Hands-on Introduction to Quantum Computing

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Quantum Computing … why?

Quantum computers can solve problems that are intractable for even the most powerful classical supercomputers (e.g., simulation, search and optimization)

New materials (aerospace, automotive, energy)

New drugs (chemistry and pharma)

Better products and services (logistics, healthcare, finance)

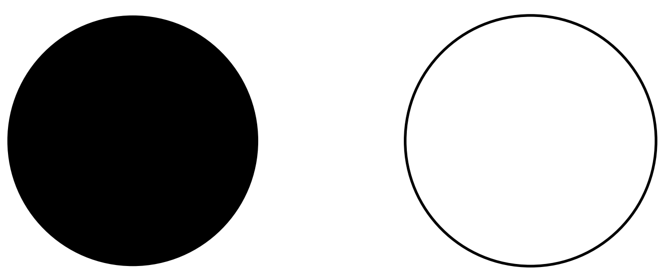
What is different in QC?

Basic unit of information

Bit

Exclusive state

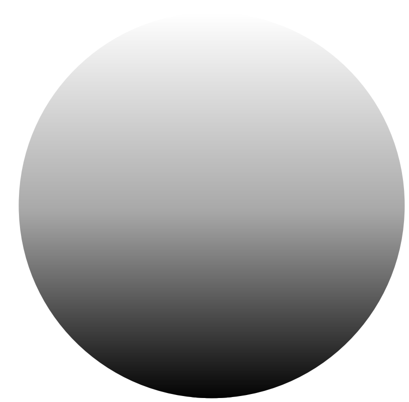
0 1



Quantum bit (Qubit)

$|0\rangle$ and $|1\rangle$

Superposition

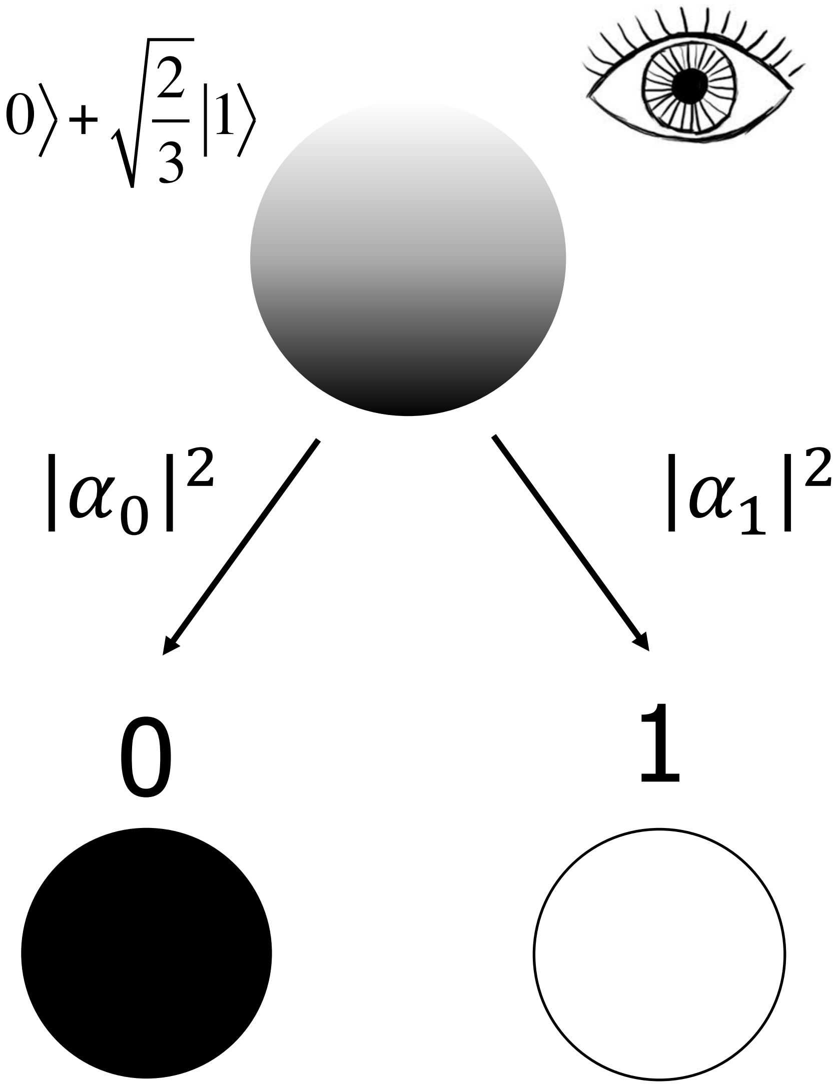


$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$$

What is different in QC?

Reading out information

$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$$



Memory

0	1	0	0
0	0	1	1
1	1	0	1
1	1	1	0
0	0	0	1

$$|\alpha_0|^2 + |\alpha_1|^2 = 1$$

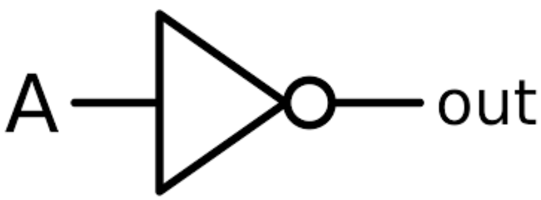
Probabilistic process

What is different in QC?

Operations

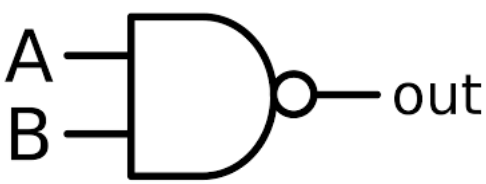
Classical gates

NOT



A	out
0	1
1	0

NAND

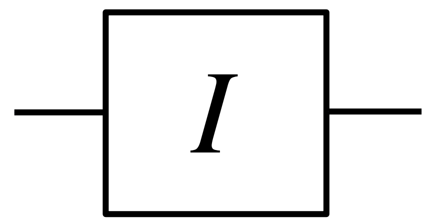


A	B	out
0	0	1
0	1	1
1	0	1
1	1	0

- Logical operation
- Truth tables

Quantum gates

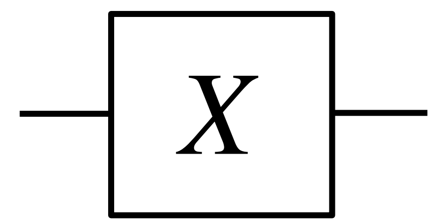
Identity



$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$|0\rangle \rightarrow |0\rangle$$
$$|1\rangle \rightarrow |1\rangle$$

Pauli X



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

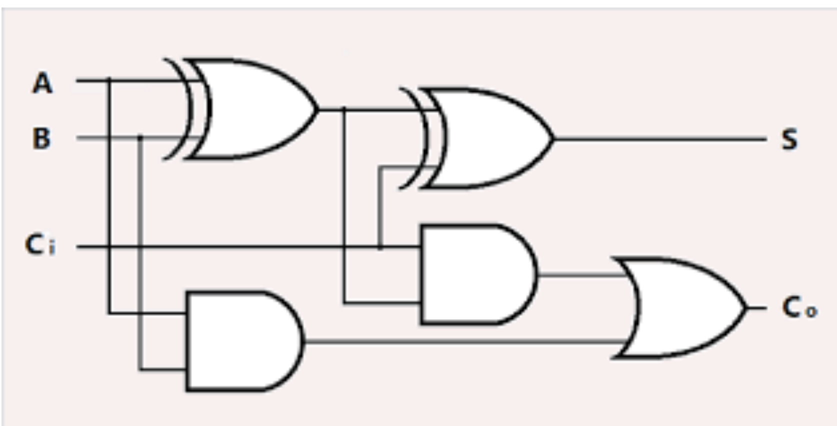
$$|0\rangle \rightarrow |1\rangle$$
$$|1\rangle \rightarrow |0\rangle$$

- Unitary operation
- Unitary matrix

What is different in QC?

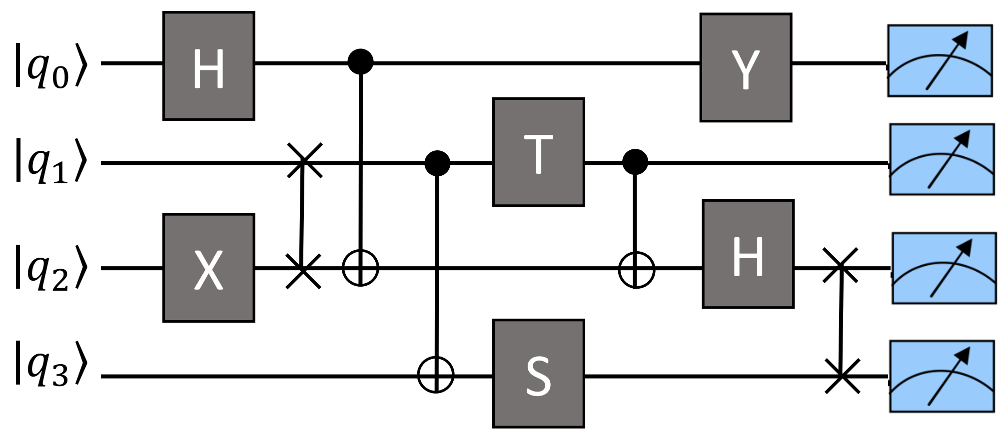
Computation

Classical circuits



Quantum circuits

Circuit model of computation



- Single-qubit gates: H, X, T, S, Y
- Two-qubit gates: CNOT and SWAP

What is different in QC?

