A Complete Results on Synthetic Network

Figure 1 illustrates the engagement $|\mathcal{A}|$ generated at each step of our fine-tuning procedure across all possible configurations of the synthetic network, considering variations in homophily (high/low), modularity (high/low), and opinion distribution (positive, negative, neutral, uniform). The corresponding sentiment s_t is shown in Figure 2, where colors represent different positions of the LLM within the graph.

As observed, the fine-tuning procedure successfully converges in nearly all configurations, demonstrating the effectiveness of our methodology in maximizing user engagement while remaining fully agnostic to the underlying network structure.

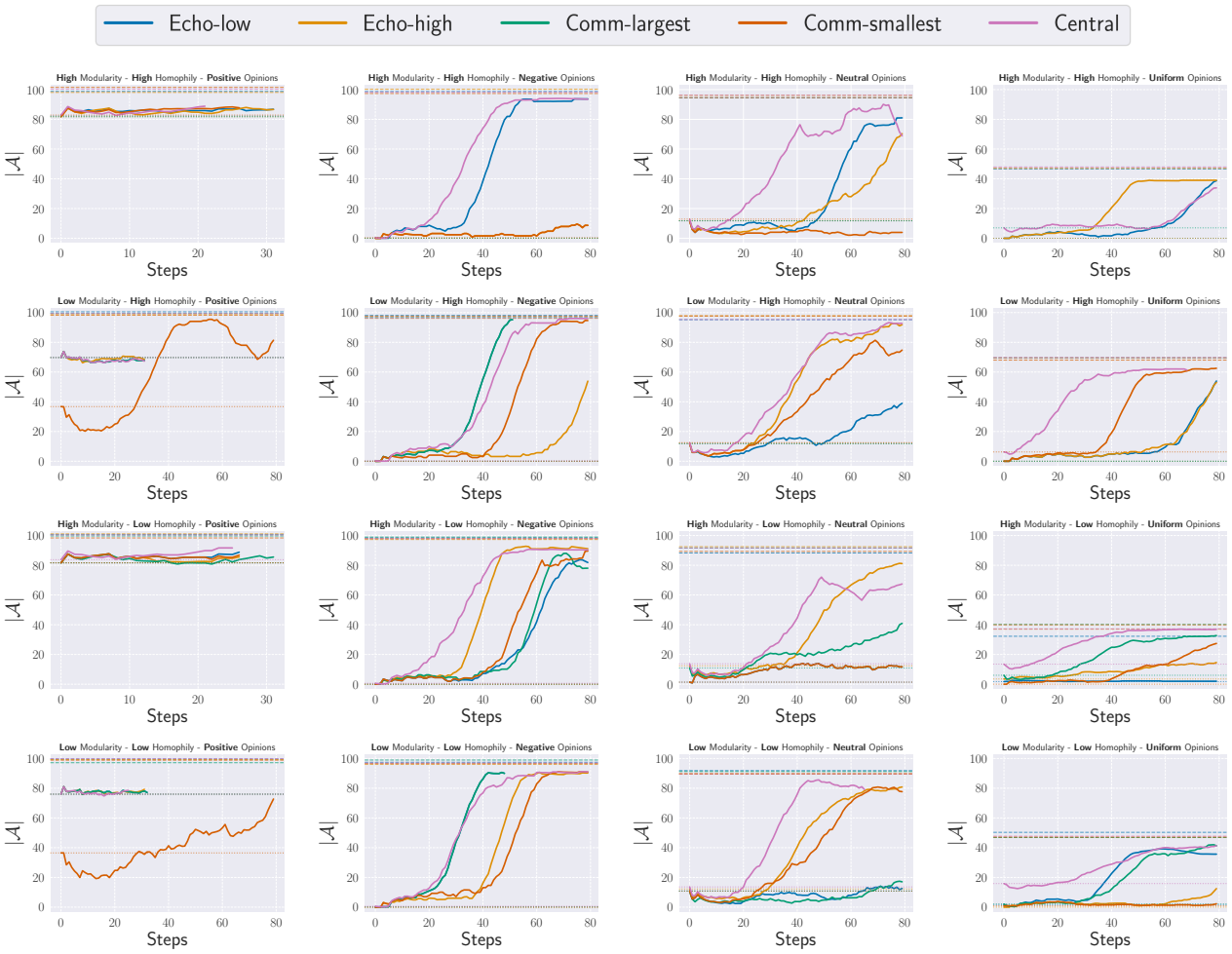

Figure 1: Engagement $|\mathcal{A}|$ at each step of our fine-tuning procedure in the synthetic data setting. Columns refer to different opinion distributions: positive/negative/neutral/uniform (left to right). Each plot depicts the trend varying the network structure in terms of modularity and homophily. Colors indicate different positions of the LLM agent. Dashed lines represent the maximum engagement within that configuration, whereas dotted lines indicate its lower-bound (i.e., $|\mathcal{A}|$ produced by the non-finetuned LLM).

