Duration Dependence and Unemployment Persistence

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Abstract

The unemployment rate is persistent over the business cycle. Its persistence is particularly apparent after recessions as the unemployment rate decreases slowly to its previous level. However, standard search models struggle to generate realistic persistence quantitatively. I embed duration dependence in a basic search model and show that duration dependence helps to generate accurate unemployment persistence over the business cycle. Intuitively, after recessions, the composition of the unemployment pool shifts to the long-term unemployed, and since they have lower job finding rates, this decreases the aggregate job finding rate and slows recovery. The effect is mitigated if (a) unobserved heterogeneity drives the appearance of duration dependence or (b) a job separation shock characterizes the recession.

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The unemployment rate is persistent over the business cycle. This is particularly true after recessions when unemployment slowly declines back to its previous level (Dupraz et al., 2019). Meanwhile, negative duration dependence in job finding implies that individuals who have been unemployed for a longer period of time are less likely to find a job (Kroft et al., 2013). In this paper, I show that the existence of negative duration dependence can help explain unemployment persistence.

The mechanism behind this result is as follows. During a recession, all job finding rates are low, so the composition of the unemployment pool shifts towards the long-term unemployed. Due to their longer unemployment spell, these individuals' chances of finding a job are lower. Duration dependence thus drags down the job finding probability of the unemployment pool as a whole, and the unemployment rate recovers more slowly.

The finding addresses a well-known the problem that search models such as Mortensen and Pissarides (1994) struggle to generate realistic unemployment persistence. Rather, these models predict that, after shocks, unemployment quickly snaps back to its steady state level. Like others (Gorry et al., 2020; Pries, 2004), I find that a notion of worker heterogeneity can reconcile the facts; in my case, I use heterogeneity in job finding rates arising from the length of time unemployed.

I draw these conclusions using a calibrated search and matching model with heterogeneous unemployment and duration dependence. The model economy features two states of unemployment, high and low, where those in the low state have a lower probability of finding a job. Because of duration dependence, workers who have been unemployed for a longer period of time are more likely to be in the low state. My calibration strategy requires the model to match job finding rates given unemployment duration and the calibration separates the effects of pure duration dependence from unobserved heterogeneity, allowing me to experiment with apportioning different levels of blame to pure duration dependence and unobserved heterogeneity.

I find two important caveats to my main result. First, "pure" (causal) duration dependence has stronger macroeconomic consequences than observed duration dependence which appears as a result of fixed heterogeneity in job finding rates. In other words, I find weaker aggregate effects if the observed decline in job finding rates over unemployment duration is entirely due to inherent characteristics of workers. Thus, this paper points out that distinguishing between "pure" duration dependence and unobserved heterogeneity is of some importance in macroeconomics.

Second, duration dependence slows recovery after productivity shocks but not after job destruction shocks. The reason is that large inflows into unemployment improve the composition of the unemployment pool since, by definition, newly unemployed workers are short-term unemployed and more likely to find a job.

This paper relates closest the two branches of literature. The first branch deals with the fact that the DMP model fails to generate realistic persistence of unemployment, as is pointed out in Pries (2004)[1] and Shimer (2005).[2] My solution is similar to Gorry et al. (2020), which points out that changes in composition in the labor force can amplify shocks and increase persistence. However, instead of heterogeneity in skill, my solution merely requires that workers lose job finding probability over the course of their unemployment spell.

The second branch of literature seeks to connect duration dependence with the aggregate labor market. One general message from this branch is that a notion of heterogeneity and/or duration dependence is crucial for understanding long-term unemployment. For example, Kroft et al. (2016) shows that duration dependence led to decreased job finding rates during the Great Recession.[3] The driving force behind my results, which is that unemployment composition shifts during recessions, is also discussed in Ferraro (2018), Ravenna and Walsh (2012), and Wiczer (2015), though duration dependence is not a key mechanism in those papers.[4] My result is closest to Pissarides (1992), which finds that the loss of skills during

[1]In a way, this paper is the inverse of Pries (2004). Whereas Pries (2004) focuses on the high risk of job loss for new workers, I focus on the low job finding rate for the long-term unemployed. Both generate more persistence in unemployment.

[2]The same is true in Merz (1995), which combines DMP labor market search with a real business cycle model.

[3]Elsby et al. (2010) and Aaronson et al. (2010) also suggested that duration dependence may slow recovery after the Great Recession.

[4]Though I often mention unemployment recovery, my analysis is about unemployment persistence over

unemployment generates greater persistence in unemployment.[5]

Finally, my model speaks to another well-known problem with DMP models, namely that these model struggles to match business cycle variation in matching efficiency unless the matching efficiency multiplier (typically denoted by μ) varies substantially (Barnichon and Figura, 2015; Lubik, 2009). My model incorporates unemployment composition, which provides an endogenous channel through which matching efficiency decreases during recessions.

Despite being difficult to disentangle, the concepts in this paper have already entered policy conversation. A 2014 White House report titled "Addressing the Negative Cycle of Long-Term Unemployment" claims that "the cycle of long-term unemployment hampers the economy at large, depressing aggregate demand and resulting in the underutilization of productive resources" (White House, 2014). After the Great Recession, a Congressional Budget Report stated that duration dependence "currently accounts for about a quarter of a percentage point of the increase in unemployment during and following the recession" (Congressional Budget Office, 2012). This paper takes a step toward understanding the relationship between duration dependence and economic recovery. More generally, it addresses the effects of a large group of long-term unemployed workers on the aggregate labor market.

The remainder of this paper is as follows. Section 1 establishes the empirical facts which motivate my analysis. I describe and calibrate the model in Sections 2 and 3. Section 4 presents my results. I conclude in Section 5.

1 Empirical Facts

My analysis is driven by two empirical facts, one macro and one micro. The macro fact is that unemployment is persistent. The micro fact is that individuals are more likely to find

the entire business cycle, not unemployment asymmetry after recessions. However, my illustrative focus will be with negative shocks. With a model of only negative shocks (Dupraz et al., 2019), an analysis of unemployment persistence is simultaneously an analysis of slow recoveries.

[5]Other related paper include Ahn and Hamilton (2020) and Hornstein (2012), which account for duration dependence in analyzing outflows from unemployment to employment, and Jarosch and Pilossoph (2019), which provides a counter-argument to this paper that duration dependence is not relevant in the aggregate.

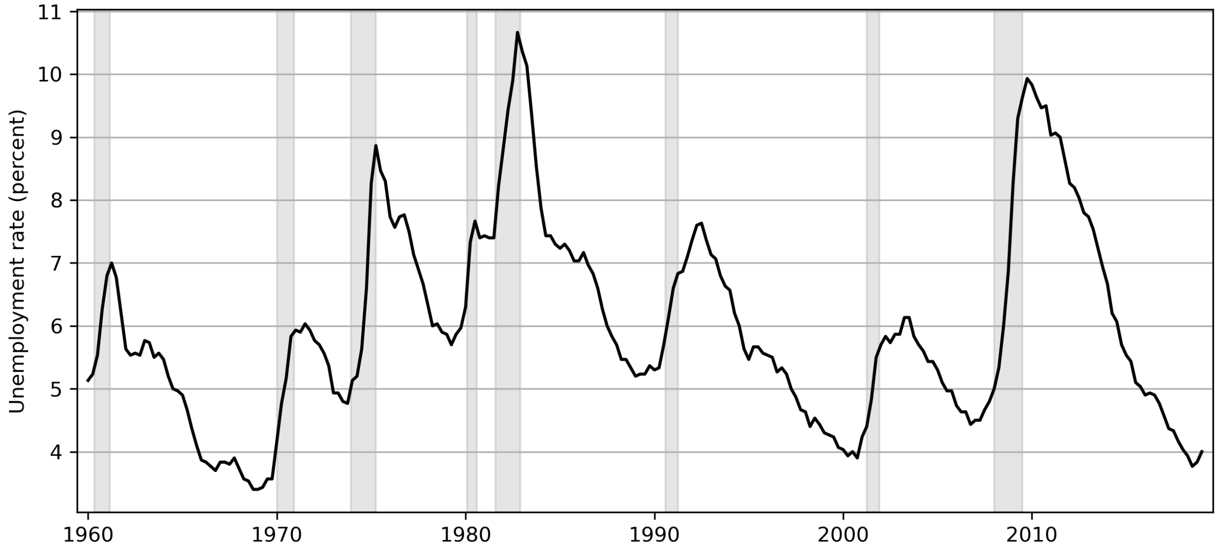

Figure 1: Unemployment rate over time

Source: US Bureau of Labor Statistics (BLS). Unemployment rate is quarterly. Gray bars denote NBER recessions.

jobs earlier in their unemployment spell. In this paper, I show how the micro fact helps explain the macro fact.

First, unemployment is persistent. As illustrated in Figure 1, unemployment appears particularly persistent after recessions, which leads to slow recoveries.[6] The autocorrelation of quarterly unemployment is 0.975.[7]

Second, Figure 2 shows that the probability of finding a job decreases over unemployment duration, especially early in the unemployment spell.[8] On average, an individual who has been unemployed for less than a month has a 50% chance of finding a job, while an individual who has been unemployed for eight months has a 22% chance of finding a job.

Negative duration dependence is partially responsible for this fact. In the unemployment context, negative duration dependence refers to the notion that unemployment duration negatively affects an individual's probability of finding a job.[9] In an experimental study,

[6]During recovery, the unemployment rate decreases at half the speed that it increases during recessions (Dupraz et al., 2019).

[7]See Table 4.

[8]Monthly job finding rates are calculated as described in Appendix A.

