



Output, unemployment and Okun's law: Some evidence from the G7

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ABSTRACT

An economy may move between 'high-effort' and 'low-effort' states. Our estimates of the threshold unemployment rate which separates these states are significantly positive for G7 countries; only German data exhibit tendency to being persistently in the high-effort state.

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1. Introduction

Okun's law predicts that a percentage fall in output below trend is normally accompanied by a significant but smaller rise in unemployment. This prediction and its policy implications are straightforward when output and unemployment exhibit a systematic negative relationship with each other beyond trend and cyclical variations, but they are less clear when these variables are positively related. In this paper we examine time-series data from the G7 countries to explore whether there is any evidence of a systematic deviation from the negative relationship between output and unemployment. Our motivation stems from the fact that some well-established theoretical results, e.g., the existence of a negative relationship between output and unemployment, may break down when the underlying conditions are modified to accommodate some rigidity or distortion, e.g., 'efficiency wages', 'unionisation', 'wage contracts', 'unemployment insurance', etc. For instance, [Acemoglu and Shimer \(2000\)](#) focus on the effect of unemployment insurance within a search model and conclude that a rise in the latter can increase output and welfare despite increasing unemployment¹. Such theoretical possibilities suggest that in general the temporal relationship between output and unemployment may not be monotonic. Section 2 highlights the

policy repercussions of a non-monotonicity which may occur when labour productivity is induced by the extent of unemployment. Data from the G7 countries is then used to examine whether the relationship between output and unemployment significantly exhibits such non-linearities. We find that only German data show support for a monotonic relationship between output and employment. Section 3 concludes the paper.

2. The relationship between output and unemployment

To illustrate the problem, consider the following version of temporal aggregate production function,

$$y = y(q, l); y'_q > 0, y'_l > 0, \quad (1)$$

which, at any point in time, traces the combinations of employment l and output y for the corresponding level of labour productivity q which is, *ceteris paribus*, determined by the level of workers' effort. If we postulate that workers' effort supply is positively related to the unemployment rate u , i.e., the higher is u the larger is the effort supplied and hence $dq/du > 0$ (see, for example, [Shapiro and Stiglitz, 1984](#)), we can use

$$\frac{dy}{du} = y'_q \frac{dq}{du} + y'_l \frac{dl}{du} \quad (2)$$

to deduce the sign of dy/du as u varies in the positive unit interval. In particular, because $dl/du < 0$ by definition, the relationship between y and u on the temporal production function would resemble that depicted in [Fig. 1](#) if, at very low levels of u , $y'_q(dq/du)$ is sufficiently

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¹ See [Blanchard \(1998\)](#), [Caballero and Hammour \(1998\)](#) and [Gordon \(1997a\)](#) for examples of a link between unemployment and productivity; none of these studies directly explores the link between unemployment and output.

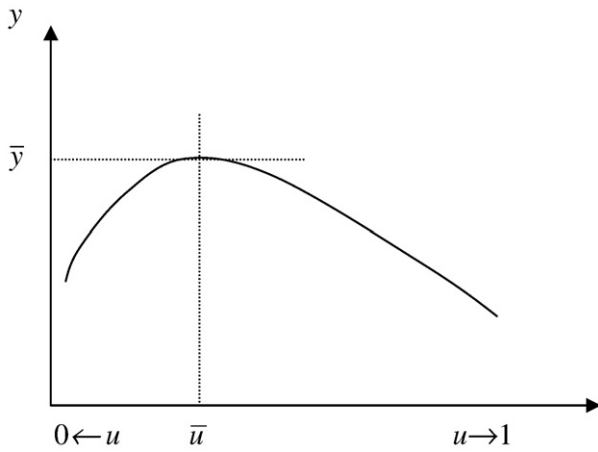


Fig. 1. The relationship between output and unemployment.

large and dominates $y_t'(dl/du)$ so as to make $dy/du > 0$. In other words, it is possible that dy/du changes sign from negative to positive as unemployment rate falls below a certain threshold, \bar{u} .