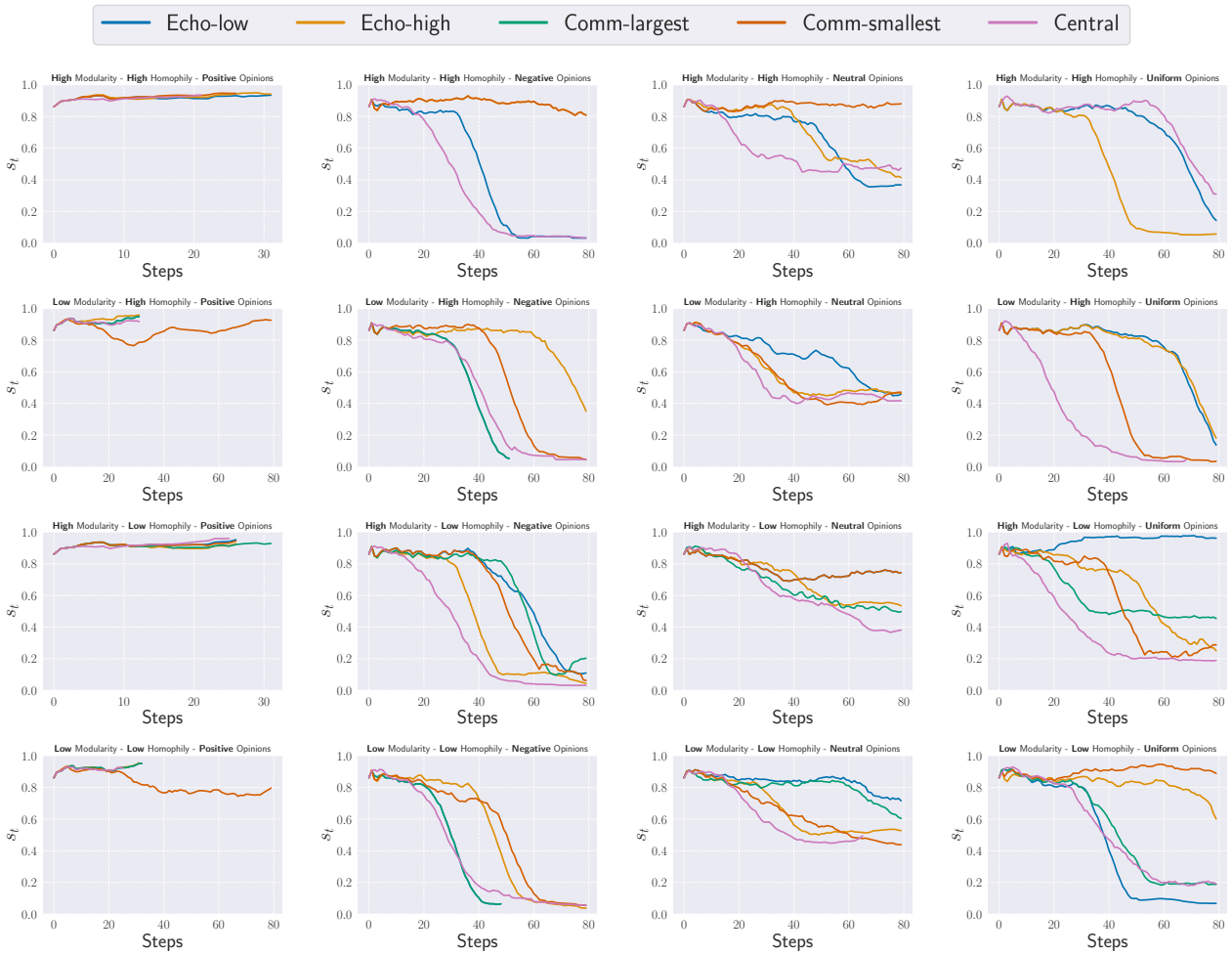

Figure 2: Sentiment content s_t produced at each step of our fine-tuning procedure in the synthetic data setting. Each column refers to a different opinion distribution: positive/negative/neutral/uniform (from left to right). Each plot depicts the trend varying the network structure in terms of modularity and homophily. Colors indicate different positions of the LLM agent.