# Problem Set 21
## 11.4: The Comparison Tests

*Please indicate the members who are present. Also indicate the group coordinator.*

| | |
|---|---|
| Group Number: | KEY |
| Members: | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

**Problem 1**

Use the comparison test to determine whether the series is convergent or divergent $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

$b_n = \frac{1}{\sqrt{n}}$

$$\frac{1}{\sqrt{n}} < \frac{1}{\sqrt{n}-1}$$

Since $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ is divergent ($p$-series with $p=\frac{1}{2}<1$)

$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ is also divergent by the comparison test

## Problem 2

Determine whether the series is convergent or divergent $\sum_{n=1}^{\infty} \frac{7^n}{6^n - 1}$

$$b_n = \frac{7^n}{6^n} = \left(\frac{7}{6}\right)^n$$

$$\frac{7^n}{6^n} < \frac{7^n}{6^n - 1}$$

Since $\sum_{n=1}^{\infty} \frac{7^n}{6^n}$ is divergent (geometric Series with $r = 2 > 1$)

$\sum_{n=1}^{\infty} \frac{7^n}{6^n - 1}$ is also divergent

by the Comparison test.

## Problem 3

Determine whether the series is convergent or divergent $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3+4n+3}}$

$$b_n = \frac{\sqrt[3]{n}}{\sqrt{n^3}} = \frac{n^{1/3}}{n^{3/2}} = \frac{1}{n^{7/6}}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n^{1/3}}{\sqrt{n^3+4n+3}}}{n^{-7/6}} = \lim_{n\to\infty} \frac{n^{\frac{3}{2}}}{(n^3+4n+3)^{1/2}}$$

$$= \lim_{n\to\infty} \frac{n^{\frac{3}{2}}}{n^{3/2}\left(1+\frac{4}{n^2}+\frac{3}{n^3}\right)^{1/2}} = 1 > 0$$

Since $\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$ is Convergent ($p$-series with $p=\frac{7}{6}>1$)

$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3+4n+3}}$ is also convergent

by L.C.T

## Problem 4

Determine whether the series is convergent or divergent $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2 - 1}}$

$$b_n = \frac{1}{n^2}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{n\sqrt{n^2-1}}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2-1}}$$

$$= \lim_{n\to\infty} \frac{n}{n\sqrt{1-\frac{1}{n^2}}} = 1$$

$\sum_{n=2}^{\infty} \frac{1}{n^2}$ is Convergent ( P-Series with P=2>1 )

$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$ is also convergent by L.C.T

## Problem 5

Determine whether the series is convergent or divergent $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

$$a_n = \frac{n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1}{n \cdot n \cdot n \cdots n \cdot n \cdot n} \qquad b_n = \frac{2}{n^2}$$

$$\frac{n!}{n^n} < \frac{2}{n^2}$$

Since $\sum_{n=1}^{\infty} \frac{2}{n^2}$ is Convergent (P-Series with $p = 2 > 1$)

$\sum_{i=1}^{\infty} \frac{n!}{n^n}$ is also convergent by the Comparison test.

## Problem 6

Determine whether the series is convergent or divergent $\sum_{n=1}^{\infty} \sin^2\left(\frac{1}{n}\right)$

$b_n = \frac{1}{n^2}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sin^2\left(\frac{1}{n}\right)}{\frac{1}{n^2}} = \lim_{n\to\infty} \left(\frac{\sin\frac{1}{n}}{\frac{1}{n}}\right)^2$$

$$\boxed{\begin{array}{c} \text{let } u = \frac{1}{n} \\ n\to\infty \quad u\to 0 \end{array}}$$

$$= \lim_{u\to 0} \left(\frac{\sin u}{u}\right)^2 = 1$$

Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent (P-Series with $n = 2 > 0$)

$\sum_{n=1}^{\infty} \sin^2\left(\frac{1}{n}\right)$ is also convergent

by L.C.T