Cyclical output, cyclical unemployment, and Okun's coefficient
A structural time series approach

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Abstract

Okun's coefficient is estimated from U.S. quarterly data covering the period 1947:1–1992:2. The cyclical components of unemployment and output are extracted by smoothing using the Kalman filter as applied to Harvey's structural time series model. The estimated Okun's coefficient is around -0.38 irrespective of the whether the model used is static or dynamic and irrespective of the lag length in the dynamic model. © 1999 Elsevier Science Inc. All rights reserved.

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1. Introduction

Empirical studies of the relationship between unemployment and real output growth have produced a range of values for the coefficient on real output growth, or what has become to be known as Okun's coefficient. The range of estimates is wide because of several factors that have given rise to different interpretations of the coefficient. These factors include: (1) using dynamic versus static specifications, which brings about the question whether the relationship is contemporaneous or lagged,[1] (2) allowing, or otherwise, for the effect of other variables such as capacity utilization, hours per worker and labor force participation,[2] (3) the method used to extract the cyclical components of unemployment and output; (4) distinguishing between demand and

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supply shocks; (5) the econometric method used to estimate that final model; (6) distinguishing between long-run and short-run effects; and (7) the sample period.

In general, more recent studies have produced results suggesting that output growth exerts greater impact on unemployment than what was implied by Okun's results. The main reason for this conclusion seems to be the use of dynamic specifications which allow the estimation of the long-run effect. For example, while Okun (1962) produced a value of -0.32 using a static regression, Gordon (1984) and Evans (1989) used an autoregressive distributed lag (ADL) model to estimate the lagged effect and produced values of the coefficient exceeding 0.4 in magnitude. On the other hand, allowing for other factors seems to reduce the value of the coefficient. For example, by incorporating other factors in his model, Prachowny (1993) has shown that the coefficient is much smaller than what was obtained by Gordon (1984), Evans (1989), and Okun (1962). The reason for this finding is a combination of incorporating other variables and not allowing for the lagged effects.

More recently, Weber (1995) utilised the results of Perron (1989, 1990) and Evans (1989) to estimate the post-war cyclical components of the U.S. real GNP and unemployment rates, and used them to estimate Okun's coefficient by employing four different methods and three sample periods.[3] The results derived from the static regression support Okun's (1962) original results. And while the results obtained from the static regression indicated some sort of a structural break in 1973, the dynamic results revealed no such a break. Weber suggested that is would be interesting to find out if the results obtained from different estimates of the cyclical components are different from his. This paper makes such a comparison possible.

The objective of this paper is to estimate Okun's coefficient in the post-war U.S. economy using a different method for extracting the cyclical components of output and unemployment. For this purpose, a set of quarterly data is used covering the period extending between 1947:1 and 1992:2. The starting point is an outline of the methodology, which is followed by a presentation and a discussion of the results. The paper ends with some concluding remarks.

2. Methodology

The structural time series model of Harvey (1985, 1989) is used to decompose the observed output and unemployment series into their unobserved components. The rationale for using this methodology, as opposed the techniques employed in similar studies, is provided by Harvey and Jaeger (1993). Several arguments are put forward for using the structural time series model. First, this approach allows investigators to deal explicitly with seasonal and irregular movements, which may distort the cyclical component. Second, it provides the most useful framework within which to present stylized facts on time series, as it is explicitly based on the stochastic properties of the data. Third, it provides useful information and serves as a basis for exposing the limitations of other techniques. As such, it provides a basis for exposing the limitations of autoregressive integrated moving average (ARIMA) models and models based on deterministic trends with single breaks. For example, Harvey and Jaeger (1993) argue

that mechanical detrending techniques such as the HP filter can lead to spurious cyclical behavior.

The structural time series model may be written as Eq. (1)

$$z_t = \mu_t + \varphi_t + \gamma_t + \epsilon_t \tag{1}$$

where z_t is the observed value of the series, μ_t is the trend component, φ_t is the cyclical component, γ_t is the seasonal component and ϵ_t is the irregular component. The trend, cycle, and seasonal components are assumed to be uncorrelated, while ϵ_t is assumed to be white noise.

