Two classes of spectral three-term derivative-free method for solving nonlinear equations with application

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Detailed Numerical Results

Table 1: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in ENGVAL1Grad*

ENGVAL1Grad																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
DIM	INP	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
1000	x_0^1	83	348	0.170251	6.27266E-12	61	256	0.039711	7.9914E-12	160	1920	0.04697	5.08023E-12	36	322	0.029459	5.08725E-12	43	427	0.032163	5.41684E-12
	x_0^2	148	584	0.0425269	9.83775E-12	189	719	0.011635	8.3598E-12	226	2977	0.049286	5.21365E-12	669	5471	0.08823	9.24679E-12	115	1079	0.020052	8.60829E-12
	x_0^3	187	752	0.0236343	9.21219E-12	185	725	0.018132	9.8016E-12	118	1099	0.018483	7.94536E-12	698	5708	0.083597	8.48142E-12	136	1269	0.022194	8.21844E-12
	x_0^4	179	724	0.02171	5.99226E-12	202	785	0.018604	9.3775E-12	141	1281	0.023715	5.61117E-12	708	5788	0.089491	8.48443E-12	134	1251	0.018243	6.25975E-12
	x_0^5	227	911	0.0185836	9.43381E-12	203	777	0.01614	8.5171E-12	164	2126	0.0265	4.99018E-12	715	5844	0.087157	8.39903E-12	135	1260	0.018592	6.5548E-12
	x_0^6	161	635	0.0126765	9.82134E-12	170	664	0.015482	4.5894E-12	148	1830	0.028966	6.29587E-12	703	5749	0.083791	8.37516E-12	134	1252	0.017828	9.46069E-12
	x_0^7	221	876	0.0171659	9.40806E-12	198	763	0.018036	9.8189E-12	112	1090	0.016718	3.43567E-12	705	5766	0.087422	9.09106E-12	143	1346	0.022672	9.98484E-12
	x_0^8	189	758	0.0150974	9.01653E-12	200	772	0.017156	9.919E-12	198	2893	0.041651	9.58545E-12	655	5364	0.0798	9.33572E-12	137	1281	0.017984	7.79394E-12
	x_0^9	196	780	0.0159042	9.4115E-12	195	759	0.018308	7.9408E-12	164	1853	0.027913	4.25447E-12	699	5718	0.08459	9.14347E-12	139	1314	0.019986	9.67551E-12
	x_0^{10}	137	563	0.0096911	9.66033E-12	111	461	0.007555	9.037E-12	270	3712	0.054607	4.45453E-12	418	3431	0.062501	9.61582E-12	46	459	0.006091	5.3876E-12
	x_0^{11}	240	965	0.0231056	9.45027E-12	210	833	0.018493	9.4547E-12	180	2346	0.033777	7.20983E-12	682	5586	0.083634	9.51749E-12	140	1312	0.024945	6.51291E-12
	x_0^{12}	137	552	0.0094106	8.12888E-12	136	535	0.012756	7.9508E-12	164	1953	0.041669	9.71017E-12	428	3511	0.053159	8.52904E-12	45	442	0.006114	8.9978E-12
	x_0^{13}	81	351	0.0082148	6.86658E-12	74	313	0.00443	7.1965E-12	146	1487	0.028041	2.71521E-12	38	340	0.007144	7.81672E-12	36	355	0.004275	7.63998E-12
	x_0^{14}	242	974	0.0187274	9.35372E-12	188	730	0.015689	8.9558E-12	220	3855	0.054182	8.87848E-12	682	5586	0.08708	9.56062E-12	140	1312	0.022992	6.45555E-12
10000	x_0^1	70	292	0.045975	7.68093E-12	80	334	0.050725	9.6112E-12	175	1971	0.232248	6.32847E-12	37	331	0.038745	4.47553E-12	42	420	0.049087	6.66445E-12
	x_0^2	213	843	0.1504736	7.47409E-12	185	713	0.108107	7.6931E-12	115	1215	0.152756	7.82858E-12	657	5374	0.635148	9.41105E-12	108	1016	0.118062	6.40213E-12
	x_0^3	133	543	0.0961941	8.51329E-12	182	722	0.105694	9.3925E-12	163	2237	0.272835	6.10655E-12	687	5619	0.686741	8.87349E-12	135	1260	0.14218	7.34157E-12
	x_0^4	192	775	0.1387053	9.06063E-12	198	764	0.134417	7.8369E-12	216	3248	0.362836	7.90495E-12	691	5651	0.673806	9.12815E-12	129	1208	0.13753	9.13491E-12
	x_0^5	216	864	0.1405472	9.6831E-12	195	744	0.117332	9.0835E-12	139	1286	0.14788	3.89277E-12	683	5585	0.685966	9.96838E-12	117	1099	0.128467	6.61802E-12
	x_0^6	190	761	0.1643459	9.98719E-12	188	728	0.110874	8.9282E-12	130	1306	0.171415	6.17162E-12	692	5660	0.685848	8.45322E-12	130	1216	0.156155	6.58387E-12
	x_0^7	213	854	0.1832063	9.88552E-12	200	773	0.1162	8.8185E-12	148	2067	0.23717	5.65781E-12	688	5629	0.678254	9.95346E-12	142	1336	0.157444	7.19394E-12
	x_0^8	193	764	0.132412	9.76179E-12	198	762	0.118329	8.0187E-12	137	1258	0.16115	9.86845E-12	721	5906	0.705982	8.99307E-12	137	1281	0.144989	8.36029E-12
	x_0^9	214	858	0.1410929	9.40619E-12	169	632	0.111239	9.0347E-12	143	1909	0.231369	7.75354E-12	688	5629	0.685119	9.95347E-12	138	1298	0.152524	5.98622E-12
	x_0^{10}	113	474	0.0848578	9.33623E-12	111	460	0.070648	9.034E-12	159	1822	0.243875	5.43271E-12	300	2472	0.292672	8.98255E-12	42	418	0.048371	6.45226E-12
	x_0^{11}	236	951	0.175587	9.39171E-12	202	795	0.119506	9.433E-12	230	4027	0.441638	8.74433E-12	676	5537	0.669292	8.72518E-12	131	1231	0.151988	9.60002E-12
	x_0^{12}	109	447	0.0766534	9.2353E-12	79	327	0.050995	9.8811E-12	138	1225	0.139917	8.57673E-12	308	2536	0.302534	9.8739E-12	41	409	0.045308	4.98783E-12
	x_0^{13}	101	436	0.0692791	7.87695E-12	78	353	0.050574	6.7712E-12	281	4478	0.507398	6.67165E-12	37	331	0.038912	8.2364E-12	44	440	0.057446	5.43578E-12
	x_0^{14}	237	955	0.1645599	9.49709E-12	182	718	0.106602	9.193E-12	133	1512	0.178789	8.34875E-12	676	5537	0.684784	8.72676E-12	131	1231	0.14183	9.57503E-12
100000	x_0^1	79	335	0.5914944	5.2961E-12	76	321	0.517002	4.8677E-12	137	1523	1.972852	7.9112E-12	38	340	0.437734	3.86319E-12	41	412	0.497639	4.13487E-12
	x_0^2	211	845	1.611918	8.74563E-12	199	769	1.354225	8.9757E-12	158	1592	2.116809	7.88159E-12	651	5325	6.822158	9.56446E-12	103	973	1.2	8.97092E-12
	x_0^3	173	703	1.3152719	5.77705E-12	201	798	1.380495	7.7656E-12	126	1436	1.784808	5.10775E-12	672	5498	7.149216	8.57992E-12	130	1215	1.499284	6.52554E-12
	x_0^4	190	784	1.4222491	9.36323E-12	205	835	1.392768	9.2847E-12	195	2647	3.322391	4.90602E-12	2000	16119	20.94318	1.88587E-11	102	961	1.203014	6.74785E-12
	x_0^5	189	743	1.3960489	9.06948E-12	190	739	1.252762	7.1853E-12	193	2276	2.977787	6.74518E-12	680	5560	7.144622	9.56443E-12	108	1020	1.232727	5.80222E-12
	x_0^6	170	680	1.3380962	7.66697E-12	183	712	1.206049	9.4263E-12	239	3115	4.236914	9.78698E-12	2000	16117	20.90791	3.40124E-11	130	1216	1.601046	9.08238E-12
	x_0^7	187	748	1.3868081	9.78687E-12	188	725	1.315373	8.6458E-12	173	2068	2.689648	7.12687E-12	667	5460	7.145277	9.82417E-12	143	1362	1.691828	9.12536E-12
	x_0^8	170	668	1.2508546	9.85327E-12	181	705	1.209741	9.4987E-12	213	3503	4.159242	3.47283E-12	705	5773	7.384159	8.09542E-12	132	1236	1.610706	8.55344E-12
	x_0^9	174	696	1.3297856	9.87818E-12	193	745	1.286028	8.6347E-12	200	3056	3.720686	9.01505E-12	667	5460	7.025734	9.82417E-12	141	1343	1.68278	8.83844E-12
	x_0^{10}	99	466	0.8303602	9.73451E-12	95	427	0.706558	8.9528E-12	191	2361	3.051151	4.45064E-12	171	1426	1.831488	8.48577E-12	39	393	0.473664	9.39465E-12
	x_0^{11}	216	871	1.6413838	9.96849E-12	203	803	1.378676	9.2862E-12	135	1643	2.01967	8.60599E-12	651	5336	6.839253	8.71648E-12	125	1178	1.459519	8.55529E-12
	x_0^{12}	82	346	0.5977986	9.50177E-12	105	434	0.767035	9.4032E-12	137	1541	2.044481	6.26697E-12	189	1570	2.011022	9.28713E-12	41	413	0.520061	7.35795E-12
	x_0^{13}	88	435	0.7196789	6.94657E-12	89	427	0.714496	6.4497E-12	128	1678	2.103194	8.16208E-12	39	349	0.436007	4.47612E-12	30	302	0.364805	3.94714E-12
	x_0^{14}	226	912	1.6553533	9.61912E-12	173	682	1.256372	9.9755E-12	213	2808	3.397369	7.41462E-12	651	5336	7.020791	8.72945E-12	125	1178	1.456469	8.55042E-12

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Table 2: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in ExponetialSineCosine*