[9]As is common, I use the terms "negative duration dependence" and "duration dependence" interchange-

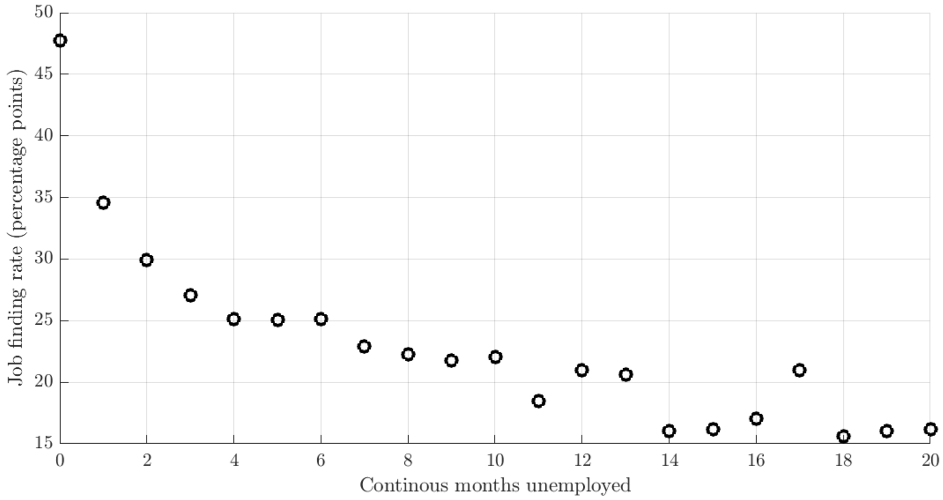

Figure 2: Job finding rate by unemployment duration

Source: Author's calculations using CPS micro data for 1978-2019. The y-intercept is the probability that a worker finds a job before being unemployed for a full month.

Kroft et al. (2013) find that a job applicant with under one month of unemployment is 45% more likely to receive a callback for a job interview than an applicant who has been unemployed for eight months with an otherwise identical resume.[10]

The causes of negative duration dependence are unclear. One hypothesis is that workers lose skills while unemployed, rendering them less productive upon returning to the workforce (Ljungqvist and Sargent, 1998; Edin and Gustavsson, 2008). Another theory is that firms may interpret long duration as a signal of inefficiency and statistically discriminate against such workers (Blanchard and Diamond, 1994; Lockwood, 1991). And long unemployment duration may be associated with worker discouragement and lower search intensity (Krueger and Mueller, 2011).

In my model, I take productivity to be constant across workers, so explanations of duration dependence which rely upon productivity are incompatible with my framework. One possible interpretation of my model relates to recall hiring. Over 40% of unemployed work-

ably, though "negative duration dependence" is more precise.

[10]For more experimental evidence, see Eriksson and Rooth (2014), Farber et al. (2019), and Oberholzer-Gee (2008).

ers who separate into unemployment return to previous employer, and the probability of getting recalled declines sharply over the unemployment spell. In fact, the rate of exit from unemployment to a different employer is only slightly decreasing over unemployment duration (Fujita and Moscarini, 2017). Another interpretation has to do with exhausting job opportunities in one's social network.[11]

However, Figure 2 does not necessarily imply the existence of negative duration dependence. The downward-sloping curve could merely be a result of worker heterogeneity.[12] If job searchers are heterogeneous in job finding rates regardless of unemployment length, workers who are more likely to find work are also more likely to find work earlier in their unemployment spell. Thus, workers with a higher unemployment duration are more likely to be those with a worse job finding probability at the beginning. This explanation does not require the existence of "pure " duration dependence.[13] My model accounts for both pure duration dependence and heterogeneity, and I experiment with assigning different levels of blame to the two factors. In any case, understanding which is more responsible for the downward slope in Figure 2 is crucial for understanding long term unemployment.

The facts above are related in Figure 3 in two respects. First, an increase in mean unemployment duration is associated with a decrease in the aggregate job finding rate. In a way, this is simply the aggregate version of Figure 2. Second, like the unemployment rate as a whole, job finding (outflows from unemployment) are slow to recover after recessions.

Figure 3 also summarizes the mechanism in my model. As more unemployed individuals become long-term unemployed, negative duration dependence puts downward pressure on

[11] In Calvó-Armengol and Jackson (2004) social networks contribute to duration dependence in because those within the same social network are more likely to be unemployed at the same time.

[12] This heterogeneity is referred to as "unobserved heterogeneity" in the duration dependence literature, of which a key result is that even when controlling for observable factors, the curve in Figure 2 still appears. The term is confusing in this paper because differences in job finding are the only source of heterogeneity in my model.

[13] Distinguishing between pure state dependence and unobserved heterogeneity is a well-known puzzle in econometrics (Heckman, 1991). Some studies suggest that the decrease in job finding rates over unemployment duration is mostly explained by unobserved heterogeneity (Alvarez et al., 2019; Abbring et al., 2002; Machin and Manning, 1999). On the other hand, this view is difficult to reconcile with studies which find that observable differences between workers are not important predictors of unemployment duration (Elsby et al., 2010; Krueger et al., 2014).

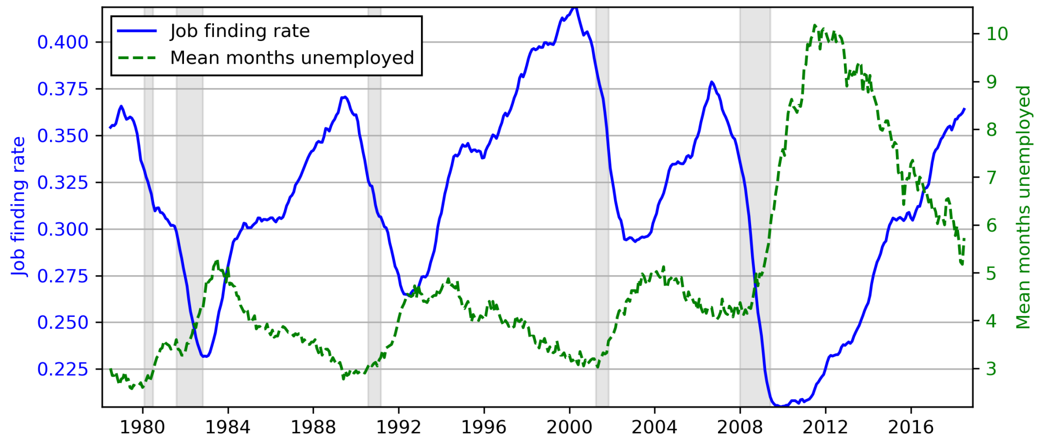

Figure 3: Aggregate job finding rate and mean unemployment duration

Mean unemployment duration source: BLS. Job finding rate source: Author's calculations using CPS data. Both series are quarterly. Gray bars denote NBER recessions. The job finding rate is monthly.

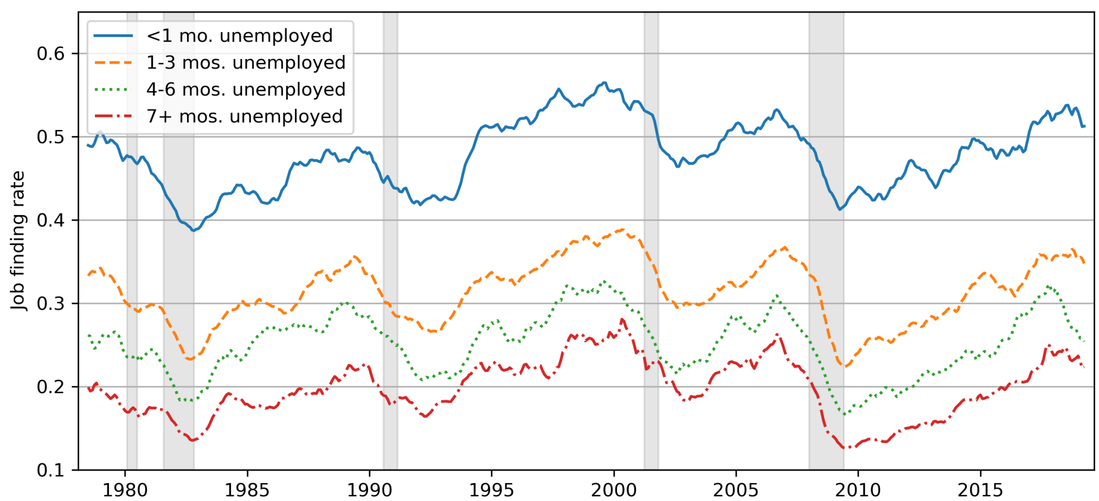

Figure 4: Job finding rate by unemployment duration over time

Source: Author's calculations using CPS data. Gray bars denote NBER recessions. Job finding rates are monthly. All series are smoothed to quarterly.

the aggregate job finding rate, making the job finding rate slower to recover. The slow recovery of the job finding rate then drives the slow recovery of unemployment.[14]

The gap in job finding probability by unemployment duration is relatively constant over time. As Figure 4 shows, over the business cycle, job finding rates by different unemployment durations move roughly in parallel. In addition to motivating a key assumption of the model, it suggests that overall job finding rates can fluctuate as a result changes in the composition of unemployment duration.

2 Model

I analyze duration dependence using a basic DMP search model with unemployment heterogeneity. There are two states of unemployment, the high state and the low state, where the only distinguishing feature is that those in the low state are less likely to find a job than those in the high state. In each period of unemployment, there is a probability that a unemployed worker in the high state transitions to the low state. Thus, duration dependence enters the model because workers who have been unemployed longer are more likely to be in the low state and therefore less likely to find a job. Following negative shocks, the composition of the unemployment pool shifts toward the low state, decreasing the aggregate job finding rate, and slowing recovery.

2.1 Environment

The model environment closely resembles the standard DMP search model in discrete time (Pissarides, 2000). There is a measure one of workers and a continuum of firms, both of which discount the future by discount factor β. Workers and firms are risk neutral and infinitely lived. The sole market is for labor. Firms post v_t vacancies in time t to maximize

[14] The idea that recovery is driven more by job finding (outflow) than job separations (inflow) is empirically supported. For the view that unemployment fluctuations are mostly driven by job finding (outflow), see Shimer (2012), Hall (2005), and Elsby et al. (2009). For the view that separations are important for fluctuations in unemployment but that recovery is still mostly driven by outflows, see Elsby et al. (2013), Barnichon (2012), Fujita (2011), Elsby et al. (2010), and Fujita and Ramey (2009).

expected future profit, are randomly matched to workers, and use hired labor to produce a single output good, the price of which is normalized to one. Each employed worker produces A output in exchange for a wage w_t. Workers are either employed or unemployed. Jobs are destroyed exogenously with probability λ, whereupon workers become unemployed.

My model differs from the standard search model by allowing for heterogeneous unemployment and duration dependence. Unemployed workers are either in the high state or the low state. The total unemployment rate is $u_t = u_t^L + u_t^H$, the sum of unemployed workers in the high state, u_t^H, and the low state, u_t^L. The employment rate is $n_t = 1 - u_t$.

