Two classes of spectral three-term derivative-free method for solving nonlinear equations with application

Abdulkarim Hassan Ibrahim[1], Mohammed Alshahrani*[2], and Suliman Al-Homidan[3]

[1,2,3]*Interdisciplinary Research Center for Smart Mobility and Logistics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia*
[1,3]*Department of Mathematics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia*

Detailed Numerical Results

Table 1: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in ENGVAL1Grad*

		ENGVAL1Grad																			
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
x_0	DIM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
x_0^1	1000	76	319	0.1587676	7.28725E-12	69	286	0.032394	8.6392E-12	160	1920	0.054325	5.08023E-12	36	322	0.022706	5.08725E-12	43	427	0.032784	5.41684E-12
x_0^2		186	743	0.0270061	9.09696E-12	194	741	0.017835	9.9473E-12	226	2977	0.042157	5.21365E-12	669	5471	0.093482	9.24679E-12	115	1079	0.02515	8.60829E-12
x_0^3		130	530	0.0209917	8.32138E-12	202	785	0.019192	9.0144E-12	118	1099	0.01666	7.94536E-12	698	5708	0.081056	8.48142E-12	136	1269	0.023869	8.21844E-12
x_0^4		237	958	0.0227215	9.12291E-12	199	775	0.019028	9.0844E-12	141	1281	0.022322	5.61117E-12	708	5788	0.085776	8.48443E-12	134	1251	0.018565	6.25975E-12
x_0^5		212	847	0.020277	9.97509E-12	203	777	0.017422	8.5171E-12	164	2126	0.029965	4.99018E-12	715	5844	0.083534	8.39903E-12	135	1260	0.018007	6.5548E-12
x_0^6		171	683	0.0142996	7.22446E-12	179	694	0.016256	9.7017E-12	148	1830	0.025403	6.29587E-12	703	5749	0.083996	8.37516E-12	134	1252	0.018294	9.46069E-12
x_0^7		221	876	0.0209504	9.40806E-12	198	763	0.016212	9.8189E-12	112	1090	0.016985	3.43567E-12	705	5766	0.081545	9.09106E-12	143	1346	0.023306	9.98484E-12
x_0^8		178	710	0.0141392	9.10214E-12	213	827	0.01853	8.9469E-12	198	2893	0.043249	9.58545E-12	655	5364	0.088255	9.33572E-12	137	1281	0.018737	7.79394E-12
x_0^9		196	780	0.0159082	9.4115E-12	195	759	0.013739	7.9408E-12	164	1853	0.027298	4.25447E-12	699	5718	0.086223	9.14347E-12	139	1314	0.019381	9.67551E-12
x_0^{10}		144	589	0.016892	9.99548E-12	141	568	0.013515	9.1701E-12	270	3712	0.049403	4.45453E-12	418	3431	0.05024	9.61582E-12	46	459	0.006141	5.3876E-12
x_0^{11}		240	965	0.0184942	9.45027E-12	210	833	0.017698	9.4547E-12	180	2346	0.031521	7.20983E-12	682	5586	0.083189	9.51749E-12	140	1312	0.023321	6.51291E-12
x_0^{12}		138	561	0.0168241	7.14892E-12	118	469	0.011142	9.3219E-12	164	1953	0.030195	9.71017E-12	428	3511	0.050415	8.52904E-12	45	442	0.004951	8.9978E-12
x_0^{13}		80	335	0.0054112	4.9595E-12	85	357	0.005547	4.8525E-12	146	1487	0.023252	2.71521E-12	38	340	0.003957	7.81672E-12	36	355	0.005163	7.63998E-12
x_0^{14}		242	974	0.0214821	9.35372E-12	188	730	0.014392	8.9558E-12	220	3855	0.049441	8.87848E-12	682	5586	0.083713	9.56062E-12	140	1312	0.021019	6.45555E-12
x_0^1	10000	79	333	0.0550551	8.74816E-12	59	242	0.037389	7.6384E-12	175	1971	0.241277	6.32847E-12	37	331	0.051132	4.47553E-12	42	420	0.049459	6.66445E-12
x_0^2		215	858	0.1330208	9.48144E-12	196	754	0.118111	8.5229E-12	115	1215	0.145187	7.82858E-12	657	5374	0.715145	9.41105E-12	108	1016	0.122181	6.40213E-12
x_0^3		163	664	0.1070565	9.17568E-12	190	760	0.117602	9.8629E-12	163	2237	0.268596	6.10655E-12	687	5619	0.718593	8.87349E-12	135	1260	0.156357	7.34157E-12
x_0^4		193	766	0.1218971	9.08669E-12	205	805	0.125094	9.2233E-12	216	3248	0.379063	7.90495E-12	691	5651	0.699582	9.12815E-12	129	1208	0.150553	9.13491E-12
x_0^5		178	700	0.1128475	8.92441E-12	194	758	0.124815	9.967E-12	139	1286	0.151864	3.89277E-12	683	5585	0.725386	9.96838E-12	117	1099	0.136788	6.61802E-12
x_0^6		142	568	0.0908694	8.18777E-12	185	720	0.116956	9.0191E-12	130	1306	0.155051	6.17162E-12	692	5660	0.706982	8.45322E-12	130	1216	0.150607	6.58387E-12
x_0^7		213	854	0.1334169	9.88552E-12	200	773	0.120824	8.8185E-12	148	2067	0.239173	5.65781E-12	688	5629	0.704721	9.95346E-12	142	1336	0.152703	7.19394E-12
x_0^8		216	860	0.1406281	9.4491E-12	187	727	0.126127	9.0243E-12	137	1258	0.160991	9.86845E-12	721	5906	0.747868	8.99307E-12	137	1281	0.153646	8.36029E-12
x_0^9		214	858	0.1345058	9.40619E-12	169	632	0.108439	9.0347E-12	143	1909	0.226054	7.75354E-12	688	5629	0.708066	9.95347E-12	138	1298	0.156207	5.98622E-12
x_0^{10}		135	559	0.0878785	9.85169E-12	112	447	0.071516	8.1527E-12	159	1822	0.225472	5.43271E-12	300	2472	0.30789	8.98255E-12	42	418	0.048314	6.45226E-12
x_0^{11}		236	951	0.1482719	9.39171E-12	202	795	0.124835	9.433E-12	230	4027	0.467828	8.74433E-12	676	5537	0.680419	8.72518E-12	131	1231	0.156607	9.60002E-12
x_0^{12}		116	465	0.0732762	5.60319E-12	104	419	0.066433	9.1764E-12	138	1225	0.165668	8.57673E-12	308	2536	0.311611	9.8739E-12	41	409	0.049525	4.98783E-12
x_0^{13}		82	351	0.0541222	8.67559E-12	64	267	0.040561	9.3015E-12	281	4478	0.542008	6.67165E-12	37	331	0.042192	8.2364E-12	44	440	0.055186	5.43578E-12
x_0^{14}		237	955	0.1668894	9.49709E-12	182	718	0.111215	9.193E-12	133	1512	0.192747	8.34875E-12	676	5537	0.700005	8.72676E-12	131	1231	0.149069	9.57503E-12
x_0^1	100000	82	345	0.6105675	9.04151E-12	71	297	0.462737	8.7678E-12	137	1523	1.998058	7.9112E-12	38	340	0.420932	3.86319E-12	41	412	0.481557	4.13487E-12
x_0^2		150	601	1.0232906	8.92569E-12	193	746	1.270597	9.084E-12	158	1592	2.290932	7.88159E-12	651	5325	6.894198	9.56446E-12	103	973	1.142415	8.97092E-12
x_0^3		181	737	1.2578894	7.60029E-12	179	720	1.215281	9.971E-12	126	1436	1.786511	5.10775E-12	672	5498	7.125333	8.57992E-12	130	1215	1.433747	6.52554E-12
x_0^4		197	788	1.4188977	8.69963E-12	205	805	1.475873	9.8733E-12	195	2647	3.050073	4.90602E-12	2000	16119	20.50817	1.88587E-11	102	961	1.150292	6.74785E-12
x_0^5		170	681	1.2079056	9.63412E-12	185	720	1.244666	8.7345E-12	193	2276	2.629548	6.74518E-12	680	5560	7.607655	9.56443E-12	108	1020	1.183683	5.80222E-12
x_0^6		160	632	1.2137653	7.82796E-12	167	648	1.142925	8.1822E-12	239	3115	3.696096	9.78698E-12	2000	16117	20.50738	3.40124E-11	130	1216	1.442187	9.08238E-12
x_0^7		187	748	1.3681992	9.78687E-12	188	725	1.272768	8.6458E-12	173	2068	2.475139	7.12687E-12	667	5460	6.885035	9.82417E-12	143	1362	1.64168	9.12536E-12
x_0^8		167	665	1.1858527	9.14507E-12	182	710	1.288348	9.8997E-12	213	3503	3.981833	3.47283E-12	705	5773	7.383313	8.09542E-12	132	1236	1.448838	8.55344E-12
x_0^9		174	696	1.2375088	9.87818E-12	193	745	1.319114	8.6347E-12	200	3056	3.590529	9.01505E-12	667	5460	6.949001	9.82417E-12	141	1343	1.577849	8.83844E-12
x_0^{10}		108	462	0.777358	9.84634E-12	87	368	0.641514	9.5543E-12	191	2361	2.911667	4.45064E-12	171	1426	1.793839	8.48577E-12	39	393	0.468477	9.39465E-12
x_0^{11}		216	871	1.5265109	9.96849E-12	203	803	1.344443	9.2862E-12	135	1643	1.916908	8.60599E-12	651	5336	6.767742	8.71648E-12	125	1178	1.391772	8.55529E-12
x_0^{12}		105	436	0.7783658	9.64372E-12	98	404	0.73374	9.5163E-12	137	1541	1.847055	6.26697E-12	189	1570	2.025288	9.28713E-12	41	413	0.489988	7.35795E-12
x_0^{13}		85	368	0.687449	4.45099E-12	75	318	0.569117	8.8984E-12	128	1678	1.966147	8.16208E-12	39	349	0.433505	4.47612E-12	30	302	0.366097	3.94714E-12
x_0^{14}		226	912	1.6630938	9.61912E-12	173	682	1.122146	9.9755E-12	213	2808	3.288956	7.41462E-12	651	5336	6.904258	8.72945E-12	125	1178	1.383605	8.55042E-12

