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GPS/IMU data fusion using multisensor Kalman filtering: introduction of contextual aspects

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Abstract

The aim of this article is to develop a GPS/IMU multisensor fusion algorithm, taking context into consideration. Contextual variables are introduced to define fuzzy validity domains of each sensor. The algorithm increases the reliability of the position information. A simulation of this algorithm is then made by fusing GPS and IMU data coming from real tests on a land vehicle. Bad data delivered by GPS sensor are detected and rejected using contextual information thus increasing reliability. Moreover, because of a lack of credibility of GPS signal in some cases and because of the drift of the INS, GPS/INS association is not satisfactory at the moment. In order to avoid this problem, the authors propose to feed the fusion process based on a multisensor Kalman filter directly with the acceleration provided by the IMU. Moreover, the filter developed here gives the possibility to easily add other sensors in order to achieve performances required.

Keywords: Global positioning system; Inertial measurement unit; Kalman filter; Data fusion; Multisensor system

1. Introduction

Autonomous land vehicles (ALV) have different potential applications (goods transport, autonomous taxi, automatic highways,...) and are the subject of intensive researches through the world. ALV need continuous and precise positioning information. Integrity of the positioning system is one of the key factors of such systems.

Two types of sensors are able to give position of a mobile vehicle: absolute sensors (GPS, radar) which take their information in the environment outside the mobile and get the position in an absolute reference frame, and dead-reckoning sensors, which take their information on the mobile itself. In this last case, the position is derived from the last point and the positioning error is therefore drifting with time.

At the present time, the global positioning system (GPS), which is an absolute sensor, is the basic component of a land positioning system. In differential mode, it can reach centimeter precision [11]. However, the lack of credibility of GPS in some cases, due to multipath or mask effects, often leads to mix it with other sensors, such as dead-reckoning ones. These sensors, as for instance inertial sensors (gyroscopes and accelerometers), have the advantage of giving continuous positioning information, independent of the external environment. A package of inertial sensors may be classified into two groups [1]: inertial measurement unit (IMU) which delivers raw data from gyroscopes and accelerometers, corrected from scale factors and biases, and inertial

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navigation system (INS), which is an IMU however the output is sent to navigation algorithms to provide position, velocity and attitude of the vehicle.

Many research works have been led on the GPS/INS data fusion, especially using a Kalman filter [1,3,5]. Structures of GPS/INS fusion have been investigated in [1]. However, experimental results show [2,4,14] that, in case of extended loss or degradation of the GPS signal (more than 30 s), positioning errors quickly drift with time. So GPS/INS association is not a satisfactory association and the solution could be to add other absolute or dead-reckoning sensors, in order to have precise positioning information in any environment.

Fault detection of the GPS signal has also been investigated in [2,3,6]. Sukkarieh [2] introduced a threshold derived from a statistical reasoning to determine whether the GPS data is valid, McNeil [6] proposed weightings on GPS and INS measurements according to fuzzy rules and Stephen [3] introduced a condition on the GDOP (geometric dilution of precision, delivered by the GPS sensor) value.

In this paper is developed a multisensor Kalman filter (KF), which is suitable to integrate a high number of sensors, without rebuilding the whole structure of the filter. By introducing contextual information in the KF, validity domains of each sensor are defined in order to reject bad data when detected, thus increasing the reliability of the data fusion. Reliability is defined here as the robustness to system failures. Integrity of a navigation system is the ability to provide reliable navigation information while also monitoring the health of navigation data and either correct or reject bad data [1].

Basics of multisensor Kalman filtering are exposed in Section 2. Section 3 introduces contextual information as a way to define validity domains of the sensors and so to increase reliability. A basic development of the multisensor KF using contextual information is made in Section 4 with two sensors, a GPS and an IMU. Simulation of the algorithm presented in Section 4 is made in Section 5 with data coming from real experiments. Results are compared in terms of accuracy with a structure based on [1] and specifically developed for the fusion of GPS and INS. First results about the integrity of the filter in case of degradation of the GPS signal are also given.

2. Multisensor Kalman filtering

Consider a discrete-time linear stationary signal model (1) [8–10]:

$$x(k+1) = Fx(k) + w(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $w(k) \in \mathbb{R}^n$ is a sequence of zero mean white gaussian noise of assumed known covariance matrix $Q(k) = E[w(k)w(k)^{\mathrm{T}}]$. $F \in \mathbb{R}^{n\times n}$ is the known state transition matrix. In the simplest case, measurements are expressed as a linear relation with respect to the state space variables and are corrupted by noise. The following relation (2) describes the measurements for a set of N sensors:

$$z_i(k) = H_i\, x(k) + b_i(k), \quad i = 1, \ldots, N \tag{2}$$

with $z_i(k) \in \mathbb{R}^l$ the measurement vector of the sensor i, $b_i(k) \in \mathbb{R}^l$ the white gaussian observation noise for the sensor i with zero mean and with assumed known covariance matrix $R_i(k) = E[b_i(k)b_i(k)^{\mathrm{T}}]$, $H_i \in \mathbb{R}^{l\times n}$ is the measurement matrix associated to the sensor i and N is the number of sensors. Given the model described by Eqs. (1) and (2), the multisensor KF can be computed as an estimation stage and a prediction stage [12,13,15].

- The estimation stage

$$\hat{x}(k \mid k) = \hat{x}(k \mid k-1) + \sum_{i=1}^{N} K_i(k)[z_i(k) - H_i\, \hat{x}(k \mid k-1)] \tag{3}$$

with

$$K_i(k) = P(k \mid k)H_i^{\mathrm{T}} R_i^{-1}(k) \tag{4}$$

the Kalman gain for the data fusion associated to the sensor i, the quantity $z_i(k) - H_i\, \hat{x}(k \mid k-1) = v_i(k)$ is called the innovation associated to the observation from the sensor i. The uncertainty on the estimate is given by the matrix

$$P^{-1}(k \mid k) = P^{-1}(k \mid k-1) + \sum_{i=1}^{N} H_i^{\mathrm{T}} R_i^{-1}(k) H_i \tag{5}$$

Proofs of these equations from the derivation of the multisensor information filter are given in Appendix A.