Some workers begin unemployment in the low state while others flow from the high state to the low state due to duration dependence. With probability ζ, newly separated workers begin their unemployment spell in the low state. With probability ϕ, high-state workers who are unable to find a job flow to the low state. ζ represents heterogeneity and ϕ represents duration dependence.

The probability of finding a job in the low state, f_t^L, is a constant fraction $\gamma \in (0, 1)$ of the probability of finding a job in the high state f_t^H, $f_t^L = \gamma f_t^H$. I refer to γ as the low state penalty.

This heterogeneous unemployment framework nests the standard search model when $\zeta = \phi = 0$. I refer to this benchmark case as the DMP model.

I simulate the model response to exogenous shocks to productivity, A, or the separation rate, λ. Both are given by AR(1) processes in deviation from the steady state,

$$s_{t+1} - s = \rho_s (s_t - s) + \varepsilon_s, \quad \varepsilon_s \sim N(0, \sigma_s^2)$$

for $s \in \{A, \lambda\}$ where ρ_s is persistence and σ_s is volatility.

2.2 Laws of Motion

The inflows and outflows of unemployment are summarized by two laws of motion. Individuals flow out of the high state of unemployment either by finding a job with probability

f_t^H or, if they do not find a job, by moving to the low state with probability ϕ. The total number of workers flowing into unemployment is λn_t, and a fraction $1-\zeta$ of workers who separate from jobs begin unemployment in the high state. Combining inflows and outflows, the law of motion for high-state unemployment is

$$u_{t+1}^H = \left(1-f_t^H\right)u_t^H - \phi\left(1-f_t^H\right)u_t^H + (1-\zeta)\lambda n_t. \tag{1}$$

Workers in the low state of unemployment cannot move to the high state, so workers in the low state can only flow out by finding a job with probability f_t^L. The low state receives inflows of workers from a fraction ζ of workers who lose their job as well as those who flow from the high state to the low state, $\phi\left(1-f_t^H\right)u_t^H$. The law of motion for low-state unemployment is

$$u_{t+1}^L = u_t^L\left(1-f_t^L\right) + \phi\left(1-f_t^H\right)u_t^H + \zeta\lambda n_t. \tag{2}$$

2.3 Matching Functions

I use a matching technology which generates a constant gap in job finding probabilities between the high state and low state, $f_t^L = \gamma f_t^H$,[15] and nests the standard Cobb-Douglas matching function. The following scheme with two Cobb-Douglas-esque matching functions, one for each unemployment state, is intuitive and satisfies both requirements.

In time t, the number of matches formed between open job vacancies and high-state unemployed workers is

$$m^H\left(u_t^H, u_t^L, v_t\right) = \mu\frac{u_t^H}{u_t}\left(u_t^H + \gamma u_t^L\right)^\alpha v_t^{1-\alpha} \tag{3}$$

where μ is matching efficiency and α is an elasticity parameter. For intuition, contrast this matching function with the standard Cobb-Douglas matching function, $\tilde{m}\left(u_t, v_t\right) = \mu u_t^\alpha v_t^{1-\alpha}$. My function includes two new terms. The first, u_t^H/u_t, represents the portion of the unemployment pool to which this matching function applies (a necessary element since

[15]The assumption of a constant wedge between the finding rates is motivated by the data in Figure 4.

v_t is common to both worker types). The second, $u_t^H + \gamma u_t^L$, replaces u_t and represents the weighted "matchability" of the unemployment pool.

The number of matches between open job vacancies and unemployed workers in the low state is

$$m^L\left(u_t^H, u_t^L, v_t\right) = \gamma\mu\frac{u_t^L}{u_t}\left(u_t^H + \gamma u_t^L\right)^{\alpha} v_t^{1-\alpha}, \tag{4}$$

the same expression as (3) but scaled down by γ and applied to u_t^L/u_t.

Let $\theta_t \equiv v_t/u_t$ denote labor market tightness and $x_t \equiv u_t^H/u_t$ denote the fraction of the unemployment pool in the high state. The job finding rate for unemployed workers in the high state is

$$f_t^H \equiv \frac{m^H\left(u_t^H, u_t^L, v_t\right)}{u_t^H} = \mu\left(x_t\left(1-\gamma\right)+\gamma\right)^{\alpha}\theta_t^{1-\alpha}. \tag{5}$$

Similarly, the job finding rate for unemployed workers in the low state is

$$f_t^L \equiv \frac{m^L\left(u_t^H, u_t^L, v_t\right)}{u_t^L} = \gamma\mu\left(x_t\left(1-\gamma\right)+\gamma\right)^{\alpha}\theta_t^{1-\alpha}. \tag{6}$$

These matching functions successfully generate $f_t^L = \gamma f_t^H$.

Unemployment composition x_t is they key element will generates persistence beyond the DMP model. Define the aggregate, or average, job finding rate as

$$f_t \equiv x_t f_t^H + \left(1-x_t\right) f_t^L = \mu\left(x_t\left(1-\gamma\right)+\gamma\right)^{\alpha+1}\theta_t^{1-\alpha}. \tag{7}$$

Note that f_t is increasing in x_t as well as θ_t.[16] If x_t varies over the business cycle and is persistent, then f_t will be more persistent. x_t decreases during recessions because unemployment spells get longer, which puts negative pressure on the aggregate job finding rate and slows unemployment recovery.

From the firm's perspective, the probability that an open position is filled by an unemployed worker from the high state is $h_t^H \equiv m^H\left(u_t^H, u_t^L, v_t\right)/v_t$, and the probability that an open position is filled by a worker from the low state is $h_t^L \equiv m^L\left(u_t^H, u_t^L, v_t\right)/v_t$. The

[16] In the DMP model, $\tilde{f}_t = \mu\theta_t^{1-\alpha}$.

aggregate hiring rate is

$$h_t \equiv h_t^L + h_t^H = \mu \left(x_t(1-\gamma) + \gamma \right)^{\alpha+1} \theta_t^{-\alpha}. \tag{8}$$

Like the aggregate finding rate (7), the aggregate hiring rate is increasing in composition x_t.[17]

2.4 Wages

I first present Bellman equations. For a worker, the value of unemployment in the high state is

$$U_t^H = z + \beta \left[f_t^H E_{t+1} + \left(1 - f_t^H\right) \left(\phi U_{t+1}^L + (1-\phi) U_{t+1}^H \right) \right] \tag{9}$$

where E_t is the worker's value of employment and z is the flow utility of unemployment.[18] With probability f_t^H, the worker finds a job and begins work in the next period; otherwise, the worker remains unemployed. Conditional on not finding a job, the worker begins the next period in the low state of unemployment with probability ϕ; otherwise, the worker remains in the high state. The value of unemployment in the low state is

$$U_t^L = z + \beta \left[f_t^L E_{t+1} + \left(1 - f_t^L\right) U_{t+1}^L \right]. \tag{10}$$

An unemployed worker in the low state cannot move to the high state and has a lower probability of finding a job because $f_t^L < f_t^H$.

Next, the value of working is

$$E_t = w_t + \beta \left[(1-\lambda) E_{t+1} + \lambda \left(\zeta U_{t+1}^L + (1-\zeta) U_{t+1}^H \right) \right]. \tag{11}$$

With probability λ, the worker is separated from their job; otherwise, the worker remains employed. Upon separation, the worker begins the unemployment spell in the low state with

[17] In the DMP model, $\tilde{h}_t = \mu \theta_t^{-\alpha}$.

[18] z includes both unemployment benefits and the relative value of nonwork, so z is the opportunity cost of employment.

probability ζ; otherwise, the worker begins unemployment in the high state.

From the firm's perspective, the value of a filled job is

$$J_t = A - w_t + \beta \left[(1-\lambda) J_{t+1} + \lambda V_{t+1} \right].$$

where V_t is the value of an open job vacancy. In the current period, the firm earns the value of the worker's production minus the wage. With probability $1-\lambda$, the job stays intact for another period; with probability λ, the match is destroyed, and the job becomes an open vacancy. The value of an open job vacancy is

$$V_t = -\kappa + \beta \left[h_t J_{t+1} + (1-h_t) V_{t+1} \right]$$

where κ is the cost of posting a vacancy.

I assume free entry of firms in the labor market. In equilibrium, profit maximization requires that the total discounted value of another vacancy equals zero, or $V_t = 0$ for all t. So, in equilibrium, the previous two equations become

$$J_t = A - w_t + \beta (1-\lambda) J_{t+1} \tag{12}$$

and

$$\kappa = \beta h_t J_{t+1}. \tag{13}$$

Equation (13) (the free entry condition) determines the number of vacancies posted in equilibrium. Combining (13) and (8), one can see how unemployment composition will affect job posting; all else equal, if the unemployment pool includes many workers in the low state (x_t is low), fewer jobs will be posted (v_t and θ_t will be low).

Wages are determined by Nash bargaining where the worker has weight ψ. Therefore, the worker earns ψ of the total surplus - worker surplus plus firm surplus - generated from the match. Firm surplus is the value of a filled job minus the value of a vacancy, $J_t - V_t = J_t$; worker surplus is the value of working, E_t, minus the value of unemployment for that worker.

However, a worker's value of unemployment (i.e., their threat level) depends upon their status in the current period. A worker exists in one of three categories: currently employed, high-state unemployed, or low-state unemployed. These categories determine a worker's value of unemployment and thus their threat level. For example, since low-state unemployed workers have a lower value of unemployment than high-state unemployed workers, low-state workers are more desperate and willing to work for a lower wage.[19]

Using the Bellman equations above, if a worker is currently employed, their value of unemployment is the value of losing their job, $\zeta U_t^L + (1-\zeta) U_t^H$. If a worker is high-state unemployed, their value of remaining unemployed is $\phi U_t^L + (1-\phi) U_t^H$. And if a worker is low-state unemployed, their value of remaining unemployed is U_t^L.

I assume that all workers are paid the same wage.[20] For all workers to earn the same wage, I must assume that firms cannot discern between workers. (Either worker status is unobservable for firms or firms are constrained to pay the same wage to all workers.) So, job vacancies are posted for the unemployment pool as a whole and firms are randomly matched with workers.

