网格参数化

Mesh Parameterization

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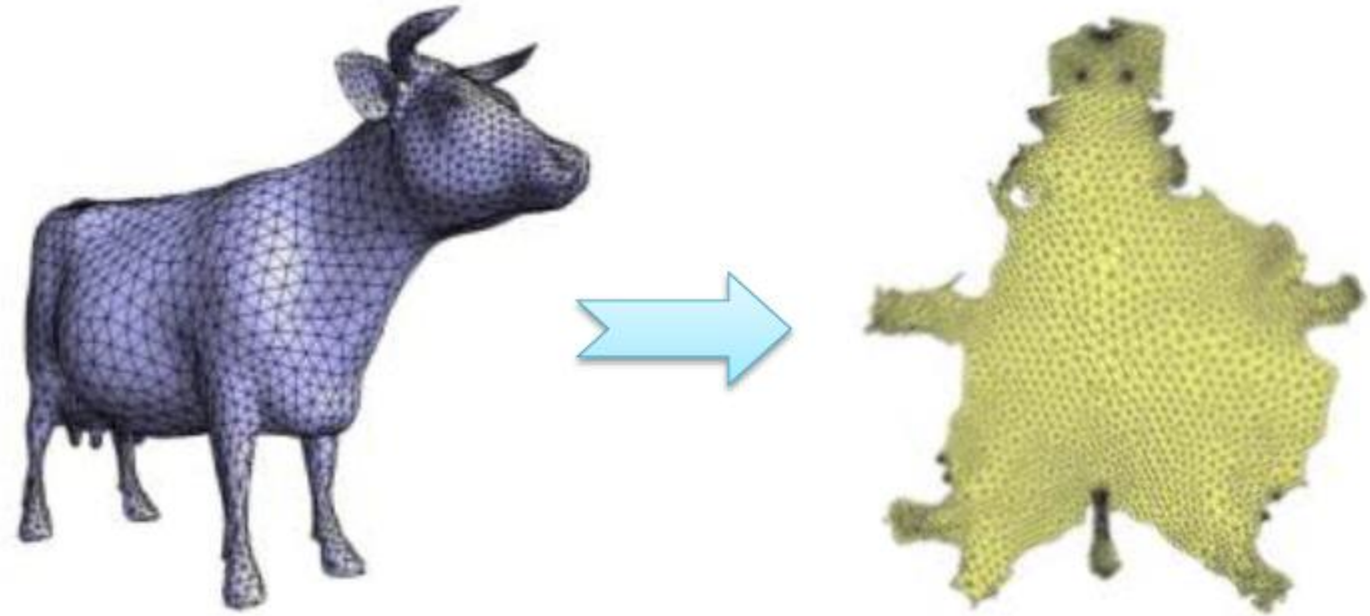
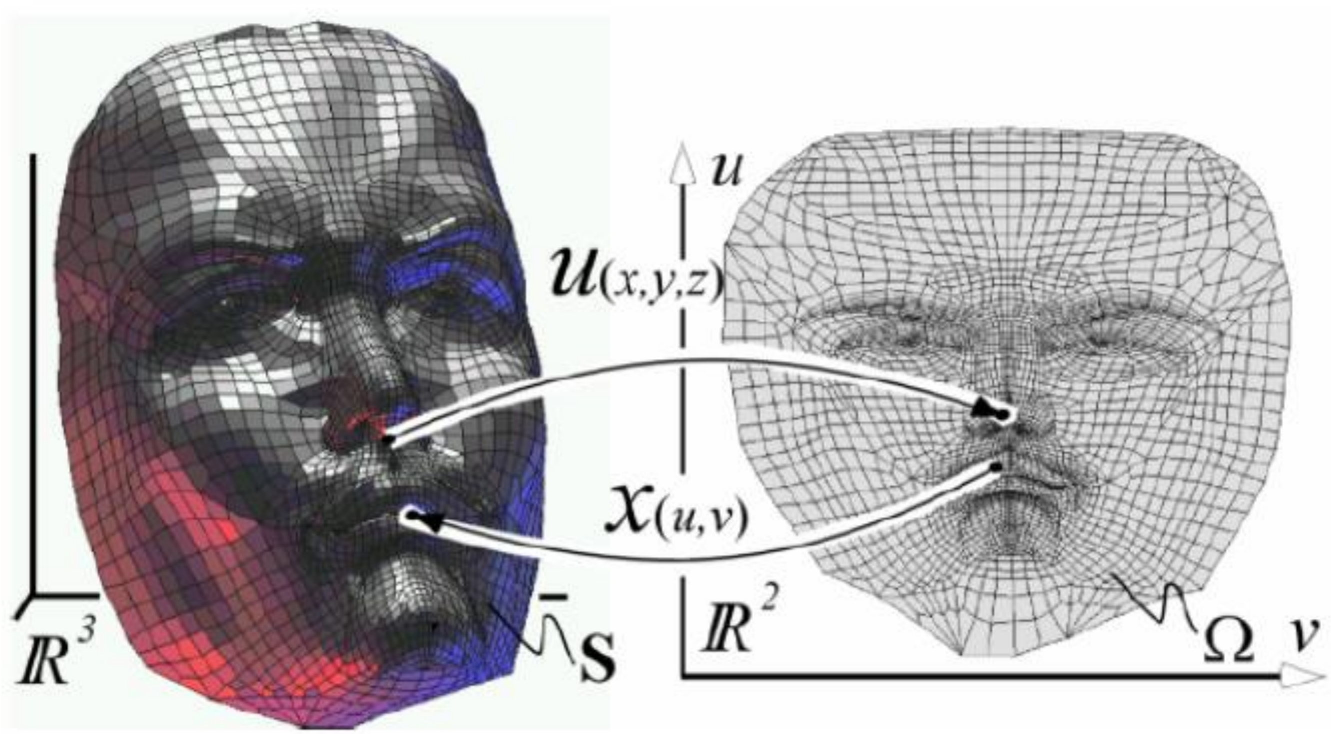
Mesh Parameterization



