

The Okun curve is non-linear

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Abstract

This paper presents cross-country evidence on a non-linear Okun curve. The analysis is based on a simple error-correction model of unemployment. The model is estimated using a threshold model estimator. Evidence from 20 OECD countries for the period 1960–1997 provides support for the existence of non-linearities in terms of the output growth effects. Output growth has a strong effect on unemployment when unemployment is low and output is high, and vice versa. Thus, in bad times, the effect of output growth on unemployment can be close to zero. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

This paper analyses the relationship between output growth and unemployment (i.e. the Okun curve). The aim is to find out whether this much debated relationship is linear – especially in terms of the cyclical situation of the economy. The discovery that the relationship is non-linear would change our thinking in many respects. First of all, non-linearities would provide an explanation for the varying effectiveness of unemployment policies. Second, non-linearities would have important implications for aggregation, say in the context of EMU countries which differ considerably in terms of their cyclical situation. Finally, if the Okun curve turns out to be non-linear, that would suggest that the issue is important for other economic relationships as well.

The empirical analysis is based on a simple (non-linear) error-correction model for unemployment. The long-run level of unemployment is related to the working-age population and to the structure of the labour market, which is proxied by time trends. Thus, this equation takes the form:

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$$u_t = \alpha_0 + \alpha_{1n}n_t + \alpha_3t + \alpha_4t^2 + \mu_t, \quad (1)$$

where u is the log of unemployment U (U is the number of unemployed persons), $n = \log(N)$, which is the working age population, and t is the time trend.¹ The corresponding error–correction model is then of the following form:

$$\Delta u_t = \beta_0 + \beta_1 EC_{t-1} + \beta_2 \Delta n_t + \beta_3 \Delta y_t^- + \beta_4 \Delta y_t^+ + \varepsilon_t, \quad (2)$$

where Δ denotes the first-difference operator, EC the error–correction term (derived from Eq. (1)) and y output. y^- denotes the values of y which correspond to the observation in which $z < z^*$ where z is the threshold variable and z^* the estimated fixed threshold value used in sorting the data. Accordingly, y^+ is related to the observations in which $z \geq z^*$ (see Granger and Teräsvirta (1993) and Hansen (1997) for details of the methodology).

The basic idea of Eq. (2) is simple: in the short run, unemployment reacts to both population and output shocks. The effect of population shocks is assumed to be invariant with respect to the cyclical situation (to keep the number of estimated parameters at a reasonable level) while the output effect is assumed to differ in booms and depressions. This paper does not explain why the output effect differs but simple Keynesian models provide a rationalization for possible regime shifts.

Population shocks can be reasonably well characterized as exogenous but output may not have this property. Given the data (and the threshold model), we cannot really do very much as regards this possible simultaneity problem.

2. Empirical results

The data used in this analysis are annual and cover 20 OECD countries over the period 1960–1997 (with minor exceptions). The data are derived from the recent (December 1998) OECD CD (for more details of the data, see Kiander and Virén (1998)). The data can be characterized by the following average values (over time and all countries): $\Delta u = 4.9\%$, $\Delta n = 0.8\%$, $\Delta y = 3.2\%$, $U/N = 3.5\%$, $U/(E + U) = 5.2\%$, where E is total employment. Thus, $U/(E + U)$ is the unemployment rate.

The following alternative threshold variables are experimented in the analysis: $y - y^*$, Δy , $u - u^*$ and Δu where y^* denotes the output trend (constructed by the HP filter) and u^* the ‘equilibrium level’ of unemployment which was constructed using Eq. (1). For the sake of comparison, Eq. (2) is also estimated in a linear form and, moreover, using a simple fixed zero threshold for Δy .

A summary of estimation results of Eq. (2) is presented in Table 1. We report a comparison of the coefficient estimates and the R^2 's for the linear model (which corresponds to the parameter restriction $\beta_3 = \beta_4$ in Eq. (2)) and for different threshold models. Detailed estimation results are presented in Table 2 just for one threshold variable. On the basis of Table 1, we have chosen the unemployment gap $u - u^*$ as this representative threshold variable.²

¹Unit root tests suggest that the time series of u , e , p , and y are of $I(1)$ type. Also the unemployment/population ratio turns out to be non-stationary. For reasons of space, the test results are not reported here but they are available upon request from the author.

