$$X_{k+2} - 9X_{k+1} + 20X_k = 4k+5 \text{ ; } X(0)=4 \quad X(1)=5$$

$$X_k = C_1(k)4^k + C_2(k)5^{-k} \quad ①$$

$$X_{k+1} = C_1(k+1)4^{k+1} + C_2(k+1)5^{-k+1} = C_1(k+1)4^{k+1} - C_1(k)4^{k+1} + C_1(k)4^{k+1} + C_2(k+1)5^{-k+1} - C_2(k)5^{-k+1} + C_2(k)5^{-k+1}$$

$$= \Delta C_1(k)4^{k+1} + C_1(k)4^{k+1} + \Delta C_2(k)5^{-k+1} + C_2(k)5^{-k+1}$$

$$\Delta C_1(k)4^{k+1} + \Delta C_2(k)5^{-k+1} = 0 \quad ②$$

$$X_{k+1} = C_1(k)4^{k+1} + C_2(k)5^{-k+1} \quad ③$$

$$X_{k+2} = C_1(k+1)4^{k+2} + C_2(k+1)5^{-k+2} = C_1(k+1)4^{k+2} \mp C_1(k)4^{k+2} + C_1(k)4^{k+2} \mp C_2(k)5^{-k+2} + C_2(k+1)5^{-k+2} - C_2(k)5^{k+2}$$

$$X_{k+2} = \Delta C_1(k)4^{k+2} + C_1(k)4^{k+2} + \Delta C_2(k)5^{-k+2} + C_2(k)5^{-k+2} \quad ④$$

$$\Delta C_1(k)4^{k+2} + C_1(k)4^{k+2} + \Delta C_2(k)5^{-k+2} + C_2(k)5^{-k+2} - 9(C_1(k)4^{k+1} + C_2(k)5^{-k+1}) + 20(C_1(k)4^k + C_2(k)5^{-k}) =$$
$$= 4k+5$$

$$\Delta C_1(k)4^{k+2} + C_1(k)4^k(4^2 - 9\cdot 4^1 + 20) + \Delta C_2(k)5^{-k+2} + C_2(k)5^{k}(5^2 - 9\cdot 5^1 + 20) = 4k+5$$

$$f_k = 4k+5 \qquad \Delta C_1(k)4^{k+2} + \Delta C_2(k)5^{-k+2} = f_k \quad ⑤$$

$$\begin{cases} \Delta C_1(k)4^{k+1} + \Delta C_2(k)5^{-k+1} = 0 \\ \Delta C_1(k)4^{k+2} + \Delta C_2(k)5^{-k+2} = f_k \end{cases} \qquad \Delta C_1(k) = -4^{-k-1}f_k \quad \Delta C_2(k) = 5^{-k-1}f_k$$

$$\sum_{k=0}^{n-1}\Delta C_1(k) = -\sum_{k=0}^{n-1}4^{-k-1}f_k \qquad C_1(n) - C_1(0) = -\sum_{k=0}^{n-1}4^{-k-1}f_k \text{ ; } C_1(n) = C_1(0) - \frac{1}{4}\sum_{m=0}^{n-1}4^{-m}f_m$$

$$\sum_{k=0}^{n-1}\Delta C_2(k) = +\sum_{k=0}^{n-1}5^{-k-1}f_k \qquad C_2(n) = C_2(0) + \frac{1}{5}\sum_{m=0}^{n-1}3^{-m}f_m$$

$$X_k = C_1(0)4^k - 4^{k-1}\sum_{m=0}^{k-1}4^{-m}f_m + C_2(0)5^k + 5^{k-1}\sum_{m=0}^{k-1}5^{-m}f_m$$

$$X_0 = 4 \qquad X_1 = 5$$

$$\begin{cases} C_1(0) + C_2(0) = 4 \\ 4C_1(0) - f_0 + 5C_2(0) + f_0 = 5 \end{cases} \qquad \begin{cases} C_1(0) = 15 \\ C_2(0) = -11 \end{cases}$$

