

47. 1 способ

Кладковей

$$y''(t) - 9y'(t) + 20y(t) = 4t + 5; \quad y(0) = 4; \quad y'(0) = 5$$

$$y(t) = y^{(4,4)}(t) + \sum_{i=1}^2 C_i y_i(t) = y^{(4,4)}(t) + C_1 y_1(t) + C_2 y_2(t)$$

$$y''(t) - 9y'(t) + 20y(t) = 0$$

$$y'(t) = e^{\lambda t} \Rightarrow \lambda^2 - 9\lambda + 20 = 0$$

$$D = 81 - 80 = 1; \quad \lambda_{1,2} = \frac{9 \pm 1}{2} = \begin{cases} 5 \\ 4 \end{cases} \Rightarrow \begin{cases} y_1(t) = e^{5t} \\ y_2(t) = e^{4t} \end{cases}$$

$$\text{Получаем: } y(t) = y^{(4,4)}(t) + C_1 e^{5t} + C_2 e^{4t}$$

$$y(t) = At + B \Rightarrow y'(t) = A \Rightarrow y''(t) = 0$$

$$y''(t) - 9y'(t) + 20y(t) = 4t + 5 \Rightarrow 0 - 9A + 20A + 20B = 4t + 5 \Rightarrow$$

$$\begin{cases} 20A = 4 \\ 20B - 9A = 5 \end{cases} \Rightarrow \begin{cases} A = \frac{1}{5} \\ B = \frac{17}{50} \end{cases} \Rightarrow y^{(4,4)}(t) = \frac{1}{5}t + \frac{17}{50}$$

$$y(t) = \frac{1}{5}t + \frac{17}{50} + C_1 e^{5t} + C_2 e^{4t}$$

$$\text{Найдем } C_1 \text{ и } C_2: \quad y(0) = 4 \quad y'(0) = 5$$

$$\begin{cases} \frac{17}{50} + C_1 + C_2 = 4 \\ \frac{1}{5} + 5C_1 + 4C_2 = 5 \end{cases} \Rightarrow \begin{cases} C_1 + C_2 = \frac{183}{50} \\ 5C_1 + 4C_2 = \frac{24}{5} \end{cases} \Rightarrow \begin{cases} C_1 = \frac{-246}{25} \\ C_2 = \frac{27}{2} \end{cases}$$

$$y(t) = \frac{1}{5}t + \frac{17}{50} - \frac{246}{25}e^{5t} + \frac{27}{2}e^{4t}$$

47. II способ

Кладников

$$y'''(t) - 9y'(t) + 20y(t) = 4t + 5; y(0) = 4; y'(0) = 5$$

$$y''(t) - 9y'(t) + 20y(t) = 0; y'(t) = e^{\lambda t} \rightarrow \text{Характерист. ур.} = \lambda^2 - 9\lambda + 20 = 0$$

$$D = 81 - 80 = 1 \quad \lambda_{1,2} = \frac{9 \pm 1}{2} = \begin{cases} 5 & y_1(t) = 5 \\ 4 & y_2(t) = 4 \end{cases}$$

$$y(t) = C_1(t)e^{5t} + C_2(t)e^{4t}$$

$$y'(t) = 5C_1(t)e^{5t} + 4C_2(t)e^{4t} + C_1'(t)e^{5t} + C_2'(t)e^{4t}; C_1'(t)e^{5t} + C_2'(t)e^{4t} = 0 \quad (1)$$

$$y''(t) = 25C_1(t)e^{5t} + 16C_2(t)e^{4t} + 5C_1'(t)e^{5t} + 4C_2'(t)e^{4t}$$

$$y'''(t) = 25C_1(t)e^{5t} + 16C_2(t)e^{4t} + 5C_1'(t)e^{5t} + 4C_2'(t)e^{4t}$$

$$25C_1(t)e^{5t} + 16C_2(t)e^{4t} + 5C_1'(t)e^{5t} + 4C_2'(t)e^{4t} = 9(5C_1(t)e^{5t} + 4C_2(t)e^{4t}) + 20C_1(t)e^{5t} + 20C_2(t)e^{4t} = 4t + 5$$

$$5C_1'(t)e^{5t} + 4C_2'(t)e^{4t} = 4t + 5 \quad (2)$$

$$\begin{cases} 5C_1'(t)e^{5t} + 4C_2'(t)e^{4t} = 4t + 5 \\ C_1'(t)e^{5t} + C_2'(t)e^{4t} = 0 \end{cases} \Rightarrow \begin{cases} C_1'(t)e^{5t} = 4t + 5 \\ C_2'(t)e^{4t} = -C_1'(t)e^{5t} \end{cases} \Rightarrow \begin{cases} C_1'(t) = \frac{4t+5}{e^{5t}} \\ C_2'(t) = -\frac{4t+5}{e^{4t}} \end{cases}$$

$$C_1(t) = C_1(0) + \int_0^t \frac{4t+5}{e^{5t}} dt = C_1(0) + \frac{20t}{25e^{5t}} - \frac{29}{25e^{5t}} + \frac{29}{25}$$

$$C_2(t) = C_2(0) + \int_0^t -\frac{4t+5}{e^{4t}} dt = C_2(0) + \frac{2t}{2e^{4t}} + \frac{3}{2e^{4t}} - \frac{3}{2}$$

$$y(t) = e^{5t} \left(C_1(0) + \frac{20t}{25e^{5t}} - \frac{29}{25e^{5t}} + \frac{29}{25} \right) + e^{4t} \left(C_2(0) + \frac{2t}{2e^{4t}} + \frac{3}{2e^{4t}} - \frac{3}{2} \right) =$$

$$= e^{5t} C_1(0) + \frac{20t}{25} - \frac{29}{25} + \frac{29}{25} e^{5t} + e^{4t} C_2(0) + \frac{2t}{2} + \frac{3}{2} - \frac{3}{2} e^{4t} =$$

$$= e^{5t} C_1(0) + e^{4t} C_2(0) + \frac{1}{5}t + \frac{17}{50} + \frac{29}{25} e^{5t} - \frac{3}{2} e^{4t}$$

$$y(0) = C_1(0) + C_2(0) + \frac{17}{50} + \frac{29}{25} - \frac{3}{2} = 4$$

$$y'(0) = 5C_1(0) + 4C_2(0) + \frac{1}{5} + \frac{145}{25} - 6 = 5$$

$$\begin{cases} C_1(0) + C_2(0) = 4 \\ 5C_1(0) + 4C_2(0) = 3 \end{cases} \quad \begin{cases} C_1(0) = -11 \\ C_2(0) = 15 \end{cases}$$

$$y(t) = -11e^{5t} + 15e^{4t} + \frac{1}{5}t + \frac{17}{50} + \frac{29}{25}e^{5t} - \frac{3}{2}e^{4t}$$

$$y(t) = \frac{1}{5}t + \frac{17}{50} - \frac{246}{25}e^{5t} + \frac{27}{2}e^{4t}$$
