

Problem set :

①

Evaluate the Riemann sum for $f(x) = x^2$ over $[0, 2]$, taking the sample points to be the Midpoints with n approximating rectangles of equal widths.

②

* Show that $R_n \rightarrow \frac{8}{3}$ as $n \rightarrow \infty$

③

Find $\lim_{n \rightarrow \infty} n \left[\frac{1}{n^2+1^2} + \frac{1}{n^2+2^2} + \dots + \frac{1}{2n^2} \right]$

④

Problem 11

11. Some computer algebra systems have commands that will draw approximating rectangles and evaluate the sums of their areas, at least if x_i^* is a left or right endpoint. (For instance, in Maple use `leftbox`, `rightbox`, `leftsum`, and `rightsum`.)

- (a) If $f(x) = 1/(x^2 + 1)$, $0 \leq x \leq 1$, find the left and right sums for $n = 10, 30$, and 50 .
- (b) Illustrate by graphing the rectangles in part (a).
- (c) Show that the exact area under f lies between 0.780 and 0.791 .