I use the Nash bargaining with asymmetric information solution from Harsanyi and Selten (1972). The firm knows the composition of the workforce but does not know the type of worker with which it is negotiating. So, the firm knows that the probability that a worker is currently working, unemployed in the high state, or unemployed in the low state is n_t, u_t^H, and u_t^L, respectively. Writing E_t and J_t as functions of the wage, the wage solves

$$\begin{aligned} w_t = &\arg\max_{\hat{w}_t} \Bigg\{ \Bigg(\left[E_t(\hat{w}_t) - \left(\zeta U_t^L + (1-\zeta) U_t^H\right)\right]^{n_t} \\ &\times \left[E_t(\hat{w}_t) - \left(\phi U_t^L + (1-\phi) U_t^H\right)\right]^{u_t^H} \left[E_t(\hat{w}_t) - U_t^L\right]^{u_t^L} \Bigg)^{\psi} J_t(\hat{w}_t)^{1-\psi} \Bigg\}. \end{aligned}$$

[19]There is empirical evidence which suggests that workers with longer unemployment spells have lower reservation wages (Krueger and Mueller, 2016) and earn lower wages once they find a job (Schmieder et al., 2016).

[20]My results are robust to this assumption. Other wage bargaining schemes where firms can pay different types of workers different wages generate very similar predictions. The most extreme example is a model with separate labor markets for high- and low-state workers, and this type of model has similar results as well.

Table 1: External parameters

Parameter	Meaning	Value
β	Discount factor	0.9967
z	Flow utility of unemployment	0.73
κ	Vacancy creation cost	0.3
α	Matching function elasticity parameter	0.6
ψ	Worker bargaining weight	0.6
ρ_A	Productivity shock persistence	0.965
σ_A	Productivity shock standard deviation	0.007
ρ_λ	Separation shock persistence	0.875
σ_λ	Separation shock standard deviation	0.042

External parameter choices.

Taking logs and maximizing, the equilibrium wage satisfies

$$(1-\psi)\frac{1}{J_t(w_t)}$$
$$= \psi\left(\frac{n_t}{E_t(w_t) - \left[\zeta U_t^L + (1-\zeta)\, U_t^H\right]} + \frac{u_t^H}{E_t(w_t) - \left[\phi U_t^L + (1-\phi)\, U_t^H\right]} + \frac{u_t^L}{E_t(w_t) - U_t^L}\right). \tag{14}$$

Again, the model collapses to DMP with standard Nash bargaining if $\zeta = \phi = 0$.

3 Calibration

3.1 External Calibration

External parameters choices are listed in in Table 1. The model period is one month. I set β to 0.9967 in accordance with a risk-free real interest rate of 4%. The flow utility of unemployment z is set to 0.73 according to the calculations in Mortensen and Nagypal (2007) and the vacancy creation cost κ is set to 0.3 according to Michaillat (2012).[21] I set α and ψ equal to 0.6.[22] I follow Coles and Moghaddasi Kelishomi (2018) for shock process

[21]Michaillat (2012) uses micro literature to suggest that κ is around 0.32 times the steady state wage.

[22]With $\alpha = 0.6$, I seek to anchor to the macro standard of 0.5 and acknowledge the larger estimates in micro literature such as Petrongolo and Pissarides (2001) and Lange and Papageorgiou (2020). Similarly, ψ is set to 0.6 by combining the macro standard of 0.5 with the larger results in Jäger et al. (2020).

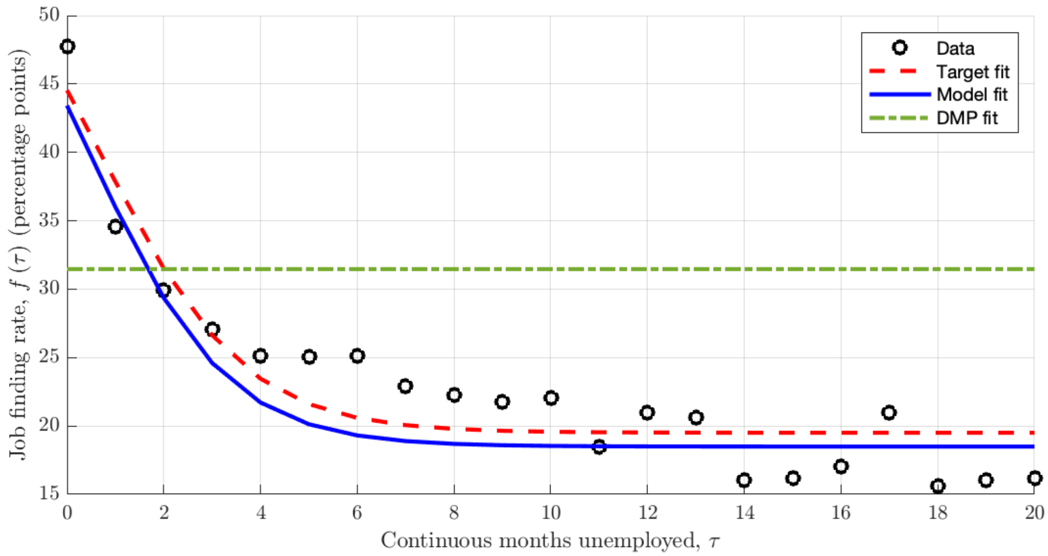

Figure 5: Simulated fit of job finding rate by unemployment duration

Data is identical to Figure 2. Target fit is the curve which fits the data under the functional form of $f(\tau)$ implied by the model of this paper. Model fit is the resulting $f(\tau)$ curve after calibration. The fit looks identical for model variant. DMP fit refers to the fit of $f(\tau)$ in the standard DMP model where $\phi = \zeta = 0$.

parameters $\{\rho_A, \sigma_A, \rho_\lambda, \sigma_\lambda\}$. A is normalized to 1.

3.2 Internal Calibration

I internally calibrate λ, γ, ϕ, ζ, and μ by targeting the unemployment rate u, job finding rate f, and the job finding rate as a function unemployment duration $f(\tau)$ in the data. First, I first calibrate λ according to the steady state relationship

$$u = u(1-f) + \lambda(1-u), \tag{15}$$

which results in $\lambda = 0.02$. The four remaining parameters are chosen to minimize the distance between moments in the data and the model in the steady state.[23]

In addition to u and f, I calibrate my parameters to fit the job finding rate as a function of unemployment duration, $f(\tau)$, as illustrated in Figure 2. My strategy consists of fitting

[23]For the DMP model, I set μ so that $f = \mu\theta^{1-\alpha}$ where θ is the same as the heterogeneous model. In that case, $\phi = \zeta = 0$ and γ is redundant.

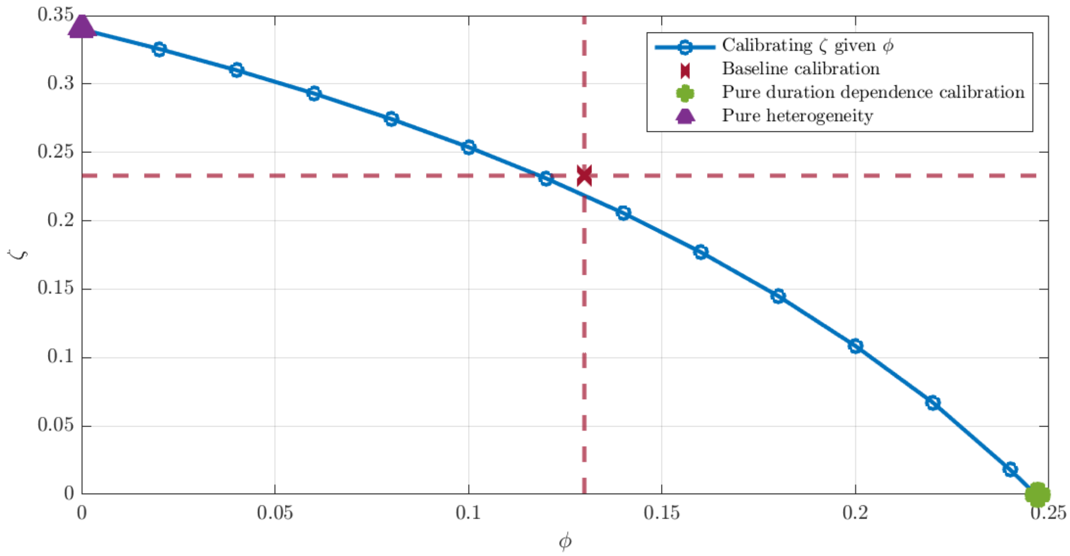

Figure 6: Joint identification of duration dependence and unobserved heterogeneity

Each point on the curve is a different calibration of the no information model. The line plots calibrated values of ζ when restricting ϕ. ζ represents unobserved heterogeneity and ϕ represents pure duration dependence. The baseline point is the calibrated (ϕ, ζ) pair when additional moments from van den Berg and van Ours (1996) are targeted. This calibration represents a realistic relationship between ϕ and ζ. The other two points are extreme counterfactual calibrations. If $\zeta = 0$, only pure duration dependence generates observed duration dependence; if $\phi = 0$, only heterogeneity generates observed duration dependence.

a curve to the data for $f(\tau)$ and targeting the coefficients of that curve. For details, see Appendix B.1. Figure 5 illustrates the $f(\tau)$ curve I target as well as the fit of the model. The figure also highlights a strength of my model. The standard DMP model assumes that the job finding rate is constant across all unemployment durations, relegating $f(\tau)$ to a horizontal line, whereas my model matches captures the downward sloping job finding rate.

However, with these targets, ϕ and ζ are jointly identified, but not separately identified. Recall that it is generally unclear whether the downward-sloping $f(\tau)$ curve observed in the data is caused by pure duration dependence (ϕ) or heterogeneity (ζ). The same is true in my model. In fact, the $f(\tau)$ curve can be generated entirely by negative duration dependence, entirely by unobserved heterogeneity, or by infinite combinations of the two.

The lack of separate identification between pure duration dependence (ϕ) and unobserved heterogeneity (ζ) is illustrated in Figure 6. The downward-sloping line is the locus of (ϕ, ζ)

pairs which fit the data, and each point on the curve fits the data equally well.

I exploit this ambiguity to separate the aggregate effects of duration dependence from heterogeneity. I consider three calibrations. For the first, I assume that pure duration dependence is the only cause of the downward-sloping job finding rate and call it the "pure duration dependence calibration." To do so, I set ζ equal to zero, cutting off heterogeneity as a channel of duration dependence, and calibrate the remaining parameters. This calibration corresponds to the green circle on the horizontal axis in Figure 6. I then assume the opposite, that pure duration dependence is nonexistent and the downward-sloping $f(\tau)$ curve is due entirely to heterogeneity. In this case, I set ϕ equal to zero. This calibration, called the "pure heterogeneity calibration," corresponds to the purple triangle in Figure 6.

