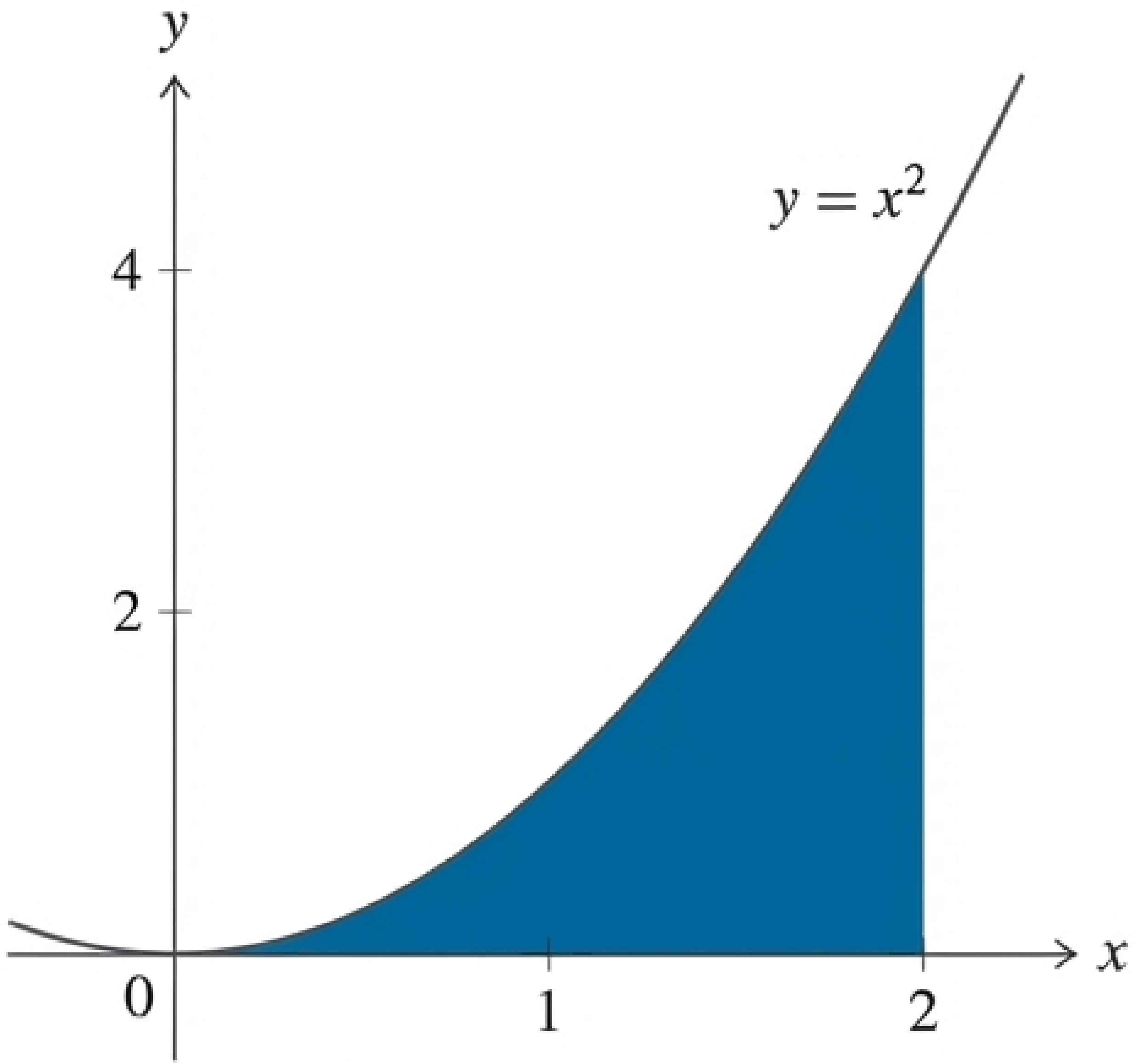
The Concept of Area – Section 5.2

MATH102 – Calculus II

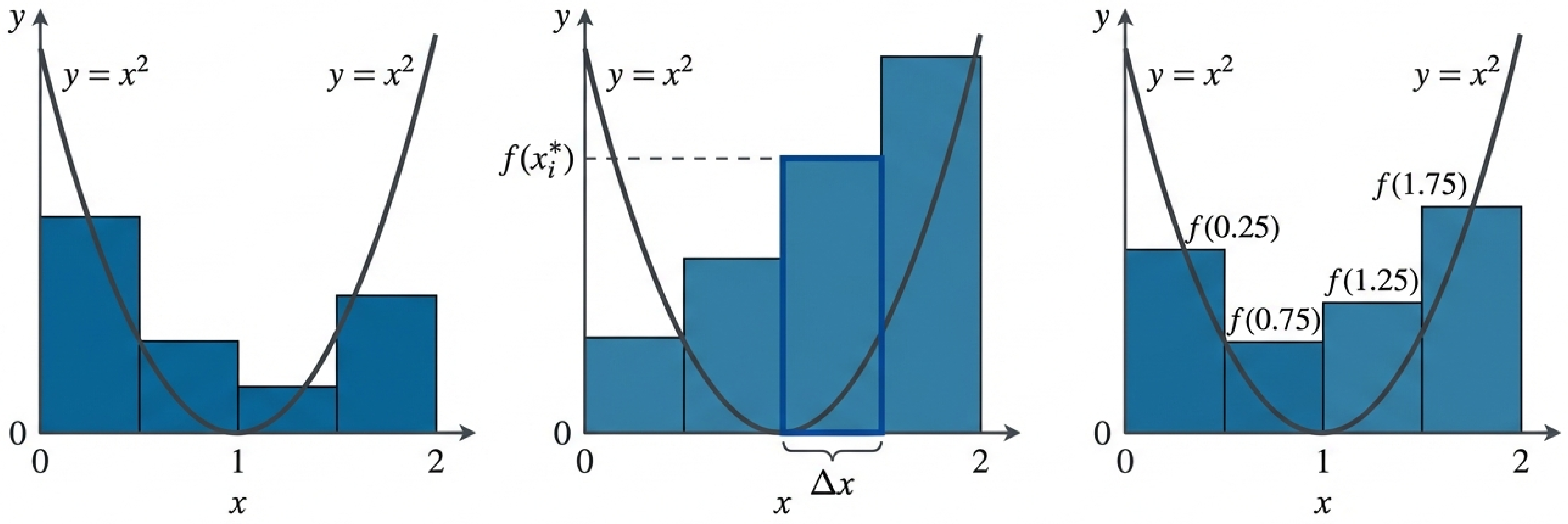
Based on Larson & Edwards, *Calculus: Early Transcendental Functions* (7th ed.)

How do we find the area of a region bounded by a curve?



For simple shapes like triangles or rectangles, the answer is easy.
But what about the area of this curved region?

The Core Idea: Approximation with Rectangles

