Problem Set 5-Monday

Please indicate the members who are present. Also indicate the group coordinator.

Group Number:	
Members:	

Problem 1

Find the integral

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left(x \tan(x^4) + x^2 \right) dx$$

$$= \underbrace{\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} x \tan x^4 \, dx}_{\text{odd}} + \underbrace{\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} x^2 \, dx}_{\text{even}}$$

$$= 0 + 2\int_0^{\frac{\pi}{3}} x^2 \, dx = \frac{2}{3} x^3 \Big|_0^{\frac{\pi}{3}} = \frac{2}{3} \frac{\pi^3}{27} = \frac{2\pi^3}{81}$$

Problem 2

Find the integral

$$\int_0^{\frac{1}{2}} \frac{\cos\left(\sin^{-1} x\right)}{\sqrt{1-x^2}} dx$$

$$= \int_0^{\frac{\pi}{6}} \cos u \, du$$

$$= \sin u \Big|_0^{\frac{\pi}{6}} = \frac{1}{2}$$

Let $u = \sin^{-1} x$

$du = \dfrac{dx}{\sqrt{1-x^2}}$

$x = 0 \Rightarrow u = 0$

$x = \frac{1}{2} \Rightarrow u = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

Problem 3

Find

$$\int \sqrt[3]{x^3+1}\;\; x^5 dx$$

$u = x^3 + 1$

$du = 3x^2 dx$

$x^3 = u - 1$

$$= \frac{1}{3}\int u^{\frac{1}{3}} (u-1)\, du$$

$$= \frac{1}{3}\int \left(u^{\frac{4}{3}} - u^{\frac{1}{3}}\right) du$$

$$= \frac{1}{3}\cdot\frac{3}{7} u^{\frac{7}{3}} - \frac{3}{4} u^{\frac{4}{3}} + C$$

$$= \frac{1}{7}(x^3+1)^{\frac{7}{3}} - \frac{3}{4}(x^3+1)^{\frac{4}{3}} + C$$

Problem 4

Find the integral

$$\int_0^{\frac{\pi}{4}} (1 + \tan x)^3 \sec^2 x \, dx$$

$u = 1 + \tan x$

$du = \sec^2 x \, dx$

$x = 0 \Rightarrow u = 1$

$x = \frac{\pi}{4} \Rightarrow u = 2$

$$= \int_1^2 u^3 \, du = \frac{u^4}{4} \Big|_1^2 = \frac{1}{4}(16 - 1) = \frac{15}{4}$$

Problem 5

Find the integral

$$\int_0^1 4e^x \sinh x \, dx$$

$$= \int_0^1 4e^x \left(\frac{e^x - e^{-x}}{2} \right) dx$$

$$= \int_0^1 (2e^{2x} - 2) \, dx$$

$u = 2x$
$du = 2dx$
$x = 0 \Rightarrow u = 0$
$x = 1 \Rightarrow u = 2$

$$= \int_0^2 (e^u - 1) \, du = e^u - u \Big|_0^2 = e^2 - 2 - 1 = e^2 - 3$$