² Δu and $u - u^*$ are the best threshold variables in terms of explanatory power. Δu , although representing the best fit, is, however, in some cases (slightly) non-stationary, and obviously using Δu we would create some sort of a simultaneity problem. Therefore, we report here the detailed results for $u - u^*$.

Table 1
Summary of estimation results for all countries^a

	Constant	EC_{t-1}	Δn	Δy^-	Δy^+	R^2
None	0.12	−0.27	8.7	−4.0	−4.0	0.455
$\Delta y = 0$	0.12	−0.26	7.8	−10.0	−3.8	0.497
$y - y^*$	0.10	−0.28	9.4	−1.2	−5.0	0.563
Δy	0.15	−0.25	8.9	−6.5	−4.1	0.547
$u - u^*$	0.11	−0.43	10.2	−7.1	−1.8	0.665
Δu	0.11	−0.22	8.9	−6.2	−1.1	0.672

^a Numbers are average values for coefficient estimates and R^2 's for all countries.

The results are qualitatively very similar for all threshold variable specifications. Thus, population shocks increase and output shocks decrease unemployment. The coefficient of the error correction terms suggest that it takes about three years to reach the 'equilibrium' level of unemployment. The important point is, however, the fact that although the explanatory power of the alternative specifications differs somewhat, as can be seen from Table 1, threshold models clearly outperform a linear specification in all cases. Thus, for most countries, there appears to be a statistically significant regime shift in the output effect of our Okun curve specification Eq. (2). In the case of the $u - u^*$ threshold variable (Table 2), only in the case of Finland and Iceland can we not reject the hypothesis of no-threshold at the conventional 5% level of significance when the bootstrapped P -values are used.³

This shift can be characterized in the following way: output growth has a negative effect on unemployment (growth) when output is high (above the long-run trend) or unemployment is low (again, relative to the 'natural level'). By contrast, in the case of depression, the magnitude of the output effect is much smaller or completely negligible (see Table 2).⁴ The coefficient estimated for different threshold variables (say, between Δy and $y - y^*$, and similarly between Δu and $u - u^*$) differ somewhat but that is not particularly surprising because these variables are far from perfectly correlated with each other (reflecting slightly different cyclical situations).⁵

The results are nevertheless so clear and systematic that we should pay more attention to this non-linearity (regime shift) possibility both when modelling unemployment policy and carrying out econometric analysis in this area.

³In addition to the 20 countries mentioned above we had also data for Germany. Unfortunately, because of unification, the sample size was so small that threshold estimation did not make sense. If, however, a more straightforward methodology with a fixed zero threshold for Δy is used, the results turn out to be very similar to those in other countries. Thus, the following parameter estimates are obtained for $\beta_3 = -14.0$ and $\beta_4 = -9.1$.

⁴If we look at the unemployment rate effects instead of the unemployment growth effects, we could say that (on an average) a 1% increase in output growth lowers the unemployment rate from, say, 5% to 4.9–4.4 depending on the cyclical situation. A 1% increase in working-age population growth would increase the unemployment rate to 5.5% (i.e. one-third of additional population would be unemployed in the short-run).

⁵The following average values of correlation coefficients can be computed for the sample period: $\{y - y^*, \Delta y\}$ 0.41, $\{y - y^*, u - u^*\}$ −0.60, $\{y - y^*, \Delta u\}$ −0.531 $\{\Delta y, u - u^*\}$ −0.218 $\{\Delta y, \Delta u\}$ −0.55 and $\{u - u^*, \Delta u\}$ 0.36.

