A Model of Wages with Endogenous Human Capital and Job Search

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Abstract

There are two paradigms which explain why wages grow over the lifecycle: human capital accumulation, and on-the-job search in an environment with search frictions. I reconcile both theories in a model of wages which includes endogenous human capital accumulation, endogenous search effort, endogenous job posting, and a lifecycle. The model is calibrated to the US using SIPP data. In the benchmark economy, both human capital and search contribute significantly to lifecycle wage growth and dispersion. To demonstrate the usefulness of the model, I perform a policy experiment where I increase tax progressivity. Increasing tax progressivity decreases wages, and the negative effect is driven more by a decrease in search effort than by a decrease in human capital accumulation. In contrast to the prediction of a pure human capital model, the model predicts that increasing tax progressivity has little effect on cross-sectional wage inequality.

Economists have long been interested in wage levels, dispersion, and lifecycle wage growth. In this paper, I will focus on two paradigms which explain why wages grow over the lifecycle.[1] The first involves human capital accumulation, embodied by Ben-Porath (1967). In this theory, workers invest in accumulating skills. As workers accumulate human capital, they become more productive, which increases their wage. Such a theory does not require any heterogeneity in firms or search frictions to generate lifecycle wage growth. However, to generate the lifecycle profile in wage dispersion, the human capital theory relies on idiosyncratic shocks, heterogeneity in learning ability, and correlation between initial human capital and learning ability Huggett et al. (2011).

Another paradigm involves on-the-job search as embodied by Burdett and Mortensen (1998). In this theory, workers jump to progressively more productive firms over time. As workers move to more productive firms, they become more productive, and their wages increase. Such a theory relies on search frictions; if workers could start in the most productive firm, there would be no wage growth. There are potential channels of endogeneity in this theory in the form of worker search intensity and firm job postings.

In this paper, I develop a theory which combines both paradigms. I write a macro labor model of wages with combines endogenous human capital accumulation, endogenous search effort, endogenous job posting, and a lifecycle. To my knowledge, a quantitative model with all of these elements has not been studied. Combining these channels gives me rich interactions and endogeneity in evaluating policy.

Why does it matter if both human capital accumulation and search effort are endogenous? The first reason is that it is clear that the real world involves both channels. Both channels are important for accounting for the level of wages in an economy as well as lifecycle wage growth. It therefore follows that both channels may respond to changes in policy. For a more accurate prediction, it is important to have both.

Second, my paper allows us to consider which channel is more important for accounting for wage levels, as well as which channel drives the response to policy changes. Such results illuminate which channels are the most important, or if both are important when doing counterfactual analyses of wages.

In my model, workers are heterogeneous in their level of human capital, their stage in the lifecycle, and in fixed learning ability. Firms are heterogeneous in productivity. Workers choose how much effort to invest in human capital accumulation or in searching for a new job. Both activities are costly in the sense that the worker experiences disutility. Both activities benefit the worker because they lead to wage growth. In the case of human capital investment, workers accumulate human capital according to a law of motion of human capital as in Ben-Porath (1967); thus, increases in wages from human capital accumulation are continuous. In the case of job search effort, workers increase their probability of meeting an outside firm by investing more in search effort. Upon meeting an outside firm, if the worker can earn a greater wage at the new firm, he will leave his current firm to work for the new firm. Thus, increases in wages from job search are continuous and stochastic.

The labor market is characterized by random search. When workers and firms meet, they

[1] There are more possible paradigms, such as learning about type.

bargain over wages according to a surplus sharing rule as in Cahuc et al. (2006). Workers face negative risk from exogenous unemployment shocks. Thus, the model includes a thick left tail of wage shocks.

Firms choose how many jobs vacancies to post. In equilibrium, a free entry condition requires that the number of job vacancies is such that firms are indifferent between posting another job. From the firm's perspective, the optimal number of job postings depends on how workers invest in human capital and job search. If workers have more human capital, then matches with workers will be more profitable, incentivizing more job posting. Similarly, if workers invest more effort into search, then firms have a greater probability of converting a vacant job to a filled job, increasing the benefit of job posting and incentivizing more job posting.

I calibrate the model to replicate key features in the data. I use moments from the Survey of Income and Program Participation going back to 1990. Importantly, the model generates accurate lifecycle profiles for mean wages, wage dispersion, and job-to-job transitions. Furthermore, I replicate the level of wage increases for those who stay in the same job and those who switch jobs, which helps to disentangle wage growth from human capital versus job search. My successful calibration illustrates that my theory of endogenous shocks can successfully match the data.

With my calibrated benchmark model, I first demonstrate how wage levels are equal parts attributable to human capital and firm productivity. Lifecycle wage growth is also equally attributable to both channels. However, the increase in wage dispersion over the lifecycle is entirely attributable to an increase in dispersion in human capital.

I demonstrate the usefulness of the model with a simple policy experiment: I increase tax progressivity.

In a model with only the human capital channel, an increase in tax progressivity will lead to a decrease in wage dispersion. The reason is that increasing tax progressivity disincentivizes wage accumulation for those with higher wages. The increase in tax progressivity decreases wages for all groups, but particularly those with high learning ability. This decreases wage dispersion.

1 Model

The key features of the model are endogenous human capital accumulation, endogenous job search intensity, endogenous job posting, a lifecycle, and progressive taxes and transfers. For computational efficiency, I use continuous time. In general, I make modeling assumptions in line with current literature; my aim is to evaluate the effects of a handful of new elements in an otherwise-standard model.

1.1 Life Cycle

The model uses a stochastic lifecycle. All workers have $I+1$ stages of life; they work for I equally-spaced stages of life, then retire. Workers transition from stage i to $i{+}1$, $i \in \{1, ..., I\}$, with arrival rate ζ. In the retirement stage, workers die with probability $\overline{\zeta}$, then are replaced by newborns in the first working stage. To fix ideas, when I calibrate the model, I set $I = 4$ and set the working years from ages 23 through 65, so each stage of life is approximately a decade. Upon retirement, the worker earns a flat social security payment, SS.