The trend component, which represents the long-term movement of a series, is assumed to be stochastic and linear. This component can be represented by Eqs. (2) and (3):

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \tag{2}$$

$$\beta_t = \beta_{t-1} + \zeta_t \tag{3}$$

where $\eta_t \sim NID(0, \sigma_\eta^2)$, and $\zeta_t \sim NID(0, \sigma_\zeta^2)$. μ_t is a random walk with a drift factor, β_t, which follows a first order autoregressive process as represented by Eq. (3). This process collapses to a simple random walk with drift if $\sigma_\zeta^2 = 0$, and to a deterministic linear trend if $\sigma_\eta^2 = 0$ as well. If, on the other hand, $\sigma_\eta^2 = 0$ while $\sigma_\zeta^2 \neq 0$, the process will have a trend which changes relatively smoothly.[4]

The cyclical component, which is assumed to be a stationary linear process, may be represented by Eq. (4):

$$\varphi_t = a\cos\theta t + b\sin\theta t \tag{4}$$

where t is time and the amplitude of the cycle is given by $(a^2 + b^2)^{1/2}$. To make the cycle stochastic, the parameters a and b are allowed to evolve over time, while preserving continuity is achieved by writing down a recursion for constructing φ before introducing the stochastic components. By introducing disturbances and a damping factor we obtain Eqs. (5) and (6),

$$\varphi_t = \rho(\varphi_{t-1} \cos\theta + \varphi_{t-1}^* \sin\theta) + \omega_t \tag{5}$$

$$\varphi_t^* = \rho(-\varphi_{t-1} \sin\theta + \varphi_{t-1}^* \cos\theta) + \omega_t^* \tag{6}$$

where φ_t^* appears by construction such that ω_t and ω_t^* are uncorrelated white noise disturbances with variances σ_ω^2 and $\sigma_{\omega^*}^2$ respectively. The parameters $0 \leq \theta \leq \pi$ and $0 \leq \rho \leq 1$ are the frequency of the cycle and the damping factor on the amplitude respectively. In order to make numerical optimisation easier, the constraint $\sigma_\omega^2 = \sigma_{\omega^*}^2$ is imposed.

While there are a number of different specifications for the seasonal component (see Harvey, 1989, Chapter 2), we use the trigonometric specification which, for an even s, is written as Eq. (7),

$$\gamma_t = \sum_{j=1}^{s/2} \gamma_{j,t} \tag{7}$$

$\gamma_{j,t}$ is given by Eqs. (9) and (10):

$$\gamma_{j,t} = \gamma_{j,t-1} \cos\lambda_j + \gamma_{j,t-1}^* \sin\lambda_j + \kappa_{j,t} \tag{8}$$

$$\gamma_{j,t}^* = \gamma_{j,t-1} \sin\lambda_j + \gamma_{j,t-1}^* \cos\lambda_j + \kappa_{j,t}^* \tag{9}$$

where $j = 1, \ldots, s/2 - 1$, $\lambda_j = 2\pi j/s$ and

$$\gamma_{j,t} = -\gamma_{j,t-1} + \kappa_{j,t}, \quad j = s/2 \tag{10}$$

where $\kappa_{j,t} \sim NID(0, \sigma_\kappa^2)$ and $\kappa_{j,t}^* \sim NID(0, \sigma_{\kappa*}^2)$. Again, the assumption $\sigma_\kappa^2 = \sigma_{\kappa*}^2$ is imposed. One advantage of this specification is that it allows for smoother changes in the seasonals (Harvey & Scott, 1994, p. 1328).

The extent to which the trend, seasonal, and cyclical components evolve over time depends on the values of $\sigma_\eta^2, \sigma_\zeta^2, \sigma_\kappa^2, \sigma_\omega^2, \theta$, and ρ which are known as the hyperparameters. The hyperparameters as well as the components can be estimated by maximum likelihood using the Kalman filter to update the state vector. This procedure requires writing the model is state space form. Related smoothing algorithms can be used to obtain the estimates of the state vector at any point in time within the sample period (Harvey, 1989, Chapter 4).

