

## Hands-on Introduction to Quantum Computing with LibKet

IEEE Quantum Week 2021

October 17-22, 2021

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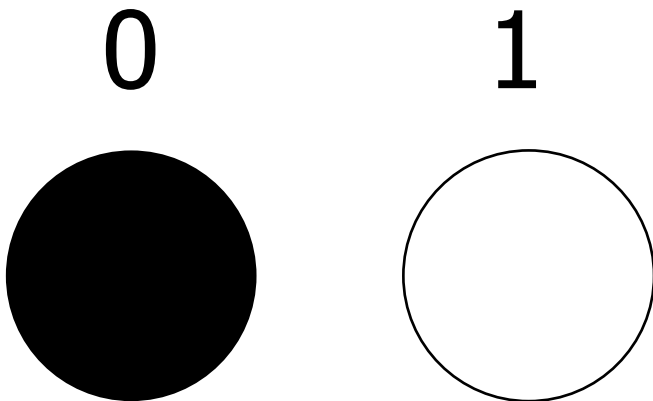
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# What is different in QC?

Basic unit of information

Bit

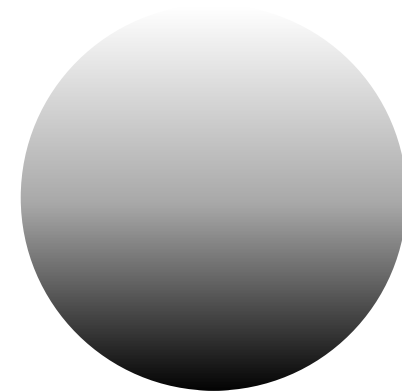
Exclusive state



Quantum bit (Qubit)

$|0\rangle$  and  $|1\rangle$

Superposition



$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$$

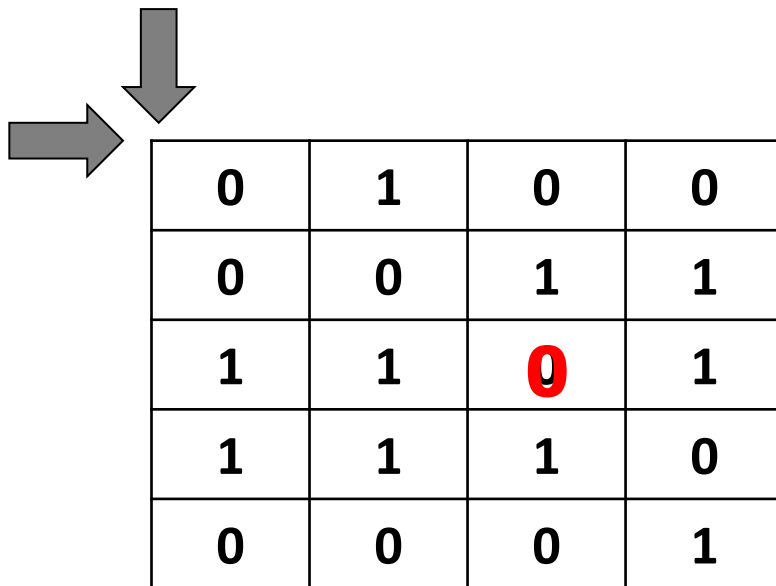
# What is different in QC?

Reading out information

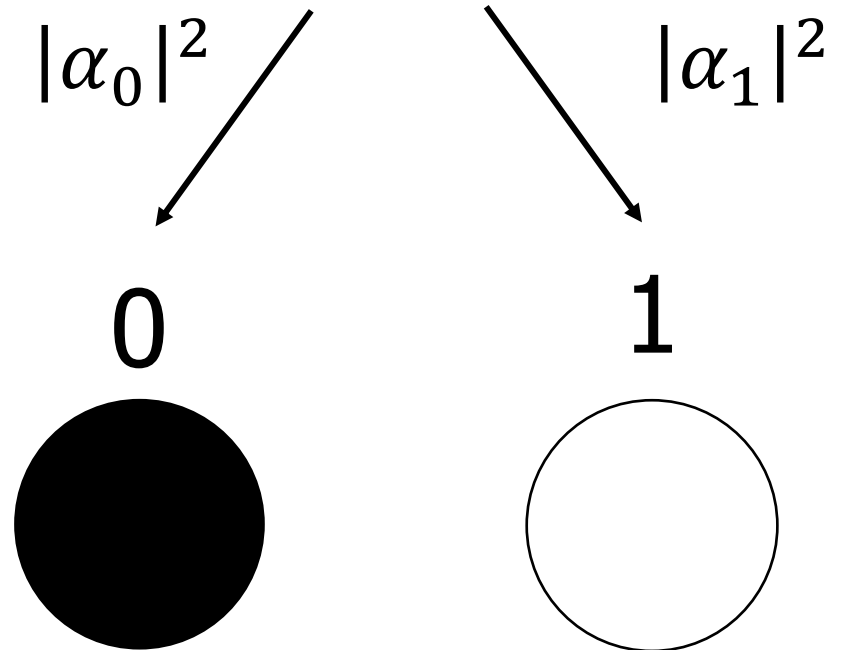
$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$$



Memory



0	1	0	0
0	0	1	1
1	1	0	1
1	1	1	0
0	0	0	1



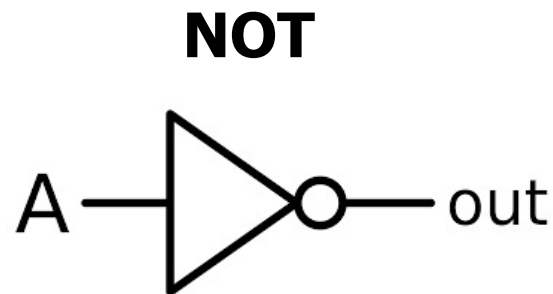
$$|\alpha_0|^2 + |\alpha_1|^2 = 1$$

Probabilistic process

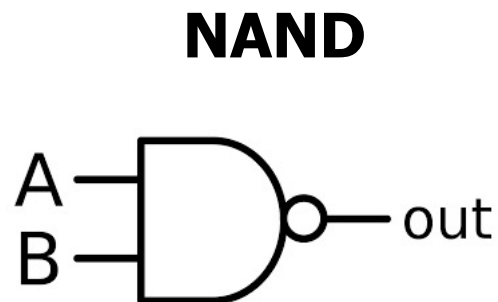
# What is different in QC?

