Middle East Technical University | Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall '2021-2022

Take Home Exam 5

Due date: January 21 2022, Friday , 23:55

Question 1

Use either Kruskal's or Prim's algorithm to find a minimum spanning tree for the graph G given below (Figure 1). Please state the algorithm you choose at the beginning of your solution.

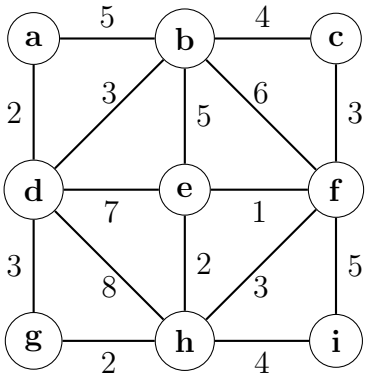


Figure 1: Graph G in Q1.

a) Write the order in which the edges are added to the tree.

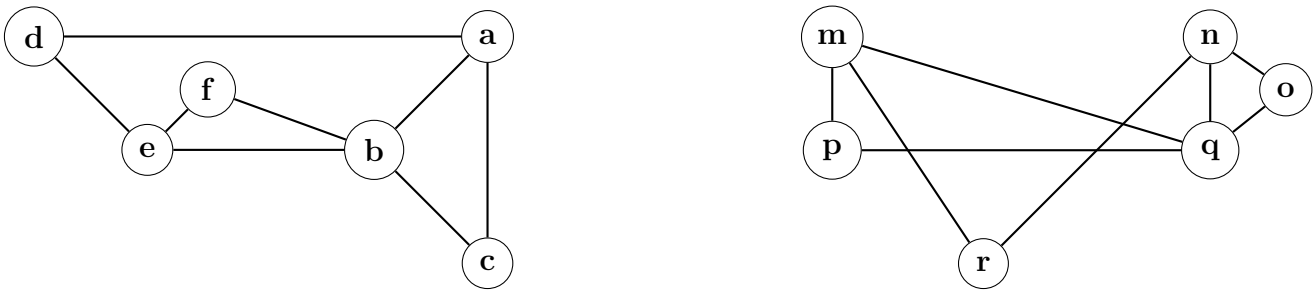
b) Draw the minimum spanning tree.

c) Is the minimum spanning tree unique for the graph G in Figure 1? In general, is the minimum spanning tree unique for any connected edge-weighted undirected graph? Justify your answer.

d) Prove that if the minimum-weight edge of a graph is unique, then this edge is included in any minimum spanning tree for that graph.

Question 2

Given the graphs G and H in Figure 2.



(a) G (b) H

Figure 2: Graph G and H in Q2.

Determine whether G and H are isomorphic, or not. Explain your answer.

Question 3

Answer the following questions using the binary tree T in Figure 3. Vertices of T are marked with <**identifier:key**> annotations. Note that T has the vertex p as its root. Use the notational conventions in your textbook to decide whether a vertex is left or right child of some vertex whenever applicable.

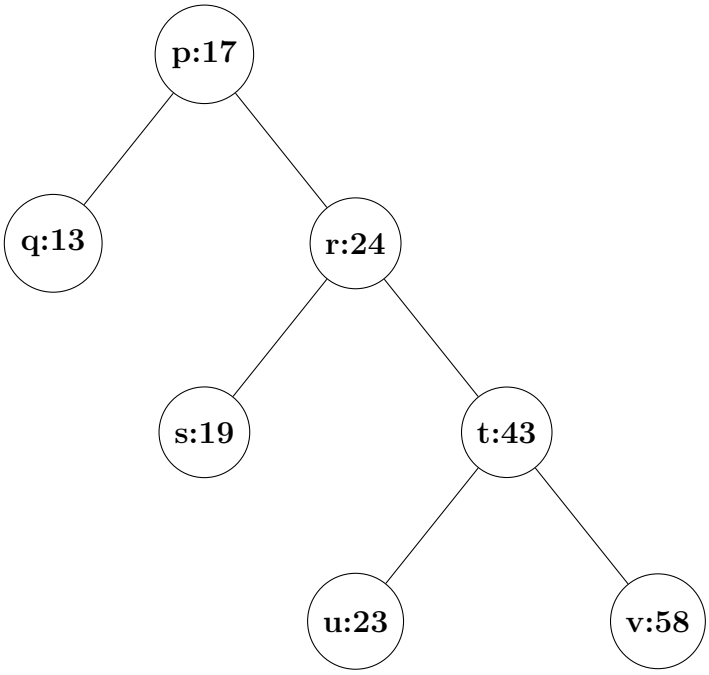


Figure 3: Tree T in Q3.

