d.) $y_i \mid x_i, \beta \sim N(x_i \beta, \sigma^2)$

$$\beta \sim N\left(0, \tau^2 / \sum_{i=1}^{n} x_i^2\right).$$

Let $\lambda = (\sigma^2)^{-1}$. $\lambda_0 = \left(\tau^2 / \sum x_i^2\right)^{-1}$.

$$p(\beta \mid y_{1:n}) \underset{\beta}{\propto}$$

$$\exp\left\{-\frac{\lambda}{2}\sum_{i=1}^{n}(y_i - x_i\beta)^2\right\} \times \exp\left\{-\frac{\lambda_0}{2}\beta^2\right\}$$

$$= \exp\left\{-\frac{\lambda}{2}\sum_{i=1}^{n}\left(y_i^2 - 2y_i x_i \beta + (x_i\beta)^2\right\}\right.$$

$$\times \exp\left\{-\frac{\lambda_0}{2}\beta^2\right\}$$

$$\underset{\beta}{\propto} \exp\left\{-\frac{\lambda}{2}\sum_{i=1}^{n}\left(-2y_i x_i\beta + x_i^2\beta^2\right\}\right.$$

$$\times \exp\left\{-\frac{\lambda_0}{2}\beta^2\right\}$$

$$= \exp\left\{-\frac{1}{2}\left[2\lambda\left(\sum_{i=1}^{n} x_i y_i\right)\beta + \left(\lambda_0 + \lambda\sum x_i^2\right)\beta^2\right]\right.$$

$$= \exp\left\{-\frac{1}{2}\left[2\frac{1}{\sigma^2}\left(\sum_{i=1}^{n} x_i y_i\right)\beta + \left(\frac{\sum x_i^2}{\tau^2} + \lambda\sum x_i^2\right)\beta^2\right]\right.$$

$$= \exp\left\{-\frac{1}{2}\left[2\frac{1}{\sigma^2}\left(\sum_i x_i y_i\right)\beta + \sum x_i^2\left(\frac{1}{\tau^2} + \frac{1}{\sigma^2}\right)\beta^2\right]\right.$$

$$= \ldots \frac{1}{} \left(\sum x_i y_i\right)\beta + \sum x_i^2 / \frac{\sigma^2\tau^2}{} \ldots \Big)\beta^2\Big]$$

$$= \exp\left\{ -\frac{1}{2} \left[2 \frac{1}{\sigma^2} \left(\sum x_i y_i \right) \beta \; [illegible] \; \tau^2 + \sigma^2 \right) \right.$$

By prop of the normal,

$$L^{-1} = \frac{\sigma^2 \tau^2}{\tau^2 + \sigma^2}$$

$$L^{-1} M = L^{-1} \times \Big(\underbrace{\frac{\sum x_i y_i}{\sigma^2}}_{M} \Big)$$

Then, $\beta \mid y_{1:n} \sim N(L^{-1}M, L^{-1})$.