which is necessary to match the large difference in unemployment-to-employment rates compared to job switching rates. Third, my calibrated value for the dispersion of learning ability,  $\sigma_a$ , is relatively large compared to the literature. I attribute this to the fact — described above — that endogenous search acts as an equalizer for wages. Compared to a model without endogenous search, in order to generate the same level of wage dispersion, I need more dispersion in learning ability.

<sup>&</sup>lt;sup>33</sup>For discussions of this parameter, see Blandin (2018) and Browning et al. (1999).

<sup>&</sup>lt;sup>34</sup>See Badel et al. (2020) and Esfahani (2020).

Table 2: Fit between data and model, non-lifecycle moments

| Moment                               | Target | Model |
|--------------------------------------|--------|-------|
| Unemployment to employment rate      | 0.283  | 0.327 |
| Wage growth, stayers                 | 0.008  | 0.007 |
| Wage growth, switchers               | 0.106  | 0.107 |
| Wage growth, switchers with increase | 0.350  | 0.421 |
| Log wage skewness                    | 0.412  | 0.408 |

Model values are from calibrated benchmark model. Targets are calculated from SIPP data. Unemployment to employment rate and wage growth rates are monthly. Unemployment to employment rate is a continuous-time arrival rate.

Table 3: Internally calibrated parameters

| Parameter            | Meaning                                   | Value  |
|----------------------|---|--------|
| φ                    | Disutility level                          | 6.56   |
| $\gamma_E$           | Disutility curvature, employed            | 1.53   |
| $\gamma_U$           | Disutility curvature, unemployed          | 3.84   |
| $\omega$             | Human capital investment curvature        | 0.48   |
| ξ                    | Meeting efficiency                        | 1.69   |
| $\mu_p$              | Firm productivity distribution, level     | 0.23   |
| $\lambda_p$          | Firm productivity distribution, tail      | 0.36   |
| $\mu_a$              | Learning ability distribution, level      | 0.0044 |
| $\sigma_a$           | Learning ability distribution, dispersion | 0.67   |
| $\lambda_a$          | Leaning ability distribution, tail        | 0.06   |
| $eta_1$              | Correlation between $a$ and $h_0$         | 0.0025 |
| $\sigma_{arepsilon}$ | $h_0$ dispersion                          | 0.10   |
| ψ                    | Godfather shock rate                      | 0.0003 |

Values for parameters which generate best model fit with SIPP data.

As illustrated in Panel (c) of Figure 2, the job switching rate decreases over the lifecycle. This feature in the data is also a feature in my on-the-job search environment. In my framework, workers move to more productive firms as they age. But since the firm productivity distribution is fixed, an older the worker is less likely to meet an outside firm which is more productive than his current firm. So, young workers are more likely to switch jobs. This is sometimes referred to as the "job shopping" stage.

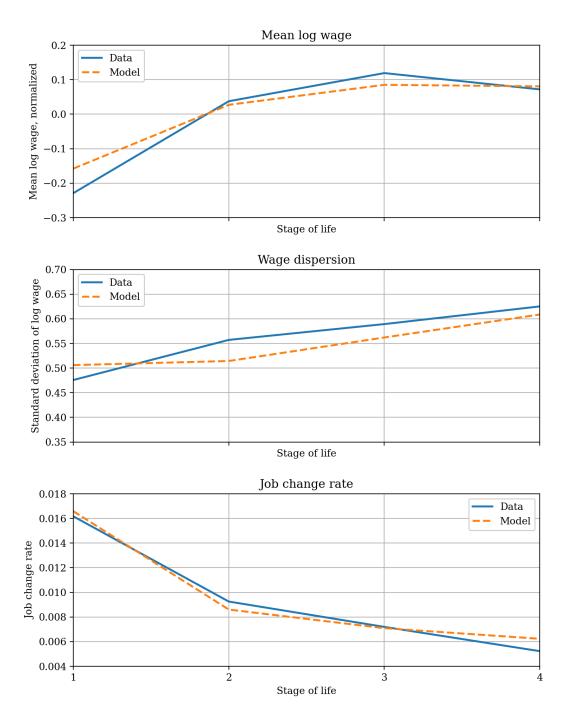


Figure 2: Fit between data and model, lifecycle moments

Model values are from calibrated benchmark model. Data source: SIPP.

# 6 Properties of Benchmark Model

Before experimenting with taxes, I describe properties of the benchmark model that are vital for understanding how the model works and how it will respond to policy changes.

## 6.1 Mean Wages

Recall that the wage is hpr. Given this structure, I can easily decompose log wages between human capital h, firm productivity p, and the bargained piece rate r. The log wage is the sum of the log of the three components,<sup>35</sup>

$$\ln(wage) = \ln(h) + \ln(p) + \ln(r).$$

The mean log wage follows,

$$E[\ln(wage)] = E[\ln(h)] + E[\ln(p)] + E[\ln(r)]. \tag{21}$$

Taken together, p and r comprise the search channel, and h represents the human capital channel.

In the benchmark model, the growth of wages over the lifecycle is mostly driven by workers moving up the job ladder to more productive firms. Figure 3 plots the growth of mean log wages over the lifecycle along with its three additive components. All three components are significant in accounting for lifecycle wage growth. Approximately 2/3 of lifecycle wage growth comes from the search channel and 1/3 comes from human capital accumulation.<sup>36</sup>

However, the averages mask significant heterogeneity in the ways that workers grow wages. A worker's choice of *l* and *s* depend on his state, age, and fixed learning ability.

 $<sup>^{35}</sup>$ The decomposition is somewhat compromised by the fact that h, p, and r all interact with each other. However, I show later that the implications of this decomposition are the same as when I take a more careful approach in Appendix 7.5, and this additive decomposition is far more intuitive.

<sup>&</sup>lt;sup>36</sup>Bayer and Kuhn (2019) find a similar result.

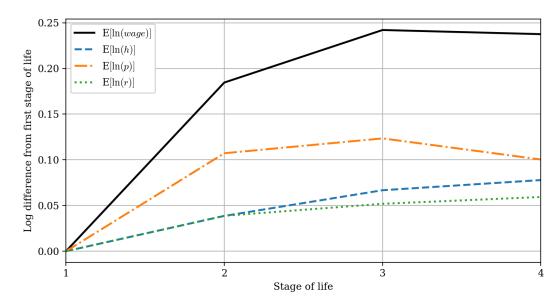


Figure 3: Mean log wage lifecycle decomposition

Additive decomposition of mean log wage as in Equation (21). Mean log wage is the sum of mean log human capital h, firm productivity p, and piece rate r. Dashed colored lines sum to black line. Computed in benchmark calibrated model.

In practice, learning ability is especially important. As described in Section 4, workers with high learning ability will substitute away from search effort toward human capital accumulation, and vice versa.

