# What can economists estimate and predict being Economics a Non-experimental Science?

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#### Abstract

In this paper we develop procedures to calculate estimation intervals for some given combinations of parameters of the General Linear Model (GLM) associated with each of the regressors that are easily interpretable and that can be estimated with accuracy. We also provide prediction intervals for future values of the dependent variable conditional on the information on one of the regressors. These intervals are calculated using, on the one hand, the estimate of each individual coefficient from the Simple Linear Model, and, on the other hand, the estimation of the variance of the disturbance from a GLM whose variables are determined by means of the adjusted  $\mathbb{R}^2$ .

**Keywords:** non-experimental, estimation, prediction.

### 1 Introduction

Although as Varian (1996, p. 246) writes ... "experimental economics has been one of the great success of the last 20 years" (see also Sugden (2005) and the references cited there) it is clear that the scope for experimental economics is yet rather limited. And the scope for experiment in economics is limited because "many important economic phenomena (from the impact of market structure to most of macroeconomics) cannot be investigated experimentally and because if the subjects of economic experiments are aware that they are participating in an experiment it is hard to be sure that their behaviour is not affected". (Backhouse (1997, p. 79)).

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The result of this non-experimental framework, is that the data of real life are necessarily the product of many influences other than those which it is desired to isolate. Economic data being largely historical, are therefore uninformative and incomplete making difficult to separate the individual contributions of explanatory factors. The main consequence of this non-experimental character of economic data is that it is very difficult to provide an structural causal account to the observed data in the sense of Hoover (2001).

Are the economists concious of the non-experimental character of their data? Does this non-experimental character influence their decisions when building a model? It seems that when economists set the object of the estimation process, they are hardly worried about the non-experimentality of their data. They propose to estimate structural parameters or reduced form parameters close to what a natural scientist would do. A structural parameter is thought to give the direct and individual effect-the causal effect- of a variable on another variable. A reduced form parameter is a complex combination of structural parameters that summarises the interrelationships that are implicit in a given structural form. In many situations, economists think that they can estimate structural parameters and that they can assess the validity of the estimation by considering a priori information about the sign and size of these parameters. However, unless there is a coincidence between the frequency of the causal process and the frequency of the observed data, the possibility of estimating a structural parameter does not exist. In that case, what economists are estimating is a complex combination of effects that is difficult to interpret. The same happens when the objective is to estimate reduced form parameters. They are considered as multipliers collecting all the relationships implicit in the structural form.

Whatever be what we are estimating the second relevant question to ask is: can we estimate it with precision? With respect to this question, economists seem to be aware of the non-experimental character of their data because when they are estimating an individual parameter they take into account the effect of other factors considering a General Linear Model. However, the joint consideration of a group of variables leads to the multicollinearity problem which, in many situations, makes difficult to estimate with precision individual parameters. So, we can say that when we are estimating an individual parameter we face, at least, two serious problems: first, it is difficult to interpret the parameter we are estimating and second, we cannot estimate it with a reasonable precision.

In this paper, we propose to estimate some linear combinations of the parameters of the General Linear Model that are easily interpretable and that can be estimated with a high level of accuracy. We propose interval estimates whose centre are the estimations of the parameters calculated from a Simple Model while the limits of the intervals are determined using the standard deviation obtained from a GLM whose variables have been determined by means of the adjusted coefficient of determination. The results are extended to predict the value of a dependent variable in that particular situation in which we know only the future value of one of the regressors and, for the rest of the regressors, we assume that their future values equal the conditional expectation of those regressors given the value of the known regressor.

Although, in this paper we adopt a very simple framework to motivate and develop the proposals, the same questions can be dealt with considering more complex and general settings. See Aznar and Domingo (2006) where the model considered is a model with cointegration and Error Correction Mechanism. See also Smith (2000), Hoover (2000) and Favero (2000).

The structure of the rest of the paper is as follows. In Section 2, we motivate the proposal commenting two examples. The estimated intervals for linear combinations of the coefficients are presented in Section 3.In Section 4, we present the prediction intervals and the results of an empirical application are presented in Section 5.The main conclusions are collected in Section 6.

