3.36pt

Mathematic Analysis with Matlab

Lecture 2: Limits, Continuity and Derivative of a Function

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Outline

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Commands for product of a sequence and summation of a sequence

- Given vector $x = [x_1, x_2, ..., x_i, ..., x_n]$
- 'sum(x)' returns $\sum_{i=1}^{n} x_i$
- 'prod(x)' returns $\prod_{i=1}^{n} x_i$
- Try following commands and check the output:
  1. sum(1./(1:100))
  2. prod(1:100)

Commands for product of a sequence and summation of a sequence

- Given matrix $x = [x_1, x_2, ..., x_i, ..., x_n]$, x_i a column vector:
- 'sum(x)' returns a vector of $\sum_{j=1}^{m} x_{ij}$
- 'prod(x)' returns a vector of $\prod_{j=1}^{m} x_{ij}$
- Try following commands and check the output:
- x=rand(5,8)
  1. sum(x)
  2. prod(x)

Commands for taking limit of a function

- Given function f(x):
- Study the trend of f(x) when **x** approximates to **a**
- **a** could be constant or 'inf', or '-inf'
- Basic form: limit(f(x),x,a)

1. Right limit ($x \rightarrow a+0$): limit(f(x), x, a, 'right')
2. Left limit ($x \rightarrow a-0$): limit(f(x), x, a, 'left')
3. Limit to indefinite large ($x \rightarrow +\infty$): limit(f(x), x, +inf)
4. Limit to indefinite small ($x \rightarrow -\infty$): limit(f(x), x, -inf)

Taking limit of a function

- Given following function:

$$\lim_{n\to\infty} \frac{2n^3+1}{5n^3+1}$$

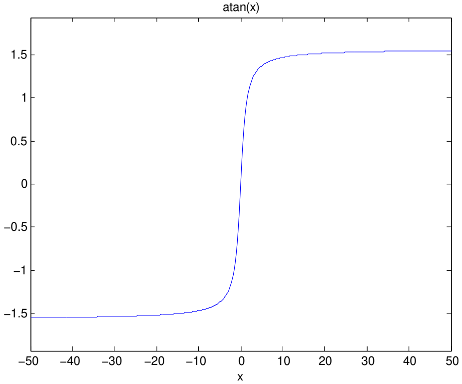
1. syms n
2. limit((2*n^3+1)/(5*n^3+1), n, inf)

Taking limit of a function

- Given following function:

$$\lim_{x \to \infty} arctan(x)$$

- try: ezplot('atan(x)',[-50,50])



Taking limit of a function

- Given following function:

$$\lim_{x \to \infty} arctan(x)$$

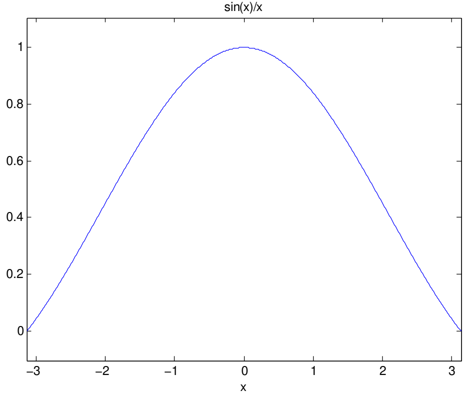
1. syms x
2. limit(atan(x), x, +inf)
3. limit(atan(x), x, -inf)

Taking limit of a function

- Given following function:

$$\lim_{x \to 0} \frac{sin(x)}{x}$$

- try: ezplot('sin(x)/x',[-pi,pi])



Taking limit of a function

- Given following function:

$$\lim_{x \to 0} \frac{sin(x)}{x}$$

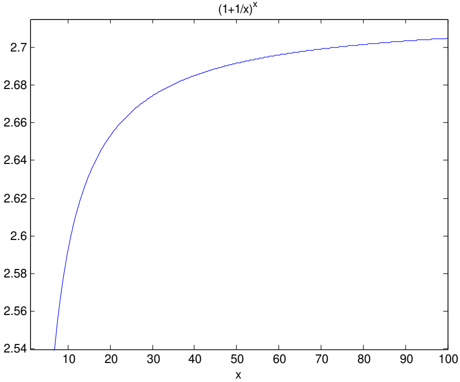
1. syms x
2. limit(sin(x)/x, x, 0)

Taking limit of a function

- Given following function:

$$\lim_{x \to \infty} (1 + \frac{1}{x})^x$$

- try: ezplot('(1+1/x)^x',[1,100])



Taking limit of a function

- Given following function:

$$\lim_{x \to \infty} (1 + \frac{1}{x})^x$$

1. syms x
2. limit((1+1/x)^x, x, inf)