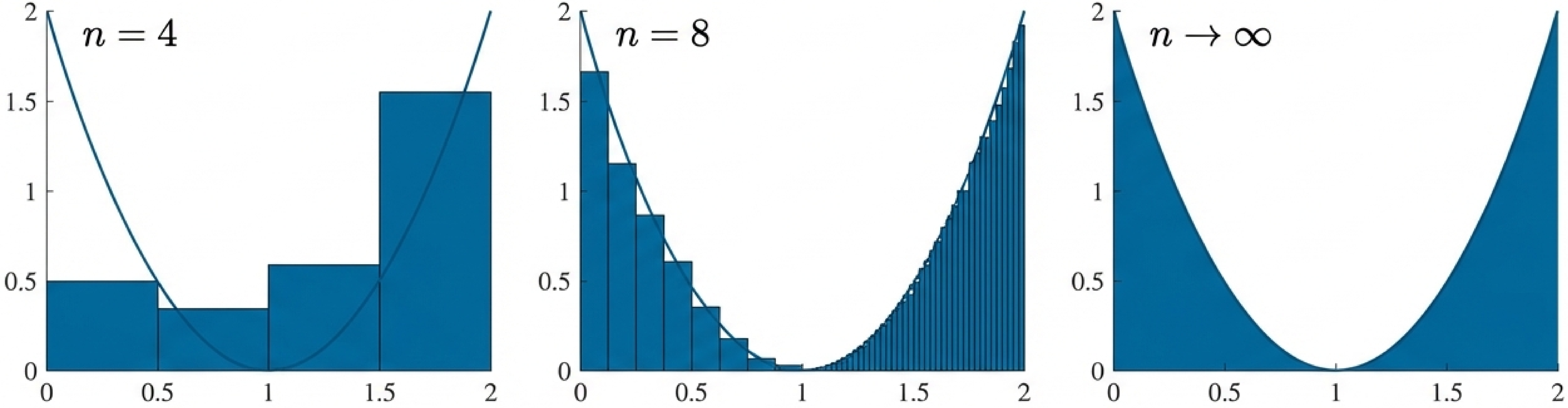


$$\text{Area} \approx \sum \text{Area of Rectangles} = \sum (\text{height}) \times (\text{width})$$

$$A \approx \sum f(x_i^*)\Delta x$$

NotebookLM

From Approximation to Exactness: The Limit



Definition:

The area A of the region bounded by the graph of a continuous function f, the x-axis, and the lines $x=a$ and $x=b$ is the limit of the sum of the areas of approximating rectangles.