### 2 Motivation

We begin this section by considering two examples, the first from Leamer (1978) and the second from Allen (1992). In the presentation that follows we respect the notation used by the two authors.

The aim of Leamer is to estimate a model to explain the demand for oranges. The demand for oranges, D, is thought to depend negatively on the price of oranges P, positively on the price of grapefruit  $\pi$ , and positively on money income Y. Using logs, the estimated results are the following (with standard errors in parentheses)

$$\log D = 7.0 + .1 \log P + .2 \log \pi + .6 \log Y \tag{1}$$

Commenting these results, Leamer says that, unhappily, the direct price elasticity-the coefficient of log P- has the wrong sign and that neither coefficient is significantly different from zero. Why does Leamer think that the sign of the estimation of the parameter of log P is wrong? Because he is thinking that that coefficient is a structural coefficient that informs us about the direct effect of log P on log D. However, as we indicated in the Introduction, this interpretation can be accepted only if the frequency of the causal process coincides with that of the observed data. Otherwise, the coefficient of log P accounts for a complex collection of different effects whose final sign is unpredictable.

The example from Allen (1992), is dedicated to specify a model that explains the real rate of interest. Allen considers the following IS-LM-AS model

$$IS: r_t = a_0 + a_1 D_t + a_2 (M_t - P_t) - a_3 Q_t - a_4 S S_t + a_5 V_{\pi t}$$
(2)

$$LM: Q_t = b_0 + b_1(M_t - P_t) + b_2 i_t (1 - T_t)$$
(3)

$$AS: P_t = P_t^e + c_1 Q_t + c_2 S S_t (4)$$

$$i_t(1 - T_t) = r_t + \pi_t^e \tag{5}$$

where all the coefficients are positive, Q is the output gap, D is a federal debt variable, M is the log of M1, P is the log of the price level so that (M-P)

are real balances,  $V_{\pi}$  represents the dispersion of inflationary expectations, SS is the external supply shock, T is the marginal tax rate on interest income,  $P^{e}$  is the log of the expected price level, i is the nominal interest rate and r is the real interest rate.

First, using (3), (4) and (5), we obtain

$$Q_t = W \left[ b_0 + b_1 (M_t - P_t^e) - b_1 c_2 S S_t + b_2 r_t + b_2 \pi_t^e \right]$$
 (6)

where

$$W = \frac{1}{1 + b_1 c_1}$$

On the other hand, from (4) we obtain

$$M_t - P_t = M_t - P_t^e - c_1 Q_t - c_2 S S_t$$

Now, substituting  $(M_t-P_t)$  and (6) into (2) yields

$$r_t = Z(a_0 + b_0 VW) + a_1 D_t + (a_2 + b_1 VW)(M_t - P_t^e) -$$

$$(b_1 c_2 VW + a_2 c_2 + a_4) SS_t + b_2 VW \pi^e + a_5 V_{\pi t}$$
(7)

Where  $V = -(a_2c_1 + a_3)$  and  $Z = \frac{1}{1 - b_2VW}$ . In a compact form, this relation can be written as

$$r_t = \beta_0 + \beta_1 D_t + \beta_2 (M_t - P_t^e) + \beta_3 S S_t + \beta_4 \pi^e + \beta_5 V_{\pi t}$$
 (8)

where

$$\beta_4 = b_2 ZVW = \frac{-b_2(a_2c_1 + a_3)}{(1 + b_1c_1) + b_2(a_2c_1 + a_3)} \tag{9}$$

Using quarterly data, Allen reports the following OLS estimate of (8),(t-statistics are in parentheses)

$$r_t = -.89 + 13.94D_t + 1.47(M_t - P_t^e) - 5.75SS_t + .33\pi^e - .04V_{\pi t}$$
(10)

It is seen that the only variable that is statistically significant is the debt variable, D. No other coefficient is significant. Allen notes that although one would expect that the sign of the expectations variable to be negative it can be seen that the estimation is positive. He thinks that this dissappointing result is a consequence of "degrading levels of multicollinearity". In order to improve the results, he proposes to estimate the model using the first differences of the variables instead of their levels.