*Corresponding Author mshahrani@kfupm.edu.sa

Table 2: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in ExponetialI*

ExponetialI																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
x_0	DIM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
x_0^1	1000	1	1	0.109382	0	1	2	0.02748	0	1	1	0.019507	0	1	1	0.018029	0	1	1	0.021166	0
x_0^2		3	9	0.000173	0	7	22	0.000322	2.3172E-13	257	514	0.008055	9.85842E-12	16	49	0.000621	1.95202E-12	1	4	4.61E-05	0
x_0^3		4	11	0.000157	0	1	4	4.93E-05	0	263	526	0.008456	9.42308E-12	16	49	0.00063	2.6261E-12	1	4	0.000044	0
x_0^4		4	11	0.000156	0	1	4	4.85E-05	0	265	530	0.015545	9.05795E-12	16	49	0.000695	2.71036E-12	1	4	4.32E-05	0
x_0^5		4	11	0.000152	0	1	4	0.000047	0	266	532	0.008315	9.29669E-12	16	49	0.000653	2.66121E-12	1	4	4.45E-05	0
x_0^6		4	11	0.000151	0	1	4	4.69E-05	0	268	536	0.008615	9.07902E-12	16	49	0.000674	1.89585E-12	1	4	4.26E-05	0
x_0^7		4	11	0.000158	0	1	4	0.000049	0	269	538	0.015332	9.27562E-12	17	54	0.000726	3.05443E-12	1	5	4.81E-05	0
x_0^8		4	11	0.000161	0	1	4	4.79E-05	0	269	538	0.008407	9.72501E-12	17	54	0.009641	3.0474E-12	1	5	4.79E-05	0
x_0^9		4	11	0.000158	0	1	4	4.62E-05	0	269	538	0.014923	9.2686E-12	17	54	0.000674	3.0474E-12	1	5	4.87E-05	0
x_0^{10}		39	115	0.001786	8.24407E-12	51	152	0.00215	7.9172E-12	240	480	0.007939	9.304E-12	49	150	0.00198	8.19133E-12	28	114	0.001145	3.34401E-12
x_0^{11}		56	167	0.002606	6.49627E-12	56	167	0.002367	7.655E-12	265	530	0.008328	9.26891E-12	62	189	0.002532	5.75779E-12	14	58	0.000628	0
x_0^{12}		4	12	0.000179	0	6	19	0.00025	3.3002E-13	208	416	0.010048	9.59862E-12	13	40	0.000534	3.55296E-12	1	4	0.000045	0
x_0^{13}		27	80	0.001125	8.54152E-12	1	4	4.77E-05	2.2204E-16	229	458	0.007213	9.95237E-12	48	145	0.00183	5.66004E-12	1	4	0.000047	2.22045E-16
x_0^{14}		56	167	0.002368	6.57743E-12	56	167	0.002321	7.7707E-12	265	530	0.008609	9.27942E-12	61	187	0.00241	9.01408E-12	15	63	0.000742	0
x_0^1	10000	1	1	0.000188	0	1	2	0.000187	0	1	1	6.65E-05	0	1	1	5.11E-05	0	1	1	0.000112	0
x_0^2		3	9	0.001199	0	7	22	0.002725	7.3275E-13	268	536	0.083862	9.79217E-12	16	49	0.005387	6.17284E-12	1	4	0.000445	0
x_0^3		5	14	0.001917	0	1	4	0.000424	0	274	548	0.086078	9.34808E-12	16	49	0.005569	8.32667E-12	1	4	0.000447	0
x_0^4		5	14	0.002086	0	1	4	0.000406	0	275	550	0.083997	9.99201E-12	16	49	0.007548	8.57092E-12	1	4	0.000526	0
x_0^5		5	14	0.002078	0	1	4	0.000409	0	277	554	0.092155	9.21485E-12	16	49	0.006004	8.39329E-12	1	4	0.000552	0
x_0^6		5	14	0.002122	0	1	4	0.00042	0	279	558	0.091937	9.01501E-12	16	49	0.005835	5.9952E-12	1	4	0.000477	0
x_0^7		5	14	0.00189	0	1	4	0.000443	0	280	560	0.090681	9.19265E-12	17	54	0.006026	9.63674E-12	1	5	0.000576	0
x_0^8		5	14	0.001904	0	1	4	0.000438	0	280	560	0.088174	9.65894E-12	17	54	0.008129	9.63674E-12	1	5	0.000497	0
x_0^9		5	14	0.002152	0	1	4	0.000457	0	280	560	0.085318	9.19265E-12	17	54	0.006003	9.63674E-12	1	5	0.000479	0
x_0^{10}		39	115	0.017833	7.42665E-12	51	152	0.021848	7.6521E-12	240	480	0.076448	9.3088E-12	46	141	0.016313	5.1914E-12	17	70	0.010095	0
x_0^{11}		58	173	0.024763	9.47136E-12	64	192	0.026528	6.4912E-13	276	552	0.097128	9.2028E-12	65	198	0.023793	9.7581E-12	9	38	0.0041	0
x_0^{12}		4	12	0.001492	0	6	18	0.002198	4.5297E-12	197	394	0.06233	9.68114E-12	12	37	0.004524	7.03881E-12	1	4	0.000425	0
x_0^{13}		27	80	0.011098	8.54152E-12	1	4	0.000565	4.9651E-16	229	458	0.071386	9.95237E-12	48	145	0.018596	5.66004E-12	1	4	0.000422	4.44089E-16
x_0^{14}		58	173	0.024951	9.48227E-12	64	192	0.028909	6.5045E-13	276	552	0.086309	9.20384E-12	58	178	0.021656	9.95918E-12	13	54	0.005429	0
x_0^1	100000	1	1	0.002978	0	1	2	0.001936	0	1	1	0.000575	0	1	1	0.000853	0	1	1	0.000634	0
x_0^2		3	9	0.01985	0	8	24	0.04844	1.0533E-12	279	558	0.971393	9.6899E-12	17	52	0.082358	3.08953E-12	1	4	0.0052	0
x_0^3		5	14	0.022015	0	1	4	0.007493	0	285	570	1.017698	9.2686E-12	17	52	0.080391	4.213E-12	1	4	0.004639	0
x_0^4		5	14	0.028356	0	1	4	0.007264	0	286	572	1.035779	9.90055E-12	17	52	0.076765	4.35343E-12	1	4	0.004517	0
x_0^5		5	14	0.028216	0	1	4	0.00657	0	288	576	1.091419	9.12817E-12	17	52	0.074587	4.28322E-12	1	4	0.005258	0
x_0^6		5	14	0.023615	0	1	4	0.006616	0	289	578	1.0746	9.90055E-12	17	52	0.07452	3.01932E-12	1	4	0.006174	0
x_0^7		5	14	0.025448	0	1	4	0.006479	0	291	582	1.025266	9.12817E-12	18	57	0.074131	4.91517E-12	1	5	0.007024	0
x_0^8		5	14	0.029923	0	1	4	0.006719	0	291	582	1.024741	9.54947E-12	18	57	0.092419	4.84495E-12	1	5	0.005232	0
x_0^9		5	14	0.027699	0	1	4	0.005995	0	291	582	1.10511	9.12817E-12	18	57	0.069582	4.84495E-12	1	5	0.006618	0
x_0^{10}		39	115	0.228362	7.12533E-12	51	152	0.214068	7.626E-12	240	480	0.89138	9.30941E-12	44	135	0.189527	7.56343E-12	18	74	0.088677	0
x_0^{11}		61	182	0.3364	6.9232E-12	66	198	0.309866	8.0143E-13	287	574	1.059495	9.13318E-12	66	201	0.28335	7.93308E-12	16	66	0.082629	7.98013E-12
x_0^{12}		4	12	0.025018	0	6	18	0.022324	1.4746E-12	186	372	0.69576	9.76012E-12	12	37	0.062532	2.24693E-12	1	4	0.004834	0
x_0^{13}		27	80	0.131095	8.54152E-12	1	4	0.004377	1.5223E-15	229	458	0.844008	9.95237E-12	48	145	0.202096	5.66004E-12	1	4	0.004096	1.15378E-15
x_0^{14}		61	182	0.325281	6.92429E-12	66	198	0.326596	8.0141E-13	287	574	1.062315	9.13319E-12	65	198	0.295699	9.40401E-12	14	58	0.07005	4.36422E-12

Table 3: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in ExponetialSineCosine*

ExponetialSineCosine																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
x_0	DIM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
x_0^1		1	1	0.151578	0	1	2	0.030054	0	1	1	0.020678	0	1	1	0.018606	0	1	1	0.030068	0
x_0^2		37	112	0.002603	7.66064E-12	27	83	0.012301	5.4769E-12	42	84	0.002012	6.17907E-12	13	79	0.001245	2.9491E-12	48	289	0.005463	8.35578E-12
x_0^3		38	115	0.004478	5.13986E-12	28	86	0.001908	4.213E-12	43	86	0.008917	5.36455E-12	13	79	0.001278	3.92511E-12	50	301	0.004846	6.87421E-12
x_0^4		38	112	0.015728	5.17848E-12	28	83	0.001811	4.49387E-12	43	86	0.00218	6.24226E-12	15	86	0.001496	6.42834E-12	52	308	0.004864	6.35461E-12
x_0^5		38	115	0.002675	5.37509E-12	28	86	0.001851	5.23114E-12	43	86	0.002176	6.91634E-12	13	79	0.001296	4.1147E-12	51	307	0.004803	6.56526E-12
x_0^6		37	112	0.002636	4.95379E-12	27	83	0.001814	7.8116E-12	43	86	0.002169	7.57638E-12	15	87	0.001477	6.42834E-12	52	309	0.01211	6.38972E-12
x_0^7	1000	38	115	0.003133	4.81686E-12	29	89	0.002041	3.70042E-12	43	86	0.002159	6.81804E-12	14	87	0.001497	4.14278E-12	48	290	0.004613	7.79405E-12
x_0^8		35	107	0.002806	6.83208E-12	23	72	0.001619	4.97836E-12	43	86	0.002124	5.5401E-12	15	88	0.00151	6.42834E-12	48	290	0.004666	7.79405E-12
x_0^9		38	115	0.002787	4.90815E-12	29	89	0.001966	3.74255E-12	43	86	0.002134	6.8391E-12	14	87	0.001516	4.14278E-12	48	290	0.004757	7.6466E-12
x_0^{10}		75	250	0.005391	6.30549E-12	62	203	0.004147	7.95818E-12	39	78	0.001912	7.08177E-12	61	377	0.005967	9.65331E-12	63	441	0.006754	4.42195E-12
x_0^{11}		82	275	0.007031	7.22636E-12	73	240	0.005056	9.10481E-12	43	86	0.002184	5.89291E-12	69	426	0.007048	4.81024E-12	61	430	0.006733	8.84692E-12
x_0^{12}		30	91	0.003214	6.30195E-12	22	68	0.00141	3.77064E-12	34	68	0.001721	9.19838E-12	11	67	0.001074	2.63313E-12	38	229	0.003602	8.3277E-12
x_0^{13}		68	227	0.009019	7.13332E-12	60	196	0.003677	5.89442E-12	38	76	0.001734	5.12202E-12	56	345	0.00488	9.72235E-12	57	398	0.005341	8.92389E-13
x_0^{14}		82	275	0.016851	8.87518E-12	74	245	0.005156	3.78447E-12	43	86	0.002154	5.89916E-12	59	365	0.005919	6.77147E-12	57	402	0.006278	8.71251E-12
x_0^1		1	1	0.000306	0	1	2	0.000308	0	1	1	9.36E-05	0	1	1	0.000106	0	1	1	0.000106	0
x_0^2		38	115	0.024485	6.20615E-12	28	86	0.017436	6.05072E-12	43	86	0.021824	9.76996E-12	13	79	0.012682	9.32587E-12	50	301	0.048377	8.23785E-12
x_0^3		38	115	0.025436	8.81517E-12	29	89	0.020455	4.66294E-12	44	88	0.019918	8.4599E-12	14	85	0.015066	1.13243E-12	52	313	0.050588	6.79456E-12
x_0^4		38	112	0.028168	9.00391E-12	29	86	0.017732	4.996E-12	44	88	0.022661	9.85878E-12	16	92	0.015212	1.88738E-12	54	320	0.052704	6.30607E-12
x_0^5		38	115	0.024343	9.59233E-12	29	89	0.019702	5.77316E-12	45	90	0.022183	5.4623E-12	14	85	0.013509	1.22125E-12	53	319	0.051295	6.53921E-12
x_0^6		38	115	0.02724	5.9952E-12	28	86	0.017436	8.63754E-12	45	90	0.020939	5.9952E-12	16	93	0.017431	1.89848E-12	54	321	0.051978	6.30607E-12
x_0^7	10000	39	118	0.028433	5.88418E-12	30	92	0.020798	4.10783E-12	45	90	0.022791	5.41789E-12	15	93	0.015086	1.19904E-12	50	302	0.048182	7.68274E-12
x_0^8		35	107	0.022743	7.33857E-12	24	75	0.014866	5.50671E-12	44	88	0.020346	8.77076E-12	16	94	0.015636	1.88738E-12	50	302	0.046542	7.68274E-12
x_0^9		39	118	0.026777	5.87308E-12	30	92	0.020601	4.10783E-12	45	90	0.023542	5.41789E-12	15	93	0.014534	1.22125E-12	50	302	0.048972	7.66054E-12
x_0^{10}		74	247	0.049176	6.42282E-12	65	213	0.041308	8.21086E-12	39	78	0.018937	7.08487E-12	59	364	0.056437	8.76246E-12	62	434	0.064003	5.94514E-12
x_0^{11}		84	282	0.060591	9.04336E-12	74	245	0.049598	6.0465E-12	44	88	0.020267	9.32165E-12	67	413	0.069416	9.59801E-12	66	465	0.071328	9.07394E-13
x_0^{12}		28	85	0.01697	9.00391E-12	20	62	0.014077	9.71445E-12	33	66	0.016696	5.79536E-12	10	61	0.010554	8.9706E-12	36	217	0.033388	8.43769E-12
x_0^{13}		68	227	0.042607	7.13332E-12	60	196	0.035672	5.89442E-12	38	76	0.015838	5.12202E-12	56	345	0.050834	9.72235E-12	57	398	0.05249	8.9239E-13
x_0^{14}		84	282	0.059653	9.00351E-12	71	233	0.047962	9.31973E-12	44	88	0.021683	9.32275E-12	69	426	0.070789	3.69813E-12	64	451	0.067968	6.12528E-12
x_0^1		1	1	0.005089	0	1	2	0.004753	0	1	1	0.001112	0	1	1	0.00084	0	1	1	0.001664	0
x_0^2		39	118	0.323667	4.98538E-12	29	89	0.215516	6.7408E-12	45	90	0.300724	7.72383E-12	14	85	0.160883	2.73845E-12	52	313	0.587519	8.14513E-12
x_0^3		39	118	0.305046	7.65362E-12	30	92	0.260487	5.12582E-12	46	92	0.314344	6.7408E-12	14	85	0.162494	3.58105E-12	54	325	0.667154	6.67058E-12
x_0^4		39	115	0.304692	7.79405E-12	30	89	0.215307	5.4769E-12	46	92	0.280719	7.79405E-12	16	92	0.186759	6.00353E-12	56	332	0.660461	6.03863E-12
x_0^5		39	118	0.32464	8.49622E-12	30	92	0.21598	6.45993E-12	46	92	0.310595	8.70687E-12	14	85	0.166368	3.86192E-12	55	331	0.639671	6.28439E-12
x_0^6		39	118	0.294262	5.61733E-12	29	89	0.211071	9.61968E-12	46	92	0.297979	9.47925E-12	14	85	0.172036	3.51083E-12	56	333	0.708792	6.24928E-12
x_0^7	100000	40	121	0.330479	5.68755E-12	31	95	0.2268	4.52898E-12	46	92	0.304891	8.49622E-12	15	93	0.222762	3.86192E-12	52	314	0.618663	7.65362E-12
x_0^8		35	107	0.278233	5.61733E-12	25	78	0.19979	6.03863E-12	46	92	0.290471	6.95145E-12	17	99	0.221965	5.96842E-12	52	314	0.622921	7.65362E-12
x_0^9		40	121	0.294285	5.8982E-12	31	95	0.248421	4.52898E-12	46	92	0.27405	8.49622E-12	15	93	0.216377	3.86192E-12	52	314	0.625848	7.65362E-12
x_0^{10}		74	247	0.597182	6.64307E-12	64	212	0.480691	8.54496E-12	39	78	0.235065	7.08502E-12	60	370	0.657306	8.17058E-12	68	476	0.837599	9.11549E-12
x_0^{11}		87	292	0.735773	7.6501E-12	79	261	0.636613	5.14741E-12	46	92	0.310964	7.36989E-12	72	444	0.885054	8.96535E-12	68	479	0.888263	9.61665E-12
x_0^{12}		27	82	0.213333	6.03863E-12	19	59	0.144157	8.77708E-12	31	62	0.193569	7.37275E-12	10	61	0.13437	2.87888E-12	34	205	0.42941	8.426E-12
x_0^{13}		68	227	0.569215	7.13332E-12	60	196	0.438686	5.89442E-12	38	76	0.200148	5.12202E-12	56	345	0.572933	9.72235E-12	57	398	0.638386	8.92413E-13
x_0^{14}		87	292	0.793219	7.64699E-12	72	238	0.614841	9.62268E-12	46	92	0.266107	7.37053E-12	72	444	0.863561	8.17398E-12	69	486	0.906792	8.85652E-12