The third calibration, which I call the "baseline" calibration, approximates a realistic mix of pure duration dependence and heterogeneity. To do so, I target additional moments estimated in van den Berg and van Ours (1996), a paper which quantifies the separate effects of pure duration dependence and heterogeneity. I explain how I incorporate these targets in Appendix B.2. This calibration corresponds to the red X in Figure 6.[24] Both pure duration dependence and heterogeneity are significant in this case.

In the Appendix, I consider one more calibration which I call the "Hagedorn-Manovkii" calibration. It is identical to the baseline calibration but uses the parameter values for the flow utility of unemployment, z, and the worker bargaining weight, ψ, from Hagedorn and Manovskii (2008). These parameter values do not change my persistence results but, as explained in Hagedorn and Manovskii (2008), they significantly increase unemployment volatility.

3.3 Calibration Results

Table 2 lists the resulting parameters for all calibrations. Each calibration fits the data well. See Appendix B.3 for model fit and steady state endogenous variables across calibrations.

First note that the job finding penalty in the low state is large. In the baseline model,

[24]The point is not perfectly in line with the curve, which is expected given that I target additional moments.

Table 2: Calibrated parameters

		Calibration			
Parameter	Meaning	DMP	Baseline	Pure duration dependence	Pure heterogeneity
γ	Low state penalty		0.34	0.43	0.34
ϕ	Transition rate	0.00	0.13	0.29	0.00
ζ	Initial low state	0.00	0.30	0.00	0.38
μ	Match efficiency	0.40	0.97	0.67	1.05

The baseline calibration includes moments from van den Berg and van Ours (1996). These moments are not targeted in the other three calibrations; in these calibrations, either ζ, ϕ, or both are restricted to zero.

the estimate for γ reveals that those in low state are 35% as likely to find a job as those in the high state.

Second, many unemployed workers are in the low state. Given ζ in the baseline model, workers have a 23% chance of beginning their unemployment spell in the low state, and given ϕ, those that begin in the high state have a 13% change of moving to the low state each period. Compare this result with the estimated steady state fraction of unemployed workers in the high state, $x = 0.43$ (see Table 5 in Appendix B.3). Less than a quarter of workers begin unemployment in the low state, but at a typical point in time, over half of unemployed workers are in the low state. In words, most workers find jobs quickly, but most of the unemployment pool consists of workers who will probably not find jobs quickly.[25] This paper suggests that the relative size of the latter group has significant business cycle implications.

[25]This notion is empirically compatible with Morchio (2020) which finds that 2/3 of prime-age unemployment is accounted for by 10% of workers.

Table 3: Unemployment impulse response function statistics

Statistic	DMP	Baseline	Pure duration dependence	Pure heterogeneity
Peak t	8	13	15	10
Half peak t	31	40	44	37
Half t - peak t	23	27	29	27
Peak	0.00046	0.00046	0.00058	0.00037

Statistics corresponding to unemployment IRFs in Figure 7 Panel A. The first two rows show how many time periods transpire after the shock until the unemployment rate reaches its peak or half-peak. IRFs are measured in deviations from the steady state.

4 Results

4.1 Main Results: Duration Dependence and Unemployment Persistence

In this section, I show that duration dependence increases unemployment persistence over the business cycle. To do so, I compare the persistence of unemployment under different calibrations after a productivity shock.

I begin by showing impulse response functions (IRFs) because they are intuitive. Figure 7 plots impulse response functions under different calibrations from a negative productivity shock, and Table 3 reports statistics associated with the unemployment IRF in Panel A. The four lines correspond to the four calibration assumptions: the baseline calibration with a mix of pure duration dependence and heterogeneity, the standard DMP model ($\phi = \zeta = 0$), the pure duration dependence calibration ($\zeta = 0$), and the pure unobserved heterogeneity calibration ($\phi = 0$).

Panel A shows that unemployment recovers more slowly duration dependence. First compare the baseline calibration with DMP. Whereas the DMP calibration peaks 8 months after the shock, the baseline calibration does not peak until 15 months after the shock. Furthermore, the number of months from peak until the half-peak level is 36 in the baseline calibration and 30 in the homogeneous model, implying that unemployment also declines at

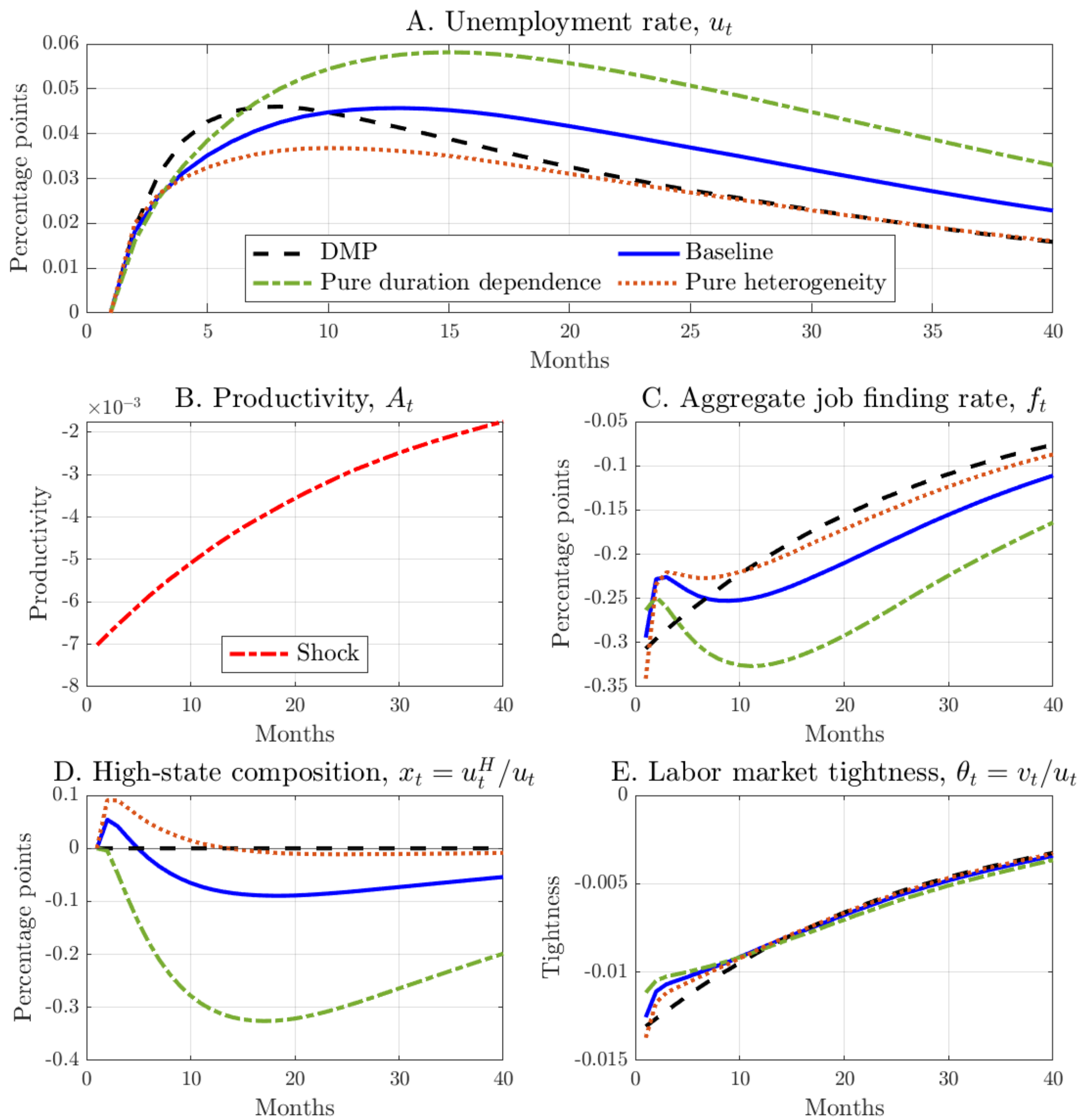

Figure 7: Impulse response functions by calibration after a negative productivity shock

All impulse responses are measured in deviations from the steady state. DMP refers to the the model without unemployment heterogeneity. The baseline calibration is a realistic combination of pure duration dependence and heterogeneity; the pure duration dependence calibration attributes all differences in job finding to duration dependence; and the pure heterogeneity calibration attributes all differences in job finding to heterogeneity. Panel B plots the exogenous shock; all other panels plot endogenous responses.

a slower rate. Both the later peak and the slower recovery rate imply more persistence.

The effect is significantly stronger if pure duration dependence, rather than heterogeneity, is the source of differences in job finding rates. Under the pure duration dependence calibration, the unemployment rate peaks 17 months after the shock compared 12 months after the shock with pure heterogeneity. The rate of recovery is also slower under pure duration dependence, and its peak is much higher.

The intuition for why duration dependence increases persistence is apparent from the other panels in Figure 7. The negative productivity shock is in Panel B. λ is fixed and thus the inflow rate into unemployment is fixed, so unemployment fluctuations are driven entirely by the job finding rate, or the outflows of unemployment. The size and shape of the aggregate finding rate in Panel C is therefore responsible for the size and shape of the unemployment rate in Panel A. Aggregate job finding, in turn, is a function of (and increasing in) unemployment composition x_t (Panel D) and labor market tightness θ_t (Panel E). There is little difference in tightness, so the differences between IRFs are driven by unemployment composition.

