计算机图形学

B样条曲线、曲面

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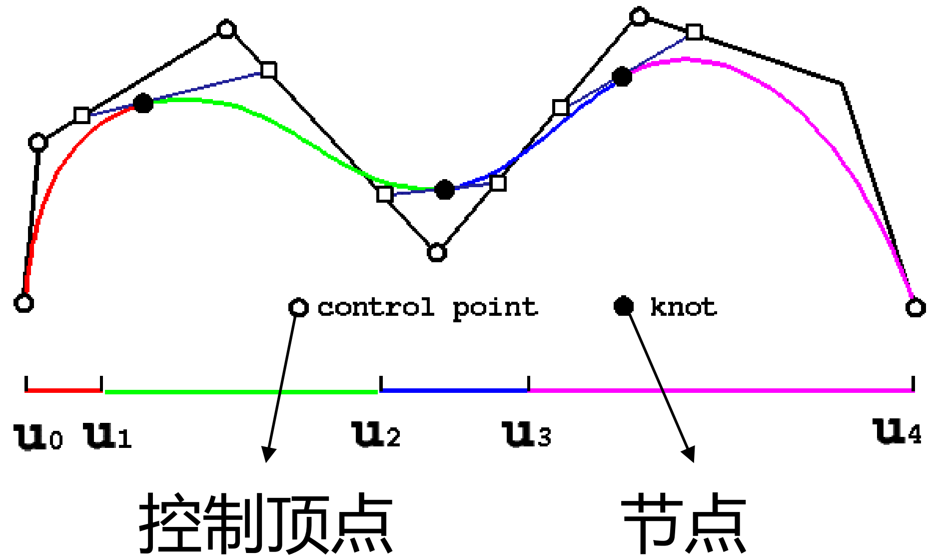
第三节 B样条曲线、曲面

- B样条曲线
- B样条曲面
- 有理B样条曲线曲面（NURBS）

B样条曲线

- 分段多项式曲线
- 参数表达
- 通过控制点生成



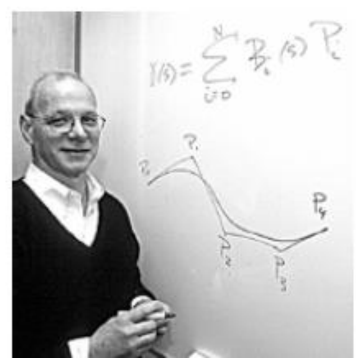
样条（spline）

源于生产实践，是富有弹性的细长条。用压铁使样条通过指定的型值点，并调整样条使它具有满意的形状，然后沿样条画出曲线。

B样条曲线历史

- Schoenberg: 样条曲线(1946)
- de Boor: B样条曲线的递归算法(1966)
- Riesenfeld: B样条曲线用作几何设计(1970s)
- Versprille: 第一次讨论有理B样条曲线(1975)
- NUBRS: 成为工业标准(1990s)






B样条曲线定义

- 控制点 $\boldsymbol{c}_i$ (i=0,…,*n*) 称为de Boor 点
- 次数 *k*（阶数*k*+1）
- 节点向量 T= {u_0, …, u_{n+k+1}}

$$\boldsymbol{p}(u)=\sum_{i=0}^{n}\boldsymbol{c}_i N_i^k(u), \quad u\in[\mathrm{u}_k,\mathrm{u}_{n+1}]$$

de Boor-Cox递推定义

k+1阶(k次)B样条基函数

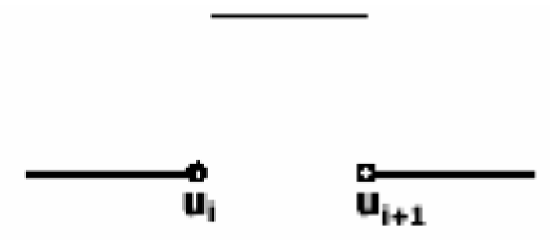
$$N_i^0(u)=\begin{cases}1, & u\in[\mathrm{u}_i,\mathrm{u}_{i+1})\\ 0, & otherwise\end{cases}$$

$$N_i^k(u)=\frac{u-\mathrm{u}_i}{\mathrm{u}_{i+k}-\mathrm{u}_i}N_i^{k-1}(u)+\frac{\mathrm{u}_{i+k+1}-u}{\mathrm{u}_{i+k+1}-\mathrm{u}_i}N_{i+1}^{k-1}(u)$$