Why does Allen expect that the sign of the coefficient of the expectations variable is negative? Because assuming, as Allen does, that all coefficients of the structural form, (2)-(5), are positive then it is easily seen that the reduced form coefficient written in (9) is negative. However, this apriori determination of the sign is doubtful because, for the reasons already given in the text, the parameters of the model written in (2)-(5) are not really structural parameters so that their sign is difficult to specify.

From these two examples we can conclude that when we are estimating individual parameters, either considered as structural or as reduced form parameters, in many situations it is difficult to provide an easy interpretation of these coefficients and even more difficult to say something about their sign and size. Given the non-experimental character of economic data, economists try to estimate complex combinations of effects whose interpretation is beyond their scope.

In the two examples we have just commented both authors, Leamer and Allen, explain the disappointing results as a consequence of the poor quality of the data. Many of the causes of the poor quality of the data derive from the non-experimental character of those data: small sample size, collinearity between the regressors, measurement errors, etc.... In what follows, we are going to focus on the multicollinearity problem.

The multicollinearity problem refers to a situation where the explanatory variables are highly intercorrelated and makes difficult to disentangle the separate effects of each of the explanatory variables on the dependent variable. Consider the standard general linear model

$$y = X\beta + u \tag{11}$$

where y is the vector of the T observations of the dependent variable, X is the matrix of T observations of the k regressors,  $\beta$  is a vector of k coefficients and u is the column vector of T disturbances. We assume that u follows a process with a zero mean vector and a scalar covariance matrix  $\sigma^2 I_T$ . Let  $\overset{\wedge}{\beta}$  the k-vector of OLS estimators. It is well known that  $E(\overset{\wedge}{\beta}) = \beta$  and  $Var(\overset{\wedge}{\beta}) = \sigma^2(X'X)^{-1}$ . For the estimator of the first regressor we have

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{\sum x_{1t}^2 (1 - R_{1.2..k}^2)}$$
 (12)

where  $R_{1,2...k}^2$  is the coefficient corresponding to the regression of  $\mathbf{x}_1$  on the remaining regressors. It is seen that the variance of the estimator depends on three factors: the variance of the disturbance, the sum of squares of the regressor and the linear dependence of the regressor on the rest of the regressors. Given the first two factors, the precision of the estimation is determined by  $R_{1,2...k}^2$ . When this  $\mathbf{R}^2$  takes a value close to one, then the variance is large and the parameter is estimated with no precision. Values of  $R_{1,2...k}^2$  not far from one are not so strange if the number of regressors is large.

We have seen in this section that when one tries to estimate individual parameters using the General Linear Model two problems arise. First, in general, they are reduced form parameters that collect the effects from many influences and it is extremely difficult to give an interpretation of these parameters. Secondly, in many situations we cannot estimate precisely those parameters because of the multicollinearity problem. In what follows of the paper, we propose to switch the objective of the estimation process in such a way that the linear combination is easily interpretable and can be estimated with precision.

## 3 Estimated Intervals for Linear Combinations of Parameters—

In this section, we propose estimated intervals for some combinations of the coefficients that are easily interpretable and that can be estimated with precision.

We begin by considering the same GLM written in (11). First, we extract the k principal components corresponding to the k regressors. Then, using an orthogonal rotation-see Rummel (1970) for the concept of orthogonal rotation-we obtain k new orthogonal principal components such that the first component coincides with the first regressor in the model. Let  $P_1$ ,  $P_2$ ,....,  $P_k$  be these k principal components. We have

$$x_k = \alpha_{k1}P_1 + \alpha_{k2}P_2 + \dots + \alpha_{kk}P_k$$

Let  $\mathbf{x}^{+\prime} = (x_2, .....x_k)$ . We can write

$$x^{+} = \alpha_{\cdot 1} P_1 + A \begin{pmatrix} P_2 \\ P_3 \\ \vdots \\ P_k \end{pmatrix} \tag{14}$$

where

$$\alpha_{\cdot 1} = \begin{pmatrix} \alpha_{21} \\ \alpha_{31} \\ \vdots \\ \alpha_{k1} \end{pmatrix} \text{ and } A = \begin{pmatrix} \alpha_{22} & \cdots & \alpha_{2k} \\ \vdots & \ddots & \vdots \\ \alpha_{k2} & \cdots & \alpha_{kk} \end{pmatrix}$$