Table 4: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in Logarithmic*

Logarithmic																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
x_0	DIM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
x_0^1	1000	1	1	0.160448	0	1	2	0.02811	0	1	1	0.019389	0	1	1	0.018169	0	1	1	0.021571	0
x_0^2		10	29	0.000528	3.26181E-12	9	26	0.000376	5.68189E-13	260	520	0.009169	9.1962E-12	15	45	0.000589	4.62265E-12	34	102	0.00131	8.52279E-12
x_0^3		11	31	0.000591	9.49081E-12	10	28	0.000422	7.29521E-13	267	534	0.016465	9.78543E-12	16	48	0.000599	3.03032E-12	35	105	0.009336	6.20796E-12
x_0^4		12	33	0.000581	5.83268E-12	10	28	0.000416	2.23767E-12	270	540	0.009465	9.39261E-12	16	48	0.00063	4.88921E-12	35	105	0.001529	6.95502E-12
x_0^5		12	33	0.000592	7.93707E-12	10	28	0.000437	4.65071E-12	272	544	0.016118	9.60305E-12	16	48	0.000667	7.34433E-12	35	105	0.001656	7.38642E-12
x_0^6		12	34	0.000601	5.68187E-13	10	29	0.000479	2.52526E-13	276	552	0.009754	9.28739E-12	17	51	0.000718	2.32886E-12	35	105	0.001458	7.48112E-12
x_0^7		12	34	0.000635	7.85637E-13	10	29	0.000449	3.99831E-13	279	558	0.016593	9.3365E-12	17	51	0.00072	4.1176E-12	35	105	0.001441	6.72354E-12
x_0^8		12	34	0.000598	8.62799E-13	10	29	0.000445	4.41919E-13	280	560	0.010069	9.69424E-12	17	51	0.000695	5.32412E-12	35	105	0.001605	6.07468E-12
x_0^9		12	34	0.000584	7.92657E-13	10	29	0.000453	3.99834E-13	279	558	0.010115	9.31545E-12	17	51	0.000682	4.10356E-12	35	105	0.001462	6.72705E-12
x_0^{10}		49	145	0.002468	9.6202E-12	51	152	0.002319	8.20092E-12	247	494	0.012139	9.78739E-12	51	153	0.009029	6.25269E-12	54	216	0.002645	8.47039E-12
x_0^{11}		56	166	0.002849	9.71739E-12	57	170	0.00261	8.82237E-12	273	546	0.009691	9.00527E-12	58	174	0.002405	5.00044E-12	58	232	0.002874	8.88842E-12
x_0^{12}		8	23	0.000408	1.75366E-13	7	20	0.000343	1.16794E-12	208	416	0.0108	9.8205E-12	10	30	0.000407	3.40911E-12	28	84	0.001192	6.39736E-12
x_0^{13}		49	145	0.00252	9.67769E-12	50	149	0.009254	7.36431E-12	233	466	0.008378	9.33893E-12	47	141	0.001826	6.7036E-12	52	207	0.002655	9.37614E-12
x_0^{14}		56	166	0.002828	9.70857E-12	57	170	0.002597	8.80883E-12	273	546	0.01002	9.02227E-12	58	174	0.002397	4.75282E-12	58	232	0.002969	8.8942E-12
x_0^1	10000	1	1	0.000254	0	1	2	0.000242	0	1	1	7.56E-05	0	1	1	0.000066	0	1	1	8.12E-05	0
x_0^2		10	29	0.006433	6.15002E-12	9	26	0.003715	1.68737E-12	270	540	0.092446	9.88E-12	16	48	0.006402	2.17582E-12	36	108	0.015552	5.88359E-12
x_0^3		11	31	0.004935	8.54786E-12	10	28	0.004002	2.26463E-12	278	556	0.095552	9.45815E-12	16	48	0.006739	8.8809E-12	36	108	0.016276	9.68018E-12
x_0^4		12	33	0.005439	6.12782E-12	10	28	0.00434	6.9271E-12	281	562	0.099673	9.08072E-12	17	51	0.009975	2.30903E-12	37	111	0.015719	4.80679E-12
x_0^5		12	33	0.005349	9.70238E-12	10	29	0.006038	3.33035E-13	283	566	0.108511	9.28054E-12	17	51	0.007223	3.46355E-12	37	111	0.013943	5.11762E-12
x_0^6		12	34	0.007623	1.26553E-12	10	29	0.004538	7.99281E-13	286	572	0.100384	9.96881E-12	17	51	0.008592	6.81609E-12	37	111	0.016139	5.19532E-12
x_0^7		12	34	0.005307	1.79838E-12	11	31	0.004874	5.55056E-13	290	580	0.100408	9.01411E-12	18	54	0.0088	1.93159E-12	37	111	0.014015	4.66247E-12
x_0^8		12	34	0.005166	1.90939E-12	11	31	0.004482	5.9946E-13	291	582	0.102411	9.36935E-12	18	54	0.010549	2.48665E-12	36	108	0.013651	9.52476E-12
x_0^9		12	34	0.00555	1.79838E-12	11	31	0.004588	5.55056E-13	290	580	0.10033	9.01411E-12	18	54	0.008562	1.93159E-12	37	111	0.01585	4.68467E-12
x_0^{10}		49	145	0.023437	9.18362E-12	51	152	0.025131	8.36886E-12	247	494	0.085204	9.55425E-12	51	153	0.023049	9.40378E-12	54	216	0.025871	9.21529E-12
x_0^{11}		59	175	0.027901	8.91352E-12	60	179	0.026538	6.89673E-12	283	566	0.100804	9.67617E-12	60	180	0.024215	7.8949E-12	61	244	0.027042	6.50823E-12
x_0^{12}		8	23	0.00338	2.22025E-14	6	18	0.002711	7.9928E-13	197	394	0.067454	9.70238E-12	8	24	0.005622	4.04081E-12	27	81	0.012579	4.9511E-12
x_0^{13}		49	145	0.023008	9.76458E-12	50	149	0.02281	7.49451E-12	233	466	0.079199	9.1306E-12	47	141	0.019256	8.3219E-12	53	211	0.028064	5.26184E-12
x_0^{14}		59	175	0.027761	8.91242E-12	60	179	0.02678	6.89561E-12	283	566	0.096502	9.67798E-12	60	180	0.025987	7.9146E-12	61	244	0.030126	6.50823E-12
x_0^1	100000	1	1	0.002138	0	1	2	0.002189	0	1	1	0.001043	0	1	1	0.000994	0	1	1	0.000742	0
x_0^2		11	31	0.0526	5.33641E-12	9	26	0.043417	5.33641E-12	281	562	1.136531	9.76002E-12	16	48	0.08812	6.81095E-12	37	111	0.150897	8.39081E-12
x_0^3		12	33	0.075886	4.59915E-12	10	28	0.047871	7.16203E-12	289	578	1.162347	9.33872E-12	17	51	0.076942	4.49382E-12	38	114	0.189416	6.10879E-12
x_0^4		12	33	0.075058	9.47916E-12	10	29	0.067192	4.91512E-13	291	582	1.170154	9.97067E-12	17	51	0.079317	7.23224E-12	38	114	0.163933	6.88117E-12
x_0^5		12	34	0.065547	1.7554E-12	11	31	0.057618	4.21295E-13	294	588	1.213766	9.19829E-12	18	54	0.08537	1.7554E-12	38	114	0.193424	7.23224E-12
x_0^6		12	34	0.069701	2.10648E-12	11	31	0.061381	1.12346E-12	297	594	1.218966	9.90045E-12	18	54	0.088559	3.44058E-12	38	114	0.17335	7.37268E-12
x_0^7		12	34	0.065784	2.52778E-12	11	31	0.051751	1.7554E-12	300	600	1.247476	9.90045E-12	18	54	0.084894	6.03857E-12	38	114	0.169483	6.67052E-12
x_0^8		12	34	0.070457	2.66821E-12	11	31	0.051014	1.96605E-12	302	604	1.210567	9.26851E-12	18	54	0.084326	7.79397E-12	38	114	0.16808	5.96836E-12
x_0^9		12	34	0.063618	2.52777E-12	11	31	0.055375	1.7554E-12	300	600	1.288567	9.90045E-12	18	54	0.088917	6.03857E-12	38	114	0.210125	6.67052E-12
x_0^{10}		49	145	0.326998	9.14021E-12	51	152	0.287059	8.38617E-12	247	494	0.970095	9.53123E-12	51	153	0.243813	9.71004E-12	54	216	0.3082	9.29292E-12
x_0^{11}		62	184	0.371274	6.97413E-12	62	185	0.355018	8.50549E-12	294	588	1.233688	9.57676E-12	62	186	0.283555	9.4468E-12	63	252	0.361214	8.21123E-12
x_0^{12}		8	22	0.048841	5.05555E-12	6	18	0.028551	2.45756E-13	186	372	0.742545	9.76002E-12	6	18	0.032439	4.98533E-12	25	75	0.114701	8.00462E-12
x_0^{13}		49	145	0.28073	9.77314E-12	50	149	0.269271	7.50769E-12	233	466	0.950301	9.11003E-12	47	141	0.240365	8.50206E-12	53	211	0.280625	5.30795E-12
x_0^{14}		62	184	0.358166	6.97404E-12	62	185	0.34987	8.50539E-12	294	588	1.165306	9.57695E-12	62	186	0.303714	9.45137E-12	63	252	0.356514	8.2109E-12

Table 5: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in Modified-NonsmoothSine*

