

# Algorithms SV worksheet 1

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## 1 Complexity

1. Write down an incorrect definition for  $o(n)$  by taking the definition of  $O(n)$  and replacing  $\leq$  by  $<$ . Then find values for  $k$  and  $N$  that, by this definition, would allow us to claim that  $f(3n^2) \in o(n^2)$ .
2. Prove the following equalities/inequalities:
  - $|\sin(n)| = O(1)$
  - $|\sin(n)| \neq \Theta(1)$
  - $200 + \sin(n) = \Theta(1)$
  - $n^{100} = o(2^n)$
3. By drawing its recursion tree, solve the following recurrence relation:  $T(n) = 3T(n/2) + \Theta(n)$ .

## 2 Sorting

1. What is the smallest number of pairwise comparisons you need to perform to find the smallest of  $n$  items? Justify your solution.
2. And to find the second smallest?
3. Can picking the pivot at random really make any difference to the expected performance? How will it affect the average case? The worst case? Discuss.
4. Explain how you can achieve an  $O(n)$  worst-case time complexity for finding the element with rank  $k$ .
5. Write pseudocode for the bottom-up mergesort.