The relationship between the cyclical components of unemployment and output, u^c and y^c, may be written in a stochastic form as

$$\mu_t^c = \alpha y_t^c + \mu_t \tag{11}$$

Eq. (11) assumes that the relationship is totally contemporaneous, which may not be plausible theoretically. It may also be inadequate empirically owing to the omission of short-run dynamics. Following Hendry, Pagan, and Sargan (1984) the dynamic model used here is the ADL model

$$\mu_t^c = \sum_{i=1}^m \delta_i \mu_{t-i}^c + \sum_{i=0}^m \alpha_i y_{t-i}^c + \upsilon_t \tag{12}$$

where the contemporaneous (impact or short-run) effect of output on unemployment is measured by the coefficient α_0 while the long-run effect is measured by calculating a function of the coefficients, ω, which is given by Eq. (13):

$$\omega = \frac{\sum\limits_{i=0}^m \alpha_i}{1 - \sum\limits_{i=1}^m \delta_i} \tag{13}$$

The question that arises here is whether Okun's coefficient is α_0 or ω. The tendency is to define Okun's coefficient as measuring the long-run effect, because the relationship between unemployment and output is not necessarily contemporaneous. Hence, Okun's coefficient is taken to be ω.[5]

3. Data and empirical results

The methodology outlined in the previous section is applied to a sample of U.S. quarterly data covering the period 1947:1–1992:2 as reported in Gordon (1993), Table

Table 1
The estimated parameters of cyclical components

Parameter	Unemployment	Output
σ_η^2	0.060	0.000
σ_ς^2	0.000	2.11×10^{-6}
σ_ω^2	0.059	5.58×10^{-5}
θ	0.355	0.328
ρ	0.950	0.911
$\tilde{\sigma}$	0.380	0.0098
R^2	0.95	0.99
R_s^2	0.21	0.109
N	33.38	22.89
H	0.54	0.66
$r(1)$	0.51	0.11
$r(12)$	−0.08	−0.12
DW	0.98	1.78
Q	72.04	17.02

A-2. The output variable is real GDP (measured at 1987 prices). The cyclical components of unemployment and output are extracted by the *Structural Time Series Analyser, Modeller and Predictor* (STAMP) version 5.0. The procedure requires the estimation of the univariate time series model represented by Eq. (1) for the level of unemployment and the logarithm of output. For details, see Koopman, Harvey, Doornik, and Shephard (1995).