B样条曲线

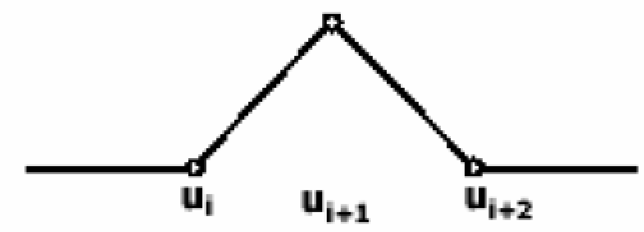
- B样条基函数取代Bernstein基函数
  - 1阶（0次）基函数

$$N_i^0(u) = \begin{cases} 1, & u \in [\mathrm{u}_i, \mathrm{u}_{i+1}) \\ 0, & otherwise \end{cases}$$



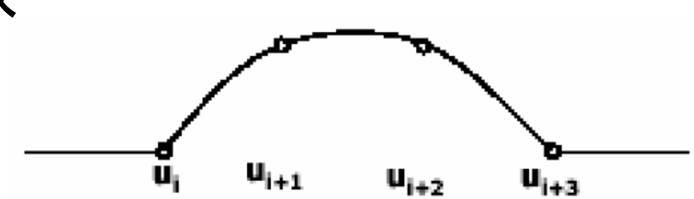
  - 2阶（1次）基函数

$$N_i^1(u) = \begin{cases} \dfrac{u - \mathrm{u}_i}{\mathrm{u}_{i+1} - \mathrm{u}_i}, & u \in [\mathrm{u}_i, \mathrm{u}_{i+1}) \\[2ex] \dfrac{\mathrm{u}_{i+2} - u}{\mathrm{u}_{i+2} - \mathrm{u}_{i+1}}, & u \in [\mathrm{u}_{i+1}, \mathrm{u}_{i+2}) \end{cases}$$