1.2 Production

Workers are heterogeneous in human capital h and firms are heterogeneous in productivity p. A match between a worker with human capital h and a firm with productivity p produces hp of the numeraire consumption good. Importantly, note that human capital and firm productivity are complements in production. Of the total production of the match, workers earn a piece-rate $r \in [0, 1]$, so the worker earns the pre-tax-and-transfer wage hpr. Firms earn the pre-tax profit $(1 - r)hp$. Workers and firms negotiate r as described below.

1.3 Worker Decisions

Workers are either employed, E, or unemployed, U. Employed workers chooses how much effort to invest in human capital accumulation, l (for "learning"), and job search, s. Unemployed workers can only invest in job search.

Workers experience utility from consumption, $u(c)$, and disutility of effort, $d_j(l + s)$ for $j \in \{E, U\}$. Flow utility is the sum of each, $u(c)+d_j(l+s)$. Consumption equals the worker's after-tax-and-transfer wage.[2] Workers discount the future at discount rate ρ.

1.4 Human Capital Accumulation

Workers accumulate human capital as in Ben-Porath (1967). At the time of birth, workers are heterogeneous in fixed learning ability, a. Human capital evolves according to the law of motion

$$\frac{dh}{dt} = a(lh)^{\omega} - \delta h \tag{1}$$

where $\omega \in (0, 1)$ governs the curvature to the return to learning, $\delta \in [0, 1]$ is the human capital depreciation rate, and $a > 0$ determines the efficiency of human capital accumulation. According to equation (1), all else equal, workers with a higher a will have greater returns to human capital accumulation. So, workers with higher learning ability will choose a higher l and accumulate more human capital.

1.5 Labor Market

All workers, employed or unemployed, meet open vacancies at rate $sm(\theta)$, where s is search effort as described above, $m(\theta)$ is a meeting function, and θ is the labor market tightness ratio. θ is defined as the number of vacancies v per unit of aggregate search effort S, or

[2] In other words, workers cannot save.

$\theta = v/S$.[3] Search is random in the sense that workers cannot direct their search to firms with particular productivity levels and firms cannot post jobs for specific types of workers.

Upon meeting an open vacancy, the worker observes the associated firms' productivity, p', drawn from a distribution $F(p')$ over $[\underline{p}, \overline{p}]$. The worker and new firm then bargain over the wage in the form of the piece rate r, and will form a match if the worker is better off with the new firm.

Each firm has a maximum of one worker. So, if a match is destroyed, the firm's production and profit drops to zero. When bargaining, the value of the firm's outside option, the absence of a match, is always 0. Firms post jobs at cost κ in each period. From the firm's perspective, an open job vacancy meets a worker with probability $m_f(\theta)$.

Matches are subject to job destruction shocks which are a decreasing function of wage, $\Lambda(hpr)$. I assume that the job destruction rate depends on the wage for two reasons: (1) it is a clear feature in the data, and (2) the increased risk of unemployment for low-wage workers is key for accounting for increasing wage dispersion (Karahan et al., 2022).

Unemployed workers earn unemployment benefits bw where b is the replacement rate (typically 0.5 in the US) and w is the most recent wage the worker was earning before becoming unemployed. Unemployment benefits expire with probability χ, after which the worker gets a transfer payment T_0. I will denote workers without unemployment benefits with $w = 0$.

1.6 Wage Bargaining

I use the wage bargaining protocol of Cahuc et al. (2006) as applied in Bagger et al. (2014). The protocol determines the value of r when a worker and a firm form a match. In summary, the protocol uses a surplus sharing rule which generates endogenous values of r which depend on characteristics of the firm and the worker.

Let $E_{ai}(h, p, r)$ be the value of employment for a worker with learning ability a, in stage of life i, with human capital h, and currently working for a firm with productivity p at the piece rate r. I will define this value function mathematically below. The worker is currently earning the wage hpr. The greatest wage that the worker can earn while working at the current firm is hp, the case where $r = 1$. If $r = 1$, then the firm makes zero profit, and since the value of the firm's outside option is 0, the firm is indifferent between maintaining the match or not.

First, consider the case of a meeting between an employed worker and an outside firm, the result of on-the-job search. The incumbent firm and the outside firm will commence Bertrand competition for the worker, making alternating bids on the worker's wage. As the firms offer progressively higher wages, they reach a point where the firm with lower p cannot pay the worker a higher wage without earning negative profit; for the firm with lower productivity, the wage is given by $r = 1$. At this point, the firm with the higher productivity wins the worker.

Suppose that the outside firm has greater productivity that the incumbent firm, $p' > p$.

[3] S is mathematically defined in Appendix A Equation (24).

In this case, the worker will be poached and will make a job-to-job transition to the outside firm. The poaching firm will pay the worker a piece rate $R^P_{ai}(h, p, p')$ which solves

$$E_{ai}\left(h, p', R^P_{ai}(h, p, p')\right) = E_{ai}(h, p, 1) + \eta\left[E_{ai}(h, p', 1) - E_{ai}(h, p, 1)\right] \tag{2}$$

where $\eta \in [0, 1]$ is the worker's bargaining weight. At the new firm (p'), the worker will earn $r = R^P_{ai}(h, p, p')$ where the worker's utility the maximum the incumbent firm could have paid, $E_{ai}(h, p, 1)$, plus a fraction η of the additional worker surplus created from the match.[4]

Next, suppose that the outside firm has lower productivity than the incumbent firm, $p' < p$. In this case, the incumbent firm will retain the worker because it can pay the worker a greater wage while remaining profitable. However, if the outside firm has a high enough p, it is possible that the firm could pay the worker a greater wage than it earns now. In this case, the outside firm triggers a renegotiation between the worker and the incumbent firm. The worker will stay at the current firm but get wage increase; the worker will earn a greater piece rate $R^R_{ai}(h, p, p')$ which solves

$$E_{ai}\left(h, p, R^R_{ai}(h, p, p')\right) = E_{ai}(h, p,' 1) + \eta\left[E_{ai}(h, p, 1) - E_{ai}(h, p', 1)\right]. \tag{3}$$

The worker earns a piece rate such that worker earns the entirety of the what the outside firm could offer, $E_{ai}(h, p', 1)$, plus a fraction of the additional worker surplus.