The maximum likelihood estimates of the parameters are reported in Table 1. Also reported are some measures of goodness of fit and diagnostic test statistics. The goodness of fit measures include the coefficient of determination (R^2) and the modified coefficient of determination (R_s^2). The latter is calculated on the basis of the seasonal mean. Also reported is the standard error ($\tilde{\sigma}$), which is calculated as the square root of the one-step ahead predictive error variance. Diagnostic statistics are reported for normality, heteroscedasticity, and serial correlation. The diagnostic for normality (N) is the Bowman-Shenton (1975) test for normality, which is based on the departure of the third and fourth moments from their expected values under normality. It is distributed as $\chi^2(2)$. The diagnostic for heteroscedasticity (H) is calculated as the ratio of the squares of the last h residuals to the squares of the first h residuals where h is the closet integer to one-third of the sample size. It is distributed as $F(h,h)$. The autocorrelation coefficients of the residuals with lags one and 12 are also reported. For serial correlation, the diagnostics include the DW statistic and the Ljung-Box Q-statistic (Ljung & Box, 1978) calculated on the basis of the first 12 autocorrelation coefficient. It is distributed as $\chi^2(7)$. The results show that the frequency of the unemployment and output cycles are rather close and so are the damping factors on the amplitude of the two cycles. These estimates imply that the cycle period for unemployment is 17.66 (4.416 years) while the cycle period for output is 19.15 (4.78 years). Moreover, both models seem to be well determined and pass the diagnostics.
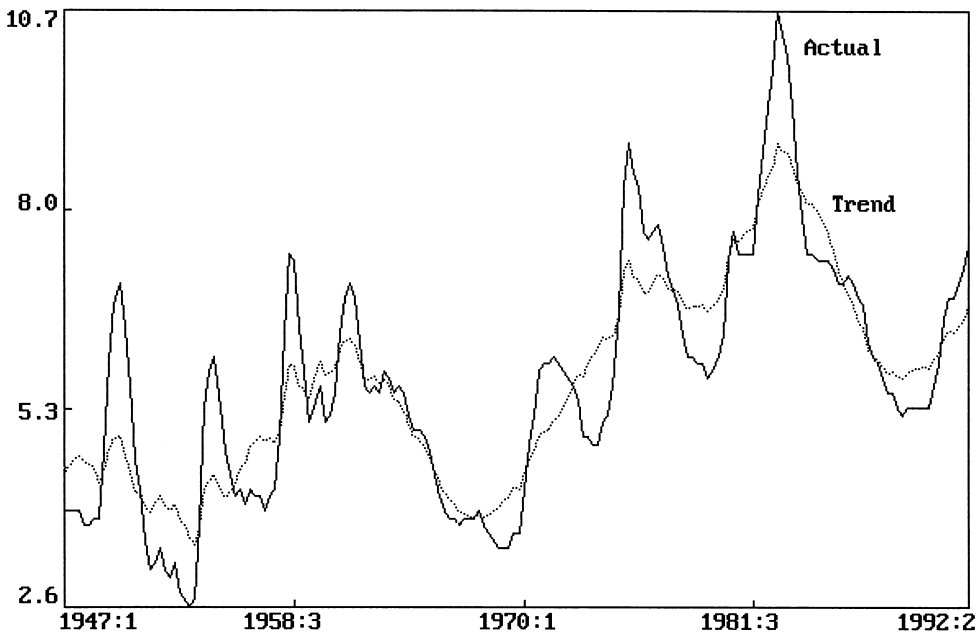
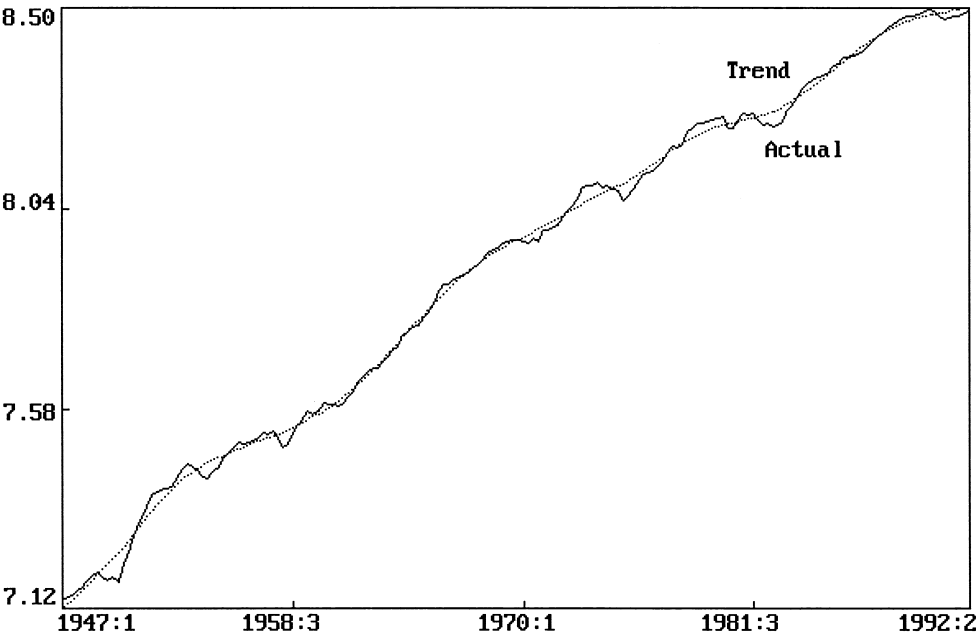Fig. 1. Unemployment rate (actual and trend).
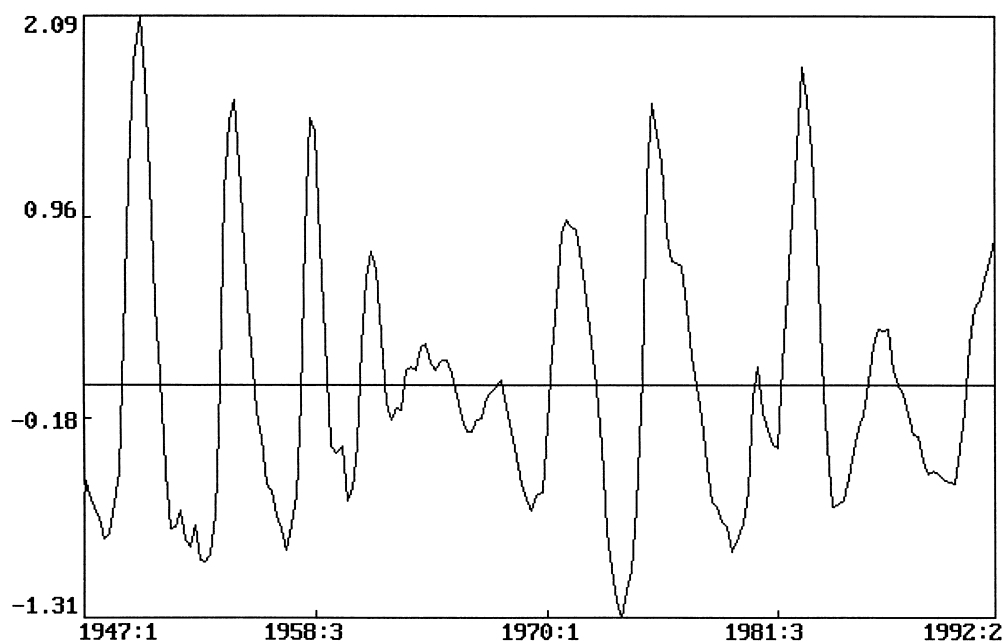