ModifiedNonsmoothSine																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
x_0	DIM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
x_0^1	1000	15	45	0.130036	9.27211E-12	13	39	0.029872	1.32183E-12	139	278	0.024473	9.73554E-12	11	45	0.018683	5.36631E-12	56	225	0.02295	8.27855E-12
x_0^2		15	45	0.000607	6.76713E-12	13	39	0.000491	7.86427E-13	137	274	0.003994	8.34525E-12	11	45	0.000399	2.48216E-12	56	225	0.0021	8.0872E-12
x_0^3		15	44	0.000553	8.53484E-12	12	36	0.000413	3.45466E-12	131	262	0.01095	8.59452E-12	11	45	0.000388	6.1264E-13	54	217	0.002037	8.60856E-12
x_0^4		14	41	0.000582	8.00119E-12	11	33	0.000402	5.97368E-12	121	242	0.003478	8.28381E-12	10	41	0.007911	1.20246E-12	50	201	0.001799	7.86778E-12
x_0^5		15	45	0.00063	3.2721E-12	12	36	0.000439	3.84787E-12	132	264	0.003957	8.43653E-12	11	45	0.000496	6.19662E-13	55	221	0.001983	7.98715E-12
x_0^6		15	45	0.000605	4.67467E-12	12	36	0.000476	7.55531E-12	137	274	0.004139	8.16093E-12	11	45	0.000413	1.44822E-12	57	229	0.002073	9.64426E-12
x_0^7		15	44	0.000592	8.33823E-12	12	36	0.00046	1.90989E-12	139	278	0.004096	8.66825E-12	11	45	0.000437	2.00118E-12	59	237	0.002216	6.47573E-12
x_0^8		16	47	0.000636	6.12114E-12	13	39	0.000493	2.80165E-12	140	280	0.004146	8.66649E-12	10	40	0.000553	3.61265E-12	59	237	0.002216	6.47573E-12
x_0^9		15	44	0.000599	8.39791E-12	12	36	0.000783	1.98011E-12	139	278	0.011095	8.64718E-12	11	45	0.000522	1.99942E-12	59	237	0.002401	6.46344E-12
x_0^{10}		49	146	0.002044	7.20634E-12	58	174	0.003885	9.31125E-12	139	278	0.004208	9.60664E-12	60	263	0.00255	5.43798E-12	56	289	0.002503	9.04621E-12
x_0^{11}		56	168	0.002401	8.65555E-12	66	199	0.015573	8.43546E-12	136	272	0.004084	9.80226E-12	64	281	0.002552	5.95588E-12	60	311	0.003934	7.18995E-12
x_0^{12}		15	45	0.000618	9.27387E-12	13	39	0.000475	1.31656E-12	139	278	0.00428	9.71097E-12	11	45	0.000416	5.34875E-12	56	225	0.013826	8.29083E-12
x_0^{13}		46	137	0.002657	8.44788E-12	51	153	0.001946	9.74792E-12	139	278	0.004094	9.72788E-12	56	244	0.002248	2.81979E-12	55	282	0.002312	4.6777E-12
x_0^{14}		56	168	0.004195	8.59324E-12	66	199	0.002531	8.40785E-12	136	272	0.010554	9.79869E-12	59	260	0.002427	7.8906E-12	61	313	0.002609	3.75265E-12
x_0^1	10000	16	47	0.005973	6.98885E-12	13	39	0.004928	4.17999E-12	145	290	0.042245	8.86513E-12	12	49	0.004529	9.49241E-13	59	237	0.022744	6.28386E-12
x_0^2		16	47	0.007798	5.16254E-12	13	39	0.006421	2.4869E-12	142	284	0.041649	9.36473E-12	11	45	0.006623	7.84928E-12	58	233	0.020456	9.88098E-12
x_0^3		15	45	0.006128	6.2117E-12	13	39	0.00433	7.43849E-13	136	272	0.039134	9.63118E-12	11	45	0.00418	1.93734E-12	57	229	0.022022	6.53366E-12
x_0^4		14	42	0.005274	8.26561E-12	12	36	0.004594	1.29896E-12	126	252	0.035326	9.28146E-12	10	41	0.003608	3.80251E-12	52	209	0.019802	9.59233E-12
x_0^5		15	45	0.005554	6.42819E-12	13	39	0.004392	8.38218E-13	137	274	0.039165	9.45355E-12	11	45	0.003868	1.95954E-12	57	229	0.019977	9.75331E-12
x_0^6		15	45	0.008185	9.93094E-12	13	39	0.005105	1.63203E-12	142	284	0.040694	9.15379E-12	11	45	0.003965	4.57967E-12	60	241	0.023154	7.32747E-12
x_0^7		15	45	0.005574	3.93019E-12	12	36	0.006341	6.03961E-12	144	288	0.041401	9.72E-12	11	45	0.004289	6.32827E-12	61	245	0.024743	7.92699E-12
x_0^8		16	48	0.006177	4.35763E-12	13	39	0.004402	8.85958E-12	145	290	0.04563	9.72E-12	11	44	0.005909	6.38378E-13	61	245	0.026765	7.92699E-12
x_0^9		15	45	0.00576	3.91909E-12	12	36	0.004204	6.05627E-12	144	288	0.041322	9.72E-12	11	45	0.003928	6.32827E-12	61	245	0.025042	7.92699E-12
x_0^{10}		47	140	0.01961	6.081E-12	53	159	0.019807	7.50205E-12	145	290	0.042908	8.85985E-12	64	279	0.026254	6.33963E-12	59	304	0.030094	8.60121E-12
x_0^{11}		59	177	0.024267	9.31694E-12	69	208	0.02586	7.68251E-12	142	284	0.043191	8.93433E-12	65	279	0.026051	3.04317E-12	65	334	0.029794	6.79745E-12
x_0^{12}		16	47	0.006106	6.98885E-12	13	39	0.00454	4.17999E-12	145	290	0.042196	8.86513E-12	12	49	0.004821	9.49241E-13	59	237	0.022615	6.28386E-12
x_0^{13}		43	128	0.017744	9.7228E-12	47	141	0.01796	9.75965E-12	145	290	0.047068	8.86444E-12	58	255	0.024829	2.52358E-12	52	265	0.02271	8.56123E-12
x_0^{14}		59	177	0.023199	9.3141E-12	69	208	0.025479	7.6799E-12	142	284	0.044317	8.93406E-12	68	301	0.027951	7.85136E-12	63	324	0.030477	8.33633E-12
x_0^1	100000	16	48	0.080335	5.61733E-12	14	42	0.054226	8.95263E-13	150	300	0.496525	9.95321E-12	12	49	0.051704	3.00176E-12	61	245	0.273554	7.67117E-12
x_0^2		16	48	0.076156	3.75659E-12	13	39	0.051307	7.84671E-12	148	296	0.489397	8.51377E-12	12	49	0.059795	1.40433E-12	61	245	0.271816	7.46052E-12
x_0^3		16	47	0.073151	5.14337E-12	13	39	0.049872	2.35226E-12	142	284	0.488875	8.77708E-12	11	45	0.057712	6.1264E-12	59	237	0.268235	7.98715E-12
x_0^4		15	45	0.080381	3.24752E-12	12	36	0.051983	4.10768E-12	132	264	0.443914	8.47866E-12	11	45	0.041868	7.02167E-13	55	221	0.234999	7.30253E-12
x_0^5		16	47	0.092126	5.49445E-12	13	39	0.05557	2.65068E-12	143	286	0.507697	8.60154E-12	11	45	0.041908	6.19662E-12	60	241	0.278206	7.3903E-12
x_0^6		16	47	0.090308	8.6191E-12	13	39	0.063077	5.16093E-12	148	296	0.513407	8.32068E-12	12	49	0.045688	8.07492E-13	62	249	0.260506	8.97018E-12
x_0^7		15	45	0.075742	7.7765E-12	13	39	0.056347	1.28145E-12	150	300	0.530479	8.86485E-12	12	49	0.056135	1.10591E-12	63	253	0.297506	9.6899E-12
x_0^8		16	48	0.080222	8.58399E-12	14	42	0.060904	1.93096E-12	151	302	0.497725	8.86485E-12	11	44	0.042402	2.01873E-12	63	253	0.268494	9.6899E-12
x_0^9		15	45	0.068464	7.7765E-12	13	39	0.067725	1.28145E-12	150	300	0.534222	8.86485E-12	12	49	0.053984	1.10591E-12	63	253	0.294618	9.6899E-12
x_0^{10}		45	134	0.223304	9.60526E-12	50	150	0.230688	7.84062E-12	150	300	0.488381	9.94308E-12	58	249	0.309889	8.01934E-12	68	346	0.336195	5.56148E-12
x_0^{11}		62	186	0.279024	8.21811E-12	72	217	0.337208	6.69621E-12	148	296	0.489515	8.14528E-12	68	295	0.311395	7.80284E-12	72	368	0.396887	4.76195E-12
x_0^{12}		16	48	0.072401	5.61733E-12	14	42	0.065995	8.95263E-13	150	300	0.49756	9.95321E-12	12	49	0.047372	3.00176E-12	61	245	0.304155	7.67117E-12
x_0^{13}		42	125	0.196735	7.46065E-12	44	132	0.18244	7.96574E-12	150	300	0.549222	9.95313E-12	62	274	0.298722	6.88801E-12	55	281	0.275779	6.57513E-12
x_0^{14}		62	186	0.288853	8.21776E-12	72	217	0.30556	6.69639E-12	148	296	0.50283	8.14525E-12	69	301	0.327376	4.61915E-12	63	326	0.361593	5.0452E-12

Table 6: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in ModifiedTridiagonal*

ModifiedTridiagonal																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
x_0	DIM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
x_0^1	1000	3	8	0.231721	85.99124	2	7	0.028736	85.99124	2000	4001	0.191195	85.99414	2000	4001	0.178032	85.99414	2000	4001	0.184164	85.99414
x_0^2		4	30	0.000698	85.95962	2	22	0.000471	85.99124	2000	4001	0.160345	85.99414	2000	4001	0.154001	85.99414	2000	4001	0.150185	85.99414
x_0^3		4	33	0.000828	85.95962	2	22	0.000469	85.99124	2000	4001	0.160933	85.99414	2000	4001	0.149932	85.99414	2000	4001	0.147074	85.99414
x_0^4		4	33	0.001027	85.95962	2	22	0.000499	85.99124	2000	4001	0.158964	85.99414	2000	4001	0.15258	85.99414	2000	4001	0.146543	85.99414
x_0^5		4	33	0.001012	85.95962	2	22	0.000502	85.99124	2000	4001	0.165145	85.99414	2000	4001	0.15031	85.99414	2000	4001	0.147414	85.99414
x_0^6		4	34	0.000956	85.95962	2	22	0.000508	85.99124	2000	4001	0.160625	85.99414	2000	4001	0.152122	85.99414	2000	4001	0.150519	85.99414
x_0^7		4	34	0.000964	85.95962	2	22	0.000492	85.99124	2000	4001	0.160334	85.99414	2000	4001	0.149569	85.99414	2000	4001	0.155071	85.99414
x_0^8		4	34	0.000966	85.95962	2	22	0.000494	85.99124	2000	4001	0.160368	85.99414	2000	4001	0.148782	85.99414	2000	4001	0.146725	85.99414
x_0^9		4	34	0.00104	85.95962	2	22	0.000511	85.99124	2000	4001	0.160144	85.99414	2000	4001	0.151788	85.99414	2000	4001	0.151293	85.99414
x_0^{10}		6	12	0.000645	85.99631	3	7	0.000423	85.99588	2000	4001	0.160276	85.99705	2000	4001	0.150997	85.99705	2000	4001	0.14664	85.99705
x_0^{11}		6	12	0.000568	85.99016	4	9	0.000402	85.99173	2000	4001	0.156587	85.99705	2000	4001	0.155277	85.99705	2000	4001	0.146365	85.99705
x_0^{12}		3	6	0.00029	85.99124	2	5	0.000178	85.99124	2000	4001	0.164713	85.99414	2000	4001	0.150494	85.99414	2000	4001	0.149068	85.99414
x_0^{13}		6	12	0.000526	85.99309	3	7	0.000269	85.99274	2000	4001	0.162256	85.99705	2000	4001	0.152952	85.99705	2000	4001	0.149828	85.99705
x_0^{14}		6	12	0.000527	85.99016	4	9	0.000389	85.99173	2000	4001	0.162393	85.99705	2000	4001	0.160727	85.99705	2000	4001	0.15486	85.99705
x_0^1	10000	3	6	0.002576	271.8382	2	5	0.002014	271.8382	2000	4001	1.47443	271.8391	2000	4001	1.391867	271.8391	2000	4001	1.358381	271.8391
x_0^2		4	24	0.005559	271.8282	2	20	0.004593	271.8382	2000	4001	1.465424	271.8391	2000	4001	1.401598	271.8391	2000	4001	1.364914	271.8391
x_0^3		4	26	0.006256	271.8282	2	22	0.004887	271.8382	2000	4001	1.470509	271.8391	2000	4001	1.396984	271.8391	2000	4001	1.354855	271.8391
x_0^4		4	26	0.010122	271.8282	2	22	0.004957	271.8382	2000	4001	1.461524	271.8391	2000	4001	1.399792	271.8391	2000	4001	1.377035	271.8391
x_0^5		4	27	0.006108	271.8282	2	22	0.006404	271.8382	2000	4001	1.469177	271.8391	2000	4001	1.395878	271.8391	2000	4001	1.377051	271.8391
x_0^6		4	27	0.006066	271.8282	2	22	0.004736	271.8382	2000	4001	1.461611	271.8391	2000	4001	1.409148	271.8391	2000	4001	1.373888	271.8391
x_0^7		4	28	0.006122	271.8282	2	22	0.004977	271.8382	2000	4001	1.476393	271.8391	2000	4001	1.400263	271.8391	2000	4001	1.356988	271.8391
x_0^8		4	28	0.008468	271.8282	2	22	0.004657	271.8382	2000	4001	1.487894	271.8391	2000	4001	1.386006	271.8391	2000	4001	1.357263	271.8391
x_0^9		4	28	0.006341	271.8282	2	22	0.004761	271.8382	2000	4001	1.46721	271.8391	2000	4001	1.399912	271.8391	2000	4001	1.362222	271.8391
x_0^{10}		6	12	0.017924	271.8398	3	7	0.019	271.8396	2000	4001	1.451933	271.84	2000	4001	1.554331	271.84	2000	4001	1.386372	271.84
x_0^{11}		5	10	0.008488	271.838	4	9	0.010188	271.8382	2000	4001	1.464018	271.84	2000	4001	1.469192	271.84	2000	4001	1.356186	271.84
x_0^{12}		2	4	0.001721	271.8382	2	5	0.001698	271.8382	2000	4001	1.497376	271.8391	2000	4001	1.395733	271.8391	2000	4001	1.355177	271.8391
x_0^{13}		6	12	0.004591	271.8388	3	7	0.002457	271.8387	2000	4001	1.465711	271.84	2000	4001	1.411824	271.84	2000	4001	1.371163	271.84
x_0^{14}		5	10	0.008435	271.838	4	9	0.008501	271.8382	2000	4001	1.471108	271.84	2000	4001	1.477684	271.84	2000	4001	1.382669	271.84
x_0^1	100000	3	6	0.025477	859.5994	2	5	0.015872	859.5994	2000	4001	17.54576	859.5996	2000	6000	20.98722	859.5996	2000	4001	16.04822	859.5996
x_0^2		4	20	0.057026	859.5962	2	17	0.044786	859.5994	2000	4001	18.45236	859.5996	2000	6000	20.72763	859.5996	2000	4001	15.99819	859.5996
x_0^3		4	22	0.065216	859.5962	2	19	0.042054	859.5994	2000	4001	17.78341	859.5996	2000	6000	20.70359	859.5996	2000	4001	16.00872	859.5996
x_0^4		4	22	0.070612	859.5962	2	19	0.045763	859.5994	2000	4001	18.69586	859.5996	2000	6001	20.75626	859.5996	2000	4001	16.04131	859.5996
x_0^5		4	23	0.075782	859.5962	2	20	0.040514	859.5994	2000	4001	18.6064	859.5996	2000	6001	20.66175	859.5996	2000	4001	16.02138	859.5996
x_0^6		4	23	0.062159	859.5962	2	20	0.042123	859.5994	2000	4001	17.68645	859.5996	2000	6001	20.53634	859.5996	2000	4001	16.20727	859.5996
x_0^7		4	23	0.054602	859.5962	2	20	0.050739	859.5994	2000	4001	17.89573	859.5996	2000	6001	20.68192	859.5996	2000	4001	16.14094	859.5996
x_0^8		4	24	0.073548	859.5962	2	21	0.039144	859.5994	2000	4001	18.05967	859.5996	2000	6001	20.63041	859.5996	2000	4001	16.08834	859.5996
x_0^9		4	23	0.074717	859.5962	2	20	0.046138	859.5994	2000	4001	17.75099	859.5996	2000	6001	20.57291	859.5996	2000	4001	16.37621	859.5996
x_0^{10}		5	10	1.48764	859.5999	3	7	1.529416	859.5998	2000	4001	19.0808	859.5999	2000	6000	33.34328	859.5999	2000	4001	16.34733	859.5999
x_0^{11}		6	12	0.947087	859.5994	4	9	0.627514	859.5994	2000	4001	18.18503	859.5999	2000	6001	31.32929	859.5999	2000	4001	16.332	859.5999
x_0^{12}		1	2	0.010932	859.5994	1	3	0.009011	859.5994	2000	4001	17.90768	859.5996	2000	6000	20.83506	859.5996	2000	4001	16.2013	859.5996
x_0^{13}		5	10	0.050858	859.5995	3	7	0.028582	859.5995	2000	4001	17.72982	859.5999	2000	6000	20.8514	859.5999	2000	4001	16.17115	859.5999
x_0^{14}		6	12	0.92695	859.5994	4	9	0.622294	859.5994	2000	4001	18.27345	859.5999	2000	6001	31.43937	859.5999	2000	4001	16.35037	859.5999