- The prediction stage
  The prediction stage is defined by Eqs. (6) and (7)

$$\hat{x}(k+1 \mid k) = F\hat{x}(k \mid k) \tag{6}$$

$$P(k+1 \mid k) = FP(k \mid k)F^{\mathrm{T}} + Q(k) \tag{7}$$

3. Contextual information

Nimier [7] developed a theoretic framework on multisensor data fusion taking context into consideration. He proposed a method to combine symbolic and numerical information, in order to have a supervised fusion process. The supervision is realized by a level of treatment which analyses the context using contextual variables, so that the estimation process is adapted to this context. The result is to favor measurements provided by the sensors well-adapted to the context and to minimize the importance of those that are not well-adapted. GPS sen-

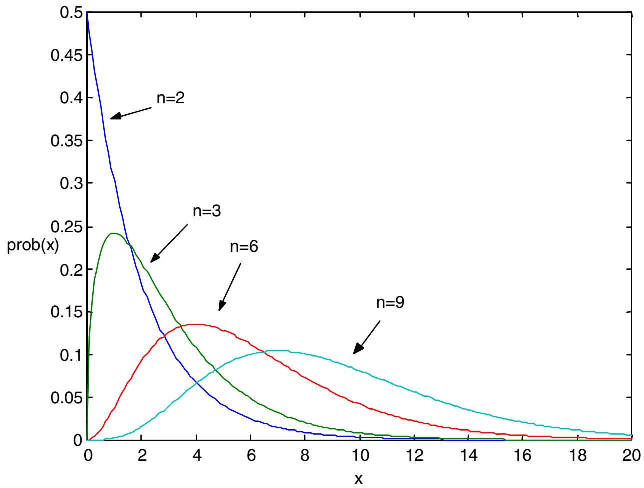

Fig. 1. Pdf of a χ^2 distribution with n degrees of freedom.

sor, whose signal quality depends on the environment, is suitable to this framework.

Giving a sensor i and its innovation $v_i(k) \in \mathbb{R}^l$ associated, the following relation is formed (8):

$$q_i = v_i^{\mathrm{T}}(k) S_i^{-1}(k) v_i(k) \tag{8}$$

with

$$S_i(k) = H_i\, P(k \mid k-1) H_i^{\mathrm{T}} + R_i(k) \tag{9}$$

the measurement prediction covariance matrix also called the innovation covariance matrix. The quadratic form $q_i \in \mathbb{R}^+$ defined by Eq. (8) is theoretically a χ^2 distribution with l degrees of freedom [11], whose probability density function (pdf) is drawn in Fig. 1.

From standard χ^2 tables and usual statistical tests [16], it is possible to define validity domains of the sensors based on the confidence level required: if the value of q_i is beyond a predefined threshold t_i, then the sensor is assumed unusable [1] and data from this sensor are ignored by the fusion process. [2] For example, considering a 95% confidence level and an innovation $v_i(k) \in \mathbb{R}^3$, then $t_i = 7.8$. The value of q_i defines the validity domain of the sensor i and is named a contextual variable.

Consider a set of N sensors. A context $q = (q_1, \ldots, q_N)$, $q \in (\mathbb{R}^+)^N$ is determined by N values of each contextual variable q_i, $i \in N$. $(\mathbb{R}^+)^N$, which is the definition domain of q, is called the contextual space. A sensor i is valid for a set of contexts represented by a subset of $(\mathbb{R}^+)^N$, named C_i, expressed by (10)

$$C_i = \{q \in (\mathbb{R}^+)^N \mid q_i < t_i\} \tag{10}$$

To every subset $J \in 2^{\{1,\ldots,N\}}$ correspond the group of sensors whose indices are contained in J. Given $J \in 2^{\{1,\ldots,N\}}$, c_J is called the exclusive validity domain; it represents the subsets of contexts where the sensors J are the only valid sensors. The expression of c_J is given by Eq. (11) where $\complement_{\{1,\ldots,N\}}(J)$ is the complementary of J into $\{1, \ldots, N\}$.

$$c_J = \{q \in (\mathbb{R}^+)^N \mid q_l < t_l \text{ and } q_m > t_m,\ l \in J,\ m \in \complement_{\{1,\ldots,N\}}(J)\} \tag{11}$$

A partition of the contextual space is defined by

$$A = \{c_J,\ J \in 2^{\{1,\ldots,N\}}\} \tag{12}$$

4. GPS/IMU data fusion

4.1. Definition of the state and measurement models

- State model

Considering the standard state model of a KF defined by Eq. (1), the state model chosen is a Wiener process acceleration model [12]. It is a basic model giving a good compromise between complexity and performance in the modelling of a land vehicle dynamics. $x(k) \in \mathbb{R}^9$ is the state vector representing position, velocity and acceleration in North, East and Down directions. In such a model, F and w are equal to:

$$F = \begin{pmatrix} I_3 & TI_3 & \frac{T^2}{2} I_3 \\ 0_3 & I_3 & TI_3 \\ 0_3 & 0_3 & I_3 \end{pmatrix} \quad \text{and} \quad w(k) = \begin{pmatrix} \frac{T^3}{6} B \\ \frac{T^2}{2} B \\ TB \end{pmatrix} \gamma(k)$$

with $\gamma(k) \in \mathbb{R}$ a zero mean white gaussian noise of assumed known covariance,

$$B = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad 0_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

So $w(k)$ is a zero mean white gaussian noise with assumed known covariance

$$E[w(k)w(j)^{\mathrm{T}}] = Q(k)\Delta(k,j) \quad \text{with } \Delta(k,j) = \begin{cases} 1 & \text{if } k = j \\ 0 & \text{if } k \neq j \end{cases}$$

- Measurement models

Observations given by the sensors are, in the absolute frame, position for the GPS and acceleration for the IMU. This latter is obtained by transforming data given by accelerometers (corrected from biases and scale

[1] In fact, if the quadratic form of the innovation exceeds the threshold, this suggests that either the physical model of the system is invalid, or the measurements are invalid. Here the latter is assumed, the physical model being always considered as correct.

