Middle East Technical University | Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall '2021-2022

Take Home Exam 4

Due date: January 9 2022, Sunday , 23:55

Question 1

Use generating functions(no partial credit to any solution not involving generating functions) to solve the following recurrence relation:

$$a_n = a_{n-1} + 2^n, \quad n \geq 1$$

with the initial condition $a_0 = 1$.

Question 2

Given the relation $R = \{(a, b) | a \text{ divides } b\}$ on $A = \{1, 2, 3, 9, 18\}$, solve the following questions.

a) Draw the Hasse diagram of R.

b) Give the matrix representation for R.

c) Is (A, R) a lattice? Explain your answer.

d) Give the matrix representation for R_s, where R_s is the symmetric closure of R. Explain your answer.

e) In (A, R), are the integers 2 and 9 are comparable? Are 3 and 18 comparable? Explain your answer.

Question 3

Let A be a set with n elements

a) How many different binary relations on A are anti-symmetric? Explain your answer.

b) How many different binary relations on A are both reflexive and anti-symmetric? Explain your answer.

Question 4(self-study, ungraded)

Find the generating function (in closed form) for the sequence $< 1, 4, 7, 10, 13, \cdots >$. Show all the steps clearly.

1 Regulations

1. Your submission should be a single vector-based PDF document with the name "the4.pdf". Do not submit solutions for ungraded questions.
2. **Late Submission:** Not allowed!
3. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.
4. **Newsgroup:** You must follow the newsgroup (odtuclass.metu.edu.tr) for discussions and possible updates on a daily basis.
5. **Evaluation:** Your pdf file will be checked for plagiarism automatically using "black-box" technique and manually by assistants.

2 Submission

Submission will be done via odtuclass. For those who prefer to use LaTeX to generate the vector-based pdf file, a template answer file "the4.tex" will be provided in odtuclass. You need to compile the filled template yourselves and submit the generated pdf file only.