$$X_k = 15\cdot 4^k - 4^{k-1}\sum_{m=0}^{n-1}4^{-m}f_m - 11\cdot 5^k + 5^{k-1}\sum_{m=0}^{k-1}5^{-m}f_m$$

$$\sum_{m=0}^{k-1} 4^{-m}(4m+5) = 4\sum_{m=0}^{k-1} 4^{-m}m + 5\sum_{m=0}^{k-1} 4^{-m}; \quad \sum_{m=0}^{k-1} 5^{-m}(4m+5) = 4\sum_{m=0}^{k-1} 5^{-m}m + 5\sum_{m=0}^{k-1} 5^{-m}$$

$$\sum_{m=0}^{k-1} 4^{-m} = \frac{4^{-k}-1}{4^{-1}-1} = \frac{4}{3}(1-4^{-k}) \quad \sum_{m=0}^{k-1} 5^{-m} = \frac{5^{-k}-1}{5^{-1}-1} = \frac{5}{4}(1-5^{-k})$$

$$\sum_{m=0}^{k-1} 4^{-m}m = \frac{4^{-k}}{4^{-1}-1}k - \sum_{m=0}^{k-1}\frac{4^{-k-1}}{4^{-1}-1} = \frac{4^{-k}}{4^{-1}-1}k - \frac{4^{-1}}{4^{-1}-1}\sum_{k=0}^{k-1} 4^{-k} = \frac{4^{-k}}{4^{-1}-1}k - \frac{4^{-1}}{4^{-1}-1}\frac{4^{-k}-1}{4^{-1}-1}$$

$$\sum_{m=0}^{k-1} 5^{-m}m = \frac{5^{-k}}{5^{-1}-1}k - \frac{5^{-1}}{5^{-1}-1}\frac{5^{-k}-1}{5^{-1}-1}$$

$$\sum_{m=0}^{k-1} 4^{-m}(4m+5) = 4\left(\frac{4^{-k}}{4^{-1}-1}k - \frac{4^{-1}}{4^{-1}-1}\frac{4^{-k}-1}{4^{-1}-1}\right) + 5\left(\frac{4}{3}(1-4^{-k})\right) = \cancel{-\frac{8(4^{-k+1}+2)}{9} + \frac{25}{4}(1-5^{-k})}$$

$$\sum_{m=0}^{k-1} 5^{-m}(4m+5) = \cancel{-\frac{4k \cdot 5^{-k+1} \ldots + 5}{\ldots}} \cancel{+\frac{25}{4}(1-5^{-k})} = 4\left(\frac{5^{-k}}{5^{-1}-1} - \frac{5^{-k+1}-5}{16}\right) + \frac{25}{4}(1-5^{-k})$$

$$= \cancel{-\frac{4(3k \cdot 4^{-k+1} - 4^{-k+1} + 4)}{9} + \frac{20}{3}(1-4^{-k})} = 4\left(\frac{4^{-k}}{4^{-1}-1} - \frac{4^{-k+1}-4}{9}\right) + \frac{20}{3}(1-4^{-k})$$

$$X_k = 15 \cdot 4^k - 4^{k-1}\left(4\left(\frac{4^{-k}}{4^{-1}-1} - \frac{4^{-k+1}-4}{9}\right) + \frac{20}{3}(1-4^{-k})\right) - 11 \cdot 5^k + 5^{k-1}\left(4\left(\frac{5^{-k}}{5^{-1}-1} - \frac{5^{-k+1}-5}{16}\right) + \frac{25}{4}(1-5^{-k})\right)$$

$$X_k = 15 \cdot 4^k - \left(\frac{4^k(19 - 4^{-k+2} - 15 \cdot 4^{-k})}{9}\right) - 11 \cdot 5^k + \left(\frac{5^{k-1}(15 - 5^{-k+2})}{2}\right)$$