Table 7: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in ModifiedTrigI*

ModifiedTrigI																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
x_0	DIM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
x_0^1	1000	15	44	0.253689	8.33823E-12	12	36	0.02901	1.90287E-12	139	278	0.032687	8.66123E-12	11	45	0.020213	2.00118E-12	59	237	0.031553	6.47398E-12
x_0^2		15	45	0.000939	4.67643E-12	12	36	0.000696	7.55531E-12	137	274	0.006698	8.15918E-12	11	45	0.00101	1.44997E-12	57	229	0.004376	9.64075E-12
x_0^3		15	45	0.000938	3.2721E-12	12	36	0.000961	3.8549E-12	132	264	0.006437	8.43302E-12	11	45	0.009715	6.21418E-13	55	221	0.003744	7.99768E-12
x_0^4		14	41	0.001113	8.0047E-12	11	33	0.007671	5.97895E-12	121	242	0.013225	8.28557E-12	10	41	0.000712	1.20773E-12	50	201	0.003436	7.87129E-12
x_0^5		15	44	0.000993	8.53835E-12	12	36	0.000683	3.45466E-12	131	262	0.006489	8.60154E-12	11	45	0.000774	6.17907E-13	54	217	0.012213	8.61559E-12
x_0^6		15	45	0.00099	6.76187E-12	13	39	0.000761	7.93448E-13	137	274	0.01342	8.34876E-12	11	45	0.000836	2.48567E-12	56	225	0.003762	8.08896E-12
x_0^7		15	45	0.000971	9.27211E-12	13	39	0.00086	1.32007E-12	139	278	0.00674	9.73905E-12	11	45	0.000878	5.37158E-12	56	225	0.003915	8.2645E-12
x_0^8		15	45	0.000998	9.95672E-12	13	39	0.000967	1.5869E-12	140	280	0.006881	9.7531E-12	11	45	0.000912	7.36573E-12	56	225	0.004355	6.39674E-12
x_0^9		15	45	0.000948	9.27913E-12	13	39	0.000814	1.32007E-12	139	278	0.010294	9.71097E-12	11	45	0.000863	5.35051E-12	56	225	0.004034	8.27855E-12
x_0^{10}		61	187	0.014753	9.1182E-12	62	189	0.004038	7.51869E-12	139	278	0.006831	8.54851E-12	39	227	0.005762	9.79427E-12	23	152	0.009009	4.04517E-12
x_0^{11}		69	212	0.004693	9.63474E-12	75	225	0.004646	9.32561E-12	136	272	0.010439	9.78284E-12	45	249	0.006059	4.64113E-12	27	176	0.00294	5.16739E-12
x_0^{12}		15	44	0.000998	8.39791E-12	12	36	0.000747	1.97309E-12	139	278	0.006821	8.64718E-12	11	45	0.001107	2.00118E-12	59	237	0.004306	6.45993E-12
x_0^{13}		62	191	0.004204	7.94932E-12	60	183	0.01146	4.90182E-12	139	278	0.006857	8.58951E-12	38	221	0.013745	5.84461E-12	25	163	0.002399	4.18735E-12
x_0^{14}		67	212	0.013716	9.0129E-12	68	204	0.004139	6.70264E-12	136	272	0.009906	9.78441E-12	45	249	0.003986	4.39087E-12	27	176	0.002643	5.23728E-12
x_0^1	10000	15	45	0.00865	3.90799E-12	12	36	0.006747	6.01741E-12	144	288	0.067664	9.70335E-12	11	45	0.008757	6.32827E-12	61	245	0.040979	7.90479E-12
x_0^2		15	45	0.011625	9.9476E-12	13	39	0.009527	1.63203E-12	142	284	0.065361	9.15934E-12	11	45	0.007945	4.58522E-12	60	241	0.041826	7.30527E-12
x_0^3		15	45	0.008374	6.43929E-12	13	39	0.010384	8.21565E-13	137	274	0.063796	9.448E-12	11	45	0.006903	1.96509E-12	57	229	0.039405	9.76996E-12
x_0^4		14	42	0.011119	8.23785E-12	12	36	0.006886	1.28786E-12	126	252	0.063191	9.28146E-12	10	41	0.009053	3.81917E-12	52	209	0.035472	9.61453E-12
x_0^5		15	45	0.00899	6.20615E-12	13	39	0.009584	7.32747E-13	136	272	0.063806	9.63674E-12	11	45	0.007151	1.95399E-12	57	229	0.039457	6.53921E-12
x_0^6		16	47	0.011101	5.15143E-12	13	39	0.007487	2.4869E-12	142	284	0.065769	9.37028E-12	11	45	0.009602	7.86038E-12	58	233	0.043891	9.88098E-12
x_0^7		16	47	0.010756	6.99441E-12	13	39	0.009997	4.17444E-12	145	290	0.070778	8.88178E-12	12	49	0.007936	9.76996E-13	59	237	0.051667	6.23945E-12
x_0^8		16	47	0.011478	7.87148E-12	13	39	0.007548	5.01821E-12	146	292	0.071402	8.88178E-12	12	49	0.010325	1.33227E-12	58	233	0.042953	7.81597E-12
x_0^9		16	47	0.011089	6.99441E-12	13	39	0.010019	4.17444E-12	145	290	0.072073	8.88178E-12	12	49	0.008233	9.76996E-13	59	237	0.043414	6.23945E-12
x_0^{10}		66	204	0.042849	9.789E-12	58	175	0.038455	7.94784E-12	144	288	0.077122	9.70032E-12	39	228	0.035679	6.26064E-12	23	152	0.02314	6.52648E-12
x_0^{11}		72	224	0.050444	9.05486E-12	69	206	0.045839	8.46758E-12	142	284	0.069843	8.93234E-12	40	219	0.03482	7.0859E-12	27	175	0.031286	8.55582E-12
x_0^{12}		15	45	0.010664	3.93019E-12	12	36	0.007025	6.08402E-12	144	288	0.06742	9.70335E-12	11	45	0.006885	6.32827E-12	61	245	0.041975	7.90479E-12
x_0^{13}		64	197	0.043565	5.01664E-12	66	198	0.044141	6.02041E-12	144	288	0.072154	9.71887E-12	40	233	0.038221	3.86368E-12	26	169	0.025983	8.0889E-12
x_0^{14}		65	199	0.043485	5.89843E-12	71	212	0.047871	7.77468E-12	142	284	0.067649	8.93267E-12	40	219	0.036146	7.85739E-12	27	175	0.027145	8.89696E-12
x_0^1	100000	15	45	0.111241	7.86427E-12	13	39	0.082986	1.33412E-12	150	300	0.876609	8.8473E-12	12	49	0.092147	1.15858E-12	63	253	0.456045	9.61968E-12
x_0^2		16	47	0.113416	8.63665E-12	13	39	0.092044	5.16093E-12	148	296	0.851621	8.39089E-12	12	49	0.102897	8.426E-13	62	249	0.482202	8.91752E-12
x_0^3		16	47	0.123126	5.54712E-12	13	39	0.086588	2.59802E-12	143	286	0.765465	8.63665E-12	11	45	0.094004	6.21418E-12	60	241	0.429559	7.44297E-12
x_0^4		15	45	0.106168	3.30018E-12	12	36	0.092519	4.07257E-12	132	264	0.747858	8.46111E-12	11	45	0.089078	7.02167E-13	55	221	0.439663	7.30253E-12
x_0^5		16	47	0.10667	5.09071E-12	13	39	0.090723	2.31715E-12	142	284	0.782434	8.77708E-12	11	45	0.093629	6.17907E-12	59	237	0.49467	8.0047E-12
x_0^6		16	48	0.12026	3.65127E-12	13	39	0.100109	7.86427E-12	148	296	0.801307	8.49622E-12	12	49	0.104081	1.40433E-12	61	245	0.470889	7.51318E-12
x_0^7		16	48	0.127362	5.61733E-12	14	42	0.114355	9.83033E-13	150	300	0.829999	9.97077E-12	12	49	0.109371	3.08953E-12	61	245	0.475779	7.51318E-12
x_0^8		16	48	0.122854	6.24928E-12	14	42	0.114995	1.08836E-12	151	302	0.871129	9.97077E-12	12	49	0.089752	4.213E-12	60	241	0.456637	9.61968E-12
x_0^9		16	48	0.12199	5.54712E-12	14	42	0.123974	9.83033E-13	150	300	0.896348	9.97077E-12	12	49	0.100988	3.08953E-12	61	245	0.462257	7.51318E-12
x_0^{10}		59	185	0.447258	7.90408E-12	66	199	0.526486	5.55961E-12	150	300	0.829623	8.83395E-12	36	209	0.376461	5.02617E-12	28	180	0.293541	2.11448E-12
x_0^{11}		73	229	0.565117	8.89686E-12	64	193	0.467156	8.08534E-12	148	296	0.840023	8.14632E-12	40	215	0.430485	8.17414E-12	31	199	0.350826	3.68213E-12
x_0^{12}		15	45	0.121085	7.86427E-12	13	39	0.104742	1.33412E-12	150	300	0.79169	8.8473E-12	12	49	0.106555	1.15858E-12	63	253	0.50189	9.61968E-12
x_0^{13}		62	187	0.477985	8.04222E-12	72	217	0.56553	5.44355E-12	150	300	0.827569	8.84666E-12	40	231	0.432716	5.39097E-12	23	149	0.248798	7.36273E-12
x_0^{14}		72	223	0.579242	8.8292E-12	61	184	0.483482	9.89609E-12	148	296	0.831448	8.14763E-12	40	215	0.396363	8.20563E-12	31	199	0.341468	3.68222E-12