The result is that wage growth is significantly heterogeneous, and the heterogeneity is driven by human capital. In Figure 4 and going forward, I split my distribution of a values into two bins: the lower 60% and the upper 40%.<sup>37</sup> Figure 4 replicates the same decomposition of mean log wages for each of the two learning ability bins. Clearly, human capital growth differentiates the two groups. In fact, the growth of p and r is quite similar across groups. However, those with high learning ability rely mostly on human capital for wage growth, while those with low learning ability rely entirely on search. (In fact, those with low learning ability experience a net loss of human capital over the life cycle. In general, this finding supports the conclusions of Ozkan et al. (2023) and Bag-

<sup>&</sup>lt;sup>37</sup>These values translate conveniently to how I discretize the distribution of a. See Appendix D.2.

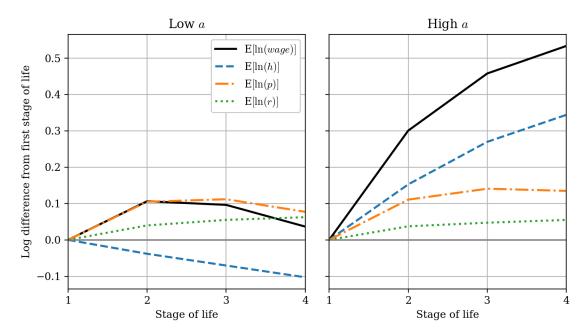


Figure 4: Mean log wage lifecycle decomposition by learning ability

Additive decomposition of mean log wage as in Equation (21) and Figure 3 split by learning ability. Learning ability a is split between lower 60% and upper 40% of a values. Computed in benchmark calibrated model.

ger et al. (2014) which argue that the search channel is the largest determinant of wage growth for the lower part of the wage distribution, while human capital growth is the largest determinant of wage growth for the upper part of the wage distribution.

## 6.2 Wage Dispersion

### 6.2.1 Lifecycle Wage Dispersion in the Benchmark Model

As with mean log wages, the structure of the model implies a straightforward decomposition for the variance of log wages. The variance of the log wage is the sum of the variance of each log component plus an interaction term for each pair of log components,

$$Var[ln(wage)] = Var[ln(h)] + Var[ln(p)] + Var[ln(r)]$$

$$+ 2Cov[ln(h), ln(p)] + 2Cov[ln(h), ln(r)] + 2Cov[ln(p), ln(r)].$$
(22)

Using this decomposition, I find that the increase in wage dispersion over the lifecycle is completely driven by human capital. In Figure 5, I decompose wage dispersion over the lifecycle into the six terms in Equation (22). In the cross section, variance in human capital accounts for about half of the variance of log wages. But over the lifecycle, the increase in variance of human capital is solely responsible for the increase in wage dispersion.

As Huggett et al. (2011) shows, two key ingredients are required to generate increasing wage dispersion in a human capital model: heterogeneity in learning ability and positive correlation between learning ability and initial levels of human capital. With heterogeneity in learning ability, those with high learning ability accumulate human capital faster than those with low learning ability. If initial human capital and learning ability are positively correlated, then the gap between workers with high and low learning ability is guaranteed in increase over the lifecycle.

The benchmark model can be summarized as follows: wage growth is generally a function of the job ladder, while inequality is generally a function of human capital.

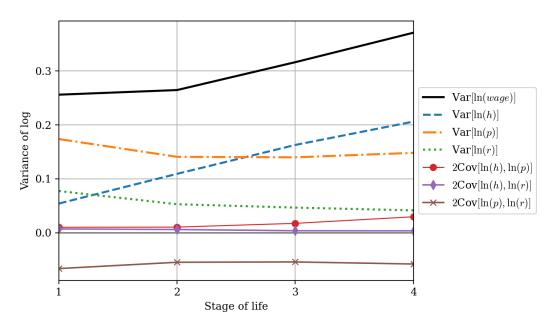


Figure 5: Variance of log wage lifecycle decomposition

Additive decomposition of the variance of log wage as in Equation (22). Colored lines sum to black line. Computed in benchmark calibrated model.

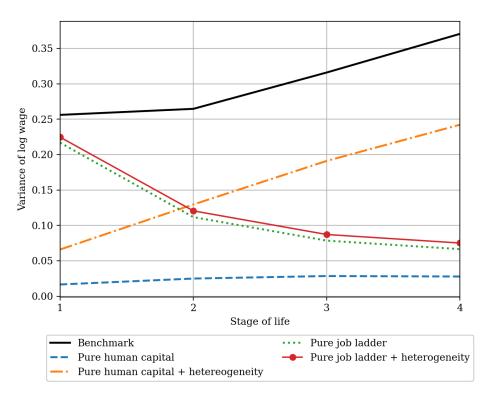


Figure 6: Lifecycle wage dispersion when channels are shut down

Lifecycle variance of log wage in models where channels are shut down. All models are computed using the benchmark calibration. Pure human capital models shut down the search channel, and pure job ladder models shut down the human capital channel. In both cases, "heterogeneity" refers to heterogeneity in initial human capital  $h_0$  and learning ability a. (In the pure job ladder model with heterogeneity, learning ability a is irrelevant, so heterogeneity refers heterogeneity in fixed human capital.)

#### 6.2.2 Lifecycle Wage Dispersion in Restricted Models

I now introduce "restricted models" where I shut down certain channels in order to compare different frameworks. For now, I use the same benchmark parameters for all models. By shutting down channels, I will show how different features of the model generate different results with respect to lifecycle wage inequality. (Later, in Section 7.6, I recalibrate each model and perform a "horse race" experiment.)

There are four restricted models. First, I shut down the search channel but keep the rest of the model the same. I refer to this as the "pure human capital + heterogeneity" model. Then, I shut down all heterogeneity in learning and refer to this model as the

"pure human capital" model. Returning to the benchmark model, I then shut down the human capital channel but keep the rest the same. This is the "pure job ladder + heterogeneity" model. Then, I shut down all heterogeneity human capital and refer to it as the "pure job ladder" model.<sup>38</sup>

First, note how lifecycle wage dispersion behaves when I shut down the search channel. Clearly, the search channel adds a significant amount of wage dispersion to the model. Also note how a pure human capital without heterogeneity has far less wage dispersion as well as hardly any increase in wage dispersion over the lifecycle. Again, as pointed out in Huggett et al. (2011), in a human capital model, heterogeneity in learning ability and initial human capital are vital ingredients for generating an increase in wage dispersion over the lifecycle.

When I shut down the human capital channel (and I am left with a 11pure job ladder" model), wage dispersion *decreases* over the lifecycle. To understand why, recall how onthe-job search works in the model. At the beginning of life, workers randomly meet a firm with productivity p from the fixed distribution F(p). Thus, the initial distribution of p will roughly mirror F(p). As workers search while on the job, they meet more firms with productivity drawn from F(p). If they meet a firm with higher productivity than their current firm, they switch. As workers move up the job ladder, workers tend to bunch more at the top of the wage distribution, and the variance of firm productivity (for filled jobs) decreases. One implication is that a pure job ladder model will have great difficulty in generating an increase in wage dispersion over the lifecycle.