The Riemann Sum Formula:

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

A More Powerful Notation: The Definite Integral

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \qquad \int_a^b f(x)\,dx$$

The limit of the Riemann sum is called the definite integral.

Integral Symbol

Upper Limit of Integration

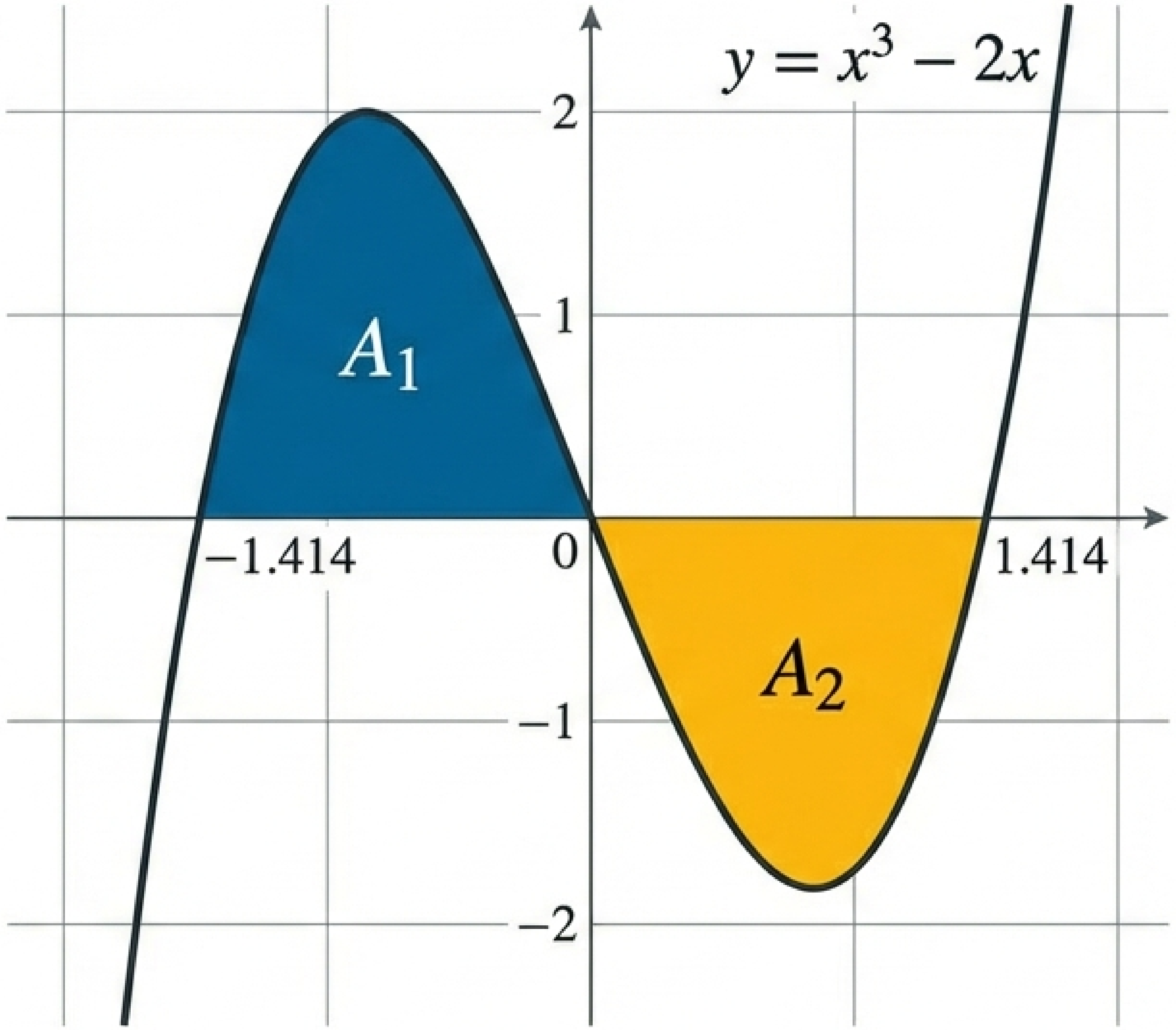
$$\int_a^b f(x)\,dx$$

Integrand

Lower Limit of Integration

Variable of Integration

What is "Net" Area?