Mesh Parameterization

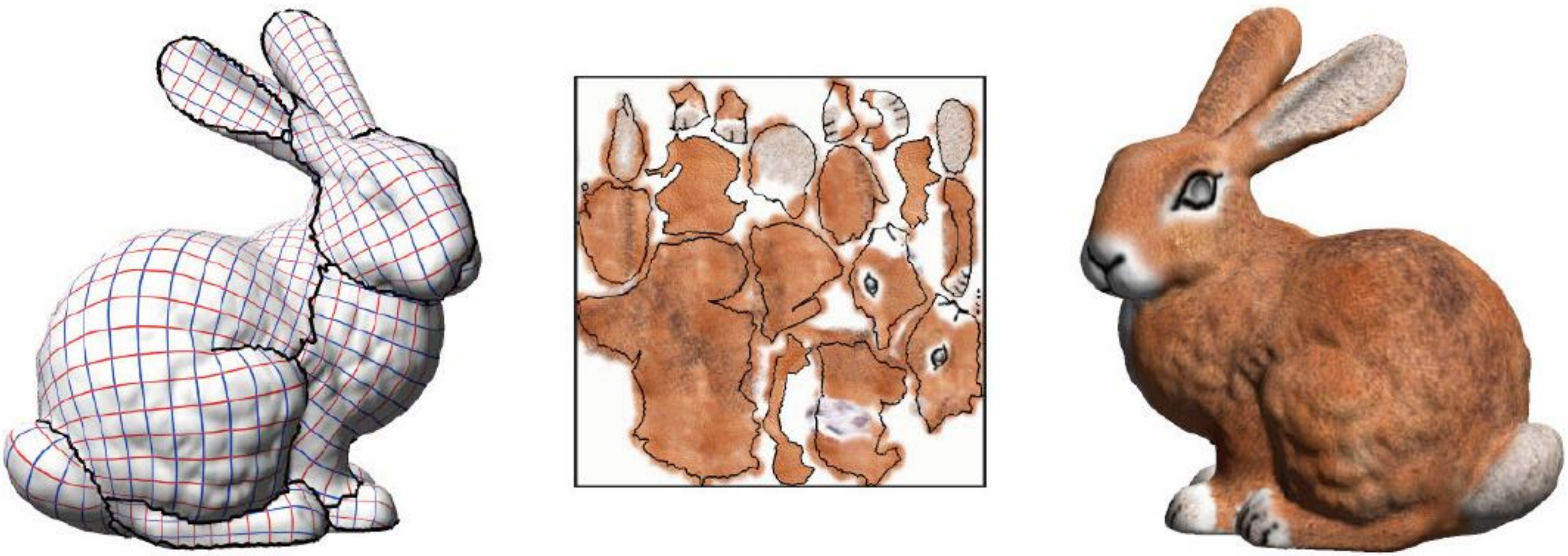
Problem Definition

- Input: a 3D triangular mesh surface
- Output: a 2D isomorphic triangulation

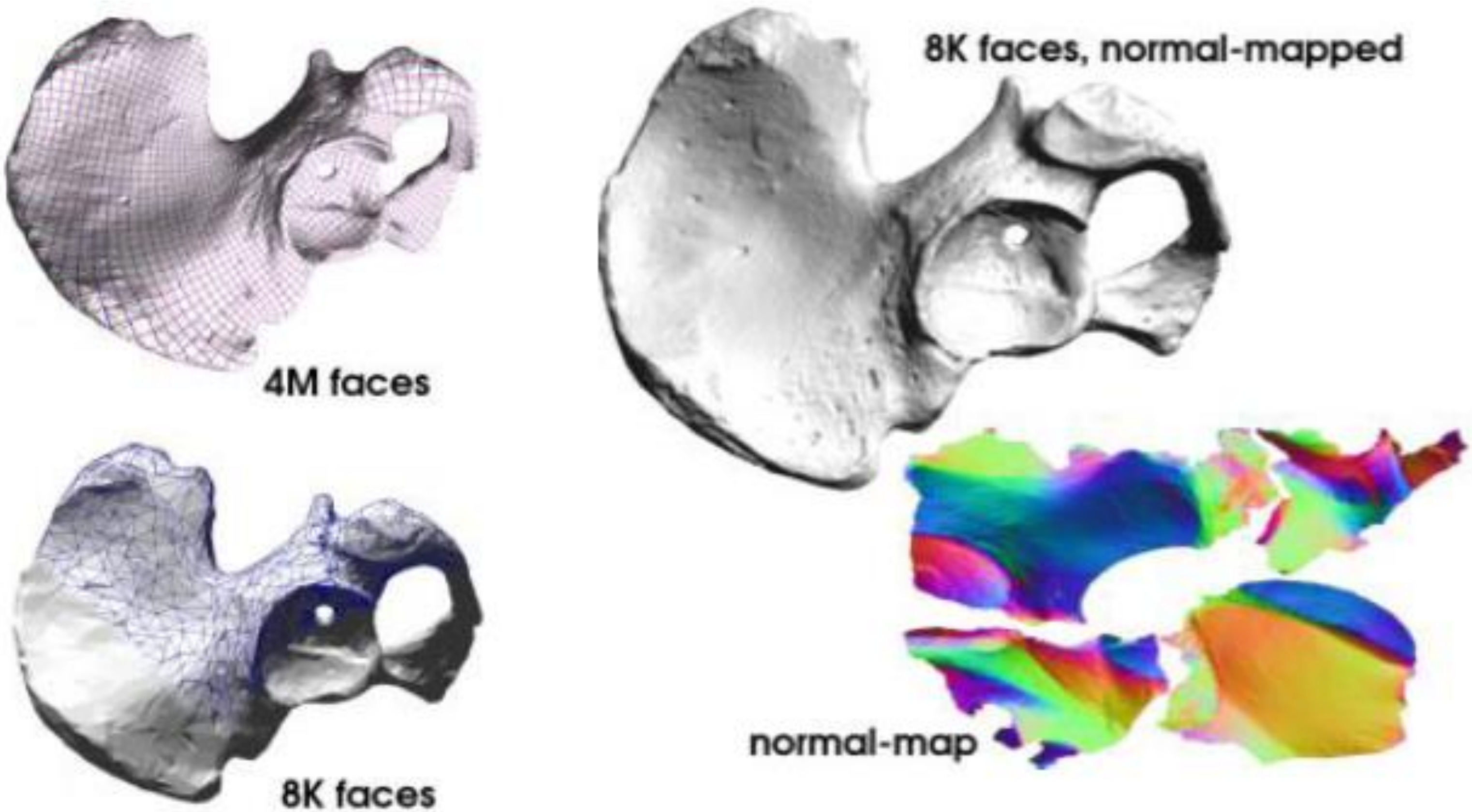


Applications

- Texture mapping

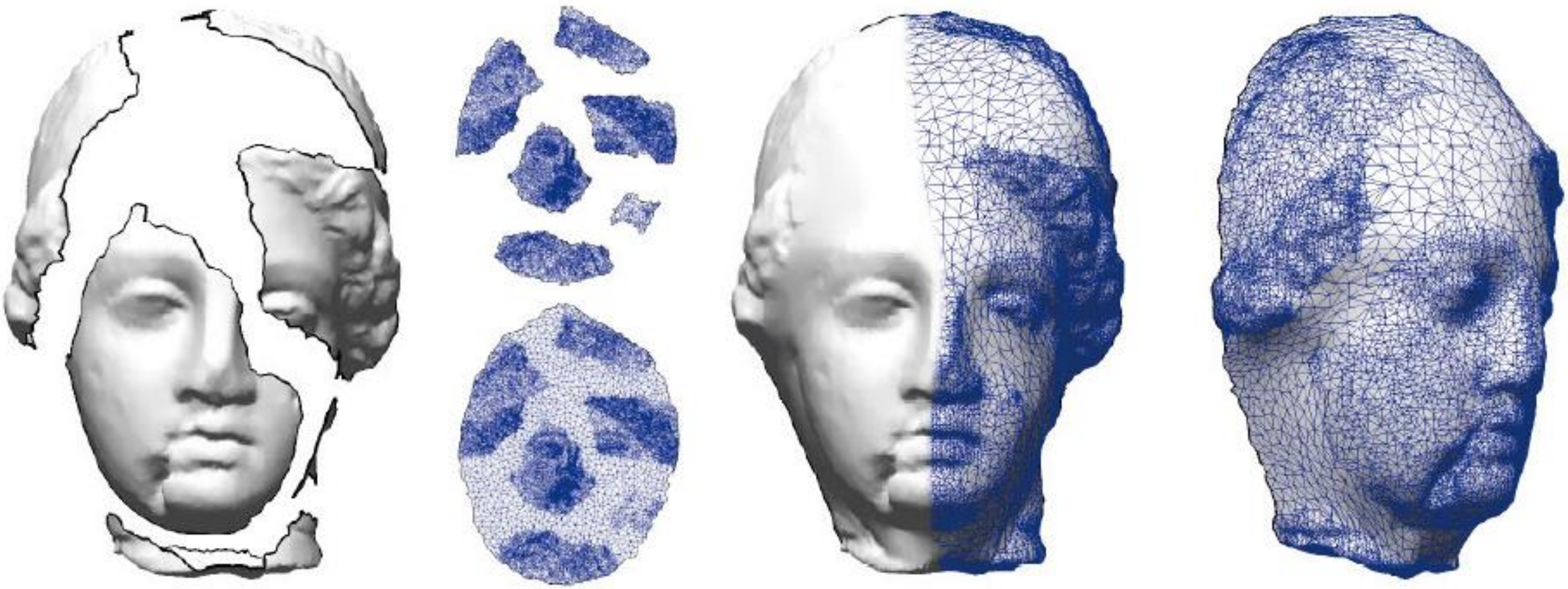


Normal Mapping



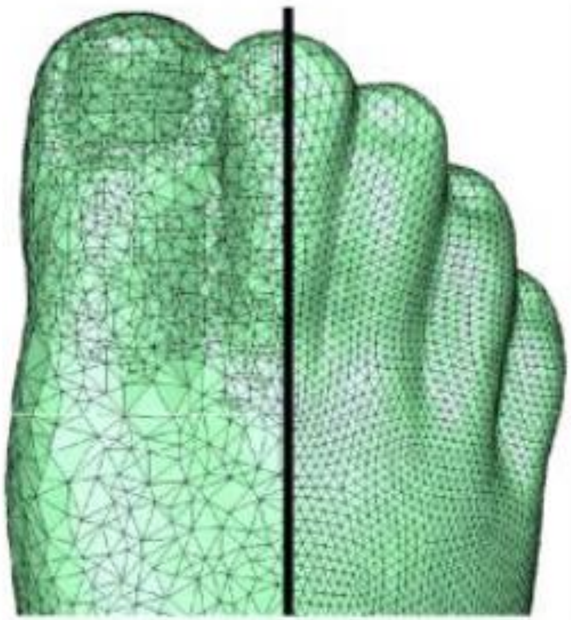
Applications

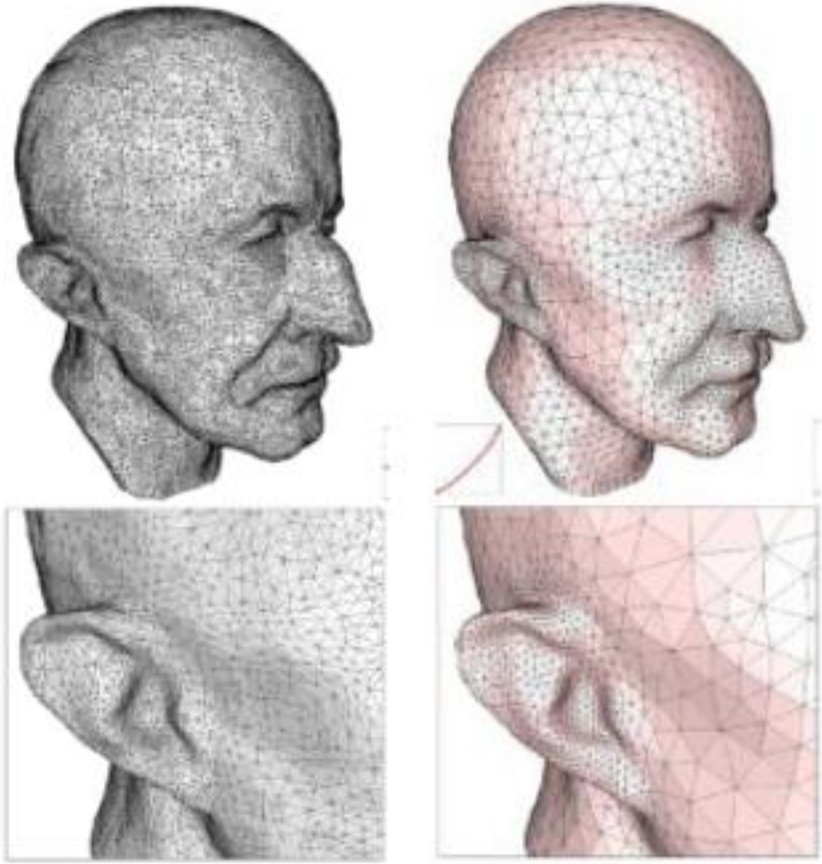
- Many operations are simpler on planar domain