B样条曲线

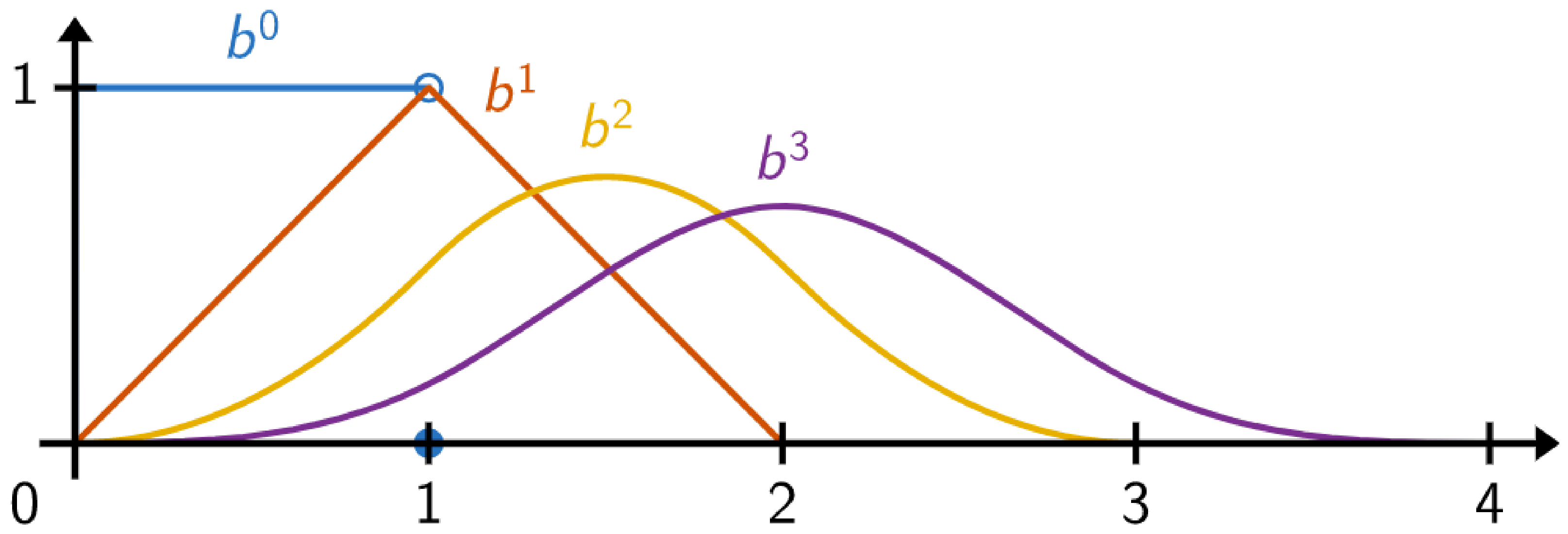
- B样条基函数取代Bernstein基函数
  - 3阶（2次）基函数



$$N_i^2(u)=\begin{cases}\dfrac{u-u_i}{u_{i+2}-u_i}\cdot\dfrac{u-u_i}{u_{i+1}-u_i}, & u\in[u_i,u_{i+1})\\[2ex] \dfrac{u_{i+2}-u}{u_{i+2}-u_{i+1}}\cdot\dfrac{u-u_i}{u_{i+2}-u_i}+\dfrac{u_{i+3}-u}{u_{i+3}-u_{i+1}}\cdot\dfrac{u-u_{i+1}}{u_{i+2}-u_{i+1}}, & u\in[u_{i+1},u_{i+2})\\[2ex] \dfrac{u_{i+3}-u}{u_{i+3}-u_{i+1}}\cdot\dfrac{u_{i+3}-u_i}{u_{i+1}-u_{i+2}}, & u\in[u_{i+1},u_{i+2})\end{cases}$$