## Operations

### Classical gates



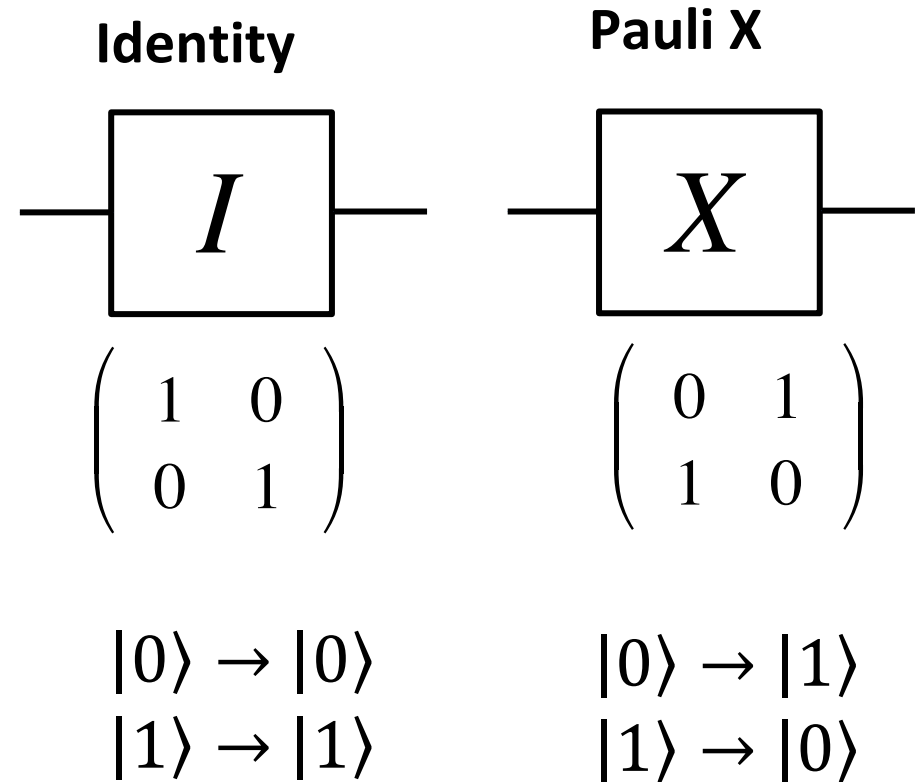
A	out
0	1
1	0



A	B	out
0	0	1
0	1	1
1	0	1
1	1	0

- Logical operation
- Truth tables
- Most of them only run forward

### Quantum gates

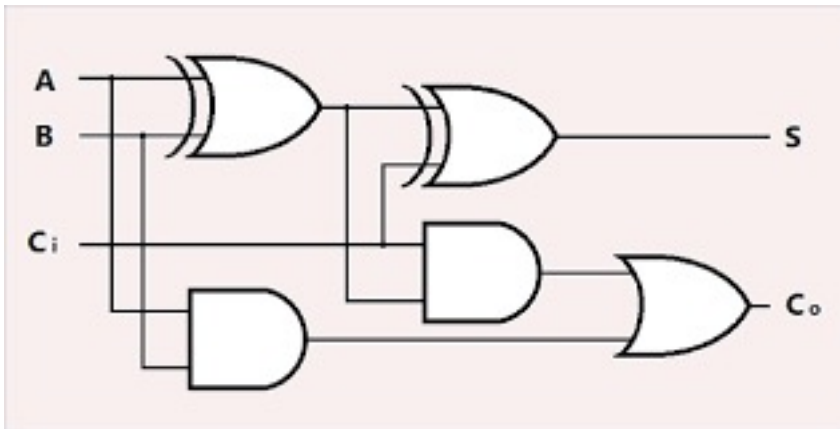


- Unitary operation
- Unitary matrix
- All quantum gates are reversible

# What is different in QC?

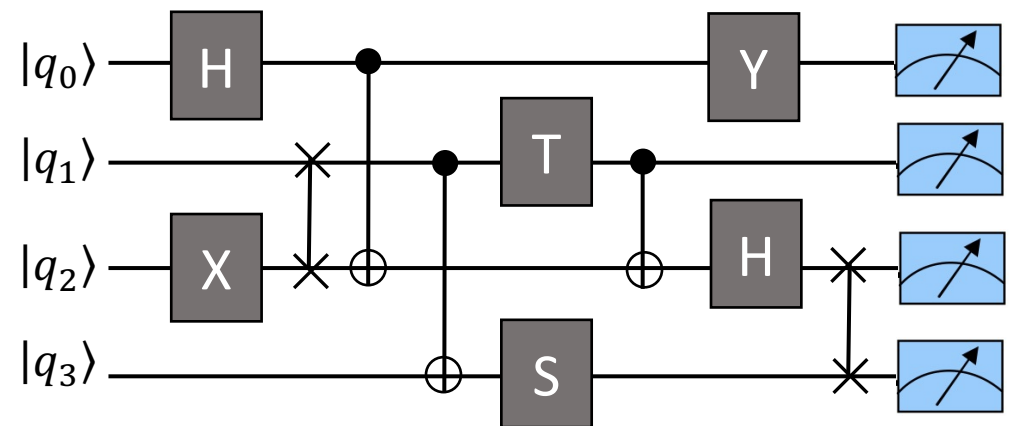
## Computation

### Classical circuits



### Quantum circuits

#### Circuit model of computation



- Single-qubit gates: H, X, T, S, Y
- Two-qubit gates: CNOT and SWAP

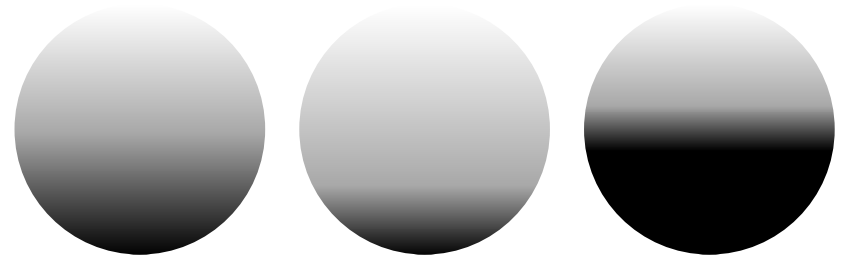
# What is different in QC?

3 bits

000 **or** 001 **or** 010 **or** 011 **or**  
100 **or** 101 **or** 110 **or** 111

- n bits hold 1 value: from 0 to  $2^n - 1$

3 qubits



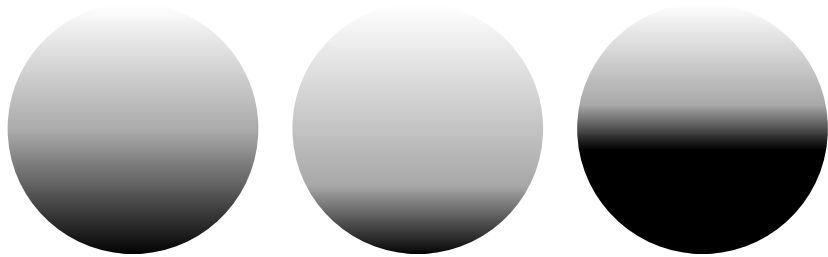
$$\alpha_{000}|000\rangle + \alpha_{001}|001\rangle + \alpha_{010}|010\rangle + \alpha_{011}|011\rangle \\ + \alpha_{100}|100\rangle + \alpha_{101}|101\rangle + \alpha_{110}|110\rangle + \alpha_{111}|111\rangle$$

000 **and** 001 **and** 010 **and** 011 **and**  
100 **and** 101 **and** 110 **and** 111

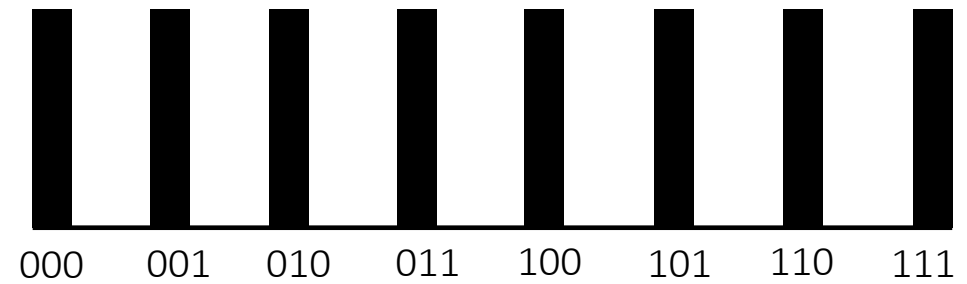
- n qubits can hold  $2^n$  values (50 qubits,  $2^{50}$  complex amplitudes)
- All states (amplitudes) can be manipulated at the same time

# What is different in QC?

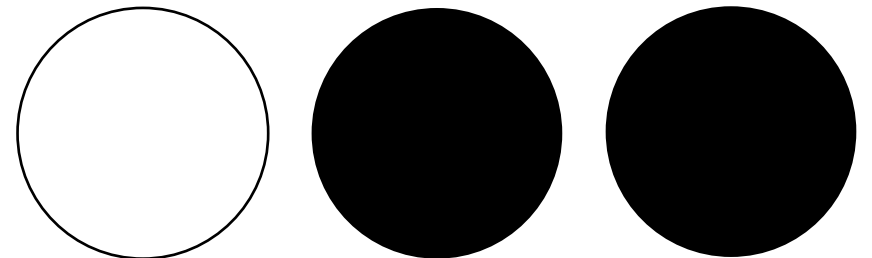
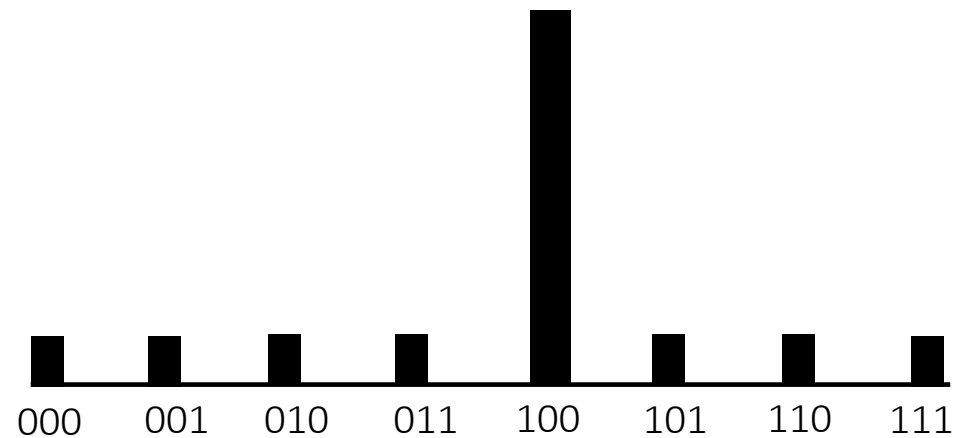
Superposition and entanglement



$$\alpha_{000}|000\rangle + \alpha_{001}|001\rangle + \alpha_{010}|010\rangle + \alpha_{011}|011\rangle \\ + \alpha_{100}|100\rangle + \alpha_{101}|101\rangle + \alpha_{110}|110\rangle + \alpha_{111}|111\rangle$$



Interference

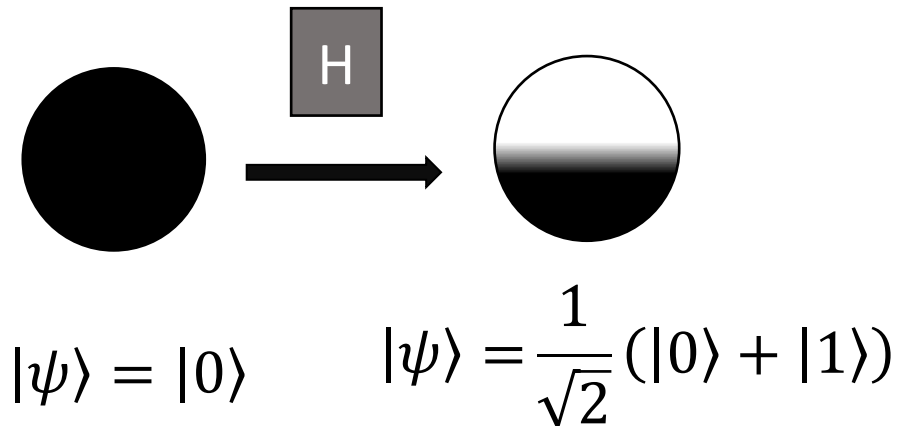


- Probabilistic computation
- Result is a binary value

# What is different in QC?

## In-memory computing

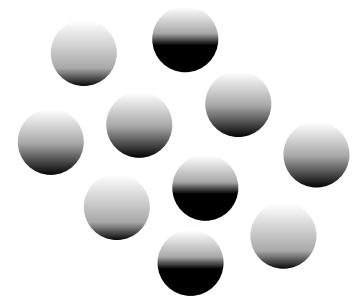
- The qubits hold the information in a form of a quantum state which is modified by applying an operation on them.



## Qubits and gates are error prone

- Qubits have short coherence time
- Imperfect operations
  - Gate error rates:  $10^{-2}$ - $10^{-3}$