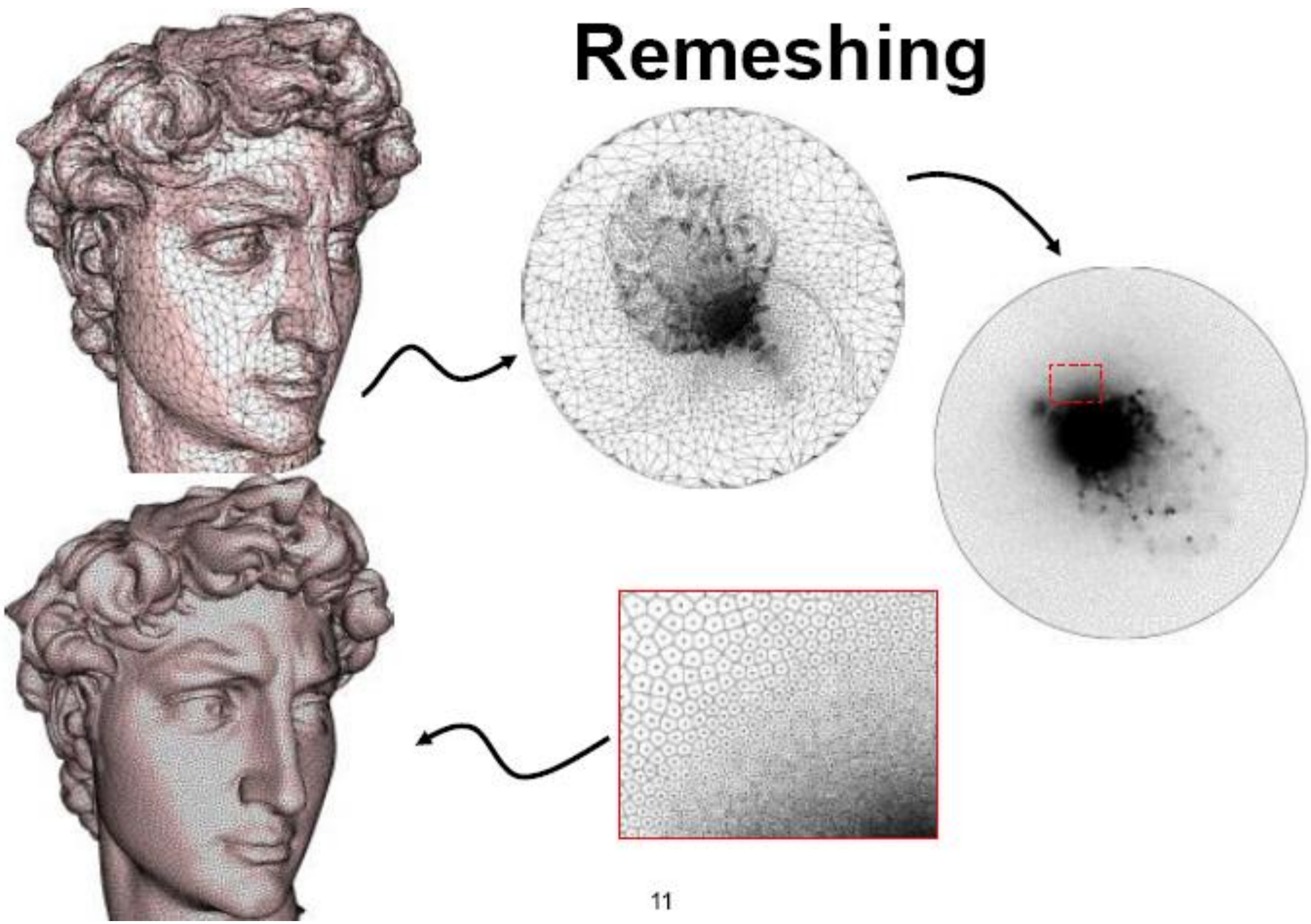
Applications

- Remeshing





Applications

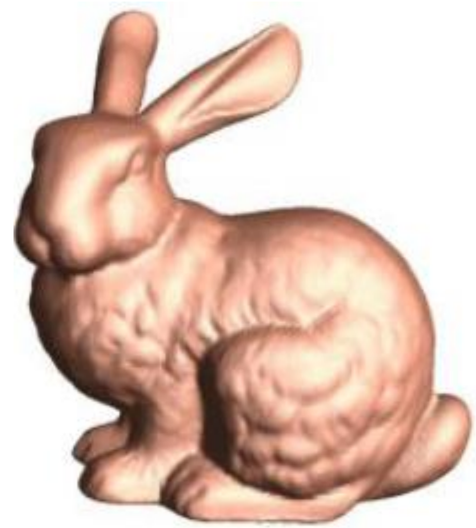


Applications

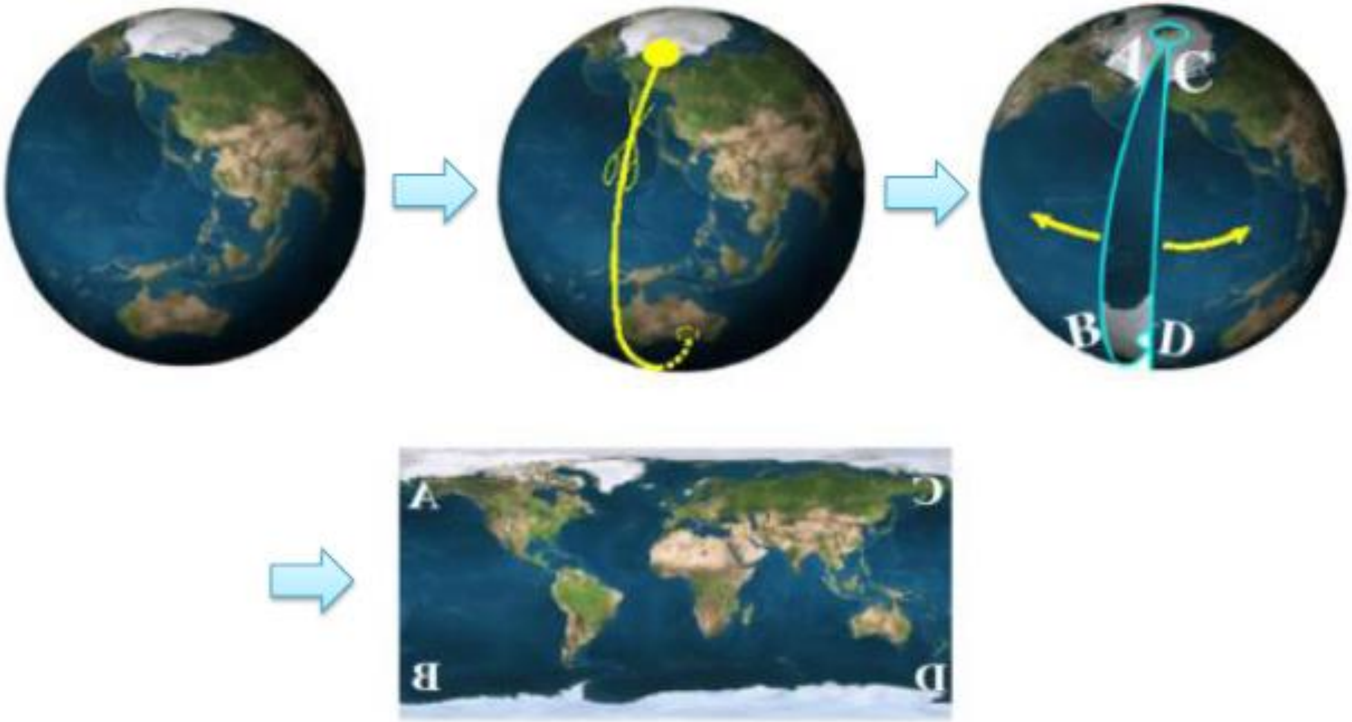
- Compression

Stanford Bunny

=



Unfolding the World



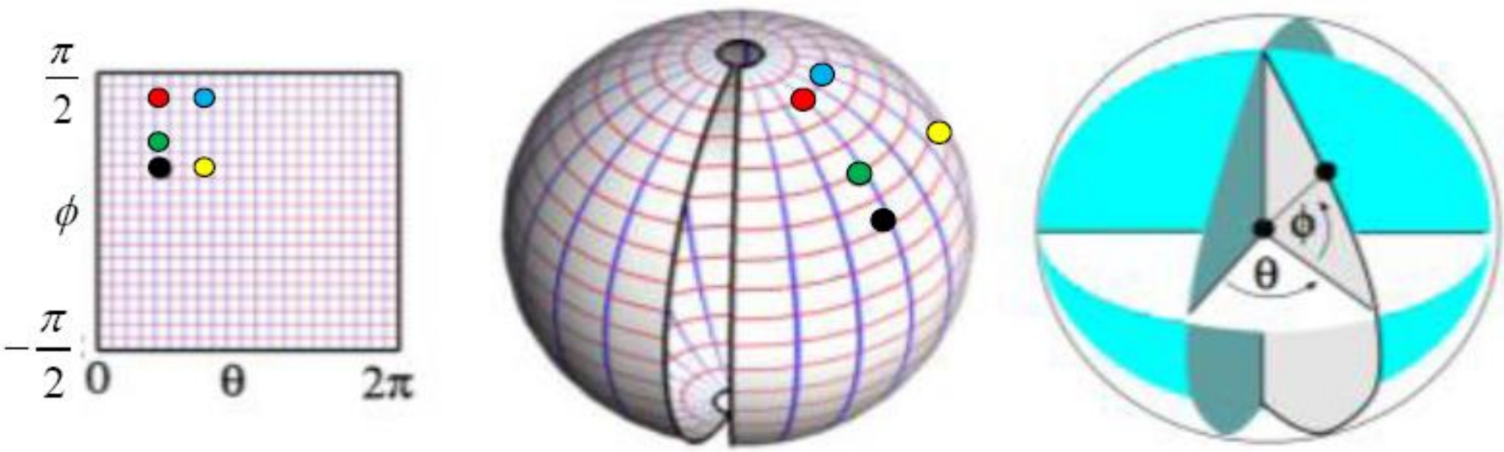
Spherical Coordinates

$$\theta \in [0, 2\pi), \phi \in [-\pi/2, \pi/2)$$

$$x(\theta, \phi) = R \cos\theta \cos\phi$$

$$y(\theta, \phi) = R \sin\theta \cos\phi$$

$$z(\theta, \phi) = R \sin\phi$$



Standard Map Projections

orthographic

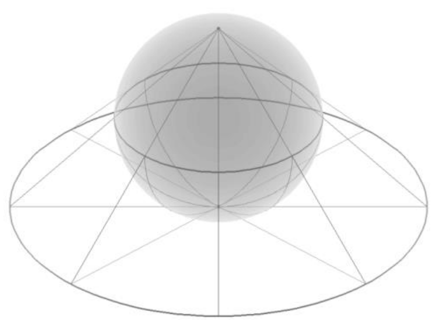
stereographic

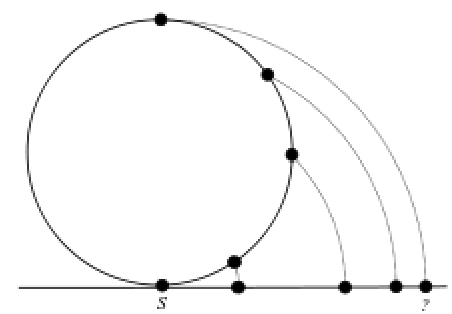
Mercator

Lambert

preserves angles = **conformal**

preserves area = **equiareal**





Mesh Parameterization

- *Q*: What is a **good** parameterization?
- ***A***: One that preserves all the basic geometry length, angles, area, …

  $\rightarrow$ **Isometric parameterization**

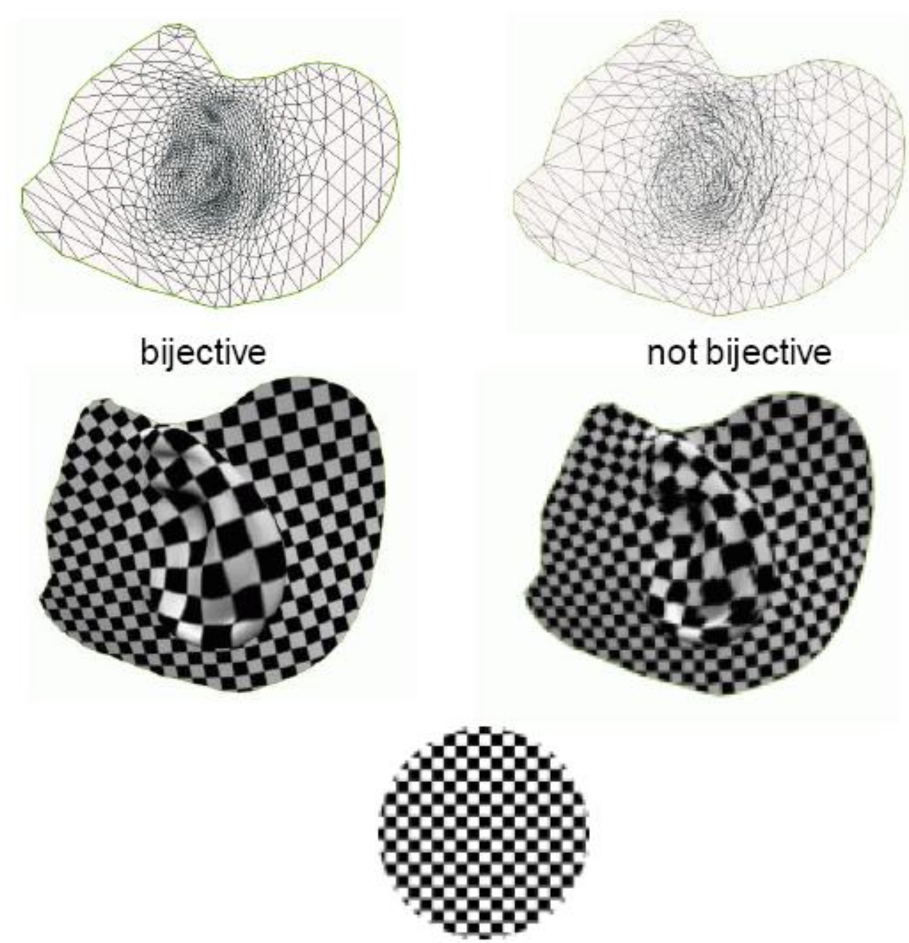
  But: possibly only for developable surfaces

  e.g. there will always be distortions!

  Try to keep the distortion as small as possible

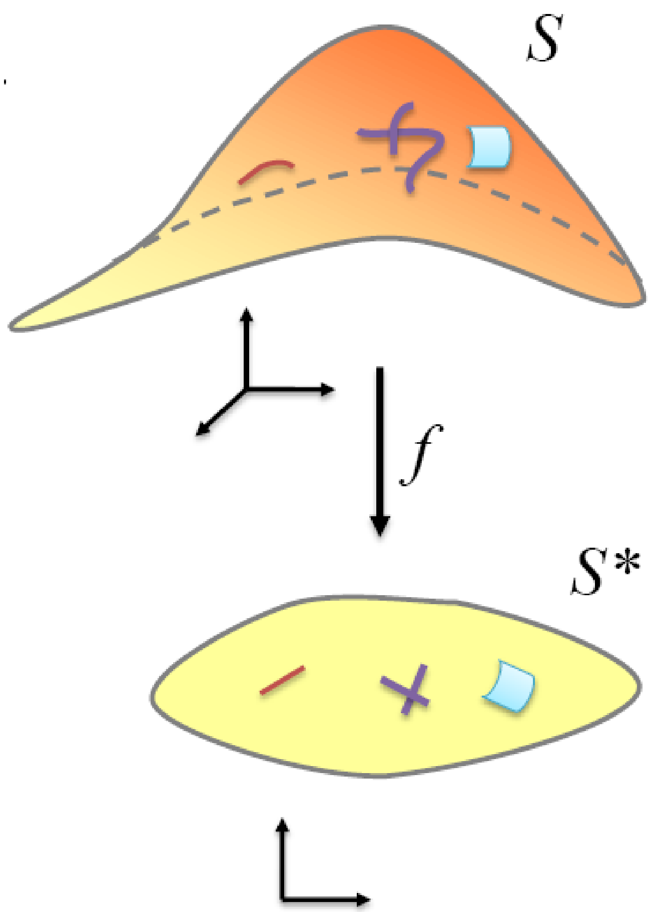
Desirable Properties

- Low distortion
- Bijective mapping
- Efficiently computable

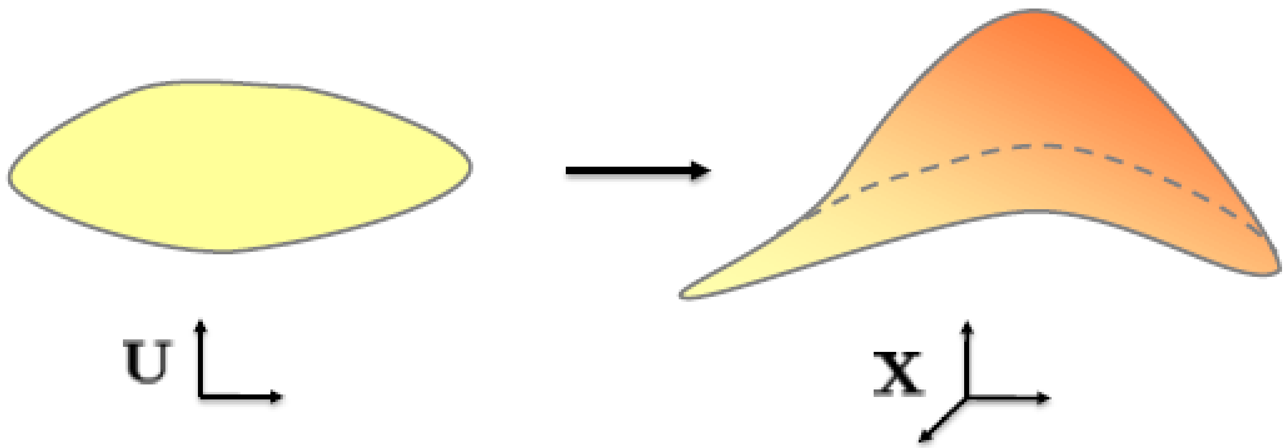


Definitions

- f is ***isometric*** (length preserving), if the *length* of any arc on S is preserved on S^*.
- f is ***conformal*** (angle preserving), if the *angle* of intersection of every pair of intersecting arcs on S is preserved on S^*.
- f is ***equiareal*** (area preserving) if the *area* of an area element on S is preserved on S^*.