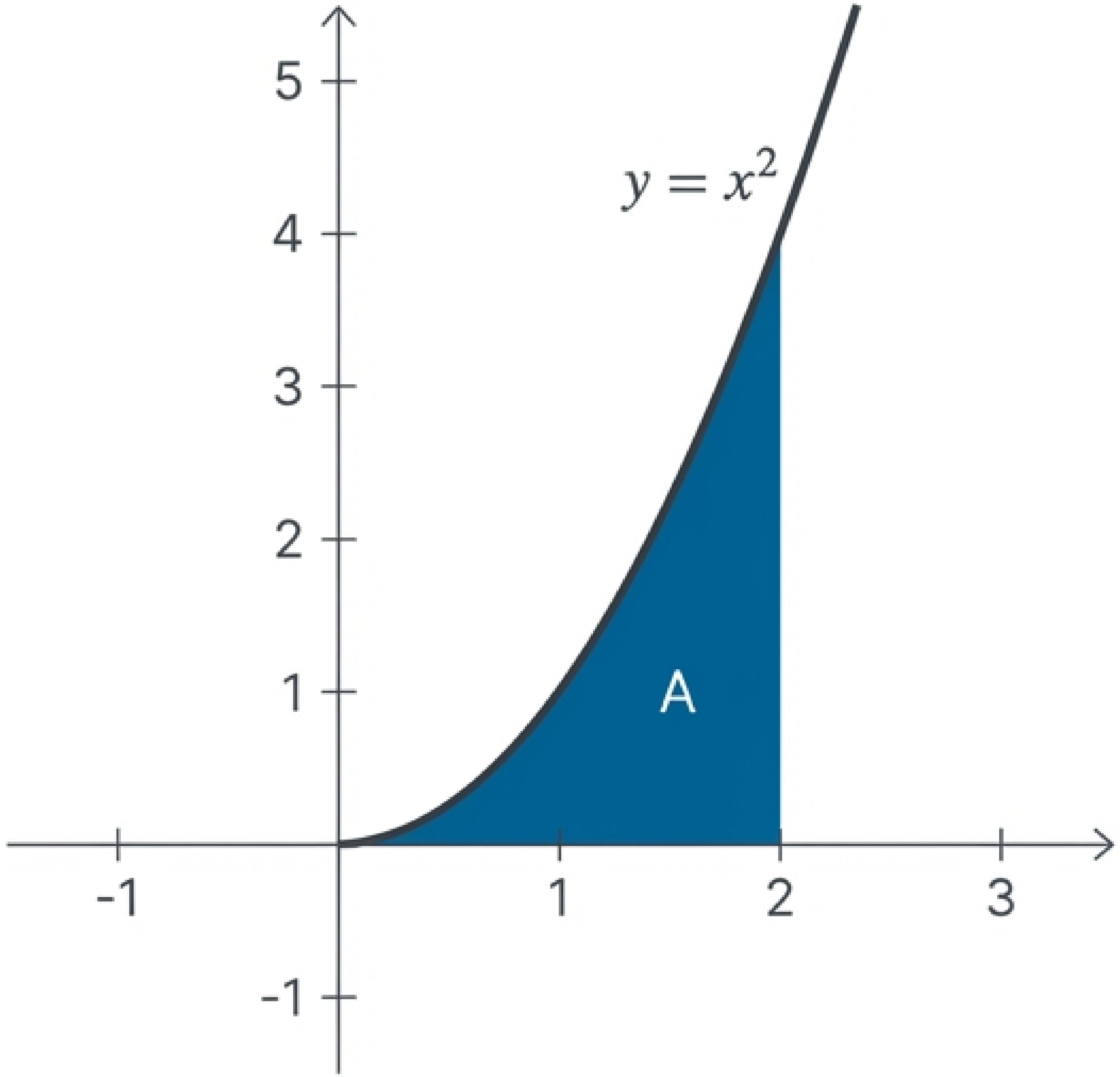
- Regions above the x-axis contribute positively to the integral.
- Regions below the x-axis contribute negatively to the integral.

$$\text{Definite Integral} = \int_a^b f(x)\,dx$$

$$= (\text{Area above axis}) - (\text{Area below axis})$$

$$= A_1 - A_2$$

Example 1: Solving Our Original Problem



Find the area under the curve $y = x^2$ from $x = 0$ to $x = 2$.

