

2022

Certificate Authority Cup International Mathematical Contest Modeling
<http://mcm.tzmcm.cn>

Problem A (MCM)

How Pterosaurs Fly

Pterosaurs is an extinct clade of flying reptiles in the order, Pterosauria. They existed during most of the Mesozoic: from the Late Triassic to the end of the Cretaceous. Pterosaurs are the earliest vertebrates known to have evolved powered flight. Their wings were formed by a membrane of skin, muscle, and other tissues stretching from the ankles to a dramatically lengthened fourth finger[1].

There were two major types of pterosaurs. Basal pterosaurs were smaller animals with fully toothed jaws and long tails usually. Their wide wing membranes probably included and connected the hind legs. On the ground, they would have had an awkward sprawling posture, but their joint anatomy and strong claws would have made them effective climbers, and they may have lived in trees. Basal pterosaurs were insectivores or predators of small vertebrates. Later pterosaurs (pterodactyls) evolved many sizes, shapes, and lifestyles. Pterodactyls had narrower wings with free hind limbs, highly reduced tails, and long necks with large heads. On the ground, pterodactyls walked well on all four limbs with an upright posture, standing plantigrade on the hind feet and folding the wing finger upward to walk on the three-fingered “hand”. The fossil trackways show at least some species were able to run and wade or swim[2].

Pterosaurs sported coats of hair-like filaments known as pycnofibers, which covered their bodies and parts of their wings[3]. In life, pterosaurs would have had smooth or fluffy coats that did not resemble bird feathers. Earlier suggestions were that pterosaurs were largely cold-blooded gliding animals, deriving warmth from the environment like modern lizards, rather than burning calories. However, later studies have shown that they may be warm-blooded (endothermic), active animals. The respiratory system had efficient unidirectional “flow-through” breathing using air sacs, which hollowed out their bones to an extreme extent. Pterosaurs spanned a wide range of adult sizes, from the very small anurognathids to the largest known flying creatures, including *Quetzalcoatlus* and *Hatzegopteryx*[4][5], which reached wingspans of at least nine metres. The combination of endothermy, a good oxygen supply and strong

muscles made pterosaurs powerful and capable flyers.

The mechanics of pterosaur flight are not completely understood or modeled at this time. Katsufumi Sato did calculations using modern birds and concluded that it was impossible for a pterosaur to stay aloft[6]. In the book *Posture, Locomotion, and Paleoecology of Pterosaurs* it is theorized that they were able to fly due to the oxygen-rich, dense atmosphere of the Late Cretaceous period[7]. However, both Sato and the authors of *Posture, Locomotion, and Paleoecology of Pterosaurs* based their research on the now-outdated theories of pterosaurs being seabird-like, and the size limit does not apply to terrestrial pterosaurs, such as azhdarchids and tapejarids. Furthermore, Darren Naish concluded that atmospheric differences between the present and the Mesozoic were not needed for the giant size of pterosaurs[8].

Another issue that has been difficult to understand is how they took off. If pterosaurs were cold-blooded animals, it was unclear how the larger ones of enormous size, with an inefficient cold-blooded metabolism, could manage a bird-like takeoff strategy, using only the hind limbs to generate thrust for getting airborne. Later research shows them instead as being warm-blooded and having powerful flight muscles, and using the flight muscles for walking as quadrupeds[9]. Mark Witton of the University of Portsmouth and Mike Habib of Johns Hopkins University suggested that pterosaurs used a vaulting mechanism to obtain flight[10]. The tremendous power of their winged forelimbs would enable them to take off with ease[9]. Once aloft, pterosaurs could reach speeds of up to 120 km/h and travel thousands of kilometres[10].

Your team are asked to develop a reasonable mathematical model of the flight process of at least one large pterosaur based on fossil measurements and to answer the following questions.

1. For your selected pterosaur species, estimate its average speed during normal flight.
2. For your selected pterosaur species, estimate its wing-flap frequency during normal flight.
3. Study how large pterosaurs take off; is it possible for them to take off like birds on flat ground or on water? Explain the reasons quantitatively.

References

- [1] Elgin RA, Hone DW, Frey E (2011). The Extent of the Pterosaur Flight Membrane. *Acta Palaeontologica Polonica*. 56 (1): 99-111.

- [2] Mark Witton. Terrestrial Locomotion.
https://pterosaur.net/terrestrial_locomotion.php
- [3] Laura Geggel. It's Official: Those Flying Reptiles Called Pterosaurs Were Covered in Fluffy Feathers. <https://www.livescience.com/64324-pterosaurs-had-feathers.html>
- [4] Wang, X.; Kellner, A.W.A.; Zhou, Z.; Campos, D.A. (2008). Discovery of a rare arboreal forest-dwelling flying reptile (Pterosauria, Pterodactyloidea) from China. *Proceedings of the National Academy of Sciences*. 105 (6): 1983-87.
- [5] Buffetaut E, Grigorescu D, Csiki Z. A new giant pterosaur with a robust skull from the latest cretaceous of Romania. *Naturwissenschaften*. 89 (4): 180-84.
- [6] Devin Powell. Were pterosaurs too big to fly?
<https://www.newscientist.com/article/mg20026763-800-were-pterosaurs-too-big-to-fly/>
- [7] Templin, R. J.; Chatterjee, Sankar. Posture, locomotion, and paleoecology of pterosaurs. Boulder, Colo: Geological Society of America. p. 60.
- [8] Naish, Darren. Pterosaurs breathed in bird-like fashion and had inflatable air sacs in their wings.
<https://scienceblogs.com/tetrapodzoology/2009/02/18/pterosaur-breathing-air-sacs>
- [9] Mark Witton. Why pterosaurs weren't so scary after all.
<https://www.theguardian.com/science/2013/aug/11/pterosaurs-fossils-research-mark-witton>
- [10] Jeff Hecht. Did giant pterosaurs vault aloft like vampire bats?
<https://www.newscientist.com/article/dn19724-did-giant-pterosaurs-vault-aloft-like-vampire-bats/>