Quantum information needs to be protected

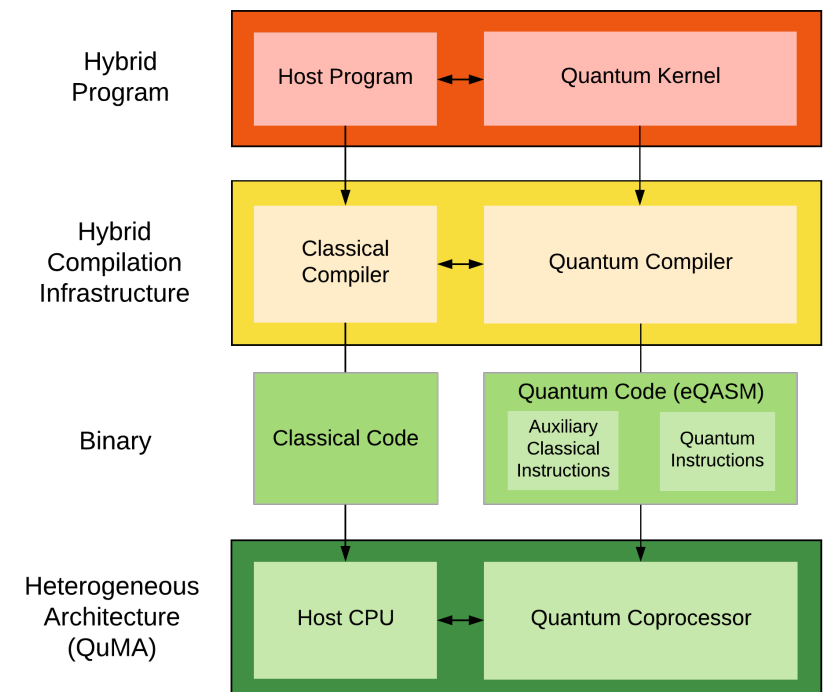
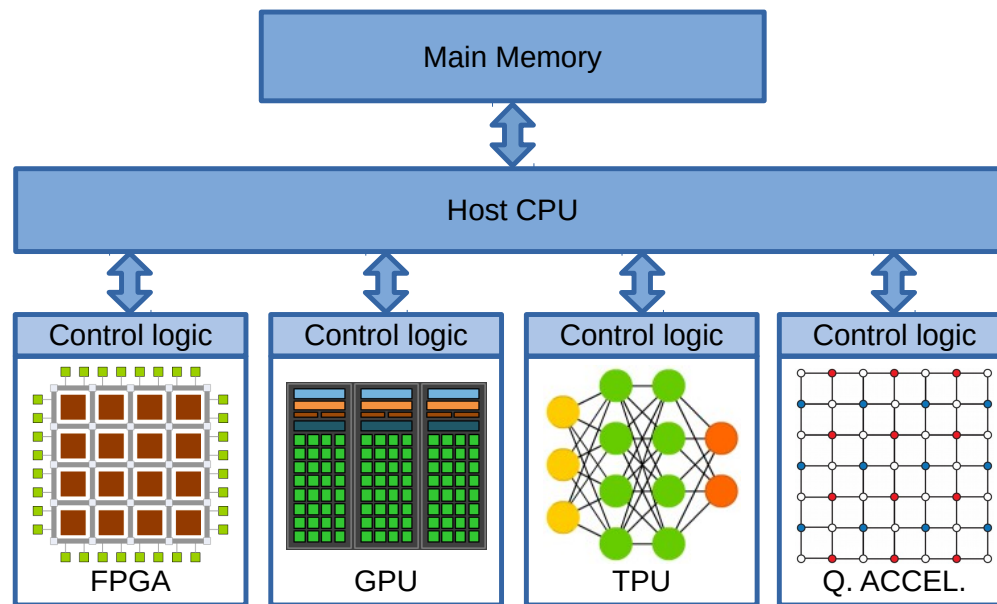


Quantum error correction



# A quantum computer is not (is)

- It is not a replacement for classical computers
- It is an in-memory-computing device
- It is a co-processor in a (heterogeneous) multi-core architecture



X. Fu et. al, "eQASM: An Executable Quantum Instruction Set Architecture", *IEEE International Symposium on High Performance Computer Architecture (HPCA)*, 2019.

Riesebos, L., et al. "Quantum Accelerated Computer Architectures." *2019 IEEE International Symposium on Circuits and Systems (ISCAS)*. IEEE, 2019.

Quantum Computation is  
based on Linear Algebra

# Reading out quantum information

Measuring a qubit (quantum state)

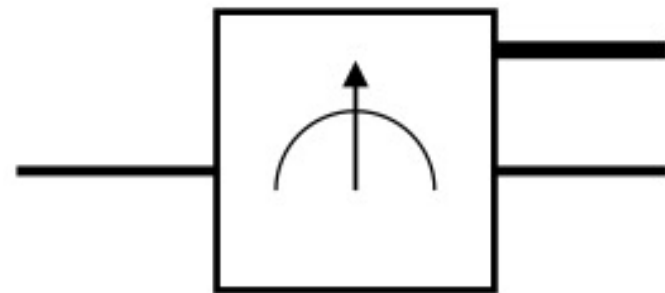
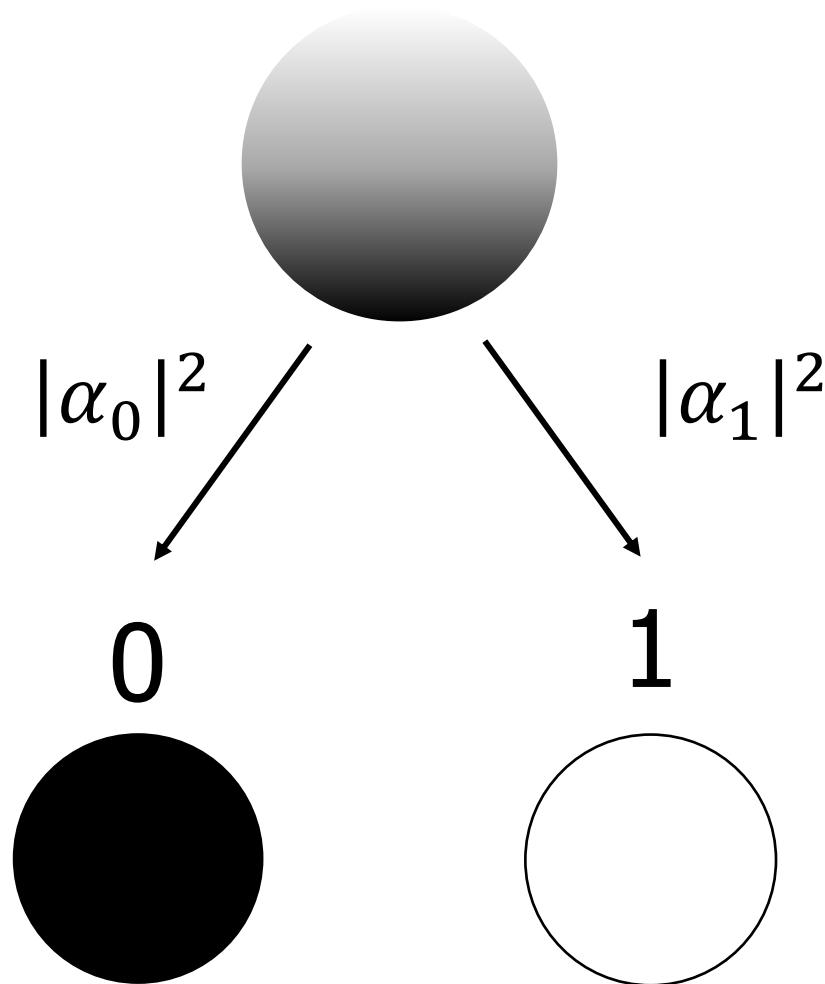
$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$



$$\alpha_0, \alpha_1 \in \mathbb{C}$$

$$|\alpha_i|^2$$

is the probability of finding the qubit in state  $|i\rangle$  when we measure it (in the computational basis)



Probabilistic process

Binary value

Projective measurement

# Measuring a qubit

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$$

$ \alpha_0 ^2$	$ \alpha_1 ^2$
Prob. result 0	Prob. result 1

$$\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

$$\left| \frac{1}{\sqrt{2}} \right|^2 = \frac{1}{2}$$

$$\left| \frac{1}{\sqrt{2}} \right|^2 = \frac{1}{2}$$

$$\frac{1}{2} |0\rangle + \frac{\sqrt{3}}{2} |1\rangle$$

$$\left| \frac{1}{2} \right|^2 = \frac{1}{4}$$

$$\left| \frac{\sqrt{3}}{2} \right|^2 = \frac{3}{4}$$

$$\frac{1}{2} |0\rangle + \frac{i\sqrt{3}}{2} |1\rangle$$

$$\left| \frac{1}{2} \right|^2 = \frac{1}{4}$$

$$\left| \frac{i\sqrt{3}}{2} \right|^2 = \frac{3}{4}$$

$$\frac{1}{2} |0\rangle + \frac{1}{2} |1\rangle$$

$$\left| \frac{1}{2} \right|^2 = \frac{1}{4}$$

$$\left| \frac{1}{2} \right|^2 = \frac{1}{4}$$

This is not a quantum state

$$|\alpha_0|^2 + |\alpha_1|^2 = 1$$

A quantum state is a vector

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle = \alpha_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$$

# Representation on the Bloch Sphere

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} \quad \alpha_0, \alpha_1 \in \mathbb{C}$$

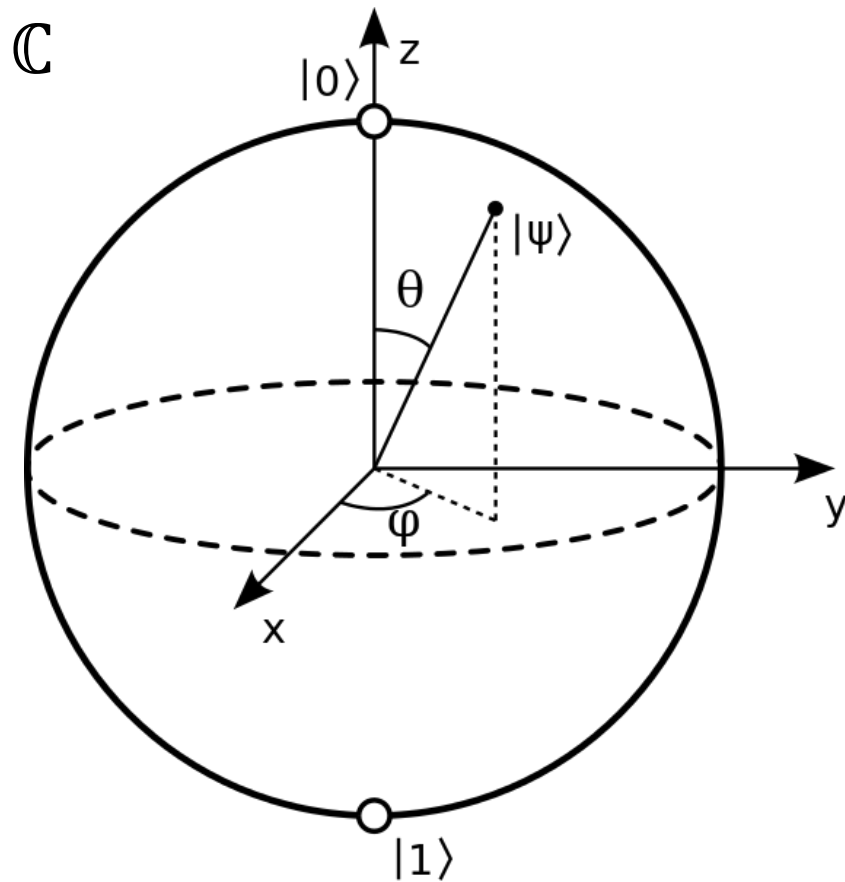
- $|\psi\rangle = \alpha_0|0\rangle + e^{i\varphi}\alpha_1|1\rangle$   
 $\alpha_0, \alpha_1 \in \mathbb{R}$