[2] For a given sensor, when the quadratic form of the innovation exceeds the threshold, the choice was made to reject all the observations from this sensor. However, subset testing of the innovations could be conducted to try to remove any erroneous measurement. This could possibly allow more observations to be used, since all observations from a given sensor would not necessarily be rejected. Such a method has not been tested yet.

factors by internal algorithms of the IMU) from the body frame to the absolute reference frame, using data delivered by gyroscopes. [3] In this very simplified context, GPS and IMU measurement models are

$$z_{\mathrm{GPS}}(k) = H_{\mathrm{GPS}}(k)x(k) + b_{\mathrm{GPS}}(k), \quad H_{\mathrm{GPS}} = \begin{pmatrix} I_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & 0_3 \end{pmatrix} \tag{13}$$

$$z_{\mathrm{IMU}}(k) = H_{\mathrm{IMU}}(k)x(k) + b_{\mathrm{IMU}}(k), \quad H_{\mathrm{IMU}} = \begin{pmatrix} 0_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & I_3 \end{pmatrix} \tag{14}$$

with $b_{\mathrm{GPS}}(k)$ and $b_{\mathrm{IMU}}(k)$ zero mean white gaussian noises of respectively assumed known covariances $R_{\mathrm{GPS}}(k)$ and $R_{\mathrm{IMU}}(k)$.

4.2. Definition of the contextual space

Using Eqs. (8) and (9), validity domains of GPS and IMU sensors are determined through the definition of contextual variables. A context $q = \{q_{\mathrm{GPS}}, q_{\mathrm{IMU}}\}$, $q \in \mathbb{R}^2$, is defined by two measures q_{GPS} and q_{IMU} defined as

$$\begin{cases} q_{\mathrm{GPS}} = (v_{\mathrm{GPS}}(k))^{\mathrm{T}}(S_{\mathrm{GPS}}(k))^{-1}v_{\mathrm{GPS}}(k) \\ q_{\mathrm{IMU}} = (v_{\mathrm{IMU}}(k))^{\mathrm{T}}(S_{\mathrm{IMU}}(k))^{-1}v_{\mathrm{IMU}}(k) \end{cases} \tag{15}$$

with

$$\begin{cases} S_{\mathrm{GPS}}(k) = H_{\mathrm{GPS}}P(k \mid k-1)(H_{\mathrm{GPS}})^{\mathrm{T}} + R_{\mathrm{GPS}}(k) \\ S_{\mathrm{IMU}}(k) = H_{\mathrm{IMU}}P(k \mid k-1)(H_{\mathrm{IMU}})^{\mathrm{T}} + R_{\mathrm{IMU}}(k) \end{cases} \tag{16}$$

the covariance matrices of the GPS and IMU innovations.

Fig. 2 summarizes the different validity domains according to the contextual variables q_{GPS} and q_{IMU}.

Thresholds are defined considering the confidence level required. For example, given that $v_{\mathrm{GPS}}(k) \in \mathbb{R}^3$ and $v_{\mathrm{IMU}}(k) \in \mathbb{R}^3$, then GPSThreshold = IMUThreshold = 7.8 for a 95% confidence level.

C_{GPS} is the subset of contexts for which the GPS sensor is valid, i.e. when q_{GPS} < GPSThreshold; C_{IMU} is the subset of contexts for which the IMU sensor is valid, i.e. when q_{IMU} < IMUThreshold; c_{GPS} is the subset of contexts for which the GPS sensor is the only sensor valid, i.e. when q_{GPS} < GPSThreshold and q_{IMU} > IMUThreshold. c_0 is the subset of contexts for which there is not any valid sensor. So it comes: $c_0 = \overline{C_{\mathrm{GPS}}} \cap \overline{C_{\mathrm{IMU}}}$, $c_{\mathrm{GPS}} = C_{\mathrm{GPS}} \cap \overline{C_{\mathrm{IMU}}}$, $c_{\mathrm{IMU}} = \overline{C_{\mathrm{GPS}}} \cap C_{\mathrm{IMU}}$ and $c_{\mathrm{GPS+IMU}} = C_{\mathrm{GPS}} \cap C_{\mathrm{IMU}}$.

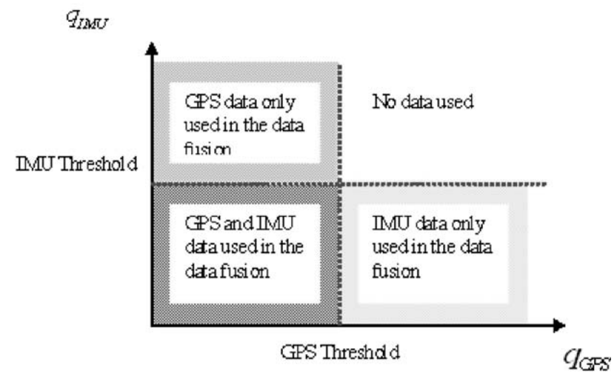

Fig. 2. Sensors validity domains.

The set $A = \{c_0, c_{\mathrm{GPS}}, c_{\mathrm{IMU}}, c_{\mathrm{GPS+IMU}}\}$ defines a partition of the contextual space.

Bounds of the validity domains are chosen according to a X% confidence level from χ^2 tables. Fuzzy logic is then introduced to bring imprecision in the definition of the contextual space. Sensors are not defined as being whether valid or invalid, but can have a certain degree of thrust between these two states. Degrees of thrust are modelling by membership functions μ_{GPS} and μ_{IMU} (Fig. 3).