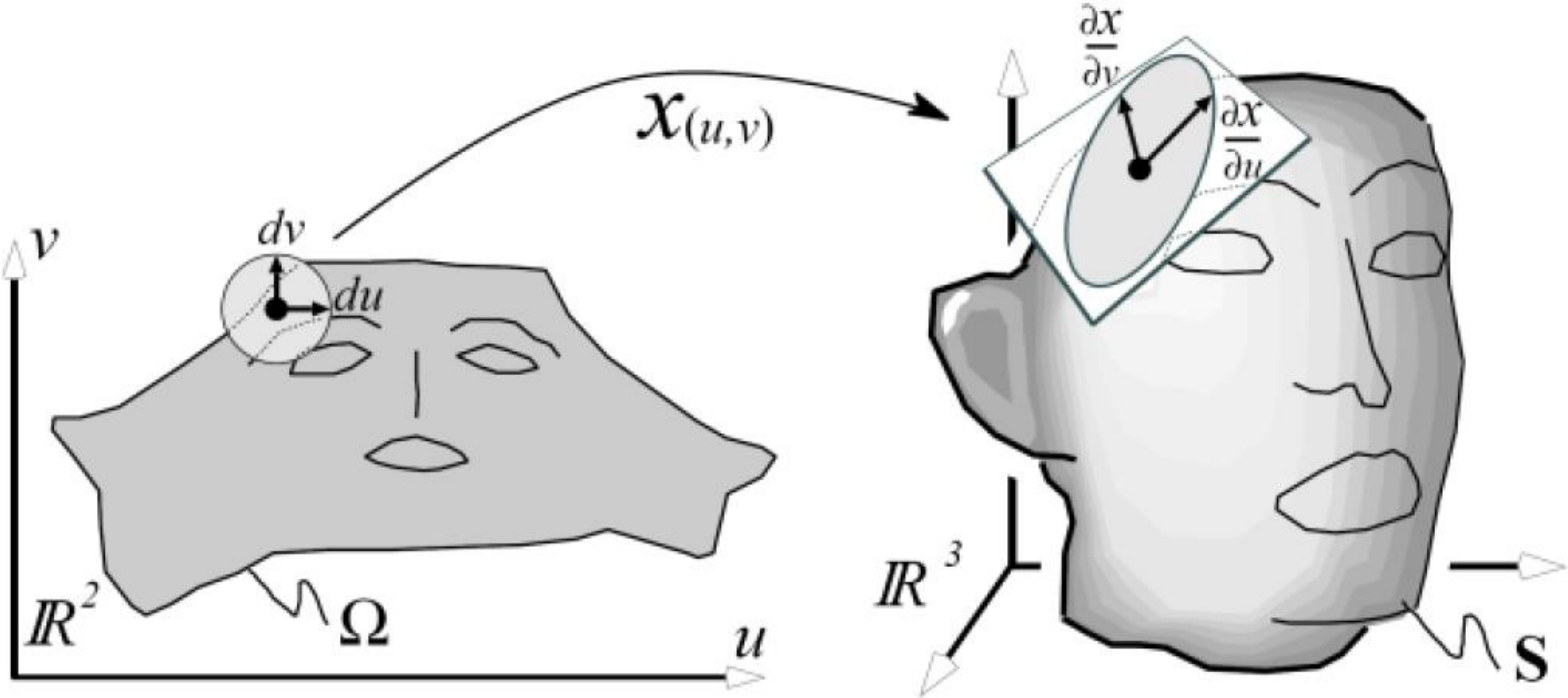
Distortion Analysis



Jacobian

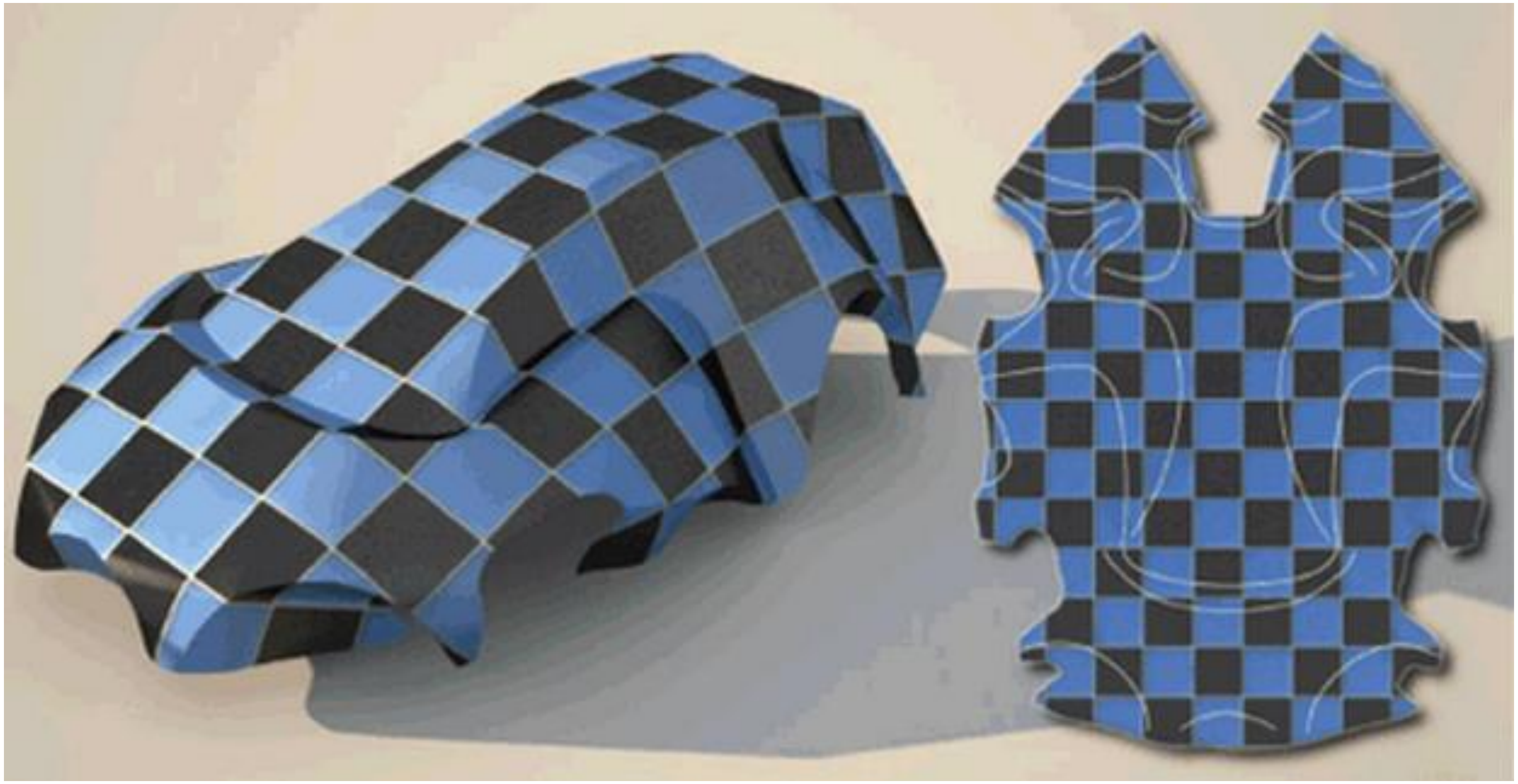
$$\mathbf{X}(u,v) = \left(\begin{array}{c} x(u,v) \\ y(u,v) \\ z(u,v) \end{array} \right) \qquad J = \left(\begin{array}{cc} x_u & x_v \\ y_u & y_v \\ z_u & z_v \end{array} \right)$$