- $\sqrt{\alpha_0^2 + \alpha_1^2} = 1 \quad (\sqrt{\sin^2 x + \cos^2 x} = 1)$

$$\alpha_0 = \cos \frac{\theta}{2} \quad \alpha_1 = \sin \frac{\theta}{2}$$

$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle$$

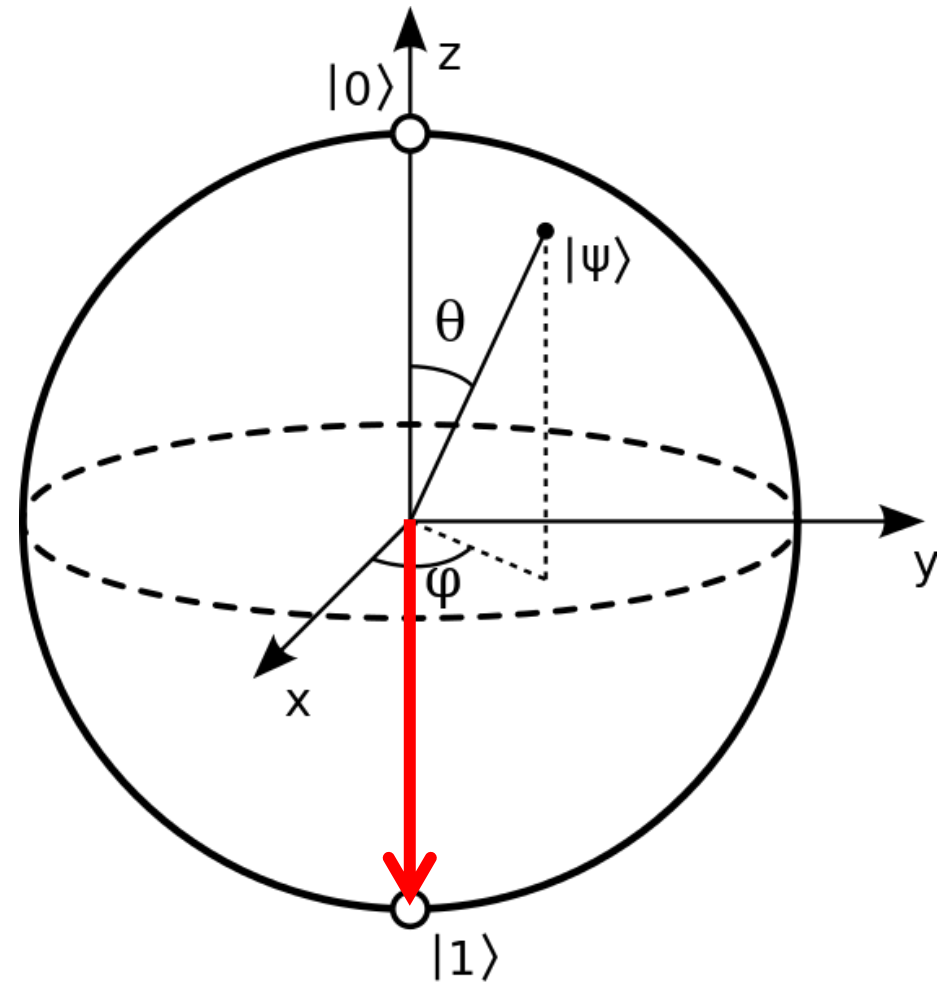
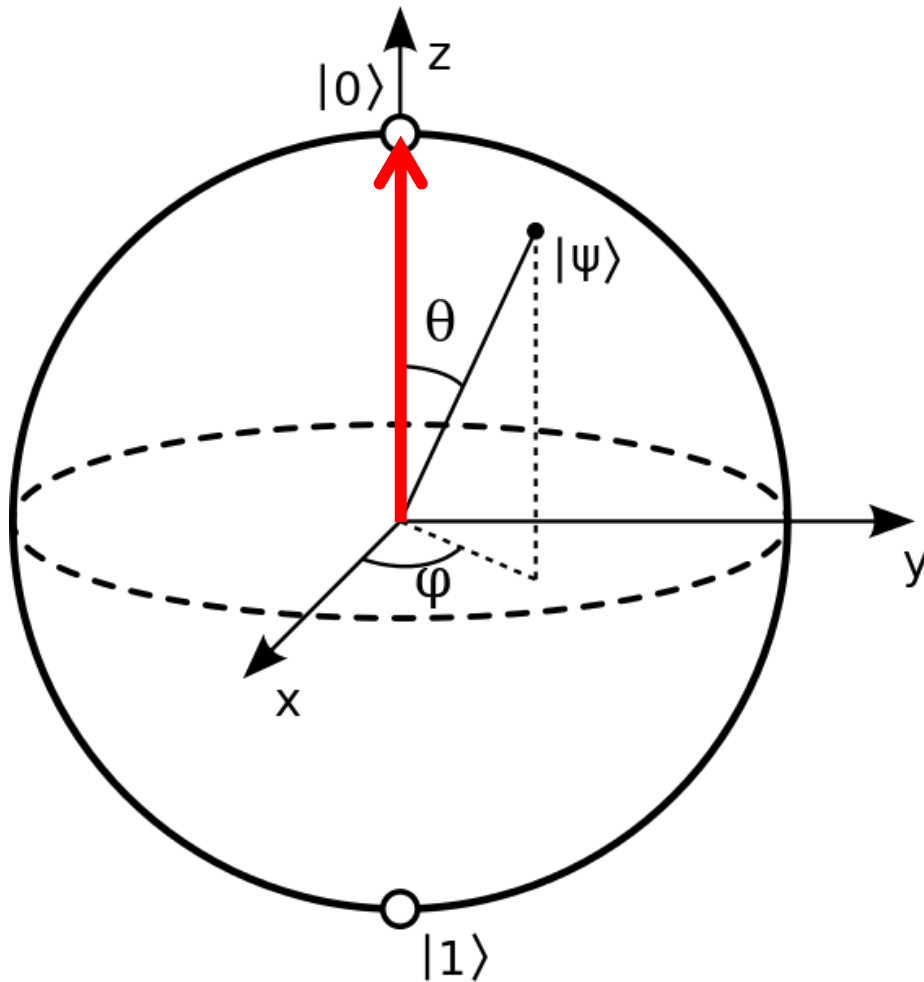
$$\theta, \varphi \in \mathbb{R}$$



$\theta$ : polar angle  $\in [0, \pi]$

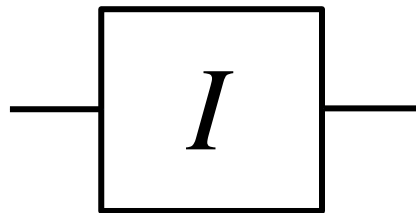
$\varphi$ : azimuthal angle  $\in [0, 2\pi]$

# Representation on the Bloch Sphere



# Single-qubit gates

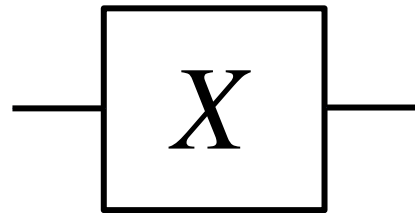
## Identity



$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{aligned} |0\rangle &\rightarrow |0\rangle \\ |1\rangle &\rightarrow |1\rangle \end{aligned}$$

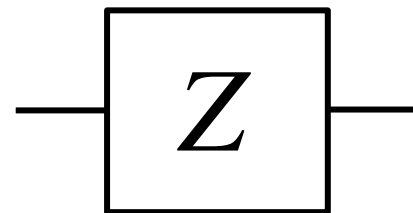
## Pauli X



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{aligned} |0\rangle &\rightarrow |1\rangle \\ |1\rangle &\rightarrow |0\rangle \end{aligned}$$

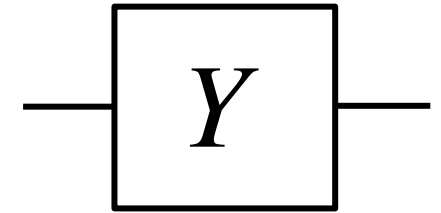
## Pauli Z



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned} |0\rangle &\rightarrow |0\rangle \\ |1\rangle &\rightarrow -|1\rangle \end{aligned}$$

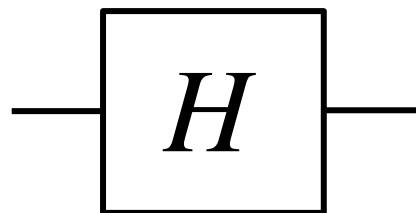
## Pauli Y



$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\begin{aligned} |0\rangle &\rightarrow i|1\rangle \\ |1\rangle &\rightarrow -i|0\rangle \end{aligned}$$