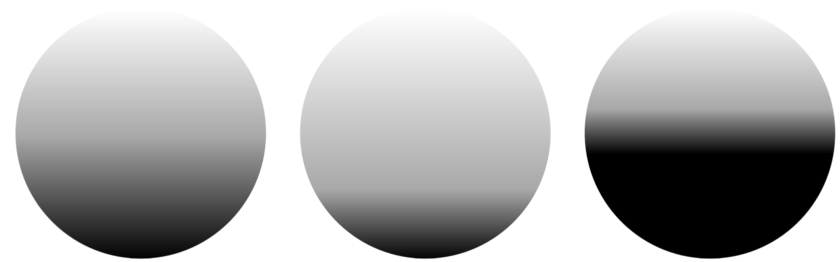
3 bits

000 **or** 001 **or** 010 **or** 011 **or**

100 **or** 101 **or** 110 **or** 111

- n bits hold 1 value: from 0 to 2^n-1

3 qubits



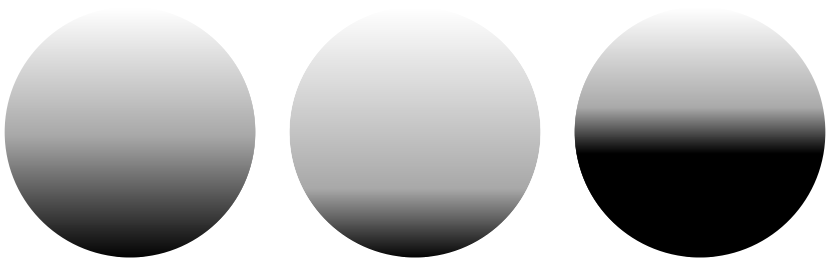
$$\alpha_{000}|000\rangle + \alpha_{001}|001\rangle + \alpha_{010}|010\rangle + \alpha_{011}|011\rangle$$
$$+\alpha_{100}|100\rangle + \alpha_{101}|101\rangle + \alpha_{110}|110\rangle + \alpha_{111}|111\rangle$$

000 **and** 001 **and** 010 **and** 011 **and**
100 **and** 101 **and** 110 **and** 111

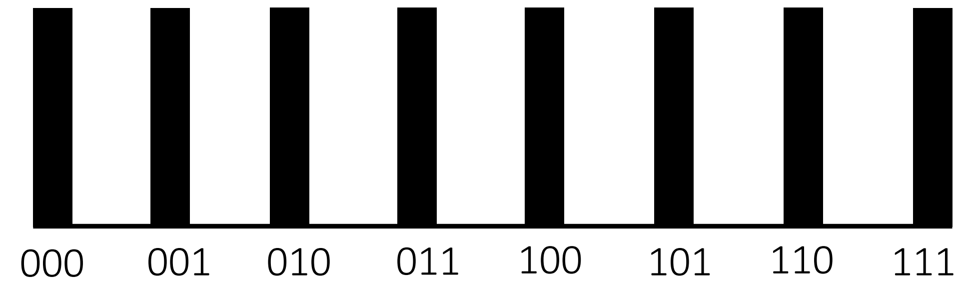
- n qubits can hold 2^n values (50 qubits, 2^{50} complex amplitudes)
- All states (amplitudes) can be manipulated at the same time

What is different in QC?

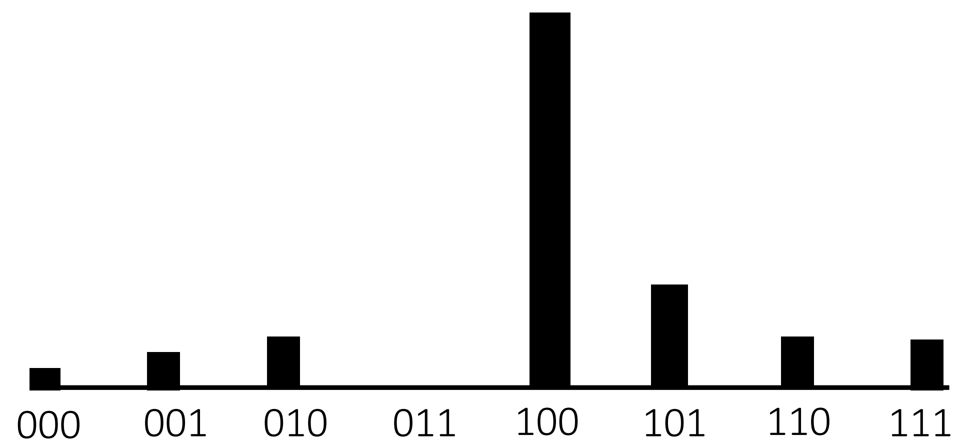
Superposition and entanglement

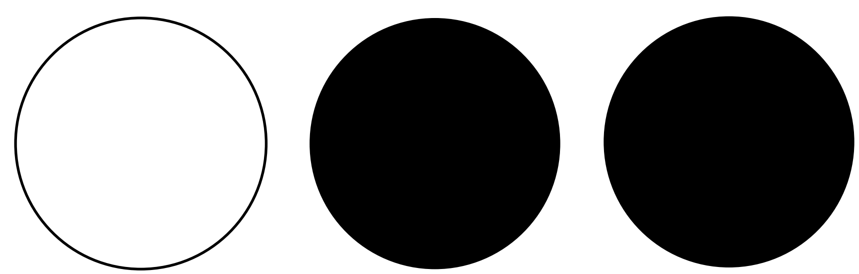


$$\alpha_{000}|000\rangle + \alpha_{001}|001\rangle + \alpha_{010}|010\rangle + \alpha_{011}|011\rangle$$
$$+\alpha_{100}|100\rangle + \alpha_{101}|101\rangle + \alpha_{110}|110\rangle + \alpha_{111}|111\rangle$$



Interference



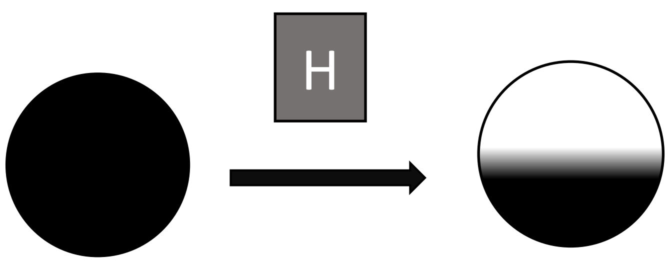


- Probabilistic computation
- Result is a binary value

What is different in QC?

In-memory computing

- The qubits hold the information in a form of a quantum state which is modified by applying an operation on them.

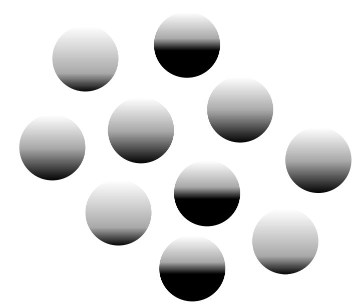


$|\psi\rangle = |0\rangle$ $\quad$ $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Qubits and gates are error prone

- Qubits have short coherence time
- Imperfect operations
  - Gate error rates: 10^{-2}-10^{-3}

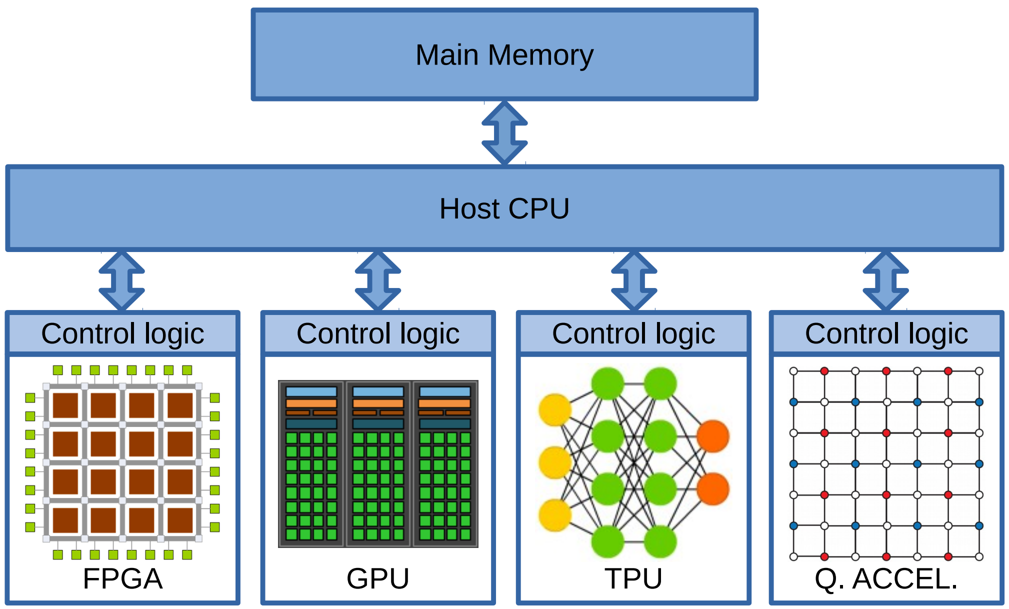
Quantum information needs to be protected

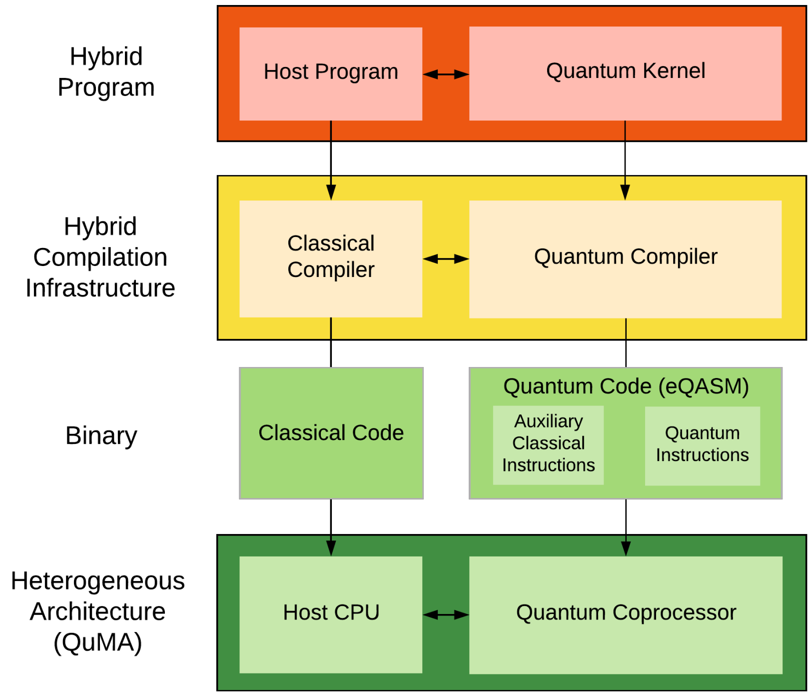


Quantum error correction

A quantum computer is not (is)

- It is not a replacement for classical computers
- It is a co-processor in a (heterogeneous) multi-core architecture





X. Fu et. al, “eQASM: An Executable Quantum Instruction Set Architecture”, *IEEE International Symposium on High Performance Computer Architecture (HPCA), 2019*.