Fig. 2. Logarithm of output (actual and trend).

Fig. 3. Cyclical unemployment.

Finally, the estimated value of σ_η^2 is high for unemployment and zero for output, implying a volatile trend for the former and a smooth trend for the latter.

The estimated trends and cyclical components are shown in Fig. 1–4. Fig. 1 and 2 show the actual time series and the extracted trends for unemployment and (the logarithm of) output respectively. Figs. 3 and 4 show the extracted cyclical components of the two series. Notice, however, that the cyclical component of output as extracted from the logarithmic univariate model is scaled by multiplying it by 100. This is because unemployment is measured in percentage terms while output is measured in natural logarithms. Since the cyclical component of output is measured as a fraction, it requires this kind of adjustment to be measured in percentage terms (see Gordon, 1993, pp. 322–323).

Now that observations on the cyclical components are available, it is possible to proceed with the estimation of Okun's coefficient. When Eq. (11) is estimated by OLS, the following results are obtained

$$\mu_t^c = -0.379 y_t^c$$
$$(-26.31)$$

$$R^2 = 0.79 \qquad DW = 0.60$$

Although Okun's coefficient is statistically significant and correctly signed, the fitted equation suffers from serial correlation, most likely due to the omission of short-run dynamics. It is widely accepted now that serial correlation should not be "corrected,"
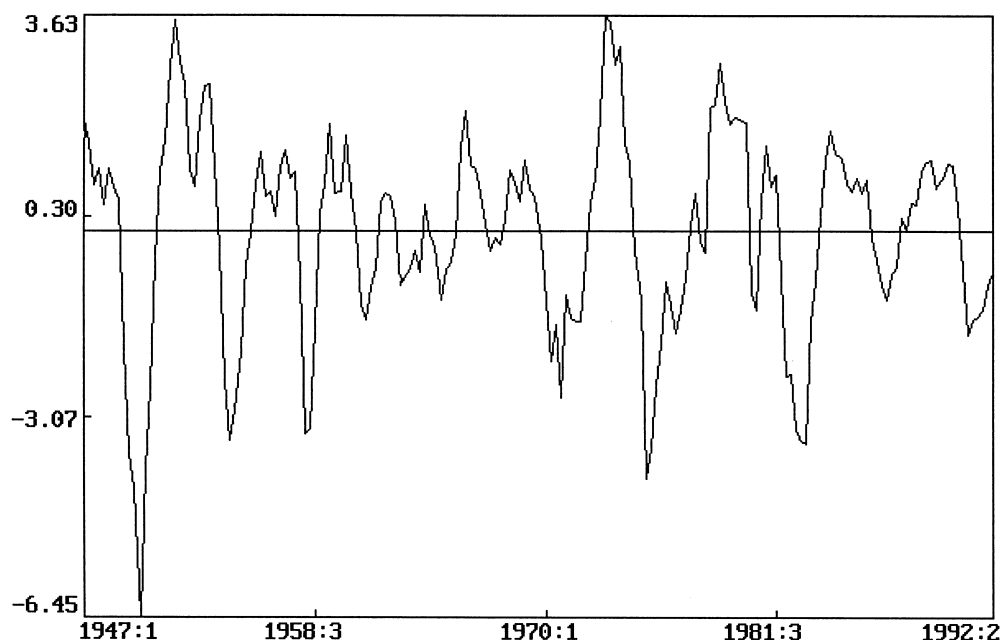
Fig. 4. Cyclical output.

