$x(k+2) - 9x(k+1) + 20x(k) = 4k+5;\ x(0) = 4 \quad x(1) = 5$

$x(k) = r^k;\ r^{k+2} - 9r^{k+1} + 20r^k = 0 \ |:r^k$

$r^2 - 9r + 20 = 0$

$D = 81 - 80 = 1 \quad r_{1,2} = \frac{9 \pm 1}{2} = \left[\begin{matrix}5\\4\end{matrix}\right.$

$x_k^{(1)} = 4^k \quad x_k^{(2)} = 5^{-k}$

$x_k = x_k^{(\text{частное})} + C_1 4^k + C_2 5^k$

$x_k^{(\text{частное})} = Ak + B$

$A(k+2) + B - 9(A(k+1) + B) + 20(Ak+B) = 4k+5$

$(2A + B - 9A - 9B + 20B) + k(\cancel{2}A - 9A + 20A) = 4k+5$

$(12B - 7A) + 12Ak = 4k+5$

$\begin{cases} 12B - 7A = 5 \\ 12A = 4 \end{cases} \quad \begin{cases} B = \frac{11}{18} \\ A = \frac{1}{3} \end{cases} \quad x_k^{(\text{частное})} = \frac{1}{3}k + \frac{11}{18}$

$x_k = \frac{1}{3}k + \frac{11}{18} + C_1 4^k + C_2 5^k$

$k = 0 - \quad x_0 = \frac{11}{18} + C_1 + C_2$

$k = 1 \quad x_1 = \frac{17}{18} + 4C_1 + 5C_2$

$\begin{cases} \frac{11}{18} + C_1 + C_2 = 4 \\ \frac{17}{18} + 4C_1 + 5C_2 = 5 \end{cases} \quad \begin{cases} C_1 = \frac{61}{18} - C_2 \\ \frac{17}{18} + 4\left(\frac{61}{18} - C_2\right) + 5C_2 = 5^{(1)} \end{cases} \quad \begin{cases} C_1 = \frac{116}{9} \\ C_2 = -\frac{19}{2} \end{cases}$

(1) $\frac{17}{18} + \frac{244}{18} + C_2 = 5;\ C_2 = -\frac{19}{2}$

Ответ: $x_k = \frac{1}{3}k + \frac{11}{18} + \frac{116}{9} \cdot 4^k - \frac{19}{2} \cdot 5^k$

$$X_{k+2}-9X_{k+1}+20X_k=4k+5\ ;\ X(0)=4\quad X(1)=5$$

$$X_k=C_1(k)4^k+C_2(k)5^{-k} \quad (1)$$

$$X_{k+1}=C_1(k+1)4^{k+1}+C_2(k+1)5^{-k+1}=C_1(k+1)4^{k+1}-C_1(k)4^{k+1}+C_1(k)4^{k+1}+C_2(k+1)5^{-k+1}-C_2(k)5^{-k+1}+C_2(k)5^{-k+1}$$

$$=\Delta C_1(k)4^{k+1}+C_1(k)4^{k+1}+\Delta C_2(k)5^{-k+1}+C_2(k)5^{-k+1}$$

$$\Delta C_1(k)4^{k+1}+\Delta C_2(k)5^{-k+1}=0 \quad (2)$$

$$X_{k+1}=C_1(k)4^{k+1}+C_2(k)5^{-k+1} \quad (3)$$

$$X_{k+2}=C_1(k+1)4^{k+2}+C_2(k+1)5^{-k+2}=C_1(k+1)4^{k+2}-C_1(k)4^{k+2}+C_1(k)4^{k+2}+C_2(k)5^{-k+2}+C_2(k+1)5^{-k+2}-C_2(k)5^{-k+2}$$

$$X_{k+2}=\Delta C_1(k)4^{k+2}+C_1(k)4^{k+2}+\Delta C_2(k)5^{-k+2}+C_2(k)5^{-k+2} \quad (4)$$

$$\Delta C_1(k)4^{k+2}+C_1(k)4^{k+2}+\Delta C_2(k)5^{-k+2}+C_2(k)5^{-k+2}-9(C_1(k)4^{k+1}+C_2(k)5^{-k+1})+20(C_1(k)4^k+C_2(k)5^{-k})=$$
$$=4k+5$$

$$\Delta C_1(k)4^{k+2}+C_1(k)4^k(4^2-9\cdot 4^1+20)+\Delta C_2(k)5^{-k+2}+C_2(k)5^{-k}(5^{-2}-9\cdot 5^1+20)=4k+5$$

$$f_k=4k+5\qquad \Delta C_1(k)4^{k+2}+\Delta C_2(k)5^{-k+2}=f_k \quad (5)$$

$$\begin{cases}\Delta C_1(k)4^{k+1}+\Delta C_2(k)5^{-k+1}=0\\ \Delta C_1(k)4^{k+2}+\Delta C_2(k)5^{-k+2}=f_k\end{cases}\qquad \Delta C_1(k)=-4^{-k-1}f_k\quad \Delta C_2(k)=5^{-k-1}f_k$$

$$\sum_{k=0}^{n-1}\Delta C_1(k)=-\sum_{k=0}^{n-1}4^{-k-1}f_k\qquad C_1(n)-C_1(0)=-\sum_{k=0}^{n-1}4^{-k-1}f_k\ ;\ C_1(n)=C_1(0)-\frac{1}{4}\sum_{m=0}^{n-1}4^{-m}f_m$$

