0/1 Knapsack problem - Dynamic programming

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References : [Javatpoint-0-1-knapsack]

Two types of KnapSack problems :

1. Fractional Knapsack
2. 0/1 knapsack

Fractional Knapsack -

- Greedy approach is used
- You can take the partial weight and profit accordingly to fill the bag

0/1 Knapsack -

- Dynamic programming
- Either pick the complete object (1) or drop out the complete object(0) to satisfy the weight of the bag .

Dynamic programming :

- Problem should be solved based on sequence of decisions
- Try all possible solutions and pick out anyone.
- Optimization - Here profit is maximum

Hints or points to remember :

- Arrange the objects in Ascending order of weights
- In the tabulation method , while filling profit , until that particular weight occurs , for previous weight put profit values from the previous cell.
- Formula : v[i,w]=max{v[i-1,w],(v[i-1,(w-w[i])]+p[i])} , where i is row no. , w is column no.

Constraints:

- $\sum xiwi <= m$
- Max $\sum xipi \rightarrow$ Profit should be maximized.

Example 1 :

Weight = { 3,4,6,5}
Profit = { 2,3,1,4}
m=8

			0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Pi	**Wi**	**0**	0	0	0	0	0	0	0	0	0
2	**3**	**1**	0	0	0	2	2	2	2	2	2
3	**4**	**2**	0	0	0	2	3	3	3	5	5
4	**5**	**3**	0	0	0	2	3	4	4	5	6
1	**6**	**4**	0	0	0	2	3	4	4	6	6

Maximum Profit = 8
Selection of object = { 3,4,6,5} = {1,0,0,1}

Example 2 :

Weight ={ 1,2,5,6,7}
Profit = {1,6,18,22,28}
m=11

			0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Pi	**Wi**	**0**	0	0	0	0	0	0	0	0	0	0	0	0
1	**1**	**1**	0	1	1	1	1	1	1	1	1	1	1	1
6	**2**	**2**	0	1	6	7	7	7	7	7	7	7	7	7
18	**5**	**3**	0	1	6	7	7	18	19	24	25	25	25	25
22	**6**	**4**	0	1	6	7	7	18	22	23	28	29	29	40
28	**7**	**5**	0	1	6	7	7	18	22	28	29	34	35	40

Maximum Profit = 40

- v[i,w]=max{v[i-1,w],(v[i-1,(w-w[i])]+p[i])}

  v[4,10] = max{ v[3,10],v{3,(10-6)+22}

$v[4,10] = max\{ 25 , v\{3,4\}+22\}$

$v[4,10] = max\{ 25 , v\{3,4\}+22\}$

$v[4,10] = max\{ 25 , v\{3,4\}+22\}$

$v[4,10] = max\{ 25 , 7+22\}$

$v[4,10] = max\{ 25 , 29 \}$

$v[4,10] =29$