Algorithms SV worksheet 5

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I understand that the order in which the topics are taught has changed slightly from previous years. Before attempting the questions, make sure you know the following concepts/definitions: DAG, Minimum Spanning Tree/Kruskal's Algorithm, Topological sort, Strongly Connected Components, Bipartite Graph Matching. You may want to use the notes from 2024.

1. If we take a DAG and reverse all the edges, do we get a DAG? Briefly justify your answer.
2. Let G be a weighted graph with positive weights. Show that if you replace all the weights with their squares, the resulting graph has the same Minimum Spanning Tree.
3. You are given an $N \times N$ grip represented y a matrix A consisting of 0s and 1s. You want to choose N cells in such a way that
   - You choose exactly one cell from every row.
   - You choose exactly one cell from every column.
   - You can only choose cells where the corresponding entry is 1.

   How would you find such a set of cells?
4. You are given a DAG $G = (V, E)$, and a list of special nodes, $[a_1, ..., a_k]$. It is guaranteed that, for all $1 \leq 1 < k$, there is a path from a_i to a_{i+1}.

   Design an algorithm that finds a topological sort of G which minimises the ranks for all a_i in the resulting list.
5. Explain how to solve this coding problem efficiently. Please include the full code in your preferred programming language, along with an explanation of your approach.
6. You have an island represented by an $N \times M$ grid, with two harbours located on the boundary (either in the first or last row or first or last column). You want to build a border checkpoint line, a set of cells such that any path between the two harbours must pass through at least one checkpoint cell. A path is a sequence of moves between neighbouring cells (left, right, up or down, no diagonal moves). Checkpoints cannot be built on harbour cells. Each cell (i, j) has an associated cost $A[i, j]$ to build a checkpoint.

   Find a set of checkpoints that satisfies the requirements at a minimal cost.

   Example