$$\sum_{k=0}^{n-1}\Delta C_2(k)=+\sum_{k=0}^{n-1}5^{-k-1}f_k\qquad C_2(n)=C_2(0)+\frac{1}{5}\sum_{m=0}^{n-1}3^{-m}f_m$$

$$X_k=C_1(0)4^k-4^{k-1}\sum_{m=0}^{k-1}4^{-m}f_m+C_2(0)5^k+5^{k-1}\sum_{m=0}^{k-1}5^{-m}f_m$$

$$X_0=4\qquad X_1=5$$

$$\begin{cases}C_1(0)+C_2(0)=4\\ 4C_1(0)-f_0+5C_2(0)+f_0=5\end{cases}\qquad \begin{cases}C_1(0)=15\\ C_2(0)=-11\end{cases}$$

$$X_k=15\cdot 4^k-4^{k-1}\sum_{m=0}^{n-1}4^{-m}f_m-11\cdot 5^k+5^{k-1}\sum_{m=0}^{k-1}5^{-m}f_m$$

$$\sum_{m=0}^{k-1} 4^{-m}(4m+5) = 4\sum_{m=0}^{k-1} 4^{-m} m + 5\sum_{m=0}^{k-1} 4^{-m}; \quad \sum_{m=0}^{k-1} 5^{-m}(4m+5) = 4\sum_{m=0}^{k-1} 5^{-m} m + 5\sum_{m=0}^{k-1} 5^{-m}$$

$$\sum_{m=0}^{k-1} 4^{-m} = \frac{4^{-k}-1}{4^{-1}-1} = \frac{4-4^{1-k}}{3}; \quad \sum_{m=0}^{k-1} 5^{-m} = \frac{5^{-k}-1}{5^{-1}-1} = \frac{5-5^{1-k}}{4}$$

$$\sum_{m=0}^{k-1} 4^{-m} m = \frac{4^{-k}}{4^{-1}-1}k - \sum_{m=0}^{k-1}\frac{4^{-k-1}}{4^{-1}-1} = \frac{4^{-k}}{4^{-1}-1}k - \frac{4^{-1}}{4^{-1}-1}\sum_{m=0}^{k-1}4^{-k} = \frac{4^{-k}}{4^{-1}-1}k - \frac{4^{-1}}{4^{-1}-1}\cdot\frac{4^{-k}-1}{4^{-1}-1} = \frac{-4^{1-k}}{3}k - \frac{4^{1-k}-4}{9}$$

$$\sum_{m=0}^{k-1} 5^{-m} m = \frac{5^{-k}}{5^{-1}-1}k - \sum_{m=0}^{k-1}\frac{5^{-k+1}}{5^{-1}-1} = \frac{5^{-k}}{5^{-1}-1}k - \frac{5^{-1}}{5^{-1}-1}\sum_{m=0}^{k-1}5^{-k} = \frac{5^{-k}}{5^{-1}-1}k - \frac{5^{-1}}{5^{-1}-1}\cdot\frac{5^{-k}-1}{5^{-1}-1} = \frac{-5^{1-k}}{4}k - \frac{5^{1-k}-5}{16}$$

$$\sum_{m=0}^{k-1} 4^{-m}(4m+5) = 4\left(\frac{-4^{1-k}}{3}k - \frac{4^{1-k}-4}{9}\right) + 5\left(\frac{4-4^{1-k}}{3}\right) = \frac{-4^{2-k}}{3}k - \frac{4^{2-k}}{9} + \frac{16}{9} + \frac{20}{3} - \frac{5\cdot 4^{1-k}}{3} =$$

$$= -\frac{16(4^{-k})}{3}k - \frac{76(4^{-k})}{9} + \frac{76}{9}$$

$$\sum_{m=0}^{k-1} 5^{-m}(4m+5) = 4\left(\frac{-5^{1-k}}{4}k - \frac{5^{1-k}-5}{16}\right) + 5\left(\frac{5-5^{1-k}}{4}\right) = -5^{1-k}k - \frac{5^{1-k}}{4} + \frac{5}{4} + \frac{25}{4} - \frac{5^{2-k}}{4} =$$

$$= 5(5^{-k})k - \frac{15(5^{-k})}{2} + \frac{15}{2}$$

$$X_k = 15\cdot 4^k - 4^{k+1}\left(\frac{-16(4^{-k})}{3}k - \frac{76(4^{-k})}{9} + \frac{76}{9}\right) - 11\cdot 5^k + 5^{k-1}\left(-5(5^{-k})k - \frac{15(5^{-k})}{2} + \frac{15}{2}\right) =$$

$$= 4^k\left(15 - \frac{19}{9}\right) + k\left(\frac{4}{3} - 1\right) + \left(\frac{19}{9} - \frac{3}{2}\right) - 5^k\left(11 - \frac{3}{2}\right) = \frac{1}{3}k + \frac{11}{18} + \frac{116}{9}\cdot 4^k - \frac{19}{2}\cdot 5^k$$

Ответ: $X_k = \frac{1}{3}k + \frac{11}{18} + \frac{116}{9}\cdot 4^k - \frac{19}{2}5^k$