Table 8: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in NonmoothLogarithmic*

NonmoothLogarithmic																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
x_0	DIM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
x_0^1	1000	1	1	0.160222	0	1	2	0.027583	0	1	1	0.019366	0	1	1	0.019438	0	1	1	0.022127	0
x_0^2		2	4	9.24E-05	0	1	3	0.000047	0	260	520	0.015908	9.1962E-12	2	5	7.78E-05	0	1	3	4.78E-05	0
x_0^3		2	4	9.91E-05	0	1	3	4.77E-05	0	267	534	0.009808	9.78543E-12	2	5	7.46E-05	0	1	3	4.47E-05	0
x_0^4		2	4	0.000102	0	1	3	5.24E-05	0	270	540	0.01744	9.39261E-12	2	5	7.36E-05	0	1	3	4.77E-05	0
x_0^5		2	4	9.87E-05	0	1	3	5.19E-05	0	272	544	0.01031	9.60305E-12	2	5	7.29E-05	0	1	3	4.87E-05	0
x_0^6		2	4	0.000108	0	1	3	5.03E-05	0	276	552	0.01672	9.28739E-12	2	5	7.25E-05	0	1	3	4.69E-05	0
x_0^7		2	4	9.89E-05	0	1	3	0.000051	0	279	558	0.010021	9.3365E-12	2	5	7.73E-05	0	1	3	0.00005	0
x_0^8		2	4	0.000092	0	1	3	5.04E-05	0	280	560	0.010092	9.69424E-12	2	5	7.73E-05	0	1	3	4.82E-05	0
x_0^9		2	4	0.000089	0	1	3	5.19E-05	0	279	558	0.013562	9.31545E-12	2	5	7.69E-05	0	1	3	4.83E-05	0
x_0^{10}		3	6	0.000115	0	2	5	9.57E-05	0	247	494	0.008874	9.78739E-12	14	29	0.000455	0	2	5	7.07E-05	0
x_0^{11}		15	30	0.00061	0	2	5	8.85E-05	0	273	546	0.013171	9.00527E-12	17	35	0.00056	0	3	7	0.000131	0
x_0^{12}		2	4	7.97E-05	0	1	3	4.57E-05	0	208	416	0.007998	9.8205E-12	2	5	6.99E-05	0	1	3	4.11E-05	0
x_0^{13}		5	10	0.000197	4.95915E-16	2	5	8.15E-05	5.43264E-16	233	466	0.009036	9.33893E-12	6	13	0.000209	3.13782E-16	2	5	7.18E-05	1.48536E-19
x_0^{14}		18	36	0.000738	1.68708E-21	2	5	8.14E-05	0	273	546	0.013814	9.02227E-12	17	35	0.000603	0	2	5	7.69E-05	0
x_0^1	10000	1	1	0.000219	0	1	2	0.000287	0	1	1	7.12E-05	0	1	1	7.82E-05	0	1	1	8.73E-05	0
x_0^2		2	4	0.000707	0	1	3	0.000442	0	270	540	0.094221	9.88E-12	2	5	0.000648	0	1	3	0.000442	0
x_0^3		2	4	0.000725	0	1	3	0.000438	0	278	556	0.098352	9.45815E-12	2	5	0.000635	0	1	3	0.000544	0
x_0^4		2	4	0.000827	0	1	3	0.000499	0	281	562	0.101057	9.08072E-12	2	5	0.000716	0	1	3	0.000492	0
x_0^5		2	4	0.000854	0	1	3	0.000478	0	283	566	0.098951	9.28054E-12	2	5	0.000802	0	1	3	0.000536	0
x_0^6		2	4	0.000758	0	1	3	0.000488	0	286	572	0.10061	9.96881E-12	2	5	0.000738	0	1	3	0.000455	0
x_0^7		2	4	0.002951	0	1	3	0.000577	0	290	580	0.104126	9.01411E-12	2	5	0.000779	0	1	3	0.000454	0
x_0^8		2	4	0.000821	0	1	3	0.00047	0	291	582	0.102912	9.36935E-12	2	5	0.000779	0	1	3	0.0029	0
x_0^9		2	4	0.00083	0	1	3	0.000463	0	290	580	0.104974	9.01411E-12	2	5	0.000778	0	1	3	0.000452	0
x_0^{10}		3	6	0.001079	0	2	5	0.000754	0	247	494	0.08647	9.55425E-12	14	29	0.007709	0	2	5	0.00065	0
x_0^{11}		3	6	0.001425	0	2	5	0.000826	0	283	566	0.098162	9.67617E-12	21	43	0.009453	0	3	7	0.001033	0
x_0^{12}		2	4	0.000726	0	1	3	0.000411	0	197	394	0.068407	9.70238E-12	2	5	0.000915	0	1	3	0.000404	0
x_0^{13}		5	10	0.001781	5.87407E-16	2	5	0.00074	6.6606E-16	233	466	0.084741	9.1306E-12	6	13	0.002439	4.96462E-16	2	5	0.00069	2.89109E-20
x_0^{14}		3	6	0.001133	0	2	5	0.000816	0	283	566	0.101921	9.67798E-12	21	43	0.00694	0	3	7	0.001034	0
x_0^1	100000	1	1	0.002156	0	1	2	0.004002	0	1	1	0.000664	0	1	1	0.00099	0	1	1	0.000719	0
x_0^2		2	4	0.007766	0	1	3	0.008162	0	281	562	1.153429	9.76002E-12	2	5	0.009287	0	1	3	0.00403	0
x_0^3		2	4	0.007926	0	1	3	0.008244	0	289	578	1.213562	9.33872E-12	2	5	0.011328	0	1	3	0.004606	0
x_0^4		2	4	0.008762	0	1	3	0.009485	0	291	582	1.222619	9.97067E-12	2	5	0.010364	0	1	3	0.004496	0
x_0^5		2	4	0.01227	0	1	3	0.005827	0	294	588	1.20401	9.19829E-12	2	5	0.01018	0	1	3	0.004526	0
x_0^6		2	4	0.008647	0	1	3	0.005301	0	297	594	1.173985	9.90045E-12	2	5	0.010238	0	1	3	0.004858	0
x_0^7		2	4	0.009384	0	1	3	0.005088	0	300	600	1.270771	9.90045E-12	2	5	0.011062	0	1	3	0.004968	0
x_0^8		2	4	0.010739	0	1	3	0.006267	0	302	604	1.26708	9.26851E-12	2	5	0.010554	0	1	3	0.004627	0
x_0^9		2	4	0.011037	0	1	3	0.005022	0	300	600	1.227618	9.90045E-12	2	5	0.010786	0	1	3	0.005907	0
x_0^{10}		3	6	0.017154	0	2	5	0.009114	0	247	494	1.078886	9.53123E-12	14	29	0.058437	0	2	5	0.009649	0
x_0^{11}		3	6	0.01621	0	2	5	0.009169	0	294	588	1.214786	9.57676E-12	25	51	0.086107	0	3	7	0.012721	0
x_0^{12}		2	4	0.008691	0	1	3	0.004791	0	186	372	0.764234	9.76002E-12	2	5	0.008177	0	1	3	0.004038	0
x_0^{13}		5	10	0.021448	1.15376E-15	2	5	0.011426	1.04147E-15	233	466	0.979461	9.11003E-12	6	13	0.019595	9.67861E-16	2	5	0.009865	7.76105E-21
x_0^{14}		3	6	0.012198	0	2	5	0.012526	0	294	588	1.18263	9.57695E-12	25	51	0.09638	0	3	7	0.012166	0

Table 9: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in NonsmoothSine*

NonsmoothSine																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
x_0	DIM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
x_0^1		1	1	0.393398	0	1	2	0.029079	0	1	1	0.020382	0	1	1	0.018349	0	1	1	0.020998	0
x_0^2		13	39	0.000488	4.2241E-12	8	24	0.000254	4.438E-13	258	516	0.014623	9.86079E-12	16	49	0.000477	2.87132E-12	1	4	4.14E-05	0
x_0^3		14	41	0.000489	5.9382E-12	8	24	0.000261	1.53518E-12	265	530	0.007672	9.32543E-12	16	49	0.000455	5.48346E-12	1	4	3.85E-05	0
x_0^4		14	41	0.000609	9.0607E-12	8	24	0.000248	2.17007E-12	267	534	0.007478	9.36307E-12	16	49	0.000468	6.62617E-12	1	4	3.74E-05	0
x_0^5		14	41	0.000669	5.8567E-13	8	24	0.000247	2.52902E-12	269	538	0.014446	9.00993E-12	16	49	0.00045	7.6356E-12	1	4	3.85E-05	0
x_0^6		13	37	0.000698	8.9672E-12	8	24	0.00033	7.58649E-13	271	542	0.007588	9.4944E-12	16	49	0.000497	9.2168E-12	1	4	4.14E-05	0
x_0^7	1000	12	36	0.000591	7.0744E-12	1	4	4.51E-05	0	273	546	0.007948	9.33169E-12	17	52	0.00052	1.63156E-12	1	4	0.015898	0
x_0^8		14	43	0.000635	3.3852E-12	1	4	4.26E-05	0	274	548	0.014416	9.08776E-12	17	52	0.000539	1.67438E-12	1	4	5.71E-05	0
x_0^9		12	36	0.000498	4.7587E-12	1	4	4.26E-05	0	273	546	0.007823	9.32385E-12	17	52	0.000526	1.63102E-12	1	4	4.46E-05	0
x_0^{10}		22	64	0.000796	5.6159E-12	27	85	0.000901	4.1995E-12	243	486	0.010422	9.19795E-12	58	176	0.0021	6.29127E-12	1	4	4.05E-05	0
x_0^{11}		27	82	0.001049	8.8341E-12	27	84	0.000922	5.42909E-12	268	536	0.007567	9.4316E-12	58	177	0.002564	9.71883E-12	1	4	4.29E-05	0
x_0^{12}		11	31	0.000389	6.6196E-12	6	19	0.000205	3.16233E-13	208	416	0.005955	9.60377E-12	13	40	0.000417	3.56041E-12	1	4	3.92E-05	0
x_0^{13}		21	67	0.000751	5.5855E-12	13	39	0.000413	9.62459E-12	231	462	0.009653	9.42525E-12	46	139	0.001309	3.42995E-12	1	4	3.65E-05	0
x_0^{14}		26	74	0.000969	7.7965E-12	27	84	0.000912	5.45535E-12	268	536	0.007855	9.44481E-12	72	219	0.002165	8.73831E-12	1	4	4.12E-05	0
x_0^1		1	1	0.000129	0	1	2	0.000134	0	1	1	0.000037	0	1	1	2.91E-05	0	1	1	4.85E-05	0
x_0^2		15	46	0.007143	9.8254E-13	8	24	0.002424	1.40342E-12	269	538	0.073484	9.78542E-12	16	49	0.004573	9.07992E-12	1	4	0.000333	0
x_0^3		13	38	0.004933	4.1007E-12	8	24	0.002969	4.85466E-12	276	552	0.072998	9.25415E-12	17	52	0.005173	2.77443E-12	1	4	0.00038	0
x_0^4		13	39	0.004374	1.888E-12	8	24	0.002676	6.86235E-12	278	556	0.076281	9.2915E-12	17	52	0.004745	3.3526E-12	1	4	0.000355	0
x_0^5		15	45	0.005011	1.6526E-12	8	24	0.002518	7.99745E-12	279	558	0.076489	9.93451E-12	17	52	0.004617	3.86334E-12	1	4	0.000356	0
x_0^6		13	39	0.004794	3.2372E-12	8	24	0.002557	2.39906E-12	282	564	0.076397	9.42183E-12	17	52	0.007141	4.66337E-12	1	4	0.000395	0
x_0^7	10000	12	37	0.006261	6.6355E-12	1	4	0.00044	0	284	568	0.082458	9.26036E-12	17	52	0.004951	5.15945E-12	1	4	0.000399	0
x_0^8		12	37	0.004252	9.5326E-12	1	4	0.000412	0	285	570	0.085831	9.0183E-12	17	52	0.00495	5.29487E-12	1	4	0.000549	0
x_0^9		12	37	0.004785	6.6323E-12	1	4	0.000425	0	284	568	0.079354	9.25959E-12	17	52	0.005085	5.15928E-12	1	4	0.000386	0
x_0^{10}		30	87	0.009743	2.8679E-12	27	85	0.011048	4.22075E-12	243	486	0.073986	9.20076E-12	46	139	0.014513	4.99978E-12	1	4	0.000348	0
x_0^{11}		33	94	0.013227	4.3608E-12	29	90	0.009257	3.13029E-12	279	558	0.074864	9.36541E-12	67	202	0.020579	6.9959E-12	1	4	0.000367	0
x_0^{12}		10	29	0.003351	4.6507E-12	6	18	0.002137	4.5E-12	197	394	0.053444	9.67775E-12	12	37	0.0034	7.03687E-12	1	4	0.000347	0
x_0^{13}		21	67	0.007507	5.5855E-12	13	39	0.005856	9.62459E-12	231	462	0.060799	9.42525E-12	46	139	0.013973	3.42995E-12	1	4	0.000348	0
x_0^{14}		37	111	0.014986	7.7619E-12	29	90	0.009276	3.13204E-12	279	558	0.078407	9.36672E-12	60	183	0.016698	9.3905E-12	1	4	0.000365	0
x_0^1		1	1	0.001227	0	1	2	0.001241	0	1	1	0.000522	0	1	1	0.000442	0	1	1	0.000434	0
x_0^2		14	43	0.064428	2.3984E-12	8	24	0.034522	4.438E-12	280	560	0.861423	9.71062E-12	17	52	0.053442	4.59412E-12	1	4	0.003696	0
x_0^3		13	39	0.061335	6.9057E-12	8	25	0.029846	3.41151E-13	287	574	0.901692	9.18341E-12	17	52	0.069269	8.77353E-12	1	4	0.003899	0
x_0^4		14	42	0.066773	6.174E-12	8	25	0.0357	4.82237E-13	289	578	0.898445	9.22048E-12	18	55	0.05764	1.6963E-12	1	4	0.003617	0
x_0^5		14	42	0.053384	8.99E-12	8	25	0.039101	5.62004E-13	290	580	0.89413	9.85857E-12	18	55	0.06426	1.95471E-12	1	4	0.003624	0
x_0^6		12	35	0.044556	9.7458E-12	8	24	0.034032	7.58649E-12	293	586	0.864506	9.34981E-12	18	55	0.066086	2.3595E-12	1	4	0.005761	0
x_0^7	100000	14	42	0.058188	7.0101E-13	1	4	0.00488	0	295	590	0.956043	9.18958E-12	18	55	0.059855	2.6105E-12	1	4	0.006157	0
x_0^8		12	37	0.05549	8.5352E-12	1	4	0.006071	0	295	590	0.991897	9.94373E-12	18	55	0.072051	2.67902E-12	1	4	0.006523	0
x_0^9		14	42	0.054901	7.0099E-13	1	4	0.006184	0	295	590	1.009185	9.1895E-12	18	55	0.067439	2.61049E-12	1	4	0.005312	0
x_0^{10}		22	67	0.098313	2.9158E-12	27	85	0.101133	4.2228E-12	243	486	0.70884	9.20105E-12	50	152	0.17805	8.87208E-12	1	4	0.004881	0
x_0^{11}		23	69	0.109171	9.0624E-12	29	90	0.12223	9.90137E-12	290	580	0.862648	9.2944E-12	66	203	0.24112	9.77579E-12	1	4	0.004668	0
x_0^{12}		10	29	0.037791	1.4707E-12	6	18	0.03558	1.42302E-12	186	372	0.560526	9.75229E-12	12	37	0.047214	2.22526E-12	1	4	0.004805	0
x_0^{13}		21	67	0.08694	5.5855E-12	13	39	0.049714	9.62459E-12	231	462	0.678275	9.42525E-12	46	139	0.147847	3.42995E-12	1	4	0.004461	0
x_0^{14}		23	69	0.100204	5.332E-12	29	90	0.130214	9.90193E-12	290	580	0.922228	9.29454E-12	60	183	0.234141	1.64094E-12	1	4	0.005978	0