B样条曲线

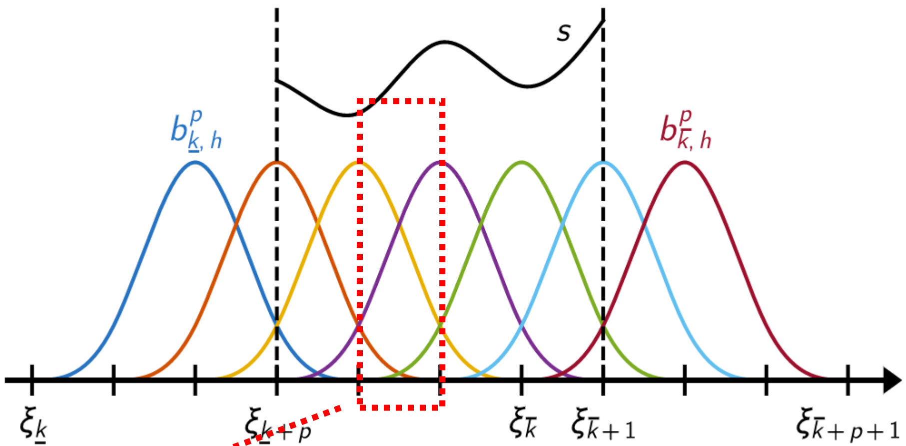
- B样条基函数取代Bernstein基函数

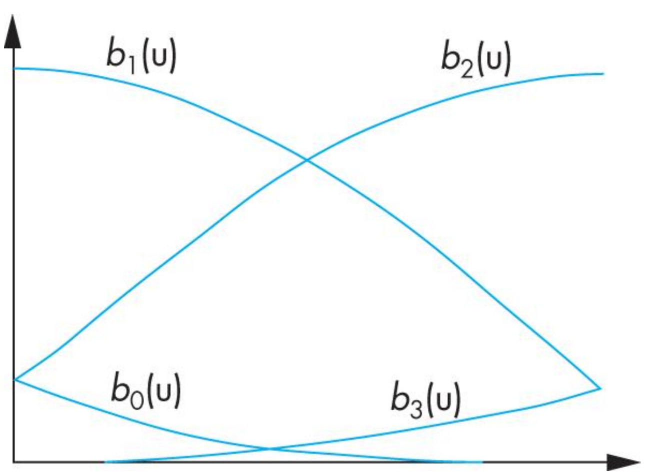


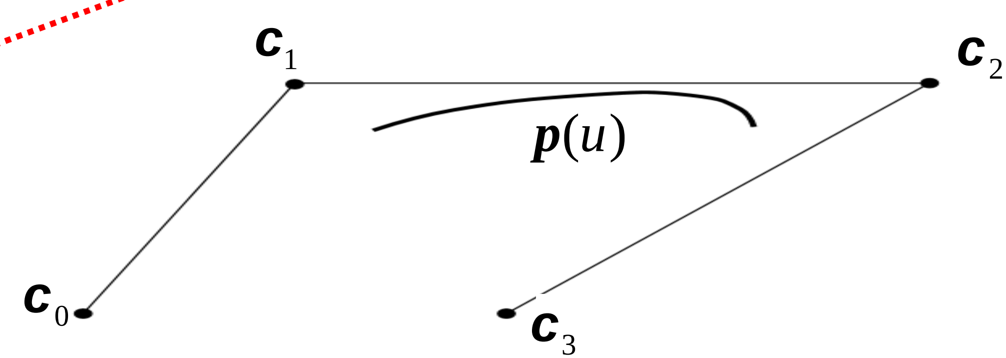
B样条曲线

- 三次均匀B样条曲线

$$\boldsymbol{b}(u)=\boldsymbol{M}_S^T\boldsymbol{u}=\frac{1}{6}\begin{bmatrix}(1-u)^3\\4-6u^2+3u^3\\1+3u+3u^2-3u^3\\u^3\end{bmatrix}$$