Probabilities of validity of each sensor according to the contextual variables can be calculated using membership functions μ_{GPS} and μ_{IMU}:

$$\begin{cases} P(C_{\mathrm{GPS}} \mid \{q_{\mathrm{GPS}}, q_{\mathrm{IMU}}\}) = \mu_{\mathrm{GPS}}(q_{\mathrm{GPS}}) \\ P(C_{\mathrm{IMU}} \mid \{q_{\mathrm{GPS}}, q_{\mathrm{IMU}}\}) = \mu_{\mathrm{IMU}}(q_{\mathrm{IMU}}) \\ P(C_{\mathrm{GPS}} \cap C_{\mathrm{IMU}} \mid \{q_{\mathrm{GPS}}, q_{\mathrm{IMU}}\}) \\ \quad = \mu_{\mathrm{GPS}}(q_{\mathrm{GPS}})\mu_{\mathrm{IMU}}(q_{\mathrm{IMU}}) \end{cases} \tag{17}$$

Probabilities of exclusive validity β are

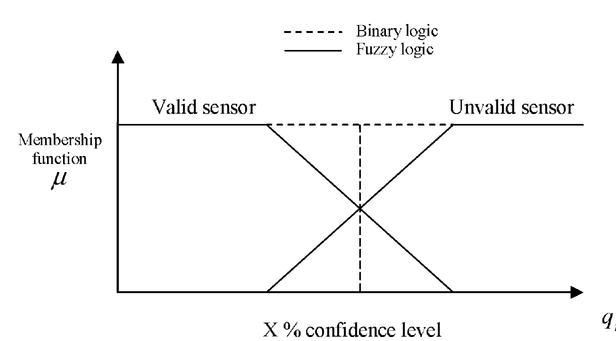

Fig. 3. Fuzzy rules.

[3] Gyroscopes data are assumed to have a negligible error. A simple linear model is chosen in this paper to illustrate the method but more sophisticated models could be implemented to model inertial sensors dynamics in a more realistic way.

$$\begin{cases} \beta_{\mathrm{GPS}} = P(c_{\mathrm{GPS}}) = P(C_{\mathrm{GPS}}) - P(C_{\mathrm{GPS}} \cap C_{\mathrm{IMU}}) \\ \beta_{\mathrm{IMU}} = P(c_{\mathrm{IMU}}) = P(C_{\mathrm{IMU}}) - P(C_{\mathrm{GPS}} \cap C_{\mathrm{IMU}}) \\ \beta_{\mathrm{GPS+IMU}} = P(c_{\mathrm{GPS+IMU}}) = P(C_{\mathrm{GPS}} \cap C_{\mathrm{IMU}}) \\ \beta_0 = P(c_0) \\ \quad = 1 - P(C_{\mathrm{GPS}}) - P(C_{\mathrm{IMU}}) + P(C_{\mathrm{GPS}} \cap C_{\mathrm{IMU}}) \end{cases} \tag{18}$$

β coefficients verify

$$\beta_0 + \beta_{\mathrm{GPS}} + \beta_{\mathrm{IMU}} + \beta_{\mathrm{GPS+IMU}} = 1 \tag{19}$$

4.3. Equations of the Kalman Filter

Estimate at time k is obtained by weighting, using the β masses defined by the contextual variables, the estimates obtained with the different sensor associations. The following equations are derived from Eqs. (3)–(5).

The estimate obtained only taking the GPS data is:

$$\hat{x}_{\mathrm{GPS}}(k \mid k) = \hat{x}(k \mid k-1) + K_{\mathrm{GPS}}(k)(z_{\mathrm{GPS}}(k) - H_{\mathrm{GPS}}\hat{x}(k \mid k-1))$$

$$K_{\mathrm{GPS}}(k) = P_{\mathrm{GPS}}(k \mid k)(H_{\mathrm{GPS}})^{\mathrm{T}}(R_{\mathrm{GPS}})^{-1}$$

$$(P_{\mathrm{GPS}}(k \mid k))^{-1} = P^{-1}(k \mid k-1) + (H_{\mathrm{GPS}})^{\mathrm{T}}(R_{\mathrm{GPS}})^{-1}H_{\mathrm{GPS}}$$

In the same way, the estimate obtained only taking the INS data is:

$$\hat{x}_{\mathrm{IMU}}(k \mid k) = \hat{x}(k \mid k-1) + K_{\mathrm{IMU}}(k)(z_{\mathrm{IMU}}(k) - H_{\mathrm{IMU}}\hat{x}(k \mid k-1))$$

$$K_{\mathrm{IMU}}(k) = P_{\mathrm{IMU}}(k \mid k)(H_{\mathrm{IMU}})^{\mathrm{T}}(R_{\mathrm{IMU}})^{-1}$$

$$(P_{\mathrm{IMU}}(k \mid k))^{-1} = P^{-1}(k \mid k-1) + (H_{\mathrm{IMU}})^{\mathrm{T}}(R_{\mathrm{IMU}})^{-1}H_{\mathrm{IMU}}$$

The estimate obtained by fusing the GPS and IMU data is:

$$\begin{aligned}\hat{x}_{\mathrm{GPS+IMU}}(k \mid k) = {} & \hat{x}(k \mid k-1) + K_{\mathrm{GPS|(GPS+IMU)}}(k)(z_{\mathrm{GPS}}(k) \\ & - H_{\mathrm{GPS}}\hat{x}(k \mid k-1)) + K_{\mathrm{IMU|(GPS+IMU)}}(k) \\ & \times (z_{\mathrm{IMU}}(k) - H_{\mathrm{IMU}}\hat{x}(k \mid k-1))\end{aligned}$$

$$K_{\mathrm{GPS|(GPS+IMU)}}(k) = P_{\mathrm{GPS+IMU}}(k \mid k)(H_{\mathrm{GPS}})^{\mathrm{T}}(R_{\mathrm{GPS}})^{-1}$$

$$K_{\mathrm{IMU|(GPS+IMU)}}(k) = P_{\mathrm{GPS+IMU}}(k \mid k)(H_{\mathrm{IMU}})^{T}(R_{\mathrm{IMU}})^{-1}$$

$$\begin{aligned}(P_{\mathrm{GPS+IMU}}(k \mid k))^{-1} = {} & P^{-1}(k \mid k-1) + (H_{\mathrm{GPS}})^{\mathrm{T}}(R_{\mathrm{GPS}})^{-1}H_{\mathrm{GPS}} \\ & + (H_{\mathrm{IMU}})^{\mathrm{T}}(R_{\mathrm{IMU}})^{-1}H_{\mathrm{IMU}}\end{aligned}$$

β weightings are then calculated from values of q_{GPS} and q_{IMU}, using membership functions μ_{GPS} and μ_{IMU} (see Fig. 4 and Eqs. (17) and (18)). Table 1 summarizes the different estimates and their associated weighting β.