Clearly, \bar{u} is a critical level of unemployment since it acts as a threshold separating $dy/du < 0$ from $dy/du > 0$. Within the $u > \bar{u}$ region, the economy can be said to be in a 'high-effort' state in which a *ceteris paribus* increase in output can only be achieved by a rise in employment. In contrast, the $u < \bar{u}$ region corresponds to the 'low-effort' state where a higher level of output is achievable at a lower level of employment because a rise in unemployment induces a sufficiently large increase in effort of the employed workers which more than compensates the effect on output of the fall in employment. Thus, unlike the high-effort state in which a *ceteris paribus* negative aggregate demand shock is expected to raise unemployment and hence Okun's law has a straightforward interpretation, in the low-effort state Okun's law can only be observed if the negative shock is accompanied by an appropriate shift in the temporal production function. Therefore, the interpretation of Okun's law and its consequences for macroeconomics policy differ drastically depending on which state prevails. In particular, unless $u > \bar{u}$, the observation that a fall in y is accompanied by a rise in u (and hence Okun's law holds) could only have been caused by a structural change which shifted the production function down and/or to the right. In such circumstances, the standard macroeconomic policies are unlikely to yield the expected result. More specifically, an exogenous stimulation of aggregate demand is unlikely to lead to a reduction in unemployment.

In the rest of the paper we examine whether time-series data over 1960Q1–2001Q4 from G7 countries (Canada, France, Germany, Italy, Japan, UK and US) support the existence of a non-monotonicity, similar to that described above, in the relationship between y and u beyond their trend and cyclical fluctuations. We also seek to ascertain if $u > \bar{u}$ holds systematically over the sample period. To do so, for each country we use the level of output and the unemployment rate to estimate a state-space local linear trend model represented by the measurement equation,

$$y_t = x_t' \pi_t + \varepsilon_t. \quad (3)$$

To capture the proposed non-linearity and obtain a robust estimate of \bar{u} , we assume that deviation of output from a random disturbance, $\varepsilon_t \sim \text{iidn}(0, \sigma_\varepsilon^2)$, is a quadratic function² of the unemployment rate and let $x_t' = (1, u_t, u_t^2)$. Also, to account for any secular and/or

Table 1

Selected Results from Estimation of Eq. (3) over 1964Q1–2001Q4

	(I) $\hat{\phi}_T$	(II) $\hat{\phi}_T > 0$	(III) $\hat{\delta}_T$	(IV) $\hat{\delta}_T > 0$	(V) \hat{u}_t	(VI) $\hat{u}_t > 0$	(VII) $\hat{u}_t > \bar{u}_t$
US	2.49 (0.232)	100%	-0.250 (0.036)	100%	5.62 (1.21)	100%	0% [3%]
Canada	2.26 (0.223)	100%	-0.124 (0.022)	100%	9.14 (2.26)	92%	0% [0%]
UK	2.47 (0.640)	100%	-0.158* (0.178)	100%	8.02* (5.03)	95%	22% [26%]
France	4.73 (0.776)	100%	-0.258 (0.124)	100%	9.13 (2.13)	80%	6% [25%]
Germany	21.06 (4.60)	100%	-2.47 (0.593)	100%	4.26 (0.07)	100%	80% [80%]
Italy	4.75 (0.560)	100%	-0.224 (0.082)	100%	10.58 (2.77)	98%	0% [0%]
Japan	4.26 (1.04)	100%	-0.585 (0.231)	96%	3.65 (0.59)	85%	0% [0%]

Notes: The initial 4 years were used to allow the filtered estimates to stabilise. The local linear trend components were not significant for German data and were excluded for that country. The statistical significances of $\hat{\phi}_T$ and $\hat{\delta}_T$ in columns (I) and (III) are based on asymptotic standard errors (numbers in parentheses). To assess the statistical significance of \hat{u}_t on a period-by-period basis we have conducted a parametric bootstrap using 2000 replications for each quarter. The numbers in parentheses are the bootstrapped standard errors for the final period. An asterisk indicates not significant at the 5% level.

cyclical fluctuations in y_t and u_t , we allow the (state) parameters $\pi_t' = (\alpha_t, \phi_t, \delta_t)$ to evolve as follows:

$$\alpha_t = \alpha_{t-1} + \beta_{t-1} + \zeta_t; \quad \zeta_t \sim \text{iidn}(0, \sigma_\zeta^2), \quad (4)$$

$$\beta_t = \theta \beta_{t-1} + \xi_t; \quad \xi_t \sim \text{iidn}(0, \sigma_\xi^2), \quad 0 < \theta < 1, \quad (5)$$

$$\phi_t = \phi_{t-1} + \eta_t; \quad \eta_t \sim \text{iidn}(0, \sigma_\eta^2), \quad (6)$$

$$\delta_t = \delta_{t-1} + \psi_t; \quad \psi_t \sim \text{iidn}(0, \sigma_\psi^2). \quad (7)$$

The extent of generality allowed by this set up is particularly useful when it is applied to bivariate relationships which involve variables that have strong secular patterns and/or are subject to cyclical fluctuations³, and are by construction restricted and fail to condition explicitly on a host of other potentially relevant variables. The underlying state-space representations are very flexible since the non-stationary processes generating ϕ_t and δ_t are allowed to evolve in a manner that capture any fundamental changes which may have occurred in the historical relationship between y_t and u_t . Moreover, to account for trends in y_t and u_t over the estimation period, we have allowed for local linear trends since both the α_{t-1} and β_{t-1} in Eq. (4) vary over time⁴.