Distortion Analysis

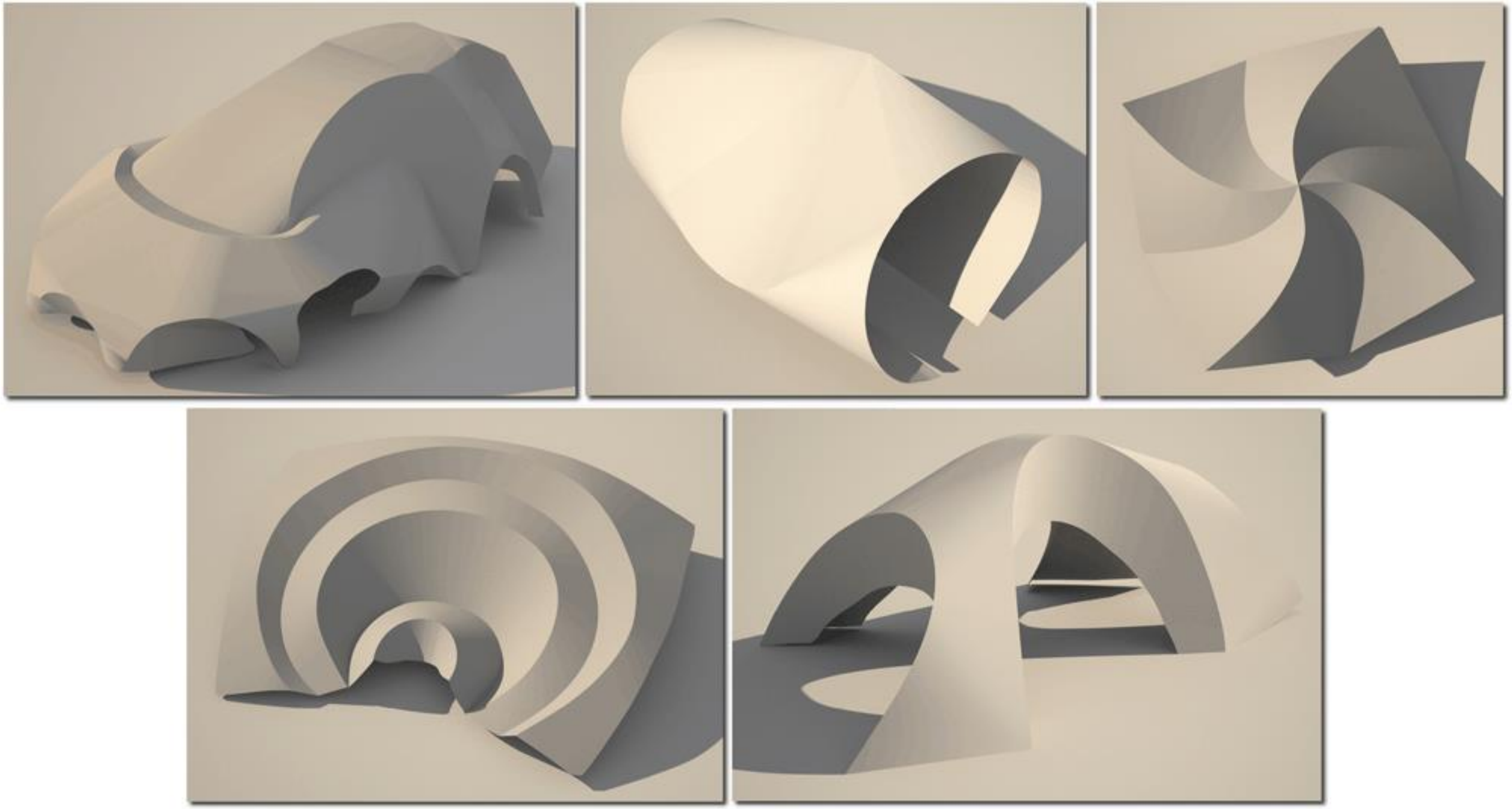


Isometric Maps

- Developable surface

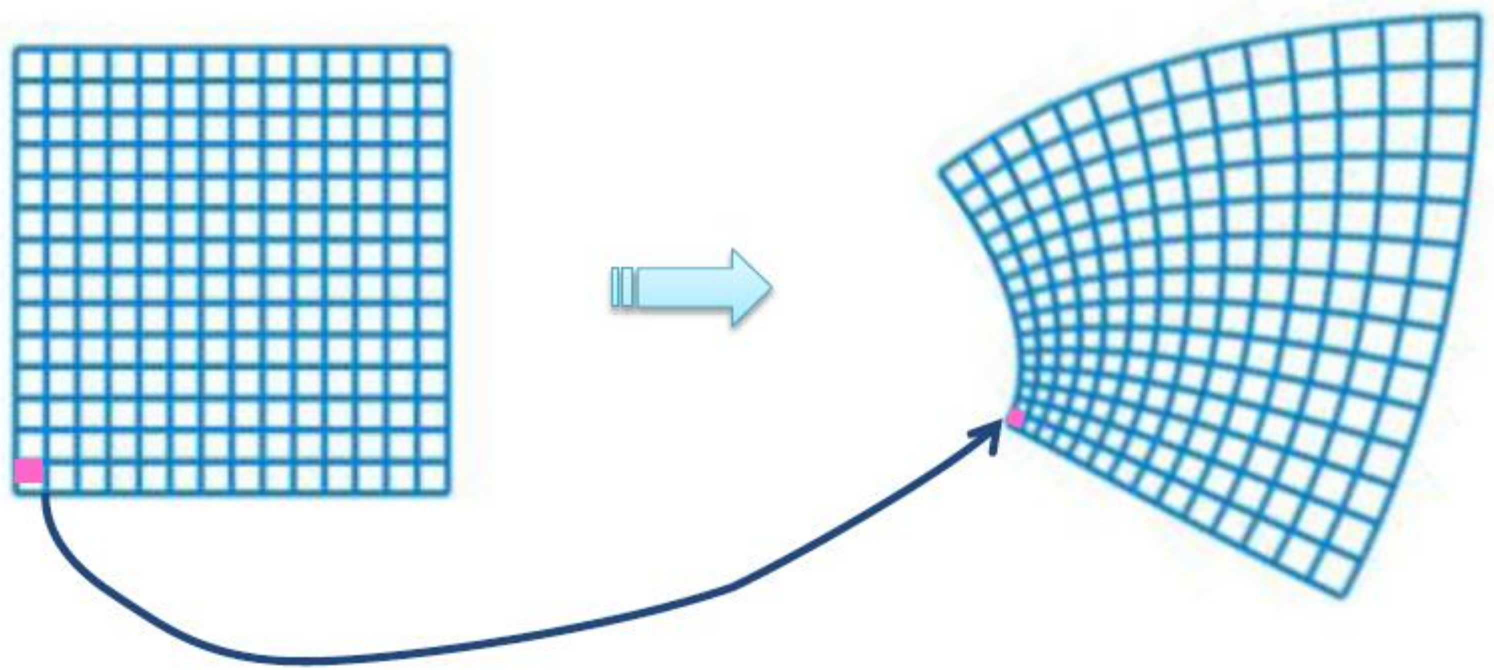


Isometric Maps



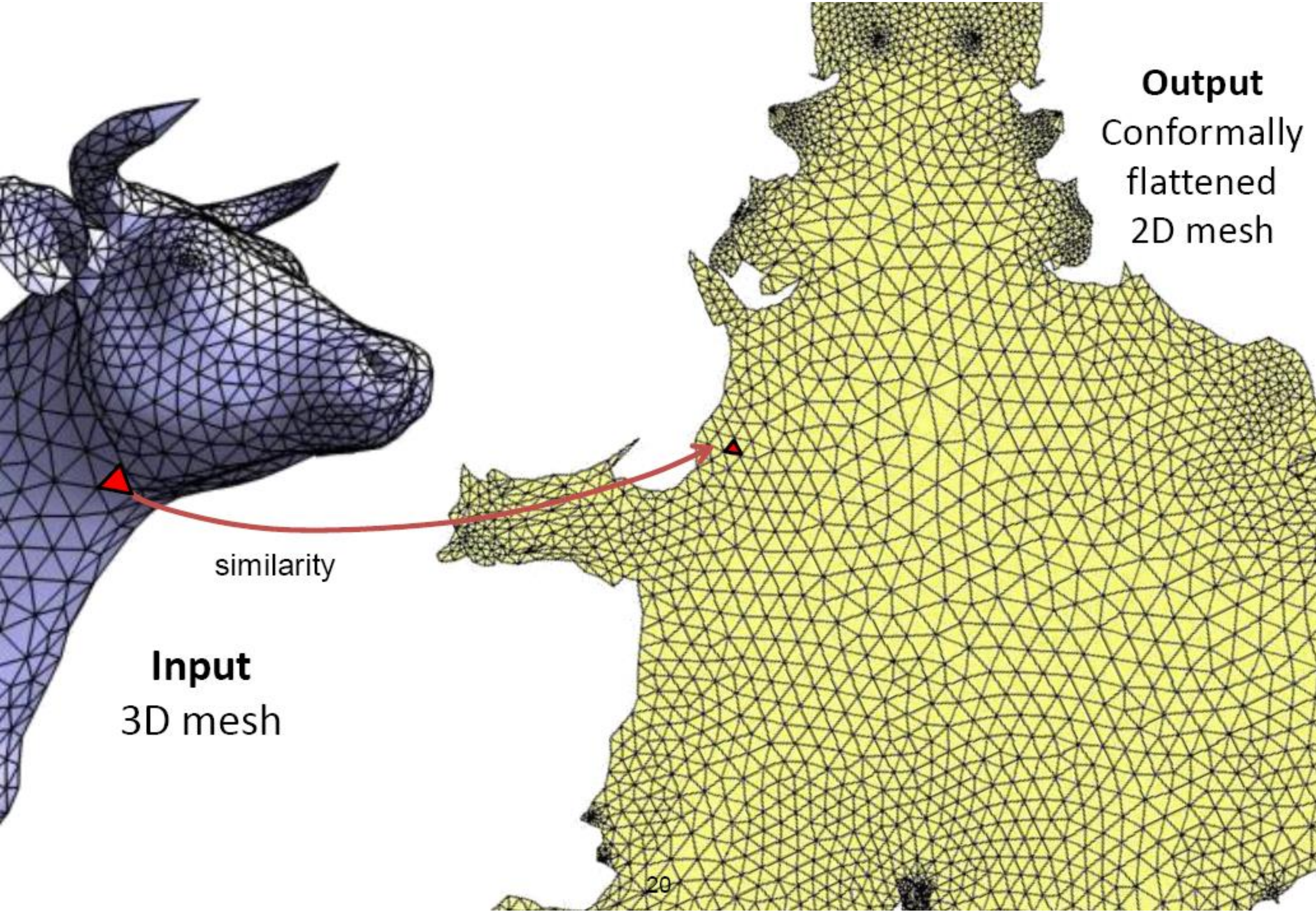
- Martin Kilian, Simon Floery, **Zhonggui Chen**, Niloy J. Mitra, Alla Sheffer, Helmut Pottmann, **CURVED FOLDING,** ACM SIGGRAPH 2008

Conformal Map

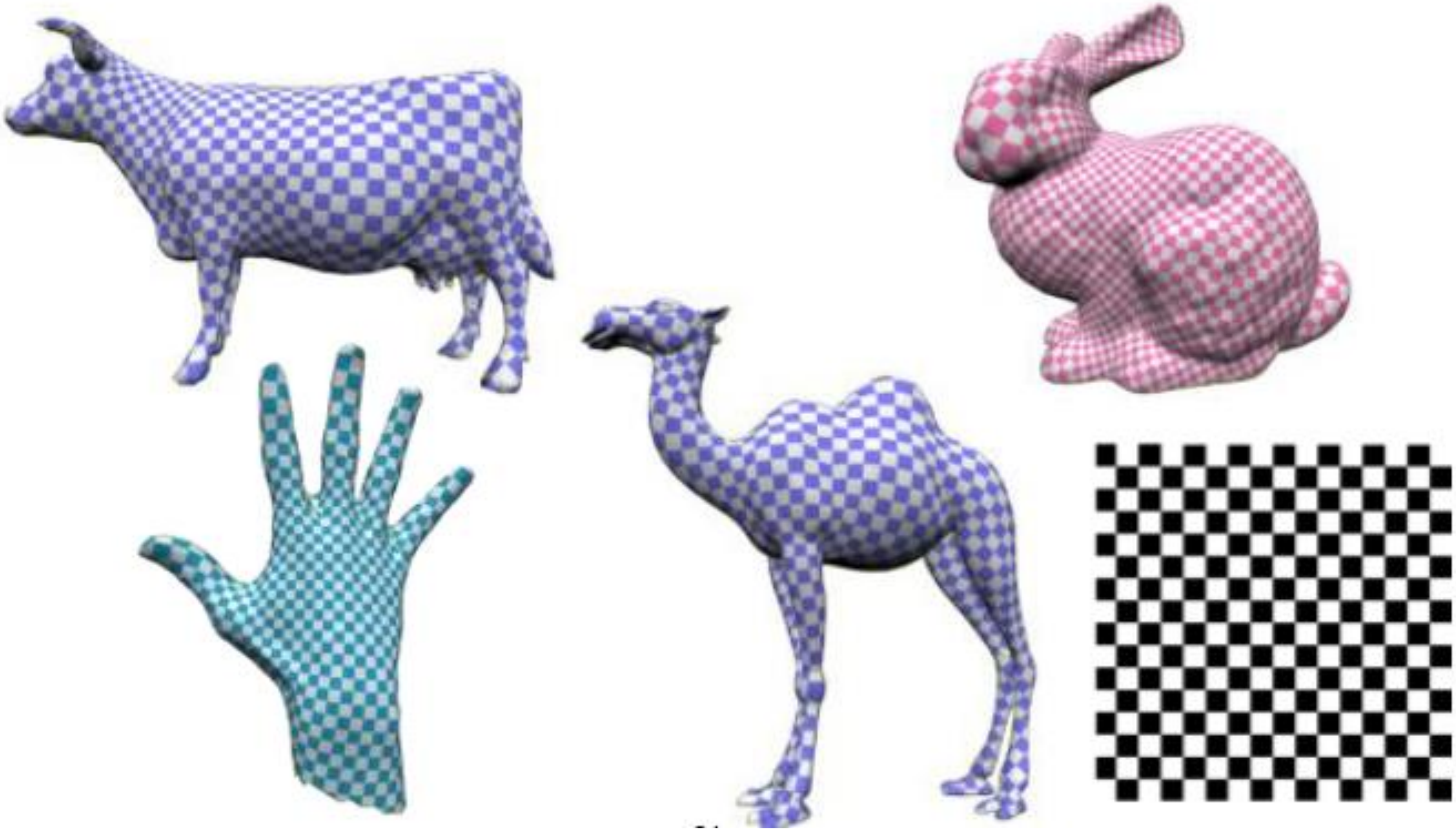


Similarity = Rotation + Scale

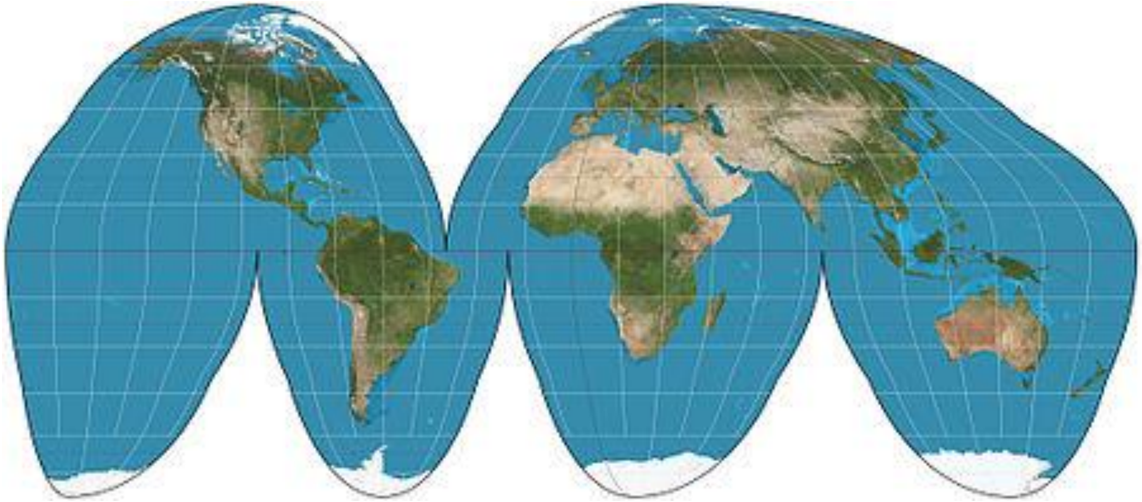
Preserves angles

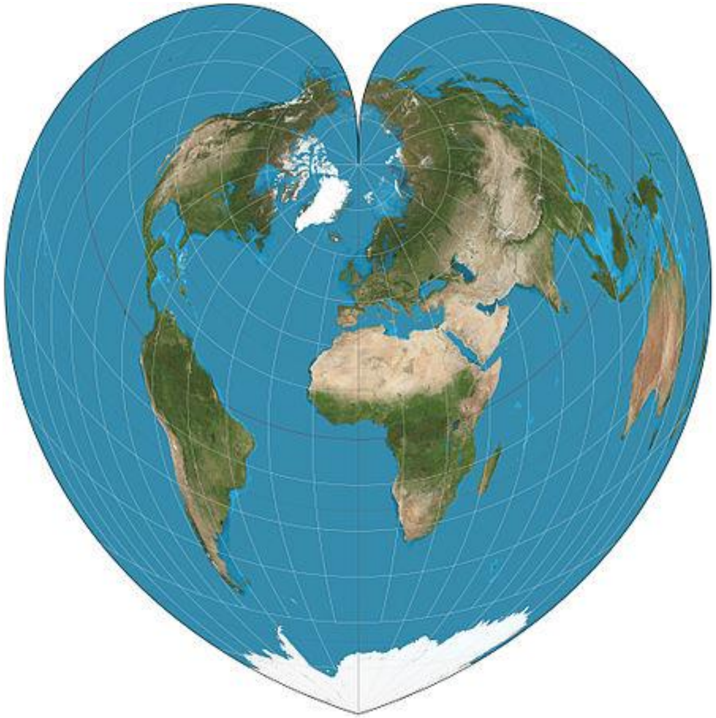

Output
Conformally flattened 2D mesh
similarity
Input
3D mesh

Conformal Parameterization



Equiareal Maps





Relationships

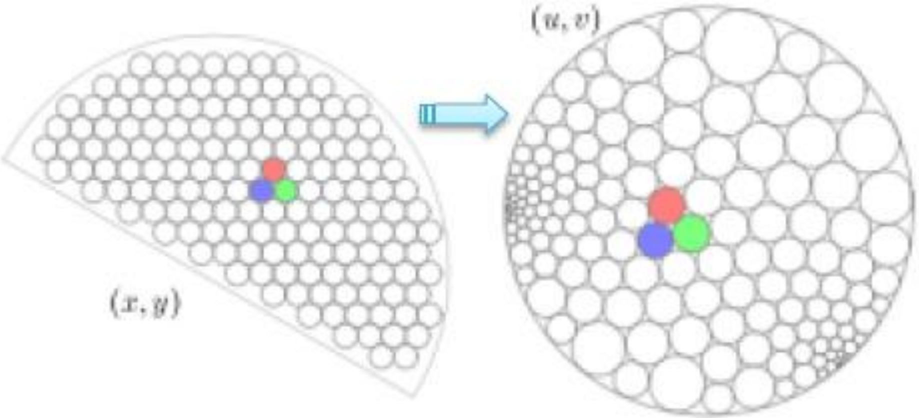
- **Theorem:** Every isometric mapping is conformal and equiareal, and vice versa.