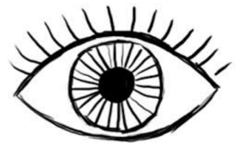
Riesebos, L., et al. "Quantum Accelerated Computer Architectures." *2019 IEEE International Symposium on Circuits and Systems (ISCAS)*. IEEE, 2019.

Quantum Computation is based on Linear Algebra

Reading out quantum information

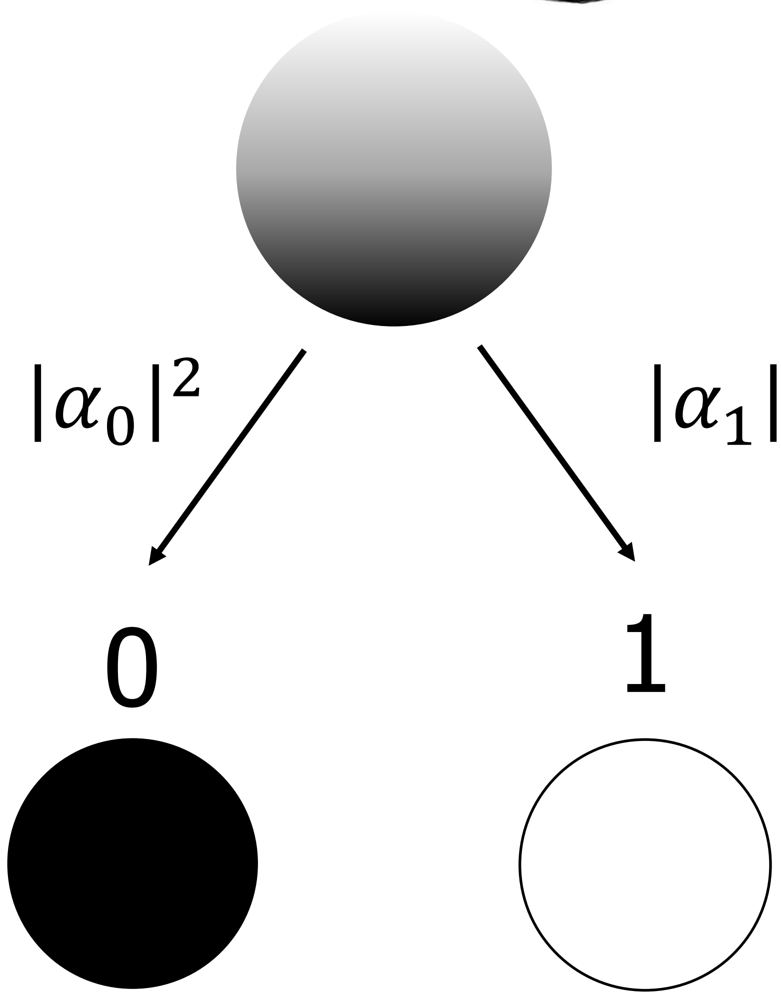
Measuring a qubit (quantum state)

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$

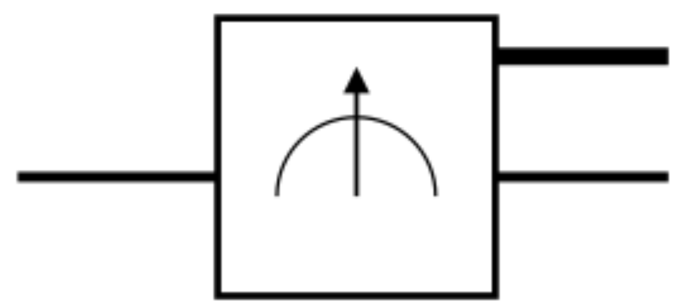


$\alpha_0, \alpha_1 \in \mathbb{C}$

$|\alpha_i|^2$ is the probability of finding the qubit in state $|i\rangle$ when we measure it (in the computational basis)



$$|\alpha_0|^2 + |\alpha_1|^2 = 1$$



Probabilistic process
Binary value
Projective measurement

Measuring a qubit

$\lvert\psi\rangle = \alpha_0\lvert 0\rangle + \alpha_1\lvert 1\rangle$	$\lvert\alpha_0\rvert^2$ Prob. result 0	$\lvert\alpha_1\rvert^2$ Prob. result 1
$\frac{1}{\sqrt{2}}\lvert 0\rangle + \frac{1}{\sqrt{2}}\lvert 1\rangle$	$\left\lvert\frac{1}{\sqrt{2}}\right\rvert^2 = \frac{1}{2}$	$\left\lvert\frac{1}{\sqrt{2}}\right\rvert^2 = \frac{1}{2}$
$\frac{1}{2}\lvert 0\rangle + \frac{\sqrt{3}}{2}\lvert 1\rangle$	$\left\lvert\frac{1}{2}\right\rvert^2 = \frac{1}{4}$	$\left\lvert\frac{\sqrt{3}}{2}\right\rvert^2 = \frac{3}{4}$
$\frac{1}{2}\lvert 0\rangle + \frac{i\sqrt{3}}{2}\lvert 1\rangle$	$\left\lvert\frac{1}{2}\right\rvert^2 = \frac{1}{4}$	$\left\lvert\frac{i\sqrt{3}}{2}\right\rvert^2 = \frac{3}{4}$
$\frac{1}{2}\lvert 0\rangle + \frac{1}{2}\lvert 1\rangle$	$\left\lvert\frac{1}{2}\right\rvert^2 = \frac{1}{4}$	$\left\lvert\frac{1}{2}\right\rvert^2 = \frac{1}{4}$

This is not a quantum state

$$\lvert\alpha_0\rvert^2 + \lvert\alpha_1\rvert^2 = 1$$

A quantum state is a vector

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\boxed{|\psi\rangle = \alpha_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}}$$

Representation on the Bloch Sphere

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} \qquad \alpha_0, \alpha_1 \in \mathbb{C}$$

- $|\psi\rangle = \alpha_0|0\rangle + e^{i\varphi}\alpha_1|1\rangle$

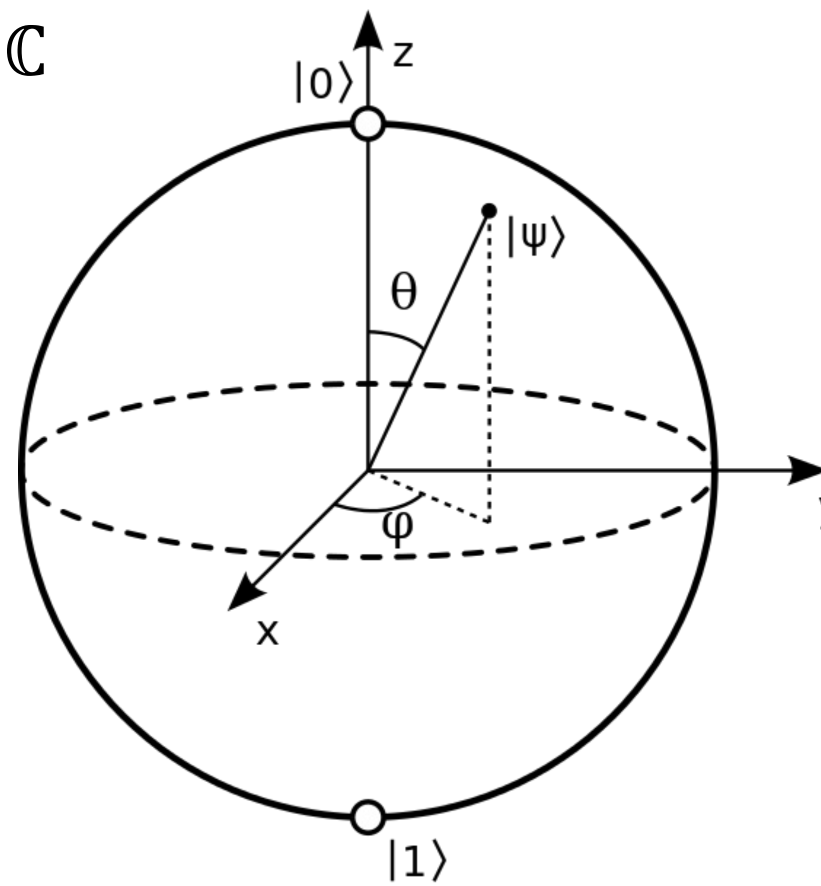
  $\alpha_0, \alpha_1 \in \mathbb{R}$

- $\sqrt{\alpha_0^2 + \alpha_1^2} = 1 \qquad (\sqrt{sin^2 x + cos^2 x = 1})$

  $\alpha_0 = cos\frac{\theta}{2} \qquad \alpha_1 = sin\frac{\theta}{2}$

  $|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\varphi} sin\frac{\theta}{2}|1\rangle$