Thus, the human capital and search channels have opposite predictions for lifecycle wage dispersion. Following this logic, when I increase tax progressivity in the model, the two channels will have opposite effects.

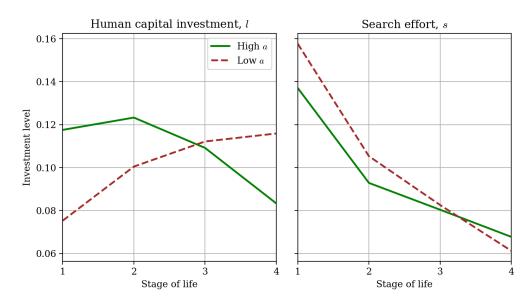


Figure 7: Mean policy functions by learning ability

Mean policy functions for human capital investment l and search effort s over life cycle. Learning ability a is split between lower 60% and upper 40% of the a distribution.

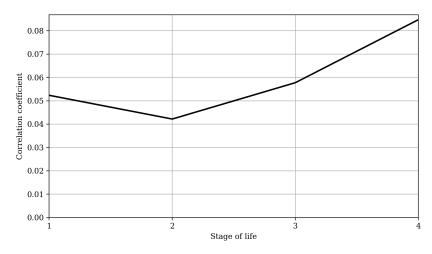


Figure 8: Correlation of ln(h) and ln(p)

Weighted correlation coefficient of log human capital and log firm productivity over the life cycle. Computed in benchmark calibrated model.

### 6.3 Interaction of Human Capital and On-the-Job Search

In Section 4, I show that there are forces in the model that imply that human capital and job search may move together or in opposite directions. Quantitatively, the benchmark model suggests that the two channels are substitutes within the individual worker's decision problem, but positively correlated on an aggregate level.

In Figure 7, I plot mean human capital investment l and search effort s for workers in the two learning ability bins from earlier. Overall, the figure suggests that workers substitute between l or s. Workers with learning learning ability tend to invest more in job search while workers with high learning ability tend to invest more in human capital accumulation. Furthermore, there is evidence of substitution within the lifecycle, as workers with lower learning ability start life by investing heavily in job search, but shift their investment mix toward human capital over the course of the life cycle.

Despite the substitutability of human capital and search, Figure 8 shows that, at the aggregate level, the correlation between h and p is positive.<sup>39</sup> There are at least two forces at work. First, as shown in Section 4, since h and p are complements in production, there may be positive correlation between h and p arising from worker decisions. Second, workers are heterogeneous in unemployment risk; because low-wage workers are more likely to lose their job, they do not climb the job ladder as quickly (Jarosch, 2023). Thus, the correlation between h and p increases over the life cycle, which is true in Figure 8.

Why does the correlation between h and p matter? The extent to which high-skill workers tend to work at high-productivity firms can help explain the extent of wage inequality we observe in the data. For example, in a search-and-matching model using the same on-the-job search apparatus as my model, Bagger and Lentz (2019) find that a substantial portion of wage dispersion is due to positive sorting. My findings support this argument even with many potentially confounding features in the model. Furthermore, using the lifecycle feature in my model, I show that the increasing correlation of h and p

<sup>&</sup>lt;sup>38</sup>Heterogeneity in learning ability is immaterial in a model without human capital accumulation.

<sup>&</sup>lt;sup>39</sup>The literature following Abowd et al. (1999) has tended to find a positive correlation between estimated worker and firm productivity (though the original study did not); see Card et al. (2018) for a discussion.

over the life cycle is a contributing factor to increasing wage dispersion over the life cycle.

# 7 Tax Progressivity Experiments

To demonstrate the usefulness of the model, I perform counterfactual experiments on tax progressivity. From Section 4 alone, we know that an increase in tax progressivity will decrease wages. With higher tax progressivity, if a worker increases their wage, then they take home a smaller part of the wage. So, since the benefit of investment decreases while the cost of investment is unchanged, *l* and *s* will decrease, and wages decrease as well. What is unclear is the magnitude of the effect and how the channels interact quantitatively.

In counterfactual experiments, I feed the model new values of tax progressivity,  $\tau_1$ . There are two objects that adjust so that model is in equilibrium under the new  $\tau_1$ : labor market tightness  $\theta$ , which adjusts such that the free entry condition (17) holds, and the tax level  $\tau_0$ , which adjusts such that government budget constraint (18) holds.

In this section, I first provide an overview how the model responds when I change tax-and-transfer progressivity. Then, I investigate the machinery behind these results, focusing on the mean wage level, wage dispersion, and lifecycle wage growth. I then compare my results with re-calibrated restricted models where I turn off certain channels.

### 7.1 Overview of Results

In the benchmark model, the level of tax progressivity,  $\tau_1=0.12$ , reflects the combined progressivity of the tax system and means-tested transfers in the US. I compare this level of progressivity with flat taxes,  $\tau_1=0$ , and with twice as much the progressivity,  $\tau_1=0.24$ . Setting  $\tau_1=0.24$  puts the level of tax-and-transfer progressivity in line with countries such as Denmark, Finland, Germany, the Netherlands, and Sweden.<sup>40</sup>

I plot the average tax rate for each tax system in Panel (a) Figure 9.<sup>41</sup> As  $\tau_1$  increases,

<sup>&</sup>lt;sup>40</sup>See the appendix of Holter et al. (2019).

<sup>&</sup>lt;sup>41</sup>For  $\tau_1 \neq 0.12$ , I use the new equilibrium values of  $\tau_0$ .

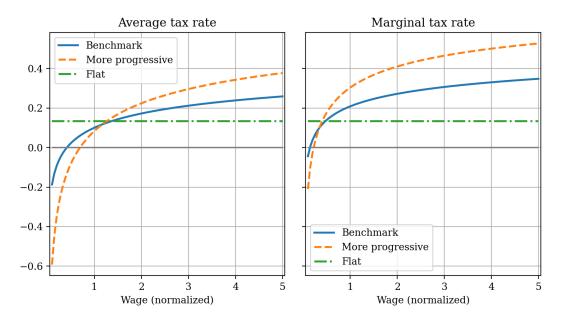


Figure 9: Tax rates by wage

The benchmark model uses progressivity parameter  $\tau_1=0.12$ , the "more progressive" model uses  $\tau_1=0.24$ , and the "flat" model uses  $\tau_1=0$ . Wage is normalized such that the mean wage equals one in the benchmark model. Each model is in equilibrium; thus, the tax level parameter  $\tau_0$  is set such that the government budget constraint holds.