$$\text{isometric} \Longleftrightarrow \text{conformal} + \text{equiareal}$$

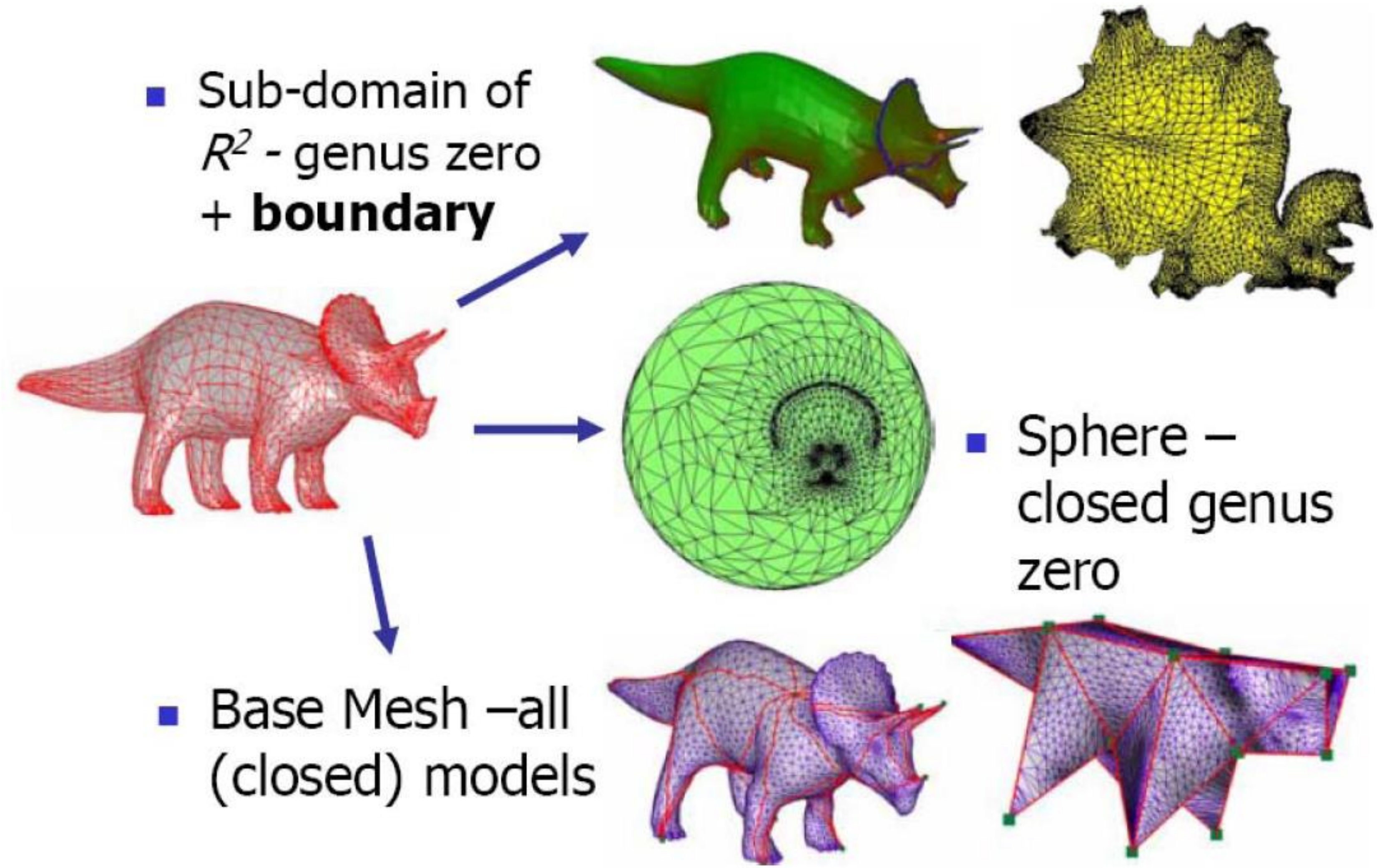
- Isometric is ideal… but rare. In practice, we use:
  - –conformal
  - –equiareal
  - –some balance between the two

Riemann Conformal Mapping Theorem

Any two simply connected compact planar regions can be mapped conformally onto each other.

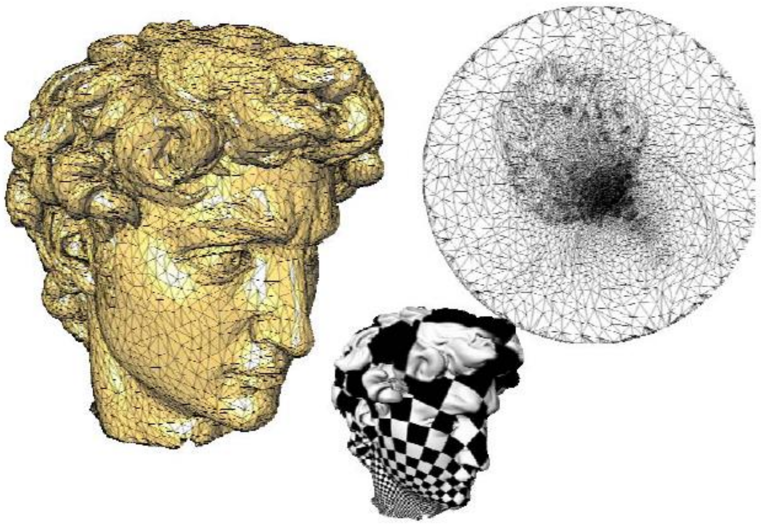


Mesh Parameterization
-- Typical Domains

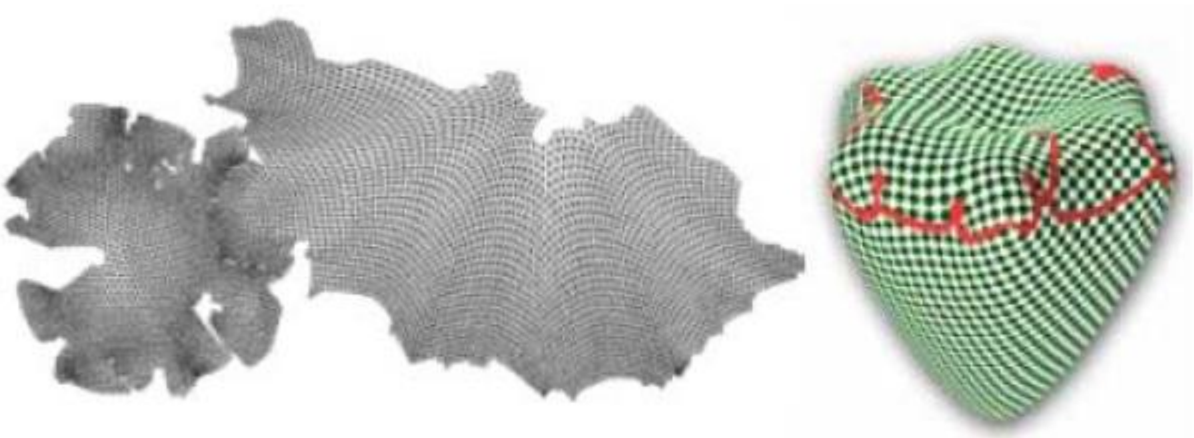


Planar Parameterization

- Fixed boundary

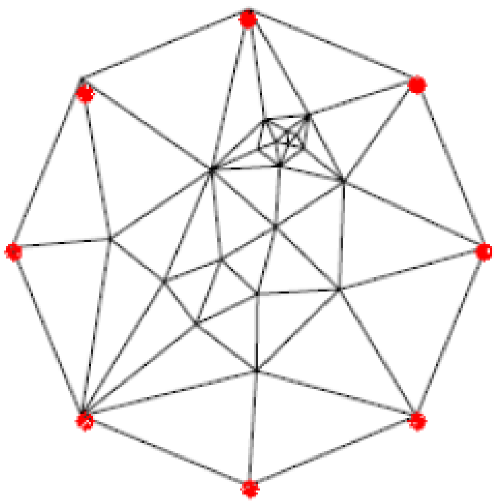


- Non-fixed boundary

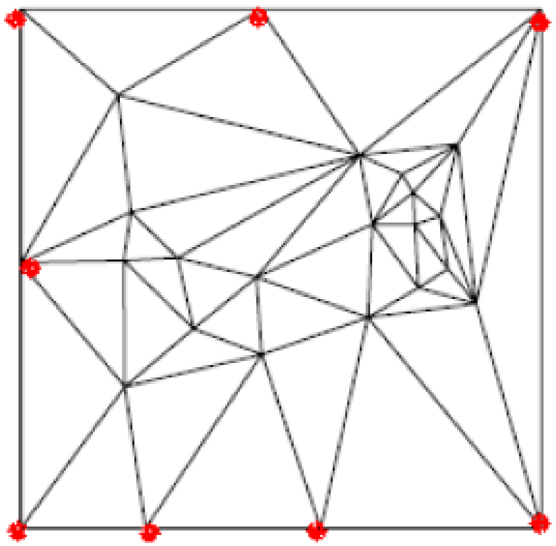


Fixed Boundary

- Fixing the boundary of the mesh onto

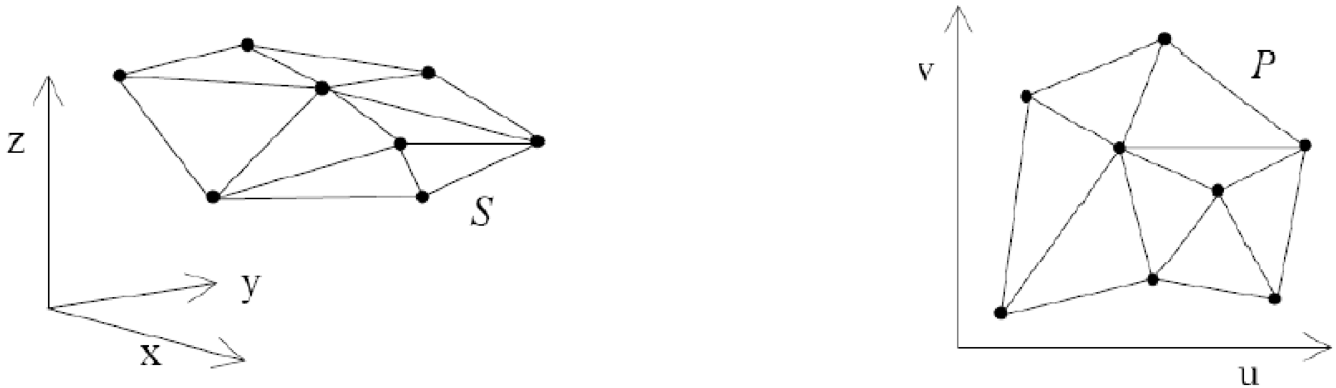


an unit circle



an unit square

Linear Methods: Idea



- For interior mesh points:

$$p_i = \sum_{\{j:(i,j)\in \text{edges}\}} \lambda_{i,j_k} p_k, \quad \sum_{k=1}^{d_i} \lambda_{i,j_k} = 1, \quad \lambda_{i,j_k} > 0$$

=> Forming a sparse linear system

2D Barycentric Embeddings

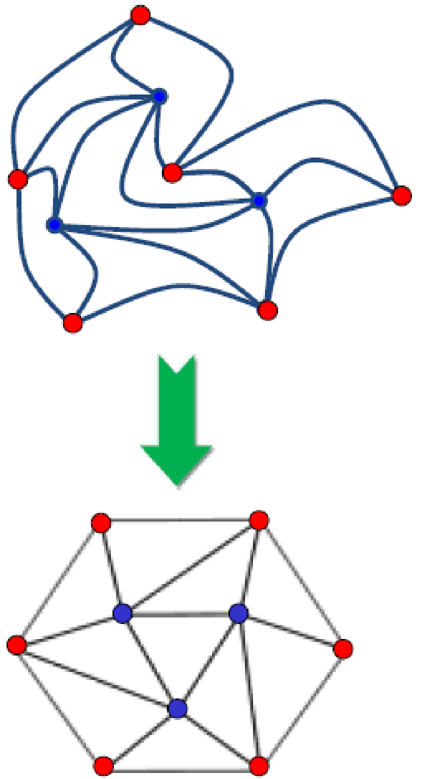
- Fix 2D boundary to convex polygon
- Define embedding as a solution of

$$Wx = b_x \qquad Wy = b_y$$

$$w_{ij} = \begin{cases} \boxed{> 0} & (i,j) \in E \\ -\sum_{j \neq i} w_{ij} & (i,i), i \notin B \\ 1 & (i,i), i \in B \\ 0 & otherwise \end{cases}$$

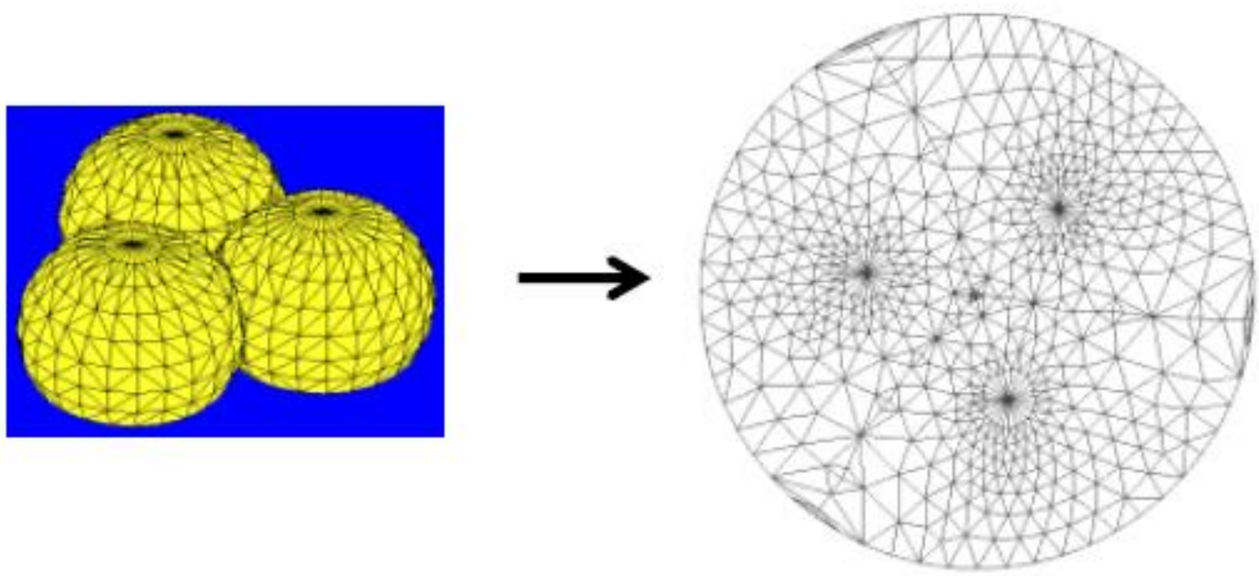
$$W \text{ is } symmetric: \; w_{ij} = w_{ji}$$

