$$X_1, \dots, X_n \overset{iid}{\sim} \text{Poisson}(\theta)$$

$$\theta \sim \text{Gamma}(a, b)$$

①

$$p(x_1, \dots, x_n \mid \theta) = \prod_{i=1}^{n} p(x_i \mid \theta)$$

$$= \prod_{i=1}^{n} \frac{e^{-\theta} \theta^{x_i}}{x_i!} = e^{-\sum_{i=1}^{n} \theta} \theta^{\sum x_i} \underbrace{\boxed{\prod_{i=1}^{n} \frac{1}{x_i!}}}_{\text{constant}}$$

$$\boxed{\underset{\theta}{\propto} e^{-n\theta} \theta^{\sum_i x_i}}$$

" prop wrt θ "

② Derive the $p(\theta \mid x_1, \dots, x_n)$.

$$p(\theta \mid x_1, \dots, x_n) \propto \underline{p(x_1, \dots, x_n \mid \theta)} \, p(\theta)$$

$$\underset{\theta}{\propto} e^{-n\theta} \theta^{\sum_{i=1}^{n} x_i} \times \boxed{\frac{b^a}{\Gamma(a)}} \theta^{a-1} e^{-b\theta} I(\theta > 0)$$

drop constant, group like terms

$$\underset{\theta}{\propto} e^{-n\theta - b\theta} \theta^{\sum x_i + a - 1} I(\theta > 0)$$

$$= e^{-\theta(n+b)} \theta^{\sum x_i + a - 1} I(\theta > 0) \; ⊛$$

kernel of the

$$\text{Gamma}(\sum x_i + a, \; n + b).$$

$\theta \mid x_1, \dots, x_n$

$\boxed{p(x)}$

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