Middle East Technical University Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall 2021-2022

Take Home Exam 2

Due date: Dec 5 2021, Sunday, 23:55

Question 1 (25 pts)

Given the sets A and B, prove that

$$(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$$

using set membership notation and logical equivalences. Show each step clearly.

Question 2 (25 pts)

Prove that the set

$$\{f \mid f : \mathbb{N} \to \{0,1\}, f \text{ is a function}\} \setminus \{f \mid f : \{0,1\} \to \mathbb{N}, f \text{ is a function}\}$$

is uncountable.

Question 3 (25 pts)

Prove that the function $f(n) = 4^n + 5n^2 \log n$ is **not** $O(2^n)$.

Question 4 (25 pts)

Given two positive integers x and n such that $x > 2$ and $n > 2$, and the congruence relation

$$(2x-1)^n - x^2 \equiv -x - 1 \pmod{(x-1)}$$

determine the value of x.

Question 5 (self-study, ungraded)

Given the function f such that $f : \mathbb{R} \to [0,1)$ with

$$f(x) = \lceil x \rceil - x$$

determine whether f is one-to-one and onto. Prove your answer.

Question 6 (self-study, ungraded)

Given any natural number $n \geq 2$, and a set $P = \{x_i \,|\, x_i = 100 + i,\ 0 \leq i < n,\ i \in \mathbb{N}\}$, prove that exactly one member of the set P is divisible by n.

Regulations

1. Your submission should be a single vector-based PDF document with the name "the2.pdf". Do not submit solutions for ungraded questions.
2. **Late Submission:** Not allowed.
3. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.
4. **Updates & Announces:** You must follow the odtuclass for discussions and possible updates. You can ask your questions freely in the Student Forum on the course page in odtuclass.
5. **Evaluation:**Your .pdf file will be checked for plagiarism automatically using "black-box" technique and manually by assistants.

Submission

Submission will be done via odtuclass. For those who prefer to use LaTeXto generate the vector-based pdf file, a template answer file "the2.tex" will be provided in odtuclass. You need to compile the filled template yourselves and submit the generated .pdf file only.