After a negative productivity shock, firms post fewer jobs, which decreases tightness and thus decreases the aggregate job finding rate. Due to duration dependence, those who do not find work may flow to the low state. Since those in the low state find jobs at a lower rate, flows to the low state build over the course of the recession, resulting in the negative hump shape of x_t. Even though θ_t is recovering, this composition effect drags down the recovery of the aggregate finding rate, and unemployment recovers more slowly. The effect is much stronger in the pure duration dependence calibration. So, to the extent that duration dependence is pure, the unemployment rate is more persistent.[26]

Note that f_t mirrors the path of A_t in the DMP model. Herein lies the reason why the standard DMP model fails to generate persistence in unemployment. Since vacancies v_t is a jump variable, θ_t adjusts to A_t each period according to the free entry condition. So, θ_t follows the same path as A_t, and since the aggregate finding rate is a function only of θ_t,

[26]For every line except for the case of pure duration dependence, x_t increases in the period after the shock. This has to do with the different levels values of γ, ϕ, and the steady state x across different calibrations.

the aggregate finding rate follows the same path as A_t as well. The result is that the finding rate recovers as quickly as the shock.[27] The model of this paper avoids this result because f_t also a function of x_t. One can see how f_t initially recovers quickly before getting "caught" by the slow-developing drop in x_t

Even without any duration dependence, unemployment still recovers slower with heterogeneity. In the case of only heterogeneity, after an initially positive effect on composition, the high-state workers find jobs at a faster rate than low-state workers, worsening the pool and slightly slowing recovery.

Table 4 shows autocorrelations from simulating the model using a stochastic path for productivity A_t. The reported statistics are quarterly and are in log deviations from an HP-filtered trend with smoothing parameter 10^5. More simulation results like standard deviations and correlations between variables are in Appendix D.

Unemployment is significantly more persistent in the baseline model, rising from an autocorrelation of 0.937 in the homogeneous case to 0.964 in the baseline case. Since the autocorrelation of unemployment is about 0.97 in the data, this is a marked improvement. Though seemingly small, these autocorrelations represent a large different in persistence. In terms of half-lives from an AR(1) process,[28] the data implies an unemployment half-life of 27 compared to a half-life of 10 in the DMP model. As is illustrated above, unemployment is more persistent with more pure duration dependence because the aggregate job finding rate is more persistent.

In Appendix D, I also include simulation results under an alternative external calibration from Hagedorn and Manovskii (2008) which addresses the lack of volatility in unemployment, and the persistence result holds. Also see Appendix E Figure 9 for a more detailed rendering of the IRFs in this section.

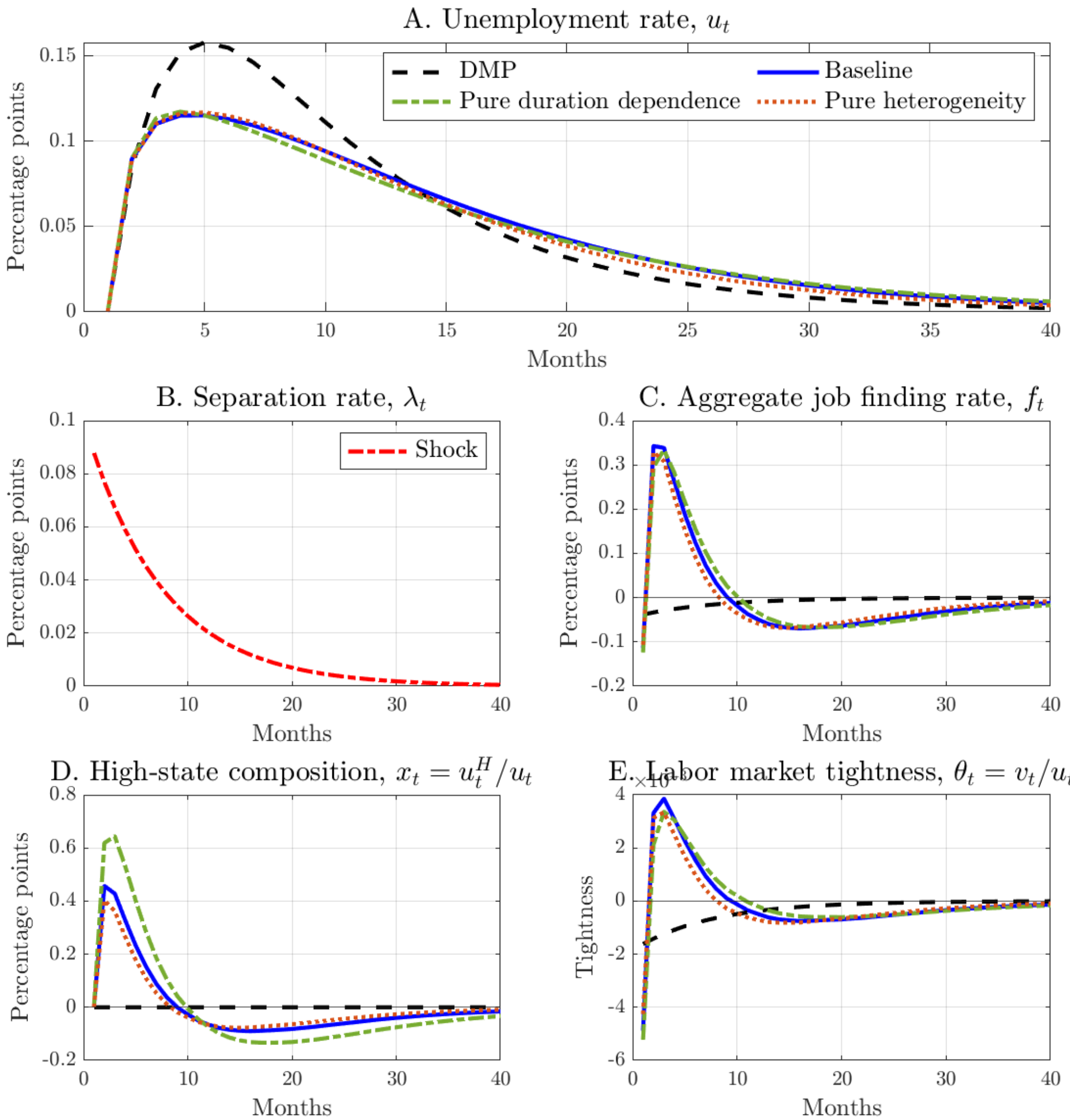

Figure 8: Impulse response functions after separation rate shock

Impulse response functions following a positive separation rate shock. All impulse responses are measured in deviations from the steady state. DMP refers to the the model without unemployment heterogeneity. The baseline calibration is a realistic combination of pure duration dependence and heterogeneity; the pure duration dependence calibration attributes all differences in job finding to duration dependence; and the pure heterogeneity calibration attributes all differences in job finding to heterogeneity. Panel B plots the exogenous shock; all other panels plot endogenous responses.

Table 4: Simulation autocorrelations

		Model			
	Data	DMP	Baseline	Pure duration dependence	Pure heterogeneity
$AC(u_t)$	0.975	0.930	0.957	0.965	0.944
$AC(\theta_t)$	0.969	0.878	0.892	0.904	0.883
$AC(f_t)$	0.980	0.878	0.929	0.948	0.901

First column is data, rest of columns are simulations of the model under different calibrations. Vacancy data source: Barnichon (2010). All other data: CPS. All series are quarterly and HP filtered with smoothing parameter 10^5.

4.2 The Case of Separation Shocks

Duration dependence does not slow unemployment recovery after a recession if the recession is driven by a separation rate shock. Figure 8 plots impulse response function from a positive separation rate shock.

Unlike a productivity shock, a positive separation shock induces a *positive* composition effect which offsets the negative effects of the shock. Since $\zeta < 0.5$, most workers enter the high state of unemployment upon separation from their previous job.[29] So, as Panel C of Figure 8 illustrates, once the separation rate shock causes a wave of workers to enter unemployment, the composition of the unemployment pool quickly shifts toward the high state. The positive shift in composition spurs a job posting spree which mitigates the rise in unemployment. By (8), the improved composition of the unemployment pool increases the probability that firms successfully find new employees. So, by (13), the large increase in x_t incentivizes firms to post more job vacancies, and θ_t spikes upward despite initially dropping. Since both x_t and θ_t increase, aggregate job finding increases in Panel B, generating an increase in unemployment outflows which offsets the increase in inflows.

In the long run, unemployment composition eventually decreases as those who lost their jobs shift to the low state. But the effect at that point is small since the shock has largely

[27] In Table 7 in the Appendix, the correlation between A_t and f_t is 0.999 in the DMP model.

[28] Calculated using $\log(0.5)/\log(\rho)$ where ρ is the autocorrelation coefficient (Gorry et al., 2020).

[29] $\zeta < 0.5$ in all calibrations.

subsided at that point.

5 Conclusion

In this paper, I use a search and matching model to show that duration dependence contributes to the persistence of unemployment. Thus, duration dependence helps to explain the slow recovery of unemployment after recessions.

I embed duration dependence into an otherwise standard search and matching model. My calibration scheme allows me to experiment with different mixes of pure duration dependence and fixed heterogeneity. Pure duration dependence generates significantly more persistence than unobserved heterogeneity. Thus, an additional implication of this paper is that distinguishing between pure duration dependence and unobserved heterogeneity is of some importance in macroeconomics.

Appendix

A Data

I use CPS data for January 1978 - June 2020 to construct the moments I target in calibration. Job vacancies are calculated using JOLTS and, before 2000, using the composite help-wanted index from Barnichon (2010).

Job finding rates are calculated using CPS microdata. They are derived by dividing unemployment to employment transitions over transitions from unemployment back to unemployment or to employment.

B Calibration Details

B.1 Targeting $f(\tau)$

I calibrate parameters to match $f(\tau)$ in the data. My strategy consists of fitting a curve to the data for $f(\tau)$ and targeting the resulting coefficients for that curve. I take this approach because the data on $f(\tau)$ in Figure 2 is noisy and targeting each point would be internally inconsistent.