## Hadamard



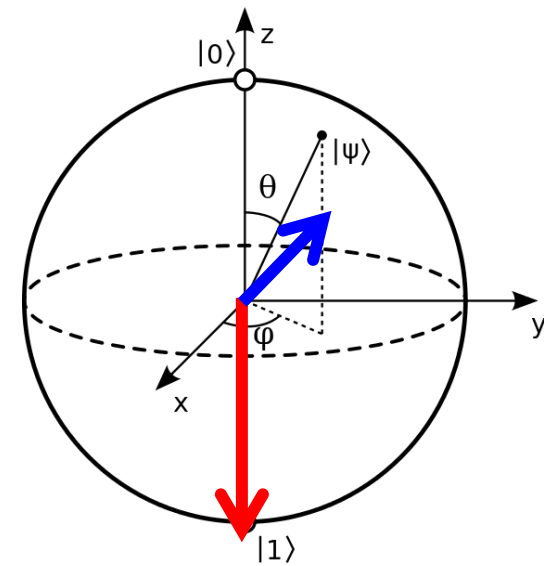
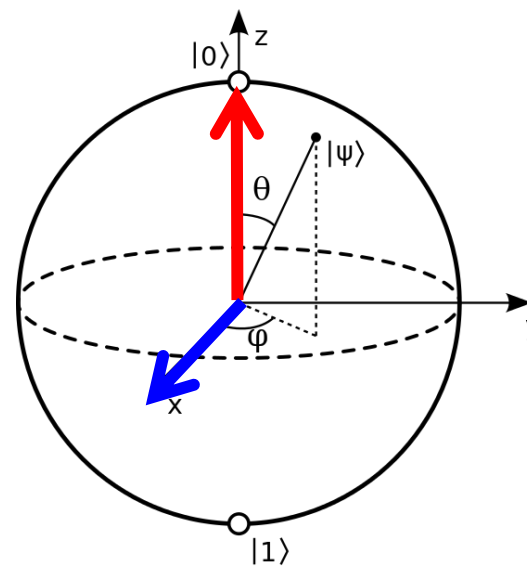
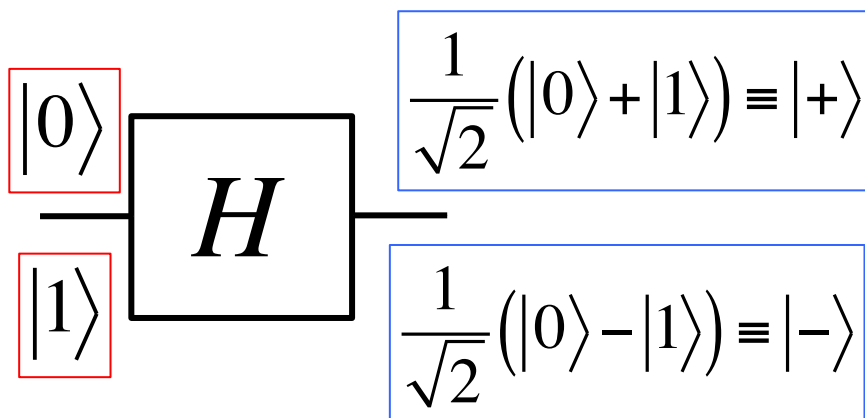
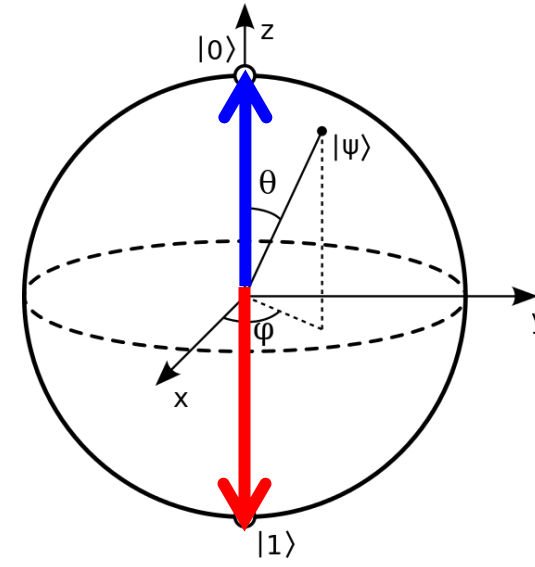
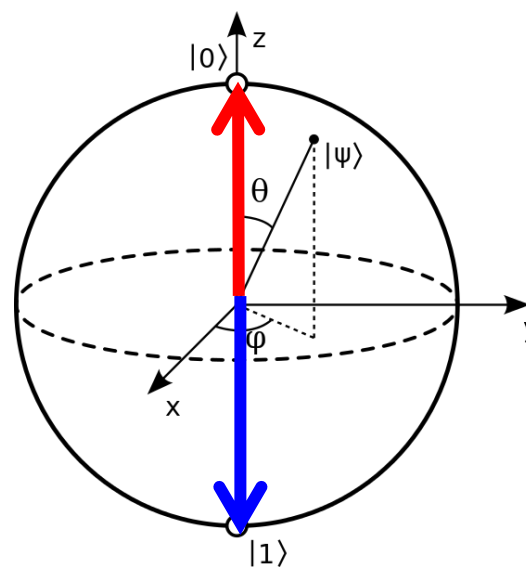
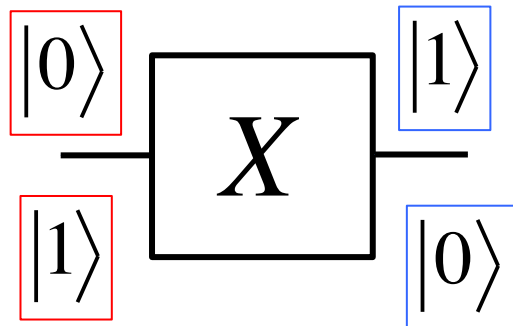
$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

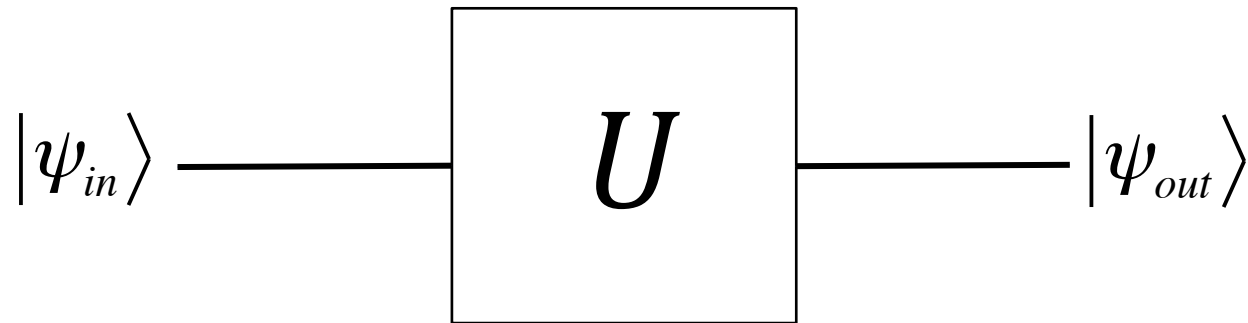
$$|1\rangle \rightarrow \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$



# Single-qubit gates



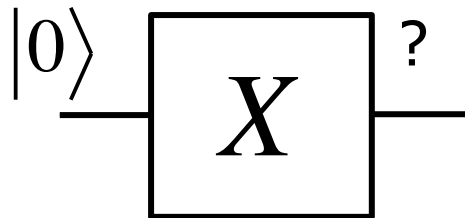
# Single-qubit gates



$$|\psi_{out}\rangle = U|\psi_{in}\rangle$$

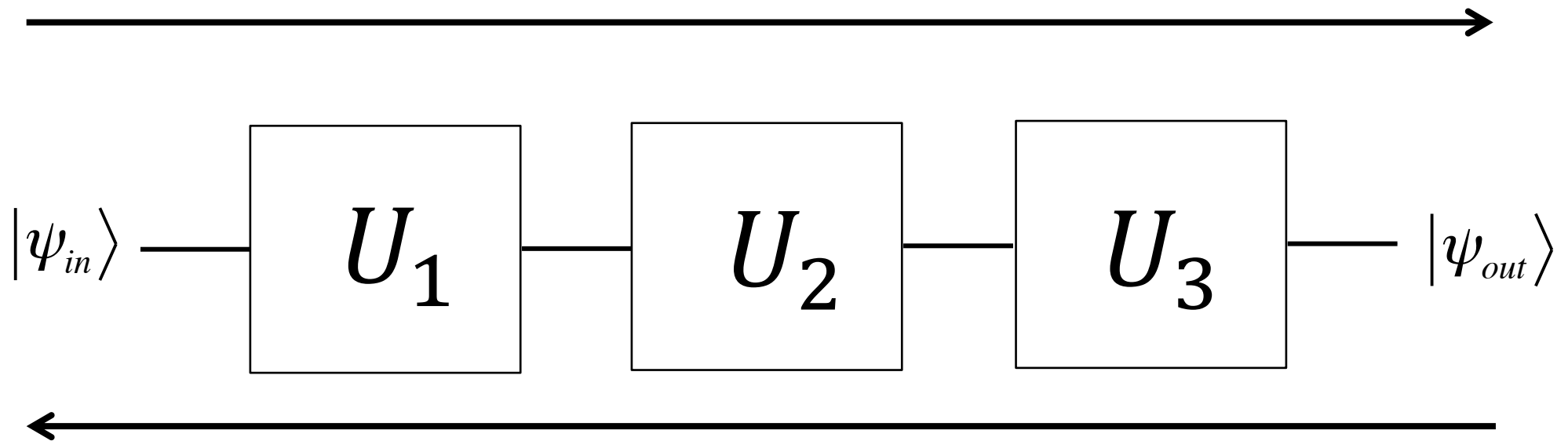
$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



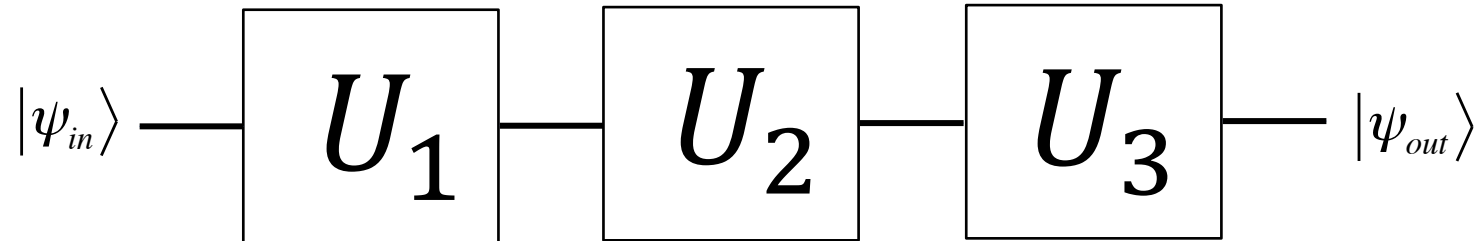
$$|\psi_{out}\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

# Quantum circuit (single qubit)

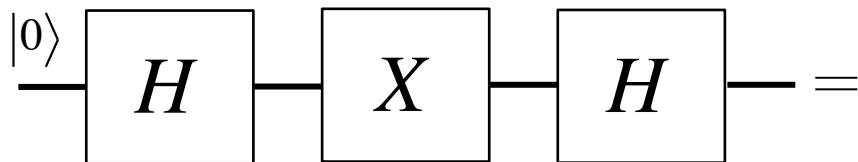
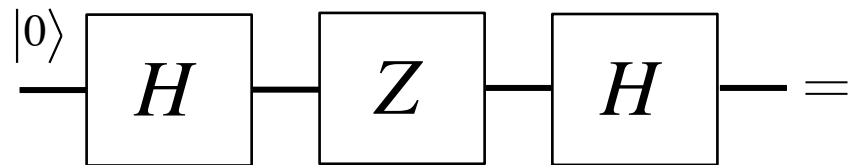


$$|\psi_{out}\rangle = U_3 U_2 U_1 |\psi_{in}\rangle$$

# Quantum circuit (single qubit)




$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



# Multi-qubit state

$$|\psi_0\rangle = \alpha_0|0\rangle + \beta_0|1\rangle$$

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$


$$|\psi\rangle = |\psi_0\rangle \otimes |\psi_1\rangle$$

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle = \begin{pmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{pmatrix}$$