but rather the equation should be re-specified.[6] Thus, the estimate of the coefficient on the basis of a static regression is not reliable.

Because it is more appropriate to estimate the short-run and long-run effect from a properly specified dynamic model, Eq. (12) is estimated by OLS. The equation is estimated for a number of large lengths to find out if the results are robust with respect to the lag selection. The choice here is to use lags between 2 and 5.[7] The estimation results are reported in Table 2 which also reports the diagnostics for serial correlation, S, functional form, F, heteroscedasticity, H, and normality, N.[8] These test statistics are distributed as χ^2 with 4, 1, 1, and 2 degrees of freedom respectively. The marginal significance levels are given in parentheses underneath the estimated values of the test statistics.

The first thing to notice about these results is the absence of serial correlation as a result of introducing short-run dynamics, even if the lag is two. Moreover, the results are robust with respect to the lag length, implying that a lag of 2 is sufficient to capture all of the dynamics. The estimate of the short-run impact coefficient is -0.160, which is substantially lower than the value obtained by Okun (1962) or that obtained from a static regression. As previously pointed out, Okun's coefficient is more appropriately defined as the long-run coefficient. The estimated value of the long-run coefficient is around -0.38, which is, perhaps surprisingly, what is obtained from the static regression. It is interesting to note that the estimates reported in Table 2 are close to the estimates obtained by Weber from his static and cointegrating regressions. The estimates obtained by Weber from the dynamic models are much smaller because he

Table 2
OLS estimates of the ADL model [Eq. (12)][a]

m	α_0	ω	R^2	$S(4)$	$F(1)$	$H(1)$	$N(2)$
2	−0.160	−0.366	0.97	7.61	2.28	2.09	0.21
	(−12.21)	(−12.10)		(0.10)	(0.13)	(0.12)	(0.90)
3	−0.162	−0.373	0.97	7.90	2.03	2.47	0.56
	(−12.03)	(−13.06)		(0.09)	(0.15)	(0.14)	(0.76)
4	−0.162	−0.386	0.97	6.69	1.79	2.42	0.95
	(−12.31)	(−14.59)		(0.23)	(0.18)	(0.14)	(0.62)
5	−0.160	−0.385	0.97	2.54	1.51	2.43	0.62
	(−12.07)	(−15.40)		(0.63)	(0.22)	(0.15)	(0.73)

[a] The t statistics are given in parentheses underneath the estimated coefficients α_0 and ω. The marginal significance levels are given in parentheses underneath the diagnostic statistics S, F, H, and N.

defined Okun's coefficient to exclude the impact coefficient. This explains the difference between Weber's two sets of results.[9]

