# Problem Set 15
# 7.8: Improper Integrals

*Please indicate the members who are present. Also indicate the group coordinator.*

| Group Number: | KEY |
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| Members: | |
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## Problem 1

Determine whether the integral $\int_0^9 \frac{1}{x\sqrt{x}}dx$ is convergent or divergent. If it is convergent, find its value

$$= \lim_{t\to 0^+} \int_t^9 \left[\frac{1}{x^{3/2}}\right] dx$$

$$= \lim_{t\to 0^+} \left[-2x^{-\frac{1}{2}}\right]_t^9$$

$$= \lim_{t\to 0^+} \left[\frac{-2}{3} + \frac{2}{\sqrt{t}}\right] = \infty$$

the integral is divergent.

## Problem 2

Determine whether the integral $\int_0^\infty xe^{-10x}dx$ is convergent or divergent. If it is convergent, find its value

$$= \lim_{t\to\infty} \int_0^t \left(x e^{-10x}\right) dx$$

| d | | I |
|---|---|---|
| $x$ | $+$ | $e^{-10x}$ |
| $1$ | $-$ | $\frac{-1}{10}e^{-10x}$ |
| $0$ | | $\frac{1}{100}e^{-10x}$ |

$$= \lim_{t\to\infty} \left[\frac{-x}{10} e^{-10x} - \frac{1}{100} e^{-10x}\right]_0^t$$

$$= \lim_{t\to\infty} \left[\frac{-t}{10} e^{-10t} - \frac{1}{100} e^{-10t} + \frac{1}{100}\right]$$

$$= \lim_{t\to\infty} \left[\frac{-t}{10 e^{10t}}\right] + \lim_{t\to\infty} \left[-\frac{1}{100 e^{10t}} + \frac{1}{100}\right]$$

$$= \lim_{t\to\infty} \left[\frac{-1}{100 e^{10t}}\right] + \lim_{t\to\infty} \left[\frac{-1}{100 e^{10t}} + \frac{1}{100}\right]$$

$$= \frac{1}{100}$$ , Convergent.

## Problem 3

Evaluate $\int_{-1}^{3} \frac{1}{\sqrt{|x-1|}} dx$.

$$= \int_{-1}^{1} \frac{1}{\sqrt{|x-1|}} dx + \int_{1}^{3} \frac{1}{\sqrt{|x-1|}} dx$$

$$= \int_{-1}^{1} \frac{1}{\sqrt{1-x}} dx + \int_{1}^{3} \frac{1}{\sqrt{x-1}} dx$$

$$= \lim_{t \to 1^-} \int_{-1}^{t} \frac{1}{\sqrt{1-x}} dx + \lim_{t \to 1^+} \int_{t}^{3} \frac{1}{\sqrt{x-1}} dx$$

$$= \lim_{t \to 1^-} \left[-2\sqrt{1-x}\right]_{-1}^{t} + \lim_{t \to 1^+} \left[2\sqrt{x-1}\right]_{t}^{3}$$

$$= \lim_{t \to 1^-} \left[-2\sqrt{1-t} + 2\sqrt{2}\right] + \lim_{t \to 1^+} \left[2\sqrt{2} - 2\sqrt{t-1}\right]$$

$$= 2\sqrt{2} + 2\sqrt{2}$$

$$= 4\sqrt{2}$$

## Problem 4

Evaluate (if possible) $\int_{-1}^{\infty} \frac{dx}{(4+3x)^{3/2}}$.

$$= \lim_{t\to\infty} \int_{-1}^{t} \frac{1}{(4+3x)^{3/2}}\,dx$$

$$= \lim_{t\to\infty} \int_{-1}^{t} \frac{1}{3u^{3/2}}\,du$$

$$\boxed{\begin{aligned} u &= 4+3x \\ du &= 3\,dx \end{aligned}}$$

$$= \lim_{t\to\infty} \frac{1}{3}\left[-2(4+3x)^{-\frac{1}{2}}\right]_{-1}^{t}$$

$$= \lim_{t\to\infty} \frac{1}{3}\left[-2(4+3t)^{-\frac{1}{2}}+2\right]$$

$$= \frac{2}{3}$$

## Problem 5

Evaluate (if possible) $\int_{-\infty}^{0} \frac{x}{(x^2+2)^{3/2}} dx$.

$$= \lim_{t \to -\infty} \int_{t}^{0} \frac{x}{(x^2+2)^{3/2}} dx$$

$$\lim_{t \to -\infty} \frac{1}{2} \left[ -2 (x^2+2)^{-\frac{1}{2}} \right]_{t}^{0}$$

$$= \lim_{t \to -\infty} \frac{1}{2} \left[ \frac{-2}{\sqrt{2}} + \frac{2}{\sqrt{t^2+2}} \right]$$

$$= \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$