Table 2

Estimation results for the $u - u^*$ threshold model^a

Country	Constant	EC_{t-1}	Δn_t	Δy^-	Δy^+	R^2/SEE	dw/lm	$F/(p)$	z^*
Australia	0.163 (1.72)	-0.367 (2.49)	4.800 (0.80)	-6.738 (5.16)	-3.117 (2.16)	0.560 0.123	1.570 0.602	44.18 (0.001)	-0.0652
Austria	0.109 (3.12)	-0.496 (4.99)	5.298 (1.87)	-5.466 (6.67)	-1.177 (1.78)	0.771 0.071	1.708 0.587	51.02 (0.002)	-0.0139
Belgium	0.059 (2.07)	-0.418 (5.20)	29.988 (6.31)	-5.929 (7.94)	-1.058 (1.26)	0.761 0.085	1.126 8.155	82.01 (0.000)	0.0025
Canada	0.023 (0.68)	-0.323 (2.45)	8.058 (4.66)	-4.987 (7.47)	-2.862 (3.67)	0.732 0.073	1.469 1.487	38.51 (0.002)	-0.0448
Denmark	0.267 (3.46)	-0.370 (2.06)	-10.575 (0.79)	-17.075 (4.58)	-4.680 (2.61)	0.593 0.191	1.328 4.616	31.03 (0.016)	-0.1620
Finland	0.213 (4.18)	-0.078 (0.78)	13.057 (2.05)	-8.434 (6.36)	-6.118 (5.39)	0.714 0.155	1.589 1.483	21.51 (0.084)	-0.2670
France	0.065 (1.16)	-0.696 (5.03)	10.096 (1.56)	-7.581 (4.42)	-0.703 (0.60)	0.813 0.110	1.966 0.042	92.04 (0.000)	-0.2203
Greece	0.017 (0.36)	-0.434 (5.65)	9.430 (2.32)	-3.092 (4.73)	-0.248 (0.33)	0.691 0.104	1.385 2.774	30.77 (0.013)	-0.0248
Iceland	0.253 (1.91)	-0.272 (3.02)	13.577 (1.55)	-11.008 (6.03)	-7.548 (6.32)	0.740 0.243	1.692 0.784	15.25 (0.200)	-0.6104
Ireland	0.071 (2.09)	-0.428 (3.50)	3.650 (1.49)	-2.978 (4.96)	-0.288 (0.43)	0.634 0.078	1.626 (1.841)	46.53 (0.004)	0.0215
Italy	0.055 (1.64)	-0.333 (3.51)	10.239 (2.66)	-7.796 (5.28)	-1.512 (1.75)	0.540 0.109	1.469 2.239	42.52 (0.002)	-0.2247
Japan	0.069 (3.40)	-0.660 (4.76)	-6.334 (3.02)	-0.178 (0.38)	2.056 (3.02)	0.612 0.060	1.513 2.332	56.87 (0.000)	0.0487
Netherlands	0.016 (0.20)	-0.407 (2.73)	14.874 (2.32)	-5.349 (2.97)	0.998 (0.45)	0.608 0.131	1.089 19.53	55.71 (0.002)	0.0100
New Zealand	0.266 (1.13)	-0.527 (2.74)	10.121 (0.54)	-22.580 (3.72)	-3.181 (0.76)	0.531 0.448	1.639 1.581	51.86 (0.001)	-0.0155
Norway	0.250 (2.18)	-0.593 (4.17)	14.791 (0.79)	-6.215 (4.31)	-0.536 (0.33)	0.663 0.118	1.735 0.393	64.00 (0.001)	-0.0158
Portugal	0.051 (0.92)	-0.619 (5.71)	1.987 (0.84)	-2.376 (2.24)	2.816 (2.07)	0.602 0.156	1.858 0.299	33.96 (0.008)	0.0774
Spain	-0.115 (3.00)	-0.524 (7.27)	27.246 (7.95)	-2.932 (8.09)	0.092 (0.17)	0.859 0.055	1.319 4.629	88.46 (0.000)	-0.0130
Sweden	0.164 (3.79)	-0.190 (1.83)	17.309 (1.80)	-12.562 (6.53)	-5.510 (3.63)	0.653 0.141	1.330 4.886	41.72 (0.002)	-0.1845
UK	-0.066 (1.41)	-0.743 (4.55)	14.871 (2.15)	-2.452 (1.87)	4.680 (2.97)	0.609 0.113	1.447 3.540	89.74 (0.000)	-0.0329
USA	0.060 (1.65)	-0.082 (0.93)	10.749 (4.13)	-6.526 (11.03)	-4.739 (5.58)	0.840 0.064	2.205 0.485	35.59 (0.030)	-0.0662

^a Numbers inside parentheses under coefficient estimates are t -ratios. LM is a test statistic for first order autocorrelation (with χ_1^2 distribution under H_0), F denotes a test statistic for the hypothesis of no threshold. Numbers inside parentheses are bootstrapped probability values (1000 replications are used). z^* is the threshold value of $u - u^*$. When computing the LM test we utilized Chan (1993) in which it is shown that the threshold parameter is superconsistent and thus can be treated as a known parameter.

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