THE1 - CENG223 - Fall 2021

Student Information

Full Name : Mohammad Mahdi Khosravi
Id Number : 2489235

Q. 1

$$
\begin{aligned}
&\neg(p \wedge q) \leftrightarrow (\neg q \rightarrow p)\\
&\equiv (\neg p \vee \neg q) \leftrightarrow (\neg q \rightarrow p) \quad De\ Morgan's\ law\\
&\equiv (\neg p \vee \neg q) \leftrightarrow (\neg(\neg q) \vee p)) \quad conditional\ statement\ logical\ equivalence(table\ 7\ row\ 1)\\
&\equiv (\neg p \vee \neg q) \leftrightarrow (q \vee p) \qquad double\ negation\ law\\
&\equiv [(\neg p \vee \neg q) \rightarrow (q \vee p)] \wedge [(q \vee p) \rightarrow (\neg p \vee \neg q)] \quad biconditional\ statement\ logical\ equivalence(table\ 8\ row\ 1)\\
&\equiv [\neg(\neg p \vee \neg q) \vee (q \vee p)] \wedge [\neg(q \vee p) \vee (\neg p \vee \neg q)] \quad conditional\ statement\ logical\ equivalence(table\ 7\ row\ 1)\\
&\equiv [(p \wedge q) \vee (q \vee p)] \wedge [(\neg q \wedge \neg p) \vee (\neg p \vee \neg q)] \quad De\ Morgan's\ law\\
&\equiv [(p \vee q \vee p) \wedge (q \vee q \vee p)] \wedge [(\neg q \vee \neg p \vee \neg q) \wedge (\neg p \vee \neg p \vee \neg q)] \quad distributive\ laws\\
&\equiv [(p \vee q) \wedge (p \vee q)] \wedge [(\neg p \vee \neg q) \wedge (\neg p \vee \neg q)] \quad idempotent\ law\\
&\equiv (p \wedge q) \wedge (\neg p \vee \neg q) \quad idempotent\ law
\end{aligned}
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Q. 2

a.

$\forall_x \forall_a \forall_y (I(x, y) \wedge I(a, y) \wedge x \neq a \rightarrow \exists_z \exists_b (E(x, z) \wedge E(a, b) \wedge b \neq z))$
where domain of x and a is all people, domain of y is all faculties and domain of b,z is all employee IDs

b.

$\forall_y \exists_x (I(x, y) \wedge S(x, x))$
where domain of y is all faculties and domain of x is all people

c.

$\forall_x \big(J(x, medicine) \rightarrow \forall_a \forall_b \forall_c (A(a, x) \wedge I(a, medicine) \wedge A(b, x) \wedge I(b, medicine) \wedge A(c, x) \wedge I(c, medicine)) \rightarrow (a = b \vee a = c \vee b = c)\big)$
where domain of x is all job positions, domain of a,b,c is all people

Q. 3

a.

$$p \lor \neg q, p \lor r \vdash (r \to q) \to p$$

1. $p \lor \neg q$	premise
2. $p \lor r$	premise
3. $r \to q$	assumption
4. $p \lor r$	copy 2
5. p	assumption
6. r	assumption
7. q	$\to e$ 3
8. $p \lor \neg q$	copy 1
9. p	disjunctive syllogism* 7,8
10. p	$\lor e$ 4,5,6-9
11. $(r \to q) \to p$	

* proved in last page

b.

$$\vdash ((q \to p) \to q) \to q$$

1. $(q \to p) \to q$	assumption
2. $\neg q$	assumption
3. q	assumption
4. $\neg p$	assumption
5. $\neg q$	copy 2
6. q	copy 3
7. $\bot$	$\neg e$ 5,6
8. $\neg\neg p$	$\neg i$ 4-7
9. p	$\neg\neg e$ 8
10. $q \to p$	$\to i$ 3,9
11. q	$\to e$ 1,10
12. $\bot$	$\neg e$ 2,11
13. $\neg\neg q$	$\neg i$ 2-12
14. q	$\neg\neg e$ 13
15. $((q \to p) \to q) \to q$	$\to i$ 1-14

Q. 4

a.