By using the methodology presented in [7], the final estimate is determined by (see Appendix B):

$$\begin{aligned}\hat{x}(k \mid k) = {} & \beta_0\hat{x}(k \mid k-1) + \beta_{\mathrm{GPS}}\hat{x}_{\mathrm{GPS}}(k \mid k) \\ & + \beta_{\mathrm{IMU}}\hat{x}_{\mathrm{IMU}}(k \mid k) + \beta_{\mathrm{GPS+IMU}}\hat{x}_{\mathrm{GPS+IMU}}(k \mid k)\end{aligned} \tag{20}$$

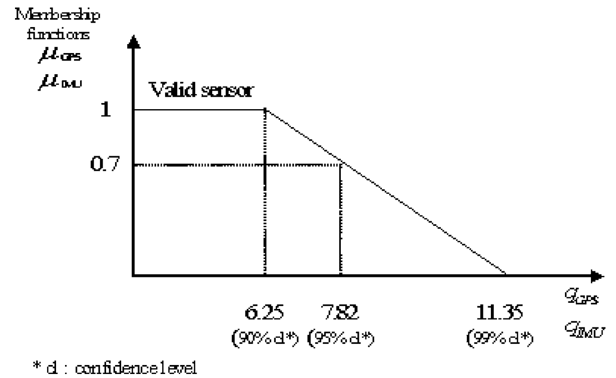

Fig. 4. Implementation of fuzzy logic to determine membership functions μ_{GPS} and μ_{IMU}.

Table 1
Estimates according to the valid sensors and their associated weightings

Sensors	Estimate	Weighting associated
None	$\hat{x}(k \mid k-1)$	β_0
GPS only	$\hat{x}_{\mathrm{GPS}}(k \mid k)$	β_{GPS}
IMU only	$\hat{x}_{\mathrm{IMU}}(k \mid k)$	β_{IMU}
GPS + IMU	$\hat{x}_{\mathrm{GPS+IMU}}(k \mid k)$	$\beta_{\mathrm{GPS+IMU}}$

The autocovariance matrix of the estimation error is given by:

$$\begin{aligned}P(k \mid k) = {} & \beta_0 P(k \mid k-1) + \beta_{\mathrm{GPS}}\Big[P_{\mathrm{GPS}}(k \mid k) + (\hat{x}(k \mid k) \\ & - \hat{x}_{\mathrm{GPS}}(k \mid k))(\hat{x}(k \mid k) - \hat{x}_{\mathrm{GPS}}(k \mid k))^{\mathrm{T}}\Big] \\ & + \beta_{\mathrm{IMU}}\Big[P_{\mathrm{IMU}}(k \mid k) + (\hat{x}(k \mid k) - \hat{x}_{\mathrm{IMU}}(k \mid k)) \\ & \times (\hat{x}(k \mid k) - \hat{x}_{\mathrm{IMU}}(k \mid k))^{\mathrm{T}}\Big] + \beta_{\mathrm{GPS+IMU}} \\ & \times \Big[P_{\mathrm{GPS+IMU}}(k \mid k) + (\hat{x}(k \mid k) - \hat{x}_{\mathrm{GPS+IMU}}(k \mid k)) \\ & \times (\hat{x}(k \mid k) - \hat{x}_{\mathrm{GPS+IMU}}(k \mid k))^{\mathrm{T}}\Big]\end{aligned} \tag{21}$$

The prediction stage is the same as for a classic KF (Eqs. (6) and (7)):

$$\hat{x}(k+1 \mid k) = F\hat{x}(k \mid k)$$

$$P(k+1 \mid k) = FP(k \mid k)F^{\mathrm{T}} + Q$$

5. Simulation

Tests have been made on a land vehicle in Nantes (France). Data have been collected from a DGPS (differential GPS) sensor and a set of inertial sensors. The DGPS is a bi-frequential RTK/OTF (real time kinematic/on the fly) sensor with centimeter precision whose data acquisition frequency is 5 Hz. Inertial sensors package is an Octans 5000, a fiber-optic gyrocompass, whose data acquisition frequency is 75 Hz. The Octans delivers roll, pitch, heading and accelerations of the vehicle in the frame of the vehicle. Data are corrected from biases and scale factors by internal algorithms.

Real position being not available, an artificial white gaussian noise is added to DGPS measurement so that to get a GPS signal with meter-level precision. So DGPS measurement is taken as the "real position" and DGPS noised data (afterwards called GPS data) are taken as the GPS observation delivered by the sensor. Improvements of positioning precision are studied between the GPS data and the GPS/IMU filtered data, the reference being the DGPS data. Figs. 5 and 6 show the histogram of the position error for the GPS only data and the GPS/IMU filtered data.

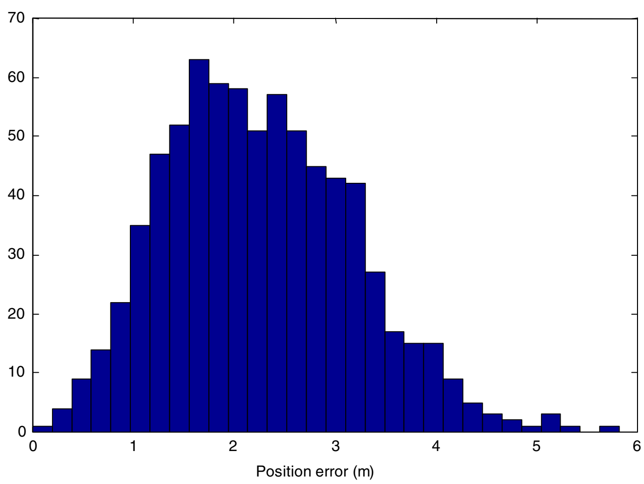

Fig. 5. Histogram of the position error of the non-filtered GPS signal.