The third possibility is that the outside firm has a lower productivity than the incumbent firm, $p' < p$, and the outside firm cannot pay the worker a greater wage even if if offers the worker $r = 1$. In this case, the worker stays at the same firm for the same r.

Whether the new firm's productivity p' is higher enough to trigger negotiation depends on $q_{ai}(h, p, r)$ as defined by

$$E_{ai}(h, p, r) = E_{ai}\left(h, q_{ai}(h, p, r), 1\right) + \eta\left[E_{ai}(h, p, 1) - E_{ai}\left(h, q_{ai}(h, p, r), 1\right)\right]. \tag{4}$$

If $p' < q_{ai}(h, p, r)$, then the new firm cannot make the worker better off even if they pay the worker $r = 1$. In sum, if $p' < q_{ai}(h, p, r)$, the new firm cannot compete and the worker stays with the same firm at the same piece rate; if $q_{ai}(h, p, r) < p' < p$, the worker will stay with their current firm but leverage the outside offer into a greater wage; and if $p' < p$, the worker is poached.

[4] A slight clarification is in order. The worker earns a fraction η of the additional potential worker surplus from the match, not the additional total surplus from the match. This is slightly different form the original scheme from Cahuc et al. (2006). The paper models the bargaining process "using a version of the Rubinstein (1982) infinite-horizon alternating-offers bargaining game." In the mofrl of Cahuc et al. (2006), workers have linear utility over the wage, as do firms; thus, $E_{ai}(h, p, 1)$ is equivalent to the total surplus of the match. In that case, we can say that the worker earns a fraction η of the additional total surplus of the match. The same is true in Engbom (2022) because workers have linear preference in that model. However, as Bagger et al. (2014) points out, when workers have curvature in utility and firms have linear utility, the total amount of surplus from the match is not independent of r and therefore not fixed. So the present scheme may not be a Nash equilibrium. My case is further complicated by the fact the workers also have utility over leisure, a new feature in this type of model. So, I elect to follow Bagger et al. (2014) (as well as Karahan et al. (2022)) and impose this wage structure even with curvature in utility. For an approach which uses total surplus but allows for curvature in utility, see Lise et al. (2016).

Finally, consider a meeting between an unemployed worker and a firm. In this case, the firm is competing not against another firm, but against the worker's outside option of remaining unemployed. Let the $U_{ai}(h, w, I_b)$ be the value of unemployment for a worker with learning ability a, in stage of life a, with human capital h, and who previously earned the pre-tax wage w. Suppose that the outside firm can make an offer which will entice the worker to leave unemployment and form a match. The worker earn a piece rate $R^U_{ai}(h, w, I_b, p')$ which solves

$$E_{ai}\left(h, p', R^U_{ai}(h, w, I_b, p')\right) = U_{ai}(h, w, I_b) + \eta\left[E_{ai}(h, p', 1) - U_{ai}(h, w, I_b)\right]. \tag{5}$$

The piece rate is set such that the worker gets the value of their outside option, $U_{ai}(h, w, I_b)$, plus a fraction η of the additional worker surplus from the match.

It is possible that a firm is cannot make an unemployment worker better off, even if the firm pays the worker $r = 1$. Let $z_{ai}(h, w, I_b)$ be the lowest possible value of p' such that the worker will leave unemployment. $z_{ai}(h, w, I_b)$ solves[5]

$$U_{ai}(h, w, I_b) = E_{ai}\left(h, z_{ai}(h, w, I_b), 1\right). \tag{6}$$

1.7 Taxes and Transfers

I assume a progressive average tax and transfer function as in Bénabou (2002). At wage hpr, workers pay the average tax rate

$$T(hpr) = 1 - \tau_0 (hpr)^{-\tau_1}. \tag{7}$$

Workers with lower wages may pay negative taxes, which means that their transfer payments exceed taxes paid. τ_1 determines the progressivity of the tax and transfer system while τ_0 determines its level. Later in the paper, I will experiment with increasing τ_1.

On the firm side, firms pay a flat rate rate τ_b.

[5]The algebra is as follows: when faced with a firm of productivity $z_{ai}(h, w, I_b)$, the unemployed worker is indifferent between remaining unemployed and working for the firm at the highest possible wage, so

$$U_{ai}(h, w, I_b) = U_{ai}(h, w, I_b) + \eta\left[E_{ai}\left(h, z_{ai}(h, w, I_b), 1\right) - U_{ai}(h, w, I_b)\right],$$

which simplifies to (6).