ExponetialSineCosine																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
DIM	INP	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
1000	x_0^1	1	1	0.1540127	0	1	2	0.031248	0	1	1	0.021361	0	1	1	0.018633	0	1	1	0.02133	0
	x_0^2	37	112	0.0025011	7.66064E-12	27	83	0.001757	5.4769E-12	42	84	0.002219	6.17907E-12	13	79	0.001252	2.9491E-12	48	289	0.00495	8.35578E-12
	x_0^3	38	115	0.0025325	5.13986E-12	28	86	0.001908	4.213E-12	43	86	0.002147	5.36455E-12	13	79	0.001284	3.92511E-12	50	301	0.012042	6.87421E-12
	x_0^4	38	112	0.0026652	5.17497E-12	28	83	0.001799	4.49387E-12	43	86	0.002172	6.24226E-12	15	86	0.00146	6.42834E-12	52	308	0.005191	6.35461E-12
	x_0^5	38	115	0.0026938	5.37509E-12	28	86	0.001871	5.22763E-12	43	86	0.002136	6.91634E-12	13	79	0.001305	4.1147E-12	51	307	0.005046	6.56526E-12
	x_0^6	37	112	0.0025668	4.95379E-12	27	83	0.001833	7.8116E-12	43	86	0.002238	7.57638E-12	15	87	0.001478	6.42834E-12	52	309	0.005188	6.38972E-12
	x_0^7	38	115	0.0030129	4.81686E-12	29	89	0.001966	3.70042E-12	43	86	0.002132	6.81804E-12	14	87	0.001488	4.14278E-12	48	290	0.005085	7.79405E-12
	x_0^8	35	107	0.013372	6.83208E-12	23	72	0.00155	4.97836E-12	43	86	0.002191	5.5401E-12	15	88	0.008984	6.42834E-12	48	290	0.004716	7.79405E-12
	x_0^9	38	115	0.0026664	4.90815E-12	29	89	0.001932	3.74255E-12	43	86	0.002168	6.8391E-12	14	87	0.001568	4.14278E-12	48	290	0.00459	7.6466E-12
	x_0^{10}	75	250	0.005493	6.30549E-12	62	203	0.0055	7.95818E-12	39	78	0.001907	7.08177E-12	61	377	0.006031	9.65331E-12	63	441	0.006938	4.42195E-12
	x_0^{11}	84	281	0.0060578	6.51805E-12	73	239	0.012786	8.66141E-12	43	86	0.002153	5.89291E-12	69	426	0.006911	4.81024E-12	61	430	0.006954	8.84692E-12
	x_0^{12}	30	91	0.0020643	6.30195E-12	22	68	0.001461	3.77064E-12	34	68	0.001658	9.19838E-12	11	67	0.001175	2.63313E-12	38	229	0.003848	8.3277E-12
	x_0^{13}	68	227	0.0046063	8.76408E-12	59	194	0.003679	7.49301E-12	38	76	0.001686	5.12202E-12	56	345	0.005041	9.72235E-12	57	398	0.013845	8.92389E-13
	x_0^{14}	83	278	0.0096373	7.68877E-12	71	235	0.005012	8.90718E-12	43	86	0.002382	5.89916E-12	59	365	0.006122	6.77147E-12	57	402	0.006461	8.71251E-12
10000	x_0^1	1	1	0.0003218	0	1	2	0.000333	0	1	1	9.75E-05	0	1	1	9.11E-05	0	1	1	0.000112	0
	x_0^2	38	115	0.0247907	6.17284E-12	28	86	0.021525	6.05072E-12	43	86	0.02011	9.76996E-12	13	79	0.01478	9.32587E-12	50	301	0.046949	8.23785E-12
	x_0^3	38	115	0.0266605	8.81517E-12	29	89	0.018614	4.66294E-12	44	88	0.022683	8.4599E-12	14	85	0.013422	1.13243E-12	52	313	0.051295	6.79456E-12
	x_0^4	38	112	0.0258925	9.00391E-12	29	86	0.0213	4.996E-12	44	88	0.020866	9.85878E-12	16	92	0.015091	1.88738E-12	54	320	0.052461	6.30607E-12
	x_0^5	38	115	0.0246641	9.60343E-12	29	89	0.020669	5.77316E-12	45	90	0.022585	5.4623E-12	14	85	0.014155	1.22125E-12	53	319	0.049648	6.53921E-12
	x_0^6	38	115	0.0266514	5.9952E-12	28	86	0.020021	8.63754E-12	45	90	0.020994	5.9952E-12	16	93	0.015531	1.89848E-12	54	321	0.052695	6.30607E-12
	x_0^7	39	118	0.0273053	5.87308E-12	30	92	0.021269	4.10783E-12	45	90	0.02354	5.41789E-12	15	93	0.018265	1.19904E-12	50	302	0.049003	7.68274E-12
	x_0^8	35	107	0.023248	7.33857E-12	24	75	0.01539	5.50671E-12	44	88	0.020752	8.77076E-12	16	94	0.015277	1.88738E-12	50	302	0.047116	7.68274E-12
	x_0^9	39	118	0.0286817	5.88418E-12	30	92	0.019071	4.08562E-12	45	90	0.023104	5.41789E-12	15	93	0.015019	1.22125E-12	50	302	0.049249	7.66054E-12
	x_0^{10}	74	247	0.0511511	6.42282E-12	65	213	0.042663	8.21086E-12	39	78	0.017305	7.08487E-12	59	364	0.055604	8.76246E-12	62	434	0.066504	5.94514E-12
	x_0^{11}	82	273	0.0596296	7.18616E-12	78	257	0.056479	6.75788E-12	44	88	0.02295	9.32165E-12	67	413	0.066103	9.59801E-12	66	465	0.07362	9.07394E-13
	x_0^{12}	28	85	0.019958	9.00391E-12	20	62	0.013966	9.71445E-12	33	66	0.015103	5.79536E-12	10	61	0.009472	8.9706E-12	36	217	0.033591	8.43769E-12
	x_0^{13}	68	227	0.0456223	8.76408E-12	59	194	0.035708	7.49301E-12	38	76	0.015767	5.12202E-12	56	345	0.049043	9.72235E-12	57	398	0.054016	8.9239E-13
	x_0^{14}	81	270	0.0573191	5.83083E-12	76	251	0.051821	8.60197E-12	44	88	0.022405	9.32275E-12	69	426	0.068037	3.69813E-12	64	451	0.069359	6.12528E-12
100000	x_0^1	1	1	0.0030981	0	1	2	0.005014	0	1	1	0.001019	0	1	1	0.001722	0	1	1	0.001791	0
	x_0^2	39	118	0.3251134	4.98538E-12	29	89	0.229663	6.7408E-12	45	90	0.23084	7.72383E-12	14	85	0.15432	2.73845E-12	52	313	0.658662	8.14513E-12
	x_0^3	39	118	0.327472	7.65362E-12	30	92	0.22739	5.12582E-12	46	92	0.289499	6.7408E-12	14	85	0.180685	3.58105E-12	54	325	0.687679	6.67058E-12
	x_0^4	39	115	0.3157349	7.79405E-12	30	89	0.218265	5.4769E-12	46	92	0.268097	7.79405E-12	16	92	0.201679	6.00353E-12	56	332	0.665571	6.03863E-12
	x_0^5	39	118	0.3805489	8.49622E-12	30	92	0.223402	6.35461E-12	46	92	0.293029	8.70687E-12	14	85	0.201566	3.86192E-12	55	331	0.665278	6.28439E-12
	x_0^6	39	118	0.3573974	5.61733E-12	29	89	0.23067	9.61968E-12	46	92	0.262586	9.47925E-12	14	85	0.161134	3.51083E-12	56	333	0.652302	6.24928E-12
	x_0^7	40	121	0.3398627	5.8982E-12	31	95	0.264461	4.52898E-12	46	92	0.248903	8.49622E-12	15	93	0.19585	3.86192E-12	52	314	0.587278	7.65362E-12
	x_0^8	35	107	0.3062105	5.61733E-12	25	78	0.198977	6.03863E-12	46	92	0.292384	6.95145E-12	17	99	0.227656	5.96842E-12	52	314	0.63941	7.65362E-12
	x_0^9	40	121	0.3222726	5.8982E-12	31	95	0.241455	4.52898E-12	46	92	0.277173	8.49622E-12	15	93	0.19262	3.86192E-12	52	314	0.589598	7.65362E-12
	x_0^{10}	74	247	0.6782202	6.64307E-12	64	212	0.495407	8.54496E-12	39	78	0.229114	7.08502E-12	60	370	0.675337	8.17058E-12	68	476	0.889066	9.11549E-12
	x_0^{11}	87	291	0.8510842	8.77271E-12	80	263	0.694551	9.70392E-12	46	92	0.274915	7.36989E-12	72	444	0.872557	8.96535E-12	68	479	0.967049	9.61665E-12
	x_0^{12}	27	82	0.2486824	6.03863E-12	19	59	0.154426	8.77708E-12	31	62	0.187253	7.37275E-12	10	61	0.108134	2.87888E-12	34	205	0.396669	8.426E-12
	x_0^{13}	68	227	0.5663099	8.76408E-12	59	194	0.422231	7.49301E-12	38	76	0.226405	5.12202E-12	56	345	0.594638	9.72235E-12	57	398	0.629989	8.92413E-13
	x_0^{14}	87	291	0.7988753	8.81471E-12	81	266	0.721837	8.01129E-12	46	92	0.274111	7.37053E-12	72	444	0.82085	8.17398E-12	69	486	1.008487	8.85652E-12

Table 3: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in Modified-NonsmoothSine*

ModifiedNonsmoothSine																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
DIM	INP	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
1000	x_0^1	15	45	0.1203594	9.2721E-12	13	39	0.029303	1.32183E-12	139	278	0.024901	9.73554E-12	11	45	0.019184	5.36631E-12	56	225	0.024049	8.27855E-12
	x_0^2	15	45	0.0006526	6.7636E-12	13	39	0.000529	7.84671E-13	137	274	0.004285	8.34525E-12	11	45	0.00047	2.48216E-12	56	225	0.002168	8.0872E-12
	x_0^3	15	44	0.000831	8.5401E-12	12	36	0.000571	3.45466E-12	131	262	0.011083	8.59452E-12	11	45	0.00046	6.1264E-13	54	217	0.002087	8.60856E-12
	x_0^4	14	41	0.0007122	8.0012E-12	11	33	0.000421	5.97368E-12	121	242	0.003773	8.28381E-12	10	41	0.0004	1.20246E-12	50	201	0.001918	7.86778E-12
	x_0^5	15	45	0.0007091	3.2721E-12	12	36	0.000465	3.84787E-12	132	264	0.004328	8.43653E-12	11	45	0.000447	6.19662E-13	55	221	0.002164	7.98715E-12
	x_0^6	15	45	0.0006293	4.6747E-12	12	36	0.000446	7.55531E-12	137	274	0.004433	8.16093E-12	11	45	0.000454	1.44822E-12	57	229	0.00256	9.64426E-12
	x_0^7	15	44	0.0132452	8.3382E-12	12	36	0.000486	1.90989E-12	139	278	0.00494	8.66825E-12	11	45	0.000557	2.00118E-12	59	237	0.002439	6.47573E-12
	x_0^8	16	47	0.0009768	6.1211E-12	13	39	0.000495	2.80165E-12	140	280	0.012389	8.66649E-12	10	40	0.000398	3.61265E-12	59	237	0.015125	6.47573E-12
	x_0^9	15	44	0.0009005	8.3979E-12	12	36	0.000446	1.98011E-12	139	278	0.004324	8.64718E-12	11	45	0.000467	1.99942E-12	59	237	0.002726	6.46344E-12
	x_0^{10}	54	160	0.0033614	9.0242E-12	64	191	0.002502	7.22603E-12	139	278	0.004395	9.60664E-12	60	263	0.002524	5.43798E-12	56	289	0.002618	9.04621E-12
	x_0^{11}	58	174	0.0025579	8.6292E-12	66	199	0.002653	8.04987E-12	136	272	0.0049	9.80226E-12	64	281	0.009833	5.95588E-12	60	311	0.002858	7.18995E-12
	x_0^{12}	15	45	0.0006656	9.2739E-12	13	39	0.000519	1.31656E-12	139	278	0.004678	9.71097E-12	11	45	0.000586	5.34875E-12	56	225	0.002851	8.29083E-12
	x_0^{13}	48	143	0.0021712	5.7127E-12	55	165	0.002186	6.94564E-12	139	278	0.012794	9.72788E-12	56	244	0.00273	2.81979E-12	55	282	0.00422	4.6777E-12
	x_0^{14}	58	174	0.0031379	6.9586E-12	66	199	0.003156	8.05033E-12	136	272	0.00424	9.79869E-12	59	260	0.002602	7.8906E-12	61	313	0.004549	3.75265E-12
10000	x_0^1	16	47	0.0065552	6.9889E-12	13	39	0.004552	4.17999E-12	145	290	0.042612	8.86513E-12	12	49	0.006527	9.49241E-13	59	237	0.026072	6.28386E-12
	x_0^2	16	47	0.0072233	5.1625E-12	13	39	0.005421	2.4869E-12	142	284	0.043324	9.36473E-12	11	45	0.004501	7.84928E-12	58	233	0.022728	9.88098E-12
	x_0^3	15	45	0.0088815	6.2117E-12	13	39	0.006272	7.43849E-13	136	272	0.04056	9.63118E-12	11	45	0.004342	1.93734E-12	57	229	0.024529	6.53366E-12
	x_0^4	14	42	0.0060561	8.2656E-12	12	36	0.00436	1.29896E-12	126	252	0.039343	9.28146E-12	10	41	0.003896	3.80251E-12	52	209	0.020466	9.59233E-12
	x_0^5	15	45	0.0061423	6.4282E-12	13	39	0.005052	8.38218E-13	137	274	0.040156	9.45355E-12	11	45	0.004534	1.95954E-12	57	229	0.024248	9.75331E-12
	x_0^6	15	45	0.0060312	9.9309E-12	13	39	0.004651	1.63203E-12	142	284	0.04471	9.15379E-12	11	45	0.004489	4.57967E-12	60	241	0.029466	7.32747E-12
	x_0^7	15	45	0.0061278	3.9302E-12	12	36	0.004323	6.03961E-12	144	288	0.042359	9.72E-12	11	45	0.00438	6.32827E-12	61	245	0.0243	7.92699E-12
	x_0^8	16	48	0.0087501	4.3576E-12	13	39	0.004422	8.85958E-12	145	290	0.044462	9.72E-12	11	44	0.004148	6.38378E-13	61	245	0.025326	7.92699E-12
	x_0^9	15	45	0.0062217	3.9191E-12	12	36	0.007337	6.05627E-12	144	288	0.042386	9.72E-12	11	45	0.006842	6.32827E-12	61	245	0.023699	7.92699E-12
	x_0^{10}	49	146	0.0215192	7.5945E-12	58	174	0.023544	6.58468E-12	145	290	0.053541	8.85985E-12	64	279	0.025776	6.33963E-12	59	304	0.02835	8.60121E-12
	x_0^{11}	59	177	0.024943	8.8053E-12	69	208	0.028393	7.34232E-12	142	284	0.045597	8.93433E-12	65	279	0.027755	3.04317E-12	65	334	0.031309	6.79745E-12
	x_0^{12}	16	47	0.0063576	6.9889E-12	13	39	0.004697	4.17999E-12	145	290	0.043595	8.86513E-12	12	49	0.004776	9.49241E-13	59	237	0.022436	6.28386E-12
	x_0^{13}	47	140	0.0206946	6.6147E-12	51	153	0.021162	8.74076E-12	145	290	0.044548	8.86444E-12	58	255	0.025679	2.52358E-12	52	265	0.024936	8.56123E-12
	x_0^{14}	59	177	0.024815	8.8081E-12	69	208	0.029564	7.34237E-12	142	284	0.041582	8.93406E-12	68	301	0.029704	7.85136E-12	63	324	0.030511	8.33633E-12
100000	x_0^1	16	48	0.0835893	5.6173E-12	14	42	0.076101	8.95263E-13	150	300	0.573819	9.95321E-12	12	49	0.063984	3.00176E-12	61	245	0.333962	7.67117E-12
	x_0^2	16	48	0.0899206	3.7566E-12	13	39	0.0672	7.86427E-12	148	296	0.563203	8.51377E-12	12	49	0.070337	1.40433E-12	61	245	0.359247	7.46052E-12
	x_0^3	16	47	0.0930652	5.1434E-12	13	39	0.076442	2.35226E-12	142	284	0.518637	8.77708E-12	11	45	0.060955	6.1264E-12	59	237	0.294052	7.98715E-12
	x_0^4	15	45	0.1023716	3.2475E-12	12	36	0.051348	4.10768E-12	132	264	0.466658	8.47866E-12	11	45	0.051678	7.02167E-13	55	221	0.273727	7.30253E-12
	x_0^5	16	47	0.0925345	5.4945E-12	13	39	0.057406	2.65068E-12	143	286	0.52992	8.60154E-12	11	45	0.057372	6.19662E-12	60	241	0.268089	7.3903E-12
	x_0^6	16	47	0.0856057	8.6191E-12	13	39	0.052317	5.16093E-12	148	296	0.544735	8.32068E-12	12	49	0.052891	8.07492E-13	62	249	0.317344	8.97018E-12
	x_0^7	15	45	0.0958666	7.7765E-12	13	39	0.052824	1.28145E-12	150	300	0.574259	8.86485E-12	12	49	0.064863	1.10591E-12	63	253	0.325522	9.6899E-12
	x_0^8	16	48	0.0901032	8.584E-12	14	42	0.064583	1.93096E-12	151	302	0.546964	8.86485E-12	11	44	0.067263	2.01873E-12	63	253	0.33593	9.6899E-12
	x_0^9	15	45	0.0868062	7.7765E-12	13	39	0.064084	1.28145E-12	150	300	0.540244	8.86485E-12	12	49	0.065994	1.10591E-12	63	253	0.332096	9.6899E-12
	x_0^{10}	48	143	0.2529833	7.9036E-12	53	159	0.280816	9.48736E-12	150	300	0.575352	9.94308E-12	58	249	0.286063	8.01934E-12	68	346	0.387014	5.56148E-12
	x_0^{11}	62	186	0.4039515	7.7176E-12	71	214	0.338685	9.98439E-12	148	296	0.577097	8.14528E-12	68	295	0.36291	7.80284E-12	72	368	0.425735	4.76195E-12
	x_0^{12}	16	48	0.0908582	5.6173E-12	14	42	0.072277	8.95263E-13	150	300	0.521524	9.95321E-12	12	49	0.067651	3.00176E-12	61	245	0.304363	7.67117E-12
	x_0^{13}	43	128	0.2170258	9.6192E-12	46	138	0.237127	9.70344E-12	150	300	0.532588	9.95313E-12	62	274	0.32701	6.88801E-12	55	281	0.311124	6.57513E-12
	x_0^{14}	62	186	0.3343659	7.7175E-12	71	214	0.345402	9.98449E-12	148	296	0.537041	8.14525E-12	69	301	0.365591	4.61915E-12	63	326	0.390964	5.0452E-12