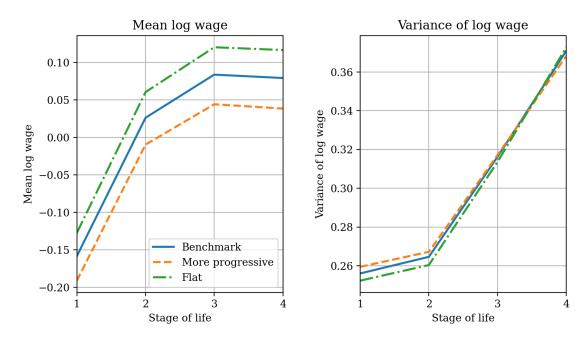


Figure 10: Effect of tax progressivity on lifecycle mean wage and wage dispersion The benchmark model uses progressivity parameter  $\tau_1 = 0.12$ , the "more progressive" model uses  $\tau_1 = 0.24$ , and the "flat" model uses  $\tau_1 = 0$ .

the average tax curve increases in curvature; those with low wages earn more transfers, and those with high wages pay more taxes. The increase in  $\tau_1$  also increases the threshold at which workers are net tax payers from about 1/3 of the mean wage to 2/3 of the mean wage. Marginal tax rates are plotted in Panel (b) of Figure 9. If  $\tau_1 = 0.12$ , the marginal tax rate at the mean wage is 21%; when  $\tau_1 = 0.24$ , it is 30%.

The counterfactual results from adjusting tax progressivity are summarized in Figure 10 and Table 4. In Figure 10, I plot the mean log wage and variance of log wage over the lifecycle for all three values of  $\tau_1$ . In Table 4 and going forward, I only compare the benchmark model ( $\tau_1 = 0.12$ ) with the more progressive model ( $\tau_1 = 0.24$ ).

In total, increasing tax progressivity decreases the mean wage by 4% and mean lifecycle growth of log wages by 4%. Interestingly, mean consumption also decreases by about 4%. Regarding inequality, increasing tax progressivity has no effect on the variance of log wages, but significantly decreases the variance of consumption.  $\theta$  adjusts as expected in

Table 4: Summary of effects from increasing tax progressivity

|  | Benchmark | More progressive | Difference |
|--|-----------|------------------|------------|
| Labor market tightness, $\theta$         | 1.00      | 0.97             | -0.03      |
| Tax level, $\tau_0$                      | 0.90      | 0.92             | 0.02       |
| Mean log wage                            | 0.01      | -0.03            | -0.04      |
| Variance of log wage                     | 0.31      | 0.31             | 0.00       |
| 90-50 interdecile wage ratio             | 2.01      | 1.96             | -0.05      |
| Lifecycle growth of mean log wage        | 0.24      | 0.23             | -0.01      |
| Lifecycle growth of variance of log wage | 0.11      | 0.11             | -0.01      |
| Unemployment rate                        | 0.0181    | 0.0166           | -0.0015    |
| Job switching rate                       | 0.0096    | 0.0100           | 0.0004     |
| Mean consumption                         | 1.02      | 0.98             | -0.05      |
| Variance of consumption                  | 0.50      | 0.28             | -0.22      |
| Mean l                                   | 0.104     | 0.099            | -0.005     |
| Mean s for employed workers              | 0.099     | 0.094            | -0.005     |

<sup>&</sup>quot;Lifecycle growth" refers to the difference between the last stage and the first stage of the life cycle. The benchmark model uses progressivity parameter  $\tau_1=0.12$  and the "more progressive" model uses  $\tau_1=0.24$ .  $\theta$  and  $\tau_0$  are solved for in equilibrium.

equilibrium; with lower levels of human capital and search effort in the labor pool, firms post fewer jobs, and labor market tightness decreases by 3%. There is no change in the unemployment rate or the job switching rate.

# 7.2 Mean Wage Level

The wage level decreases mostly because workers make fewer job-to-job transitions and, therefore, are employed at less productive firms. In Table 5, I compare log wages using the mean log wage decomposition in Equation (21). According to the overall differences of E[h], E[p], and E[r] (in the top line of each block), the decrease in log wages as a result of the tax change is mostly driven by a decrease in firm productivity, followed by human capital, and then the piece rate. In the fact, the decrease in E[ln(p)] over is twice as large as the decrease in E[ln(h)].

Earlier in the paper, I show that job-to-job transitions are primary factor behind lifecycle wage growth for the mean worker. Since job search is the most relevant margin for wage growth for most workers, the fact that job search is the driving force behind the

Table 5: Decomposing effect of tax progressivity on mean log wage

|                     | Benchmark | More progressive | Difference |
|---------------------|-----------|------------------|------------|
| E[ln(wage)]         | 0.008     | -0.030           | -0.037     |
| E[ln(wage)], low a  | -0.186    | -0.219           | -0.033     |
| E[ln(wage)], high a | 0.294     | 0.251            | -0.043     |
| E[ln(h)]            | 0.374     | 0.364            | -0.010     |
| E[ln(h)], low a     | 0.183     | 0.178            | -0.005     |
| E[ln(h)], high $a$  | 0.658     | 0.641            | -0.017     |
| E[ln(p)]            | -0.167    | -0.193           | -0.026     |
| E[ln(p)], low a     | -0.167    | -0.193           | -0.026     |
| E[ln(p)], high $a$  | -0.168    | -0.193           | -0.025     |
| E[ln(r)]            | -0.199    | -0.201           | -0.001     |
| E[ln(r)], low a     | -0.201    | -0.203           | -0.002     |
| E[ln(r)], high $a$  | -0.196    | -0.197           | -0.001     |

 $E[\ln(wage)]$  is decomposed as in Equation (21). Learning ability a is split between the lower 60% and upper 40% of the a distribution. The benchmark model uses progressivity parameter  $\tau_1=0.12$  and the "more progressive" model uses  $\tau_1=0.24$ . "Difference" is "more progressive" minus benchmark.

policy response is unsurprising.

Also, the effect on the search channel is more universal. In Table 5, the decrease in  $E[\ln(h)]$  is over three times larger for the group with high a than the group with low a. In contrast, the decrease in  $E[\ln(p)]$  is relatively similar across groups. This is related to the result in the previous section that only workers with high learning ability rely heavily on accumulating human capital for lifecycle wage growth while all workers are roughly equally successful in climbing the job ladder. Thus, taxes affect all workers through the search channel, but only some workers through the human capital channel.