Substituting (14) into (11) yields

$$y_{t} = \beta_{1}x_{1t} + P_{1t}\alpha'_{.1}\beta^{+} + (P_{2t}.....P_{kt})A'\beta^{+} + u_{t} =$$

$$= (\beta_{1} + \alpha'_{.1}\beta^{+})P_{1t} + (P_{2t}.....P_{kt})A'\beta^{+} + u_{t} =$$

$$= \beta_{1}^{*}P_{1t} + \beta_{2}^{*}P_{2t} + .... + \beta_{k}^{*}P_{kt} + u_{t}$$

$$(15)$$

where  $\beta^{+\prime}=(\beta_2,......\beta_k)$  is the vector of (k-1) elements corresponding to  $x^+.$  Note that

$$\beta_1^* = (\beta_1 + \alpha_{11}'\beta^+) = \beta_1 + \beta_2\alpha_{21} + \dots + \beta_k\alpha_{k1}$$
 (16)

Since  $P_1$  is orthogonal to the other (k-1) components, the correlation coefficient between  $x_j$  and  $x_1$  can be written as

$$r_{j1} = \frac{\sum P_{1t} x_{jt}}{(\sum P_{1t}^2)^{1/2} (\sum x_{jt}^2)^{1/2}} = \frac{\alpha_{j1} \sum P_{1t}^2}{(\sum P_{1t}^2)^{1/2} (\sum x_{jt}^2)^{1/2}} = \alpha_{j1} \frac{(\sum P_{1t}^2)^{1/2}}{(\sum x_{jt}^2)^{1/2}}$$

so that

$$\alpha_{j1} = r_{j1} \frac{(\sum x_{jt}^2)^{1/2}}{(\sum P_{1t}^2)^{1/2}} \tag{17}$$

Hence, (16) can be written as

$$\beta_1^* = \beta_1 + \frac{1}{(\sum P_{1t}^2)^{1/2}} \sum_{j=1}^k \beta_j r_{j1} (\sum x_{jt}^2)^{1/2}$$
(18)

Our proposal in this paper is to estimate  $\beta_1^*$  using the following interval estimate

$$\hat{\beta}_{1R} \pm N_{\epsilon/2} \hat{\sigma}_{\hat{\beta}_{1R}}$$
 (19)

where  $\hat{\beta}_{1R} = \frac{\sum x_{1t}y_t}{\sum x_{1t}^2}$  and  $\hat{\sigma}_{\hat{\beta}_{1R}}^2 = \frac{\hat{\sigma}^2}{\sum x_{jt}^2}$ .  $\hat{\sigma}^2$  is the OLS estimator of  $\sigma^2$  in (11). The number of variables to be included in the model to carry out the estimation is determined by using the adjusted  $R^2$ . As can be seen in Aznar et al. (1998), the adjusted  $R^2$  is the less parsimonious model selection criterion among those used to determine the number of variables to be included in a linear model. We have chosen this criterion because it provides the smallest unbiased estimation of  $\sigma^2$ .

We justify the proposal in (19) by noting that  $\beta_{1R}$  is an unbiased estimator of  $\beta^{\bullet}$  with a variance equal to  $\sigma^2/\sum x_{1t}^2$ . To see this result, note that

$$\hat{\beta}_{1R} = \frac{\sum x_{1t}y_t}{\sum x_{1t}^2} = \beta_1 + \beta_2 \frac{\sum x_{1t}x_{2t}}{\sum x_{1t}^2} + \ldots + \beta_k \frac{\sum x_{1t}x_{kt}}{\sum x_{1t}^2} + \frac{\sum x_{1t}u_t}{\sum x_{1t}^2}$$

The unbiadness follows because  $Ex_{1t}u_t = 0$  and because

$$\frac{\sum x_{1t}x_{jt}}{\sum x_{1t}^2} = \frac{(\sum x_{jt}^2)^{1/2}}{(\sum x_{1t}^2)^{1/2}}r_{j1} \qquad \text{j=1, 2, ... k}$$