We now turn to the issue of structural stability to examine whether there is a structural break in the relationship following the oil shock of 1973. To start with , the model (with five lags) seems to be structurally stable as it passes the CUSUM test as shown in Fig. 5. The model is then subjected to the Chow test and the predictive failure test (Chow's second test) taking the fourth quarter of 1973 as the breakpoint. The model passes both of these tests with test statistics of 1.21 and 0.97, distributed
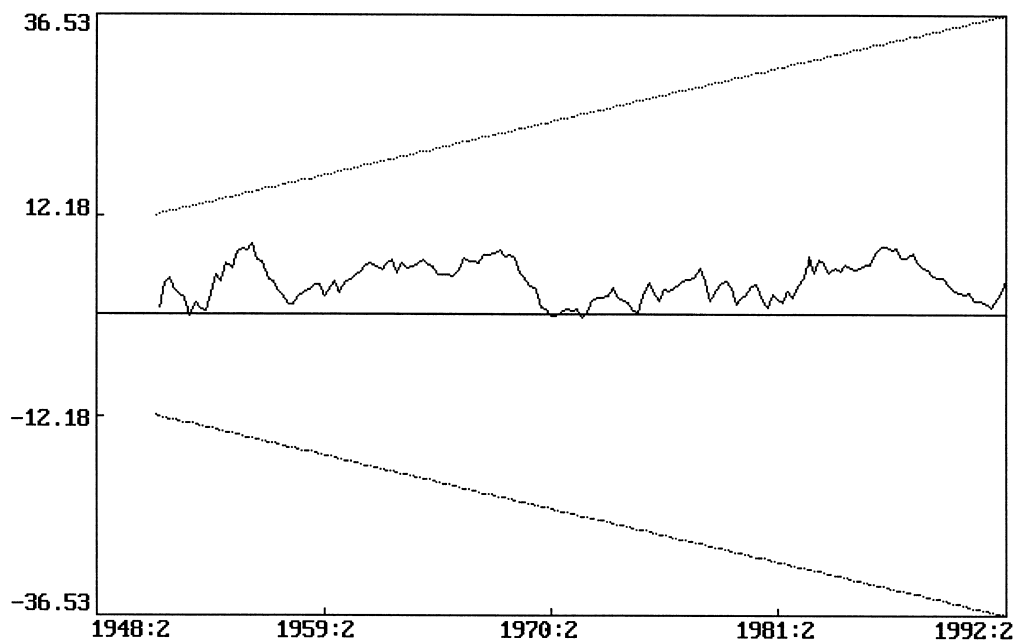

Fig. 5. Cumulative sum of recursive residuals.

as $F(11,155)$ and $F(74,92)$ respectively. Hence, the evidence shows that there is no structural break in the relationship between cyclical output and cyclical unemployment.

4. Conclusion

In this paper an attempt has been made to estimate Okun's coefficient from a dynamic ADL model relating the cyclical component of unemployment to the cyclical component of output. The cyclical component were extracted by decomposing the total time series using Harvey's structural time series model. The estimates produced a value of -0.16 for the short-run impact coefficient and a value of -0.38 for the long-run coefficient. Structural stability tests revealed no structural breaks in the relationship. Although the estimated value of the coefficient from the static regression turned out to be close to the value estimated from the dynamic regression, the former result is unreliable because the model is plagued by serial correlation.

What is more important is that the results obtained from the dynamic model are robust with respect to the lag length. The estimated value of Okun's coefficient is close to the original value estimated by Okun and more plausible than the much higher values obtained in more recent studies employing dynamic models.

Acknowledgment

Notes

1. This point boils down to whether the relationship should be tested on the basis of a static regression allowing for the contemporaneous effect only or by using a dynamic model that also allows for the lagged effect.
2. This point boils down to whether the relationship should be tested in a bivariate or a multivariate framework.
3. Weber (1995) reported eighteen estimates of Okun's coefficient using static OLS regression, cointegrating regression, ADL model with lags two and four, and Blanchard's (1989) method of innovations in the VAR. The three sample periods are 1948:1–1988:4, 1948:1–1973:3, and 1973:4–1988:4.
4. μ and β are equivalent to the constant term and the coefficient on a deterministic time trend in a conventional regression equation.
5. Weber (1995) considers Okun's coefficient to be the long-run coefficient but he calculates it by excluding the impact coefficient, α_0. This is bound to produce a smaller value of Okun's coefficient than otherwise.