## 7.3 Wage Dispersion

An increase of tax progressivity has no effect on the variance of wages. In response to an increase in tax progressivity, the human capital channel pushes wage dispersion down while the search channel pushes wage dispersion up. Together, the effects offset each other. These effects are quantified in Table 6, where I decompose the effects of increased

Table 6: Decomposing effect of tax progressivity on variance of log wage

|                                      | Benchmark | More progressive | Difference |
|--------------------------------------|-----------|------------------|------------|
| Var[ln(wage)]                        | 0.312     | 0.312            | 0.000      |
| Var[ln(h)]                           | 0.134     | 0.128            | -0.006     |
| Variance from search channel         | 0.152     | 0.158            | 0.007      |
| Var[ln(p)]                           | 0.153     | 0.158            | 0.005      |
| Var[ln(r)]                           | 0.055     | 0.059            | 0.003      |
| $2\operatorname{Cov}[\ln(p),\ln(r)]$ | -0.056    | -0.058           | -0.002     |
| Variance from interaction            | 0.026     | 0.026            | 0.000      |
| $2\operatorname{Cov}[\ln(h),\ln(p)]$ | 0.019     | 0.020            | 0.001      |
| $2\operatorname{Cov}[\ln(h),\ln(r)]$ | 0.006     | 0.006            | -0.000     |

Var[ln(wage)] decomposed as in Equation (22). The benchmark model uses progressivity parameter  $\tau_1=0.12$  and the "more progressive" model uses  $\tau_1=0.24$ . "Difference" is "more progressive" minus benchmark.

tax progressivity on the variance of log wages using Equation (22). For simplicity, I group the additive components of wage variance on the right hand side of Equation (22) into three parts: (1) the variance from the human capital channel, Var[ln(h)]; the variance from the search channel, Var[ln(p)] + Var[ln(r)] + 2 Cov[ln(p), ln(r)]; and the variance from interactions between human capital and search, 2 Cov[ln(h), ln(p)] + 2 Cov[ln(h), ln(r)].

When I increase tax progressivity, the variance from the human capital channel decreases while the variance from the search channel increases. Table 5 illustrates why the variance of human capital channel decreases. Human capital levels decrease for all workers, the but magnitude of the decrease is larger for workers with high learning ability. Therefore, the gap in human capital between workers with high *a* and low *a* decreases, which decreases the variance of human capital. Wage dispersion decreases primarily because high wage decrease; so, we can say that the human capital channel decreases wage dispersion from above.

Why do workers with high learning ability experience a larger decrease in human capital? There are two reasons. First, increased tax progressivity has more "bite" at the top of the wage distribution, so high-wage workers are more disincentivized to grow their wages. Second, those at the higher end of the distribution are the same workers

who invest more in human capital, so their relevant margin for adjustment in response to policy is human capital.

However, the decrease in variance from the human capital channel is offset by an increase in wage dispersion from the search channel. Recall from Figure 6 that, because of on-the-job search, the variance of firm productivity decreases over the lifecycle. The same logic is at work when I increase tax progressivity. When tax progressivity increases, workers exert less search effort, are less likely to meet outside firms, and do not climb the job ladder as quickly. The result is less bunching at high levels of p. Intuitively, with more tax progressivity, workers exert less search effort, so unproductive firms are more likely to retain workers. Therefore, wage dispersion increases from below. In total, with both channels, the gap between high and low wages is the same, but the level of wages has decreased.

One could argue that the last two sections are incomplete because, though we can their levels, the human capital and search channels are determined simultaneously and, as I have showed, they interact. So, it may be somewhat imprecise to assert that h represents the human capital channel while p and r represent the search channel. I address this concern in Appendix 7.5 where I analyze the effects of tax progressivity while holding the policy functions for human capital accumulation and search effort constant. Thus, I am able to isolate how the policy affects each channel one at a time. I show that the intuition developed in this section holds.

## 7.4 Lifecycle Wage Growth

The decrease in lifecycle wage growth is driven by a decrease in human capital accumulation. Table 7 illustrates this fact by taking Equation 21 one step further and decomposing log wage growth into the log growth of human capital, firm productivity, and the piece rate. Then, it compares the lifecycle growth of each of these components after the increase in tax progressivity. When progressivity increases, lifecycle wage growth decreases by 3%. The difference is entirely driven by a decrease in the lifecycle growth of human cap-

Table 7: Decomposing effect of tax progressivity on lifecycle wage growth

|                                   | Benchmark | More progressive | Difference |
|-----------------------------------|-----------|------------------|------------|
| Lifecycle growth of $E[ln(wage)]$ | 0.238     | 0.229            | -0.008     |
| Lifecycle growth of $E[ln(h)]$    | 0.078     | 0.069            | -0.009     |
| Lifecycle growth of $E[ln(p)]$    | 0.100     | 0.103            | 0.003      |
| Lifecycle growth of $E[ln(r)]$    | 0.059     | 0.057            | -0.002     |

Decomposes the log of lifecyle wage growth to the growth of its three additive components; see Equation (21). Lifecycle log growth is defined as the difference between the log value in the last stage of life and the first stage of life. The benchmark model uses progressivity parameter  $\tau_1=0.12$  and the "more progressive" model uses  $\tau_1=0.24$ . "Difference" is "more progressive" minus benchmark.

ital.<sup>42</sup> An increase in tax progressivity stymies the lifecycle growth of human capital but has little effect on the lifecycle wage growth from the search channel. In the next section, I show that my estimate of the decrease in lifecycle wage growth from an increase in progressivity relatively is small compared to a pure human capital model. So, the presence of the search channel mutes the response.

### 7.5 The Role of Interaction Between Endogenous Channels

In this section, I show how the human capital, search, and job posting channels interact by progressively shutting down the channels' endogeneity to policy changes. This section describes two main results, both of which deal with the mean log wage following an increase in tax progressivity. First, the human capital and search channels amplify one another such that the mean log wage decreases more when both channels are endogenous than when one channel is exogenous. Second, firms also respond the policy change by posting fewer jobs in equilibrium, which further decreases wages. Altogether, these results suggest that a model which does not endogenize every channel will under-predict the negative affect of tax progressivity on the wage level.

I experiment with disallowing the following objects to adjust to the increase in tax progressivity: equilibrium labor market tightness  $\theta^*$ , workers' policy functions for human

 $<sup>^{42}</sup>$ Taken together, the search channel, which includes p and r, sums to nearly zero.

capital accumulation  $l^*$ , and workers' policy functions for job search effort (for both the employed and unemployed)  $s^*$ . I consider three out-of-equilibrium models between the benchmark equilibrium and the more progressive equilibrium. For each, I impose the policy change of  $\tau_1 = 0.24$  and the new value of  $\tau_0$  which solves the government budget constraint in the new equilibrium.

In the first out-of-equilibrium model, I fix  $s^*$  and  $\theta^*$  to their values in the benchmark model, thereby only allowing  $l^*$ , human capital accumulation, to respond to the policy change. In this setup, search effort, and therefore the the probability of switching jobs, cannot respond to policy; thus, the search channel is exogenous. <sup>43,44</sup> In the second out-of-equilibrium model, I fix  $l^*$  and  $\theta^*$ . Using the same logic as above, this is a model where the path of human capital is fixed, and workers can only respond to policy changes by adjusting search effort. Finally, in the third out-of-equilibrium model, I only fix  $\theta^*$ . In this version, workers can adjust human capital and search effort in response to the change in tax policy, but firms cannot adjust the number of open vacancies.