Table 4: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in ModifiedTridiagonal*

ModifiedTridiagonal																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
DIM	INP	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
1000	x_0^1	3	8	0.2628378	85.99124179	2	7	0.028655	85.99124	2000	4001	0.191719	85.99414	2000	4001	0.179005	85.99414	2000	4001	0.175256	85.99414
	x_0^2	4	30	0.0008258	85.95961902	2	22	0.000494	85.99124	2000	4001	0.162132	85.99414	2000	4001	0.151223	85.99414	2000	4001	0.146286	85.99414
	x_0^3	4	33	0.0008518	85.95961902	2	22	0.000461	85.99124	2000	4001	0.162615	85.99414	2000	4001	0.154251	85.99414	2000	4001	0.14623	85.99414
	x_0^4	4	33	0.0007656	85.95961902	2	22	0.00047	85.99124	2000	4001	0.160786	85.99414	2000	4001	0.146883	85.99414	2000	4001	0.146983	85.99414
	x_0^5	4	33	0.0007906	85.95961902	2	22	0.000463	85.99124	2000	4001	0.162346	85.99414	2000	4001	0.153301	85.99414	2000	4001	0.148579	85.99414
	x_0^6	4	34	0.0008211	85.95961902	2	22	0.000706	85.99124	2000	4001	0.163347	85.99414	2000	4001	0.151897	85.99414	2000	4001	0.152169	85.99414
	x_0^7	4	34	0.0007953	85.95961902	2	22	0.007576	85.99124	2000	4001	0.163968	85.99414	2000	4001	0.152028	85.99414	2000	4001	0.153181	85.99414
	x_0^8	4	34	0.0007798	85.95961902	2	22	0.000512	85.99124	2000	4001	0.172441	85.99414	2000	4001	0.148032	85.99414	2000	4001	0.146643	85.99414
	x_0^9	4	34	0.0007867	85.95961902	2	22	0.000509	85.99124	2000	4001	0.192889	85.99414	2000	4001	0.148441	85.99414	2000	4001	0.145692	85.99414
	x_0^{10}	6	12	0.0006087	85.99630758	3	7	0.000402	85.99588	2000	4001	0.160813	85.99705	2000	4001	0.156855	85.99705	2000	4001	0.147913	85.99705
	x_0^{11}	6	12	0.0005362	85.99016099	4	9	0.000391	85.99173	2000	4001	0.158912	85.99705	2000	4001	0.155457	85.99705	2000	4001	0.146859	85.99705
	x_0^{12}	3	6	0.0002708	85.99124179	2	5	0.000169	85.99124	2000	4001	0.159439	85.99414	2000	4001	0.150004	85.99414	2000	4001	0.150269	85.99414
	x_0^{13}	6	12	0.0005077	85.99308704	3	7	0.000252	85.99274	2000	4001	0.159936	85.99705	2000	4001	0.155741	85.99705	2000	4001	0.147987	85.99705
	x_0^{14}	6	12	0.000559	85.99016175	4	9	0.000389	85.99173	2000	4001	0.16101	85.99705	2000	4001	0.151982	85.99705	2000	4001	0.150328	85.99705
10000	x_0^1	3	6	0.0022385	271.8381828	2	5	0.003738	271.8382	2000	4001	1.431557	271.8391	2000	4001	1.337901	271.8391	2000	4001	1.310182	271.8391
	x_0^2	4	24	0.006008	271.8281829	2	20	0.004135	271.8382	2000	4001	1.414154	271.8391	2000	4001	1.331387	271.8391	2000	4001	1.31096	271.8391
	x_0^3	4	26	0.0063286	271.8281829	2	22	0.004598	271.8382	2000	4001	1.443506	271.8391	2000	4001	1.339813	271.8391	2000	4001	1.300116	271.8391
	x_0^4	4	26	0.0061802	271.8281829	2	22	0.004532	271.8382	2000	4001	1.487275	271.8391	2000	4001	1.320174	271.8391	2000	4001	1.30359	271.8391
	x_0^5	4	27	0.006252	271.8281829	2	22	0.004538	271.8382	2000	4001	1.434849	271.8391	2000	4001	1.331131	271.8391	2000	4001	1.304608	271.8391
	x_0^6	4	27	0.0089064	271.8281829	2	22	0.004572	271.8382	2000	4001	1.459571	271.8391	2000	4001	1.33991	271.8391	2000	4001	1.294982	271.8391
	x_0^7	4	28	0.0067243	271.8281829	2	22	0.004382	271.8382	2000	4001	1.423102	271.8391	2000	4001	1.326911	271.8391	2000	4001	1.294463	271.8391
	x_0^8	4	28	0.0064616	271.8281829	2	22	0.007187	271.8382	2000	4001	1.415867	271.8391	2000	4001	1.327646	271.8391	2000	4001	1.342441	271.8391
	x_0^9	4	28	0.0063786	271.8281829	2	22	0.00463	271.8382	2000	4001	1.400542	271.8391	2000	4001	1.330458	271.8391	2000	4001	1.321446	271.8391
	x_0^{10}	6	12	0.020417	271.8397851	3	7	0.018705	271.8396	2000	4001	1.380973	271.84	2000	4001	1.468442	271.84	2000	4001	1.298259	271.84
	x_0^{11}	5	10	0.0081547	271.8379981	4	9	0.008256	271.8382	2000	4001	1.386331	271.84	2000	4001	1.404948	271.84	2000	4001	1.317584	271.84
	x_0^{12}	2	4	0.0018355	271.8381828	2	5	0.001689	271.8382	2000	4001	1.399305	271.8391	2000	4001	1.336898	271.8391	2000	4001	1.293541	271.8391
	x_0^{13}	6	12	0.0046531	271.8387727	3	7	0.002616	271.8387	2000	4001	1.367346	271.84	2000	4001	1.323041	271.84	2000	4001	1.303643	271.84
	x_0^{14}	5	10	0.0085711	271.8379981	4	9	0.008363	271.8382	2000	4001	1.428773	271.84	2000	4001	1.431437	271.84	2000	4001	1.302043	271.84
100000	x_0^1	3	6	0.0232622	859.5993523	2	5	0.034624	859.5994	2000	4001	17.04426	859.5996	2000	6000	20.50294	859.5996	2000	4001	15.5493	859.5996
	x_0^2	4	20	0.0635916	859.5961902	2	17	0.045022	859.5994	2000	4001	17.12333	859.5996	2000	6000	20.10126	859.5996	2000	4001	15.62685	859.5996
	x_0^3	4	22	0.0693673	859.5961902	2	19	0.049183	859.5994	2000	4001	18.11489	859.5996	2000	6000	20.01319	859.5996	2000	4001	15.3791	859.5996
	x_0^4	4	22	0.0629879	859.5961902	2	19	0.044557	859.5994	2000	4001	17.48771	859.5996	2000	6001	19.62867	859.5996	2000	4001	15.59981	859.5996
	x_0^5	4	23	0.0656531	859.5961902	2	20	0.037111	859.5994	2000	4001	17.1192	859.5996	2000	6001	19.74258	859.5996	2000	4001	15.2473	859.5996
	x_0^6	4	23	0.0710454	859.5961902	2	20	0.038643	859.5994	2000	4001	17.16234	859.5996	2000	6001	19.63747	859.5996	2000	4001	15.33378	859.5996
	x_0^7	4	23	0.0589353	859.5961902	2	20	0.038446	859.5994	2000	4001	17.32825	859.5996	2000	6001	19.86019	859.5996	2000	4001	15.38637	859.5996
	x_0^8	4	24	0.0659006	859.5961902	2	21	0.048591	859.5994	2000	4001	17.61502	859.5996	2000	6001	19.82709	859.5996	2000	4001	15.28186	859.5996
	x_0^9	4	23	0.081446	859.5961902	2	20	0.033323	859.5994	2000	4001	17.3032	859.5996	2000	6001	20.09854	859.5996	2000	4001	15.5886	859.5996
	x_0^{10}	5	10	1.4988516	859.599859	3	7	1.599882	859.5998	2000	4001	18.43248	859.5999	2000	6000	32.68497	859.5999	2000	4001	15.40371	859.5999
	x_0^{11}	6	12	0.9633102	859.5993575	4	9	0.620164	859.5994	2000	4001	18.07283	859.5999	2000	6001	30.45305	859.5999	2000	4001	15.53294	859.5999
	x_0^{12}	1	2	0.0092521	859.5993523	1	3	0.015711	859.5994	2000	4001	17.94464	859.5996	2000	6000	20.02029	859.5996	2000	4001	15.53272	859.5996
	x_0^{13}	5	10	0.0461032	859.599539	3	7	0.030974	859.5995	2000	4001	18.06122	859.5999	2000	6000	19.9436	859.5999	2000	4001	15.52753	859.5999
	x_0^{14}	6	12	0.9840253	859.5993575	4	9	0.611344	859.5994	2000	4001	18.6032	859.5999	2000	6001	30.37647	859.5999	2000	4001	15.6199	859.5999

Table 5: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in NonmoothLogarithmic*

NonmoothLogarithmic																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
DIM	INP	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
1000	x_0^1	1	1	0.1482352	0	1	2	0.029647	0	1	1	0.020256	0	1	1	0.017709	0	1	1	0.020931	0
	x_0^2	2	4	0.0000867	0	1	3	5.43E-05	0	260	520	0.016635	9.1962E-12	2	5	0.000074	0	1	3	4.87E-05	0
	x_0^3	2	4	0.0000822	0	1	3	4.72E-05	0	267	534	0.009587	9.78543E-12	2	5	7.18E-05	0	1	3	4.48E-05	0
	x_0^4	2	4	0.0000857	0	1	3	5.27E-05	0	270	540	0.016606	9.39261E-12	2	5	7.24E-05	0	1	3	4.72E-05	0
	x_0^5	2	4	0.0000999	0	1	3	5.29E-05	0	272	544	0.009839	9.60305E-12	2	5	7.41E-05	0	1	3	4.78E-05	0
	x_0^6	2	4	0.0000877	0	1	3	5.13E-05	0	276	552	0.009992	9.28739E-12	2	5	0.000074	0	1	3	4.76E-05	0
	x_0^7	2	4	0.0000874	0	1	3	5.11E-05	0	279	558	0.016871	9.3365E-12	2	5	8.46E-05	0	1	3	5.83E-05	0
	x_0^8	2	4	0.0000874	0	1	3	5.05E-05	0	280	560	0.010016	9.69424E-12	2	5	7.75E-05	0	1	3	4.71E-05	0
	x_0^9	2	4	0.0000876	0	1	3	0.00005	0	279	558	0.014257	9.31545E-12	2	5	7.82E-05	0	1	3	0.000053	0
	x_0^{10}	3	6	0.0001161	0	2	5	9.28E-05	0	247	494	0.008921	9.78739E-12	14	29	0.00043	0	2	5	6.96E-05	0
	x_0^{11}	14	28	0.0005512	0	2	5	0.000085	0	273	546	0.013354	9.00527E-12	17	35	0.000561	0	3	7	0.000107	0
	x_0^{12}	2	4	0.0000797	0	1	3	4.46E-05	0	208	416	0.007644	9.8205E-12	2	5	6.76E-05	0	1	3	4.03E-05	0
	x_0^{13}	5	10	0.0001913	6.65333E-16	2	5	0.000082	5.43264E-16	233	466	0.0086	9.33893E-12	6	13	0.000181	3.13782E-16	2	5	6.93E-05	1.48536E-19
	x_0^{14}	20	40	0.0007987	1.72315E-21	2	5	0.000087	0	273	546	0.013978	9.02227E-12	17	35	0.000535	0	2	5	7.61E-05	0
10000	x_0^1	1	1	0.000254	0	1	2	0.000242	0	1	1	7.07E-05	0	1	1	0.000064	0	1	1	0.000078	0
	x_0^2	2	4	0.0006713	0	1	3	0.000437	0	270	540	0.094563	9.88E-12	2	5	0.000605	0	1	3	0.00038	0
	x_0^3	2	4	0.0007036	0	1	3	0.000435	0	278	556	0.096033	9.45815E-12	2	5	0.000608	0	1	3	0.000408	0
	x_0^4	2	4	0.000839	0	1	3	0.000438	0	281	562	0.09767	9.08072E-12	2	5	0.000709	0	1	3	0.000435	0
	x_0^5	2	4	0.0007343	0	1	3	0.000446	0	283	566	0.097102	9.28054E-12	2	5	0.000666	0	1	3	0.00053	0
	x_0^6	2	4	0.0007371	0	1	3	0.000436	0	286	572	0.099834	9.96881E-12	2	5	0.000653	0	1	3	0.000436	0
	x_0^7	2	4	0.0007151	0	1	3	0.000435	0	290	580	0.098878	9.01411E-12	2	5	0.000798	0	1	3	0.000446	0
	x_0^8	2	4	0.0007381	0	1	3	0.000435	0	291	582	0.100589	9.36935E-12	2	5	0.00081	0	1	3	0.000423	0
	x_0^9	2	4	0.000747	0	1	3	0.000435	0	290	580	0.099076	9.01411E-12	2	5	0.000911	0	1	3	0.000448	0
	x_0^{10}	3	6	0.0011294	0	2	5	0.000737	0	247	494	0.083104	9.55425E-12	14	29	0.004528	0	2	5	0.000609	0
	x_0^{11}	3	6	0.0011108	0	2	5	0.000792	0	283	566	0.098835	9.67617E-12	21	43	0.006559	0	3	7	0.001004	0
	x_0^{12}	2	4	0.0006867	0	1	3	0.00041	0	197	394	0.065466	9.70238E-12	2	5	0.000626	0	1	3	0.000352	0
	x_0^{13}	5	10	0.0017746	1.5221E-15	2	5	0.000863	6.6606E-16	233	466	0.08285	9.1306E-12	6	13	0.004234	4.96462E-16	2	5	0.000647	2.89109E-20
	x_0^{14}	3	6	0.0034175	0	2	5	0.00087	0	283	566	0.111239	9.67798E-12	21	43	0.006253	0	3	7	0.001181	0
100000	x_0^1	1	1	0.0022965	0	1	2	0.003424	0	1	1	0.001059	0	1	1	0.001028	0	1	1	0.000732	0
	x_0^2	2	4	0.0084595	0	1	3	0.006076	0	281	562	1.11023	9.76002E-12	2	5	0.009127	0	1	3	0.004348	0
	x_0^3	2	4	0.0151231	0	1	3	0.005871	0	289	578	1.173077	9.33872E-12	2	5	0.009816	0	1	3	0.004156	0
	x_0^4	2	4	0.0091327	0	1	3	0.00666	0	291	582	1.1882	9.97067E-12	2	5	0.010259	0	1	3	0.00455	0
	x_0^5	2	4	0.0089859	0	1	3	0.00656	0	294	588	1.181978	9.19829E-12	2	5	0.010237	0	1	3	0.004931	0
	x_0^6	2	4	0.0109662	0	1	3	0.005208	0	297	594	1.270673	9.90045E-12	2	5	0.012228	0	1	3	0.004764	0
	x_0^7	2	4	0.0110355	0	1	3	0.005414	0	300	600	1.289264	9.90045E-12	2	5	0.010737	0	1	3	0.004893	0
	x_0^8	2	4	0.0115037	0	1	3	0.008231	0	302	604	1.318275	9.26851E-12	2	5	0.010342	0	1	3	0.006609	0
	x_0^9	2	4	0.0104918	0	1	3	0.010363	0	300	600	1.282223	9.90045E-12	2	5	0.009159	0	1	3	0.006226	0
	x_0^{10}	3	6	0.0155093	0	2	5	0.019098	0	247	494	1.083777	9.53123E-12	14	29	0.048248	0	2	5	0.010289	0
	x_0^{11}	3	6	0.0160201	0	2	5	0.011026	0	294	588	1.282916	9.57676E-12	25	51	0.092165	0	3	7	0.013726	0
	x_0^{12}	2	4	0.0140149	0	1	3	0.006093	0	186	372	0.765498	9.76002E-12	2	5	0.006867	0	1	3	0.004212	0
	x_0^{13}	5	10	0.0256413	1.81749E-15	2	5	0.010217	1.04147E-15	233	466	1.019353	9.11003E-12	6	13	0.022399	9.67861E-16	2	5	0.006929	7.76105E-21
	x_0^{14}	3	6	0.0194406	0	2	5	0.011088	0	294	588	1.165276	9.57695E-12	25	51	0.099629	0	3	7	0.011476	0

