Algorithms SV worksheet 1

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1 Complexity

1. Write down an incorrect definition for $o(n)$ by taking the definition of $O(n)$ and replacing $\leq$ by $<$. Then find values for k and N that, by this definition, would allow us to claim that $f(3n^2) \in o(n^2)$.
2. Prove the following equalities/inequalities:
   - $|\sin(n)| = O(1)$
   - $|\sin(n)| \neq \Theta(1)$
   - $200 + \sin(n) \neq \Theta(1)$
   - $n^{100} = o(2^n)$
3. By drawing its recursion tree, Solve the following recurrence relation: $T(n) = 3T(n/2) + \Theta(n)$.

2 Sorting

1. What is the smallest number of pairwise comparisons you need to perform to find the smallest of n items?
2. And to find the second smallest?
3. Can picking the pivot at random really make any difference to the expected performance? How will it affect the average case? The worst case? Discuss.
4. Consider the following method for an array of size n:

   Split the array into groups of 5 (for simplicity, assume that is a multiple of 5). Take the medians from all groups (i.e. the third largest from each group). Then let x be the median of medians. Show that there exist constants C_1 and C_2 such that the rank of x in the original array is between $C_1 n$ and $C_2 n$ for a large enough n.
5. Explain how this can be used to achieve an $O(n)$ worst-case time complexity for finding the element with rank k.
6. Write pseudocode for the bottom-up mergesort.