As is described in Appendix C, the probability of finding a job given unemployment duration τ in the steady state is

$$f(\tau) = \begin{cases} (1-\zeta) f^H + \zeta f^L, & \tau = 0 \\ \left(f^H - f^L\right)(1-\zeta)(1-\phi)^\tau \dfrac{\left(1-f^H\right)^\tau}{\prod_{i=0}^{\tau-1}\left(1-f(i)\right)} + f^L, & \tau \geq 1. \end{cases} \tag{16}$$

By (16), the functional form of $f(\tau)$ implied by the model is

$$f(\tau) = \begin{cases} a + c & \tau = 0 \\ ab^\tau \left(\dfrac{1}{1-f(0)}\right)\left(\dfrac{1}{1-f(1)}\right)\cdots\left(\dfrac{1}{1-f(\tau-1)}\right) + c, & \tau \geq 1. \end{cases} \tag{17}$$

I first estimate the coefficients a, b, and c in the data. The result is the target curve in

Figure 5. As the figure shows, this functional form of $f(\tau)$ appears to fit the data well. I then target a, b, and c in calibration.[30]

Relating the coefficients (a, b, c) to the job finding function results in

$$a = \left(f^H - f^L\right)(1 - \zeta), \tag{18}$$

$$b = (1 - \phi)\left(1 - f^H\right), \tag{19}$$

and

$$c = f^L. \tag{20}$$

These equations are intuitive. c is the horizontal asymptote of $f(\tau)$; as an individual's unemployment duration increases, their job finding probability converges downward to f^L. $a + c$ is the vertical intercept or $f(\tau)$, or the job finding probability of an individual within the month that they were separated from their previous job. So, $a + c = f(0) = (1 - \zeta) f^H + \zeta f^L$. Lastly, b is the speed of convergence. Finally, ζ and ϕ are essentially substitutable in generating the downward slope of $f(\tau)$, which is an important point for Section 3.2.

B.2 Targeting Estimates from van den Berg and van Ours (1996)

van den Berg and van Ours (1996) nonparametrically estimate η_τ, the relative decrease in job finding probability from one period of unemployment to the next which is only due to pure duration dependence. In other words, η_τ is the decrease in the job finding rate in the absence of heterogeneity. In my model, removing heterogeneity is equivalent to setting ζ equal to zero, so this moment translates to

$$\eta_\tau = \frac{f(\tau \mid \zeta = 0)}{f(\tau - 1 \mid \zeta = 0)}. \tag{21}$$

η_τ is estimated for $\tau = 1, 2, 3$ in van den Berg and van Ours (1996), and I target all of them

[30] Since I am not targeting the data directly but rather an implication of the data, this method is can be described as indirect inference. Jarosch and Pilossoph (2019) and Kroft et al. (2016) use a similar strategy to calibrate parameters to fit $f(\tau)$.

Table 5: Steady state variables across calibrations

Variable	Baseline	Pure duration dependence	Pure heterogeneity	Hagedorn-Manovskii
u	0.062	0.062	0.060	0.062
x	0.36	0.54	0.35	0.40
θ	0.54	0.54	0.53	1.12
w	0.99	0.99	0.99	0.97
f^H	0.54	0.43	0.58	0.52
f^L	0.18	0.19	0.19	0.19

Steady state variables across model calibrations.

in the baseline calibration.

B.3 Additional Calibration Results

Table 5 lists steady state variables and Table 6 describes the model fit across calibrations. The tables in this section include the Hagedorn-Manovskii calibration, which I discuss in Appendix D.

C Deriving $f(\tau)$

Let $u_t(\tau)$ denote the number of unemployed individuals who have been unemployed for τ continuous periods. Note that $\sum_\tau u_t(\tau) = u_t$. Let $u_t^H(\tau)$ denote the number of unemployed individuals who have been unemployed for τ periods and are in the high state. Finally, let $x_t(\tau)$ denote the fraction of high-state unemployed workers among all unemployed workers with τ periods of unemployment, so $x_t(\tau) = u_t^H(\tau)/u_t(\tau)$.

To derive $f(\tau)$, I must first derive $x(\tau)$. Within the period of job separation ($\tau = 0$), the number of unemployed workers is the number of workers who were just separated from their jobs. So,

$$u_t(0) = \lambda\left(1 - u_t\right),$$

Table 6: Calibration fit across model specifications

Moment	Target	Baseline	Pure duration dependence	Pure heterogeneity	Hagedorn-Manovskii
u	0.062	0.062	0.062	0.060	0.062
f	0.31	0.32	0.32	0.33	0.32
$f(\tau)$					
a	0.25	0.25	0.25	0.24	0.25
b	0.41	0.40	0.40	0.42	0.40
c	0.20	0.18	0.19	0.19	0.19
η					
η_1	0.96	0.92			0.90
η_2	0.85	0.87			0.86
η_3	0.80	0.83			0.83
SSE		0.001	0.000	0.001	0.000

Simulated moments across model specifications. SSE refers to sum of squared error.

and the unemployed in the high state when $\tau = 0$ is

$$u_t^H(0) = (1-\zeta)\,\lambda\,(1-u_t)\,.$$

These imply

$$x_t(0) = 1-\zeta.$$

In the period following separation ($\tau = 1$), the unemployment mass consists of those who were separated in the previous period and did not find a job within that period. Therefore,

$$u_t\,(1) = u_{t-1}(0)\,(1-f_{t-1}(0)) = \lambda_{t-1}\,(1-u_{t-1})\,(1-f_{t-1}(0))\,.$$

Similarly, the number of workers in the high state when $\tau = 1$ is

$$u_t^H\,(1) = u_{t-1}^H(0)\left(1-f_{t-1}^H(0)\right)(1-\phi) = (1-\zeta)\,\lambda_{t-1}\,(1-u_{t-1})\left(1-f_{t-1}^H(0)\right)(1-\phi)\,.$$

Since the job finding rate in the high state is not a function of unemployment duration,

$f_t^H(\tau) = f_t^H$ for all τ and t. So, $u_t^H(1)$ can be written as

$$u_t^H(1) = (1-\zeta)\,\lambda_{t-1}(1-u_{t-1})\left(1-f_{t-1}^H\right)(1-\phi)\,.$$

These imply

$$x_t(1) = (1-\zeta)(1-\phi)\frac{1-f_{t-1}^H}{1-f_{t-1}(0)}.$$

Using the same method for $\tau = 2$, we have

$$x_t(2) = (1-\zeta)(1-\phi)^2\frac{\left(1-f_{t-2}^H\right)\left(1-f_{t-1}^H\right)}{(1-f_{t-2}(0))(1-f_{t-1}(1))},$$

and so on.

In summary, for $\tau \geq 1$,

$$x_t(\tau) = (1-\zeta)\left(\frac{(1-\phi)\left(1-f_{t-\tau}^H\right)}{1-f_{t-\tau}(0)}\right)\left(\frac{(1-\phi)\left(1-f_{t-\tau+1}^H\right)}{1-f_{t-\tau+1}(1)}\right)\cdots\left(\frac{(1-\phi)\left(1-f_{t-1}^H\right)}{1-f_{t-1}(\tau-1)}\right).$$

Concisely written, we have

$$x_t(\tau) = \begin{cases} 1-\zeta, & \tau = 0 \\ (1-\zeta)(1-\phi)^{\tau}\prod_{i=0}^{\tau-1}\dfrac{1-f_{t-i}^H}{1-f_{t-i}(\tau-i)}, & \tau \geq 1. \end{cases} \tag{22}$$

In the steady state,

$$x(\tau) = \begin{cases} 1-\zeta, & \tau = 0 \\ (1-\zeta)(1-\phi)^{\tau}\dfrac{\left(1-f^H\right)^{\tau}}{\prod_{i=0}^{\tau-1}1-f(\tau-i)}, & \tau \geq 1. \end{cases} \tag{23}$$

Note the asymptotic properties of $x(\tau)$. Since $f^H > f(\tau)$ for all τ, $x(\tau) \to 0$ as $\tau \to \infty$. In words, the unemployment pool becomes dominated by the low state as unemployment duration increases.

The job finding probability of an individual who has been unemployed for τ periods is

$$f_t(\tau) = f_t^H x_t(\tau) + f_t^L (1 - x_t(\tau)) = x_t(\tau) \left(f_t^H - f_t^L\right) + f_t^L.$$

Since $x(\tau) \to 0$ as $\tau \to \infty$, $f(\tau) \to f^L$ as $\tau \to \infty$; as unemployment duration increases, the finding rate converges to f_t^L.

Plugging in $x_t(\tau)$ above, we have

$$f_t(\tau) = \begin{cases} (1-\zeta) f_t^H + \zeta f_t^L, & \tau = 0 \\ \left(f_t^H - f_t^L\right)(1-\zeta)(1-\phi)^\tau \prod_{i=0}^{\tau-1} \dfrac{1 - f_{t-i}^H}{1 - f_{t-i}(\tau - i)} + f_t^L, & \tau \geq 1. \end{cases} \tag{24}$$

In the steady state,

$$f(\tau) = \begin{cases} (1-\zeta) f^H + \zeta f^L, & \tau = 0 \\ \left(f^H - f^L\right)(1-\zeta)(1-\phi)^\tau \dfrac{\left(1 - f^H\right)^\tau}{\prod_{i=0}^{\tau-1} 1 - f(\tau - i)} + f^L, & \tau \geq 1. \end{cases} \tag{25}$$

D Simulation Tables

Table 7 reports simulation statistics comparing data, the simulated model with the baseline calibration, and the simulated model with the DMP calibration. For each, I list the standard deviation, autocorrelation, and correlations between variables. The autocorrelation numbers match those in Table 4.[31]

Neither of the simulations in Table 7 come close to generating the volatility of unemployment in the data (Shimer, 2005). One way to resolve this puzzle is to use significantly different parameter values for the opportunity cost of employment z and the wage bargaining parameter η. Following Hagedorn and Manovskii (2008), I set $z = 0.955$ and $\eta = 0.052$ for the simulations in Table 9. I refer to this as the Hagedorn-Manovksii calibration. The rest of the external parameters match the baseline calibration, and the internal parameters

[31] The correlation between productivity (A_t) and (u_t) is significantly weaker in the data than the model. This has to do with my time period, 1978-2019, and is pointed out in Hagedorn and Manovskii (2011).

are recalibrated as in Table 8. Changing these parameters indeed increases the volatility of unemployment, and the persistence results are unaffected.

E Additional IRFs

I include IRFs for wages, output, and vacancies from shocks to productivity (Table 9) and the separation rate (Table 10).

