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Answer 1

$a_n = a_{n-1} + 2^n,\ n \geq 1$

$a_0 = 1$

$\sum_{i=1}^{\infty} a_n x^n = \sum_{i=1}^{\infty} (a_{n-1} + 2^n) x^n$

$\sum_{i=1}^{\infty} a_n x^n = A(x) - a_0 x^0 = A(x) - 1$

$A(x) - 1 = \sum_{i=1}^{\infty} a_{n-1} x^n + \sum_{i=1}^{\infty} 2^n x^n \Rightarrow A(x) - 1 = x. \sum_{i=1}^{\infty} a_{n-1} x^{n-1} + 2x. \sum_{i=1}^{\infty} 2^{n-1} x^{n-1}$

$\Rightarrow A(x) - 1 = x.A(x) + \frac{2x}{1-2x} \Rightarrow A(x) = \frac{1}{1-x} + \frac{2x}{(1-x)(1-2x)}*$

$A(x) = \frac{1}{1-x} + \frac{-2}{1-x} + \frac{2}{1-2x}$

$(1, 1, 1, ..., a_n = 1, ...) \iff \frac{1}{1-x}$ (1)

$(-2, -2, -2, ..., a_n = -2, ...) \iff \frac{-2}{1-x}$ (2)

$(1, 2, 2^2, ..., a_n = 2^n, ...) \iff \frac{1}{1-2x}$

$\Rightarrow (2, 2^2, 2^3, ..., a_n = 2^{n+1}, ...) \iff \frac{2}{1-2x}$ (3)

From (1),(2),(3) $\Rightarrow a_n = 1 - 2 + 2^{n+1}$

$a_n = 2^{n+1} - 1$

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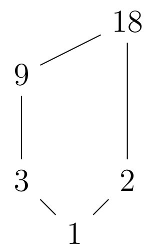
*Finding the partial functions:

$\frac{2x}{(1-x)(1-2x)} = \frac{A}{1-x} + \frac{B}{1-2x} = \frac{A-2Ax+B-Bx}{(1-x)(1-2x)} = \frac{(-2A-B)x+(A+B)}{(1-x)(1-2x)} \Rightarrow \begin{cases} \text{A+B=0} \\ \text{-2A-B=2} \end{cases}$

$\Rightarrow$A = -2, B = 2

Answer 2

a)Hasse diagram:



b)Matrix representation of R:

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c) Since every pair in the set poset (A, R) has a least upper bound and a greatest lower bound, (A, R) is a lattice. This can be also verified by observing the Hasse diagram of R.

d) The symmetric closure is:
$R_s = R \ \cup \Delta = \{(a, b)|\text{a divides b}\} \cup \{(b, a)|\text{a divides b} \wedge a \neq b\}$.

$$M_{R_s} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

For a relation to be symmetric, the matrix representation of that relation must be symmetric, which is true for M_{R_s}.

e) Based on Definition 2 Chapter 9.6:
(i) 2 and 9 are incomparable since neither 2 divides 9 nor 9 divides 2.
(ii) 3 and 18 are comparable because 3 divides 18.

Answer 3

a) If we consider the matrix representation of a binary relation on set A with n elements, we can find a way to count the number of anti-symmetric relations:

$$M_R = \begin{bmatrix} a_{11} & a_{12} & a_{13} & ... \\ a_{21} & a_{22} & a_{23} & ... \\ a_{31} & a_{32} & a_{33} & ... \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

For a anti-symmetric relation, the matrix representation should be in a way such that for all entries $a_{ij} \neq a_{ji}$ if one of them is 1 or they must both be 0 (3 possibilities).

In addition since we don't care if the relation is reflexive or not, there are n entries on the diagonal which represents whether the relation is reflexive or not.

Thus:

(i) 2^n is the number of possibilities on the diagonal.

(ii) Total entries are n^2 and we want all paired entries except n elements on the diagonal so there is $n^2 - \frac{n(n-1)}{2}$ entries for the anti-symmetric condition so the number of possibilities are $3^{n^2 - \frac{n(n-1)}{2}}$.

$\therefore$ By product rule, the number of anti-symmetric relations on A with n elements is:

$$\boxed{2^n.3^{n^2 - \frac{n(n-1)}{2}}}$$

b) Similar to (a) we use the matrix representation to find the number of possible relations except this time we must care for the diagonal of the matrix since it is representing the reflexive condition of the relation. We know for the relation to be reflexive there is only one possibility and that is all the entries on the diagonal must be 1.

For it to be anti-symmetric from (a) we know there are $3^{n^2 - \frac{n(n-1)}{2}}$ possibilities.

$\therefore$ The number of reflexive and anti-symmetric relations on a set A with n elements is:

$$\boxed{3^{n^2 - \frac{n(n-1)}{2}}}$$