  $\theta, \varphi \in \mathbb{R}$

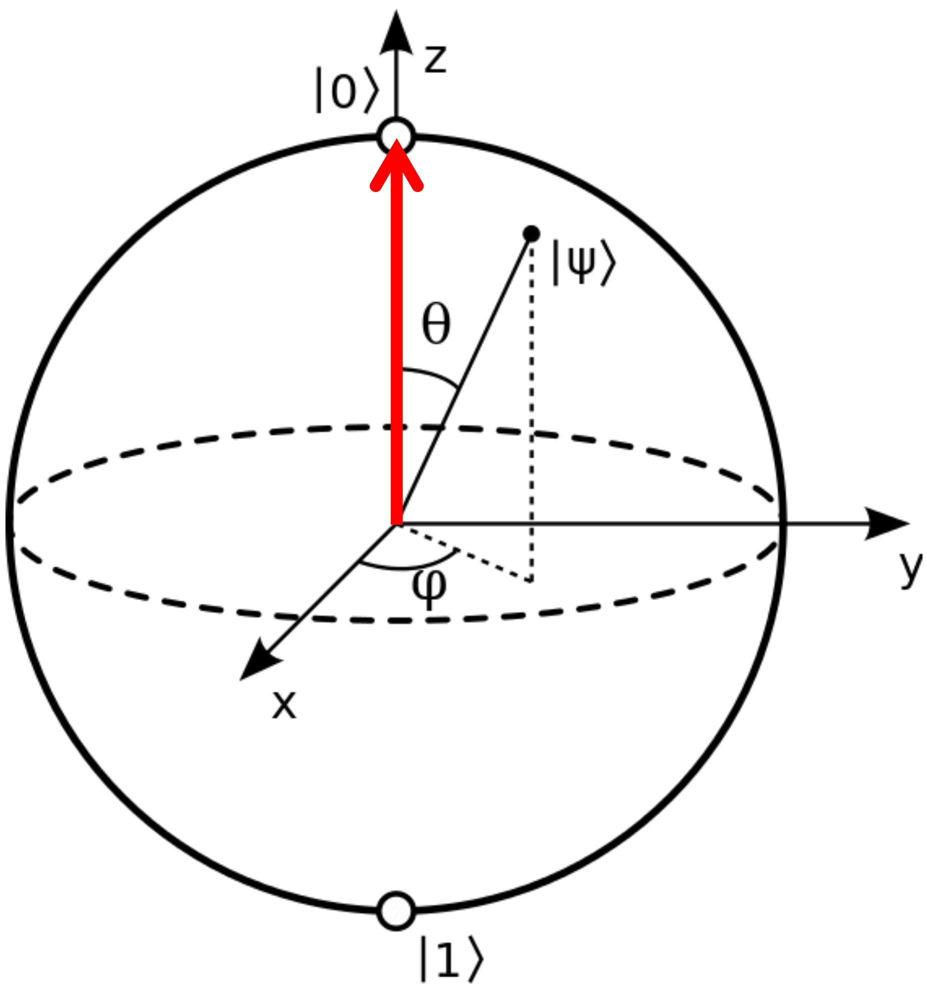


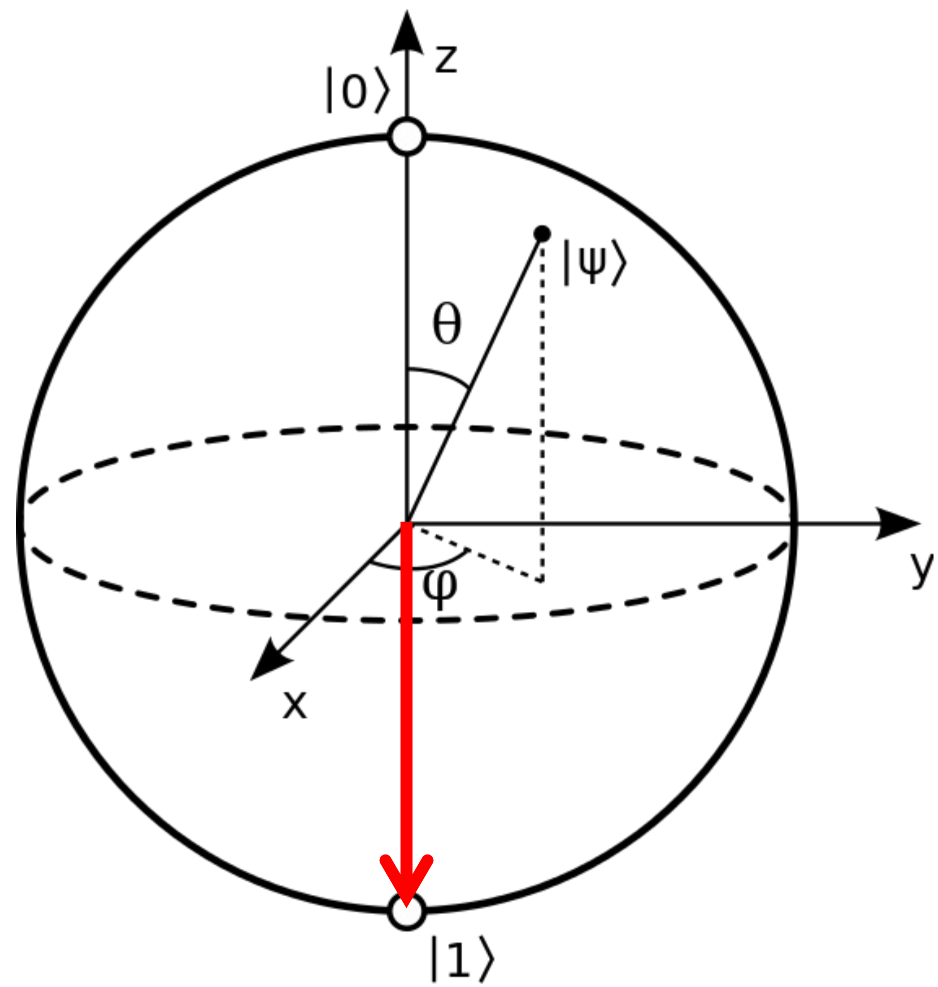
θ : polar angle $\in [0, \pi]$

φ : azimutal angle $\in [0, 2\pi]$

https://qiskit.org/textbook/ch-states/representing-qubit-states.html#bloch-sphere

Representation on the Bloch Sphere





ILib⟩

Single-qubit gates

Identity

$$-\boxed{I}-$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$|0\rangle \rightarrow |0\rangle$$
$$|1\rangle \rightarrow |1\rangle$$

Pauli X

$$-\boxed{X}-$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$|0\rangle \rightarrow |1\rangle$$
$$|1\rangle \rightarrow |0\rangle$$

Pauli Z

$$-\boxed{Z}-$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$|0\rangle \rightarrow |0\rangle$$
$$|1\rangle \rightarrow -|1\rangle$$

Pauli Y

$$-\boxed{Y}-$$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$|0\rangle \rightarrow \mathrm{i}|1\rangle$$
$$|1\rangle \rightarrow -\mathrm{i}|0\rangle$$

Hadamard

$$-\boxed{H}-$$

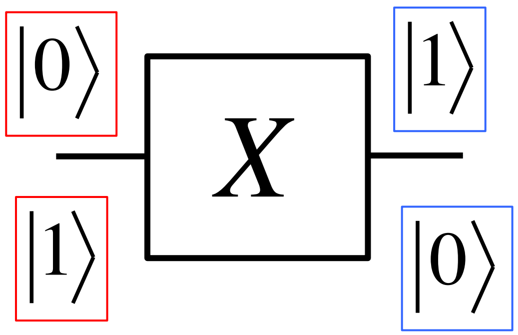
$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

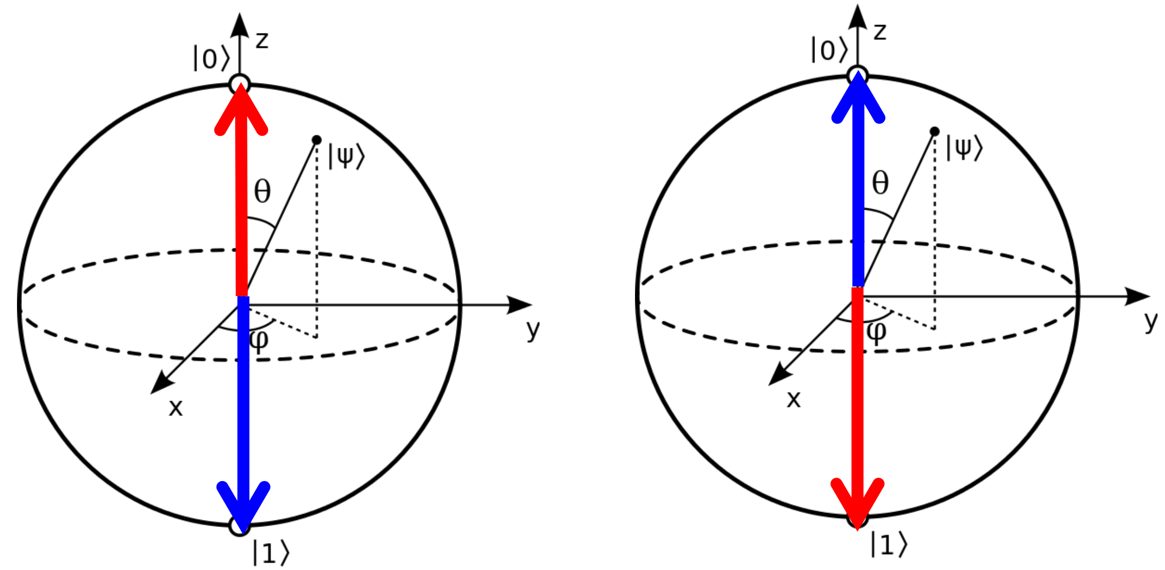
$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

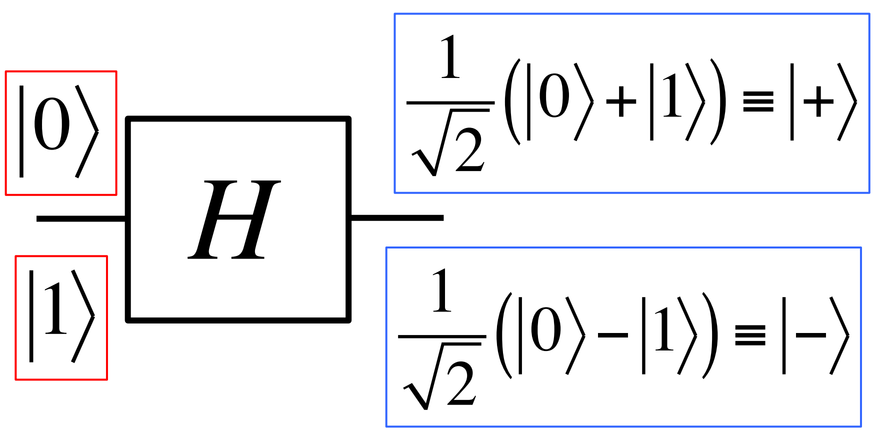
$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