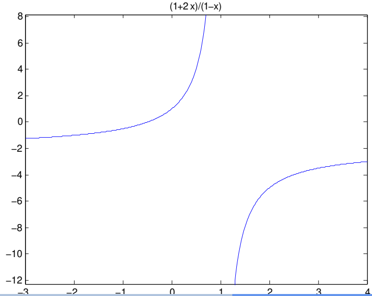
Taking limit of a function

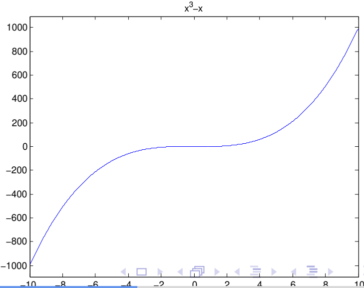
- Given following function:

$$\lim_{x\to 1+0} \frac{1+2x}{1-x}, \quad \lim_{x\to 1-0} \frac{1+2x}{1-x}$$

$$\lim_{x\to +\infty} x^3 - x, \quad \lim_{x\to -\infty} x^3 - x$$

- try: ezplot('(1+2x)/(1-x)',[-3,4])
- try: ezplot('x^3-x',[-10,10])





Taking limit of a function (1)

- Given following function:

$$\lim_{x\to 1+0} \frac{1+2x}{1-x}, \quad \lim_{x\to 1-0} \tfrac{1+2x}{1-x}$$

$$\lim_{x\to +\infty} x^3 - x, \quad \lim_{x\to -\infty} x^3 - x$$

Taking limit of a function (2)

- Given following function:

$$\lim_{x \to 1+0} \frac{1+2x}{1-x}, \quad \lim_{x \to 1-0} \tfrac{1+2x}{1-x}$$

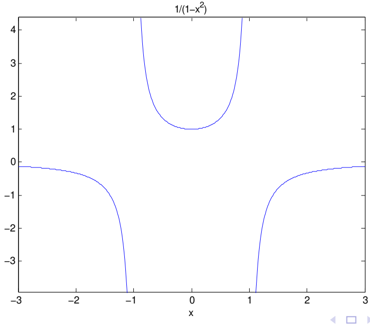
$$\lim_{x \to +\infty} x^3 - x, \quad \lim_{x \to -\infty} x^3 - x$$

Taking limit of a function (3)

- Given following function:

$$\lim_{x\to 1+0}\frac{1}{1-x^2},\quad \lim_{x\to 1-0}\frac{1}{1-x^2}$$
$$\lim_{x\to -1+0}\frac{1}{1-x^2},\quad \lim_{x\to -1-0}\frac{1}{1-x^2}$$

- try: ezplot('1/(1-x^2)',[-3,3])



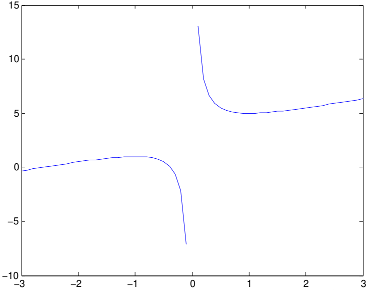
Calculate the asymptote of a function (1)

- Given following function:

$$f(x) = x + \frac{1}{x} + 3 \qquad (1)$$

1. $a = \lim_c \frac{f(x)}{x}$
2. $b = \lim_c f(x) - ax$
3. $y = ax + b$

- try: ezplot('(x+1/x+3)',[-3,3])

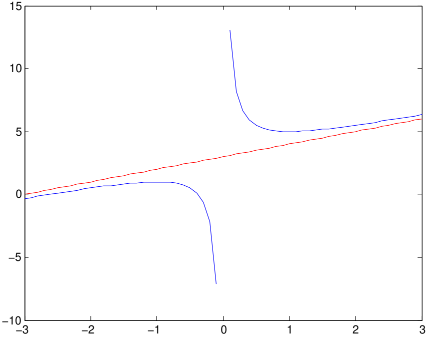


Calculate the asymptote of a function (2)

- Given following function:

$$f(x) = x + \frac{1}{x} + 3 \qquad (2)$$

- Step 1:
  1. clear; syms x;
  2. a=limit((x+1/x+3)/x, x, inf)
  3. b=limit((x+1/x+3)-a*x, x, inf)
  4. x=-3:0.1:3;
  5. f=x+1./x+3
  6. y=a*x+b
  7. plot(x,f,'b',x,y,'r')



Exercise (1)

- Take limit on following functions
- Give your answer in **5 minutes** ...

$$\lim_{x \to 0} \left(\frac{sin(x)}{x}\right)^{\frac{1}{1-cos(x)}} \tag{3}$$

$$\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - sin(x)} \tag{4}$$

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Definition of derivatives

Figure: Gottfried Wilhelm Leibniz (1646-1716)

- Given f(x) is continuous in its definition domain

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad (5)$$

- f'(x) basically reveals sensitivity to change of a quantity or variable
- Solves the problem of drawing tangent for any point on a smooth curve
- Opens up a new field: **Calculus**