Table 6: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in PolynomialI*

PolynomialI																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
DIM	INP	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
1000	x_0^1	1	1	0.6523618	0	1	2	0.028299	0	1	1	0.023851	0	1	1	0.022345	0	1	1	0.027635	0
	x_0^2	15	45	0.0315175	6.64373E-12	1	5	7.29E-05	0	80	160	0.003071	7.94192E-12	13	66	0.000836	3.08852E-12	1	6	8.94E-05	0
	x_0^3	15	46	0.001701	3.01466E-12	1	5	9.58E-05	0	82	164	0.002808	8.04228E-12	13	66	0.000922	7.00582E-12	1	6	7.38E-05	0
	x_0^4	15	46	0.0016877	3.1719E-12	1	5	7.18E-05	0	83	166	0.021606	7.15513E-12	13	66	0.000789	9.32786E-12	1	6	8.21E-05	0
	x_0^5	15	46	0.0016657	3.33293E-12	1	5	6.78E-05	0	83	166	0.002718	8.73189E-12	14	71	0.000705	1.10657E-12	1	6	7.68E-05	0
	x_0^6	15	46	0.001672	3.667E-12	1	5	6.76E-05	0	84	168	0.002754	8.43357E-12	12	60	0.00062	5.41978E-12	1	5	6.33E-05	0
	x_0^7	15	46	0.0024351	4.01806E-12	1	5	6.88E-05	0	85	170	0.004053	7.64241E-12	13	65	0.000667	1.6227E-12	1	5	6.45E-05	0
	x_0^8	15	46	0.0030289	4.20038E-12	1	5	6.92E-05	0	85	170	0.012694	8.55781E-12	13	65	0.000688	2.36699E-12	1	5	0.000063	0
	x_0^9	15	46	0.0027501	4.01626E-12	1	5	6.63E-05	0	85	170	0.003121	7.63341E-12	13	65	0.000631	1.61592E-12	1	5	6.37E-05	0
	x_0^{10}	22	67	0.0040337	4.72787E-12	36	109	0.001822	6.2004E-12	104	207	0.003851	8.65583E-12	65	325	0.020304	8.16996E-12	19	114	0.001187	3.65884E-13
	x_0^{11}	22	66	0.0037353	8.19828E-12	11	35	0.0006	1.5935E-12	100	199	0.003386	9.76623E-12	88	444	0.00449	9.16075E-12	45	270	0.002574	7.30539E-12
	x_0^{12}	12	37	0.0023773	4.50571E-12	1	5	8.29E-05	0	65	130	0.011375	8.10145E-12	11	56	0.000576	1.58276E-12	1	6	6.33E-05	0
	x_0^{13}	12	37	0.0022506	3.17242E-12	1	5	6.51E-05	6.0961E-16	96	191	0.003277	9.25591E-12	24	120	0.001388	8.12121E-12	1	5	6.18E-05	5.68259E-15
	x_0^{14}	22	66	0.0040189	8.91453E-12	11	35	0.000494	1.7158E-12	100	199	0.00333	9.77002E-12	96	481	0.018638	8.33667E-12	42	252	0.022178	7.47689E-12
10000	x_0^1	1	1	0.0002115	0	1	2	0.000205	0	1	1	7.57E-05	0	1	1	6.15E-05	0	1	1	7.92E-05	0
	x_0^2	15	45	0.0224403	9.51812E-12	1	5	0.00044	0	83	166	0.033175	8.61429E-12	13	66	0.011818	9.76677E-12	1	6	0.000492	0
	x_0^3	15	46	0.0092925	2.9975E-12	1	5	0.0005	0	85	170	0.028278	8.72315E-12	14	71	0.007426	2.05592E-12	1	6	0.000522	0
	x_0^4	15	46	0.019046	3.15414E-12	1	5	0.000485	0	86	172	0.029997	7.76089E-12	14	71	0.011532	2.73735E-12	1	6	0.000521	0
	x_0^5	15	46	0.0068451	3.31452E-12	1	5	0.005956	0	86	172	0.036956	9.47114E-12	14	71	0.007186	3.49927E-12	1	6	0.000513	0
	x_0^6	15	46	0.0172691	3.6471E-12	1	5	0.000523	0	87	174	0.031058	9.14757E-12	13	65	0.009836	1.59048E-12	1	5	0.000464	0
	x_0^7	15	46	0.0066395	3.99639E-12	1	5	0.000643	0	88	176	0.036526	8.28942E-12	13	65	0.006626	5.13144E-12	1	5	0.000459	0
	x_0^8	15	46	0.0099195	4.17771E-12	1	5	0.000556	0	88	176	0.031707	9.28232E-12	13	65	0.010385	7.48508E-12	1	5	0.000465	0
	x_0^9	15	46	0.0074135	3.99621E-12	1	5	0.000587	0	88	176	0.04304	8.28844E-12	13	65	0.006576	5.12929E-12	1	5	0.000453	0
	x_0^{10}	30	90	0.0161391	6.67132E-12	37	113	0.019003	1.0272E-12	104	207	0.048147	8.65446E-12	56	280	0.034232	6.64516E-12	19	114	0.016332	3.65048E-13
	x_0^{11}	20	61	0.0112921	4.13083E-12	3	11	0.001227	2.2702E-12	95	189	0.032281	8.57347E-12	91	459	0.052664	7.01935E-12	7	42	0.003753	0
	x_0^{12}	11	34	0.0057278	7.54656E-12	1	5	0.000485	0	62	124	0.022661	7.46804E-12	10	51	0.007123	5.39042E-12	1	6	0.000601	0
	x_0^{13}	12	37	0.0078815	3.17242E-12	1	5	0.000488	6.8771E-16	96	191	0.033308	9.25591E-12	24	120	0.012747	8.12121E-12	1	5	0.000467	6.38766E-15
	x_0^{14}	20	61	0.0122676	2.24244E-12	3	11	0.001215	2.0581E-12	95	189	0.032961	8.57382E-12	85	430	0.049313	8.43725E-12	7	42	0.004476	0
100000	x_0^1	1	1	0.0152953	0	1	2	0.002402	0	1	1	0.000743	0	1	1	0.000579	0	1	1	0.000706	0
	x_0^2	15	45	0.4712151	9.50018E-12	1	5	0.011305	0	86	172	0.459103	9.34359E-12	14	71	0.151979	2.86615E-12	1	6	0.0293	0
	x_0^3	15	46	0.2473254	2.9921E-12	1	5	0.017641	0	88	176	0.397452	9.46167E-12	14	71	0.444167	6.5014E-12	1	6	0.006502	0
	x_0^4	15	46	0.0936662	3.14855E-12	1	5	0.022797	0	89	178	0.380899	8.41793E-12	14	71	0.271882	8.65626E-12	1	6	0.008255	0
	x_0^5	15	46	0.0818547	3.30873E-12	1	5	0.013838	0	90	180	0.380661	7.19108E-12	15	76	0.100644	1.02689E-12	1	6	0.006106	0
	x_0^6	15	46	0.0803644	3.64083E-12	1	5	0.005365	0	90	180	0.374266	9.92201E-12	13	65	0.087506	5.02955E-12	1	5	0.010369	0
	x_0^7	15	46	0.0778008	3.98956E-12	1	5	0.006131	0	91	182	0.409662	8.99121E-12	14	70	0.089764	1.50587E-12	1	5	0.006243	0
	x_0^8	15	46	0.1042527	4.17056E-12	1	5	0.010647	0	92	184	0.377948	7.04772E-12	14	70	0.095798	2.19657E-12	1	5	0.007787	0
	x_0^9	15	46	0.083453	3.98955E-12	1	5	0.02721	0	91	182	0.401391	8.99111E-12	14	70	0.086913	1.5058E-12	1	5	0.005793	0
	x_0^{10}	27	83	0.1778629	6.9407E-12	38	115	0.204938	5.6451E-12	104	207	0.460681	8.65432E-12	58	290	0.36239	2.27585E-12	19	114	0.141806	3.65008E-13
	x_0^{11}	21	64	0.1252838	2.26226E-12	3	11	0.019916	2.4584E-16	91	181	0.3893	9.44375E-12	89	449	0.564995	6.03782E-12	7	42	0.053856	0
	x_0^{12}	11	33	0.0616702	8.42797E-12	1	5	0.009298	0	58	116	0.246128	9.83577E-12	10	51	0.065969	1.70451E-12	1	6	0.006355	0
	x_0^{13}	12	37	0.0732819	3.17242E-12	1	5	0.015089	3.9408E-14	96	191	0.419383	9.25591E-12	24	120	0.150898	8.12121E-12	1	5	0.007164	3.67436E-13
	x_0^{14}	21	64	0.1122553	2.07452E-12	3	11	0.023422	2.2357E-16	91	181	0.411353	9.44386E-12	88	444	0.583763	9.10381E-12	7	42	0.051836	0

Table 7: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in ModifiedTridiagonal*

