Exe 2 Week 10

1) $f(x) = e^{x_1} + x_1 \sin(x_2)$

$\vartheta_1 = e^{x_1}$

$\vartheta_2 = \sin(x_2)$

$\vartheta_3 = x_1 \vartheta_2$

$\vartheta_4 = \vartheta_1 + \vartheta_3$

2) • $P_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $D_{P_1} \vartheta_i = \frac{\partial \vartheta_i}{\partial x_1} \doteq \dot{\vartheta}_i$

$\dot{\vartheta}_1 = e^{x_1} = \vartheta_1$

$\dot{\vartheta}_2 = 0$

$\dot{\vartheta}_3 = \vartheta_2$

$\dot{\vartheta}_4 = \dot{\vartheta}_1 + \dot{\vartheta}_3 = e^{x_1} + \sin(x_2)$

• $P_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $D_{P_2} \vartheta_i = \frac{\partial \vartheta_i}{\partial x_2} = \vartheta_i'$

$\vartheta_1' = 0$

$\vartheta_2' = \cos(x_2)$

$\vartheta_3' = x_1 \vartheta_2'$

$$v_4' = v_1' + v_3' = 0 + x_1 v_2' = x_1 \cos(x_2)$$

$$\therefore \quad \frac{\partial f}{\partial x_1}(x) = \dot{v}_4 = e^{x_1} + \sin(x_2)$$

$$\frac{\partial f}{\partial x_2}(x) = v_4' = x_1 \cos(x_2)$$

3) $\bar{v}_4 = 1$

$$\bar{v}_3 = \bar{v}_4 \frac{\partial v_4}{\partial v_3} \qquad (v_4 = v_1 + v_3)$$

$$= \bar{v}_4 \cdot 1 = 1$$

$$\bar{v}_2 = \bar{v}_3 \frac{\partial v_3}{\partial v_2} + \bar{v}_4 \frac{\partial v_4}{\partial v_2} \qquad (v_3 = x_1 v_2)$$

$$= \bar{v}_3 \frac{\partial v_3}{\partial v_2} + \bar{v}_4 \cdot 0$$

$$= \bar{v}_3 \cdot x_1 = x_1$$

$$\bar{v}_1 = \bar{v}_2 \frac{\partial v_2}{\partial v_1} + \bar{v}_3 \frac{\partial v_3}{\partial v_1} + \bar{v}_4 \frac{\partial v_4}{\partial v_1}$$

$$= \bar{v}_2 \frac{\partial v_2}{\partial v_1} + \bar{v}_3 \frac{\partial v_3}{\partial v_1} + \bar{v}_4 \cdot 1$$

$$= \bar{v}_2 \frac{\partial v_2}{\partial v_1} + \bar{v}_3 \cdot 0 + 1$$

$$= \bar{v}_2 \cdot 0 + 1 \qquad (v_2 = \sin(x_2))$$

$$= 1 \qquad (v_1 = e^{x_1})$$

$$\therefore \quad \frac{\partial f}{\partial x_1}(x) = \frac{\partial v_4}{\partial x_1} = \sum_{j=1}^{4} \bar{v}_j \frac{\partial v_j}{\partial x_1}$$

$$= \bar{v}_1 \frac{\partial v_1}{\partial x_1} + \bar{v}_2 \frac{\partial v_2}{\partial x_1} + \bar{v}_3 \frac{\partial v_3}{\partial x_1} + \bar{v}_4 \frac{\partial v_4}{\partial x_1}$$

$$= \bar{v}_1 \frac{\partial v_1}{\partial x_1} + \bar{v}_2 \frac{\partial v_2}{\partial x_1} + \bar{v}_3 \frac{\partial v_3}{\partial x_1} + 1 \cdot 0$$

$$= \bar{v}_1 \frac{\partial v_1}{\partial x_1} + \bar{v}_2 \frac{\partial v_2}{\partial x_1} + 1 \cdot v_2$$

$$= \bar{v}_1 \frac{\partial v_1}{\partial x_1} + (x_1 + 1) \cdot 0 + v_2$$

$$= 1 \cdot e^{x_1} + v_2 = e^{x_1} + \sin(x_2)$$

$$\frac{\partial f}{\partial x_2}(x) = \frac{\partial v_4}{\partial x_2} = \sum_{j=1}^{4} \bar{v}_j \frac{\partial v_j}{\partial x_2}$$

$$= \bar{v}_1 \frac{\partial v_1}{\partial x_2} + \bar{v}_2 \frac{\partial v_2}{\partial x_2} + \bar{v}_3 \frac{\partial v_3}{\partial x_2} + \bar{v}_4 \frac{\partial v_4}{\partial x_2}$$

$$= \bar{v}_1 \frac{\partial v_1}{\partial x_2} + \bar{v}_2 \frac{\partial v_2}{\partial x_2} + \bar{v}_3 \frac{\partial v_3}{\partial x_2} + 1 \cdot 0$$

$$= \bar{v}_1 \frac{\partial v_1}{\partial x_2} + \bar{v}_2 \frac{\partial v_2}{\partial x_2} + 1 \cdot 0$$

$$= \bar{v}_1 \frac{\partial v_1}{\partial x_2} + x_1 \cdot \cos(x_2)$$

$$= 1 \cdot 0 + x_1 \cos(x_2)$$

$$= x_1 \cos(x_2)$$