1 TEX 101

Let us $\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$

2 PCA

Let columns of $m \times n$ matrix $\mathbf{X}$ contain n samples of m-dimensions feature.

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_n \end{bmatrix}$$

Define *covariance matrix* $\mathbf{C}_\mathbf{X}$

$$\mathbf{C}_\mathbf{X} \equiv \frac{1}{n}\mathbf{X}\mathbf{X}^\top$$

Let $\mathbf{Y} = \mathbf{P}\mathbf{X}$ be some linear transorm of $\mathbf{X}$ with orthonormal $m \times m$ matrix $\mathbf{P}$, so

$$\mathbf{P}^\top\mathbf{P} = I$$

Consider transformed covariance matrix $\mathbf{C}_\mathbf{Y}$

$$\mathbf{C}_\mathbf{Y} \equiv \frac{1}{n}\mathbf{Y}\mathbf{Y}^\top \tag{1}$$

Theorem 1 *There exists* $\mathbf{P} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \dots & \mathbf{p}_m \end{bmatrix}^\top$ *such that matrix* $\mathbf{C}_\mathbf{Y}$ *is diagonal.*

Rows $\mathbf{p}_i$ of $\mathbf{P}$ are called *principal components* (1)

$$\mathbf{C}_\mathbf{Y} = \mathbf{P}\mathbf{C}_\mathbf{X}\mathbf{P}^\top$$

References