

## Problem Set 4-Wednesday

*Please indicate the members who are present. Also indicate the group coordinator.*

Group Number:	
Members:	

## Problem 1

Find the integral

$$\begin{aligned}
 & \int_0^{\frac{\pi}{4}} \frac{1 + \sin \theta}{\cos^2 \theta} d\theta \\
 &= \int_0^{\frac{\pi}{4}} \left[ \frac{1}{\cos^2 \theta} + \frac{\sin \theta}{\cos^2 \theta} \right] d\theta \\
 &= \int_0^{\frac{\pi}{4}} (\sec^2 \theta + \tan \theta \sec \theta) d\theta \\
 &= (\tan \theta + \sec \theta) \Big|_0^{\frac{\pi}{4}} \\
 &= (1 + \sqrt{2}) - (0 + 1) = \underline{\underline{\sqrt{2}}}
 \end{aligned}$$

using Plots, SymPy, LaTeXStrings

```
1 x, theta = symbols("x, theta", real=True)
```

```
1 # problem 1
2 f1(theta) = (1+sin(theta))/(cos(theta))^2
3 I1 = integrate(f1(theta), (theta, 0, pi/4))
```

1.41421356237309

## Problem 2

Find the integral

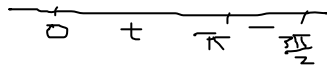
```
1 # problem 2
2 f2(x) = x*(x^(1//3)+x^(1//4))
3 f2(x)
4 I2 = integrate(f2(x),(x,0,1))
```

$\frac{55}{63}$

$$\begin{aligned}
 & \int_0^1 x (\sqrt[3]{x} + \sqrt[4]{x}) dx \\
 &= \int_0^1 x \left( x^{\frac{1}{3}} + x^{\frac{1}{4}} \right) dx \\
 &= \int_0^1 \left( x^{\frac{4}{3}} + x^{\frac{5}{4}} \right) dx = \left[ \frac{3}{7} x^{\frac{7}{3}} + \frac{4}{9} x^{\frac{9}{4}} \right]_0^1 \\
 &= \frac{3}{7} + \frac{4}{9} = \frac{27+28}{63} = \frac{55}{63}
 \end{aligned}$$

### Problem 3

Find the integral



$$\begin{aligned}
 & \int_0^{3\pi/2} |\sin x| dx \\
 &= \int_0^{\pi} \sin x dx - \int_{\pi}^{3\pi/2} \sin x dx \\
 &= -\cos x \Big|_0^{\pi} + \cos x \Big|_{\pi}^{3\pi/2} \\
 &= -(-1 - 1) + (0 + 1) \\
 &= 2 + 1 = \underline{\underline{3}}
 \end{aligned}$$

1 # problem 3  
2 I3 = integrate(abs(sin(x)), (x, 0, 3π/2))

3.0

## Problem 4

If a particle is moving along a straight line and its velocity is given by

$$v(t) = t^2 - 5t + 4, \quad 0 \leq t \leq 4.$$

Find its displacement and distance it travels on the time interval  $[0, 4]$  are:

• displacement =  $\int_0^4 (t^2 - 5t + 4) dt = \left[ \frac{1}{3}t^3 - \frac{5}{2}t^2 + 4t \right]_0^4$

$$= \frac{1}{3} \cdot 64 - \frac{5}{2} \cdot 16 + 16$$

$$= \frac{64}{3} - 40 = -\frac{8}{3}$$

• distance =  $\int_0^4 |t^2 - 5t + 4| dt$

$t^2 - 5t + 4 = (t-1)(t-4)$

Graph of  $t^2 - 5t + 4$  on  $[0, 4]$  shows the function is positive on  $[0, 1]$  and  $[4, 4]$  (a point), and negative on  $[1, 4]$ .

$$= \int_0^1 (t^2 - 5t + 4) dt + \int_1^4 -(t^2 - 5t + 4) dt$$

$$= \left[ \frac{t^3}{3} - \frac{5t^2}{2} + 4t \right]_0^1 - \left[ \frac{t^3}{3} - \frac{5t^2}{2} + 4t \right]_1^4$$

$$= \left[ \frac{1}{3} - \frac{5}{2} + 4 \right] - \left[ \left( \frac{64}{3} - \frac{5}{2} \cdot 16 + 16 \right) - \left( \frac{1}{3} - \frac{5}{2} + 4 \right) \right]$$

$$= \frac{2 - 15 + 24}{6} - \frac{8}{3} = \frac{1}{3} + 8 = \frac{25}{3}$$

```
1 # problem 4
2 displacement = integrate(x^2-5*x+4,(x,0,4))
-8/3
1 distance = integrate(abs(x^2-5*x+4),(x,0,4))
19/3
```