Table 7: Simulation statistics

Data

		u_t	v_t	θ_t	f_t	A_t
St. dev.		0.198	0.200	0.385	0.143	0.017
Autocorr.		0.975	0.959	0.969	0.980	0.927
	u_t	1	-0.873	-0.944	-0.988	0.242
	v_t		1	0.979	0.888	-0.207
Correlation	θ_t			1	0.946	-0.214
	f_t				1	-0.275
	A_t					1

Simulation statistics, baseline calibration

		u_t	v_t	θ_t	f_t	A_t
St. dev.		0.028	0.045	0.068	0.031	0.020
Autocorr.		0.957	0.802	0.892	0.929	0.877
	u_t	1	-0.720	-0.889	-0.952	-0.857
	v_t		1	0.957	0.894	0.972
Correlation	θ_t			1	0.984	0.996
	f_t				1	0.968
	A_t					1

Simulation statistics, DMP calibration

		u_t	v_t	θ_t	f_t	A_t
St. dev.		0.025	0.049	0.072	0.029	0.020
Autocorr.		0.930	0.798	0.878	0.878	0.877
	u_t	1	-0.831	-0.924	-0.924	-0.923
	v_t		1	0.980	0.981	0.979
Correlation	θ_t			1	1.000	0.998
	f_t				1	0.999
	A_t					1

Top panel is data and the rest of the panels are from simulations of the model under the baseline and DMP calibrations. The top two rows of each panel list the standard deviation and autocorrelation of each variable. The rest of the panel is the contemporaneous correlation between two variables. All series are quarterly and HP filtered with smoothing parameter 10^5.

Table 8: Hagedorn-Manovskii calibrated parameters

Parameter	Meaning	DMP	Baseline	Hagedorn-Manovskii
γ	Low state penalty		0.34	0.12
ϕ	Transition rate	0.00	0.12	0.12
ζ	Initial low state	0.00	0.31	0.31
μ	Match efficiency	0.41	0.98	0.72

Compares Hagedorn-Manovski calibrated parameters with calibrations from Table 2.

Table 9: Simulation statistics, Hagedorn-Manovski calibration

		u_t	v_t	θ_t	f_t	A_t
St. dev.		0.166	0.314	0.814	0.246	0.020
Autocorr.		0.963	0.794	0.795	0.948	0.877
Correlation	u_t	1	-0.600	-0.446	-0.781	-0.798
	v_t		1	0.406	0.876	0.895
	θ_t			1	0.406	0.527
	f_t				1	0.855
	A_t					1

Top two rows of each panel list the standard deviation and autocorrelation of each variable. The rest of the panel is the contemporaneous correlation between two variables. All series are quarterly and HP filtered with smoothing parameter 10^5.

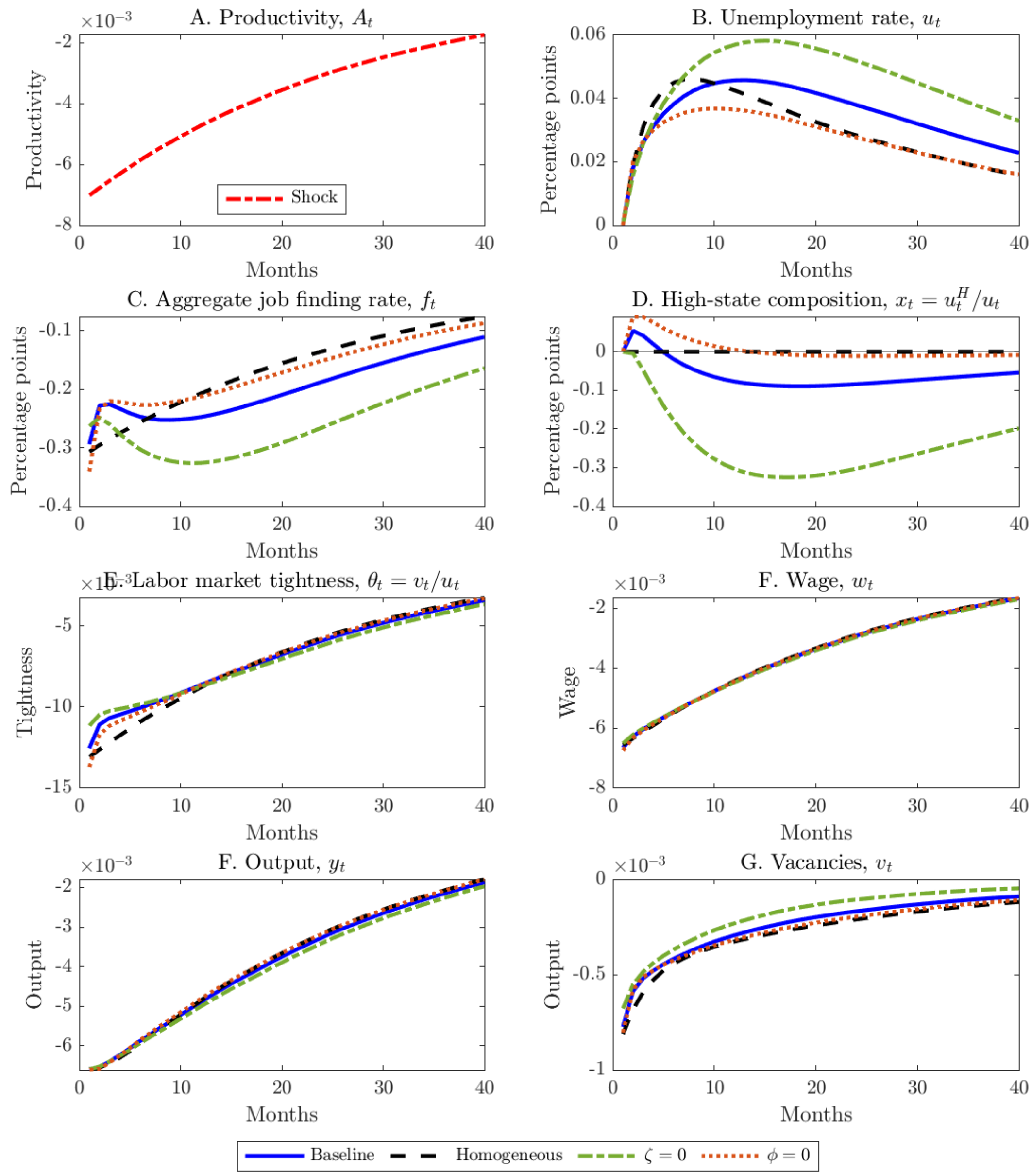

Figure 9: IRFs after negative productivity shock

Impulse response functions following a negative productivity shock. All impulse responses are measured in deviations from the steady state. Panel A plots the exogenous shock; the other panels plot endogenous responses. These IRFs build upon the IRFs in Figures 7. Output is defined as $y_t = A_t n_t$.

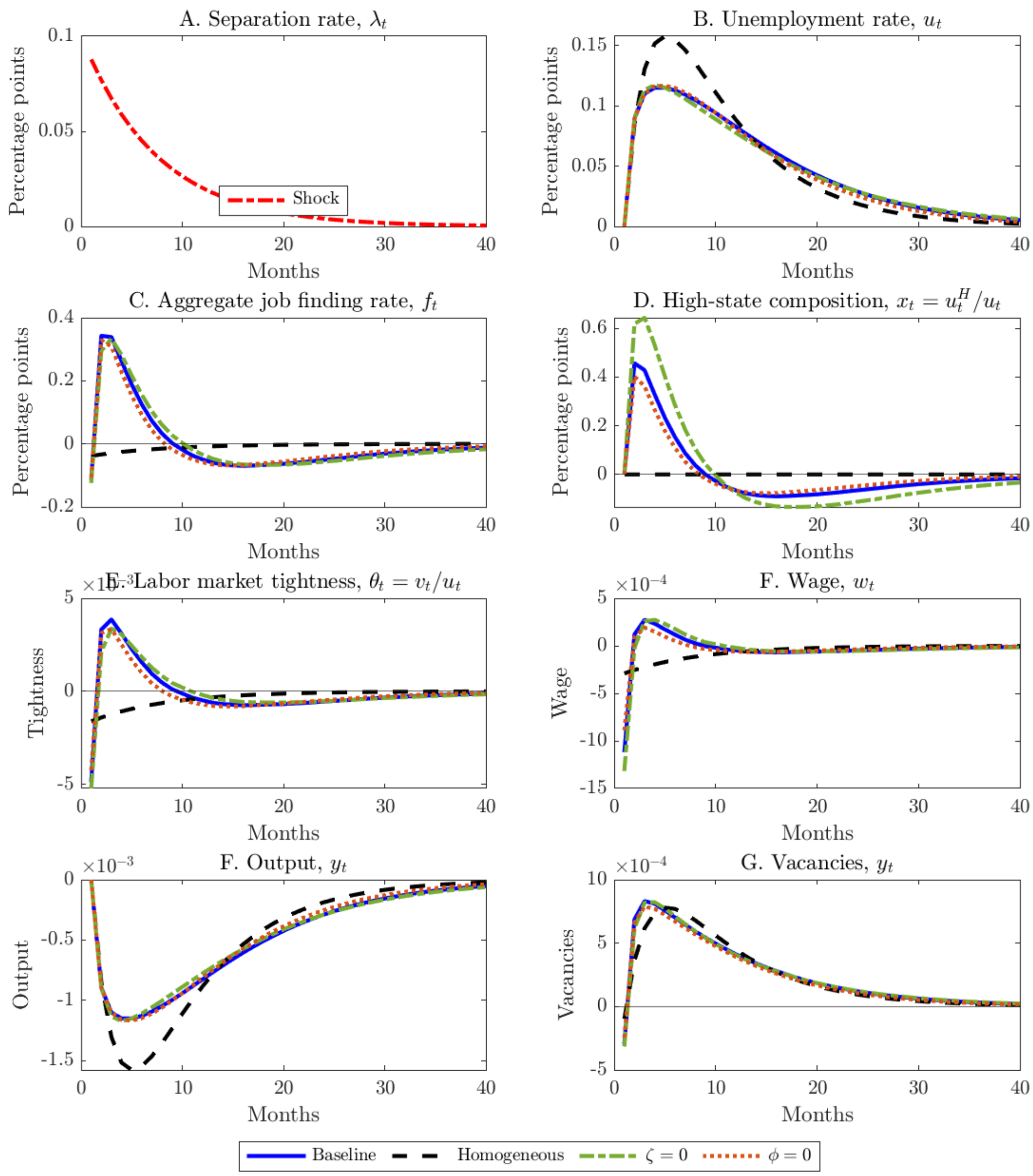

Figure 10: IRFs after positive separation rate shock

Impulse response functions following a positive separation rate shock. All impulse responses are measured in deviations from the steady state. Panel A plots the exogenous shock; the other panels plot endogenous responses. These IRFs build upon the IRFs in Figure 8. Output is defined as $y_t = A_t n_t$.

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