Command for calculating derivatives

1. syms x
2. **Matlab 2009**: diff('f(x)', x)
3. **Matlab 2013a**: diff(f(x), x)

1. syms x
2. **Matlab 2009**: diff('f(x)', x, n), where n is the expected order of derivatives with respect to x
3. **Matlab 2013a**: diff(f(x), x, n)

- **Type**: 'help diff' to check the right command

Take derivative for function of one variable

$$g(x) = x^3 - 3x^2 + x + 1$$

- Step 1:
  1. syms x
  2. diff('x^3-3*x^2+x+1')
- Step 2:
  1. x=-1:0.1:3
  2. y1=x.^3-3*x.^2+x+1;
  3. y2=3*x.^2-6*x+1;
  4. plot(x, y1,'b',x,y2,'r:')
- Think about how to get the inflection point??

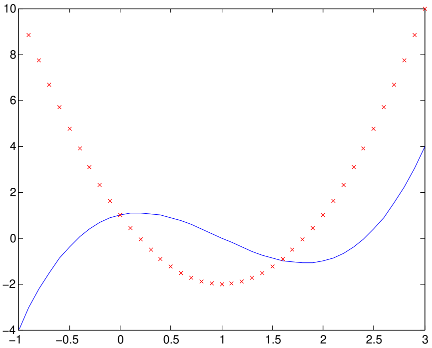

Figure: g(x) is curve in blue, g'(x) is curve decorated in red-cross.

Take derivative of higher order

$$f(x) = x^n \tag{6}$$

1. syms x
2. f=x^n;
3. df=diff(f,2);

$$g(x) = sin(x) \tag{7}$$

1. clear;
2. syms x;
3. g=sin(x)
4. dg=diff(g,5)
5. x=0.12*pi;
6. eval(dg);

Derivative of implicit function and parametric function (1)

- Given $y = f(x)$ which defines an implicite function F(x,y)=0, we have

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{F_x}{F_y}, \quad F_y \neq 0$$

- Given $x = f(t)$ and $y = g(t)$

$$\frac{dx}{dt} = \frac{f(t)}{dt}$$
$$\frac{dy}{dt} = \frac{g(t)}{dt}$$

- We have $\frac{dy}{dx} = \frac{g'(t)}{f'(t)}$

Derivative of implicit function and parametric function (2)

$$2x^2 - 2xy + y^2 + x + 2y + 1 = 0 \quad (8)$$

1. ?
2. ?;
3. ?

$$x = e^t cos(t), y = e^t sin(t) \quad (9)$$

1. clear;
2. syms t;
3. ?;
4. ?;
5. ?

Take derivative according to the definition (1)

$$f(x) = x^a \tag{10}$$

$$f(x) = a^x \tag{11}$$

- Hints: think about the defnition of function derivative
- Take limit on f(x+dx)-f(x) as ‘dx’ approaching to ‘0’
- Give your answer in **5 minutes** ...

Take derivative of a function and its tangent (1)

$$g(x) = 2x^3 + 3x^2 - 12x + 7$$

- Given g(x), tangent at $x0$ is defined as:

$$f(x) = g'(x0)(x - x0) + g(x0) \tag{12}$$

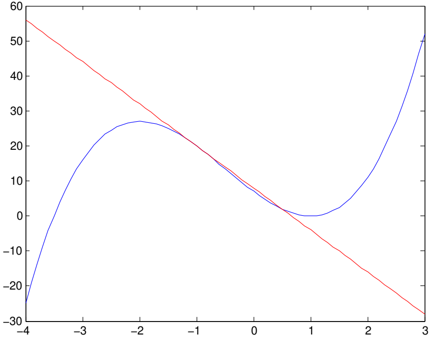

Figure: g(x) is curve in blue, tangent of g(x) at x=-1 is curve in red.

Take derivative of a function and its tangent (2)

$$g(x) = 2x^3 + 3x^2 - 12x + 7$$

- Step 1:
  1. syms x
  2. f=2*x^3+3*x^2-12*x+7;
  3. df=diff(f);
  4. x=-1;
  5. f0=eval(f);
  6. df0=eval(df);
- Step 2:
  1. x=-4:0.1:3
  2. ?
  3. ?
  4. plot(?, ?, 'b', ?, ?, 'r')

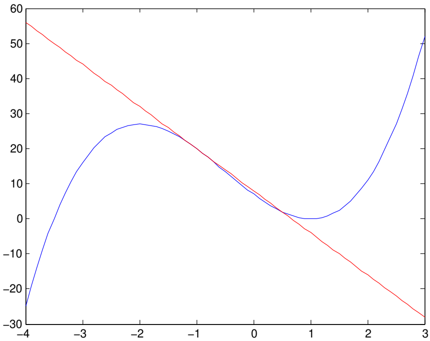

Figure: g(x) is curve in blue, tangent of g(x) at x=-1 is curve in red.

Q & A

Thanks for your attention!