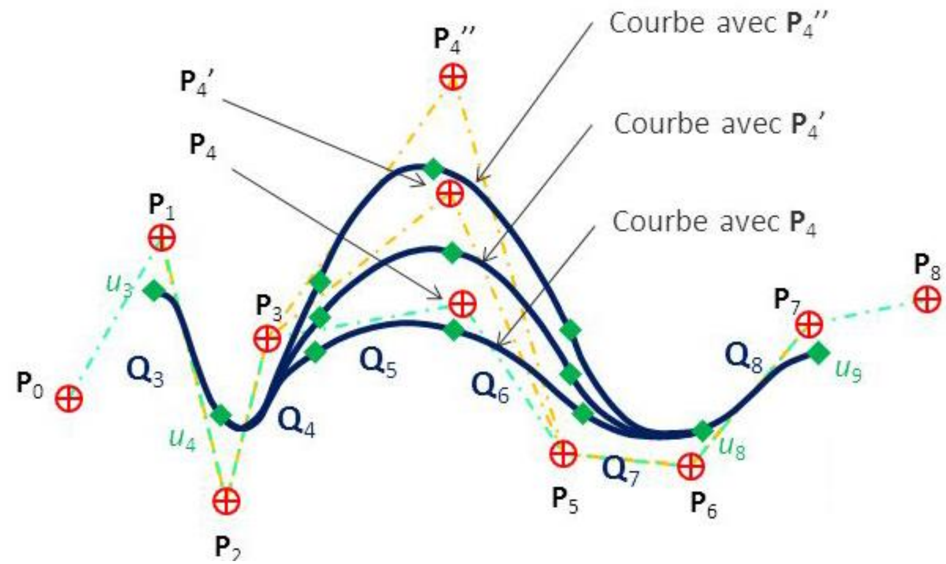




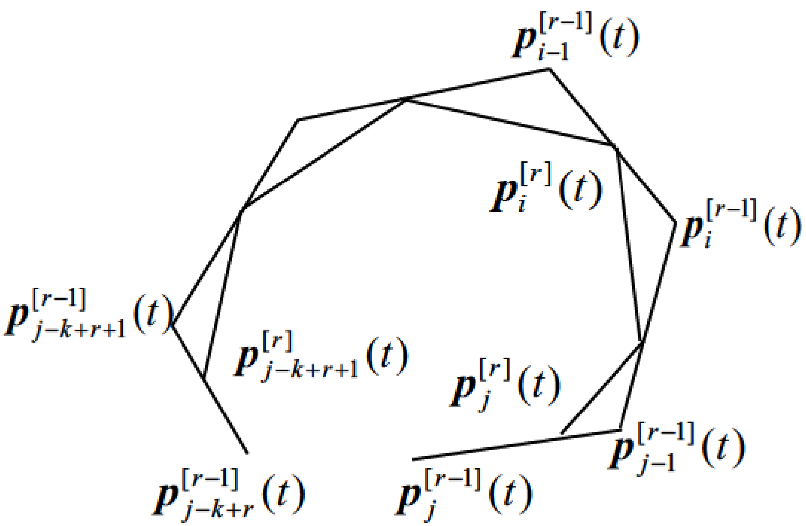


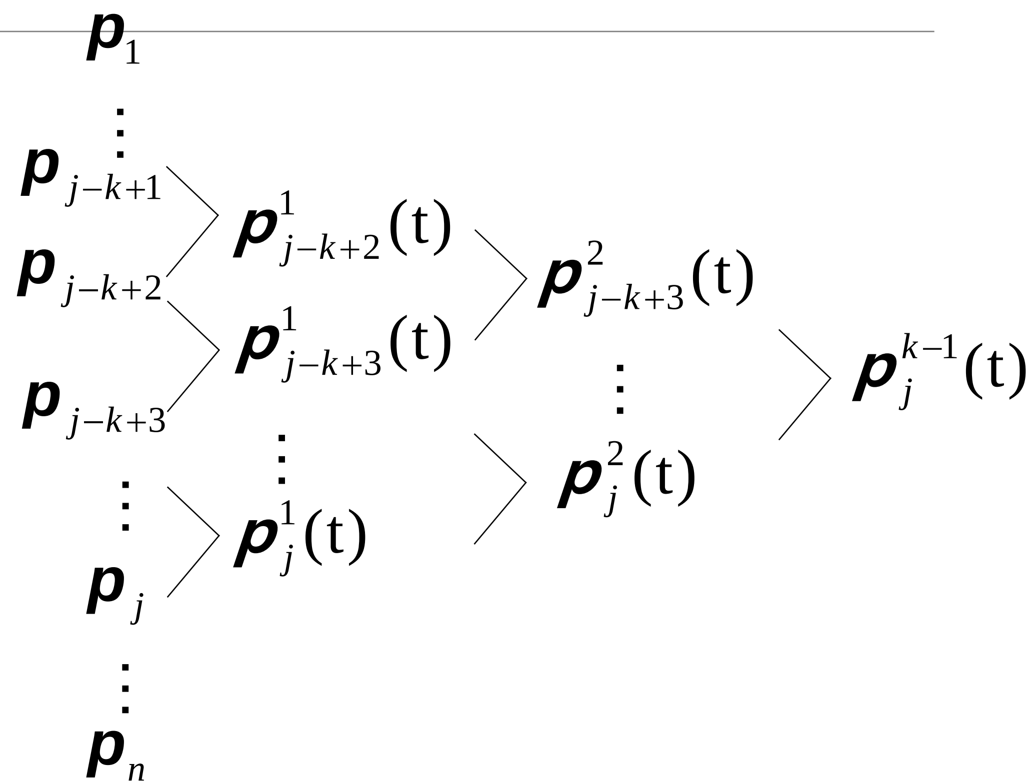
B样条曲线性质

- 保留Bezier曲线的优点
- 局部可控性：修改一个控制顶点最多会影响k+1条曲线
- 灵活拼接



de Boor递归算法



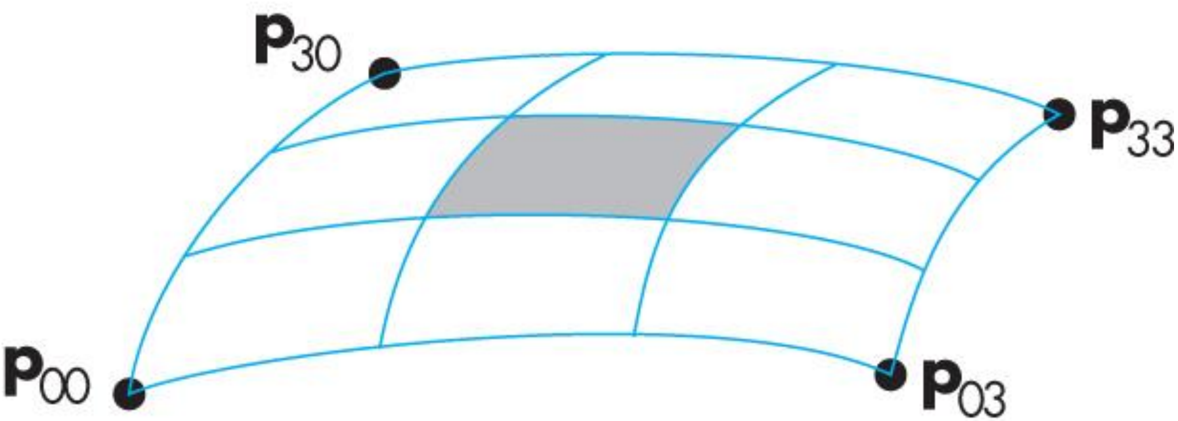


$$\boldsymbol{p}(t)=\sum_{i=j-k+1}^{j} N_{i,k}(t)\boldsymbol{p}_i=\sum_{i=j-k+2}^{j} N_{i,k-1}(t)\boldsymbol{p}_i^{[1]}(t)=\cdots=\boldsymbol{p}_j^{[k-1]}(t),\quad t_j\le t<t_{j+1},$$