ModifiedTridiagonal																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
DIM	INP	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
1000	x_0^1	3	8	0.2628378	85.99124179	2	7	0.028655	85.99124	2000	4001	0.191719	85.99414	2000	4001	0.179005	85.99414	2000	4001	0.175256	85.99414
	x_0^2	4	30	0.0008258	85.95961902	2	22	0.000494	85.99124	2000	4001	0.162132	85.99414	2000	4001	0.151223	85.99414	2000	4001	0.146286	85.99414
	x_0^3	4	33	0.0008518	85.95961902	2	22	0.000461	85.99124	2000	4001	0.162615	85.99414	2000	4001	0.154251	85.99414	2000	4001	0.14623	85.99414
	x_0^4	4	33	0.0007656	85.95961902	2	22	0.00047	85.99124	2000	4001	0.160786	85.99414	2000	4001	0.146883	85.99414	2000	4001	0.146983	85.99414
	x_0^5	4	33	0.0007906	85.95961902	2	22	0.000463	85.99124	2000	4001	0.162346	85.99414	2000	4001	0.153301	85.99414	2000	4001	0.148579	85.99414
	x_0^6	4	34	0.0008211	85.95961902	2	22	0.000706	85.99124	2000	4001	0.163347	85.99414	2000	4001	0.151897	85.99414	2000	4001	0.152169	85.99414
	x_0^7	4	34	0.0007953	85.95961902	2	22	0.007576	85.99124	2000	4001	0.163968	85.99414	2000	4001	0.152028	85.99414	2000	4001	0.153181	85.99414
	x_0^8	4	34	0.0007798	85.95961902	2	22	0.000512	85.99124	2000	4001	0.172441	85.99414	2000	4001	0.148032	85.99414	2000	4001	0.146643	85.99414
	x_0^9	4	34	0.0007867	85.95961902	2	22	0.000509	85.99124	2000	4001	0.192889	85.99414	2000	4001	0.148441	85.99414	2000	4001	0.145692	85.99414
	x_0^{10}	6	12	0.0006087	85.99630758	3	7	0.000402	85.99588	2000	4001	0.160813	85.99705	2000	4001	0.156855	85.99705	2000	4001	0.147913	85.99705
	x_0^{11}	6	12	0.0005362	85.99016099	4	9	0.000391	85.99173	2000	4001	0.158912	85.99705	2000	4001	0.155457	85.99705	2000	4001	0.146859	85.99705
	x_0^{12}	3	6	0.0002708	85.99124179	2	5	0.000169	85.99124	2000	4001	0.159439	85.99414	2000	4001	0.150004	85.99414	2000	4001	0.150269	85.99414
	x_0^{13}	6	12	0.0005077	85.99308704	3	7	0.000252	85.99274	2000	4001	0.159936	85.99705	2000	4001	0.155741	85.99705	2000	4001	0.147987	85.99705
	x_0^{14}	6	12	0.000559	85.99016175	4	9	0.000389	85.99173	2000	4001	0.16101	85.99705	2000	4001	0.151982	85.99705	2000	4001	0.150328	85.99705
10000	x_0^1	3	6	0.0022385	271.8381828	2	5	0.003738	271.8382	2000	4001	1.431557	271.8391	2000	4001	1.337901	271.8391	2000	4001	1.310182	271.8391
	x_0^2	4	24	0.006008	271.8281829	2	20	0.004135	271.8382	2000	4001	1.414154	271.8391	2000	4001	1.331387	271.8391	2000	4001	1.31096	271.8391
	x_0^3	4	26	0.0063286	271.8281829	2	22	0.004598	271.8382	2000	4001	1.443506	271.8391	2000	4001	1.339813	271.8391	2000	4001	1.300116	271.8391
	x_0^4	4	26	0.0061802	271.8281829	2	22	0.004532	271.8382	2000	4001	1.487275	271.8391	2000	4001	1.320174	271.8391	2000	4001	1.30359	271.8391
	x_0^5	4	27	0.006252	271.8281829	2	22	0.004538	271.8382	2000	4001	1.434849	271.8391	2000	4001	1.331131	271.8391	2000	4001	1.304608	271.8391
	x_0^6	4	27	0.0089064	271.8281829	2	22	0.004572	271.8382	2000	4001	1.459571	271.8391	2000	4001	1.33991	271.8391	2000	4001	1.294982	271.8391
	x_0^7	4	28	0.0067243	271.8281829	2	22	0.004382	271.8382	2000	4001	1.423102	271.8391	2000	4001	1.326911	271.8391	2000	4001	1.294463	271.8391
	x_0^8	4	28	0.0064616	271.8281829	2	22	0.007187	271.8382	2000	4001	1.415867	271.8391	2000	4001	1.327646	271.8391	2000	4001	1.342441	271.8391
	x_0^9	4	28	0.0063786	271.8281829	2	22	0.00463	271.8382	2000	4001	1.400542	271.8391	2000	4001	1.330458	271.8391	2000	4001	1.321446	271.8391
	x_0^{10}	6	12	0.020417	271.8397851	3	7	0.018705	271.8396	2000	4001	1.380973	271.84	2000	4001	1.468442	271.84	2000	4001	1.298259	271.84
	x_0^{11}	5	10	0.0081547	271.8379981	4	9	0.008256	271.8382	2000	4001	1.386331	271.84	2000	4001	1.404948	271.84	2000	4001	1.317584	271.84
	x_0^{12}	2	4	0.0018355	271.8381828	2	5	0.001689	271.8382	2000	4001	1.399305	271.8391	2000	4001	1.336898	271.8391	2000	4001	1.293541	271.8391
	x_0^{13}	6	12	0.0046531	271.8387727	3	7	0.002616	271.8387	2000	4001	1.367346	271.84	2000	4001	1.323041	271.84	2000	4001	1.303643	271.84
	x_0^{14}	5	10	0.0085711	271.8379981	4	9	0.008363	271.8382	2000	4001	1.428773	271.84	2000	4001	1.431437	271.84	2000	4001	1.302043	271.84
100000	x_0^1	3	6	0.0232622	859.5993523	2	5	0.034624	859.5994	2000	4001	17.04426	859.5996	2000	6000	20.50294	859.5996	2000	4001	15.5493	859.5996
	x_0^2	4	20	0.0635916	859.5961902	2	17	0.045022	859.5994	2000	4001	17.12333	859.5996	2000	6000	20.10126	859.5996	2000	4001	15.62685	859.5996
	x_0^3	4	22	0.0693673	859.5961902	2	19	0.049183	859.5994	2000	4001	18.11489	859.5996	2000	6000	20.01319	859.5996	2000	4001	15.3791	859.5996
	x_0^4	4	22	0.0629879	859.5961902	2	19	0.044557	859.5994	2000	4001	17.48771	859.5996	2000	6001	19.62867	859.5996	2000	4001	15.59981	859.5996
	x_0^5	4	23	0.0656531	859.5961902	2	20	0.037111	859.5994	2000	4001	17.1192	859.5996	2000	6001	19.74258	859.5996	2000	4001	15.2473	859.5996
	x_0^6	4	23	0.0710454	859.5961902	2	20	0.038643	859.5994	2000	4001	17.16234	859.5996	2000	6001	19.63747	859.5996	2000	4001	15.33378	859.5996
	x_0^7	4	23	0.0589353	859.5961902	2	20	0.038446	859.5994	2000	4001	17.32825	859.5996	2000	6001	19.86019	859.5996	2000	4001	15.38637	859.5996
	x_0^8	4	24	0.0659006	859.5961902	2	21	0.048591	859.5994	2000	4001	17.61502	859.5996	2000	6001	19.82709	859.5996	2000	4001	15.28186	859.5996
	x_0^9	4	23	0.081446	859.5961902	2	20	0.033323	859.5994	2000	4001	17.3032	859.5996	2000	6001	20.09854	859.5996	2000	4001	15.5886	859.5996
	x_0^{10}	5	10	1.4988516	859.599859	3	7	1.599882	859.5998	2000	4001	18.43248	859.5999	2000	6000	32.68497	859.5999	2000	4001	15.40371	859.5999
	x_0^{11}	6	12	0.9633102	859.5993575	4	9	0.620164	859.5994	2000	4001	18.07283	859.5999	2000	6001	30.45305	859.5999	2000	4001	15.53294	859.5999
	x_0^{12}	1	2	0.0092521	859.5993523	1	3	0.015711	859.5994	2000	4001	17.94464	859.5996	2000	6000	20.02029	859.5996	2000	4001	15.53272	859.5996
	x_0^{13}	5	10	0.0461032	859.599539	3	7	0.030974	859.5995	2000	4001	18.06122	859.5999	2000	6000	19.9436	859.5999	2000	4001	15.52753	859.5999
	x_0^{14}	6	12	0.9840253	859.5993575	4	9	0.611344	859.5994	2000	4001	18.6032	859.5999	2000	6001	30.37647	859.5999	2000	4001	15.6199	859.5999

Table 8: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in ModifiedTrigI*

ModifiedTrigI																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
DIM	INP	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
1000	x_0^1	15	44	0.2641793	8.33121E-12	12	36	0.029066	1.90287E-12	139	278	0.03368	8.66123E-12	11	45	0.020204	2.00118E-12	59	237	0.025187	6.47398E-12
	x_0^2	15	45	0.0010025	4.67643E-12	12	36	0.000711	7.55531E-12	137	274	0.006513	8.15918E-12	11	45	0.000758	1.44997E-12	57	229	0.003805	9.64075E-12
	x_0^3	15	45	0.0009522	3.2721E-12	12	36	0.000724	3.8549E-12	132	264	0.014311	8.43302E-12	11	45	0.000757	6.21418E-13	55	221	0.004128	7.99768E-12
	x_0^4	14	41	0.0008781	8.0047E-12	11	33	0.000695	5.97895E-12	121	242	0.005812	8.28557E-12	10	41	0.000693	1.20773E-12	50	201	0.010492	7.87129E-12
	x_0^5	15	44	0.0009454	8.53835E-12	12	36	0.000869	3.45466E-12	131	262	0.006548	8.60154E-12	11	45	0.000859	6.17907E-13	54	217	0.00384	8.61559E-12
	x_0^6	15	45	0.0009628	6.76187E-12	13	39	0.000855	7.86427E-13	137	274	0.01359	8.34876E-12	11	45	0.000715	2.48567E-12	56	225	0.004056	8.08896E-12
	x_0^7	15	45	0.0010266	9.28967E-12	13	39	0.000817	1.32007E-12	139	278	0.007531	9.73905E-12	11	45	0.000746	5.37158E-12	56	225	0.003642	8.2645E-12
	x_0^8	15	45	0.0010342	9.95672E-12	13	39	0.000805	1.5869E-12	140	280	0.010603	9.7531E-12	11	45	0.000811	7.36573E-12	56	225	0.011173	6.39674E-12
	x_0^9	15	45	0.0010306	9.27913E-12	13	39	0.00075	1.32007E-12	139	278	0.007007	9.71097E-12	11	45	0.009237	5.35051E-12	56	225	0.003814	8.27855E-12
	x_0^{10}	61	187	0.0045666	9.1182E-12	62	189	0.01145	7.51869E-12	139	278	0.006951	8.54851E-12	39	227	0.00373	9.79427E-12	23	152	0.002376	4.04517E-12
	x_0^{11}	69	214	0.0161446	9.10182E-12	72	220	0.005117	9.57893E-12	136	272	0.010684	9.78284E-12	45	249	0.00392	4.64113E-12	27	176	0.002784	5.16739E-12
	x_0^{12}	15	44	0.0009592	8.39791E-12	12	36	0.000933	1.97309E-12	139	278	0.00733	8.64718E-12	11	45	0.000795	2.00118E-12	59	237	0.004242	6.45993E-12
	x_0^{13}	62	191	0.0043636	7.17319E-12	63	191	0.004129	9.05541E-12	139	278	0.010642	8.58951E-12	38	221	0.003614	5.84461E-12	25	163	0.00255	4.18735E-12
	x_0^{14}	59	179	0.0040571	9.0575E-12	67	204	0.004251	8.81171E-12	136	272	0.007118	9.78441E-12	45	249	0.004535	4.39087E-12	27	176	0.010041	5.23728E-12
10000	x_0^1	15	45	0.0112586	3.90799E-12	12	36	0.006449	6.01741E-12	144	288	0.072378	9.70335E-12	11	45	0.007018	6.32827E-12	61	245	0.041936	7.90479E-12
	x_0^2	15	45	0.0086622	9.9476E-12	13	39	0.008196	1.63203E-12	142	284	0.067588	9.15934E-12	11	45	0.009152	4.58522E-12	60	241	0.04825	7.30527E-12
	x_0^3	15	45	0.0122628	6.43929E-12	13	39	0.009387	8.21565E-13	137	274	0.063333	9.448E-12	11	45	0.009743	1.96509E-12	57	229	0.039798	9.76996E-12
	x_0^4	14	42	0.011666	8.23785E-12	12	36	0.0072	1.28786E-12	126	252	0.065977	9.28146E-12	10	41	0.007521	3.81917E-12	52	209	0.034158	9.61453E-12
	x_0^5	15	45	0.0093925	6.22835E-12	13	39	0.009614	7.32747E-13	136	272	0.076971	9.63674E-12	11	45	0.010638	1.95399E-12	57	229	0.039563	6.53921E-12
	x_0^6	16	47	0.0121963	5.15143E-12	13	39	0.008408	2.4869E-12	142	284	0.0732	9.37028E-12	11	45	0.007498	7.86038E-12	58	233	0.040333	9.88098E-12
	x_0^7	16	47	0.0106598	6.99441E-12	13	39	0.008045	4.17444E-12	145	290	0.067492	8.88178E-12	12	49	0.009292	9.76996E-13	59	237	0.047693	6.23945E-12
	x_0^8	16	47	0.0114401	7.87148E-12	13	39	0.01063	5.01821E-12	146	292	0.075493	8.88178E-12	12	49	0.010954	1.33227E-12	58	233	0.054505	7.81597E-12
	x_0^9	16	47	0.009849	6.99441E-12	13	39	0.00785	4.17444E-12	145	290	0.066842	8.88178E-12	12	49	0.008408	9.76996E-13	59	237	0.054549	6.23945E-12
	x_0^{10}	63	193	0.0632282	4.61605E-12	69	209	0.046531	8.53559E-12	144	288	0.067199	9.70032E-12	39	228	0.03712	6.26064E-12	23	152	0.032781	6.52648E-12
	x_0^{11}	67	206	0.0522741	4.84985E-12	70	214	0.046625	6.45965E-12	142	284	0.064506	8.93234E-12	40	219	0.034607	7.0859E-12	27	175	0.028381	8.55582E-12
	x_0^{12}	15	45	0.011462	3.93019E-12	12	36	0.007016	6.08402E-12	144	288	0.068226	9.70335E-12	11	45	0.010064	6.32827E-12	61	245	0.042287	7.90479E-12
	x_0^{13}	67	208	0.0509592	6.01545E-12	65	196	0.047856	7.72322E-12	144	288	0.071318	9.71887E-12	40	233	0.036943	3.86368E-12	26	169	0.025975	8.0889E-12
	x_0^{14}	68	211	0.0528804	5.26251E-12	69	209	0.050783	5.13598E-12	142	284	0.065957	8.93267E-12	40	219	0.03573	7.85739E-12	27	175	0.026857	8.89696E-12
100000	x_0^1	15	45	0.1176978	7.72383E-12	13	39	0.096769	1.33412E-12	150	300	0.845763	8.8473E-12	12	49	0.111181	1.15858E-12	63	253	0.464658	9.61968E-12
	x_0^2	16	47	0.1235069	8.63665E-12	13	39	0.112473	5.16093E-12	148	296	0.860732	8.39089E-12	12	49	0.104336	8.426E-13	62	249	0.47769	8.91752E-12
	x_0^3	16	47	0.1304418	5.54712E-12	13	39	0.094769	2.59802E-12	143	286	0.829099	8.63665E-12	11	45	0.094372	6.21418E-12	60	241	0.474223	7.44297E-12
	x_0^4	15	45	0.1072809	3.30018E-12	12	36	0.083057	4.07257E-12	132	264	0.759466	8.46111E-12	11	45	0.079489	7.02167E-13	55	221	0.431471	7.30253E-12
	x_0^5	16	47	0.1402087	5.09071E-12	13	39	0.095114	2.31715E-12	142	284	0.855658	8.77708E-12	11	45	0.093873	6.17907E-12	59	237	0.461525	8.0047E-12
	x_0^6	16	48	0.1229711	3.65127E-12	13	39	0.087656	7.93448E-12	148	296	0.883141	8.49622E-12	12	49	0.103098	1.40433E-12	61	245	0.484237	7.51318E-12
	x_0^7	16	48	0.1240773	5.61733E-12	14	42	0.104072	9.12817E-13	150	300	0.859452	9.97077E-12	12	49	0.093702	3.08953E-12	61	245	0.490697	7.51318E-12
	x_0^8	16	48	0.1253594	6.24928E-12	14	42	0.102775	1.08836E-12	151	302	0.855702	9.97077E-12	12	49	0.095328	4.213E-12	60	241	0.483686	9.61968E-12
	x_0^9	16	48	0.1070273	5.54712E-12	14	42	0.098352	9.83033E-13	150	300	0.855931	9.97077E-12	12	49	0.093163	3.08953E-12	61	245	0.462936	7.51318E-12
	x_0^{10}	60	181	0.4635378	5.75489E-12	64	193	0.461315	6.94059E-12	150	300	0.883527	8.83395E-12	36	209	0.373887	5.02617E-12	28	180	0.321307	2.11448E-12
	x_0^{11}	69	216	0.5576898	7.34394E-12	68	205	0.480957	6.35048E-12	148	296	0.817777	8.14632E-12	40	215	0.388457	8.17414E-12	31	199	0.3515	3.68213E-12
	x_0^{12}	15	45	0.1138517	7.86427E-12	13	39	0.093392	1.2639E-12	150	300	0.86012	8.8473E-12	12	49	0.091321	1.15858E-12	63	253	0.508048	9.61968E-12
	x_0^{13}	72	222	0.5865181	8.09487E-12	61	186	0.474931	7.15273E-12	150	300	0.899731	8.84666E-12	40	231	0.419773	5.39097E-12	23	149	0.259051	7.36273E-12
	x_0^{14}	68	213	0.529195	9.91919E-12	69	208	0.510241	6.47765E-12	148	296	0.861445	8.14763E-12	40	215	0.374244	8.20563E-12	31	199	0.328947	3.68222E-12