1.8 Worker Value Functions

The value of employment for a worker with learning ability a, in stage of life i, with human capital h, and currently working for a firm with productivity p at the piece rate r solves

$$\begin{aligned}\rho E_{ai}(h,p,r) &= \max_{l,s}\ u(c) - d_E(l+s) + (a(lh)^{\omega} - \delta h)\,\frac{\partial E_{ai}(h,p,r)}{\partial h}\\ &+\Lambda(rhp)\left[U_{ai}(h,hpr) - E_{ai}(h,p,r)\right] + \zeta\left[E_{a,i+1}(h,p,r) - E_{ai}(h,p,r)\right]\\ &+sm(\theta)\Bigg(\int_{q_{ai}(h,p,r)}^{p}\left[E_{ai}\left(h,p,R^{R}_{ai}(h,p,p')\right) - E_{ai}(h,p,r)\right]\,dF(p')\\ &+\int_{p}^{\overline{p}}\left[E_{ai}\left(h,p',R^{P}_{ai}(h,p,p')\right) - E_{ai}(h,p,r)\right]\,dF(p')\Bigg)\end{aligned} \tag{8}$$

subject to

$$c = [1 - T(hpr)]hpr \tag{9}$$

An employed worker chooses how much effort to invest in skill accumulation and search. The worker enjoys utility from after-tax consumption and endures disutility of effort. Given l, the worker gains or loses human capital. With probability $\Lambda(hpr)$, the match is destroyed and the worker becomes unemployed, and with probability ζ, the worker ages to the next stage of life. With probability $sm(\theta)$, the worker meets an outside firm with productivity p'. If $p' \in [q_{ai}(h,p,r), p$, the worker leverages the outside offer and renegotiates a higher wage with the incumbent firm. If $p' \in (p, \overline{p}]$, the worker is poached.

The value of unemployment for a worker with learning ability a, in stage of life a, with human capital h, and who previously earned the pre-tax wage w solves

$$\begin{aligned}\rho U_{ai}(h,w) &= \max_{s}\ u(c) + d_U(s) - \delta h\frac{\partial U_{ai}(h,w)}{\partial h}\\ &+\zeta\left[U_{a,i+1}(h,w) - U_{ai}(h,w)\right] + \chi\left[U_{ai}(h,w) - U_{ai}(h,0)\right]\\ &+sm(\theta)\int_{z_{ai}(h,w)}^{\overline{p}}\left[E_{ai}\left(h,p',R^{U}_{ai}(h,w,p')\right) - U_{ai}(h,w)\right]\,dF(p')\end{aligned} \tag{10}$$

subject to

$$c = \mathbb{1}[w>0](1 - T(bw))bw + \mathbb{1}[w=0]T_0 \tag{11}$$

where $\mathbb{1}$ is an indicator function. An unemployment worker chooses how much effort to invest in search,[6] enjoys utility from consumption, and endures disutility from search effort. The worker's consumption level depends of if he still qualifies for unemployment benefits. With probability ζ, the worker ages to the next stage of life, and with probability χ, his unemployment benefits expire. With probability $sm(\theta)$, the worker meets an outside firm with productivity p', and if $p \in [z_{ai}(h,p,r), \overline{p}]$, the worker will accept a job offer.

[6] I assume that unemployed workers cannot accumulate human capital in order to better match the data.

Finally, the value of retirement, $\overline{E}$, solves

$$\left(\rho + \overline{\zeta}\right) \overline{E} = u(SS). \tag{12}$$

Both employed and unemployed workers transition to retirement as follows:

$$E_{a,I+1}(h,p,r) = U_{a,I+1}(h,w) = \overline{E}. \tag{13}$$

1.9 Firms

I assume free entry in the labor market. Firms have linear utility over profit. In equilibrium, firms will choose some amount of open vacancies, v, such that firms are indifferent between the marginal job posting. What follows builds toward this free entry condition.

The optimal level of job vacancies will depend on the distribution of workers in the economy. All else equal, if all workers in the economy were to increase their human capital, because h and p are complements, then filled jobs become more profitable, the benefit of job posting increases, and firms will post more jobs. Similarly, if workers increase search effort, then firms have a greater probability of converting an open vacancy to a filled job, the benefit of job posting increases, and firms will post more jobs.

Let $l^*_{ai}(h,p,r)$, $s^*_{Eai}(h,p,r)$, and $s^*_{Uai}(h,w)$ denote the policy functions for employed workers' human capital effort, employed workers' search effort, and unemployed workers' search effort, respectively. The value of a filled job with a worker of learning ability a, in stage of life i, with human capital h, working for a firm with productivity p at piece rate r solves

$$\begin{aligned}\rho J_{ai}(h,p,r) = hp(1-r)(1-\tau_b) + \left(a(l_{ai}(h,p,r)h)^{\omega} - \delta h\right) \frac{\partial J_{ai}(h,p,r)}{\partial h} \\ +\zeta\left[J_{a,i+1}(h,p,r) - J_{ai}(h,p,r)\right] \\ +s_{Eai}(h,p,r)m(\theta)\int_{q_{ai}(h,p,r)}^{p} \left[J_{ai}\left(h,p,R^R_{ai}(h,p,p')\right) - J_{ai}(h,p,r)\right]\, dF(p') \\ +\Big[s_{Eai}(h,p,r)m(\theta)\left(F(\overline{p}) - F(p)\right) + \Lambda(hpr)\Big](-J_{ai}(h,p,r))\end{aligned} \tag{14}$$

If the worker retires, the firm gets 0 profit:

$$J_{a,I+1}(h,p,r) = 0 \tag{15}$$

These equations closely mirror Equation (8). Note that the firms earns after-tax profit $hp(1-r)(1-\tau_b)$. There are three ways the match can be destroyed and the firm is left with a value of 0: (1) the worker is poached by a firm with higher productivity, which occurs with probability $(s_{Eai}(h,p,r)m(\theta)\left(F(\overline{p}) - F(p)\right))$; (2) the jobs is exogenously destroyed at rate $\Lambda(hpr)$; or (3) the worker retires, which occurs with probability ζ if the worker is in the last stage of working life.

