

$$4 \sum_{k=1}^n (2k-1)^2$$

$$\Delta f_k = (2k-1)^2; f_k = Bk + Ck^2 + Dk^3$$

$$\Delta(Bk + Ck^2 + Dk^3) = B + C + D + k(2C + 3D) + 3Dk^2$$

$$B + C + D + 2Ck + 3Dk + 3Dk^2 = 4k^2 - 4k + 1$$

$$\begin{cases} 3D = 4 \\ 2C + 3D = -4 \\ (B + C + D) = 1 \end{cases} \Rightarrow \begin{cases} D = \frac{4}{3} \\ C = -4 \\ B = 3\frac{2}{3} \end{cases} \Rightarrow f_k = \frac{11}{3}k - 4k^2 + \frac{4}{3}k^3$$

$$\sum_{k=1}^n (2k-1)^2 = \left(\frac{11}{3}(n+1) - 4(n+1)^2 + \frac{4}{3}(n+1)^3 \right) - \left(\frac{11}{3} - 4 + \frac{4}{3} \right) = \frac{11}{3}n + \frac{11}{3} - 4n^2 - 8n - 4 + \frac{4}{3}n^3 + 4n^2 + 4n + \frac{4}{3} - 1 = \frac{4}{3}n^3 - \frac{1}{3}n = \frac{4n^3 - n}{3} = \frac{n(4n^2 - 1)}{3}$$

$$\text{Ответ: } \frac{n(4n^2 - 1)}{3}$$