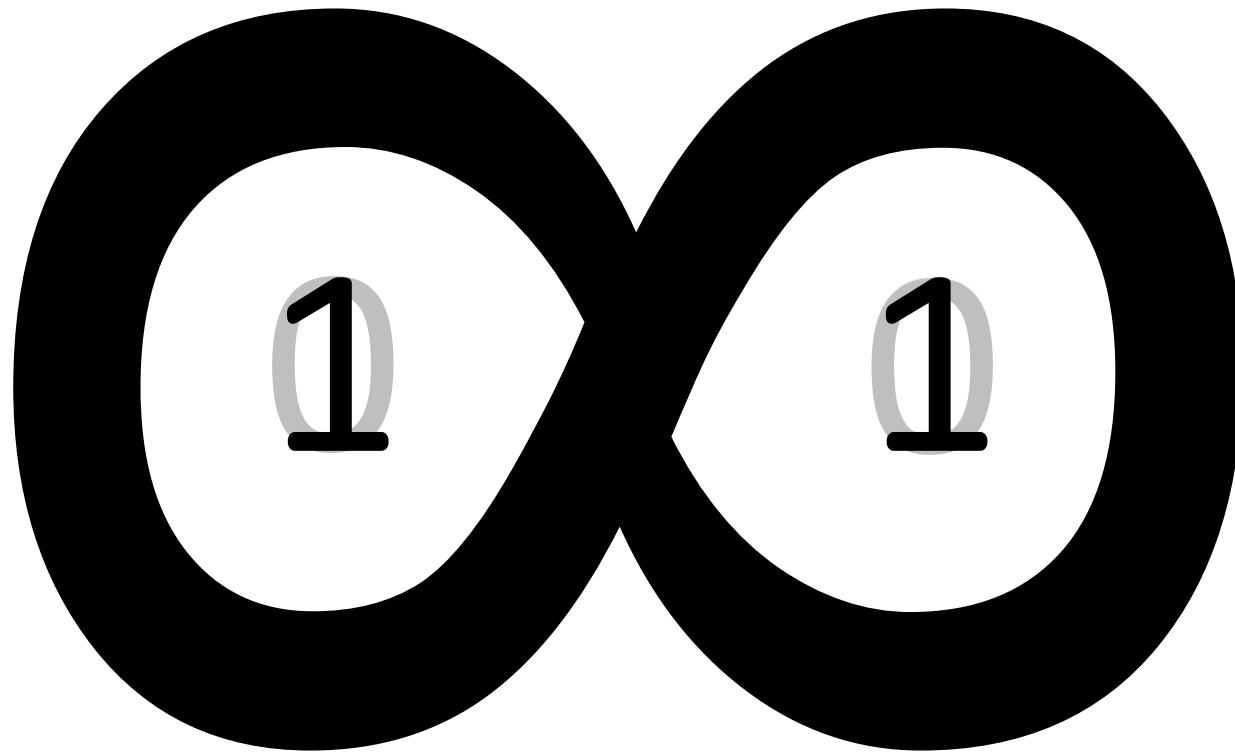
## Tensor product

$$|A\rangle \otimes |B\rangle = \begin{bmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{bmatrix}$$

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$

# Multi-qubit state

## Entanglement



Two qubits in a superposition are correlated with one another

# Multi-qubit state

## Entanglement

Quantum entanglement means that multiple particles are linked together in a way such that the measurement of one particle's quantum state determines the possible quantum states of the other particles.

This connection isn't depending on the location of the particles in space. Even if you separate entangled particles by billions of kms, changing one particle will induce a change in the other.

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

[https://www.youtube.com/watch?v=CC\\_XES4xQD4](https://www.youtube.com/watch?v=CC_XES4xQD4)

# Multi-qubit state

Two qubits are *entangled* when their joint states cannot possibly be separated into a product of individual qubit states

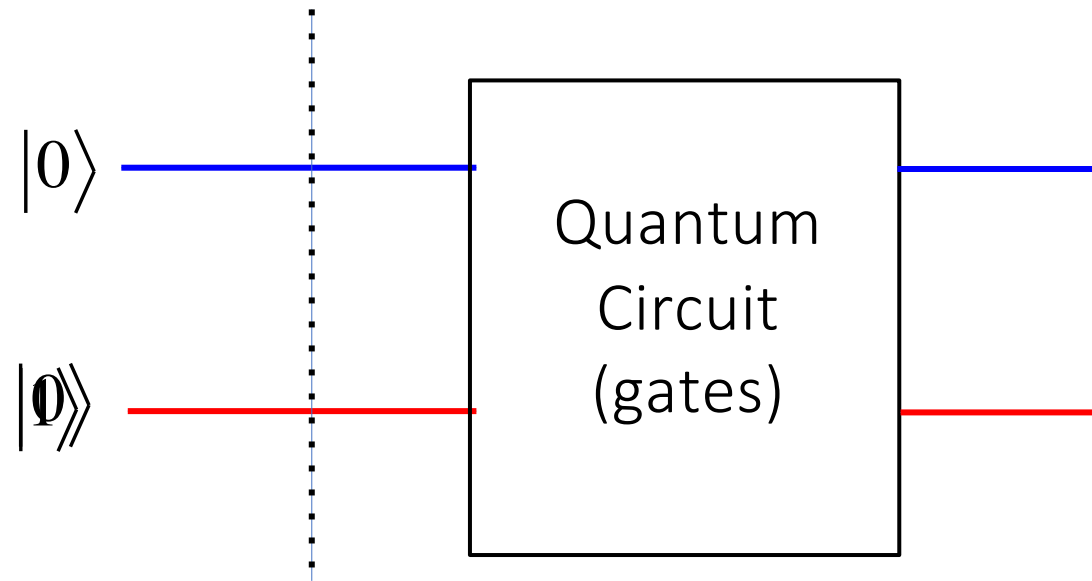
$$|\Psi\rangle = |\varphi\rangle \otimes |\psi\rangle \quad \text{vs.} \quad |\Psi\rangle = |\varphi\rangle \otimes |\psi\rangle + |\varphi'\rangle \otimes |\psi'\rangle$$

$$\frac{1}{\sqrt{2}} (|10\rangle + |11\rangle) = |1\rangle \otimes \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \quad (\text{not entangled})$$

$$\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}} |0\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}} (|1\rangle \otimes |1\rangle) \quad (\text{entangled})$$



# Quantum circuit (multiple qubits)



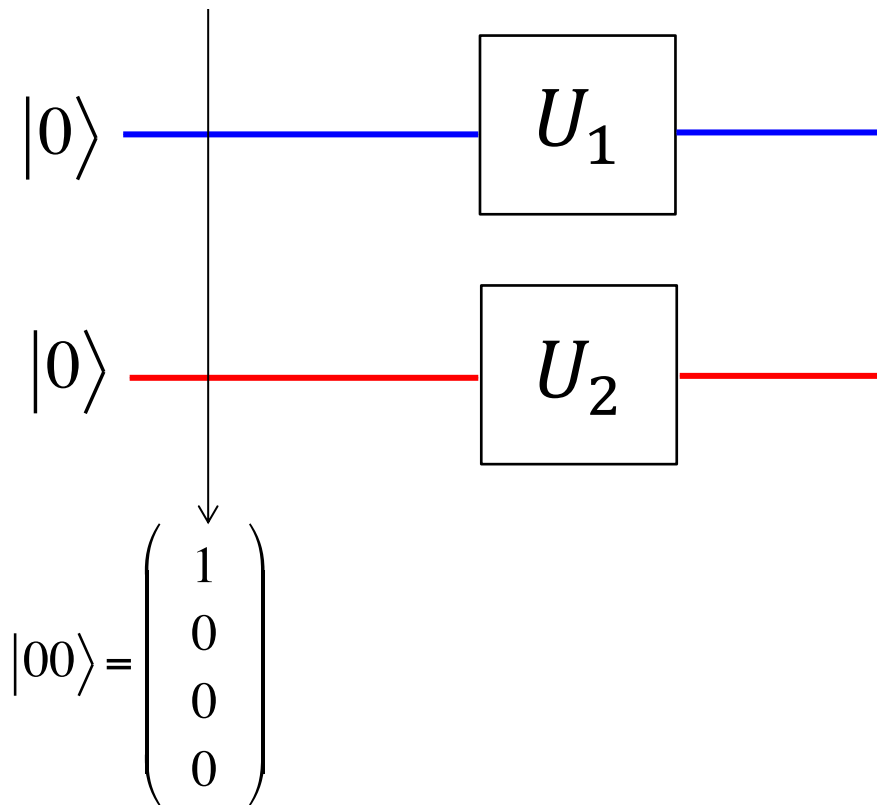
$$|0\rangle \otimes |0\rangle = |0\rangle|0\rangle = |00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$|01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$|10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

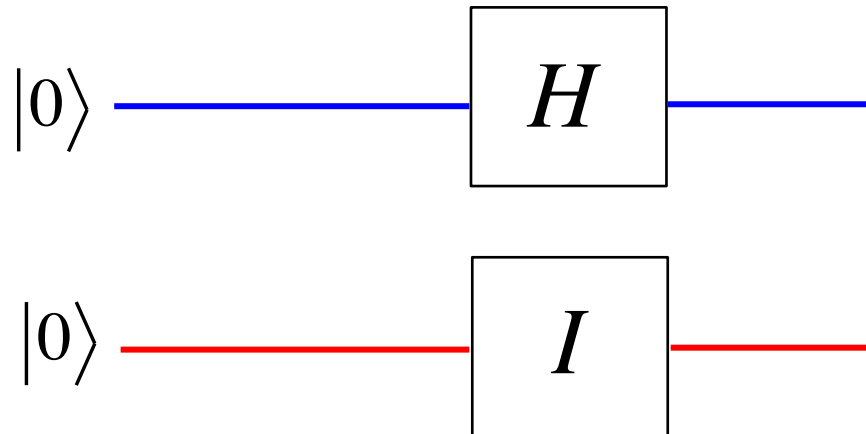
$$|11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

# Quantum circuit (multiple qubits)



$$|\psi_{out}\rangle = U_1 \otimes U_2 |\psi_{in}\rangle$$

# Quantum circuit (multiple qubits)

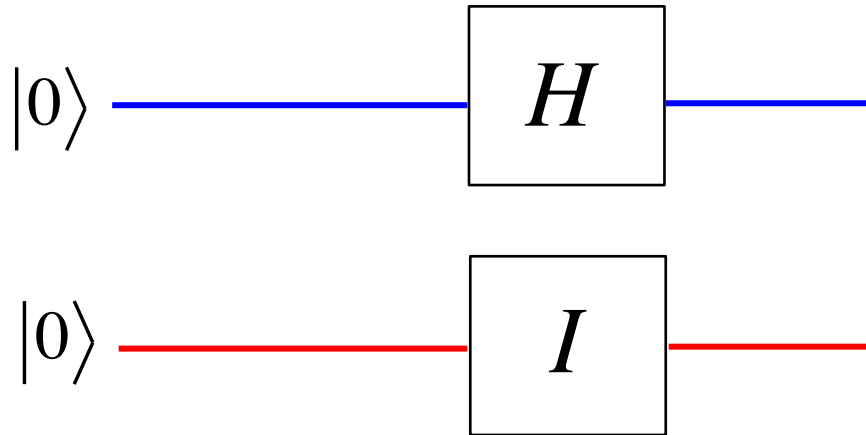


$$|\psi_{out}\rangle = H \otimes I |\psi_{in}\rangle$$

$$\hat{U}_1 \otimes \hat{U}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$|A\rangle \otimes |B\rangle = \begin{bmatrix} A_{11}B & A_{12}B \\ A_{21}B & A_{22}B \end{bmatrix}$$