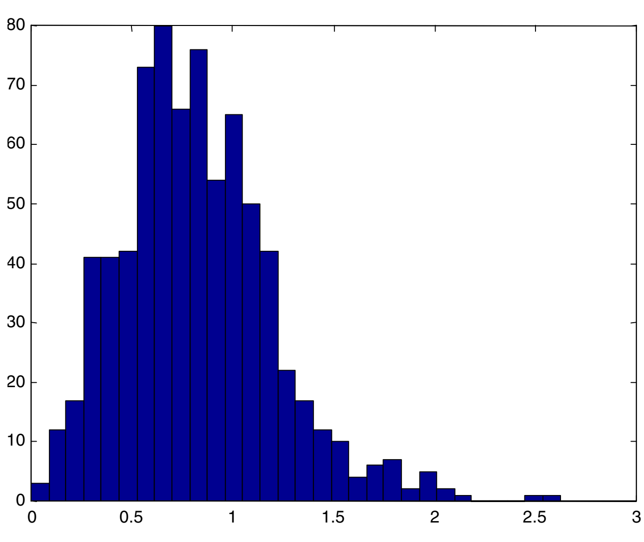

Fig. 6. Histogram of the position error of the GPS/IMU filtered signal.

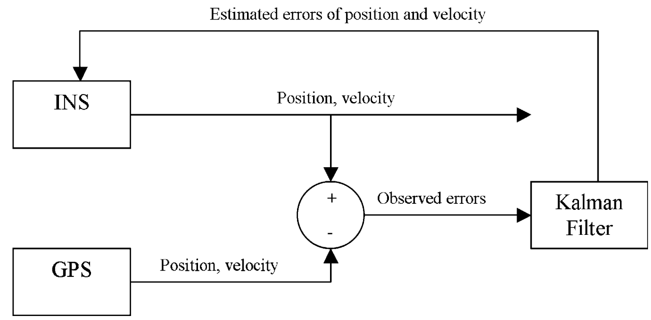

Fig. 7. Direct feedback method filter.

Table 2
Performances of the direct feedback filter

	GPS data	GPS/INS fusion	Improvement
p.e.a.[a] (m)	2.25	0.81	64.2%
p.e.s.b.[b] (m)	0.93	0.32	66.1%
95% c.l.[c] (m)	3.89	1.37	64.8%

[a] Position error average.
[b] Position error sigma bound.
[c] Confidence level.

Table 3
Performances of the multisensor filter

	GPS data	GPS/IMU fusion	Improvement
p.e.a. (m)	2.23	0.82	63.2%
p.e.s.b. (m)	0.96	0.38	60.3%
95% c.l. (m)	3.92	1.49	62.0%

In order to characterize the performances of the filter, it is compared to a structure specifically developed for the GPS/INS data fusion: the direct feedback method filter [1], illustrated in Fig. 7. In this filter, the state vector of the KF is the error of position and the velocity of the INS.

Tables 2 and 3 show that when there is no loss of GPS signal, i.e. in an ideal context, performances of the multisensor filter are slightly inferior to those of the direct feedback filter. Next simulations will show the behavior of the algorithm developed in this paper when sensor failures occur.

- Effects of multipath on the filter

One of the main interests of the filter developed in this paper lies in its ability to take into account contextual aspects and so to detect erroneous data. An environment favorable to multipath is simulated by introducing a bias on the GPS position data, as illustrated in Fig. 8. If there is no control of the health of the data, position estimate would follow biased position of the GPS signal resulting in a large error on the position estimate. With this algorithm, failure of the GPS

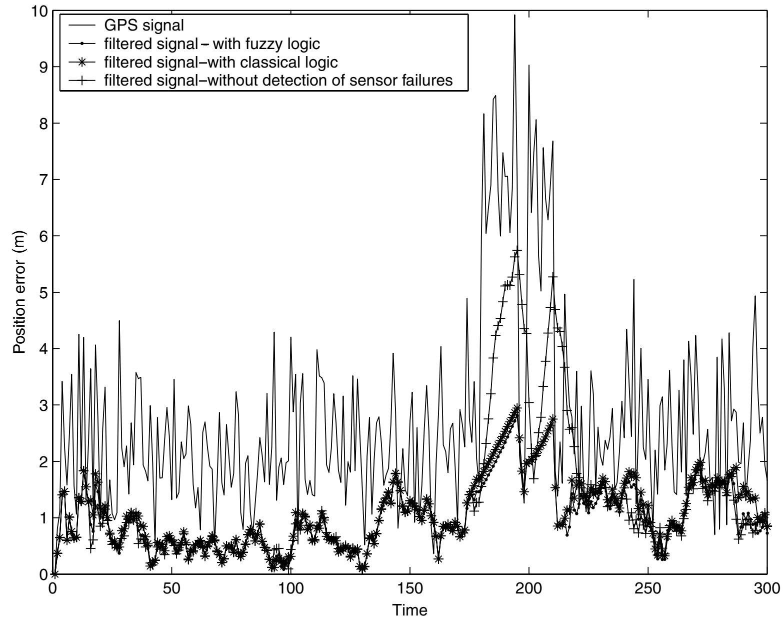

Fig. 8. Positioning error of the GPS data and the GPS/IMU filtered data.

data is detected thanks to q_{GPS} (Fig. 9) whose value is far beyond the validity threshold. GPS data are assumed bad and IMU only is used in the fusion process. As it can be seen in Fig. 11, the two sigma bound of the estimate is quickly drifting with time (from 2700 to 3100) because of the use of the dead-reckoning sensor only. Failures of the IMU sensor are also detected by the filter using q_{IMU} statistics, as illustrated in Fig. 10. Spikes are due to erroneous measurements of the IMU from time to time. A comparison is made on Fig. 8 between the positioning errors of three types of sensor failure detection: use of fuzzy logic (method developed in this paper),

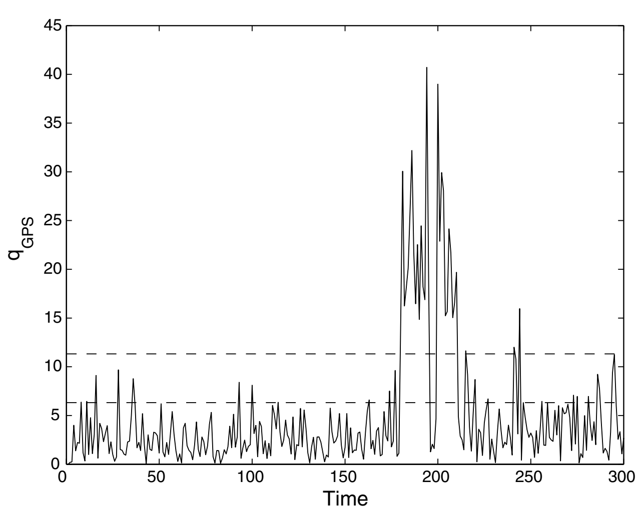

Fig. 9. q_{GPS} function of time.