a) What are the number of vertices, the number of edges and the height of T?

b) Carry out a postorder, an inorder and a preorder traversals of T and write down the order in which vertices are visited for each case.

c) Is T a full binary tree? Justify your answer.

d) Is T a complete binary tree? Justify your answer.

e) Is T a balanced binary tree? Justify your answer.

f) Is T a binary search tree? Justify your answer.

g) What is the minimum number of nodes for a full binary tree with height 5? Justify your answer.

Question 4(self-study, ungraded)

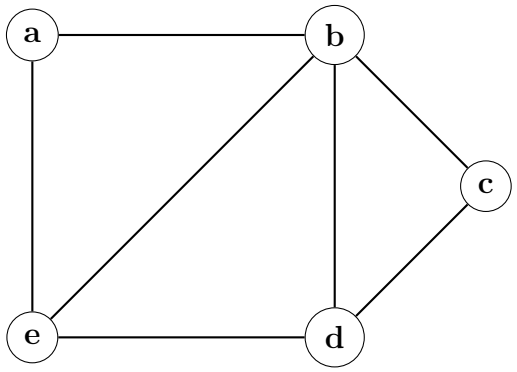


Figure 4: Graph G in Q4.

Consider the graph G in Figure 4 to answer the following questions. Explain all the answers.

a) Does G have a complete graph with at least three vertices as a subgraph? If yes, draw this subgraph.

b) Is G a bipartite graph? If yes, explain briefly; if no, remove the edges such that the resulting subgraph of G will be a bipartite graph.

c) What is the number of connected components of G? Explain your answer.

d) Is there an Euler circuit in G? If yes, give such a circuit; if no, state the reason.

e) Is there an Euler path in G? If yes, give such a path; if no, state the reason.

f) Does G have a Hamilton circuit? If yes, find such a circuit; if no, justify your answer.

g) Does G have a Hamilton path? If yes, find such a path; if no, justify your answer.

Question 5(self-study, ungraded)

Find the shortest path from vertex a to vertex j in the following weighted graph G (see Figure 5) using Dijkstra's algorithm. Describe the steps clearly.

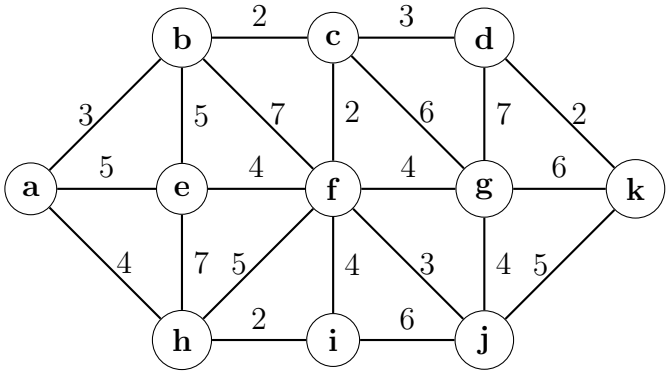


Figure 5: Graph G in Q5.

1 Regulations

1. Your submission should be a single vector-based PDF document with the name "the5.pdf". Do not submit solutions for ungraded questions.
2. **Late Submission:** Not allowed!
3. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.
4. **Newsgroup:** You must follow the newsgroup (odtuclass.metu.edu.tr) for discussions and possible updates on a daily basis.
5. **Evaluation:** Your pdf file will be checked for plagiarism automatically using "black-box" technique and manually by assistants.

2 Submission

Submission will be done via odtuclass. For those who prefer to use LaTeX to generate the vector-based pdf file, a template answer file "the5.tex" will be provided in odtuclass. You need to compile the filled template yourselves and submit the generated pdf file only.