B样条曲面

- 双三次混合多项式曲面
- 参数表达

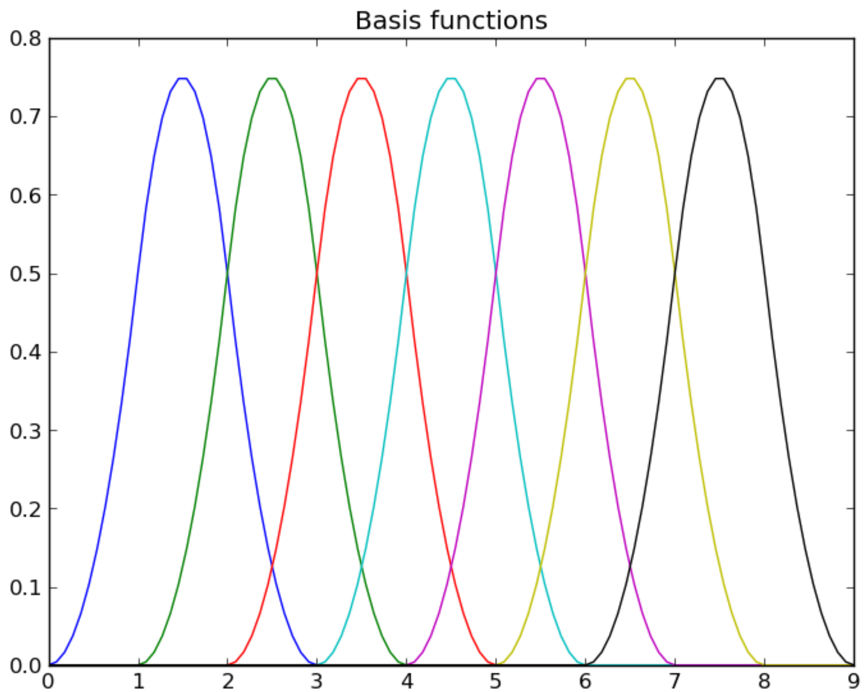
$$\boldsymbol{p}(u,v)=\sum_{i=1}^{3}\sum_{j=0}^{3}N_i^k(u)N_j^k(v)\boldsymbol{p}_{ij}$$

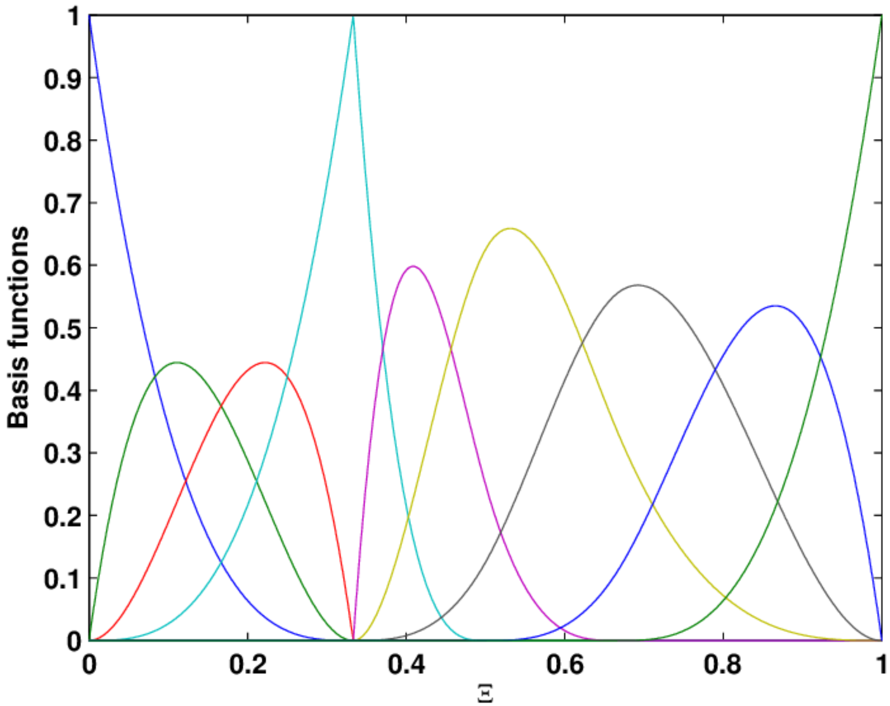


- ◆ 局部性
- ◆ 凸包性
- ◆ Bezier曲面包含性
- ◆ 变差缩减
- ◆ 。。。

非均匀有理B样条曲面（NURBS)