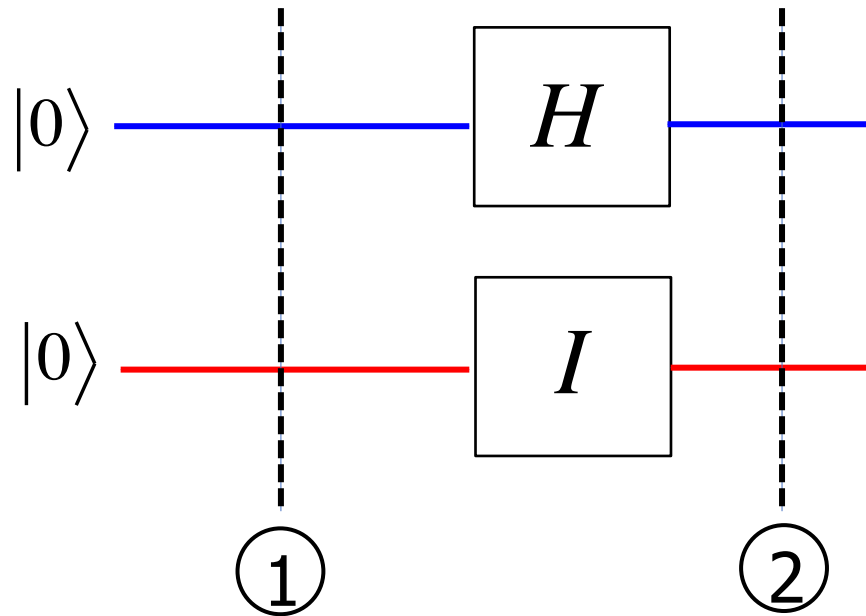
# Quantum circuit (multiple qubits)



$$|\psi_{out}\rangle = H \otimes I |\psi_{in}\rangle$$

$$|\psi_{out}\rangle =$$

# Quantum circuit (multiple qubits)



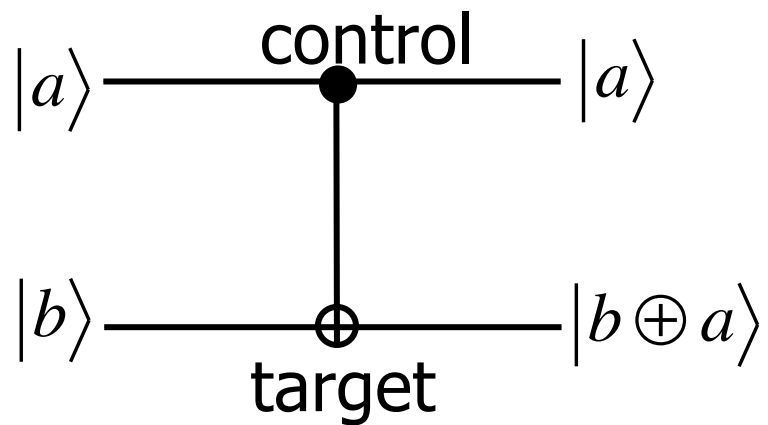
$$|\psi_{out}\rangle = H \otimes I |\psi_{in}\rangle$$

①  $|00\rangle$

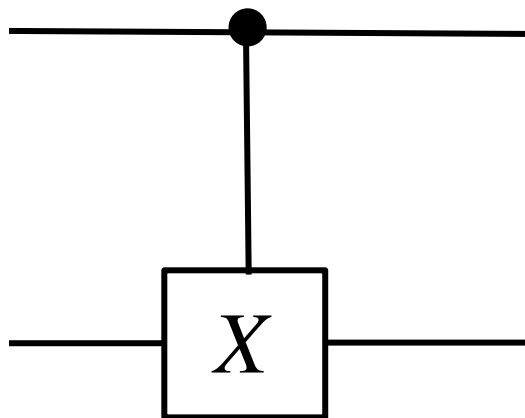
②  $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$

# Multi-qubit gates

## CNOT gate



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



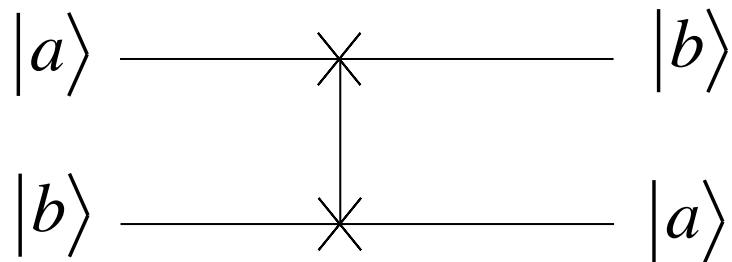
## XOR gate

a	b	out
0	0	0
0	1	1
1	0	1
1	1	0

$$a \cdot \bar{b} + \bar{a} \cdot b$$

# Multi-qubit gates

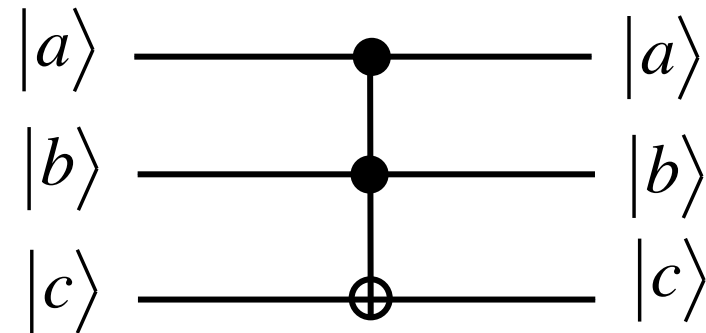
**SWAP gate**



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

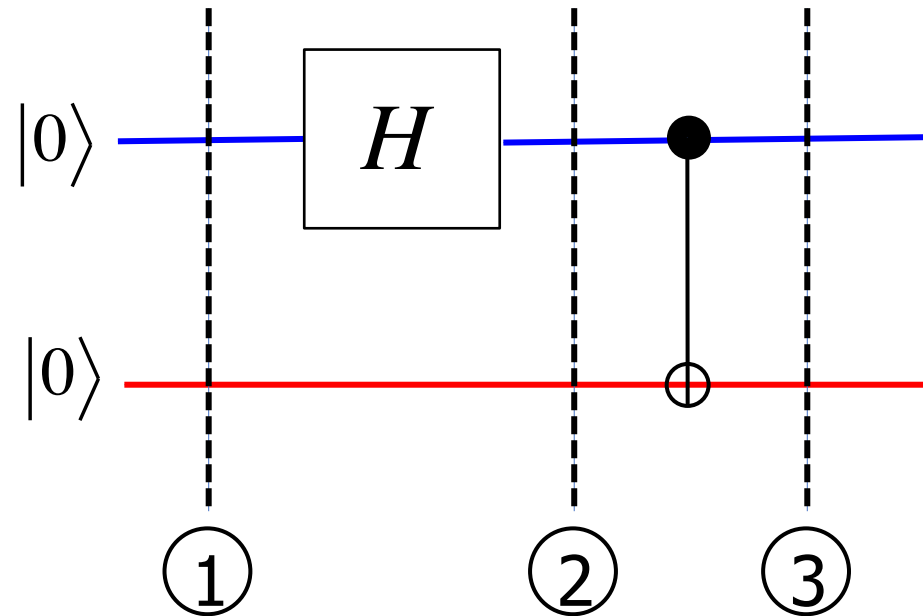
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

**Toffoli gate**



Input			Output		
0	0	<b>0</b>	0	0	<b>0</b>
0	0	<b>1</b>	0	0	<b>1</b>
0	1	<b>0</b>	0	1	<b>0</b>
0	1	<b>1</b>	0	1	<b>1</b>
1	0	<b>0</b>	1	0	<b>0</b>
1	0	<b>1</b>	1	0	<b>1</b>
1	1	<b>0</b>	1	1	<b>1</b>
1	1	<b>1</b>	1	1	<b>0</b>