Table 10: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in PolynomialI*

PolynomialI																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
x_0	DIM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
x_0^1		1	1	0.648409	0	1	2	0.02719	0	1	1	0.024425	0	1	1	0.022698	0	1	1	0.027815	0
x_0^2		15	45	0.005442	6.64373E-12	1	5	6.91E-05	0	80	160	0.003247	7.94192E-12	13	66	0.00072	3.08852E-12	1	6	0.000124	0
x_0^3		15	46	0.001859	3.01466E-12	1	5	6.21E-05	0	82	164	0.003477	8.04228E-12	13	66	0.000675	7.00582E-12	1	6	8.55E-05	0
x_0^4		15	46	0.001942	3.1719E-12	1	5	6.08E-05	0	83	166	0.00374	7.15513E-12	13	66	0.000918	9.32786E-12	1	6	0.000117	0
x_0^5		15	46	0.001859	3.33293E-12	1	5	6.01E-05	0	83	166	0.01689	8.73189E-12	14	71	0.001159	1.10657E-12	1	6	0.000108	0
x_0^6		15	46	0.002147	3.667E-12	1	5	6.02E-05	0	84	168	0.00329	8.43357E-12	12	60	0.000712	5.41978E-12	1	5	8.68E-05	0
x_0^7	1000	15	46	0.002155	4.01806E-12	1	5	5.74E-05	0	85	170	0.002987	7.64241E-12	13	65	0.00077	1.6227E-12	1	5	0.000081	0
x_0^8		15	46	0.001814	4.20038E-12	1	5	6.02E-05	0	85	170	0.002932	8.55781E-12	13	65	0.000744	2.36699E-12	1	5	5.45E-05	0
x_0^9		15	46	0.001901	4.01626E-12	1	5	6.02E-05	0	85	170	0.012452	7.63341E-12	13	65	0.018068	1.61592E-12	1	5	5.46E-05	0
x_0^{10}		19	58	0.002355	3.75764E-12	36	109	0.001607	6.2004E-12	104	207	0.00362	8.65583E-12	65	325	0.003515	8.16996E-12	19	114	0.001349	3.65884E-13
x_0^{11}		25	76	0.003135	5.86626E-12	11	35	0.000537	1.5935E-12	100	199	0.003424	9.76623E-12	88	444	0.004904	9.16075E-12	45	270	0.003515	7.30539E-12
x_0^{12}		12	37	0.001467	4.50571E-12	1	5	5.87E-05	0	65	130	0.002697	8.10145E-12	11	56	0.000657	1.58276E-12	1	6	6.34E-05	0
x_0^{13}		12	37	0.02106	3.69882E-12	1	5	6.17E-05	6.0961E-16	96	191	0.010889	9.25591E-12	24	120	0.001623	8.12121E-12	1	5	0.000106	5.68259E-15
x_0^{14}		23	70	0.001048	7.87718E-12	11	35	0.000541	1.7158E-12	100	199	0.004105	9.77002E-12	96	481	0.017915	8.33667E-12	42	252	0.003294	7.47689E-12
x_0^1		1	1	0.000211	0	1	2	0.000204	0	1	1	6.62E-05	0	1	1	6.22E-05	0	1	1	0.000113	0
x_0^2		15	45	0.021567	9.51812E-12	1	5	0.000481	0	83	166	0.037392	8.61429E-12	13	66	0.011501	9.76677E-12	1	6	0.000779	0
x_0^3		15	46	0.007518	2.9975E-12	1	5	0.000475	0	85	170	0.030422	8.72315E-12	14	71	0.013403	2.05592E-12	1	6	0.000735	0
x_0^4		15	46	0.018189	3.15414E-12	1	5	0.000551	0	86	172	0.037847	7.76089E-12	14	71	0.014698	2.73735E-12	1	6	0.000745	0
x_0^5		15	46	0.00678	3.31452E-12	1	5	0.000559	0	86	172	0.030529	9.47114E-12	14	71	0.007777	3.49927E-12	1	6	0.000805	0
x_0^6		15	46	0.016108	3.6471E-12	1	5	0.005408	0	87	174	0.03022	9.14757E-12	13	65	0.009534	1.59048E-12	1	5	0.000683	0
x_0^7	10000	15	46	0.006419	3.99639E-12	1	5	0.000534	0	88	176	0.036769	8.28942E-12	13	65	0.007494	5.13144E-12	1	5	0.004802	0
x_0^8		15	46	0.009196	4.17771E-12	1	5	0.000639	0	88	176	0.030984	9.28232E-12	13	65	0.008445	7.48508E-12	1	5	0.000589	0
x_0^9		15	46	0.006781	3.99621E-12	1	5	0.000768	0	88	176	0.032907	8.28844E-12	13	65	0.009292	5.12929E-12	1	5	0.000596	0
x_0^{10}		25	76	0.013173	2.91249E-12	37	113	0.019234	1.0272E-12	104	207	0.042242	8.65446E-12	56	280	0.032428	6.64516E-12	19	114	0.016425	3.65048E-13
x_0^{11}		22	67	0.012164	1.86695E-12	3	11	0.001311	2.2702E-12	95	189	0.033629	8.57347E-12	91	459	0.053386	7.01935E-12	7	42	0.007371	0
x_0^{12}		11	34	0.00481	7.54656E-12	1	5	0.000515	0	62	124	0.025658	7.46804E-12	10	51	0.005355	5.39042E-12	1	6	0.000614	0
x_0^{13}		12	37	0.007349	3.69882E-12	1	5	0.000614	6.8771E-16	96	191	0.035791	9.25591E-12	24	120	0.014763	8.12121E-12	1	5	0.00053	6.38766E-15
x_0^{14}		25	75	0.013028	7.09736E-12	3	11	0.001546	2.0581E-12	95	189	0.033876	8.57382E-12	85	430	0.050883	8.43725E-12	7	42	0.00443	0
x_0^1		1	1	0.010359	0	1	2	0.003012	0	1	1	0.000668	0	1	1	0.000634	0	1	1	0.001144	0
x_0^2		15	45	0.199526	9.50018E-12	1	5	0.009439	0	86	172	0.441005	9.34359E-12	14	71	0.217276	2.86615E-12	1	6	0.008656	0
x_0^3		15	46	0.402377	2.9921E-12	1	5	0.017052	0	88	176	0.371882	9.46167E-12	14	71	0.360562	6.5014E-12	1	6	0.008139	0
x_0^4		15	46	0.084895	3.14855E-12	1	5	0.029881	0	89	178	0.434725	8.41793E-12	14	71	0.253893	8.65626E-12	1	6	0.018808	0
x_0^5		15	46	0.084211	3.30873E-12	1	5	0.012051	0	90	180	0.410269	7.19108E-12	15	76	0.107671	1.02689E-12	1	6	0.006839	0
x_0^6		15	46	0.080708	3.64083E-12	1	5	0.006664	0	90	180	0.433719	9.92201E-12	13	65	0.080719	5.02955E-12	1	5	0.006904	0
x_0^7	100000	15	46	0.077076	3.98956E-12	1	5	0.006114	0	91	182	0.429725	8.99121E-12	14	70	0.086334	1.50587E-12	1	5	0.006353	0
x_0^8		15	46	0.081593	4.17056E-12	1	5	0.013086	0	92	184	0.388756	7.04772E-12	14	70	0.087138	2.19657E-12	1	5	0.009471	0
x_0^9		15	46	0.103244	3.98955E-12	1	5	0.027975	0	91	182	0.382014	8.99111E-12	14	70	0.087688	1.5058E-12	1	5	0.006485	0
x_0^{10}		30	91	0.179029	7.29861E-12	38	115	0.205526	5.6451E-12	104	207	0.46764	8.65432E-12	58	290	0.36675	2.27585E-12	19	114	0.135866	3.65008E-13
x_0^{11}		21	64	0.115766	1.96518E-12	3	11	0.015929	2.4584E-16	91	181	0.385584	9.44375E-12	89	449	0.544984	6.03782E-12	7	42	0.049822	0
x_0^{12}		11	33	0.056333	8.42797E-12	1	5	0.006102	0	58	116	0.258915	9.83577E-12	10	51	0.065846	1.70451E-12	1	6	0.009402	0
x_0^{13}		12	37	0.062588	3.69882E-12	1	5	0.015332	3.9408E-14	96	191	0.40241	9.25591E-12	24	120	0.146324	8.12121E-12	1	5	0.005825	3.67436E-13
x_0^{14}		21	64	0.123499	1.73728E-12	3	11	0.018971	2.2357E-16	91	181	0.371499	9.44386E-12	88	444	0.560744	9.10381E-12	7	42	0.045809	0

Table 11: *Comparison of the number of iterations, function evaluations and execution time of all algorithms in SmoothSine*