In equilibrium, the free entry condition implies that the benefit of a job posting equals its cost. Thus, the following free entry condition holds in equilibrium:

$$\kappa = m_f(\theta)\Bigg[\sum_{a,i}\int_{\Psi_E} s_{Eai}(h,p,r)\int_{p}^{\overline{p}} J_{ai}\left(h,p',R^P_{ai}(h,p,p')\right)dF(p')d\Psi_E(h,p,r|a,i) \\ +\sum_{a,i}\int_{\Psi_U} s_{Uai}(h,w)\int_{z_{ai}(h,w)}^{\overline{p}} J_{ai}\left(h,p',R^U_{ai}(h,w,p')\right)dF(p')d\Psi_U(h,w|a,i)\Bigg] \tag{16}$$

where Ψ is the distribution of workers. The right hand side of Equation (16) consists of two terms, both multiplied by the $m_f(\theta)$, the probability that an open job vacancy meets a worker. The first term combines the probability and benefit of poaching an employed worker. The second term combines the probability and benefit of hiring an unemployed worker.

1.10 Government Budget Constraint

When I perform counterfactual experiments with tax policy, I discipline th model such that a government budget constraint must hold in equilibrium. The government collects revenues from taxes on workers and firms. Its outlays consist of means-tested transfers (from negative average tax rates), transfers to workers without unemployment benefits, unemployment benefits, social security payments, and government spending, $\overline{g}$. I calculate $\overline{g}$ in my benchmark economy such that the government budget constraint holds, and I assume that the government must spend $\overline{g}$ in counterfactual experiments. The government budget constraint is written mathematically in Appendix A Equation (25).

2 Calibration

2.1 Data

To calibrate the model, I use data from the Survey of Income and Program Participation (SIPP). The SIPP is a panel data set with interviews every four months or every year, depending on the panel.[7] In each interview, respondents report on what occurred in the time between interviews. I use every panel which between the years 1990 and 2019, 12 panels in total.[8]

There are two features which makes the SIPP convenient for my setting. First, the SIPP has monthly wages. This feature is vital for me because it allows me to target wage between subsequent months, with or without a job change. Second, the SIPP tracks worker-job matches by assigning job IDs to workers, which allows me to observe job switches.[9]

[7] In 2018, the SIPP transitioned from interviewing respondents every four months to every year and began using an event history design.

[8] I use the 1990, 1991, 1992, 1993, 1996, 2001, 2004, 2008, 2014, 2018, 2019, and 2020 panels.

[9] I convert monthly transfer rates (unemployment to employment, employment to unemployment, job

2.2 Functional Forms and Distributions

The utility of consumption is log, $u(c) = \ln(c)$. The utility of leisure is convex, iso-elastic, and its parameters are state-dependent,

$$d_j(l+s) = \phi_j \frac{(l+1)^{1+\gamma_j}}{1+\gamma_j},\ j \in \{E, U\} \tag{17}$$

with $\phi_j > 0$ and $\gamma_j > 0$.

I use a Cobb-Douglas matching function. So, the total number of meetings in a period is $\xi S^{\alpha} v^{1-\alpha}$ with $\xi > 0$ and $\alpha \in (0,1)$. From the perspective of a worker, the probability of meeting an open vacancy per unit of search effort is

$$m(\theta) = \xi \theta^{1-\alpha}. \tag{18}$$

For the firm, the probability of meeting a worker is

$$m_f(\theta) = \xi \theta^{-\alpha}. \tag{19}$$

I assume that the distribution of firm productivity, F, is lognormally distributed, $p' \sim LN(\mu_p, \sigma_p)$. Following Badel et al. (2020), learning ability a is drawn from a Pareto lognormal distribution, $a \sim PLN(\mu_a, \sigma_a, \lambda_a)$, which is a lognormal distribution with a Pareto right tail.[10] Then, the distribution of initial human capital, h_0, is a linear function of a,

$$\ln(h_0) = \beta_0 + \beta_1 \ln(a) + \ln(\varepsilon) \tag{20}$$

with $\varepsilon \sim LN(0, \sigma_\varepsilon)$.

I make one final adjustment to the model. To help the model fit the data, I allow for "godfather shocks" (Dorn, 2018). With probability ψ, an employed worker experiences a godfather shock, which means that they meet an outside firm $p' \sim F(p')$ and must accept a job from the new firm. Though admittedly ad-hoc, godfather shocks are useful for generating one feature in the data: of all workers who switch jobs, 30% earn a lower wage in the new job. In my model, it is impossible for a worker to make a job-to-job transition without a wage increase.

2.3 Taxes and Transfers

There are two parameters to calibrate for my average tax function. τ_0 and τ_1. I estimate the average tax function such that it accurately represents the average tax that a worker in US pays while taking transfers into account. The mean wage is normalized to one.

To estimate the parameters in my tax-and-transfer function, I combine an estimate of the average tax function in the US (Guner et al., 2014) and an estimate of a function which

switches) to continuous-time arrival rates and correct for time aggregation bias using the methods developed in Mukoyama (2014).

[10]See Hajargasht and Griffiths (2013) for the equations behind the Pareto lognormal distribution.

returns the total amount of means-tested transfers in the US given the worker's income (Guner et al., 2020).[11] I apply both functions an income grid, then estimate τ_0 and τ_1 from Equation (7) on that series. As is typical, the Bénabou (2002) tax function fits the data well, even when we take transfers into account. The key difference is that including transfers significantly is necessarily accompanied with a higher estimate for progressivity, τ_1. I estimate $\tau_0 = 0.899$ and $\tau_1 = 0.120$.

For my estimate of the tax function from Guner et al. (2014), and I use the estimate for all households (married and unmarried) which takes into account the earned income tax credit (EITC).[12] For my estimate of the transfer function from Guner et al. (2020), I use an estimate for all households which takes into the following programs: WIC (the Special Supplemental Nutrition Program for Woman, Infants, and Childen), SSI (Supplemental Security Income, for those with disabilities), SNAP (Supplemental Nutrition Asisstance Program, formerly knows as Food Stamps), TANF (Temporary Assistance for Needy Families), and housing. It does not include Medicaid.