# Quantum circuit (multiple qubits)



①  $|00\rangle$

②  $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$

③  $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$



# Bell states

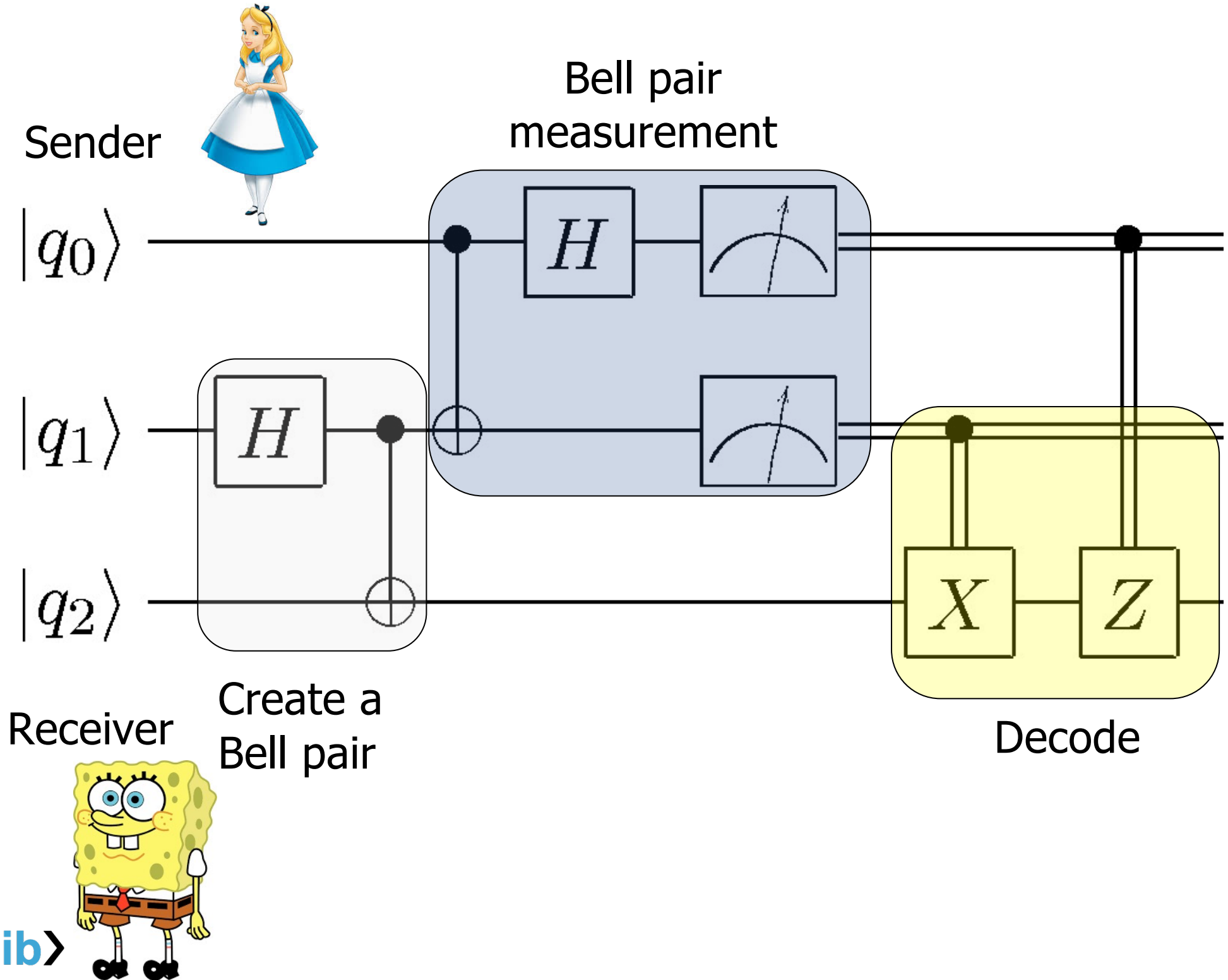
$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\beta_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

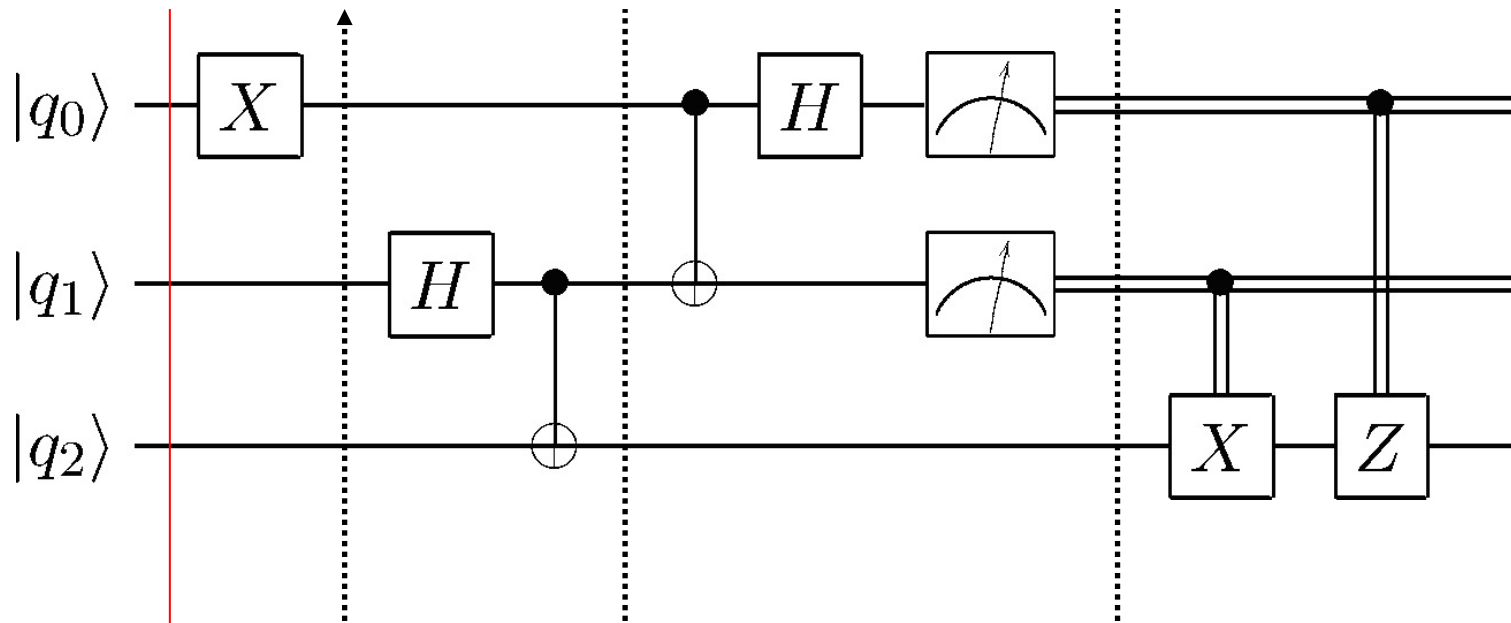
$$|\beta_{10}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

# Quantum teleportation



# Quantum teleportation



$$|q_0\rangle|q_1\rangle|q_2\rangle = |q_0q_1q_2\rangle = |000\rangle$$

**QASM-like code**

```
X q0
H q1
CNOT q1,q2
CNOT q0,q1
H q0
```

# Quantum teleportation

$$|q_0\rangle|q_1\rangle|q_2\rangle = |q_0q_1q_2\rangle = |000\rangle$$

$$\mathbf{X} \mathbf{q}_0 \rightarrow |100\rangle$$

$$\mathbf{H} \mathbf{q}_1 \rightarrow |1\rangle \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|100\rangle + |110\rangle)$$

$$\mathbf{CNOT} \mathbf{q}_1, \mathbf{q}_2 \rightarrow \frac{1}{\sqrt{2}}(|100\rangle + |111\rangle)$$

$$\mathbf{CNOT} \mathbf{q}_0, \mathbf{q}_1 \rightarrow \frac{1}{\sqrt{2}}(|110\rangle + |101\rangle)$$

$$\begin{aligned} \mathbf{H} \mathbf{q}_0 &\rightarrow \frac{1}{\sqrt{2}} \left[ \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|10\rangle + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|01\rangle \right] = \\ &= \frac{1}{2}(|010\rangle - |110\rangle + |001\rangle - |101\rangle) \end{aligned}$$

**X**  $\mathbf{q}_0$

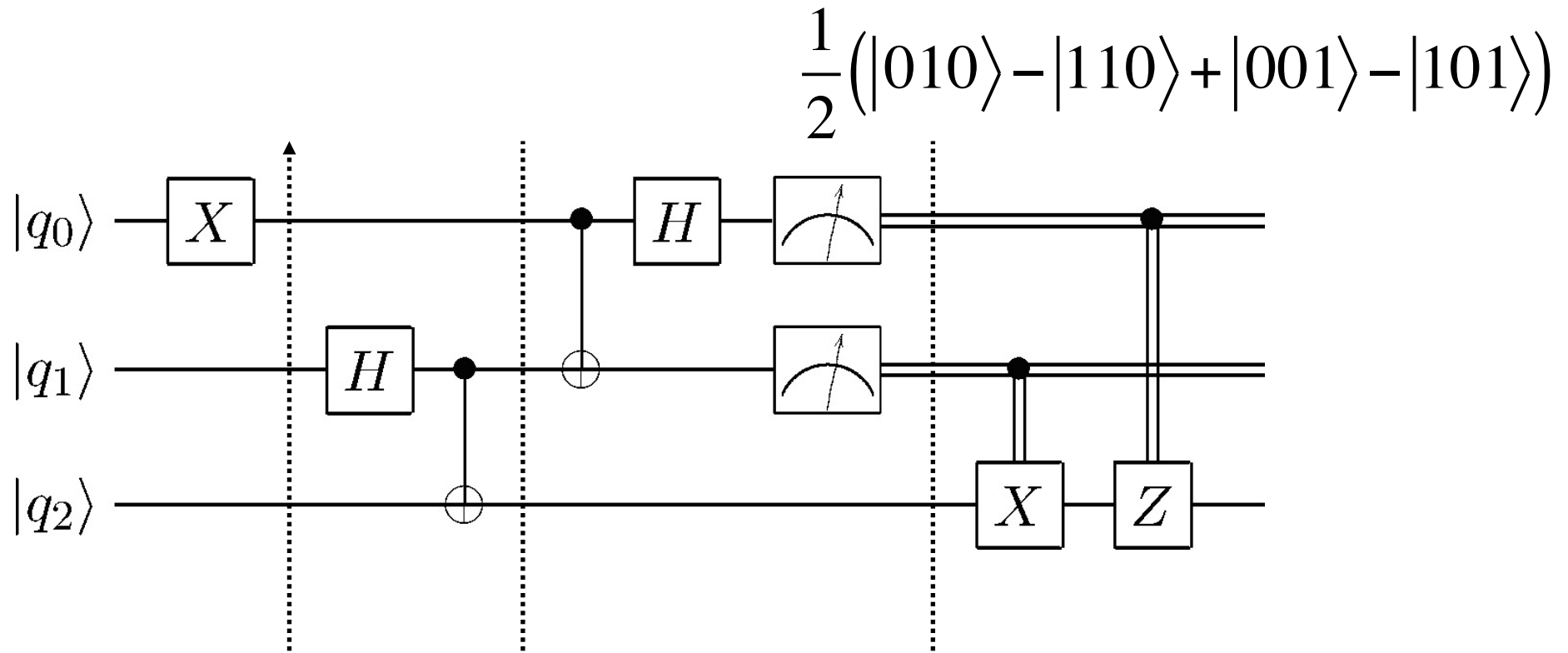
**H**  $\mathbf{q}_1$

**CNOT**  $\mathbf{q}_1, \mathbf{q}_2$

**CNOT**  $\mathbf{q}_0, \mathbf{q}_1$

**H**  $\mathbf{q}_0$

# Quantum teleportation



- Measure 00  $\rightarrow |q_2\rangle = |1\rangle \rightarrow$  no correction
- Measure 01  $\rightarrow |q_2\rangle = |0\rangle \rightarrow$  bit-flip  $q_2 \rightarrow |q_2\rangle = |1\rangle$
- Measure 10  $\rightarrow |q_2\rangle = -|1\rangle \rightarrow$  phase-flip  $q_2 \rightarrow |q_2\rangle = |1\rangle$
- Measure 11  $\rightarrow |q_2\rangle = -|0\rangle \rightarrow$  bit-flip and phase-flip  $\rightarrow |q_2\rangle = |1\rangle$