SmoothSine																					
		ISTTDFPM				STTDFPM				MOPCG				CGDFPM				AHDFPM			
x_0	DIM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM	NI	NF	CPU	NORM
x_0^1	1000	1	1	0.148719	0	1	2	0.028006	0	1	1	0.019528	0	1	1	0.017717	0	1	1	0.022242	0
x_0^2		3	9	0.000304	0	8	24	0.000668	4.438E-13	258	516	0.017442	9.86079E-12	16	49	0.001328	2.8713E-12	1	4	0.000105	0
x_0^3		3	9	0.000289	0	8	24	0.000652	1.53518E-12	265	530	0.024485	9.32543E-12	16	49	0.001322	5.4835E-12	1	4	0.00012	0
x_0^4		3	9	0.000282	0	8	24	0.000679	2.17007E-12	267	534	0.018101	9.36307E-12	16	49	0.001345	6.6262E-12	1	4	0.000101	0
x_0^5		3	9	0.000295	0	8	24	0.000703	2.52902E-12	269	538	0.018453	9.00993E-12	16	49	0.001372	7.6356E-12	1	4	0.0001	0
x_0^6		3	9	0.00028	0	8	24	0.000663	7.58649E-13	271	542	0.025038	9.4944E-12	16	49	0.001365	9.2168E-12	1	4	0.00015	0
x_0^7		5	14	0.000687	0	1	4	0.000118	0	273	546	0.01853	9.33169E-12	17	52	0.001442	1.6316E-12	1	4	0.000106	0
x_0^8		5	14	0.000514	0	1	4	0.000109	0	274	548	0.024381	9.08776E-12	17	52	0.001456	1.6744E-12	1	4	0.000189	0
x_0^9		5	14	0.000454	0	1	4	0.000108	0	273	546	0.019116	9.32385E-12	17	52	0.001448	1.631E-12	1	4	0.000176	0
x_0^{10}		3	9	0.000294	0	27	81	0.002284	7.1999E-12	243	486	0.016251	9.19795E-12	50	151	0.004249	7.8491E-12	1	4	0.000168	0
x_0^{11}		10	30	0.000971	5.18705E-12	31	93	0.022393	9.45559E-12	268	536	0.021561	9.4316E-12	52	157	0.004412	5.6226E-12	1	4	0.000149	0
x_0^{12}		4	12	0.000394	0	6	19	0.000574	3.16233E-13	208	416	0.014369	9.60377E-12	13	40	0.001142	3.5604E-12	1	4	0.000104	0
x_0^{13}		3	9	0.000293	5.02859E-18	10	31	0.000903	6.72688E-13	231	462	0.014896	9.42525E-12	44	133	0.003521	7.26E-12	1	4	0.00011	0
x_0^{14}		10	30	0.000924	5.24012E-12	31	93	0.002701	9.40655E-12	268	536	0.021149	9.44481E-12	52	157	0.004317	5.8929E-12	1	4	0.000118	0
x_0^1	10000	1	1	0.000521	0	1	2	0.000537	0	1	1	0.000178	0	1	1	0.000168	0	1	1	0.000583	0
x_0^2		4	12	0.004255	0	8	24	0.006724	1.40342E-12	269	538	0.184749	9.78542E-12	16	49	0.013599	9.0799E-12	1	4	0.001239	0
x_0^3		4	12	0.003782	0	8	24	0.006966	4.85466E-12	276	552	0.18307	9.25415E-12	17	52	0.014346	2.7744E-12	1	4	0.001157	0
x_0^4		4	12	0.003659	0	8	24	0.006504	6.86235E-12	278	556	0.182561	9.2915E-12	17	52	0.014135	3.3526E-12	1	4	0.001091	0
x_0^5		4	12	0.003664	0	8	24	0.00801	7.99745E-12	279	558	0.187732	9.93451E-12	17	52	0.01635	3.8633E-12	1	4	0.001067	0
x_0^6		3	9	0.003152	0	8	24	0.078064	2.39906E-12	282	564	0.187175	9.42183E-12	17	52	0.014165	4.6634E-12	1	4	0.001078	0
x_0^7		5	14	0.004625	0	1	4	0.001333	0	284	568	0.193423	9.26036E-12	17	52	0.014292	5.1595E-12	1	4	0.001091	0
x_0^8		5	14	0.004319	0	1	4	0.001361	0	285	570	0.204578	9.0183E-12	17	52	0.014288	5.2949E-12	1	4	0.001107	0
x_0^9		5	14	0.004391	0	1	4	0.001372	0	284	568	0.191262	9.25959E-12	17	52	0.014162	5.1593E-12	1	4	0.001096	0
x_0^{10}		3	9	0.002942	0	27	81	0.029506	7.29106E-12	243	486	0.158298	9.20076E-12	50	151	0.043159	7.8483E-12	1	4	0.001058	0
x_0^{11}		10	30	0.008933	4.42183E-12	35	105	0.038131	4.52083E-12	279	558	0.184703	9.36541E-12	54	163	0.045319	6.8048E-12	1	4	0.001069	0
x_0^{12}		4	12	0.003734	0	6	18	0.167077	4.5E-12	197	394	0.127245	9.67775E-12	12	37	0.010049	7.0369E-12	1	4	0.001075	0
x_0^{13}		3	9	0.002576	2.41358E-17	10	31	0.008198	6.72688E-13	231	462	0.150676	9.42525E-12	44	133	0.034508	7.26E-12	1	4	0.001012	0
x_0^{14}		10	30	0.011036	4.44531E-12	35	105	0.029144	4.51848E-12	279	558	0.188647	9.36672E-12	54	163	0.045585	6.8369E-12	1	4	0.001095	0
x_0^1	100000	1	1	0.008119	0	1	2	0.006736	0	1	1	0.002562	0	1	1	0.001614	0	1	1	0.001676	0
x_0^2		4	12	0.051221	0	8	24	0.09112	4.438E-12	280	560	2.231889	9.71062E-12	17	52	0.17734	4.5941E-12	1	4	0.010563	0
x_0^3		5	15	0.049488	0	8	25	0.081872	3.41151E-13	287	574	2.307577	9.18341E-12	17	52	0.184417	8.7735E-12	1	4	0.010432	0
x_0^4		5	15	0.059637	0	8	25	0.084354	4.82237E-13	289	578	2.313183	9.22048E-12	18	55	0.158569	1.6963E-12	1	4	0.010563	0
x_0^5		5	15	0.061577	0	8	25	0.072478	5.62004E-13	290	580	2.292209	9.85857E-12	18	55	0.189835	1.9547E-12	1	4	0.014265	0
x_0^6		4	12	0.05415	0	8	24	0.084532	7.58649E-12	293	586	2.351894	9.34981E-12	18	55	0.189596	2.3595E-12	1	4	0.016094	0
x_0^7		5	14	0.050472	0	1	4	0.013145	0	295	590	2.341772	9.18958E-12	18	55	0.18403	2.6105E-12	1	4	0.012164	0
x_0^8		5	14	0.065112	0	1	4	0.013095	0	295	590	2.327774	9.94373E-12	18	55	0.172351	2.679E-12	1	4	0.012677	0
x_0^9		5	14	0.067324	0	1	4	0.012599	0	295	590	2.366575	9.1895E-12	18	55	0.188774	2.6105E-12	1	4	0.013442	0
x_0^{10}		3	9	0.03926	0	27	81	0.279597	7.30023E-12	243	486	1.88388	9.20105E-12	50	151	0.512189	7.8482E-12	1	4	0.010387	0
x_0^{11}		7	21	0.075193	0	37	111	0.366113	5.56397E-12	290	580	2.288145	9.2944E-12	56	169	0.591343	8.0798E-12	1	4	0.010506	0
x_0^{12}		4	12	0.046442	0	6	18	0.055725	1.42302E-12	186	372	1.459678	9.75229E-12	12	37	0.124429	2.2253E-12	1	4	0.01107	0
x_0^{13}		3	9	0.038103	1.1121E-16	10	31	0.092046	6.72688E-13	231	462	1.723866	9.42525E-12	44	133	0.411588	7.26E-12	1	4	0.010146	0
x_0^{14}		7	21	0.07735	0	37	111	0.3805	5.56368E-12	290	580	2.277637	9.29454E-12	56	169	0.553075	8.0836E-12	1	4	0.010537	0

Table 12: *Experiment results and average result of ℓ_1-norm regularization problem via* **STTDFPM** *,* **CGDFPM** *,* **MOPCG** *and* **AHDFPM** *methods.*

MSE									
Experiments	**STTDFPM**	**AHDFPM**	**CGDFPM**	**MOPCG**	Experiments	**STTDFPM**	**AHDFPM**	**CGDFPM**	**MOPCG**
1	0.05284	0.05287	0.05405	0.05291	51	0.04206	0.04215	0.04717	0.04230
2	0.05256	0.05262	0.05375	0.05269	52	0.04499	0.04500	0.04759	0.04506
3	0.05096	0.05103	0.05388	0.05119	53	0.03398	0.03408	0.03716	0.03422
4	0.04169	0.04178	0.04388	0.04190	54	0.04476	0.04481	0.04874	0.04489
5	0.04069	0.04082	0.04202	0.04098	55	0.05324	0.05326	0.05437	0.05327
6	0.04734	0.04731	0.04812	0.04726	56	0.04135	0.04136	0.04435	0.04142
7	0.04552	0.04565	0.05000	0.04597	57	0.04020	0.04045	0.04712	0.04086
8	0.04236	0.04243	0.04505	0.04253	58	0.05260	0.05262	0.05456	0.05267
9	0.04051	0.04059	0.04416	0.04072	59	0.03338	0.03342	0.03666	0.03350
10	0.06570	0.06566	0.06482	0.06561	60	0.04219	0.04244	0.04753	0.04286
11	0.04634	0.04641	0.05008	0.04647	61	0.05566	0.05569	0.05593	0.05573
12	0.04104	0.04138	0.04659	0.04177	62	0.03025	0.03034	0.03237	0.03045
13	0.04468	0.04474	0.04717	0.04484	63	0.03813	0.03811	0.03921	0.03806
14	0.04708	0.04715	0.05091	0.04732	64	0.04286	0.04293	0.04564	0.04307
15	0.05348	0.05349	0.05704	0.05360	65	0.04216	0.04232	0.04812	0.04260
16	0.03949	0.03961	0.04402	0.03980	66	0.05871	0.05880	0.06153	0.05891
17	0.04504	0.04506	0.04680	0.04511	67	0.05396	0.05393	0.05324	0.05384
18	0.05241	0.05244	0.05513	0.05251	68	0.04694	0.04712	0.05241	0.04745
19	0.04330	0.04335	0.04565	0.04343	69	0.04378	0.04385	0.04695	0.04396
20	0.04041	0.04049	0.04300	0.04070	70	0.05371	0.05373	0.05657	0.05392
21	0.03602	0.03617	0.04368	0.03648	71	0.05019	0.05020	0.05174	0.05021
22	0.04266	0.04275	0.04627	0.04301	72	0.05210	0.05207	0.05178	0.05204
23	0.05424	0.05427	0.05837	0.05435	73	0.04313	0.04318	0.04719	0.04335
24	0.04244	0.04236	0.04333	0.04228	74	0.03706	0.03719	0.04041	0.03740
25	0.05006	0.05011	0.05224	0.05023	75	0.03853	0.03859	0.04152	0.03870
26	0.03507	0.03518	0.03966	0.03538	76	0.04341	0.04350	0.04664	0.04364
27	0.04264	0.04265	0.04466	0.04267	77	0.02827	0.02832	0.03254	0.02844
28	0.04605	0.04616	0.04941	0.04627	78	0.04593	0.04600	0.04904	0.04611
29	0.04523	0.04527	0.04888	0.04541	79	0.04472	0.04476	0.04704	0.04484
30	0.03291	0.03305	0.03708	0.03329	80	0.03862	0.03883	0.04407	0.03913
31	0.03880	0.03887	0.04119	0.03900	81	0.03274	0.03293	0.03740	0.03320
32	0.03149	0.03149	0.03403	0.03149	82	0.04847	0.04863	0.05017	0.04884
33	0.05351	0.05348	0.05465	0.05345	83	0.03386	0.03387	0.03549	0.03391
34	0.04861	0.04864	0.05129	0.04871	84	0.05266	0.05265	0.05350	0.05262
35	0.05020	0.05025	0.05393	0.05052	85	0.03691	0.03703	0.04280	0.03725
36	0.06390	0.06394	0.06654	0.06404	86	0.05090	0.05099	0.05353	0.05115
37	0.04924	0.04931	0.05223	0.04945	87	0.04659	0.04655	0.04655	0.04648
38	0.04503	0.04504	0.04542	0.04505	88	0.04078	0.04082	0.04318	0.04090
39	0.03665	0.03672	0.04055	0.03681	89	0.05044	0.05050	0.05241	0.05063
40	0.04140	0.04147	0.04393	0.04160	90	0.03858	0.03874	0.04264	0.03896
41	0.04858	0.04859	0.04997	0.04862	91	0.05956	0.05959	0.06337	0.05963
42	0.04250	0.04253	0.04335	0.04254	92	0.04417	0.04419	0.04607	0.04431
43	0.05804	0.05819	0.06132	0.05836	93	0.04371	0.04375	0.04572	0.04382
44	0.05167	0.05171	0.05323	0.05176	94	0.04024	0.04028	0.04393	0.04037
45	0.05701	0.05708	0.06141	0.05718	95	0.04405	0.04412	0.04605	0.04423
46	0.04284	0.04291	0.04619	0.04306	96	0.03023	0.03025	0.03183	0.03029
47	0.04676	0.04688	0.05041	0.04706	97	0.04784	0.04789	0.05377	0.04827
48	0.04980	0.04986	0.05215	0.04992	98	0.04602	0.04613	0.04985	0.04630
49	0.04962	0.04957	0.05146	0.04950	99	0.04266	0.04268	0.04414	0.04270
50	0.03179	0.03194	0.03540	0.03216	100	0.03843	0.03842	0.04046	0.03839
Average						0.04596	0.04603	0.04877	0.04614