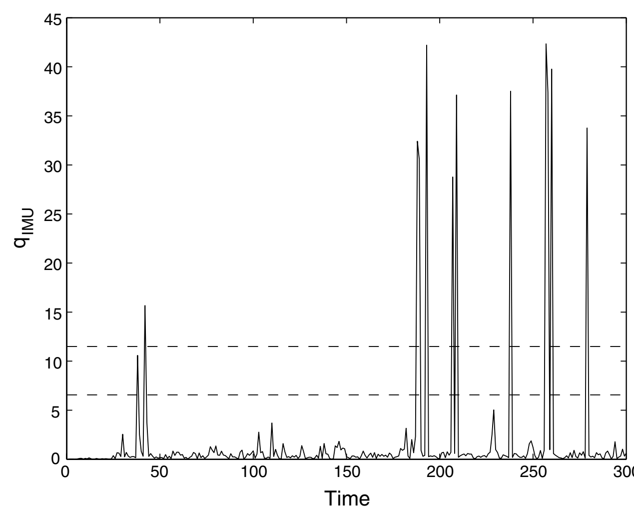

Fig. 10. q_{IMU} function of time.

use of classical logic by a 95% threshold on the normalized innovation (method developed in [2]) and non-detection of sensor failures. As shown in Fig. 8, classical and fuzzy logic give much more better results than non-detection when multipath occur. Figs. 12 and 13 give more precisely the improvement of fuzzy logic on classical one and non-detection in function of time. Simulations show an improvement of about 5% in that case between fuzzy and classical logic and 20% between fuzzy logic and non-detection. These first results have to be confirmed by tests on real experiments.

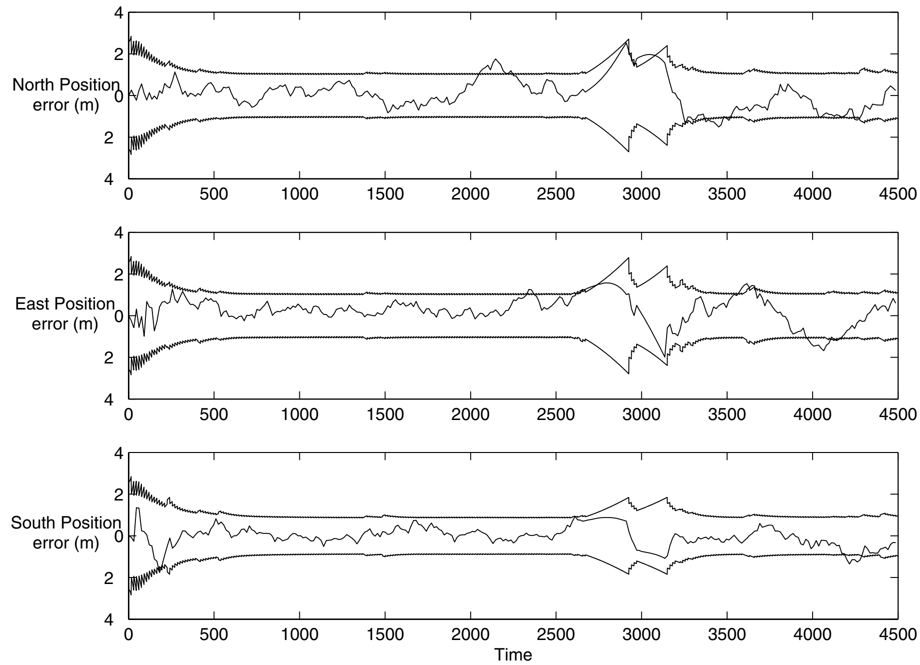

Fig. 11. North, East and South position error and 2-sigma bounds.

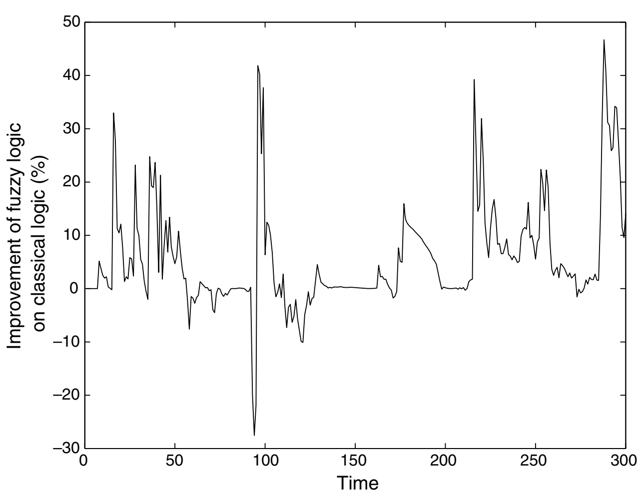

Fig. 12. Improvement of fuzzy logic on classical logic.

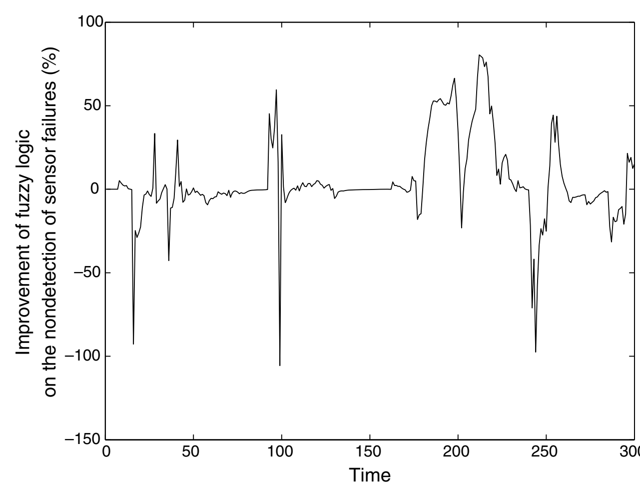

Fig. 13. Improvement of fuzzy logic on non-detection of sensor failures.