Table 9: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in NonmoothLogarithmic*

NonmoothLogarithmic																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
DIM	INP	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
1000	x_0^1	1	1	0.1482352	0	1	2	0.029647	0	1	1	0.020256	0	1	1	0.017709	0	1	1	0.020931	0
	x_0^2	2	4	0.0000867	0	1	3	5.43E-05	0	260	520	0.016635	9.1962E-12	2	5	0.000074	0	1	3	4.87E-05	0
	x_0^3	2	4	0.0000822	0	1	3	4.72E-05	0	267	534	0.009587	9.78543E-12	2	5	7.18E-05	0	1	3	4.48E-05	0
	x_0^4	2	4	0.0000857	0	1	3	5.27E-05	0	270	540	0.016606	9.39261E-12	2	5	7.24E-05	0	1	3	4.72E-05	0
	x_0^5	2	4	0.0000999	0	1	3	5.29E-05	0	272	544	0.009839	9.60305E-12	2	5	7.41E-05	0	1	3	4.78E-05	0
	x_0^6	2	4	0.0000877	0	1	3	5.13E-05	0	276	552	0.009992	9.28739E-12	2	5	0.000074	0	1	3	4.76E-05	0
	x_0^7	2	4	0.0000874	0	1	3	5.11E-05	0	279	558	0.016871	9.3365E-12	2	5	8.46E-05	0	1	3	5.83E-05	0
	x_0^8	2	4	0.0000874	0	1	3	5.05E-05	0	280	560	0.010016	9.69424E-12	2	5	7.75E-05	0	1	3	4.71E-05	0
	x_0^9	2	4	0.0000876	0	1	3	0.00005	0	279	558	0.014257	9.31545E-12	2	5	7.82E-05	0	1	3	0.000053	0
	x_0^{10}	3	6	0.0001161	0	2	5	9.28E-05	0	247	494	0.008921	9.78739E-12	14	29	0.00043	0	2	5	6.96E-05	0
	x_0^{11}	14	28	0.0005512	0	2	5	0.000085	0	273	546	0.013354	9.00527E-12	17	35	0.000561	0	3	7	0.000107	0
	x_0^{12}	2	4	0.0000797	0	1	3	4.46E-05	0	208	416	0.007644	9.8205E-12	2	5	6.76E-05	0	1	3	4.03E-05	0
	x_0^{13}	5	10	0.0001913	6.65333E-16	2	5	0.000082	5.43264E-16	233	466	0.0086	9.33893E-12	6	13	0.000181	3.13782E-16	2	5	6.93E-05	1.48536E-19
	x_0^{14}	20	40	0.0007987	1.72315E-21	2	5	0.000087	0	273	546	0.013978	9.02227E-12	17	35	0.000535	0	2	5	7.61E-05	0
10000	x_0^1	1	1	0.000254	0	1	2	0.000242	0	1	1	7.07E-05	0	1	1	0.000064	0	1	1	0.000078	0
	x_0^2	2	4	0.0006713	0	1	3	0.000437	0	270	540	0.094563	9.88E-12	2	5	0.000605	0	1	3	0.00038	0
	x_0^3	2	4	0.0007036	0	1	3	0.000435	0	278	556	0.096033	9.45815E-12	2	5	0.000608	0	1	3	0.000408	0
	x_0^4	2	4	0.000839	0	1	3	0.000438	0	281	562	0.09767	9.08072E-12	2	5	0.000709	0	1	3	0.000435	0
	x_0^5	2	4	0.0007343	0	1	3	0.000446	0	283	566	0.097102	9.28054E-12	2	5	0.000666	0	1	3	0.00053	0
	x_0^6	2	4	0.0007371	0	1	3	0.000436	0	286	572	0.099834	9.96881E-12	2	5	0.000653	0	1	3	0.000436	0
	x_0^7	2	4	0.0007151	0	1	3	0.000435	0	290	580	0.098878	9.01411E-12	2	5	0.000798	0	1	3	0.000446	0
	x_0^8	2	4	0.0007381	0	1	3	0.000435	0	291	582	0.100589	9.36935E-12	2	5	0.00081	0	1	3	0.000423	0
	x_0^9	2	4	0.000747	0	1	3	0.000435	0	290	580	0.099076	9.01411E-12	2	5	0.000911	0	1	3	0.000448	0
	x_0^{10}	3	6	0.0011294	0	2	5	0.000737	0	247	494	0.083104	9.55425E-12	14	29	0.004528	0	2	5	0.000609	0
	x_0^{11}	3	6	0.0011108	0	2	5	0.000792	0	283	566	0.098835	9.67617E-12	21	43	0.006559	0	3	7	0.001004	0
	x_0^{12}	2	4	0.0006867	0	1	3	0.00041	0	197	394	0.065466	9.70238E-12	2	5	0.000626	0	1	3	0.000352	0
	x_0^{13}	5	10	0.0017746	1.5221E-15	2	5	0.000863	6.6606E-16	233	466	0.08285	9.1306E-12	6	13	0.004234	4.96462E-16	2	5	0.000647	2.89109E-20
	x_0^{14}	3	6	0.0034175	0	2	5	0.00087	0	283	566	0.111239	9.67798E-12	21	43	0.006253	0	3	7	0.001181	0
100000	x_0^1	1	1	0.0022965	0	1	2	0.003424	0	1	1	0.001059	0	1	1	0.001028	0	1	1	0.000732	0
	x_0^2	2	4	0.0084595	0	1	3	0.006076	0	281	562	1.11023	9.76002E-12	2	5	0.009127	0	1	3	0.004348	0
	x_0^3	2	4	0.0151231	0	1	3	0.005871	0	289	578	1.173077	9.33872E-12	2	5	0.009816	0	1	3	0.004156	0
	x_0^4	2	4	0.0091327	0	1	3	0.00666	0	291	582	1.1882	9.97067E-12	2	5	0.010259	0	1	3	0.00455	0
	x_0^5	2	4	0.0089859	0	1	3	0.00656	0	294	588	1.181978	9.19829E-12	2	5	0.010237	0	1	3	0.004931	0
	x_0^6	2	4	0.0109662	0	1	3	0.005208	0	297	594	1.270673	9.90045E-12	2	5	0.012228	0	1	3	0.004764	0
	x_0^7	2	4	0.0110355	0	1	3	0.005414	0	300	600	1.289264	9.90045E-12	2	5	0.010737	0	1	3	0.004893	0
	x_0^8	2	4	0.0115037	0	1	3	0.008231	0	302	604	1.318275	9.26851E-12	2	5	0.010342	0	1	3	0.006609	0
	x_0^9	2	4	0.0104918	0	1	3	0.010363	0	300	600	1.282223	9.90045E-12	2	5	0.009159	0	1	3	0.006226	0
	x_0^{10}	3	6	0.0155093	0	2	5	0.019098	0	247	494	1.083777	9.53123E-12	14	29	0.048248	0	2	5	0.010289	0
	x_0^{11}	3	6	0.0160201	0	2	5	0.011026	0	294	588	1.282916	9.57676E-12	25	51	0.092165	0	3	7	0.013726	0
	x_0^{12}	2	4	0.0140149	0	1	3	0.006093	0	186	372	0.765498	9.76002E-12	2	5	0.006867	0	1	3	0.004212	0
	x_0^{13}	5	10	0.0256413	1.81749E-15	2	5	0.010217	1.04147E-15	233	466	1.019353	9.11003E-12	6	13	0.022399	9.67861E-16	2	5	0.006929	7.76105E-21
	x_0^{14}	3	6	0.0194406	0	2	5	0.011088	0	294	588	1.165276	9.57695E-12	25	51	0.099629	0	3	7	0.011476	0