I summarize the decomposition in Table 8. In the interest of readability, I do not report the levels of each variable for each model; I only report the benchmark values and the difference between the value from the model in question and the value in the benchmark model.

Compared to model where only l or s can respond to policy, the decrease in the wage level is larger when both channels respond simultaneously. If only human capital is endogenous, then wages decrease 1.4%; if only search effort is endogenous, then wages decrease 2.7%; and if both are endogenous, then wages decrease by 3.3%. Thus, the negative effect of tax progressivity on wages is greater when human capital and search interact with one another. However, the interaction less than additive; if the effects were additive, then the result of endogenous human capital and search would be -1.4% - 2.7% = -4.1%. The effect is smaller, -3.3%. So, if workers can only adjust

<sup>&</sup>lt;sup>43</sup>This roughly corresponds to a human capital model with exogenous shocks (Badel et al., 2020; Huggett et al., 2011).

<sup>&</sup>lt;sup>44</sup>I also hold  $z_{ai}(h, w)$  constant so that unemployment-to-employment transition rates are the same.

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Table 8: Out-of-equilibrium decomposition from increase in tax progressivity

|                                   | Benchmark | Flexible <i>l</i><br>—benchmark | Flexible <i>s</i><br>—benchmark | Flexible ( <i>l</i> , <i>s</i> ) –benchmark | Flexible $(l, s, \theta)$ —benchmark |
|-----------------------------------|-----------|---------------------------------|---------------------------------|---|--------------------------------------|
| Labor market tightness, $\theta$  | 1.000     | 0.000                           | 0.000                           | 0.000                                       | -0.029                               |
| Mean log wage                     | 0.008     | -0.014                          | -0.027                          | -0.033                                      | -0.037                               |
| $E[\ln(h)]$                       | 0.374     | -0.013                          | -0.004                          | -0.010                                      | -0.010                               |
| $E[\ln(p)]$                       | -0.167    | -0.001                          | -0.021                          | -0.021                                      | -0.026                               |
| $E[\ln(r)]$                       | -0.199    | -0.000                          | -0.002                          | -0.001                                      | -0.001                               |
| Variance of log wage              | 0.312     | -0.006                          | 0.005                           | 0.000                                       | 0.000                                |
| Lifecycle growth of mean log wage | 0.238     | -0.013                          | -0.002                          | -0.008                                      | -0.008                               |

Analyzes out-of-equilibrium models between equilibrium benchmark ( $\tau_1 = 0.12$ ) and "more progressive" ( $\tau_1 = 0.24$ ) models. Thus, the model with more tax progressivity and flexible ( $l,s,\theta$ ) is equivalent to the equilibrium "more progressive" model analyzed above. Benchmark column is in levels; all other columns are in differences relative to the benchmark.

along one channel, the workers will adjust that on that channel more dramatically. If workers can adjust along both channels, workers will make smaller adjustments along each channel, but taken together, the effect is greater.

When firms are allowed to adjust job posting, the negative effect on the wage level is further amplified. If workers have less human capital and exert less search effort, then the benefit of posting a job decreases, and firms post fewer jobs in equilibrium. Quantitatively,  $\theta$  decreases by 3%. Because there are fewer job vacancies, the marginal benefit of search decreases, and workers exert less search effort. In addition, since h and p are substitutes and workers are less likely to transition to more productive firms, then the benefit of human capital decreases as well, and workers also accumulate less human capital. Taken together, because firms post fewer jobs, the wage level in the economy will be lower. Indeed, in Table 8, when firms adjust  $\theta$ , the decrease in the wage goes from -3.3% to -3.7%.

Two additional results in Table 8 support earlier findings. First, the search channel is more responsive when *l* is exogenous than is the human capital channel when *s* is exogenous. This supports the notion that, on average, search effort is more responsive to policy changes. Second, depending on which channel is exogenous, increasing tax progressivity has opposite effects on the variance of wages. Thus, regarding wage dispersion, it is clear that the two channels offset one another.

### 7.6 Horse Race Results from Re-calibrated Restricted Models

As a final experiment, I pit my model in a horse race against the restricted models described in Section 6.2.2. For this exercise, I re-calibrate each restricted model to match the lifecycle profile of mean wages. The results of these calibrations are in Figure 12 in Appendix E. I then feed each calibrated model the same change in tax policy parameters  $(\tau_1, \tau_0)$ . I document the effect of the increase in tax-and-transfer progressivity for each model in Table 9 along two dimensions: the mean log wage level and lifecycle log wage

 $<sup>^{45}</sup>$ For this experiment, I do not find a new equilibrium  $\theta$ .

Table 9: Effect of increasing tax progressivity in re-calibrated restricted models

| Model                              | Log wage difference | Lifecycle wage growth difference |
|------------------------------------|---------------------|----------------------------------|
| Benchmark*                         | -0.031              | -0.007                           |
| Pure human capital                 | -0.029              | -0.025                           |
| Pure human capital + heterogeneity | -0.012              | -0.012                           |
| Pure job ladder                    | -0.039              | 0.030                            |
| Pure job ladder + heterogeneity    | -0.035              | -0.002                           |

The effect of increasing tax progressivity in restricted models. Restricted models are described in Section 6.2.2. "Heterogeneity" refers to heterogeneity in learning ability and initial human capital. Lifecycle wage growth is defined as the difference between the log wage in the first stage of life and the last stage of life. \*In only this table, the benchmark model is not in equilibrium with respect to  $\theta$  after the tax policy changes.

growth. Each column displays the change in each object as a result of the policy change.

My findings can be summarized as follows: As a result of increasing tax progressivity, a pure human capital model (without the search channel) will *understate* the effect on the mean wage level and *overstate* the effect on lifecycle wage growth. The opposite is true for models without a human capital channel: as a result of increasing tax progressivity, a pure job ladder model (without the human capital channel) will *overstate* the effect on the mean wage level and *understate* the effect on lifecycle wage growth.

These results are consistent with the rest of the paper. Recall that the search channel is the driving force behind the decrease in the mean wage level and that the human capital channel is the driving force behind a decrease in lifecycle wage growth. So, a model without both channels will misstate the effects of tax progressivity in the same direction. More generally, these results suggest that the rich interactions in my model are necessarily for accurately predicting the effects of policy.

## 8 Conclusion

This paper develops and studies a rich lifecycle model of wages. Existing literature has mostly consisted of models where either the human capital channel or the search channel is endogenous. I endogenize human capital accumulation, on-the-job search, and job posting. I argue that these elements interact in ways that are quantitatively important.

In the benchmark calibration, job search is the driving force of lifecycle wage growth for the average worker. However, human capital is the driving force of wage dispersion, particularly regarding the top of the wage distribution. Workers take different approaches to increasing their wages. In particular, workers who are young and have low learning ability invest more in search effort, while working with high learning ability invest more in human capital accumulation.