The transfer payment made to workers without unemployment benefits is estimated from Guner et al. (2020) as $T_0 = 0.121$. The flat business tax rate is $\tau_b = 0.243$ as estimated in Cooper et al. (2016).

2.4 Other External Parameters

External parameter choices are summarized in Table 1. The model is period is monthly. I set $I = 4$ so that there are four working stages of life. The working stages of life are from age 23 to 65. So, each stage is $(65 - 23)/4 = 10.5$ years, or 126 months. So, the rate of transitioning from one stage of life to the next is $\zeta = 1/126 = 0.0079$. I set retirement to be 10 years, so $\overline{\zeta} = 1/(10 \times 12) = 0.0093$.

The replacement rate of unemployment benefits is set $b = 0.5$ in accordance with typical UI benefits the US. The expiration rate of unemployment benefits is $\chi = 1/6$ in keeping with the standard rule that unemployment benefits can be collected for a maximum of six months. I normalize $SS = .5$.[13]

I set $\alpha = 0.5$ (Petrongolo and Pissarides, 2001) and $\eta = 0.4$ (Bagger et al., 2014). κ is set such that θ is normalized to one in the benchmark equilibrium. I set $\rho = 0.0033$ in accordance with a 4% risk-free annual real interest rate, and I set $\delta = 0.00285$ to match the decline in wages at the end of life.[14] Finally, I normalize β_0 so that the lowest possible h_0 is

[11] I adopt this strategy because I could not find an estimated tax-and-transfer function which satisfies my conditions in the literature. Maybe tax functions do not include transfers, and those than include transfers may include unemployment benefits or social security payments. I am primarily interest in mean-tested transfers.

[12] I use the power function specification in Table A5 in the appendix.

[13] The choice of SS is immaterial to my results; social security payments are equal across workers, and the level of social security payments will not affect worker decisions.

[14] Assume that workers do not invest in human capital in the last stage of life, $i = I$. Without job switching or unemployment, wage growth is $1 - \delta$ per month. If there are x months in the last stage of life, then $\frac{w_I}{w_{I-1}} = (1 - \delta)^x$. Since each working stage is 126 months on average, given that $w_I/w_{I-1} = 0.864$ in the data, we have $\delta = 0.00116$.

Table 1: Externally Calibrated Parameters

Parameter	Meaning	Value	Explanation/source
Lifecycle			
I	Stages of life	4	By choice
ζ	Transition pobability from one stage to the next	$\left(\frac{42\times 12}{I}\right)^{-1}$	Working for 42 years on average (ages 23-65)
$\overline{\zeta}$	Probability of death for the retired	$(10\times 12)^{-1}$	Retired for 10 years on average
Policy			
τ_0	Tax+transfer progressivity	0.899	Guner et al. (2014, 2020)
τ_1	Tax+transfer level	0.120	Guner et al. (2014, 2020)
T_0	Transfer for worker without UI	0.121	Guner et al. (2020)
τ_b	Business tax rate	0.243	Cooper et al. (2016)
b	Unemployment benefit replacement rate	0.5	Standard in US
χ	Unemployment benefit expiration rate	1/6	Standard US maximum of 6 months
SS	Social security payment	0.5	Normalization
$\overline{g}$	Government spending	0.084	Equalizes government budget constraint in benchmark equilibrium
Search			
α	Meeting function elasticity	0.5	Petrongolo and Pissarides (2001)
η	Worker's bargaining power	0.4	Bagger et al. (2014)
κ	Job posting cost	0.125	Normalizes benchmark equilibrium θ to 1
Other			
ρ	Discount rate	0.00330	4 percent annual interest rate
δ	Human capital depreciation rate	0.00116	Matches decline of wages in last stage of life
β_0	Initial human capital intercept	2.496	Normalized such the lowest h_0 is the bottom point on the h grid

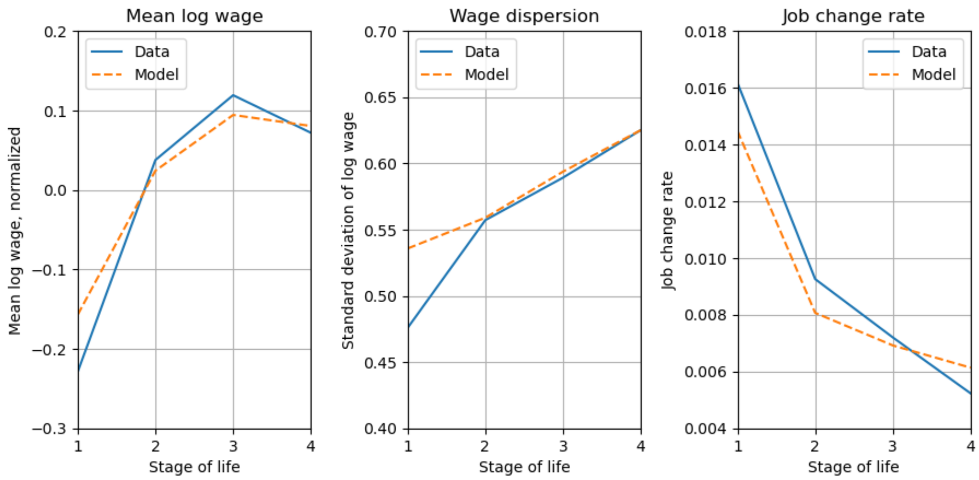

Figure 1: Fit between data and model, lifecycle moments

Table 2: Fit between data and model, nonlifecycle moments

Moment	Target	Model
U to E rate	0.283	0.269
Wage growth, stayers	0.0079	0.0068
Wage growth, switchers	0.106	0.110
Wage growth, switchers with increase	0.350	0.381
Log wage skewness	0.412	0.402

at the bottom of the human capital grid.

Finally, I estimate the job loss function $\Lambda(hpr)$ in the SIPP. The function is presented graphically in Appendix B.1.

