Local convergence in Newton's method

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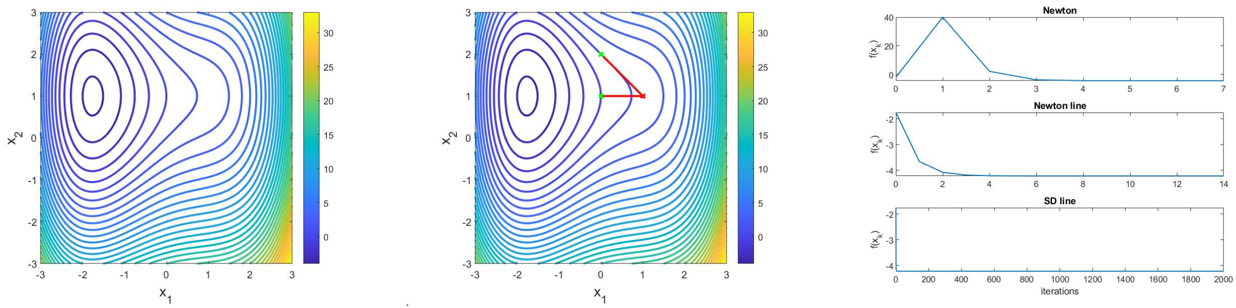
0.0.1 (a)

$$\nabla f(x) = \begin{bmatrix} x_1^3 - 2x_1 + 2 \\ 2x_2 - 2 \end{bmatrix} \tag{1}$$

$$\nabla^2 f(x) = \begin{bmatrix} 3x_1^2 - 2 & 0 \\ 0 & 2 \end{bmatrix} \tag{2}$$

0.0.2 (b)

The answer is shown in figure 1(a).



(a) (b) (c)

0.0.3 (c)

The iterates x_k is demonstrated in figure 1(b).

The method didn't converge. When in point [0,2] and [0,1], the Hession matrix is not definite. It doesn't meet the assumption that the Newton's method is based on, resulting in the wrong searching direction. That's why the method didn't converge.

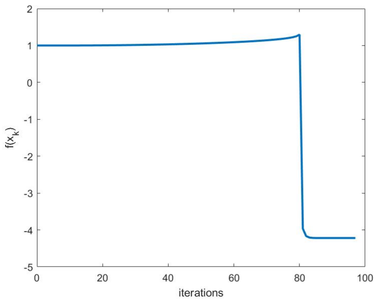
0.0.4 (d)

The result is illustrated in figure 1(d) to 1(f). The Newton's method converges now.

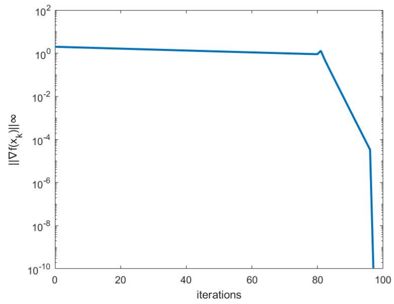
It moved at very small steps in the first 80 iterations and then jumped to a point close to the minimizer. Because in the first 80 iterations, Hession matrix was not definite, the step length had to be set in a smalll value to search slowly in a wrong direction until a definite Hession matrix appears. In initeration 81, Hession matrix was definite. The method was able to use the Hession matrix that meets the assumptions.

α_k remained abound 0.01, the minimum α value we set, in the first 81 iterations. Then it increased a lot. After 97 iterations, it raised to 1.

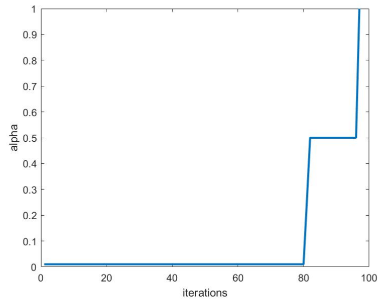
The function values $f(x_k)$ are not monotonically decreasing. At iteration 82, there was a small increase on the function value. The reason is the Newton's method doesn't promise to have a monotonic decrease in function values, even though it's on the way to the minimizer. The method only decides the correct direction to the optimal point.



(d)



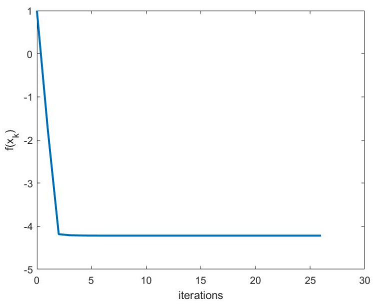
(e)



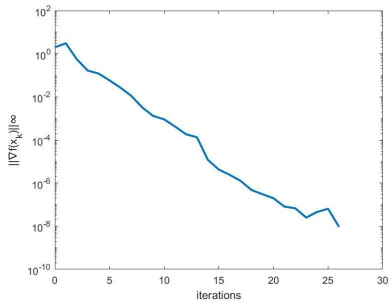
(f)

0.0.5 (e)

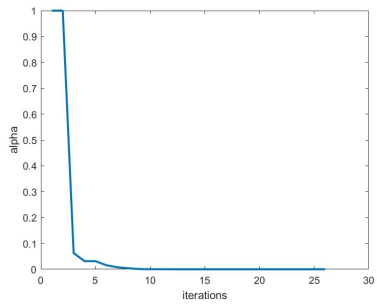
The result is presented in figure 1(g) to 1(i). It converged after 26 iterations, much faster than the Newton's method.



(g)



(h)



(i)

0.0.6 (f)

Figure 1(c) shows the function values of three methods.

The Newton's method converged much faster than that with line search. The steepest descent method didn't converge.

Due to strict prerequisites, the Newton's method sometimes doesn't work and even if we add line search, the convergence is much slower than that of steepest descend method. However, if the Hession matrix is always definite, the Newton's method is deemed to be a strong competitor of the best result. To promise convergence and efficiency, one ought to use the Newton's method with line search, since it works efficiently in most cases.