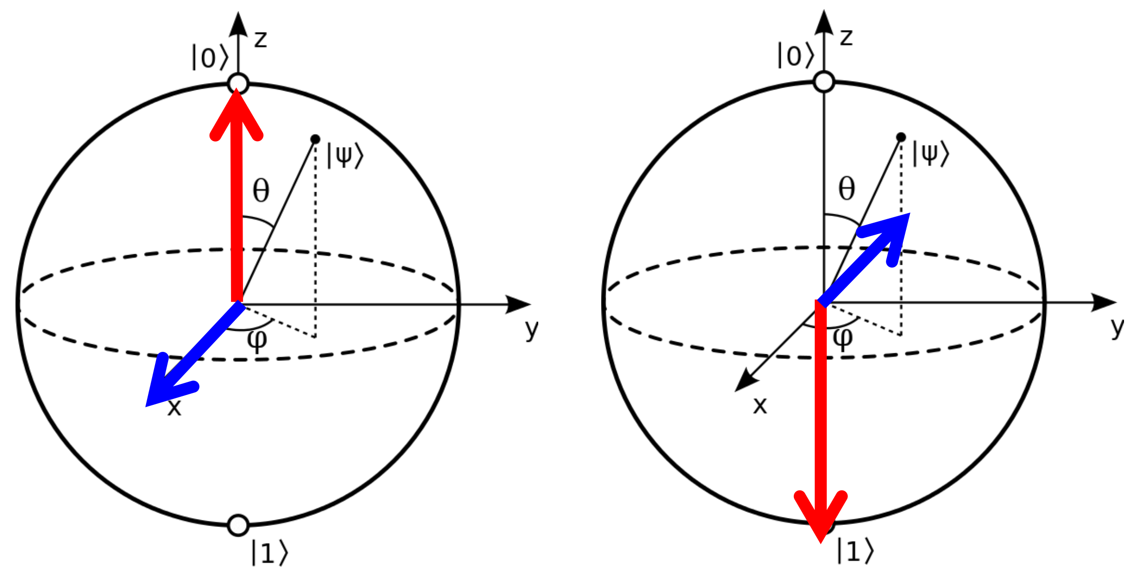
|Lib⟩

Single-qubit gates









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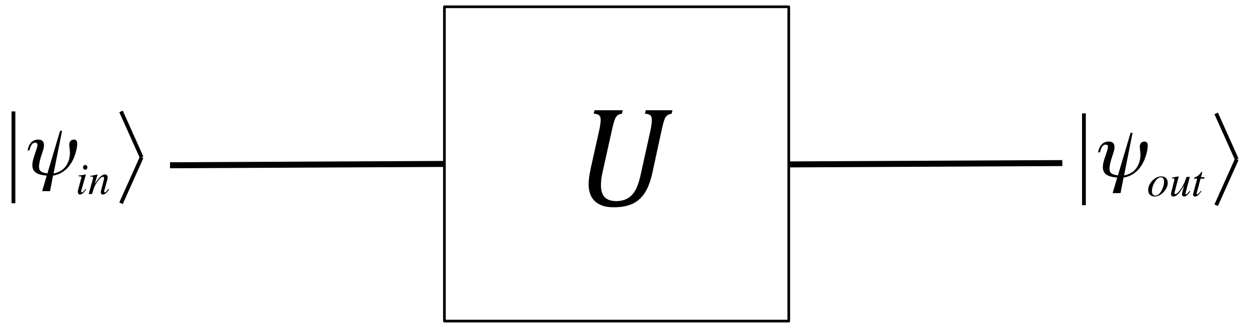
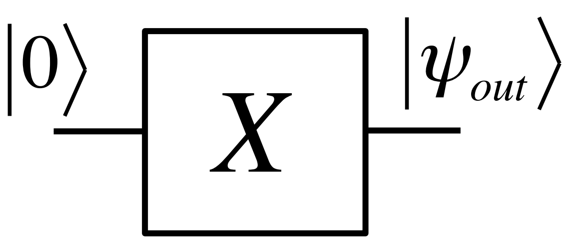
Single-qubit gates



$$|\psi_{out}\rangle = U|\psi_{in}\rangle$$

Exercise 1

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

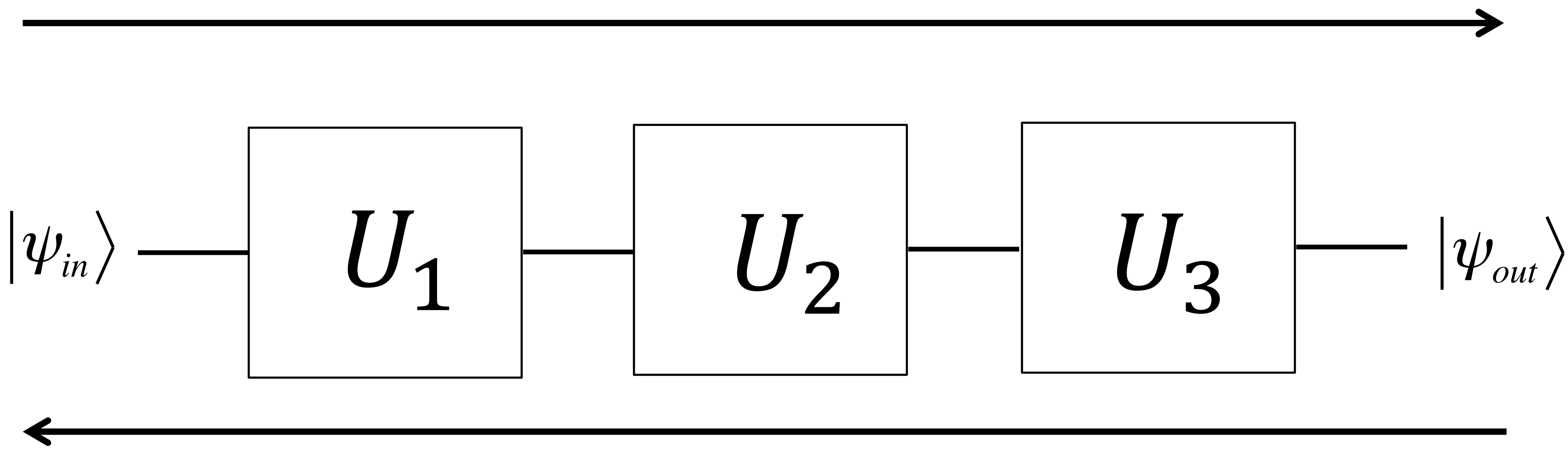


$$|\psi_{out}\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

ILib

binder

Quantum circuit (single qubit)



$$|\psi_{out}\rangle = U_3 U_2 U_1 |\psi_{in}\rangle$$

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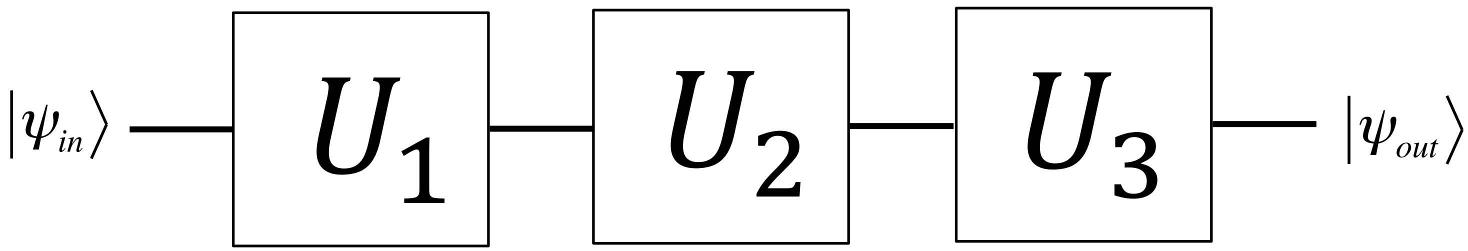
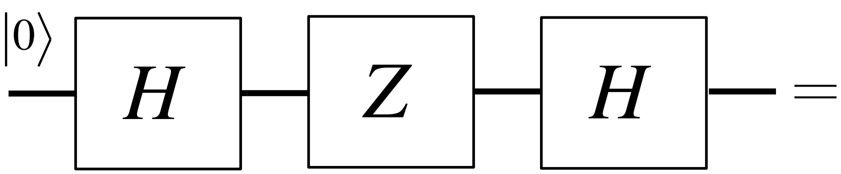
Quantum circuit (single qubit)

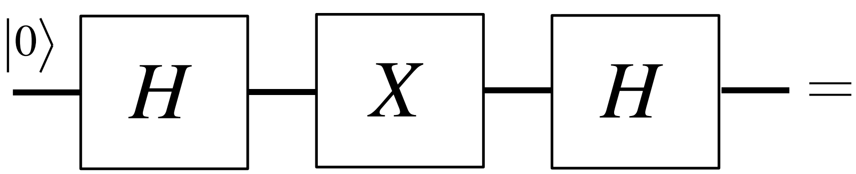


$$|\psi_{out}\rangle = U_3 U_2 U_1 |\psi_{in}\rangle$$

Exercise 2

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$





Multi-qubit state

$$|\psi_0\rangle = \alpha_0|0\rangle + \beta_0|1\rangle \qquad |\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$

$$|\psi\rangle = |\psi_0\rangle \otimes |\psi_1\rangle$$

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle = \begin{pmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{pmatrix}$$