I demonstrate the usefulness of the model by re-investigating the effects of progressive taxes and transfers on wage levels and wage inequality. I find that increasing tax progressivity from the its current level in the US to a level roughly in line with some European countries would decrease the mean wage by 4%, decrease lifecycle wage growth by 4%, and have no effect on wage dispersion. The lack of a response in the variance of wages is a unique feature arising from the combination of both channels. Finally, I show that models without both channels will misstate the effects of progressive taxes and transfers.

# **Appendix**

## A Equations

### **A.1** Aggregate Search Effort, S

Aggregate search effort is defined as

$$S = \sum_{a,i} \int_{\Psi_E} s_{Eai}(h, p, r) d\Psi_E(h, p, r|a, i) + \sum_{a,i,l} \int_{\Psi_U} s_{Uai}(h, w) d\Psi_U(h, w|a, i, l).$$
 (23)

### A.2 Equations Updated with Godfather Shocks

When a worker experiences a godfather shock, he randomly meets a firm and is forced to accept an offer with that firm. When we include godfather shocks in the employed worker's Hamilton-Jacobi-Bellman equation (8), the worker's problem adds a new line like so:

$$\rho E_{ai}(h, p, r) = \max_{l,s} u(c) - d_{E}(l+s) + (a(lh)^{\omega} - \delta h) \frac{\partial E_{ai}(h, p, r)}{\partial h} 
+ \Lambda(rhp) \left[ U_{ai}(h, hpr, \iota) - E_{ai}(h, p, r) \right] + \zeta \left[ E_{a,i+1}(h, p, r) - E_{ai}(h, p, r) \right] 
+ sm(\theta) \left( \int_{q_{ai}(h, p, r)}^{p} \left[ E_{ai} \left( h, p, R_{ai}^{R}(h, p, p') \right) - E_{ai}(h, p, r) \right] dF(p') \right) 
+ \int_{p}^{\overline{p}} \left[ E_{ai} \left( h, p', R_{ai}^{P}(h, p, p') \right) - E_{ai}(h, p, r) \right] dF(p') \right) 
+ \psi \int_{p}^{\overline{p}} \left[ E_{ai} \left( h, p', R_{ai}^{G}(h, hpr, p') \right) - E_{ai}(h, p, r) \right] dF(p')$$
(24)

subject to

$$c = [1 - T(hpr)]hpr.$$

where  $R_{ai}^G(h,hpr,p')$  is the endogenous piece rate arising from a godfather shock.

Firms have an additional risk of losing their worker to a godfather shock, which adds

 $\psi$  to the bottom line of (15) like so:

$$\rho J_{ai}(h, p, r) = hp(1 - r)(1 - \tau_b) + (a(l_{ai}(h, p, r)h)^{\omega} - \delta h) \frac{\partial J_{ai}(h, p, r)}{\partial h} + \zeta \left[ J_{a,i+1}(h, p, r) - J_{ai}(h, p, r) \right] + s_{Eai}(h, p, r)m(\theta) \int_{q_{ai}(h, p, r)}^{p} \left[ J_{ai}\left(h, p, R_{ai}^{R}(h, p, p')\right) - J_{ai}(h, p, r) \right] dF(p') + \left[ s_{Eai}(h, p, r)m(\theta) \left( F(\overline{p}) - F(p) \right) + \Lambda(hpr) + \psi \right] \left( -J_{ai}(h, p, r) \right)$$
(25)

Firms also have an additional channel for filling job vacancies; an employed worker might experience a godfather shock and land with the firm. Thus, the free entry condition gets a new term,

$$\kappa = m_{f}(\theta) \left[ \sum_{a,i} \int_{\Psi_{E}} s_{Eai}(h,p,r) \int_{p}^{\overline{p}} J_{ai} \left( h, p', R_{ai}^{P}(h,p,p') \right) dF(p') d\Psi_{E}(h,p,r|a,i) \right. \\
+ \sum_{a,i} \int_{\Psi_{U}} s_{Uai}(h,w) \int_{z_{ai}(h,w)}^{\overline{p}} J_{ai} \left( h, p', R_{ai}^{U}(h,w,p') \right) dF(p') d\Psi_{U}(h,w|a,i) \right] \\
+ \Psi \sum_{a,i} \int_{\Psi_{E}} \int_{\underline{p}}^{\overline{p}} J_{ai} \left( h, p', R_{ai}^{G}(h,hpr,p') \right) dF(p') d\Psi_{E}(h,p,r|a,i).$$
(26)

Finally, I assume that, when bargaining over wages, the outside option for workers who have been subject to a godfather shock is unemployment without unemployment benefits. It is impossible that the worker goes to this state, but this establishes the bargaining over r. So,  $R_{ai}^G(h,hpr,p')$  solves

$$E_{ai}(h, p', R_{ai}^{G}(h, hpr, p')) = U_{ai}(h, hpr, 0) + \eta \left[ E_{ai}(h, p', 1) - U_{ai}(h, hpr, 0) \right].$$
 (27)

# **B** Simple Two-Period Model

I write this model in discrete time. Agents live for two periods,  $i = \{0,1\}$ , and discount the second period at discount factor  $\beta$ . In the first period, agents work, consume, and make investments in human capital accumulation and job search effort; in the second,

agents only work and consume.

Assume that workers earn the piece rate r=1 and that there is no human capital depreciation,  $\delta=0$ . Otherwise, the human capital accumulation equation is as in the full model. If workers exert search effort s, then, with probability  $m(\theta)$ , they move from their current p to  $p + \Delta_p$ .

In the first period, a worker with learning ability a, human capital h, and firm productivity p solves

$$E_{a0}(h,p) = \max_{l,s} u(c) - d_E(l+s) + \beta E_{a1}(h',p')$$
(28)

subject to

$$c = [1 - T(hp)]hp$$
  

$$h' = h + a(lh)^{\omega}$$
  

$$p' = p + sm\Delta_p.$$

In the second period, the worker simply enjoys the fruits of his labor,

$$E_{a1}(h,p)=u(c)$$

where

$$c = [1 - T(hp)]hp.$$

When we solve for the first order conditions for Equation (28), we get Equation (19).

