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Task 1: Mathematical Truth

Proposition Every binary tree on n nodes where each has either zero or two children has precisely $\frac{n+1}{2}$ leaves.

Pf by induction

Predicate n nodes: P(n) = $\frac{n+1}{2}$ leaves , where n can only be an odd number because the root node counts as 1 and the proposition only allows us to add either 0 or 2 nodes to the binary tree and it is still be odd after adding new nodes.

Base Case P(1) = 1 leaf TRUE

Inductive Step Assume that P(i) = $\frac{k+1}{2}$ leaves is TRUE.

WTS Since P(k+0) = $\frac{k+1}{2}$ leaves has no difference from P(i), so P(k+0) is TRUE.

Let's prove that P(k+2) = $\frac{(k+2)+1}{2}$ leaves is also true.

$= \frac{k+3}{2}$

$= \frac{k+1}{2} + 1$

P(k+2) = P(k) + 1 leaves TRUE

Since, if we add 2 nodes to the current k nodes, the first added node will just change the position of its parent node to be a parent instead of a leaf and declare itself as a leaf. But the number of leaves still remains the same, while the second node we add will add up a new leaf to the binary tree. That is why we get P(k)+1 leaves after adding 2 new nodes.

Thus, by mathematical induction, every binary tree on n nodes where each has either zero or two children has precisely $\frac{n+1}{2}$ leaves.