Table 10: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in NonsmoothSine*

NonsmoothSine																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
DIM	INP	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
1000	x_0^1	1	1	0.4076834	0	1	2	0.029544	0	1	1	0.020911	0	1	1	0.01788	0	1	1	0.021491	0
	x_0^2	13	39	0.0005668	4.22405E-12	8	24	0.015189	4.438E-13	258	516	0.014996	9.86079E-12	16	49	0.000488	2.87132E-12	1	4	5.38E-05	0
	x_0^3	14	41	0.0005302	5.93824E-12	8	24	0.000327	1.53518E-12	265	530	0.008036	9.32543E-12	16	49	0.000506	5.48346E-12	1	4	4.09E-05	0
	x_0^4	14	41	0.0005292	9.06068E-12	8	24	0.00033	2.17007E-12	267	534	0.015171	9.36307E-12	16	49	0.000474	6.62617E-12	1	4	4.02E-05	0
	x_0^5	14	41	0.0005289	5.85671E-13	8	24	0.000296	2.52902E-12	269	538	0.008207	9.00993E-12	16	49	0.000496	7.6356E-12	1	4	3.79E-05	0
	x_0^6	13	37	0.0005047	8.96719E-12	8	24	0.000334	7.58649E-13	271	542	0.008436	9.4944E-12	16	49	0.000491	9.2168E-12	1	4	4.12E-05	0
	x_0^7	12	36	0.0004644	7.07445E-12	1	4	4.92E-05	0	273	546	0.017513	9.33169E-12	17	52	0.00063	1.63156E-12	1	4	0.000051	0
	x_0^8	14	43	0.0005333	3.38515E-12	1	4	4.71E-05	0	274	548	0.007997	9.08776E-12	17	52	0.000504	1.67438E-12	1	4	5.07E-05	0
	x_0^9	12	36	0.0004673	4.75872E-12	1	4	7.38E-05	0	273	546	0.009304	9.32385E-12	17	52	0.000535	1.63102E-12	1	4	4.03E-05	0
	x_0^{10}	23	73	0.0008819	8.96857E-12	27	84	0.000975	4.24406E-12	243	486	0.012184	9.19795E-12	58	176	0.001711	6.29127E-12	1	4	3.89E-05	0
	x_0^{11}	38	119	0.0015314	2.93027E-12	27	82	0.000891	4.90883E-12	268	536	0.008629	9.4316E-12	58	177	0.001801	9.71883E-12	1	4	3.98E-05	0
	x_0^{12}	11	31	0.0004525	6.61962E-12	6	19	0.000221	3.16233E-13	208	416	0.005928	9.60377E-12	13	40	0.000369	3.56041E-12	1	4	3.79E-05	0
	x_0^{13}	17	49	0.0005855	7.72743E-12	13	39	0.000418	6.23441E-12	231	462	0.010151	9.42525E-12	46	139	0.001269	3.42995E-12	1	4	3.45E-05	0
	x_0^{14}	20	60	0.0007729	9.63945E-13	27	82	0.001053	4.90899E-12	268	536	0.008404	9.44481E-12	72	219	0.002197	8.73831E-12	1	4	4.04E-05	0
10000	x_0^1	1	1	0.0001472	0	1	2	0.000128	0	1	1	3.69E-05	0	1	1	2.93E-05	0	1	1	4.94E-05	0
	x_0^2	15	46	0.0070889	9.82543E-13	8	24	0.002534	1.40342E-12	269	538	0.073079	9.78542E-12	16	49	0.004654	9.07992E-12	1	4	0.000354	0
	x_0^3	13	38	0.0046012	4.10066E-12	8	24	0.002478	4.85466E-12	276	552	0.075307	9.25415E-12	17	52	0.004861	2.77443E-12	1	4	0.000363	0
	x_0^4	13	39	0.00542	1.888E-12	8	24	0.002566	6.86235E-12	278	556	0.074917	9.2915E-12	17	52	0.004924	3.3526E-12	1	4	0.000371	0
	x_0^5	15	45	0.0049472	1.65263E-12	8	24	0.004818	7.99745E-12	279	558	0.075963	9.93451E-12	17	52	0.005371	3.86334E-12	1	4	0.000369	0
	x_0^6	13	39	0.0068127	3.23718E-12	8	24	0.002474	2.39906E-12	282	564	0.076835	9.42183E-12	17	52	0.004958	4.66337E-12	1	4	0.000454	0
	x_0^7	12	37	0.0044014	6.63546E-12	1	4	0.000375	0	284	568	0.076231	9.26036E-12	17	52	0.006967	5.15945E-12	1	4	0.000376	0
	x_0^8	12	37	0.0041501	9.53262E-12	1	4	0.000368	0	285	570	0.076613	9.0183E-12	17	52	0.005073	5.29487E-12	1	4	0.000461	0
	x_0^9	12	37	0.0040161	6.63228E-12	1	4	0.000385	0	284	568	0.077895	9.25959E-12	17	52	0.004804	5.15928E-12	1	4	0.000361	0
	x_0^{10}	24	77	0.0107462	9.82558E-13	27	84	0.008147	4.35513E-12	243	486	0.078329	9.20076E-12	46	139	0.013123	4.99978E-12	1	4	0.000345	0
	x_0^{11}	26	75	0.0087113	7.452E-12	29	88	0.009376	1.26043E-12	279	558	0.077993	9.36541E-12	67	202	0.021255	6.9959E-12	1	4	0.000365	0
	x_0^{12}	10	29	0.0036405	4.65067E-12	6	18	0.001695	4.5E-12	197	394	0.05286	9.67775E-12	12	37	0.003131	7.03687E-12	1	4	0.000336	0
	x_0^{13}	17	49	0.0054671	7.72743E-12	13	39	0.005363	6.23441E-12	231	462	0.065329	9.42525E-12	46	139	0.014072	3.42995E-12	1	4	0.000285	0
	x_0^{14}	27	78	0.0125069	1.20268E-12	29	88	0.009606	1.26237E-12	279	558	0.081686	9.36672E-12	60	183	0.017738	9.3905E-12	1	4	0.000365	0
100000	x_0^1	1	1	0.0012233	0	1	2	0.001713	0	1	1	0.000363	0	1	1	0.000293	0	1	1	0.000634	0
	x_0^2	14	43	0.0557674	2.39837E-12	8	24	0.037376	4.438E-12	280	560	0.900581	9.71062E-12	17	52	0.066051	4.59412E-12	1	4	0.004994	0
	x_0^3	13	39	0.0623224	6.90574E-12	8	25	0.034266	3.41151E-13	287	574	0.896314	9.18341E-12	17	52	0.067956	8.77353E-12	1	4	0.005233	0
	x_0^4	14	42	0.0574819	6.17402E-12	8	25	0.036552	4.82237E-13	289	578	0.938301	9.22048E-12	18	55	0.071206	1.6963E-12	1	4	0.005453	0
	x_0^5	14	42	0.0645716	8.98996E-12	8	25	0.040143	5.62004E-13	290	580	0.940139	9.85857E-12	18	55	0.067238	1.95471E-12	1	4	0.005094	0
	x_0^6	12	35	0.0523736	9.7458E-12	8	24	0.035783	7.58649E-12	293	586	0.939405	9.34981E-12	18	55	0.069958	2.3595E-12	1	4	0.005693	0
	x_0^7	14	42	0.0630572	7.01014E-13	1	4	0.006146	0	295	590	0.944738	9.18958E-12	18	55	0.075589	2.6105E-12	1	4	0.005026	0
	x_0^8	12	37	0.0556239	8.53515E-12	1	4	0.006241	0	295	590	0.943079	9.94373E-12	18	55	0.056676	2.67902E-12	1	4	0.004884	0
	x_0^9	14	42	0.0631452	7.00987E-13	1	4	0.007914	0	295	590	0.931641	9.1895E-12	18	55	0.070331	2.61049E-12	1	4	0.005673	0
	x_0^{10}	24	77	0.1205535	9.36607E-13	27	84	0.100423	4.36509E-12	243	486	0.758346	9.20105E-12	50	152	0.192963	8.87208E-12	1	4	0.004328	0
	x_0^{11}	34	105	0.1532481	9.64323E-12	29	88	0.122845	3.98867E-12	290	580	0.901521	9.2944E-12	66	203	0.255324	9.77579E-12	1	4	0.003898	0
	x_0^{12}	10	29	0.0416141	1.47067E-12	6	18	0.023303	1.42302E-12	186	372	0.598464	9.75229E-12	12	37	0.041275	2.22526E-12	1	4	0.003639	0
	x_0^{13}	17	49	0.0653237	7.72743E-12	13	39	0.048892	6.23441E-12	231	462	0.684146	9.42525E-12	46	139	0.135136	3.42995E-12	1	4	0.002998	0
	x_0^{14}	31	97	0.1338255	5.01193E-12	29	88	0.112262	3.98929E-12	290	580	0.920004	9.29454E-12	60	183	0.220246	1.64094E-12	1	4	0.004206	0

Table 11: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in PolynomialI*

PolynomialI																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
DIM	INP	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
1000	x_0^1	1	1	0.6523618	0	1	2	0.028299	0	1	1	0.023851	0	1	1	0.022345	0	1	1	0.027635	0
	x_0^2	15	45	0.0315175	6.64373E-12	1	5	7.29E-05	0	80	160	0.003071	7.94192E-12	13	66	0.000836	3.08852E-12	1	6	8.94E-05	0
	x_0^3	15	46	0.001701	3.01466E-12	1	5	9.58E-05	0	82	164	0.002808	8.04228E-12	13	66	0.000922	7.00582E-12	1	6	7.38E-05	0
	x_0^4	15	46	0.0016877	3.1719E-12	1	5	7.18E-05	0	83	166	0.021606	7.15513E-12	13	66	0.000789	9.32786E-12	1	6	8.21E-05	0
	x_0^5	15	46	0.0016657	3.33293E-12	1	5	6.78E-05	0	83	166	0.002718	8.73189E-12	14	71	0.000705	1.10657E-12	1	6	7.68E-05	0
	x_0^6	15	46	0.001672	3.667E-12	1	5	6.76E-05	0	84	168	0.002754	8.43357E-12	12	60	0.00062	5.41978E-12	1	5	6.33E-05	0
	x_0^7	15	46	0.0024351	4.01806E-12	1	5	6.88E-05	0	85	170	0.004053	7.64241E-12	13	65	0.000667	1.6227E-12	1	5	6.45E-05	0
	x_0^8	15	46	0.0030289	4.20038E-12	1	5	6.92E-05	0	85	170	0.012694	8.55781E-12	13	65	0.000688	2.36699E-12	1	5	0.000063	0
	x_0^9	15	46	0.0027501	4.01626E-12	1	5	6.63E-05	0	85	170	0.003121	7.63341E-12	13	65	0.000631	1.61592E-12	1	5	6.37E-05	0
	x_0^{10}	22	67	0.0040337	4.72787E-12	36	109	0.001822	6.2004E-12	104	207	0.003851	8.65583E-12	65	325	0.020304	8.16996E-12	19	114	0.001187	3.65884E-13
	x_0^{11}	22	66	0.0037353	8.19828E-12	11	35	0.0006	1.5935E-12	100	199	0.003386	9.76623E-12	88	444	0.00449	9.16075E-12	45	270	0.002574	7.30539E-12
	x_0^{12}	12	37	0.0023773	4.50571E-12	1	5	8.29E-05	0	65	130	0.011375	8.10145E-12	11	56	0.000576	1.58276E-12	1	6	6.33E-05	0
	x_0^{13}	12	37	0.0022506	3.17242E-12	1	5	6.51E-05	6.0961E-16	96	191	0.003277	9.25591E-12	24	120	0.001388	8.12121E-12	1	5	6.18E-05	5.68259E-15
	x_0^{14}	22	66	0.0040189	8.91453E-12	11	35	0.000494	1.7158E-12	100	199	0.00333	9.77002E-12	96	481	0.018638	8.33667E-12	42	252	0.022178	7.47689E-12
10000	x_0^1	1	1	0.0002115	0	1	2	0.000205	0	1	1	7.57E-05	0	1	1	6.15E-05	0	1	1	7.92E-05	0
	x_0^2	15	45	0.0224403	9.51812E-12	1	5	0.00044	0	83	166	0.033175	8.61429E-12	13	66	0.011818	9.76677E-12	1	6	0.000492	0
	x_0^3	15	46	0.0092925	2.9975E-12	1	5	0.0005	0	85	170	0.028278	8.72315E-12	14	71	0.007426	2.05592E-12	1	6	0.000522	0
	x_0^4	15	46	0.019046	3.15414E-12	1	5	0.000485	0	86	172	0.029997	7.76089E-12	14	71	0.011532	2.73735E-12	1	6	0.000521	0
	x_0^5	15	46	0.0068451	3.31452E-12	1	5	0.005956	0	86	172	0.036956	9.47114E-12	14	71	0.007186	3.49927E-12	1	6	0.000513	0
	x_0^6	15	46	0.0172691	3.6471E-12	1	5	0.000523	0	87	174	0.031058	9.14757E-12	13	65	0.009836	1.59048E-12	1	5	0.000464	0
	x_0^7	15	46	0.0066395	3.99639E-12	1	5	0.000643	0	88	176	0.036526	8.28942E-12	13	65	0.006626	5.13144E-12	1	5	0.000459	0
	x_0^8	15	46	0.0099195	4.17771E-12	1	5	0.000556	0	88	176	0.031707	9.28232E-12	13	65	0.010385	7.48508E-12	1	5	0.000465	0
	x_0^9	15	46	0.0074135	3.99621E-12	1	5	0.000587	0	88	176	0.04304	8.28844E-12	13	65	0.006576	5.12929E-12	1	5	0.000453	0
	x_0^{10}	30	90	0.0161391	6.67132E-12	37	113	0.019003	1.0272E-12	104	207	0.048147	8.65446E-12	56	280	0.034232	6.64516E-12	19	114	0.016332	3.65048E-13
	x_0^{11}	20	61	0.0112921	4.13083E-12	3	11	0.001227	2.2702E-12	95	189	0.032281	8.57347E-12	91	459	0.052664	7.01935E-12	7	42	0.003753	0
	x_0^{12}	11	34	0.0057278	7.54656E-12	1	5	0.000485	0	62	124	0.022661	7.46804E-12	10	51	0.007123	5.39042E-12	1	6	0.000601	0
	x_0^{13}	12	37	0.0078815	3.17242E-12	1	5	0.000488	6.8771E-16	96	191	0.033308	9.25591E-12	24	120	0.012747	8.12121E-12	1	5	0.000467	6.38766E-15
	x_0^{14}	20	61	0.0122676	2.24244E-12	3	11	0.001215	2.0581E-12	95	189	0.032961	8.57382E-12	85	430	0.049313	8.43725E-12	7	42	0.004476	0
100000	x_0^1	1	1	0.0152953	0	1	2	0.002402	0	1	1	0.000743	0	1	1	0.000579	0	1	1	0.000706	0
	x_0^2	15	45	0.4712151	9.50018E-12	1	5	0.011305	0	86	172	0.459103	9.34359E-12	14	71	0.151979	2.86615E-12	1	6	0.0293	0
	x_0^3	15	46	0.2473254	2.9921E-12	1	5	0.017641	0	88	176	0.397452	9.46167E-12	14	71	0.444167	6.5014E-12	1	6	0.006502	0
	x_0^4	15	46	0.0936662	3.14855E-12	1	5	0.022797	0	89	178	0.380899	8.41793E-12	14	71	0.271882	8.65626E-12	1	6	0.008255	0
	x_0^5	15	46	0.0818547	3.30873E-12	1	5	0.013838	0	90	180	0.380661	7.19108E-12	15	76	0.100644	1.02689E-12	1	6	0.006106	0
	x_0^6	15	46	0.0803644	3.64083E-12	1	5	0.005365	0	90	180	0.374266	9.92201E-12	13	65	0.087506	5.02955E-12	1	5	0.010369	0
	x_0^7	15	46	0.0778008	3.98956E-12	1	5	0.006131	0	91	182	0.409662	8.99121E-12	14	70	0.089764	1.50587E-12	1	5	0.006243	0
	x_0^8	15	46	0.1042527	4.17056E-12	1	5	0.010647	0	92	184	0.377948	7.04772E-12	14	70	0.095798	2.19657E-12	1	5	0.007787	0
	x_0^9	15	46	0.083453	3.98955E-12	1	5	0.02721	0	91	182	0.401391	8.99111E-12	14	70	0.086913	1.5058E-12	1	5	0.005793	0
	x_0^{10}	27	83	0.1778629	6.9407E-12	38	115	0.204938	5.6451E-12	104	207	0.460681	8.65432E-12	58	290	0.36239	2.27585E-12	19	114	0.141806	3.65008E-13
	x_0^{11}	21	64	0.1252838	2.26226E-12	3	11	0.019916	2.4584E-16	91	181	0.3893	9.44375E-12	89	449	0.564995	6.03782E-12	7	42	0.053856	0
	x_0^{12}	11	33	0.0616702	8.42797E-12	1	5	0.009298	0	58	116	0.246128	9.83577E-12	10	51	0.065969	1.70451E-12	1	6	0.006355	0
	x_0^{13}	12	37	0.0732819	3.17242E-12	1	5	0.015089	3.9408E-14	96	191	0.419383	9.25591E-12	24	120	0.150898	8.12121E-12	1	5	0.007164	3.67436E-13
	x_0^{14}	21	64	0.1122553	2.07452E-12	3	11	0.023422	2.2357E-16	91	181	0.411353	9.44386E-12	88	444	0.583763	9.10381E-12	7	42	0.051836	0