Non-uniform: 节点向量





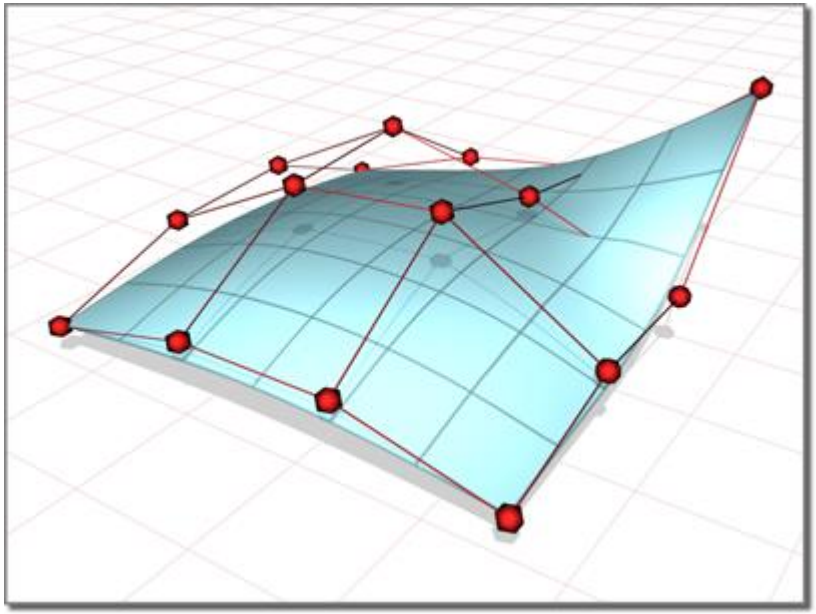
非均匀有理B样条曲面（NURBS)

Non-uniform: 节点向量

Rational:对圆锥曲线曲面等的精确表示

工业标准

$$\boldsymbol{p}(u,v)=\frac{\sum_{i=0}^{n}\sum_{j=0}^{m}\boldsymbol{c}_{i}\boxed{w_{i}}N_{i}^{k}(u)N_{j}^{k}(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m}\boxed{w_{i}}N_{i}^{k}(u)N_{j}^{k}(v)}$$

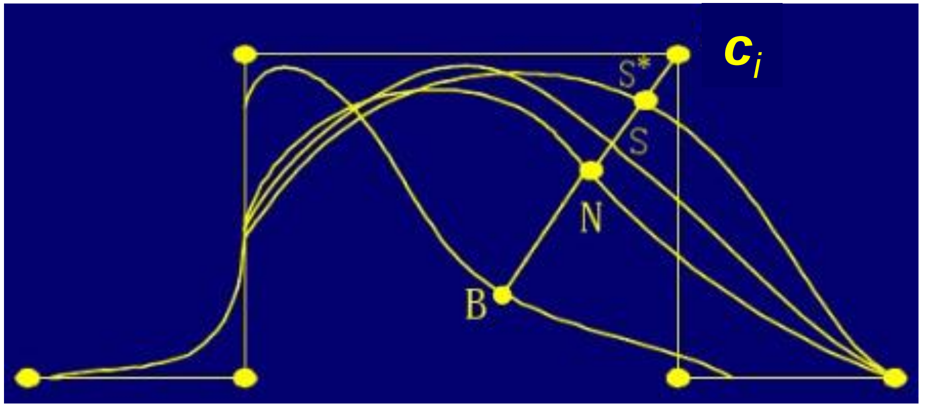


权重

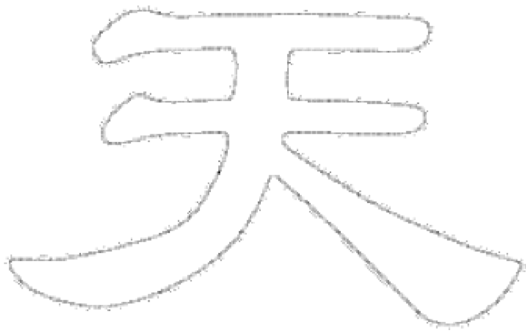
非均匀有理B样条曲面（NURBS)