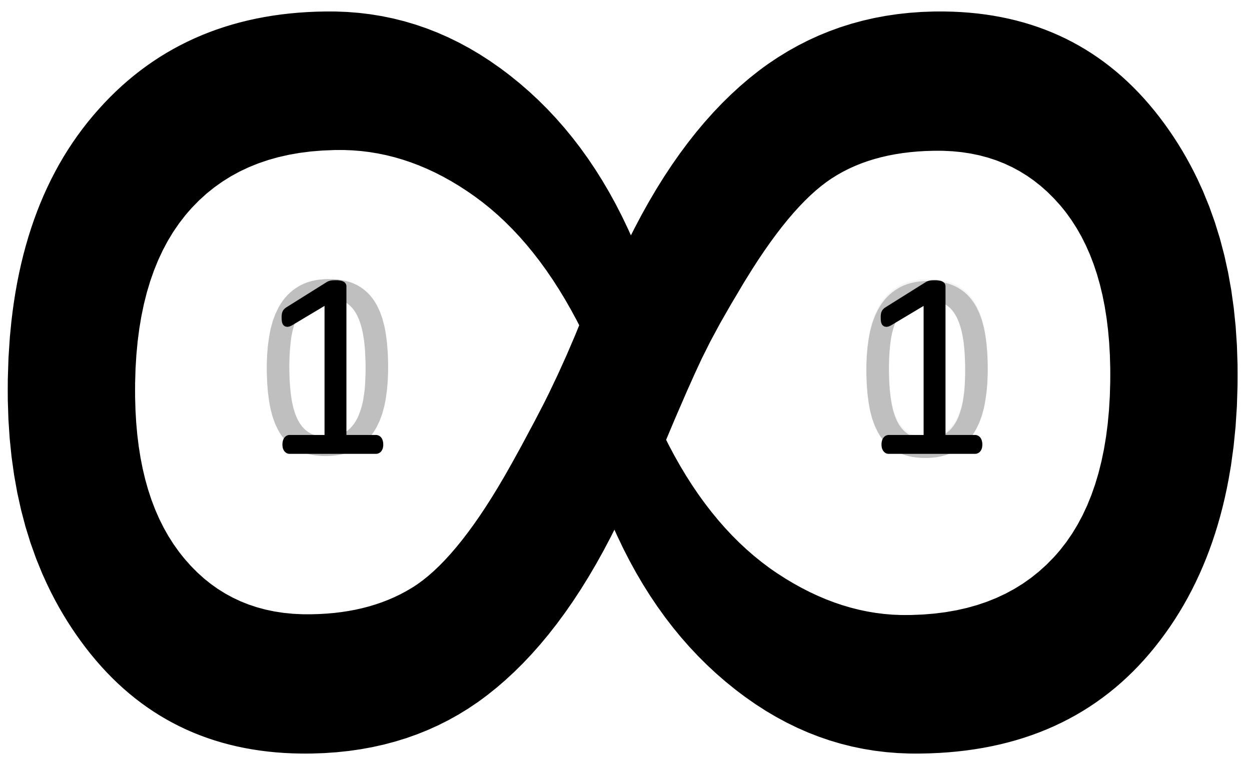
Kronecker product

$$|A\rangle \otimes |B\rangle = \begin{bmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{bmatrix}$$

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$

Multi-qubit state

Entanglement



Two qubits in a superposition are correlated with one another

Multi-qubit state

Entanglement

Quantum entanglement means that multiple particles are linked together in a way such that the measurement of one particle's quantum state determines the possible quantum states of the other particles.

This connection isn't depending on the location of the particles in space. Even if you separate entangled particles by billions of kms, changing one particle will induce a change in the other.

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

https://www.youtube.com/watch?v=CC_XES4xQD4

Multi-qubit state

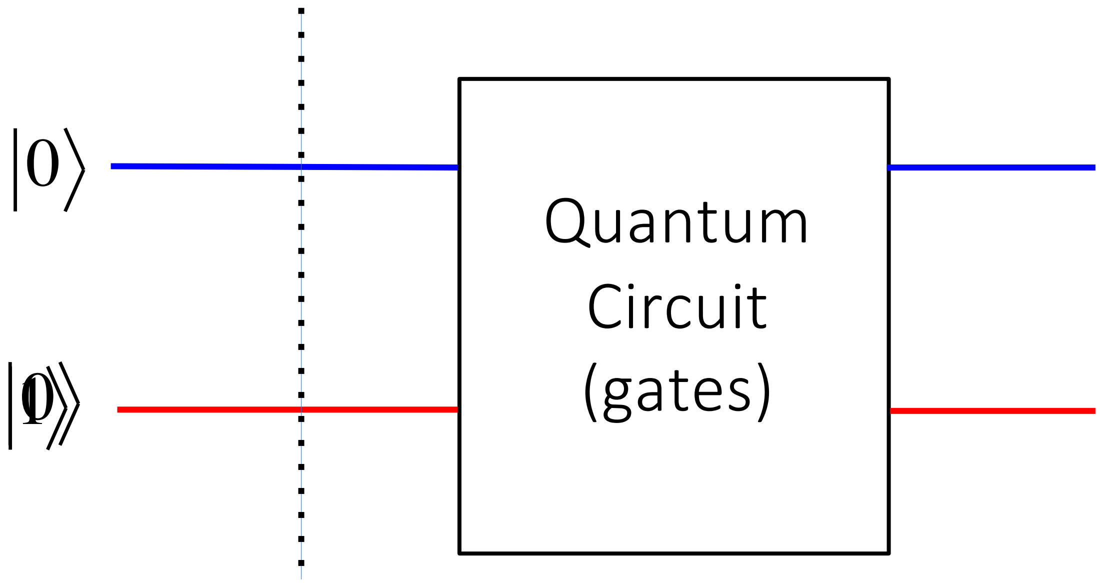
Two qubits are *entangled* when their joint states cannot possibly be separated into a product of individual qubit states

$$|\Psi\rangle = |\varphi\rangle \otimes |\psi\rangle \quad \text{vs.} \quad |\Psi\rangle = |\varphi\rangle \otimes |\psi\rangle + |\varphi'\rangle \otimes |\psi'\rangle$$

$$\frac{1}{\sqrt{2}}\left(|10\rangle + |11\rangle\right) = |1\rangle \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$ (not entangled)

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = \frac{1}{\sqrt{2}}|0\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}}\left(|1\rangle \otimes |1\rangle\right)$$ (entangled)

Quantum circuit (multiple qubits)



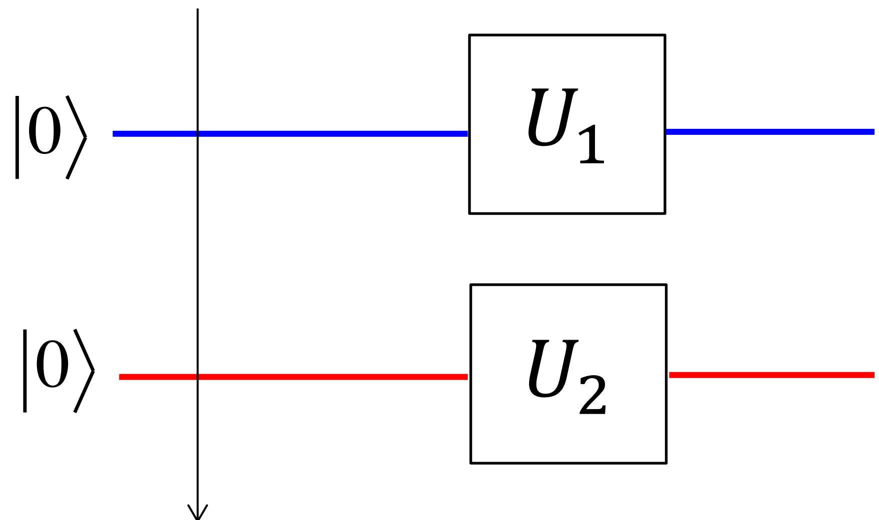
$$|01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$|10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$|0\rangle \otimes |0\rangle = |0\rangle|0\rangle = |00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$|11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Quantum circuit (multiple qubits)

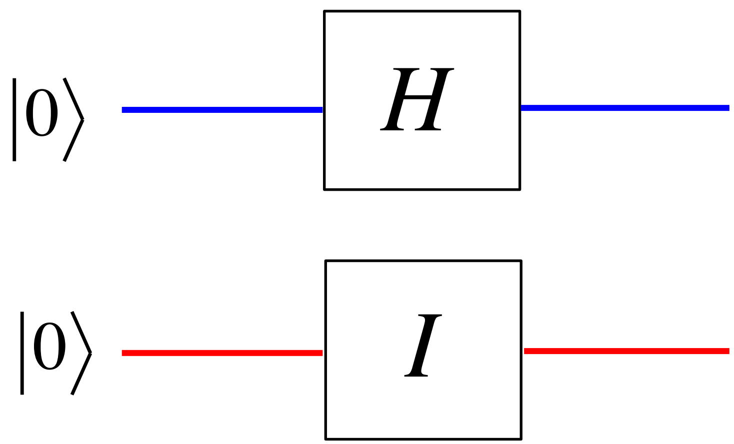


$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$|\psi_{out}\rangle = U_1 \otimes U_2 |\psi_{in}\rangle$$

ILib⟩

Quantum circuit (multiple qubits)

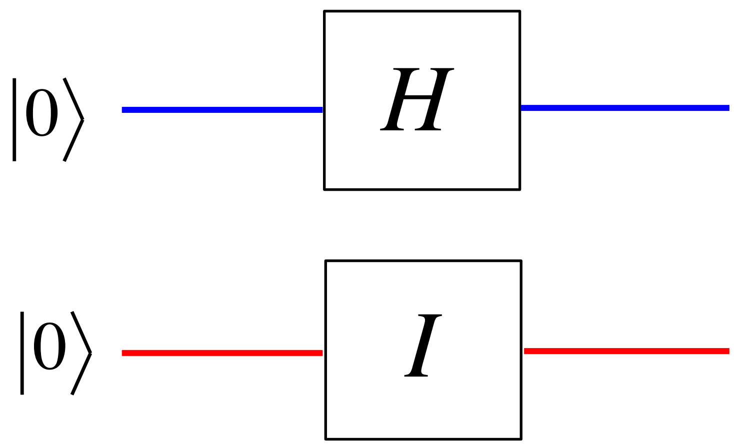


$$\left|\psi_{out}\right\rangle = H \otimes I \left|\psi_{in}\right\rangle$$

$$\hat{U}_1 \otimes \hat{U}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\left|A\right\rangle \otimes \left|B\right\rangle = \begin{bmatrix} A_{11}B & A_{12}B \\ A_{21}B & A_{22}B \end{bmatrix}$$

Quantum circuit (multiple qubits)