6. See, for example, Mizon (1995). Weber (1995, p. 440) considered first-order serial correlation in his static model to be evidence for misspecified dynamics, and so he introduced dynamic equations.
7. Lags longer than 5 have been tried but the results did not change significantly.
8. S is the LM version of Godfrey's (1978a,b) test; F is Ramsey's (1970) RESET test; H is a test for heteroscedasticity based on the auxiliary regression of squared residuals on the estimated values of the dependent variable (Koeneker, 1981); and N is the Bera and Jarque (1981) test.
9. It is possible that if Weber had estimated Okun's coefficient from a dynamic regression equation by including the impact coefficient, these estimates would have been close to those obtained from the static and cointegrating regressions.

References

Bera, A., & Jarque, C. M. (1981). An efficient large sample test for normality of observations and regression residuals. Canberra: Australian National University Working Papers, No. 40.

Blanchard, O. J. (1989). A traditional interpretation of macroeconomic fluctuations. *American Economic Review, 79*, 1146–1164.

Bowman, K. O., & Shenton, L. R. (1975). Omnibus test contours for departures from normality based on $\sqrt{b_1}$ and b_2. *Biometrika 62*, 243–250.

Evans, G. W. (1989). Output and unemployment dynamics in the United States: 1950–1985. *Journal of Applied Econometrics, 4*, 213–238.

Godfrey, L. G. (1978a). Testing against general autoregressive and moving average error models when the regressors include lagged dependent variables. *Econometrica, 46*, 1293–1302.

Godfrey, L. G. (1978b). Testing for higher order serial correlation in regression equations when the regressors include lagged dependent variables. *Econometrica, 46*, 1303–1310.

Gordon, R. J. (1984). Unemployment and potential output in the 1980's. *Brookings Papers on Economic Activity, 15*, 537–564.

Gordon, R. J. (1993). *Macroeconomics* (6th ed.). New York: Harper Collins.

Harvey, A. C. (1985). Trends and cycles in macroeconomic time series. *Journal of Business and Economic Statistics, 13*, 216–227.

Harvey, A. C. (1989). *Forecasting, Structural Time Series Models and the Kalman Filter.* Cambridge: Cambridge University Press.

Harvey, A. C., Jaeger, A. (1993). Detrending, stylized facts and the business cycle. *Journal of Applied Econometrics, 8*, 231–247.

Harvey, A. C., & Scott, A. (1994). Seasonality in dynamic regression models. *Economic Journal, 104*, 1324–1345.

Hendry, D. F., Pagan, A. R., & Sargan, J. D. (1984). Dynamic specification. In Z. Griliches & M. Intrligeter (Eds.), *Handbook of Econometrics* (vol. 2; pp 1025–1100) Amsterdam: North Holland.

Koenker, A. (1981). A note on studentizing a test for heteroskedasticity. *Journal of Econometrics, 17*, 107–112.

Koopman, S. J., Harvey, A. C., Doornik, J. A., & Shephard, N. (1995). *STAMP 5.0 Structural Time Series Analyser, Modeller and Predictor.* London: Champman and Hall.

Ljung, G. N. M., & Box, G. E. P. (1978). On a measure of lack of fit in time series models. *Biometrika, 65*, 279–303.

Mizon, G. E. (1995). A simple message for autocorrelation correctors: don't. *Journal of Econometrics, 69*, 267–288.

Okun, A. (1962). Potential GNP: Its measurement and significance. *Proceedings of the Business and Economic Statistics Section of the American Statistical Association* (pp. 89–104).

Perron, P. (1989). The great crash, the oil price shock, and the unit root hypothesis. *Econometrica, 57*, 1361–1402.

Perron, P. (1990). Testing for unit roots in a time series with a changing mean. *Journal of Business and Economics Statistics, 8*, 153–162.

Prachowny, M. F. J. (1993). Okun's law: theoretical foundations and revised estimates. *Review of Economics and Statistics, 75*, 331–336.

Ramsey, J. B. (1970). Models, specification error and inference: A discussion of some problems in econometric methodology. *Bulletin of the Oxford University Institute of Economics and Statistics, 32*, 301–318.

Weber, C. E. (1995). Cyclical output, cyclical unemployment, and Okun's coefficient: a new approach. *Journal of Applied Econometrics 10,* 433–445