$$X_1, \dots, X_n \mid \theta \overset{iid}{\sim} N(\theta, \lambda^{-1}) \qquad \lambda = (\sigma^2)^{-1}$$

$$\theta \sim N(\mu_0, \lambda_0^{-1}).$$

① Likelihood.

$$p(x \mid \theta, \lambda^{-1}) = \sqrt{\frac{\lambda}{2\pi}} \exp\left\{\frac{-\lambda}{2}(x-\theta)^2\right\}$$

$$= \boxed{\sqrt{\frac{\lambda}{2\pi}}} \exp\left\{-\frac{\lambda}{2}\left(x^2 - 2\theta x + \theta^2\right)\right\}$$

want to use prop wrt θ

$$\underset{\theta}{\propto} \exp\left\{-\frac{\lambda}{2}\left[-2\theta x + \theta^2\right]\right\}$$

$$\underset{\theta}{\propto} \exp\left\{\lambda\theta x - \frac{\lambda}{2}\theta^2\right\} \qquad (5)$$

simplified likehood

2. Prior

$$p(\theta) = N(\theta \mid \mu_0, \lambda_0^{-1}) \overset{\text{symmetry}}{=} N(\mu_0 \mid \theta, \lambda_0^{-1})$$

$$\underset{\theta}{\propto} \exp\left\{\lambda_0 \mu_0 \theta - \frac{1}{2}\lambda_0 \theta^2\right\} \qquad (6)$$

dropped norm. constants and constants that depend on θ.

Posterior

$$p(\theta \mid x_{1:n}) \propto p(x_{1:n} \mid \theta)\, p(\theta)$$

$$\underset{\theta}{\propto} \prod_{i=1}^{n} \left[\exp\left(\lambda x_i \theta - \frac{\lambda}{2}\theta^2 \right) \right] \times \exp\left\{ \lambda_0 \mu_0 \theta - \frac{\lambda_0}{2}\theta^2 \right\}$$

$$= \exp\left\{ \underline{\lambda \left(\sum_{i=1}^{n} x_i\right)\theta} - \frac{\lambda n}{2}\theta^2 + \underline{\lambda_0 \mu_0 \theta} - \frac{\lambda_0}{2}\theta^2 \right\}$$

$$= \exp\left\{ \left(\lambda_0 \mu_0 + \lambda \sum_{i=1}^{n} x_i \right)\theta - \frac{1}{2}\underbrace{\left(\lambda_0 + \lambda n\right)}_{= L}\theta^2 \right\}$$

$$M = \frac{\lambda_0 \mu_0 + \lambda \sum x_i}{\lambda_0 + \lambda n}$$

$$= \exp\left\{ (ML)\theta - \frac{1}{2} L \theta^2 \right\} \quad \text{why?}$$

$$\propto N(M \mid \theta, L^{-1}) \overset{\text{symmetry}}{=} N(\theta \mid M, L^{-1})$$

true by eqn (5)

$$M = \frac{\lambda_0 \mu_0 + \lambda \sum x_i}{\lambda_0 + n\lambda} \quad \text{(posterior mean)}$$

$$= \frac{\lambda_0}{\lambda_0 + n\lambda}\mu_0 + \frac{\lambda}{\lambda_0 + n\lambda} n\bar{x}$$

$$= \boxed{\frac{\lambda_0}{\lambda_0 + n\lambda}}(\mu_0) + \boxed{\frac{\lambda n}{\lambda_0 + n\lambda}}(\bar{x})$$

prior mean

sample mean

λ_0 = prior precision

λ = sample precision

$$L = \underline{\lambda_0 + n\lambda} \quad \text{(post prec.)}$$