2.5 Internal Calibration

I am left with 14 parameters to calibrate internally. The parameters are estimated jointly such that the simulated model hits 16 moments from the SIPP.[15]

I target the profiles of the mean log wage, variance of wage, and job change rate over the lifecycle. For each, I target the starting point, the ending point, and the midpoint. I also target five moments which are not associated with the life cycle: the unemployment to employment rate, average monthly wage growth for those who stay in the same job, average monthly wage growth for those who switch jobs, average monthly wage growth for those who switch jobs and increase their wage, and the cross-sectional log wage skewness. Finally, I target two normalizations: I normalize the mean log wage to one to be consistent with the

[15]I solve the minimization problem using MIDACO, a general-purpose ant colony optimization algorithm (Schlüter et al., 2009).

Table 3: Internally calibrated parameters

Parameter	Meaning	Value
ϕ_E	Disutility level, employed	4.717
γ_E	Disutility curvature, employed	0.799
ϕ_U	Disutility level, unemployed	26.023
γ_U	Disutility curvature, unemployed	11.887
ω	Human capital investment curvature	0.551
ξ	Meeting efficiency	0.756
μ_p	Firm productivity distribution level	0.150
λ_p	Firm productivity distribution tail	0.358
μ_a	Learning ability distribution level	0.0078
σ_a	Learning ability distribution dispersion	0.413
λ_a	Learning ability distribution tail	0.104
β_1	Correlation between learning ability a and initial h	0.350
σ_ε	Initial h dispersion conditional on a	0.227
ψ	Godfather shock rate	0.0007

tax-and-transfer function and the Λ function in the data, and I normalize the mean learning plus searching effort equal to 0.1.

The fit between the model and simulated lifecycle moments is illustrated in Figure 1, and the fit between the model and other moments is presented in Table 2. The resulting parameters are listed in Table 3.

One important fact is that the job switching rate decreases over the lifecycle, and my model is able to replicate that fact. Such a prediction is in line with a on-the-job search model as in Burdett and Mortensen (1998). In a Burdett-Mortensen framework, young workers are matched with firms of low productivity. As they age, workers move to more productive firms. An implication is that young workers are more likely to meet outside firms with higher productivity, and therefore, young workers are more likely to switch jobs.

3 Properties of Benchmark Model

Before proceeding to counterfactual experiments, I investigate some properties of the benchmark model. Recall that the worker earns a wage hpr. Given the model's structure, we can easily decompose earnings between human capital h, firm productivity p, and the piece rate r. Together, p and r comprise the contribution of the search element, and h is plainly the human capital element. Therefore, the log wage is the sum of the log of the three components,

$$\ln(wage) = \ln(h) + \ln(p) + \ln(r) \tag{21}$$

The mean log wage follows:

$$\mathrm{E}[\ln(wage)] = \mathrm{E}[\ln(h)] + \mathrm{E}[\ln(l)] + \mathrm{E}[\ln(r)] \tag{22}$$

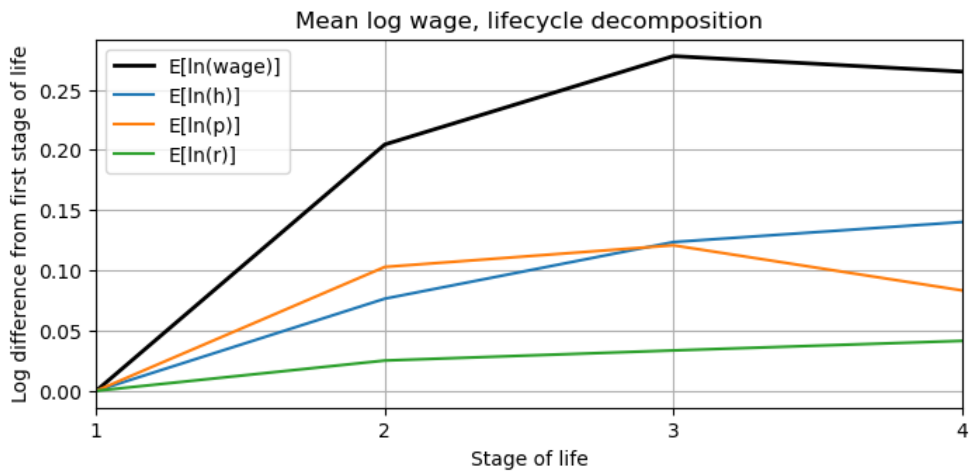

Figure 2: Simple decomposition of mean log wage

We can therefore decompose the mean log wage into three components. In Figure 2, I show that human capital and search play roughly equal roles in generating lifecycle wage growth.

We can also decompose the variance of the log wage as the sum of the variance of each component plus each interaction term as follows:

$$\begin{aligned} \mathrm{Var}[\ln(wage)] = \mathrm{Var}[\ln(h)] + \mathrm{Var}[\ln(p)] + \mathrm{Var}[\ln(r)] + 2\,\mathrm{Cov}[\ln(h), \ln(p)] \\ +2\,\mathrm{Cov}[\ln(h), \ln(r)] + 2\,\mathrm{Cov}[\ln(p), \ln(r)] \end{aligned} \tag{23}$$

In Figure 3, I decompose the increase in wage dispersion over the lifecycle into the three aforementioned categories. Within the benchmark model, both dispersion human capital and firm productivity contribute significantly to the variance of wages. However, over the lifecycle, the increase in wage dispersion is driven entirely by increased dispersion in human capital. This feature is attributable to fixed learning abilities, a, which are positively correlated with initial human capital. Thus, my findings support those of Huggett et al. (2011); even with an endogenous search componenet, wage dispersion increases over the lifecycle because workers have different learning abilities. Workers with higher learning ability will invest more in human capital accumulation and accumulate human capital faster. Over the lifecycle, the gap between workers with high learning abilities and those with low learning abilities increases.

