Problem Set 3-Monday

Please indicate the members who are present. Also indicate the group coordinator.

Group Number:	
Members:	

Solution

Problem 1

If $y = \int_{\sqrt[3]{x}}^{0} \sin(t^3)dt$. Find $\frac{dy}{dx}$.

$$y = -\int_{0}^{x^{\frac{1}{3}}} \sin(t^3)\,dt$$

$$\therefore \frac{dy}{dx} = -\sin\left[\left(x^{\frac{1}{3}}\right)^3\right] \cdot \left(\frac{1}{3} x^{-\frac{2}{3}}\right)$$

$$= \frac{-\sin x}{3x^{\frac{2}{3}}} \quad \#$$

Problem 2

Evaluate the integral

$$\int_0^4 2^x dx = \left.\frac{2^x}{\ln 2}\right|_0^4 = \frac{2^4 - 2^0}{\ln 2}$$

$$= \frac{15}{\ln 2} \#$$

Problem 3

Let $\displaystyle\int_0^{x^2} \frac{2f\left(\sqrt{t}\right)}{t^2}\,dt = x^2 - 1$. If $x > 0$, find $f'(2)$

Differentiate both sides,

$$\frac{2f(x)}{x^{\cancel{4}3}} \cdot \cancel{3x} = 2x \quad , \quad x > 0$$

$$\therefore \quad f(x) = \frac{x^4}{2}$$

$$f'(x) = 2x^3$$

hence, $f'(2) = 2 \cdot 2^3 = \underline{\underline{16}}$

Problem 4

If $G(u) = \int_1^u g(x)dx$ where $g(x) = \int_1^{x^2} \frac{\sqrt{9+t^2}}{t} dt$. Find $G''(2)$.

$G'(u) = g(u) = \int_1^{u^2} \frac{\sqrt{9+t^2}}{t} dt$ using FTC-1

$\therefore G''(u) = \frac{\sqrt{9+u^4}}{u^2} \cdot 2u$, $u > 1$

$G''(u) = \frac{2\sqrt{9+u^4}}{u}$

$\therefore G''(2) = \frac{2\sqrt{25}}{2} = \underline{\underline{5}}$

Problem (Challenge)

Show that

$$0 \leq \int_5^{10} \frac{x}{x^4 + x^2 + 1}\,dx \leq 0.6$$

(Hint: compare the integrand to a simpler function.)

integrand $= \dfrac{x}{x^4+x^2+1} \leq \dfrac{x}{x^4} = \dfrac{1}{x^3}$

since $5 \leq x \leq 10$

$$\frac{1}{100} \leq \frac{1}{x^3} \leq \frac{1}{125} = 0.008$$

$$\therefore 0 \leq \frac{x}{x^4+x^2+1} \leq \frac{1}{x^3} \leq 0.008$$

$$\therefore \quad 0 \leq \int_5^{10} \frac{x\,dx}{x^4+x^2+1} \leq 5 \times 0.008$$

$$= 0.04 < 0.6$$

other bounds are possible.