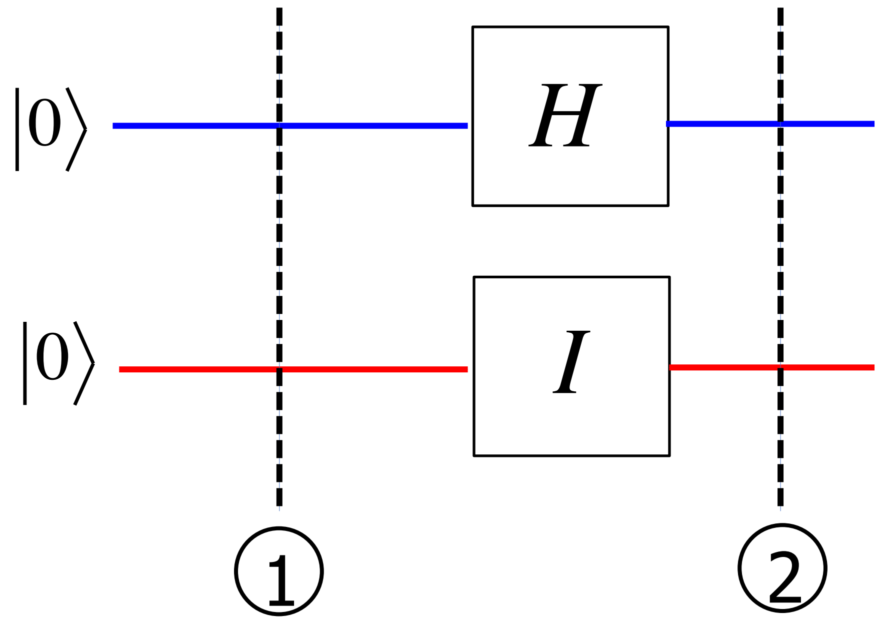


$$\left|\psi_{out}\right\rangle = H \otimes I \left|\psi_{in}\right\rangle$$

Exercise 4

$$\left|\psi_{out}\right\rangle =$$

Quantum circuit (multiple qubits)



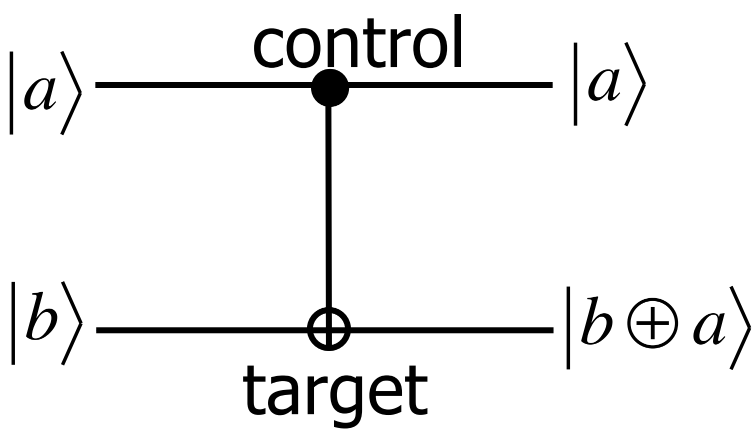
$$|\psi_{out}\rangle = H \otimes I |\psi_{in}\rangle$$

1 $|00\rangle$

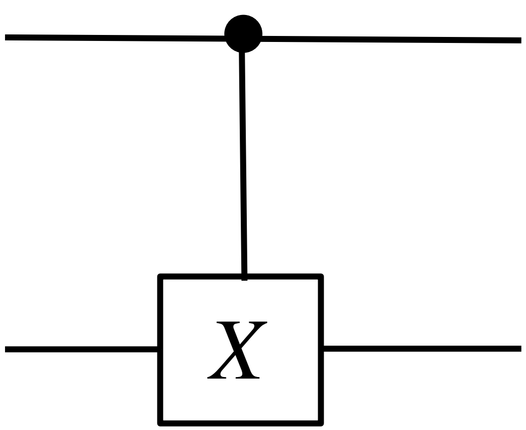
2 $\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)|0\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)$

Multi-qubit gates

CNOT gate



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



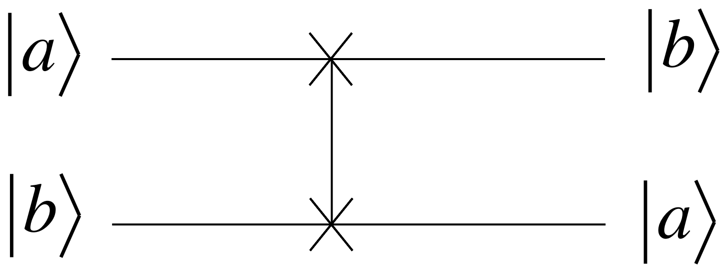
XOR gate

a	b	out
0	0	0
0	1	1
1	0	1
1	1	0

$$a \cdot \overline{b} + \overline{a} \cdot b$$

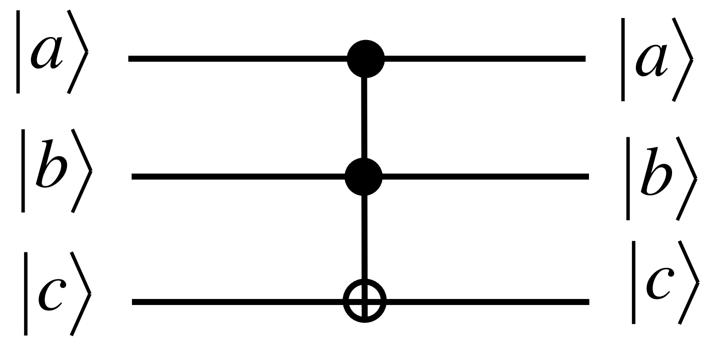
Multi-qubit gates

SWAP gate



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

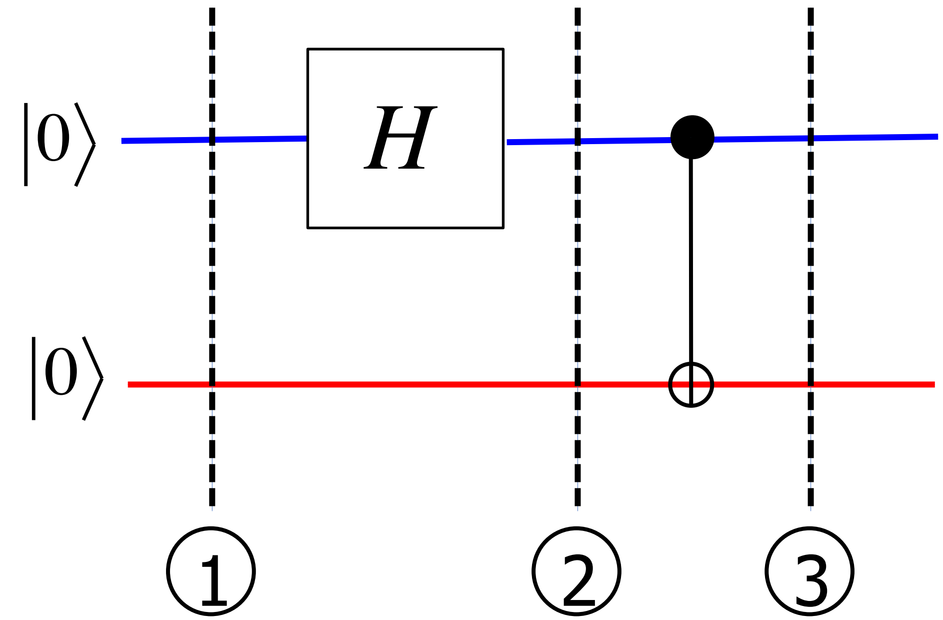
Toffoli gate



$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Input			Output		
0	0	**0**	0	0	**0**
0	0	**1**	0	0	**1**
0	1	**0**	0	1	**0**
0	1	**1**	0	1	**1**
1	0	**0**	1	0	**0**
1	0	**1**	1	0	**1**
1	1	**0**	1	1	**1**
1	1	**1**	1	1	**0**

Quantum circuit (multiple qubits)



1 $|00\rangle$

2 $\frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right)|0\rangle=\frac{1}{\sqrt{2}}\left(|00\rangle+|10\rangle\right)$

3 $\frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right)$

Bell states

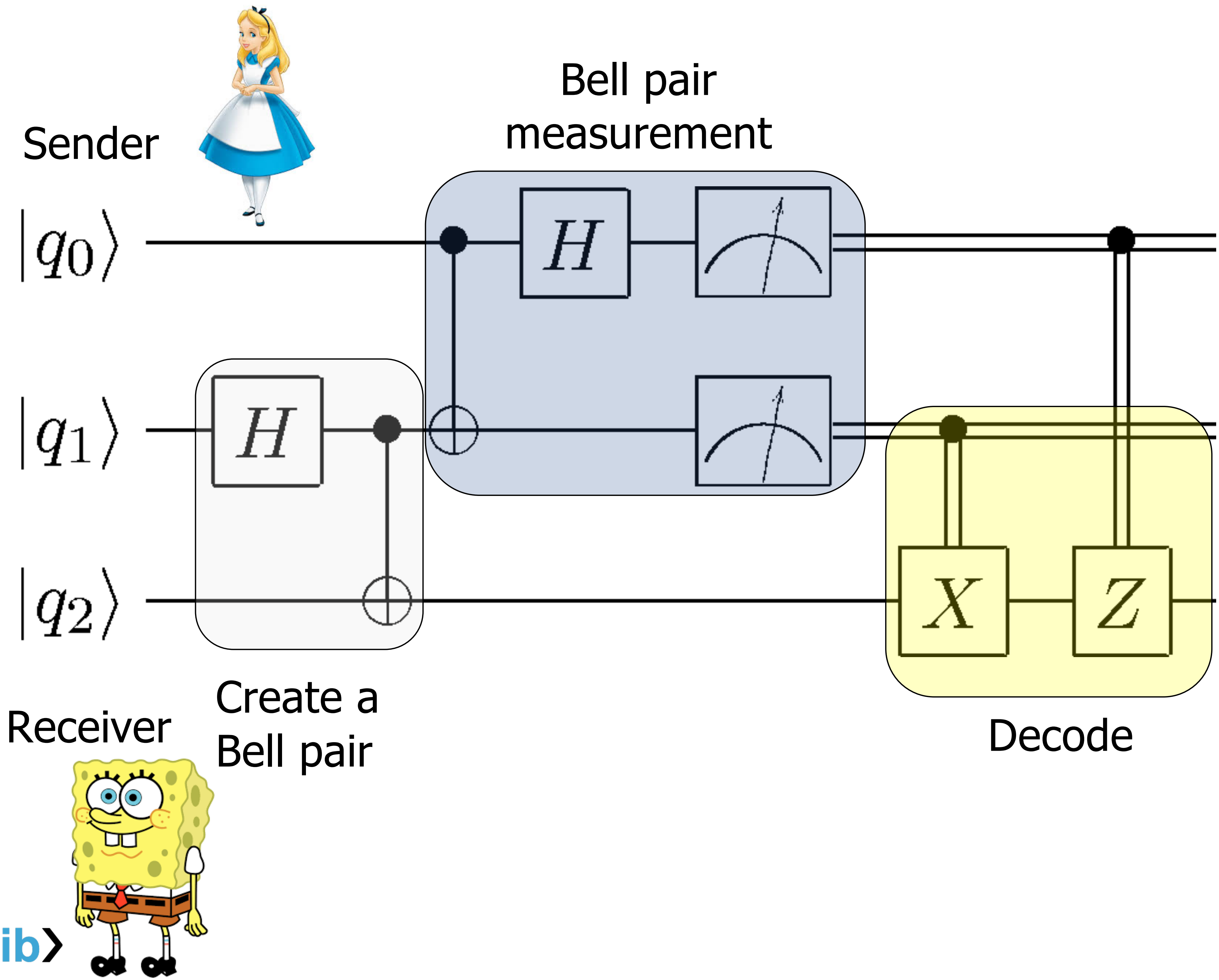
$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$|\beta_{01}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