With respect to the variance, we have

$$\operatorname{Var}(\overset{\wedge}{\beta}_{1R}) = E(\overset{\wedge}{\beta}_{1R} - \overset{\wedge}{\beta}_{1})^{2} = E(\frac{\sum x_{1t}u_{t}}{\sum x_{1t}^{2}})^{2}$$

To conclude this section we can outline the steps of the process we propose to estimate the interval written in (19). Although the analysis has been carried out for the coefficient of the first regressor, the results can be extended easily to the other coefficients. First, using any source of knowledge we specify a list of variables that are considered as potential regressors of the econometric model. Although Economic Theory will be the main source of inspiration it is worth noting that, in some cases, other sources may be usefull as well. For example, what Swann(2006) calls vernacular knowledge can provide useful insights in some situations. Second, the variables to be included in the General Linear Model are chosen using the adjusted R<sup>2</sup>; then, the variance of the disturbance is estimated using OLS. Finally, the interval written in (19) is calculated.

### 4 Prediction Intervals

In this section, we extend the results to predict a future value of the dependent variable in a particular situation in which we know the value of one regressor in the out sample period and, for the rest of the variables, we assume that their values in that period equal the conditional expectation given the known value of the regressor.

Let  $y_p, x_{1p}, \dots x_{kp}$ , be the future values taken, respectively, by the dependent variable and the regressors. The out sample period is indexed by p. We assume that the model written in (11) holds for this period so that we can write

$$y_p = x_{1p}\beta_1 + \dots + x_{kp}\beta_k + u_p \tag{20}$$

We propose to predict the conditional expectation of  $y_p$  given a future value of  $x_1$ ,  $x_{1p}$ , by means of the following interval

$$\hat{\beta}_{1R} x_{1p} \pm N_{\epsilon/2} \hat{\sigma}_{\hat{\beta}_{1R}} x_{1p} \tag{21}$$

First, we are going to show that  $\hat{\beta}_{1R}x_{1p}$  is an unbiased estimator of  $E(y_p/x_{1p})$  with a variance equal to  $\frac{\sigma^2}{\sum x_{1t}^2}x_{1p}^2$ . Note that

$$E(y_p/x_{1p}) = \beta_1 x_{1p} + \beta_2 E(x_{2p}/x_{1p}) + \dots + \beta_k E(x_{kp}/x_{1p}) = \beta_1 x_{1p} + \beta^{+\prime} E(x_p^+/x_{1p})$$

and, since  $\frac{\sum x_{1t}x_{jt}}{\sum x_{1t}^2}x_{1p} = E(x_{jp}/x_{1p})$  we have

$$E(\hat{\beta}_{1R}x_{1p}) = \beta_1 x_{1p} + \sum_{j=2}^k \beta_j \frac{\sum x_{1t} x_{jt}}{\sum x_{1t}^2} x_{1p} = \beta_1 x_{1p} + \beta^{+\prime} E(x_p^+/x_{1p})$$

so that the unbiasedness follows.

With respect to the variance, the result follows because

$$Var(\overset{\wedge}{\beta}_{1R}) = \frac{\sigma^2}{\sum x_{1t}^2} \tag{22}$$

Next, we are going to show that  $\hat{\beta}_{1R}x_{1p}$  is the same as the predictor obtained from the GLM after assuming that the values of the regressors are the conditional expectations of those regressors given  $x_{1p}$ .

The predictor of  $E(y_p/x_{1p})$  from the General Linear Model is defined as

$$\hat{y}_{1p} = E(x_p/x_{1p})'\hat{\beta} = (x_{1p}, E(x_p^+/x_{1p})') \begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix}$$

with  $\stackrel{\wedge}{\beta} = (X'X)^{-1}X'y$ . Alternatively, this predictor can be written

$$\hat{y}_{p} = x_{1p} \hat{\beta}_{1} + x_{1p} (x_{1}' x_{1})^{-1} x_{1}' X_{2} \hat{\beta}_{2}$$
(23)

The partitioned form of the normal equations of the GLM can be written as

$$x_1' x_1 \hat{\beta}_1 + x_1' X_2 \hat{\beta}_2 = x_1' y$$

$$X_{2}'x_{1}\overset{\wedge}{\beta}_{1}+X_{2}^{'}X_{2}\overset{\wedge}{\beta}_{2}=X_{2}'y$$

From the first normal equation we obtain

$$\hat{\beta}_{1} = (x_{1}^{'}x_{1})^{-1}x_{1}^{'}y - (x_{1}^{'}x_{1})^{-1}x_{1}^{'}X_{2}\hat{\beta}_{2} = \hat{\beta}_{1R} - (x_{1}^{'}x_{1})^{-1}x_{1}^{'}X_{2}\hat{\beta}_{2}$$

and substituting this expression of  $\hat{\beta}_1$  into (23) the result follows.