Table 12: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in SmoothSine*

SmoothSine																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
DIM	INP	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
1000	x_0^1	1	1	0.1500283	0	1	2	0.027986	0	1	1	0.020506	0	1	1	0.020389	0	1	1	0.020862	0
	x_0^2	3	9	0.00039	0	8	24	0.000716	4.438E-13	258	516	0.024	9.86079E-12	16	49	0.002005	2.8713E-12	1	4	0.000117	0
	x_0^3	3	9	0.0127788	0	8	24	0.000704	1.53518E-12	265	530	0.017792	9.32543E-12	16	49	0.002026	5.4835E-12	1	4	0.000122	0
	x_0^4	3	9	0.000297	0	8	24	0.000675	2.17007E-12	267	534	0.017835	9.36307E-12	16	49	0.002036	6.6262E-12	1	4	0.000104	0
	x_0^5	3	9	0.000311	0	8	24	0.000705	2.52902E-12	269	538	0.024883	9.00993E-12	16	49	0.002121	7.6356E-12	1	4	0.000102	0
	x_0^6	3	9	0.0003068	0	8	24	0.000716	7.58649E-13	271	542	0.01981	9.4944E-12	16	49	0.002073	9.2168E-12	1	4	0.000105	0
	x_0^7	5	14	0.0004655	0	1	4	0.000109	0	273	546	0.018306	9.33169E-12	17	52	0.002199	1.6316E-12	1	4	0.000106	0
	x_0^8	5	14	0.0004719	0	1	4	0.000111	0	274	548	0.025571	9.08776E-12	17	52	0.00218	1.6744E-12	1	4	0.000107	0
	x_0^9	5	14	0.0004647	0	1	4	0.000111	0	273	546	0.01847	9.32385E-12	17	52	0.002189	1.631E-12	1	4	0.000111	0
	x_0^{10}	3	9	0.0002755	0	29	87	0.002457	4.8287E-12	243	486	0.016166	9.19795E-12	50	151	0.005362	7.8491E-12	1	4	0.000146	0
	x_0^{11}	11	33	0.0009901	2.6522E-12	33	99	0.002922	6.45777E-12	268	536	0.021528	9.4316E-12	52	157	0.004271	5.6226E-12	1	4	0.000104	0
	x_0^{12}	4	12	0.0003884	0	6	19	0.000548	3.16233E-13	208	416	0.013895	9.60377E-12	13	40	0.001116	3.5604E-12	1	4	0.000104	0
	x_0^{13}	3	9	0.000282	5.02583E-18	14	43	0.001251	5.54536E-13	231	462	0.015138	9.42525E-12	44	133	0.003681	7.26E-12	1	4	0.000115	0
	x_0^{14}	11	33	0.0010063	2.72272E-12	33	99	0.003033	6.42807E-12	268	536	0.021666	9.44481E-12	52	157	0.004537	5.8929E-12	1	4	0.000105	0
10000	x_0^1	1	1	0.0005119	0	1	2	0.000499	0	1	1	0.000162	0	1	1	0.000151	0	1	1	0.000198	0
	x_0^2	4	12	0.0037445	0	8	24	0.00679	1.40342E-12	269	538	0.173951	9.78542E-12	16	49	0.012739	9.0799E-12	1	4	0.000977	0
	x_0^3	4	12	0.0040391	0	8	24	0.006643	4.85466E-12	276	552	0.178204	9.25415E-12	17	52	0.01569	2.7744E-12	1	4	0.000965	0
	x_0^4	4	12	0.0037733	0	8	24	0.007005	6.86235E-12	278	556	0.179012	9.2915E-12	17	52	0.013948	3.3526E-12	1	4	0.000967	0
	x_0^5	4	12	0.0037671	0	8	24	0.010219	7.99745E-12	279	558	0.17934	9.93451E-12	17	52	0.014135	3.8633E-12	1	4	0.000955	0
	x_0^6	3	9	0.0028901	0	8	24	0.00666	2.39906E-12	282	564	0.190966	9.42183E-12	17	52	0.014363	4.6634E-12	1	4	0.000987	0
	x_0^7	5	14	0.0048733	0	1	4	0.001054	0	284	568	0.210014	9.26036E-12	17	52	0.014162	5.1595E-12	1	4	0.000978	0
	x_0^8	5	14	0.004781	0	1	4	0.001032	0	285	570	0.199963	9.0183E-12	17	52	0.013582	5.2949E-12	1	4	0.001016	0
	x_0^9	5	14	0.0046059	0	1	4	0.001051	0	284	568	0.184888	9.25959E-12	17	52	0.015744	5.1593E-12	1	4	0.001305	0
	x_0^{10}	3	9	0.002969	0	29	87	0.024007	4.88498E-12	243	486	0.156391	9.20076E-12	50	151	0.039221	7.8483E-12	1	4	0.001311	0
	x_0^{11}	11	32	0.0098884	9.5146E-12	35	105	0.030972	7.93333E-12	279	558	0.179347	9.36541E-12	54	163	0.045416	6.8048E-12	1	4	0.001288	0
	x_0^{12}	4	12	0.0034562	0	6	18	0.00482	4.5E-12	197	394	0.128036	9.67775E-12	12	37	0.009358	7.0369E-12	1	4	0.001308	0
	x_0^{13}	3	9	0.0027227	2.41226E-17	14	43	0.012106	5.54536E-13	231	462	0.145599	9.42525E-12	44	133	0.03345	7.26E-12	1	4	0.001193	0
	x_0^{14}	11	32	0.0121892	9.5617E-12	35	105	0.029973	7.92967E-12	279	558	0.183733	9.36672E-12	54	163	0.044157	6.8369E-12	1	4	0.001293	0
100000	x_0^1	1	1	0.0050454	0	1	2	0.007549	0	1	1	0.001601	0	1	1	0.0015	0	1	1	0.002459	0
	x_0^2	4	12	0.0418928	0	8	24	0.088045	4.438E-12	280	560	2.070844	9.71062E-12	17	52	0.160866	4.5941E-12	1	4	0.011814	0
	x_0^3	5	15	0.059154	0	8	25	0.096694	3.41151E-13	287	574	2.230639	9.18341E-12	17	52	0.194295	8.7735E-12	1	4	0.01168	0
	x_0^4	5	15	0.0476069	0	8	25	0.105676	4.82237E-13	289	578	2.247195	9.22048E-12	18	55	0.17021	1.6963E-12	1	4	0.01557	0
	x_0^5	5	15	0.0603251	0	8	25	0.089624	5.62004E-13	290	580	2.22234	9.85857E-12	18	55	0.199734	1.9547E-12	1	4	0.014586	0
	x_0^6	4	12	0.0433751	0	8	24	0.089556	7.58649E-12	293	586	2.25753	9.34981E-12	18	55	0.185474	2.3595E-12	1	4	0.012765	0
	x_0^7	5	14	0.0489626	0	1	4	0.011103	0	295	590	2.319293	9.18958E-12	18	55	0.200921	2.6105E-12	1	4	0.011741	0
	x_0^8	5	14	0.0619584	0	1	4	0.012347	0	295	590	2.285059	9.94373E-12	18	55	0.19193	2.679E-12	1	4	0.015108	0
	x_0^9	5	14	0.0574898	0	1	4	0.013541	0	295	590	2.259153	9.1895E-12	18	55	0.188415	2.6105E-12	1	4	0.015992	0
	x_0^{10}	3	9	0.0342624	0	29	87	0.28137	4.89064E-12	243	486	1.808514	9.20105E-12	50	151	0.481473	7.8482E-12	1	4	0.010751	0
	x_0^{11}	7	21	0.0687001	0	37	111	0.388197	9.76414E-12	290	580	2.219649	9.2944E-12	56	169	0.573485	8.0798E-12	1	4	0.010993	0
	x_0^{12}	4	12	0.0379395	0	6	18	0.058434	1.42302E-12	186	372	1.413779	9.75229E-12	12	37	0.107919	2.2253E-12	1	4	0.011746	0
	x_0^{13}	3	9	0.036639	1.11149E-16	14	43	0.148498	5.54536E-13	231	462	1.659983	9.42525E-12	44	133	0.391031	7.26E-12	1	4	0.009632	0
	x_0^{14}	7	21	0.0745299	0	37	111	0.390614	9.76369E-12	290	580	2.215667	9.29454E-12	56	169	0.512894	8.0836E-12	1	4	0.010903	0

Table 13: *Experiment results and average result of ℓ_1-norm regularization problem via* **STTDFPM** *,* **CGDFPM** *,* **MOPCG** *and* **AHDFPM** *methods.*

MSE									
Experiments	**STTDFPM**	**AHDFPM**	**CGDFPM**	**MOPCG**	Experiments	**STTDFPM**	**AHDFPM**	**CGDFPM**	**MOPCG**
1	0.05284	0.05287	0.05405	0.05291	51	0.04206	0.04215	0.04717	0.04230
2	0.05256	0.05262	0.05375	0.05269	52	0.04499	0.04500	0.04759	0.04506
3	0.05096	0.05103	0.05388	0.05119	53	0.03398	0.03408	0.03716	0.03422
4	0.04169	0.04178	0.04388	0.04190	54	0.04476	0.04481	0.04874	0.04489
5	0.04069	0.04082	0.04202	0.04098	55	0.05324	0.05326	0.05437	0.05327
6	0.04734	0.04731	0.04812	0.04726	56	0.04135	0.04136	0.04435	0.04142
7	0.04552	0.04565	0.05000	0.04597	57	0.04020	0.04045	0.04712	0.04086
8	0.04236	0.04243	0.04505	0.04253	58	0.05260	0.05262	0.05456	0.05267
9	0.04051	0.04059	0.04416	0.04072	59	0.03338	0.03342	0.03666	0.03350
10	0.06570	0.06566	0.06482	0.06561	60	0.04219	0.04244	0.04753	0.04286
11	0.04634	0.04641	0.05008	0.04647	61	0.05566	0.05569	0.05593	0.05573
12	0.04104	0.04138	0.04659	0.04177	62	0.03025	0.03034	0.03237	0.03045
13	0.04468	0.04474	0.04717	0.04484	63	0.03813	0.03811	0.03921	0.03806
14	0.04708	0.04715	0.05091	0.04732	64	0.04286	0.04293	0.04564	0.04307
15	0.05348	0.05349	0.05704	0.05360	65	0.04216	0.04232	0.04812	0.04260
16	0.03949	0.03961	0.04402	0.03980	66	0.05871	0.05880	0.06153	0.05891
17	0.04504	0.04506	0.04680	0.04511	67	0.05396	0.05393	0.05324	0.05384
18	0.05241	0.05244	0.05513	0.05251	68	0.04694	0.04712	0.05241	0.04745
19	0.04330	0.04335	0.04565	0.04343	69	0.04378	0.04385	0.04695	0.04396
20	0.04041	0.04049	0.04300	0.04070	70	0.05371	0.05373	0.05657	0.05392
21	0.03602	0.03617	0.04368	0.03648	71	0.05019	0.05020	0.05174	0.05021
22	0.04266	0.04275	0.04627	0.04301	72	0.05210	0.05207	0.05178	0.05204
23	0.05424	0.05427	0.05837	0.05435	73	0.04313	0.04318	0.04719	0.04335
24	0.04244	0.04236	0.04333	0.04228	74	0.03706	0.03719	0.04041	0.03740
25	0.05006	0.05011	0.05224	0.05023	75	0.03853	0.03859	0.04152	0.03870
26	0.03507	0.03518	0.03966	0.03538	76	0.04341	0.04350	0.04664	0.04364
27	0.04264	0.04265	0.04466	0.04267	77	0.02827	0.02832	0.03254	0.02844
28	0.04605	0.04616	0.04941	0.04627	78	0.04593	0.04600	0.04904	0.04611
29	0.04523	0.04527	0.04888	0.04541	79	0.04472	0.04476	0.04704	0.04484
30	0.03291	0.03305	0.03708	0.03329	80	0.03862	0.03883	0.04407	0.03913
31	0.03880	0.03887	0.04119	0.03900	81	0.03274	0.03293	0.03740	0.03320
32	0.03149	0.03149	0.03403	0.03149	82	0.04847	0.04863	0.05017	0.04884
33	0.05351	0.05348	0.05465	0.05345	83	0.03386	0.03387	0.03549	0.03391
34	0.04861	0.04864	0.05129	0.04871	84	0.05266	0.05265	0.05350	0.05262
35	0.05020	0.05025	0.05393	0.05052	85	0.03691	0.03703	0.04280	0.03725
36	0.06390	0.06394	0.06654	0.06404	86	0.05090	0.05099	0.05353	0.05115
37	0.04924	0.04931	0.05223	0.04945	87	0.04659	0.04655	0.04655	0.04648
38	0.04503	0.04504	0.04542	0.04505	88	0.04078	0.04082	0.04318	0.04090
39	0.03665	0.03672	0.04055	0.03681	89	0.05044	0.05050	0.05241	0.05063
40	0.04140	0.04147	0.04393	0.04160	90	0.03858	0.03874	0.04264	0.03896
41	0.04858	0.04859	0.04997	0.04862	91	0.05956	0.05959	0.06337	0.05963
42	0.04250	0.04253	0.04335	0.04254	92	0.04417	0.04419	0.04607	0.04431
43	0.05804	0.05819	0.06132	0.05836	93	0.04371	0.04375	0.04572	0.04382
44	0.05167	0.05171	0.05323	0.05176	94	0.04024	0.04028	0.04393	0.04037
45	0.05701	0.05708	0.06141	0.05718	95	0.04405	0.04412	0.04605	0.04423
46	0.04284	0.04291	0.04619	0.04306	96	0.03023	0.03025	0.03183	0.03029
47	0.04676	0.04688	0.05041	0.04706	97	0.04784	0.04789	0.05377	0.04827
48	0.04980	0.04986	0.05215	0.04992	98	0.04602	0.04613	0.04985	0.04630
49	0.04962	0.04957	0.05146	0.04950	99	0.04266	0.04268	0.04414	0.04270
50	0.03179	0.03194	0.03540	0.03216	100	0.03843	0.03842	0.04046	0.03839
Average						0.04596	0.04603	0.04877	0.04614