Eqs. (3)–(7) were estimated using the Kalman-filter approach where, in the absence of any prior information on the initial distribution⁵, we employed a diffuse prior and set the starting values

³ Both output and unemployment have these properties. Our estimation method is a superior alternative to isolating the secular and cyclical components by pre-filtering the series. In the absence of any explicit dynamics, we employ contemporaneous values of output and unemployment. This might be expected to yield biased parameter estimates due to endogeneity problems but results from IV and GMM estimation (not reported here) suggest that any biases are quantitatively negligible.

⁴ β_t is assumed to be stationary because a non-stationary process would imply $y_t \sim I(2)$ which is against the widely acknowledged stylised fact that the growth rate of output is stationary, also supported by our data using the unit root tests (not reported here).

⁵ Given that three of the four transition equations are non-stationary, the unconditional distribution of the state vector is not defined.

² There is a wide variety of non-linear functions that capture this non-monotonicity. We have opted for the simplest and most parsimonious of these.

$(\alpha_0, \beta_0, \phi_0, \delta_0) = 0$ and $P_0 = I\lambda$ where P_t is the variance–covariance matrix of $(\alpha_t, \beta_t, \phi_t, \delta_t)$, I is the conformable unit matrix and λ is a very large number (see Harvey, 1989).

Empirical support for the existence of a non-linear relationship similar to that depicted in Fig. 1 requires that the following hypotheses cannot be rejected: $\phi_t > 0$; $\delta_t < 0$; $\bar{u}_t = -\phi_t/2\delta_t > 0$; and ε_t is stationary and unpredictable. Moreover, for the economy to be in the high-effort state, the hypothesis $u_t > \bar{u}_t$ should not be rejected. To test these hypotheses statistically, we have obtained filtered estimates of the state vector for each of the G7 countries. The results are reported in Table 1 above where columns (I), (III) and (V) give, respectively, the filtered estimates of ϕ_t and δ_t , and the implied threshold rate of unemployment $\bar{u}_t = -\phi_t/2\delta_t$, for the final observation ($t=T$). Columns (II), (IV) and (VI) report, respectively, the proportion of observations over the estimation period for which the null hypotheses $\phi_t > 0$, $\delta_t < 0$ and $\bar{u}_t > 0$ cannot be rejected at the 5% critical level. These results, together with the stationary behaviour of the estimated residuals $\hat{\varepsilon}_t$ (tests are not reported here but available on request), suggest that the quadratic specification is consistent with the data, beyond any co- and/or counter-movements due to secular and/or cyclical patterns in the underlying series. Finally, since at any point in time either $u_t > \bar{u}_t$ or $u_t \leq \bar{u}_t$, column (VII) gives the percentage of significant occurrences of $u_t > \bar{u}_t$ at 5% (and at 10% in square brackets) critical level. According to these results only German data provide a strong support for $u_t > \bar{u}_t$; US, Canada, Italy and Japan fully reject $u_t > \bar{u}_t$ while UK and to a much lesser extent France show a mild tendency towards exhibiting $u_t > \bar{u}_t$.

3. Summary and concluding remarks

One of the consequences of labour and product market imperfections is that labour productivity can vary in response to fluctuations in unemployment. As a result, it is feasible to expect an economy to move between a 'high-effort' and a 'low-effort' state. It follows that the effect

of typical demand side macroeconomic policies targeted at reducing unemployment depends on which state prevails, and that such policies are unlikely to yield the expected result when the economy is in a low-effort state. We use data from G7 countries to estimate the threshold unemployment rate that at any point in time separates the two states. We find that this threshold is significantly positive in all countries, and that only German data exhibit strong tendency to being persistently in a high-effort state. We draw attention to the relevance of these findings for interpreting Okun's law. It is worth noting that given the similarity between our definition of the threshold rate of unemployment and the NAIRU and the robustness of our empirical method for identifying and estimating a time-varying threshold rate, our approach complements those studies – e.g. see Gordon (1997b) – that estimate a time-varying NAIRU, and could throw new light on the determination of the natural rate of output.

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