Finally, we are going to outline a five step process that can be used to assess the validity of these prediction intervals. Notice that although the analysis has been carried out for the first regressor, the results can be extended to any of the other (k-1) regressors.

First, our starting point are T observations on  $k^*$  variables being  $k^*$  any arbitrary number greater than k+1. Using one of these variables as the dependent variable we form all possible models and among those which are spherical we choose the model with the highest adjusted  $R^2$ . Suppose that this model has k regressors.

Second, take the  $T_1$  first observations of the k+1 variables in the chosen model and estimate the parameter of the Simple Linear Model between the dependent variable and each of the k regressors. Let  $\beta_{1R}, \beta_{2R}, \dots, \beta_{kR}$  be the corresponding OLS estimators. Then, we define the k predictors for the future value of the dependent variable in period  $T_1 + 1$  as follows

$$\hat{\beta}_{jR} x_{jT_1+1} \pm N_{\epsilon/2} (\hat{\sigma}^2 + \frac{\hat{\sigma}^2}{(\sum x_{jt}^2)} x_{jT_1+1}^2)^{1/2}$$
 j=1,2,....k (24)

where  $x_{jT_1+1}$  is the  $T_1+1$  observation of  $x_j$  and  $\overset{\wedge}{\sigma}^2$  is the OLS estimator of the variance of the disturbance of the General Linear Model with k regressors chosen in the first stage. This estimator is calculated using the  $T_1$  first observations.

Third, let  $y_{T_1+1}$  be the observed value of the dependent variable in  $T_1 + 1$ . Then, if this value is within the limits of the interval defined in (24), we conclude by saying that the predictor corresponding to the j-th regressor has been corraborated.

Fourth, repeat 1), 2) and 3) for the following periods,  $T_1 + 2, T_1 + 3, ....T$ . using the information up to the previous period.

And fifth, chose the predictor that, among the predictors which are corroborated in all T-T<sub>1</sub> periods or, at least, in  $(1-\epsilon)$  per cent of all those periods, has the smallest length of the interval defined in (24).

Notice that the proposal we have just commented is restricted to the case where we only know the future value of one of the regressors and, for the other regressors, we assume that their future values are the conditional expectations of these regressors given the known future value. However, the approah can be extended in an straightforward manner to more general settigs. In paticular, we can apply it to the case where we know the future values corresponding to i regressors being i any number between 1 and k.Let k be k=i+j, and write the model in (11) in a partitiond form as

$$y = X_i \beta_i + X_j \beta_j + u \tag{25}$$

Where  $X_i$  is the  $T \times i$  matrix of observations of the i regressors and  $X_j$  is the  $T \times j$  matrix of observations of the remaining j regressors. We assume also that this model holds also for the future value

$$y_p = x_{ip}'\beta_i + x_{ip}'\beta_i + u_p \tag{26}$$

where  $\mathbf{x}'_{ip}$  and  $\mathbf{x}'_{jp}$  are row vectors of the future values of the corresponding regressors. In this case, the restricted predictor is given by

$$\hat{y}_{ip} = x_{ip}' \hat{\beta}_{iR} \tag{27}$$

where  $\overset{\wedge}{\beta}_{iR} = (X_i'X_i)^{-1}X_i'y$ . We are going to show that this predictor coincides with that is derived from the General Linear Model assuming that the first i regressors adopt the known future values and the remaining j regressors take values equal to the mean of the conditional distribution of these j regressors given the values of the first regressors. The predictor from the GLM is the following

$$x_{ip}^{\prime} \stackrel{\wedge}{\beta_i} + E(x_{jp}/x_{ip})^{\prime} \stackrel{\wedge}{\beta_j} \tag{28}$$

where  $\beta' = (\beta_i, \beta_j)$ . The result follows because, on the one hand,

$$E(x_{ip}/x_{ip}) = x'_{ip}(X'_iX_i)^{-1}X'_iX_i$$
(29)

and, on the other hand, using previous results related to normal equations, we can write

$$\hat{\beta}_{iR} = \hat{\beta}_i + (X_i'X_i)^{-1}X_i'X_j\hat{\beta}_j$$
 (30)