$$\neg\forall x(P(x)) \to Q(x)) \vdash \exists x(P(x) \wedge \neg Q(x))$$

1.	$\neg\forall x(P(x)) \to Q(x))$	premise
2.	$\neg\exists x(P(x) \wedge \neg Q(x))$	assumption
3.	x_0	
4.	$P(x_0) \wedge \neg Q(x_0)$	assumption
5.	$\exists x(P(x) \wedge \neg Q(x))$	$\exists i$ 3
6.	$\perp$	$\neg e$ 5,2
7.	$\neg(P(x_0) \wedge \neg Q(x_0))$	$\neg i$ 4-6
8.	$\neg P(x_0) \vee \neg\neg Q(x_0)$	De Morgan's law* 7
9.	$P(x_0)$	assumption
10.	$\neg P(x_0)$	assumption
11.	$\perp$	$\neg e$ 9,10
12.	$\neg\neg P(x_0)$	$\neg i$ 10-11
13.	$P(x_0)$	$\neg\neg e$ 12
14.	$\neg\neg Q(x_0)$	disjunctive syllogism* 8,13
15.	$Q(x_0)$	$\neg\neg e$ 14
16.	$P(x_0) \to Q(x_0)$	$\to i$ 9-15
17.	$\forall_x(P(x) \to Q(x))$	$\forall_x i$ 16
18.	$\perp$	$\neg e$ 1,17
19.	$\neg\neg\exists x(P(x) \wedge \neg Q(x))$	$\neg i$ 2-19
20.	$\exists x(P(x) \wedge \neg Q(x))$	$\neg\neg e$ 21

* proved in last page

b.

$$\forall_x \forall_y (P(x,y) \to \neg P(y,x)), \forall_x \exists_y P(x,y) \vdash \neg \exists_v \forall_z P(z,v)$$

1. $\forall_x \forall_y (P(x,y) \to \neg P(y,x))$		premise
2. $\forall_x \exists_y P(x,y)$		premise
3. a		
4. $\forall_y (P(a,y) \to \neg P(y,a))$		$\forall_x e$ 1
5. $P(a,a) \to \neg P(a,a)$		$\forall_y e$ 5
6. $\exists_y P(a,y)$		$\forall_x e$ 2
7. $P(a,a)$	assumption	
8. $\neg P(a,a)$	$\to e$ 5,7	
9. $\perp$	$\neg e$ 7,8	
10. $\neg P(a,a)$		$\neg i$ 7-9
11. $\exists_z \neg P(z,a)$		$\exists i$ 10
12. $\forall_z P(z,a)$	assumption	
13. b		
14. $\neg P(b,a)$	assumption	
15. $P(b,a)$	$\forall_z$ 12	
16. $\perp$	$\neg e$ 15,14	
17. $\perp$	$\exists e$ 11,13-16	
18. $\neg \forall_z P(z,a)$		$\neg i$ 12-17
19. $\forall_v \neg \forall_z P(z,v)$		$\forall i$ 3-18
20. $\exists_v \forall_z P(z,v)$	assumption	
21. c		
22. $\forall_z P(z,c)$	assumption	
23. $\neg \forall_z P(z,c)$	$\forall e$ 19	
24. $\perp$	$\neg e$ 22,23	
25. $\perp$	$\exists e$ 20, 21-24	
26. $\neg \exists_v \forall_z P(z,v)$		$\neg i$ 20-25

Proofs for used derived rules

a. Disjunctive Syllogism

$$P \vee Q, \neg P \vdash Q$$

1.	$P \vee Q$	premise
2.	$\neg P$	premise
3.	$\neg Q$	assumption
4.	P	assumption
5.	$\bot$	$\neg e$ 2,4
6.	Q	$\bot e$ 5
7.	Q	assumption
8.	Q	$\vee e$ 1,4-6, 7
9.	$\bot$	$\neg e$ 3, 8
10.	$\neg\neg Q$	$\neg i$ 3-9
11.	Q	$\neg\neg e$ 10

b. De Morgan's law

$$\neg(P \wedge Q) \vdash \neg P \vee \neg Q$$

1.	$\neg(P \wedge Q)$	premise
2.	$\neg(\neg P \vee \neg Q)$	assumption
3.	$\neg P$	assumption
4.	$\neg P \vee \neg Q$	$\vee i$ 3
5.	$\bot$	$\neg e 4, 2$
6.	$\neg\neg P$	$\neg i$ 3-5
7.	$\neg Q$	assumption
8.	$\neg P \vee \neg Q$	$\vee i$ 7
9.	$\bot$	$\neg e$ 8,2
10.	$\neg\neg Q$	$\neg i$ 7-9
11.	Q	$\neg\neg e$ 10
12.	$\neg\neg P$	copy 6
13.	P	$\neg\neg e$ 12
14.	$P \wedge Q$	$\wedge i$ 11,13
15.	$\bot$	$\neg e$ 1,14
16.	$\neg\neg(\neg P \vee \neg Q)$	$\neg i$ 2-15
17.	$\neg P \vee \neg Q$	$\neg\neg e$ 16