# C Equilibrium Definition

The recursive stationary equilibrium consists of a set of value functions  $\{E_{ai}(h, p, r), U_{ai}(h, w, \iota), \overline{E}, J_{ai}(h, p, r)\}$ , a human capital investment policy function  $l_{Eai}(h, p, r)$ , a set of search effort policy functions  $\{s_{Eai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, p, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, \mu, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, \mu, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, \mu, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, \mu, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, \mu, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, \mu, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, \mu, r), s_{Uai}(h, w, \iota)\}$ , a set of job search cutoff rules  $\{q_{ai}(h, \mu, r), s_{Uai}(h, w, \iota)\}$ 

 $z_{ai}(h, w, \iota)$ , a set of wage functions  $\{R_{ai}^P(h, p, p'), R_{ai}^R(h, p, p'), R_{ai}^U(h, w, \iota, p'), R_{ai}^G(h, hpr, p')\}$ , vacancies v, aggregate search effort S, labor market tightness  $\theta = v/S$ , a distribution of workers  $\Psi = \{\Psi_E(h, p, r | a, i), \Psi_U(h, w | a, i, \iota), \Psi_R(a)\}$ , and a set of government policy parameters  $\mathcal{G} = \{\tau_0, \tau_1, \tau_b, b, \chi, SS, \overline{g}\}$  which satisfy:

- 1. Employed worker optimization: Given  $R_{ai}^P(h, p, p')$ ,  $R_{ai}^R(h, p, p')$ ,  $R_{ai}^G(h, hpr, p')$ ,  $q_{ai}(h, p, r)$ ,  $\theta$ , and G,  $E_{ai}(h, p, r)$  solves (24) subject to (9) and (13) with associated decision rules  $l_{Eai}(h, p, r)$  and  $s_{Eai}(h, p, r)$ .
- 2. Unemployed worker optimization: Given  $R_{ai}^{U}(h, w, p')$ ,  $z_{ai}(h, w, \iota)$ ,  $\theta$ , and  $\mathcal{G}$ ,  $U_{ai}(h, w, \iota)$  solves (10) subject to (11) and (14) with associated decision rule  $s_{Uai}(h, w)$ .
- 3. Retired workers:  $\overline{E}$  solves (12).
- 4. Filled jobs: Given  $R_{ai}^R(h, p, p')$ ,  $l_{ai}(h, p, r)$ ,  $s_{Eai}(h, p, r)$ ,  $\theta$ , and  $\mathcal{G}$ ,  $J_{ai}(h, p, r)$  solves (25) and (16).
- 5. Wage equations: Given  $E_{ai}(h, p, r)$  and  $U_{ai}(h, w, \iota)$ ,
  - (a)  $R_{ai}^{P}(h, p, p')$  solves (3),
  - (b)  $R_{ai}^{R}(h, p, p')$  solves (4),
  - (c)  $R_{ai}^{U}(h, w, \iota, p')$  solves (6), and
  - (d)  $R_{ai}^{G}(h, hpr, p')$  solves (27).
- 6. Job search cutoff rules: Given  $E_{ai}(h, p, r)$  and  $U_{ai}(h, w, \iota)$ ,
  - (a)  $q_{ai}(h, p, r)$  solves (5) and
  - (b)  $z_{ai}(h, w, \iota)$  solves (7).
- 7. Free entry: Given  $J_{ai}(h, p, r)$ ,  $R_{ai}^P(h, p, p')$ ,  $R_{ai}^U(h, w, \iota, p')$ ,  $R_{ai}^G(h, hpr, p')$ ,  $s_{Eai}(h, p, r)$ ,  $s_{Uai}(h, w, \iota)$ , S,  $\Psi$ , and G,  $\theta$  solves (26).
- 8. Government budget constraint:  $\mathcal{G}$  satisfies (18).

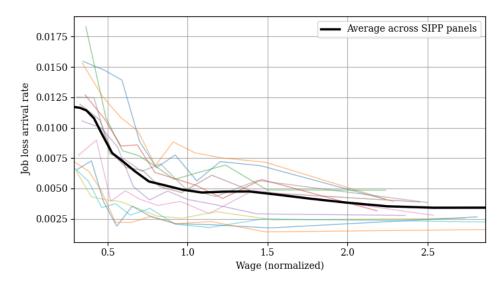


Figure 11: Estimate of exogenous job destruction function,  $\Lambda(wage)$ 

Estimates of the employment-to-unemployment rate in the SIPP by hourly wage. Each colored line represents a different SIPP panel. For each panel, I estimate the employment-to-unemployment rate for 10 hourly wage deciles, then interpolate between points. The thick black line is the resulting weighted average job loss function by wage,  $\Lambda(wage)$ . I flatly extrapolate the edges of  $\Lambda$ . Wage is normalized such that the mean hourly wage equals one. The monthly employment-to-unemployment rate is converted to a continuous time arrival rate as in Mukoyama (2014).

- 9. Aggregate search effort: Given  $s_{Eai}(h, p, r)$ ,  $s_{Eai}(h, w)$ , and  $\Psi$ , S satisfies (23).
- 10. Consistency:  $\Psi$ , as defined in (3.9), is the stationary distribution.

### **D** Calibration Details

#### D.1 $\Lambda$ Function

I estimate the exogenous job destruction function,  $\Lambda(hpr)$ , directly from SIPP data. See Figure 11 for an illustration. As with the tax function, the mean wage is normalized to one.

### D.2 Learning Ability Distribution

I discretize the learning ability distribution as follows. The grid of a points is  $\{a_1,...,a_J\}$  where  $a_j = \mathbb{E}[a|a_{P_j} \le a < a_{P_{i+1}}]$  and  $a_{P_j}$  is the  $P_j$ -th percentile of the ability distribution. I take the expectation using  $PLN(\mu_a, \sigma_a, 1/\lambda_a)$ . To calculate the PLN distribution, I use analytical expressions in Hajargasht and Griffiths (2013). Percentiles are chosen to be (0.0, 0.3, 0.6, 0.9, 0.99, 1.0).

### **E** Restricted Model Details

Figure 12 displays calibrated lifecycle profile of the mean log wage in each restricted model.

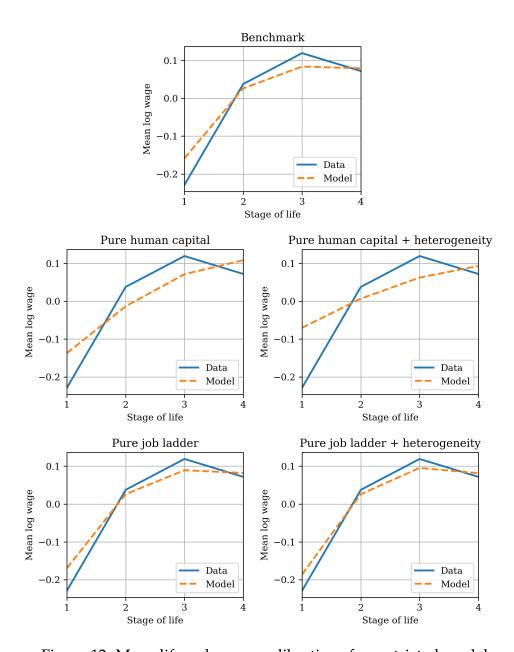


Figure 12: Mean lifecycle wage calibrations for restricted models

Calibrated lifecycle profile for mean log wage in the benchmark model and each restricted model. The restricted models are described in Section 6.2.2. The benchmark calibration in this figure is identical to Figure 5.7.

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