By introducing (28) and (29) into (27) we obtain the result.

**REMARK 1.** Note that up to now, we have considered models that are static, that is, models in which there are no lagged regressors. However, the results can be extended in a straighforward manner to dynamic models in which

lagged regressors are included. Consider the following autoregressive distributed lag model

$$y_{t} = \sum_{i=1}^{p} \phi_{i} y_{t-i} + \sum_{i=0}^{p} \sum_{j=1}^{p} \beta_{ji} x_{jt-i}^{k} + u_{t}$$
(31)

This model has p+(p+1)k regressors As with the static model the number of lags is determined by means of the adjusted  $R^2$ .

### 5 Empirical Application

In this section, we consider annual data of Gross Domestic Product (GDP) for seven countries-Denmark, Belgium, Canada, France, Italy, UK and USA. The sample period goes from 1970 to 2005 and the original data are transformed into an index form in such a way that  $GDP_{1970} = 100$  for each country. The objective is to explain the GDP of Denmark in terms of the GDP of the other six countries

Using the adjusted R<sup>2</sup> the best model we obtain is the following (standard deviations between parentheses)

$$DEN = 56.9 - .124 CAN - .481 FRAN + .36 ITA - .216 UK + .66 USA$$
 
$$\overline{R}^2 = .995 \qquad DW = 1.28 \qquad \stackrel{\wedge}{\sigma} = 2.395$$

In Table 1, we report, for each coefficient, the estimated interval obtained from the GLM (Column 2) and from the simple linear models (Column 4), respectively. In columns 3 and 5 we provide, in %, the amplitude of each interval in relation to the estimated value of the corresponding parameter. It can be seen that the amplitude of the intervals calculated using the simple models are significantly smaller than those derived from the GLM. While for the first, the percentage go around a 5%, for those obtained from the GLM the smallest value is 77%, for USA, and the largest is 392% and corresponds to UK.

Table 1. Estimated Intervals for Individual Parameters

Country	${\stackrel{\wedge}{eta}}_j{\pm}{f 2.04}{\stackrel{\wedge}{\sigma}}_{{\stackrel{\wedge}{eta}}_j}$	$rac{2.04 \hat{\sigma}_{{lpha}_j}}{\hat{eta}_j}  imes  extbf{100}$	${\stackrel{\wedge}{\beta}}_{jR}{\pm} 2.04 {\stackrel{\wedge}{\sigma}}_{{\stackrel{\wedge}{\beta}}_{jR}}$	$rac{2.04 \hat{\sigma}_{ \hat{eta}_{jR}}}{\hat{eta}_{jR}}  imes  extbf{100}$
Canada	(333; .08)	330%	(.583; .612)	4.9%
France	(89;07)	170%	(.89; .94)	5%
Italy	(.12; .6)	134%	(.87; .9)	4,9%
UK	(574; .142)	392%	(.88; .93)	4.8%
USA	(.41; .924)	77%	(.57; .59)	4.7%

Table 2 reports the results of the application of a sequential process to predict the last five values of the sample period. First, we use information until year 2000 to estimate the GLM with five regressors and then, we predict the 2001 value of the dependent variable assuming that the future values of these regressors are known. Then, we use information until 2001 to predict the 2002 value and the process follows until we calculate the 2005 value. The same process is repeated using the simple models corresponding to each regressor. The results are reported in Table 2 for the USA and in Tables 3 and 4 for the other four countries. Strictly, we cannot compare the predictions obtained from the GLM to those obtained with the simple models, because the GLM approach assumes that the future values of the five regressors are known while when we predict using the simple models we assume that we only know the future value of one of the regressors. Examining the results reported in these tables it can be seen that the only predictions that are corroborated are those corresponding to the french model.