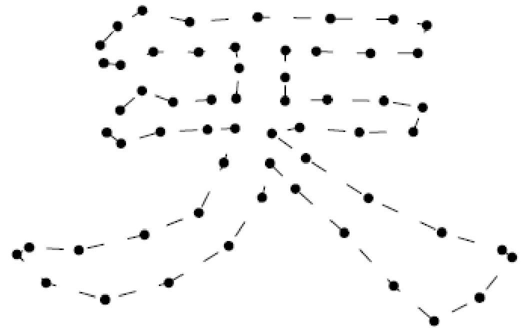
- NURBS曲线权重几何意义
  - 如果固定曲线参数u，而使权重w_i变化，则NURBS曲线变成以w_i为参数的直线，即NURBS曲线上相同的点位于同一直线上。
  - w_i增大或减小，曲线被拉向或推离$\mathbf{c}_i$点

$$\boldsymbol{p}(u)=\frac{\sum_{i=0}^{n}\mathbf{c}_i w_i N_i^k(u)}{\sum_{i=0}^{n} w_i N_i^k(u)}$$

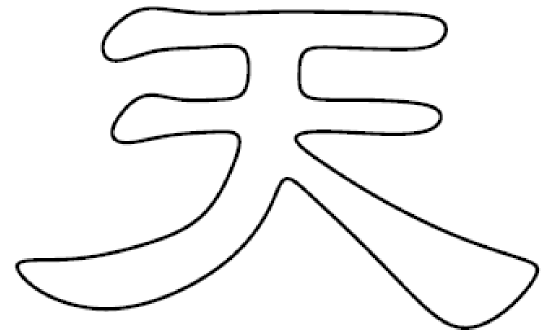


NURBS曲线/曲面建模实例

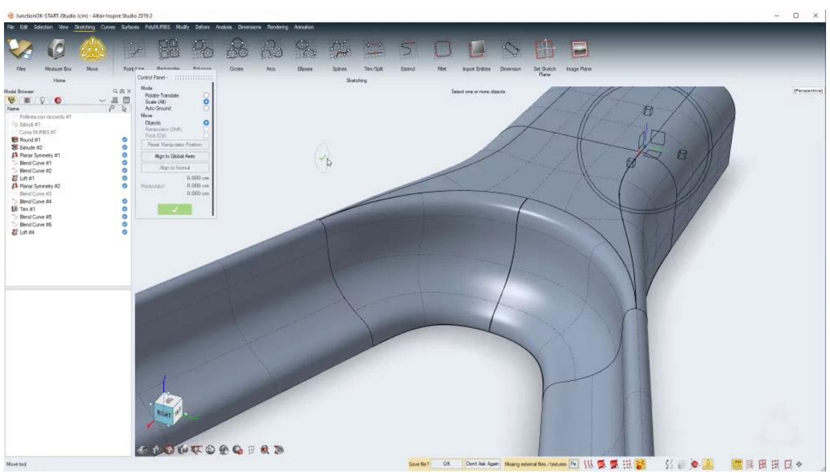


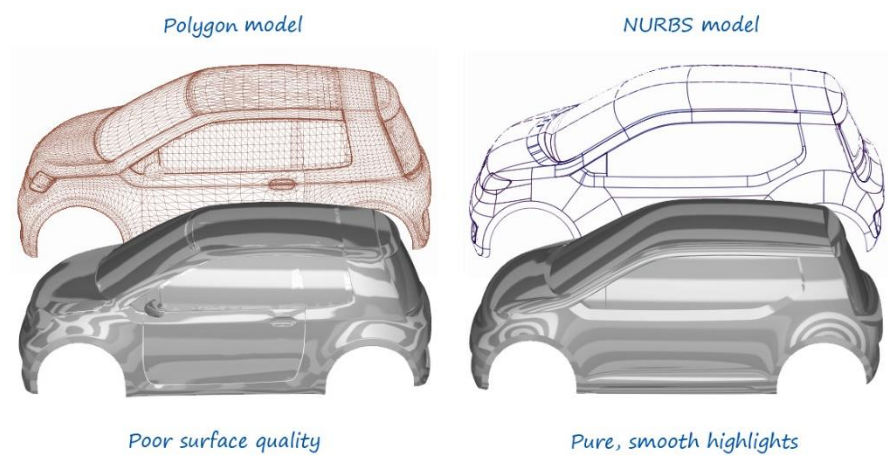


B-spline control points



B-spline curve





NURBS surface