Set up the integral

$$\text{Area} = \int_0^2 x^2\,dx$$

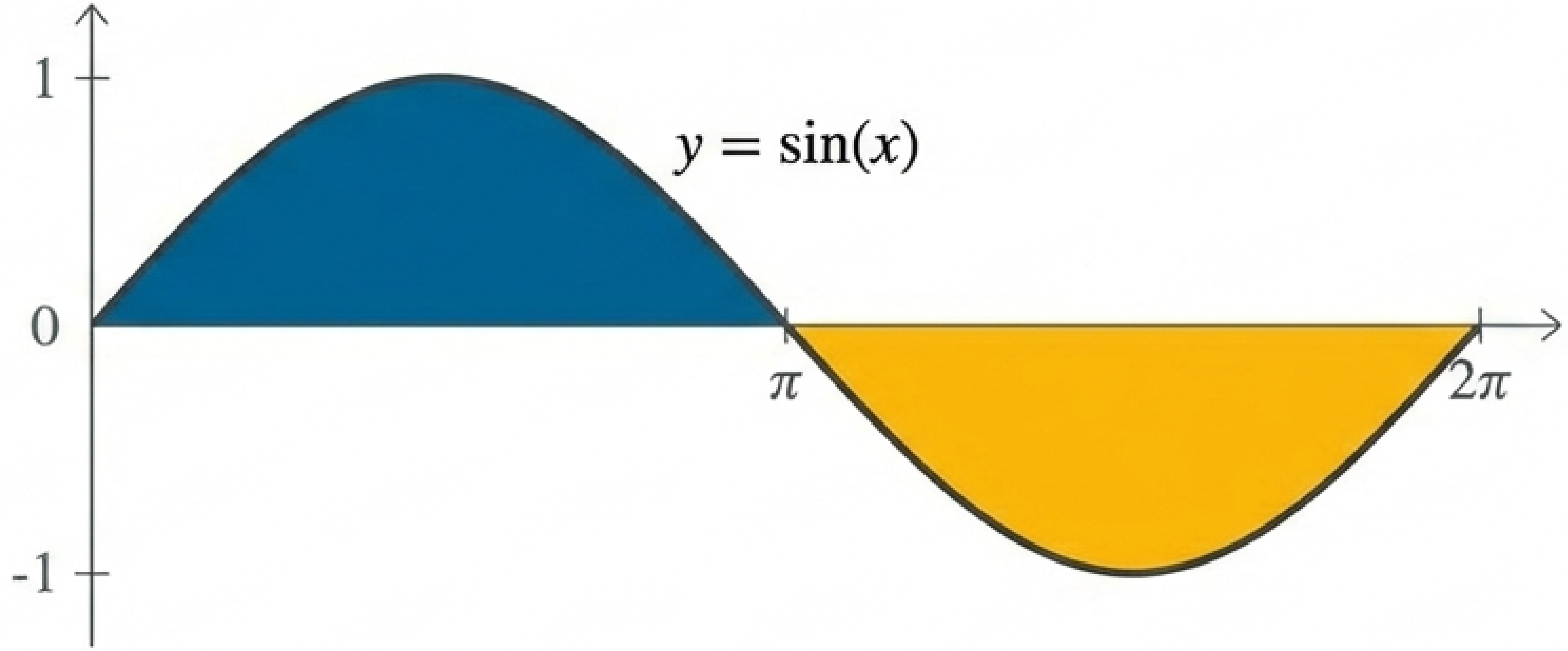
Find the antiderivative

$$= \left[\frac{x^3}{3}\right]_0^2$$

Evaluate at limits $= \left(\dfrac{2^3}{3}\right) - \dfrac{0^3}{3}\Big)$

Final Answer $\boxed{= \dfrac{8}{3}}$

Advanced Example: Cancellation in Action



Case 1

$$\int_0^{\pi} \sin(x)\,dx = 2$$

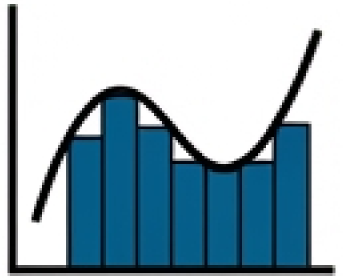
(The area is entirely positive.)

Case 2

$$\int_0^{2\pi} \sin(x)\,dx = 0$$

(Why? The positive area from $[0, \pi]$ and the negative area from $[\pi, 2\pi]$ are equal in magnitude and cancel each other out.)

Summary: The Story of Area



The problem of finding the area under a curve is solved by approximating with rectangles.

The **definite integral** is the formal definition for this area, found by taking the limit of a Riemann sum as the number of rectangles approaches infinity.

The integral calculates **net area**, where area below the x-axis is negative.

$$\text{Area} = \int_a^b f(x)\,dx$$

Reference & Course Information

Section 5.2 – Area

Calculus: Early Transcendental Functions (7th Edition)
by Ron Larson & Bruce Edwards.

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