4 Policy Experiments

To demonstrate the usefulness of the model, I perform policy counterfactuals with regards to the progressivity of taxes and transfers.

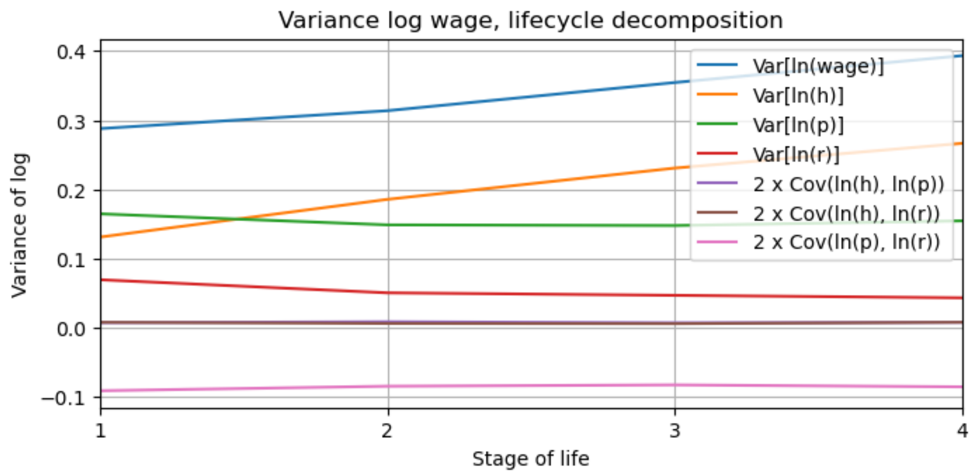

Figure 3: Simple decomposition of the variance of log wages

Table 4: Decomposition of increase in tax progressivity

	Benchmark	Fix search and θ	Fix learning and θ	Fix θ	New equilibrium
τ_1	0.120	0.241	0.241	0.241	0.241
τ_0	0.899	0.911	0.911	0.911	0.911
Tightness ratio, θ	1	1	1	1	0.949
Mean log wage	0.001	-0.021	-0.041	-0.045	-0.054
$E[\ln(h)]$	0.781	0.768	0.777	0.772	0.772
Diff. from benchmark		-0.013	-0.004	-0.009	-0.009
$E[\ln(p)]$	-0.575	-0.585	-0.612	-0.611	-0.621
Diff. from benchmark		-0.010	-0.037	-0.037	-0.046
$E[\ln(r)]$	-0.205	-0.203	-0.206	-0.205	-0.205
Diff. from benchmark		0.002	-0.001	0.000	0.000
Var log wage	0.344	0.349	0.355	0.350	0.350
Diff. from benchmark		0.005	0.010	0.006	0.006
Lifecycle log wage growth	0.265	0.257	0.266	0.260	0.259
Diff. from benchmark		-0.008	0.001	-0.005	-0.006

5 Conclusion

Appendix

A Equations

Aggregate search effort is defined as

$$S = \sum_{a,i} \int_{\Psi_E} s^*_{Eai}(h,p,r) d\Psi_E(h,p,r|a,i) + \sum_{a,i} \int_{\Psi_U} s^*_{Uai}(h,w) d\Psi_U(h,w|a,i). \tag{24}$$

The government budget constraint is

$$\begin{aligned} &\sum_{a,i} \int_{\Psi_E} T(rhp)rhp \, d\Psi_E(h,p,r|a,i) + \sum_{a,i} \int_{\Psi_E} \tau_b(1-r)hp \, d\Psi_E(h,p,r|a,i) \\ &+ \sum_{a,i} \int_{\Psi_U} T(bw)bw \, d\Psi_U(h,w|w>0,a,i) = \sum_{a,i} \int_{\Psi_U} bw \, d\Psi_U(h,w|w>0,a,i) \\ &\quad + \sum_{a,i} \int_{\Psi_U} T_0 \, d\Psi_U(h,w|w=-1,a,i) + \sum_{a} \int_{\Psi_R} SS(w) \, d\Psi_R(w|a) + \bar{g}. \end{aligned} \tag{25}$$

The left hand side of the equation sums government revenue across taxes on the employed, taxes on firms, and taxes on the unemployed, we have total taxes and transfers. The right hand side sums government outflows across unemployment benefits, transfers payments for those with unemployment benefits, social security payments, and government spending $\bar{g}$.

B Calibration Details

B.1 Estimate of Λ function

See Figure 4 for an illustration of the job destruction rate as a function of the wage, $\Lambda(hpr)$. As with the tax-and-transfer function, the mean wage is normalized to one.

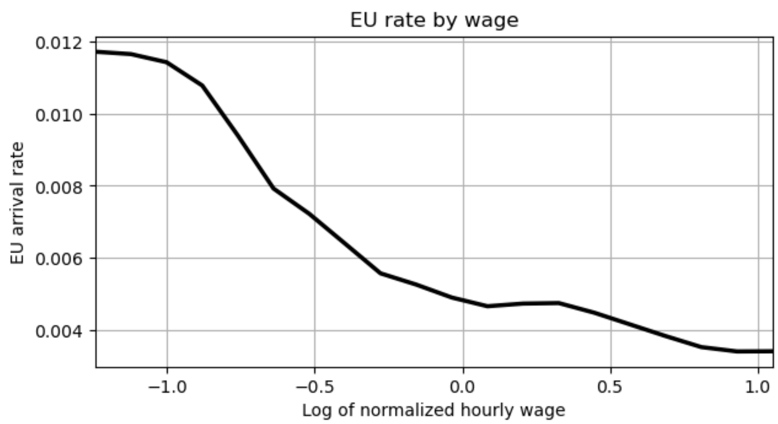

Figure 4: Estimate of exogenous job destruction function, $\Lambda(wage)$

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