Table 2. Prediction Intervals

year	${\bf Observed}$	$\operatorname{prediction}(\%)$	${\bf Interval}$	$\operatorname{prediction}(\%)$	${\bf Interval}$
		$\operatorname{GLM}$		USA	
2001	199.5	198,7(.40)	(192; 205)	200.3(.4)	194.8; 205.9
2002	200.5	201,6(.56)	(195; 208)	200.3(1.2)	194.8; 205.7
2003	201.8	204.2(1.19)	(198; 210)	201.1(2.3)	207.1; 211.7
2004	205.6	209.0(1.63)	(203; 215)	212.2(3.2)	206.8; 217.4
2005	211.7	213.1(.62)	(207; 219)	216.6(2.3)	211.3; 221.9

Table 3. Prediction Intervals from the simple models

year	$\mathbf{prediction}(\%)$	Interval	$\mathbf{prediction}(\%)$	Interval
	Canada		France	
2001	203.0(1.8)	$197.4;\ 208.6$	197.1(1.2)	$191.6;\ 202.7$
2002	207.4(3.46)	201.9; 212.9	199.6(.4)	$194.2;\ 205.0$
2003	209.4(3.7)	204; 214.8	202.0(.06)	196.7; 207.3
2004	214(4.08)	208.7; 219.3	206(.19)	200.7; 211.2
2005	217.9(2.9)	212.6; 223.3	208.4(1.59)	203.1; 213.6

Table 4. Prediction Intervals from the simple models-2

year	$\mathbf{prediction}(\%)$	${\bf Interval}$	$\mathbf{prediction}(\%)$	Interval
	Italy		UK	
2001	190.6(4.5)	185.2; 196.1	204.5(2.47)	$198.8;\ 210.1$
2002	192.3(4)	187.0; 197.7	207.7(3.6)	203.1; 213.2
2003	193.4(4.2)	188.2; 198.6	211.9(5.0)	206.5; 217.3
2004	195.8(4.7)	190.7; 201.0	217.9(5.5)	211.5; 222.2
2005	196.7(7)	191.8; 202.1	218.9(3.4)	213.6; 224.3

### 6 Conclusions

In this paper, we tried to show that the non-experimental character of Economics has important consequences about what we can estimate and what we can predict. In the first chapter of many textbooks, we are taught that Economics is a nonexperimental subject. However, in the following chapters of the book, using a highly qualified mathematical apparatus we are shown that we can answer the same questions as the natural scientists do, being these sciences truly experimental. In particular, we are said that we can estimate individual structural parameters and that we have no special difficulty with the prediction of future values of the dependent variables.

In the second section of the paper, we have shown that, in general, what we are estimating are not structural parameters, but reduced form parameters whose interpretation is a difficult task. These parameters collect all the interconnections between the variables included in a model. In general, these parameters adopt a very complex form which does not admit an a priori assessment about their size and sign. Besides, we have also shown that these reduced form parameters cannot be estimated with accuracy because, because of the multicollinearity problem.

In Section 3, we have proposed to estimate a linear combination of the parameters associated with a particular regressor in the context of the General Linear Model. We have shown that this combination is easily interpretable and that can be estimated with accuracy. We provided interval estimations in which the centre is the estimate from a Simple Linear Model and the limits of the interval are defined using the standard deviation from a General Linear Model in which the number of variables is determined using the adjusted R<sup>2</sup>.

In Section 4 the results have been extended to predict a future value of the dependent variable assuming that we only know the value of one of the regressors. For the remaining regressors we assume that their future values equal their conditional expectation given the known future value of the regressor. The advantage of the proposal relies on that the definition of the prediction needs future information only about one of the regressors. In that section we have outlined a procedure to define and validate the predictions.

The results of an empirical application have been presented in Section 5. In this example, we have considered a model in which the dependent variable is the GDP of Denmark, in an index form, and the regressors are the GDP corresponding to other five countries: Canada. France, Italy, UK and USA.

We have reported the estimated intervals for each of the coefficients and the prediction intervals from each simple model.

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