- Weights w_{ij} control triangle shapes



Why it Works

- Theorem [Tutte,63], [Maxwel,1864]
  - If $G = \langle V,E \rangle$ is a 3-connected planar graph (triangular mesh) then any **barycentric** embedding is a valid embedding

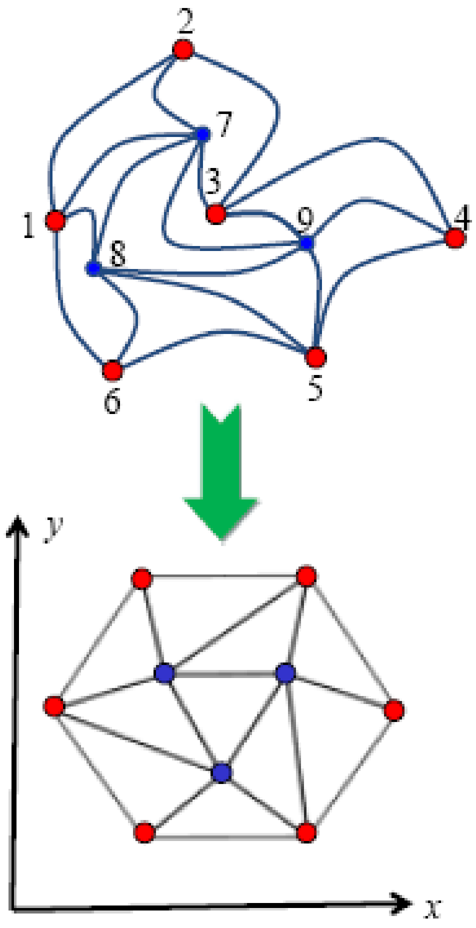


Example

$$\boxed{w_{ij} = 1}$$

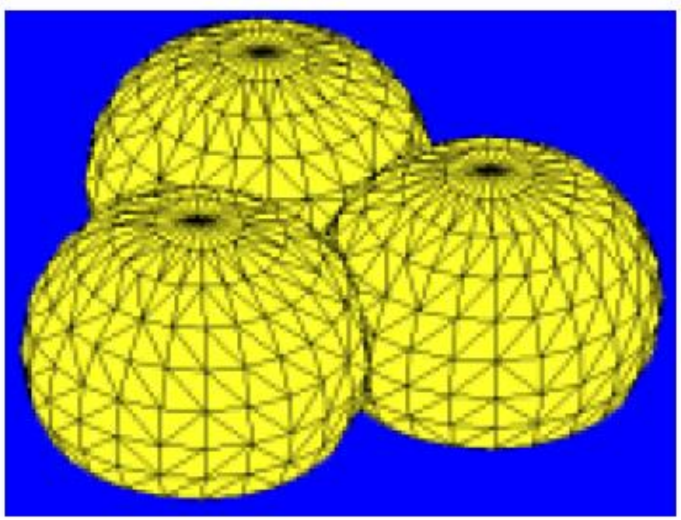
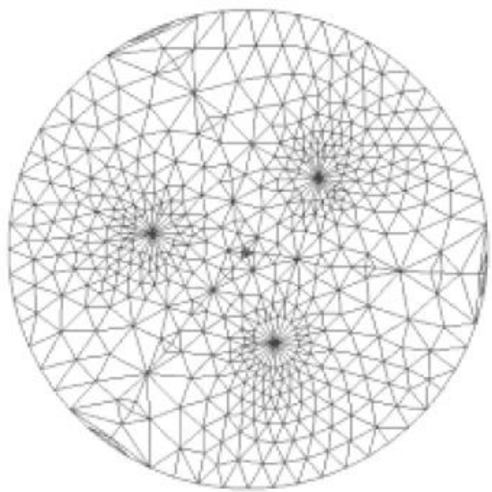
Laplacian Matrix

$$W = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & -5 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & -5 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & -5 \end{pmatrix} \quad b_x = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 3 \\ 2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad b_y = \begin{pmatrix} 2 \\ 3 \\ 3 \\ 2 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



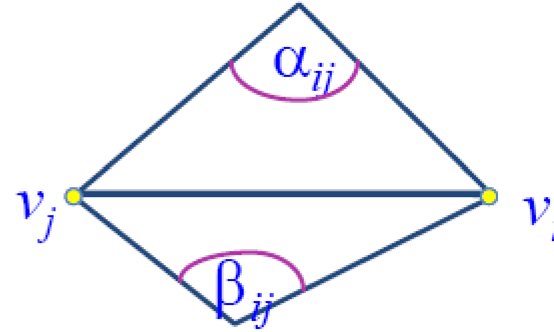
Choices of the Weights--Barycentric Formulation

- Uniform Weights
  - No shape information
  - Fastest to compute and solve

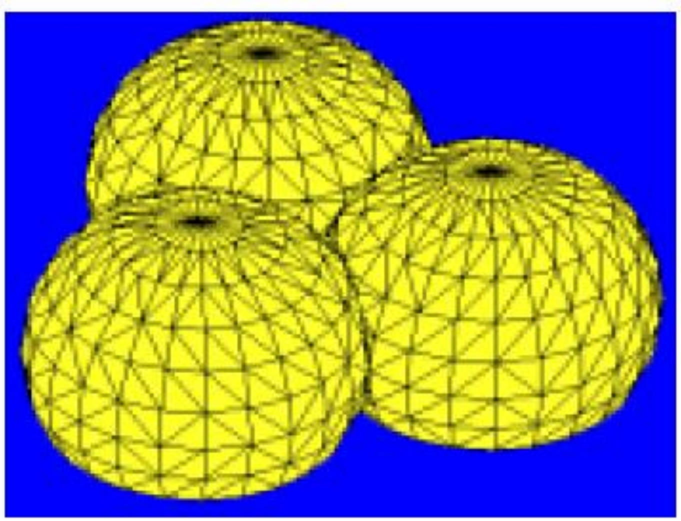



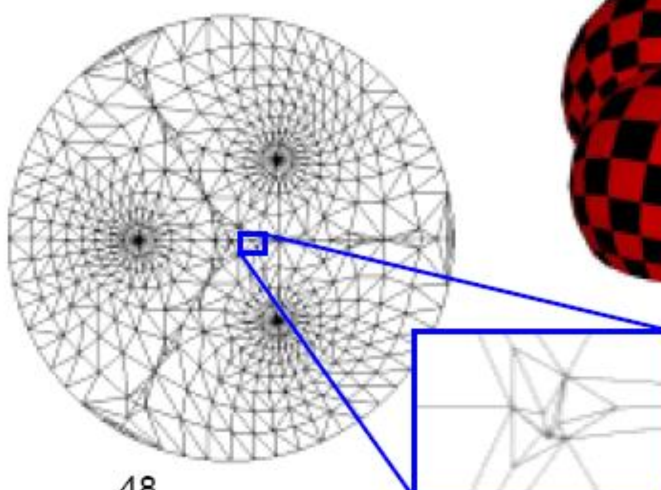

Harmonic Weights

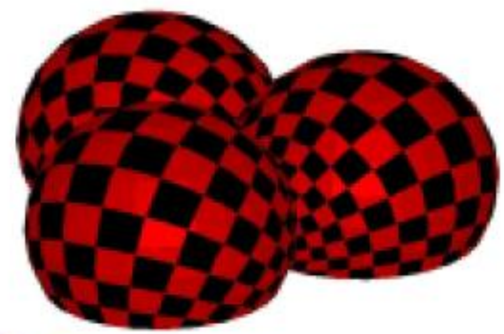
$$\boxed{w_{ij} = \frac{\cot(\alpha_{ij}) + \cot(\beta_{ij})}{2}}$$



- Weights can be negative –not always valid
- Weights depend only on angles -close to conformal

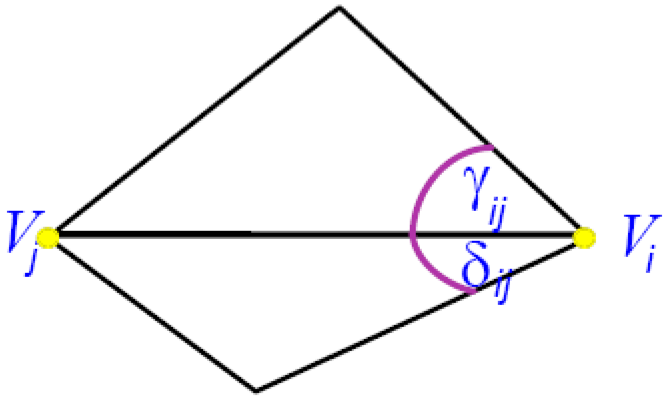




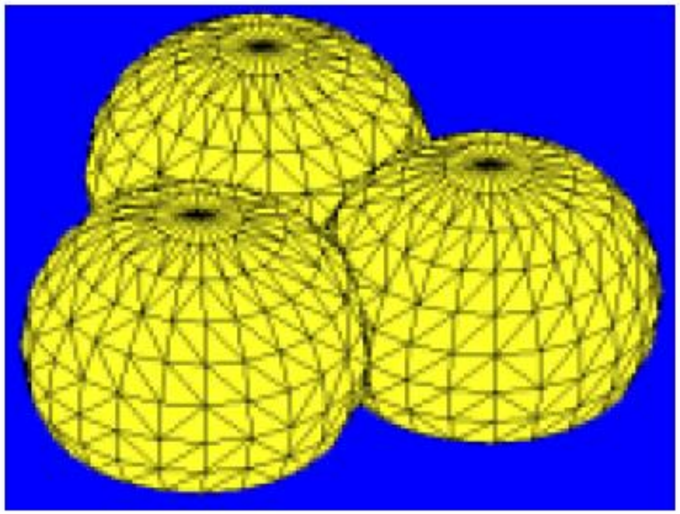


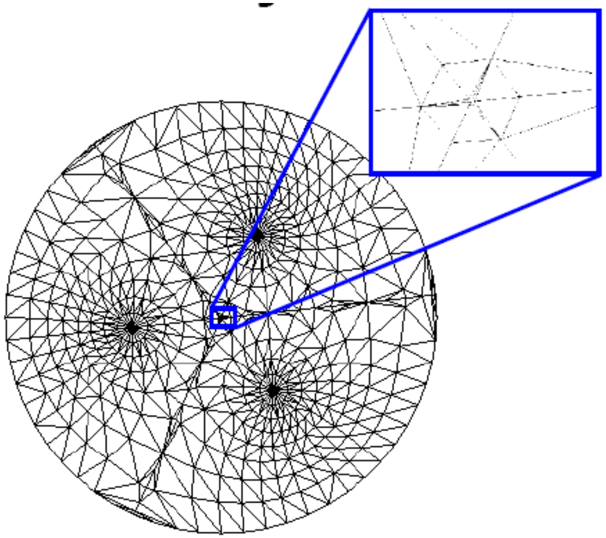
Mean-Value Weights

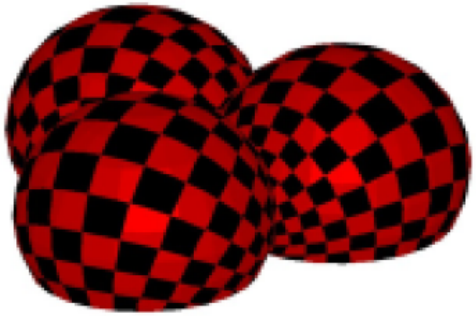
$$w_{ij} = \frac{\tan(\gamma_{ij}/2) + \tan(\delta_{ij}/2)}{2\,||V_i - V_j||}$$



