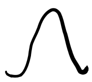
$p(x|\theta)$: likelihood
$p(\theta)$: prior

Combine likelih. (data) + prior → $\underbrace{\text{posterior}}_{p(\theta|x)}$



$p(x)$: marginal dist^{n}

Use these to derive $p(\theta|x)$

assuming data x is FIXED + KNOWN!

$$p(\theta|x) = \frac{p(\theta, x)}{p(x)} = \frac{p(x|\theta)\, p(\theta)}{p(x)}$$

→ does NOT depend on θ

$\propto$ prop θ

$$\propto \underbrace{p(\theta|x)\; p(\theta)}$$

↳ see if I recognize this as a common dist^{n}.

(Ex: Normal, Gamma, etc.).

(x is a constant wrt to θ)

$$X \sim \text{Bernoulli}(\theta), \quad 0 < \theta < 1$$

$$p(x|\theta) = \theta^{x}(1-\theta)^{1-x} \, I(0<\theta<1).$$

$$X_1, \ldots, X_n \overset{iid}{\sim} \text{Bern}(\theta)$$

$\propto$: prop to } not. shorthand.
$x_{1:n} = x_1, \ldots, x_n$

$$p(x_1, \ldots, x_n | \theta) \overset{iid}{=} \quad (1)$$

$$P(X_1 = x_1|\theta) \, P(X_2 = x_2|\theta) \cdots P(X_n = x_n|\theta) \quad (2)$$

$$= \prod_{i=1}^{n} P(X_i = x_i|\theta) \quad (3)$$

$$= \prod_{i=1}^{n} p(x_i|\theta) \quad (4)$$

$$= \prod_{i=1}^{n} \theta^{x_i}(1-\theta)^{1-x_i} \quad \text{plugging in Bern.} \quad (5)$$

$$= \underline{\theta^{\sum_{i=1}^{n} x_i}(1-\theta)^{n-\sum_{i=1}^{n} x_i}} \quad (6)$$

likelihood

Prior $\quad \theta \sim \text{Beta}(a, b), \quad a, b > 0$

$$p(\theta) = \text{Beta}(\underset{\text{r.v.}}{\theta} \mid \underbrace{a, b}_{\text{fixed}}) = \frac{1}{B(a,b)} \theta^{a-1} (1-\theta)^{b-1} \times I(0 < \theta < 1) \quad (7)$$

$$B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

Posterior

Plug in (6) + (7)

$$\boxed{p(\theta \mid x_{1:n}) \propto p(x_{1:n} \mid \theta)\, p(\theta)}$$

$$= \theta^{\Sigma x_i} (1-\theta)^{n - \Sigma x_i} \times \frac{1}{B(a,b)} \theta^{a-1} (1-\theta)^{b-1} I(0<\theta<1)$$

constant that doesn't depend on θ

$$\underset{\theta}{\propto} \theta^{\Sigma x_i} (1-\theta)^{n - \Sigma x_i} \theta^{a-1} (1-\theta)^{b-1} I(0<\theta<1)$$

group like terms

$$= \theta^{\sum_{i=1}^{n} x_i + a - 1} (1-\theta)^{n - \Sigma x_i + b - 1} I(0 < \theta < 1)$$

$$\Rightarrow \theta \mid x_{1:n} \sim \text{Beta}\left(\Sigma x_i + a,\; n - \Sigma x_i + b\right)$$

$$\int \underbrace{\theta^{a_n+1-1}(1-\theta)^{b_n-1}}_{\text{kernel of Beta}(a_n+1,\, b_n)}\, d\theta$$

$$= \int \boxed{\frac{\theta^{a_n+1-1}(1-\theta)^{b_n-1}}{B(a_n+1,\, b_n)}} \times B(a_n+1,\, b_n)\, d\theta$$

$$= \underline{B(a_n+1,\, b_n)} \,/\!/$$