$$I = E_f[h(x)] = \int_x h(x) f(x)\, dx$$

Have a prop. dist^n $g(x)$ that we can sample from.

$$= \int_x h(x) f(x) \frac{g(x)}{g(x)}\, dx$$

$$= \int \underbrace{\frac{h(x) f(x)}{g(x)}}_{\text{new } h(x)} \underbrace{g(x)}_{\text{old } f(x)}\, dx$$

$$= E_g\left[\frac{h(x) f(x)}{g(x)}\right]$$ if we cannot calc this in closed form

We can est I by $\hat{I}$

$$\hat{I} = \frac{1}{n} \sum_{i=1}^{n} \frac{h(X_i) f(X_i)}{g(X_i)}$$

where $X_1, \ldots, X_n \overset{iid}{\sim} g$ → maybe Normal, Beta, Gamma.

$$\hat{I}_{(g)} \xrightarrow{SLLN} E_f[h(x)] \qquad (\bar{h}_n)$$

$\boxed{\text{Exercise.}}$

Exercise Goal: $P(X > 5)$ $f(x) = N(0,1)$

$g(x) = N(5,1)$ → would this capture the imp't region?

$\phi_0 = f = N(0,1)$

$\phi_\theta = g = N(5,1)$

$$\begin{aligned}\hat{p} &= \int I(p > u)\, f(u)\, du \\ &= \int I(p > u)\, \frac{f(u)\, g(u)}{g(u)}\, du \\ &= \int \frac{I(p > u)\, f(u)}{g(u)}\, g(u)\, du \\ &= \int \frac{I(p > u)\, \phi_0(u)}{\phi_5(u)}\, \phi_5(u)\, du \\ &= E_{\phi_5(u)}\left[\frac{I(p > u)\, \phi_0(u)}{\phi_5(u)}\right]\end{aligned}$$

redone deriv. from before

Generate $X_1, \ldots, X_n \overset{iid}{\sim} N(5,1) = \phi_5(x)$

$$\boxed{\hat{p} = \frac{1}{n} \sum_{i=1}^{n} \frac{I(p > x_i)\, \phi_0(x_i)}{\phi_5(x_i)}}$$