- No negative weights –always

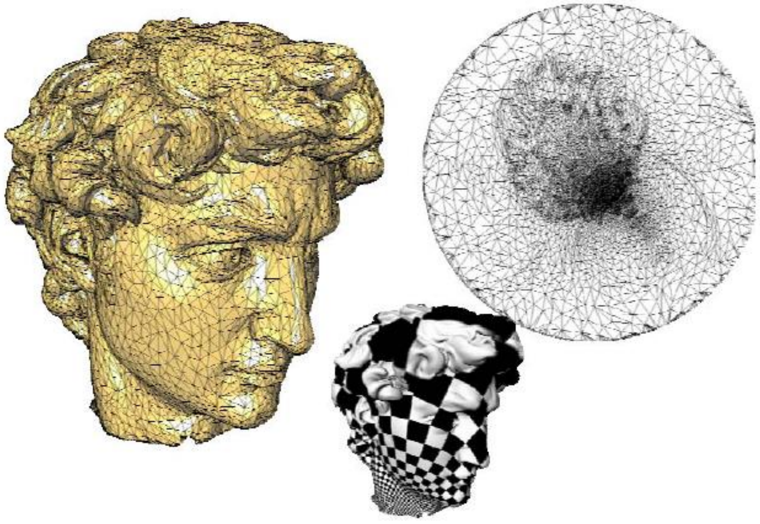


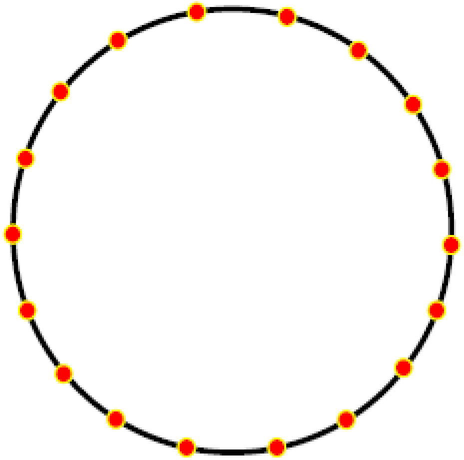




Fixing the Boundary

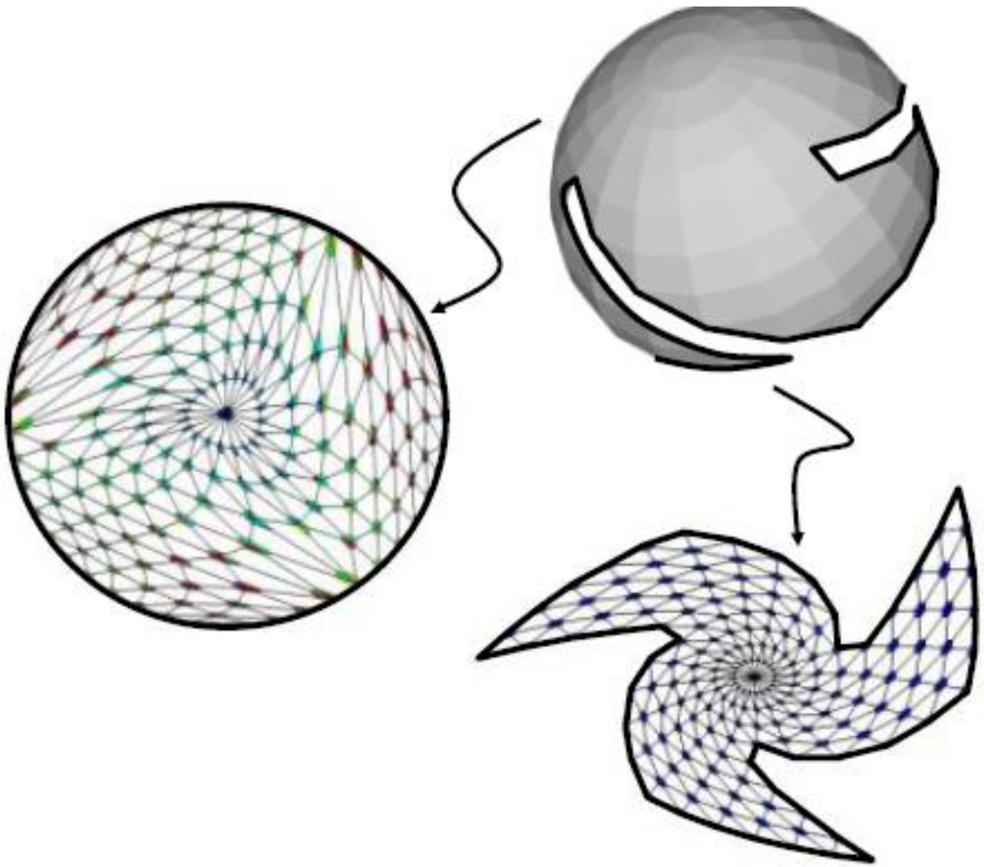
- Simple convex shape (triangle, square, circle)
- Distribute points on boundary
  - Use chord length parameterization
- Fixed boundary can create high distortion



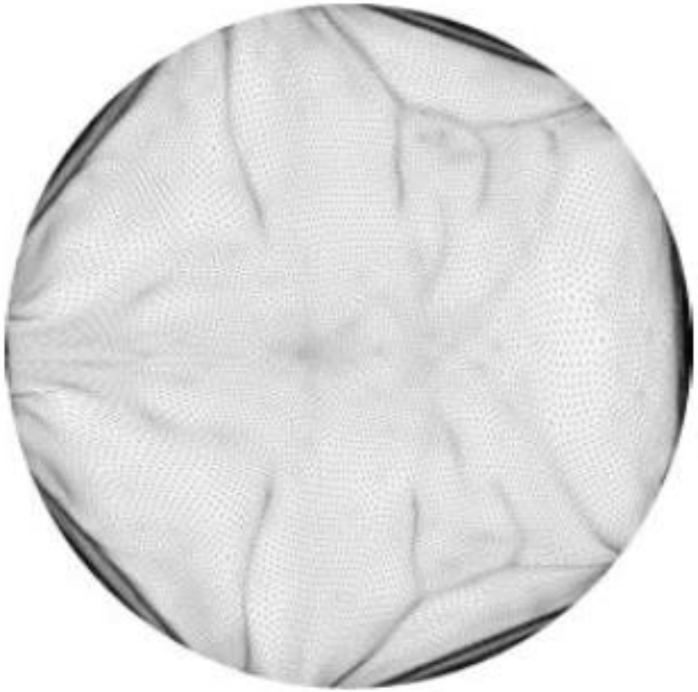


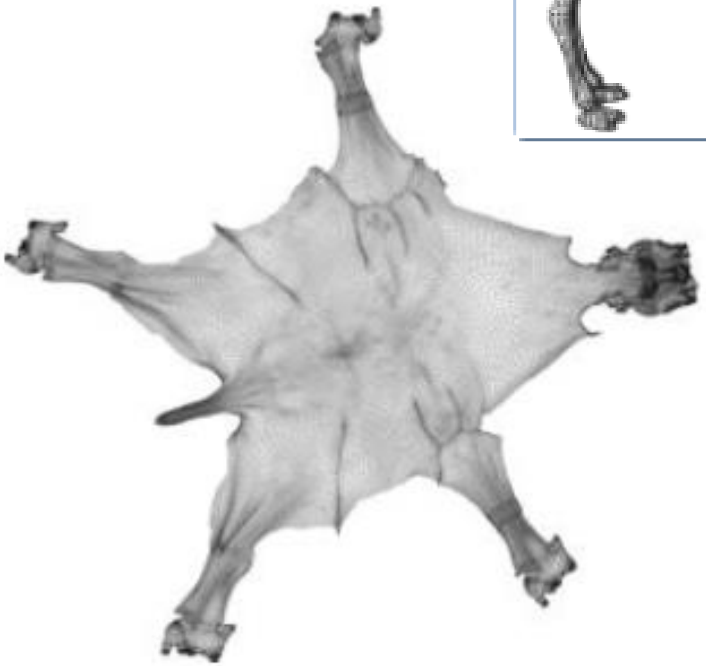
Non-Convex Boundary

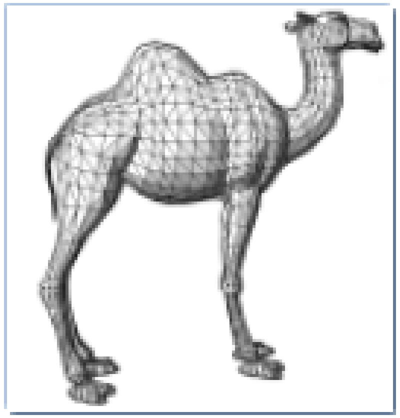
- Convex boundary creates significant distortion
- "Free" boundary is better



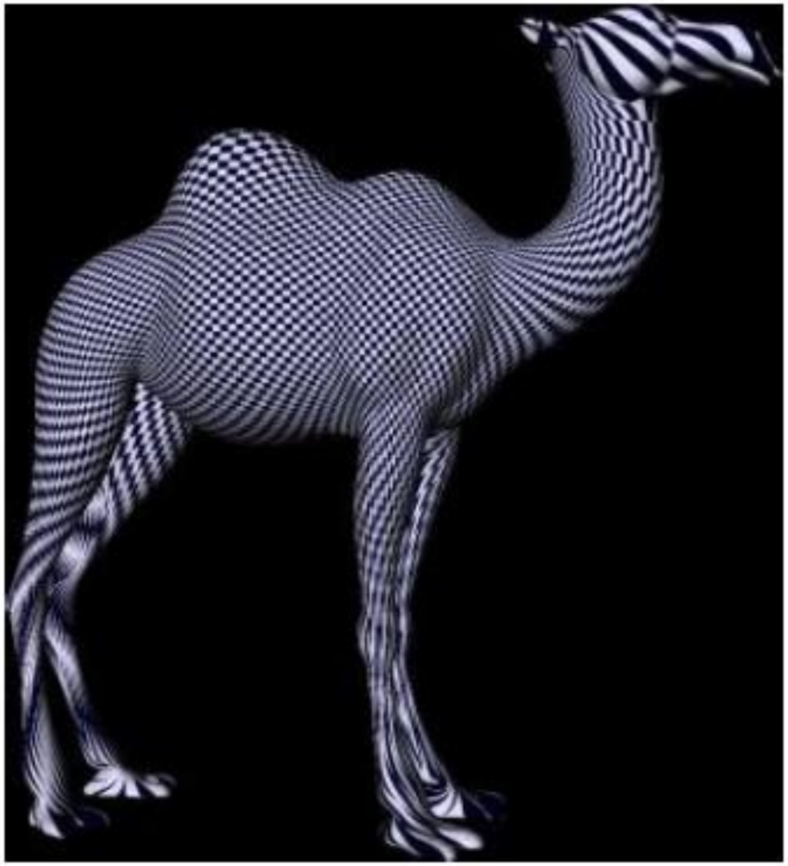
Fixed vs Free Boundary

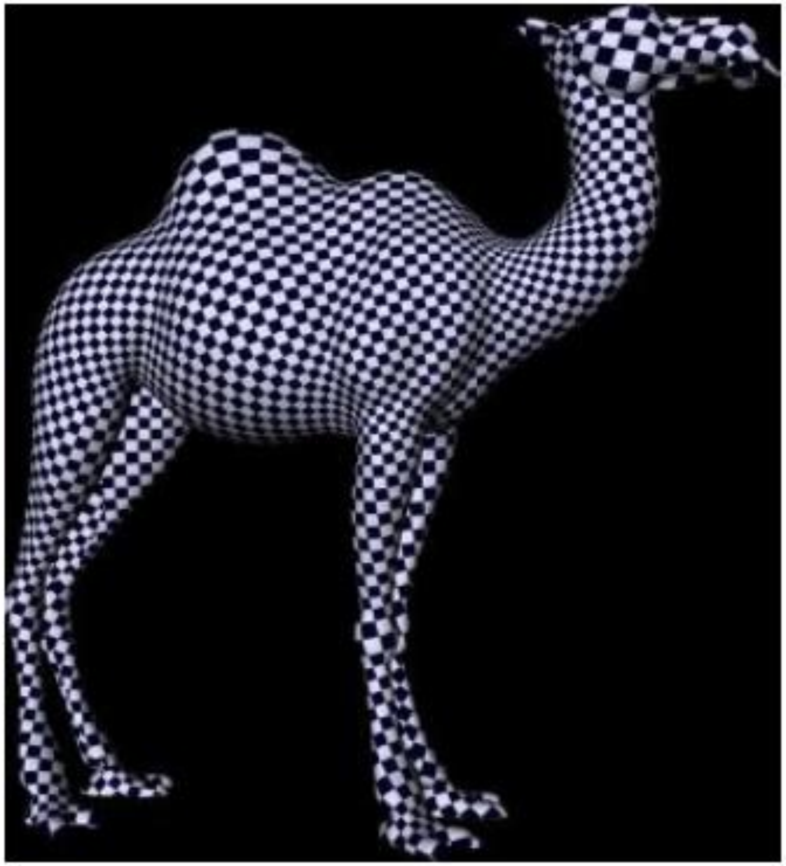






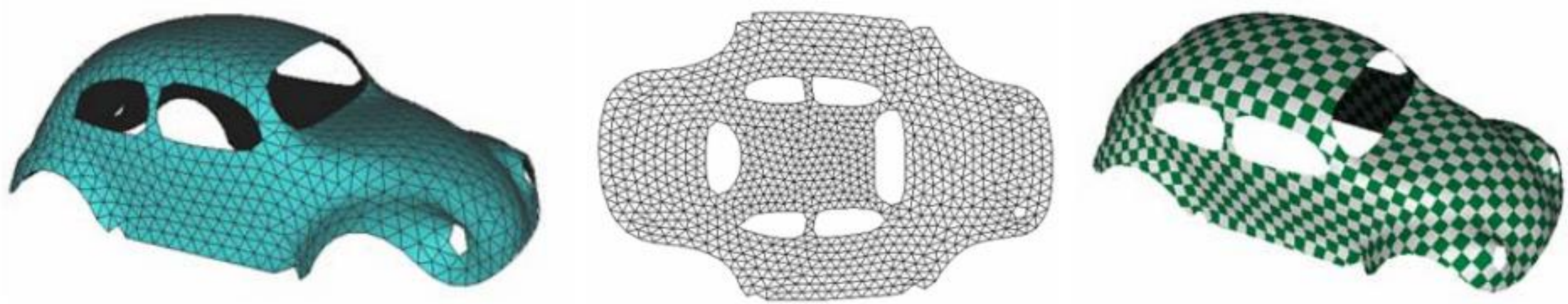
Fixed vs Free Boundary





Free Boundary

- **Zhonggui Chen**, Ligang Liu, Zhengyue Zhang, and Guojin Wang. **Surface Parameterization via Aligning Optimal Local Flattening**. Proceedings of the 2007 ACM symposium on Solid and physical modeling



Literature

- Floater & Hormann: Surface parameterization: a tutorial and survey, Springer, 2005
- Lévy, Petitjean, Ray, and Maillot: Least squares conformal maps for automatic texture atlas generation, SIGGRAPH 2002
- Desbrun, Meyer, and Alliez: Intrinsic parameterizations of surface meshes, Eurographics 2002
- Sheffer & de Sturler: Parameterization of faceted surfaces for meshing using angle based flattening, Engineering with Computers, 2000.

Questions?