Stationary points and steepest descent method

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0.1 (a)

$$\nabla f(x) = \begin{bmatrix} 4x_1 + 2x_2^2 + 4 \\ 4x_2(x_1 + x_2^2 - 1) \end{bmatrix} \tag{1}$$

$$\nabla^2 f(x) = \begin{bmatrix} 4 & 4x_2 \\ 4x_2 & 4x_1 + 12x_2^2 - 4 \end{bmatrix} \tag{2}$$

0.2 (b)

Using the first-order optimality condition, three stationary points are:

$$p1 = [-1, 0]^T$$

whose Hession matrix eigenvalues are -8 and 4.

$$p2 = [-3, -2]^T$$

whose Hession matrix eigenvalues are 1.8755 and 34.1245.

$$p3 = [-3, 2]^T$$

whose Hession matrix eigenvalues are 1.8755 and 34.1245.

According to the eigenvalues, $p2$ and $p3$ are global minimizers and $p1$ is a saddle point.

0.3 (c)

The three points are marked in red circles.

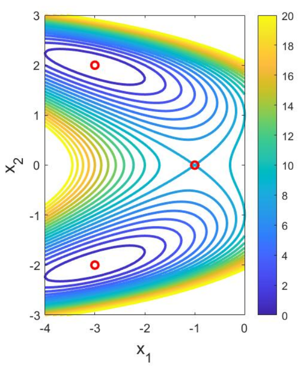

Figure 1: Contour graph

0.4 (d)

For start point $[-2, -1]^T$, after 54 iterations, the method converged to point $[-3, -2]^T$.

For start point $[-2, 0]^T$, after 1 iteration, the method converged to point $[-1, 0]^T$.

For start point $[-2, 1]^T$, after 54 iterations, the method converged to point $[-3, 2]^T$.