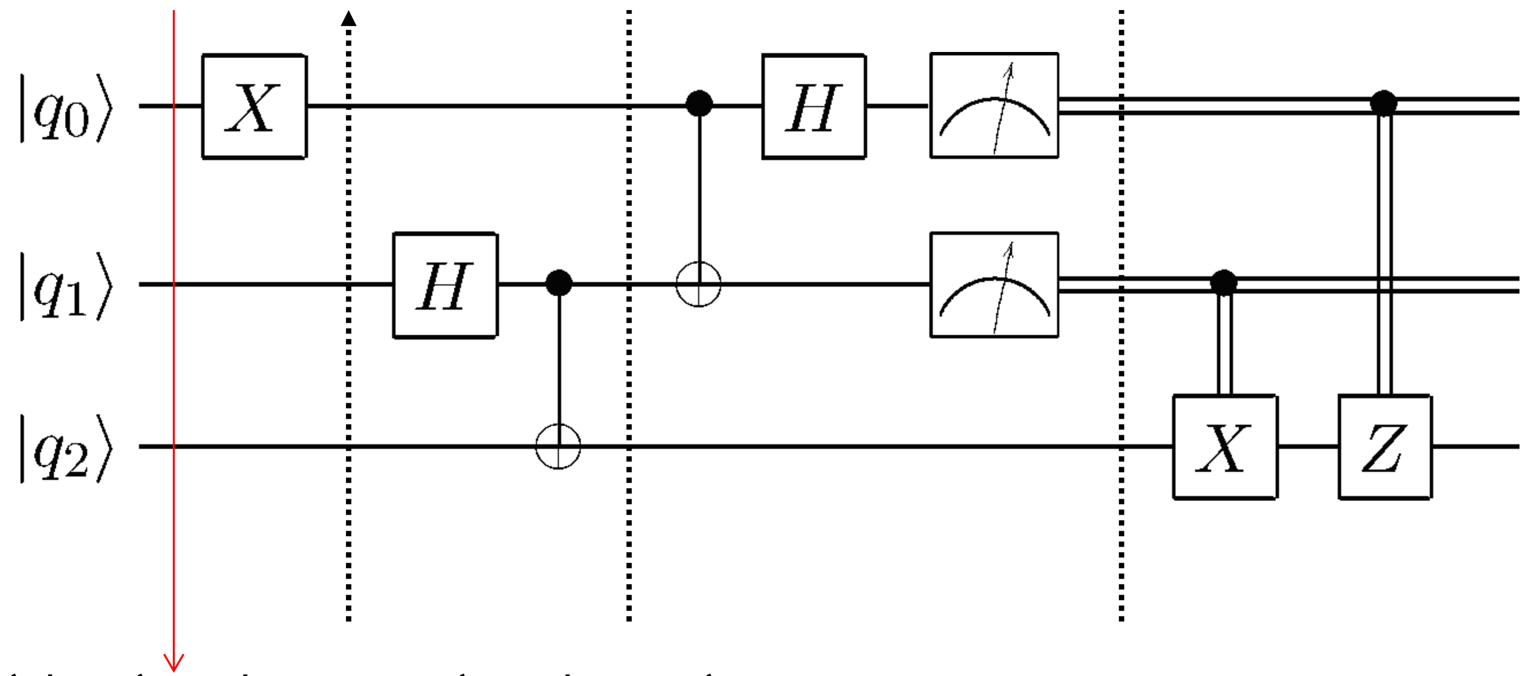
$$|\beta_{10}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

$$|\beta_{11}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

Quantum teleportation



Quantum teleportation



$$|q_0\rangle|q_1\rangle|q_2\rangle = |q_0 q_1 q_2\rangle = |000\rangle$$

QASM-like code

```
X q0
H q1
CNOT q1,q2
CNOT q0,q1
H q0
```

Quantum teleportation

```
X q0
H q1
CNOT q1,q2
CNOT q0,q1
H q0
```

$$|q_0\rangle|q_1\rangle|q_2\rangle = |q_0 q_1 q_2\rangle = |000\rangle$$

X q_0 → $|100\rangle$

H q_1 → $|1\rangle\frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right)|0\rangle = \frac{1}{\sqrt{2}}\left(|100\rangle+|110\rangle\right)$

CNOT q_1,q_2 → $\frac{1}{\sqrt{2}}\left(|100\rangle+|111\rangle\right)$

CNOT q_0,q_1 → $\frac{1}{\sqrt{2}}\left(|110\rangle+|101\rangle\right)$

H q_0 → $\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(|0\rangle-|1\rangle\right)|10\rangle+\frac{1}{\sqrt{2}}\left(|0\rangle-|1\rangle\right)|01\rangle\right]=$

$$=\frac{1}{2}\left(|010\rangle-|110\rangle+|001\rangle-|101\rangle\right)$$

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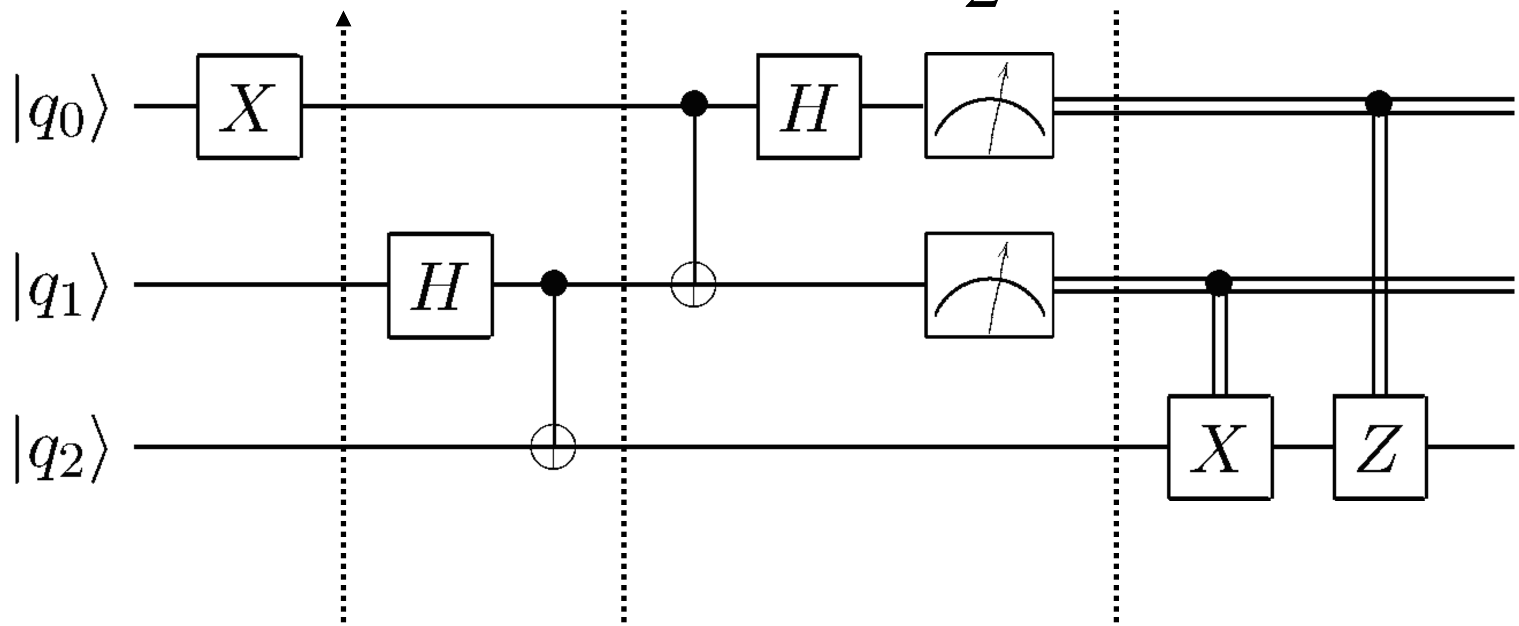
Quantum teleportation

$$\frac{1}{2}\left(|010\rangle - |110\rangle + |001\rangle - |101\rangle\right)$$



- Measure 00 → $|q_2\rangle = |1\rangle$ → no correction
- Measure 01 → $|q_2\rangle = |0\rangle$ → bit-flip q2 → $|q_2\rangle = |1\rangle$
- Measure 10 → $|q_2\rangle = -|1\rangle$ → phase-flip q2 → $|q_2\rangle = |1\rangle$
- Measure 11 → $|q_2\rangle = -|0\rangle$ → bit-flip and phase-flip → $|q_2\rangle = |1\rangle$