Algorithm Analysis Homework 9

May 23 (Due next Tuesday class)

1. Our discussion of Dijkstra's shortest-path algorithm used a heap-based priority queue. When the algorithm runs, three heap operations are used, namely, `push()`, `pop()` and `changeKey()`. Note that `changeKey()` is used when `d[v]` is updated in the following for-loop:

```
for each neighbor v (in Q) of u
  if d[u]+w(u,v) < d[v]  // if newly added vertex u makes v closer
    d[v] <- d[u]+w(u,v)  // update the distance (which updates Q)
    parent[v] <- u
```

   Explain, in less than five lines, how these three operations run in $O(\log n)$ time when a priority queue has n elements.

2. In Section 6.6.1, we discussed the matirx

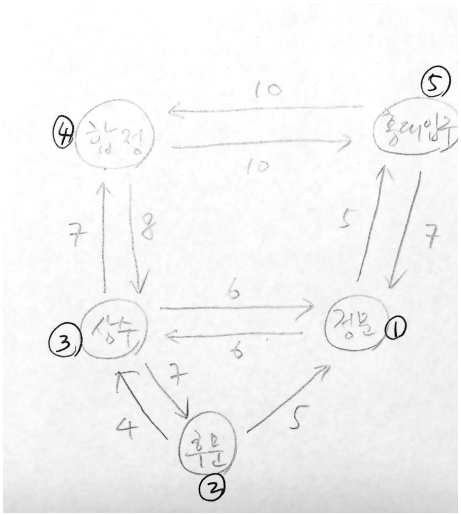
$$A^\infty = I \oplus A \oplus A^2 \oplus A^3.$$

   (a) What does (i, j)th element of A^∞ represent?

   (b) Does the usage of ∞ in A^∞ make sense? Why did we not define as follows?

$$A^\infty = I \oplus A \oplus A^2 \oplus A^3 \oplus A^4 \oplus \cdots.$$

3. Consider the following graph:



   Floyd's all-pairs shortest distance algorithm gives matrices $D^{(5)}$ and P, as we discussed in the class. Figure out the matrices $D^{(5)}$ and P without using the algorithm :)