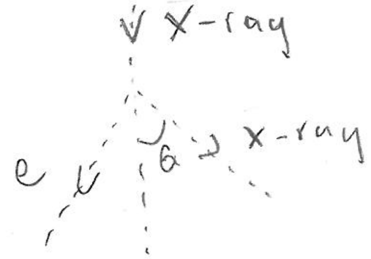
Exercise 1.   Solutions

1.



- Energy and momentum are conserved.
- We assume that $\bar{p}_{e,i}=0$, i.e., electron is not moving initially.

$$\Rightarrow \begin{cases} E_{x,inc}+E_{e,i}=E_{x,scat}+E_{e,f} & (1) \\ \bar{p}_{x,inc}=\bar{p}_{x,scat}+\bar{p}_{e,f} & (2) \end{cases}$$

Special relativity gives:

$$E^2=p^2c^2+m^2c^4 \quad (3)$$

$$\Rightarrow \begin{cases} E_{x,scat}^2=p_{x,scat}^2c^2 \\ E_{x,inc}^2=p_{x,inc}^2c^2 \\ E_{e,i}^2=m^2c^4 \\ E_{e,f}^2=p_{e,f}^2c^2+m_e^2c^4 \end{cases}$$

From (2) we get:

$$\bar{p}_{e,f}\cdot\bar{p}_{e,f}=p_{e,f}^2=(\bar{p}_{x,inc}-\bar{p}_{x,scat})(\bar{p}_{x,inc}-\bar{p}_{x,scat})$$

$$=p_{x,inc}^2+p_{x,scat}^2-2\bar{p}_{x,scat}\cdot\bar{p}_{x,inc}$$

$$=p_{x,inc}^2+p_{x,scat}^2-2\cos\theta\, p_{x,scat}\,p_{x,inc} \quad (4)$$

(1) + (3)

$$p_{x,inc}c + m_e c^2 - p_{x,scat}c = \sqrt{p_e^2 c^2 + m_e^2 c^4} \quad // (\ )^2$$

$$\Rightarrow p_{x,inc}^2 c^2 + m_e^2 c^4 + p_{x,scat}^2 c^2 + 2 p_{x,inc} c (m_e c^2 - p_{x,scat} c) + \dots$$

$$- 2 m_e c^2 p_{x,scat} c = p_e^2 c^2 + m_e^2 c^4 \overset{(4)}{=} p_{x,inc}^2 c^2 + p_{x,scat}^2 c^2 - 2\cos\theta\, p_{x,scat} p_{x,inc} c^2 + \dots$$

$$+ m_e c^4$$

$$\Rightarrow p_{x,inc} c (m_e c^2 - p_{x,scat} c) - m_e c^2 p_{x,scat}$$

$$= -\cos\theta\, p_{x,scat} p_{x,inc} c^2$$

Using again (3)
to other direction

$$\Rightarrow E_{x,scat}(E_{x,inc} - E_{x,inc}\cos\theta + m_e c^2) = E_{x,inc} m_e c^2$$

$$\Rightarrow E_{x,scat} = \frac{E_{x,inc} m_e c^2}{m_e c^2 + E_{x,inc}(1-\cos\theta)} = \frac{E_{x,inc}}{1 + \frac{E_{x,inc}}{m_e c^2}(1-\cos\theta)}$$

2,

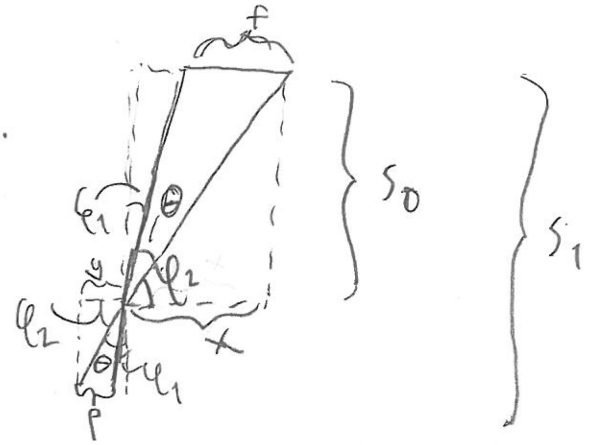
$\Rightarrow \theta = \tan^{-1}\left(\frac{d}{h}\right)$

- θ does not depend on z, because x-coordinate can be chosen arbitrarily when finding the maximum angle.

- θ is solely determined by the grid ratio,

- The grid frequency determines the probability of x-ray to find a suitable hole. to pass through.

3.



$$\begin{cases} \cot\varphi_2 = \frac{x}{s_0} \\ \tan\varphi_1 = \frac{x-f}{s_0} \end{cases} \Rightarrow \cot\varphi_2 - \tan\varphi_1 = \frac{f}{s_0}$$

$$\begin{cases} \cot\varphi_2 = \frac{y}{s_0-s_1} \\ \tan\varphi_1 = \frac{y-\rho}{s_1-s_0} \end{cases} \Rightarrow \cot\varphi_2 - \tan\varphi_1 = \frac{\rho}{s_1-s_0}$$

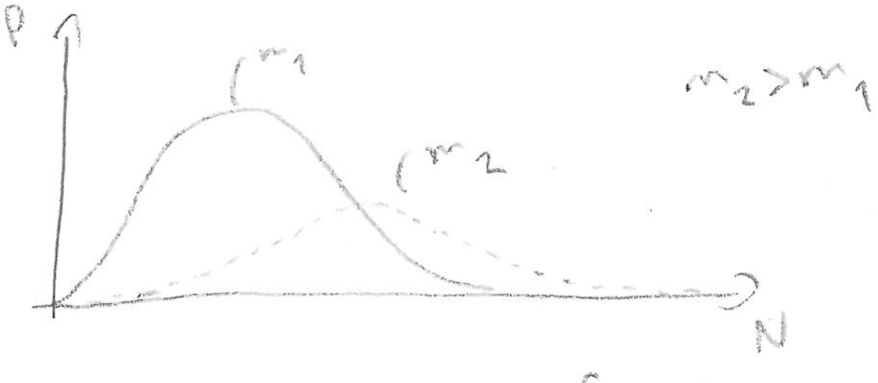
$$\Rightarrow \frac{\rho}{s_1-s_0} = \frac{f}{s_0} \Rightarrow \rho = \frac{f}{s_0}(s_1-s_0)$$

4. Poisson distribution:

$$P(N) = \frac{m^N e^{-m}}{N!}$$

, $N = 0, 1, 2, \ldots$

, m = mean value



$\Rightarrow$ The probability of getting 90% of the mean is decreased with larger m.

a) $N = \frac{35}{0,4} = 88$

$$\Rightarrow P(88) = \frac{100^{88} e^{-100}}{88!} \approx 0,02$$

b) $$P(880) = \frac{1000^{880} e^{-1000}}{880!} \approx 7,4 \cdot 10^{-6}$$

$\Rightarrow$ Increasing the dose by a factor of ten spreads the probability distribution and hence $P(880) << P(80)$