6. Conclusion and further research

This paper presents a multisensor KF taking context into consideration, based on fuzzy subsets. Simulations show that the multisensor filter has quite the same performances as the direct feedback structure developed in [1]. However, the multisensor filter is able to integrate a high number of sensors without changing its structure and the algorithm. In a first approximation, simplified linear state and observation models were proposed to illustrate the use of contextual information. Works are in progress to model vehicle and inertial sensors dynamics in a more realistic way.

Moreover, by introducing contextual information, reliability of the data fusion process is improved: bad data coming from sensors can be detected and rejected, especially GPS ones (due to multipath). First simulations show that fuzzy logic gives slightly better results than classical logic, but these results have to be confirmed with real experiments.

Moreover, it could be interesting to look further on the different contextual information to determine the nominal state of work of the GPS sensor, in order to define more precisely fuzzy validity bounds of the GPS thus increasing reliability of the fusion process.

For instance, the following data should be taken as contextual variables:

- Data given by the GPS sensor, like the GDOP (geometric dilution of precision) or the number of satellites in line of sight.
- Variables coming from external sensors, like a radar detecting objects in the vicinity of the sensor (creating multipath) or a map matching algorithm indicating if the vehicle is in an environment more or less hostile to GPS signals.

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Appendix A. Multisensor information filter

The information form of the Kalman filter is introduced because of the simple additive nature of the update stage (A.1). It makes it very attractive for multisensor estimation [13], [15]. The information state y and information matrix Y are defined as

$$\hat{y}(k \mid k) = Y(k \mid k)\hat{x}(k \mid k)$$
$$Y(k \mid k) = P(k \mid k)^{-1}$$

Equations of the prediction stage are given by

$$\hat{y}(k \mid k-1) = Y(k-1 \mid k-1)$$
$$FY(k-1 \mid k-1)^{-1}\hat{y}(k-1 \mid k-1)$$
$$Y(k \mid k-1) = (FY(k \mid k-1)^{-1}F^{\mathrm{T}} + Q(k))^{-1}$$

Each observation $z_i(k)$ (2) contributes $i_i(k)$ to the information state $\hat{y}$ and $I_i(k)$ to the information matrix Y.

$$i_i(k) = H_i^{\mathrm{T}} R_i(k)^{-1} z_i(k)$$
$$I_i(k) = H_i^{\mathrm{T}} R_i(k)^{-1} H_i$$

Under information form, the update stage of discrete-time Kalman filter reduces to

$$\begin{aligned} \hat{y}(k \mid k) &= \hat{y}(k \mid k-1) + \sum_{i=1}^{N} i_i(k) \\ Y(k \mid k) &= Y(k \mid k-1) + \sum_{i=1}^{N} I_i(k) \end{aligned} \tag{A.1}$$

So the update state estimate is obtained by

$$\begin{aligned} \hat{x}(k \mid k) &= P(k \mid k)\hat{y}(k \mid k) \\ &= P(k \mid k)\left(\hat{y}(k \mid k-1) + \sum_{i=1}^{N} i_i(k)\right) \end{aligned}$$

$$\hat{x}(k \mid k) = P(k \mid k)P(k \mid k-1)^{-1}\hat{x}(k \mid k-1) + \sum_{i=1}^{N} P(k \mid k)H_i^{\mathrm{T}} R_i(k)^{-1} z_i(k)$$

$$\begin{aligned} \hat{x}(k \mid k) &= \left(P(k \mid k)P(k \mid k-1)^{-1} + \sum_{i=1}^{N} P(k \mid k)H_i^{\mathrm{T}} R_i(k)^{-1} H_i\right) \\ &\quad \times \hat{x}(k \mid k-1) \\ &\quad + \sum_{i=1}^{N} P(k \mid k)H_i^{\mathrm{T}} R_i(k)^{-1}(z_i(k) - H_i\hat{x}(k \mid k-1)) \end{aligned}$$

$$\hat{x}(k \mid k) = P(k \mid k)Y(k \mid k)\hat{x}(k \mid k-1) + \sum_{i=1}^{N} P(k \mid k)H_i^{\mathrm{T}} R_i(k)^{-1} v_i(k)$$

$$\hat{x}(k \mid k) = \hat{x}(k \mid k-1) + \sum_{i=1}^{N} K_i(k)v_i(k)$$

The covariance matrix is then obtained by

$$P(k \mid k) = \left(P(k \mid k-1)^{-1} + \sum_{i=1}^{N} H_i^{\mathrm{T}} R_i(k)^{-1} H_i\right)^{-1}$$

Appendix B. Origin of the β_J coefficients

Using classical Bayesian estimation theory, the state estimate of the Kalman filter is obtained by:

$$\hat{x}(k \mid k) = E[x(k) \mid z(k)] = \int x(k)p(x(k) \mid z(k))\,\mathrm{d}x$$

Let us define the set $A = \{c_J, J \in 2^{\{1,\ldots,N\}}\}$ as a partition of the contextual space ((11), (12)). If we now apply the theorem of total probabilities to $p(x|z)$, it comes:

$$p(x|z) = \sum_{J \in 2^{\{1,\ldots,N\}}} p(x, c_J|z)$$
$$p(x|z) = \sum_{J \in 2^{\{1,\ldots,N\}}} p(c_J|z)p(x|z, c_J)$$

We get $\beta_J = p(c_J|z)$: indeed, β_J coefficients being calculated from values of $v_i(k) = z_i(k) - H\hat{x}(k-1 \mid k)$ (see Eqs. (15), (17) and (18)), they are conditioned by the value of the measure z.

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