Problem Set 10
7.1: Integration by Parts

Please indicate the members who are present. Also indicate the group coordinator.

Group Number:	
Members:	Key

$$\int u dv = uv - \int v du$$

$$\int_a^b u dv = uv|_a^b - \int_a^b v du$$

Problem 1

Find the integral $\int x^3 e^x dx$.

$$= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$$

diff		Int
x^3		e^x
	$+$	
$3x^2$		e^x
	$-$	
$6x$		e^x
	$+$	
6		e^x
	$-$	
0		e^x

Problem 2

Find the area of the region bounded by the curves $y = x\sec^2 x$ and the lines $x = 0, x = \frac{\pi}{4}$, and $y = 0$.

Because the function is positive, we don't have to sketch the graph of the function

$$A = \int_0^{\frac{\pi}{4}} x\sec^2 x \, dx$$

Using integration by parts

$u = x \qquad dv = \sec^2 x \, dx$

$du = dx \qquad v = \tan x$

$$= \Big[x\tan x\Big]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} (\tan x) \, dx$$

$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$

$u = \cos x$

$du = -\sin x \, dx$

$\therefore$

$\int \frac{\sin x}{\cos x} dx = \int \frac{-1}{u} du$

$= -\ln|u| = \ln\left|\frac{1}{u}\right|$

$= \ln|\sec x|$

$$= \frac{\pi}{4} - \Big[\ln(\sec x)\Big]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} - \left(\ln(\sqrt{2}) - \ln(1)\right) = \frac{\pi}{4} - \ln(\sqrt{2})$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln(2)$$

Problem 3

Find the integral $\int e^{-x} \sin 2x dx.$ $= I$

$$u = \sin 2x \qquad dv = e^{-x} dx$$
$$du = 2\cos 2x \, dx \qquad v = -e^{-x}$$

$$I = -e^{-x}\sin 2x + 2\int e^{-x}\cos 2x \, dx$$

$$u = \cos 2x \qquad dv = e^{-x}$$
$$du = -2\sin 2x \qquad v = -e^{-x}$$

$$= -e^{-x}\sin 2x + 2 + 2\left[-e^{-x}\cos 2x - \int 2e^{-x}\sin 2x \, dx\right]$$

$$I = -e^{-x}\sin 2x + 2 - 2e^{-x}\cos 2x - 4\underbrace{\boxed{\int e^{-x}\sin 2x \, dx}}_{=I}$$

$$5I = -e^{-x}\sin 2x + 2 - 2e^{-x}\cos 2x$$

$$I = \frac{1}{5}\left[-e^{x}\sin 2x + 2 - 2e^{-x}\cos 2x\right] + C$$

By substitution

Problem 4

Find the integral $\int e^{2x} \sin e^x dx$.

$w = e^x$

$dw = e^x dx$

$= \int w \sin(w)\, dw$

By part

$u = w \qquad dv = \sin(w) dw$

$du = dw \qquad v = -\cos(w)$

$= -w\cos w + \int \cos(w)\, dw$

$= -w\cos w + \sin w$

$= -e^x \cos(e^x) + \sin(e^x) + C$

Problem 5

Find the integral $\int \frac{\log_3(x^2)}{x}dx$.

By substitution

$$u = \log_3 x$$

$$du = \frac{1}{x\ln 3}dx$$

$$\ln 3\, du = \frac{1}{x}dx$$

$$I = \int \frac{\log_3(x^2)}{x}dx$$

$$I = \int \frac{2\log_3(x)}{x}dx$$

$$I = \int [2u\ln(3)]\,du$$

$$I = 2\ln(3)\int u\,du = 2\ln(3)\left(\frac{u^2}{2}\right) + C$$

$$= 2\ln(3)\cdot\frac{(\log_3(x))^2}{2} + C$$

$$= \ln(3)(\log_3(x))^2 + C$$

extra steps

$$= \ln(3)\cdot\frac{(\ln(x))^2}{(\ln(3))^2} + C = \frac{(\ln(x))^2}{\ln(3)} + C$$

Another method to solve question 5

$$I = \int \frac{\log_3(x^2)}{x}\,dx$$

$$I = \int \frac{2\log_3(x)}{x}\,dx$$

$u = 2\log_3(x)$ $\quad dv = \frac{1}{x}\,dx$

$du = \frac{2}{x\ln 3}\,dx$ $\quad v = \ln|x|$

$$= 2\log_3(x)\cdot\ln|x| - \int \frac{2\ln|x|}{x\ln 3}\,dx$$

$$= 2\log_3(x)\ln|x| - \int \frac{\ln x^2}{x\ln 3}\,dx$$

Change base Formula $\left(\log_b a = \frac{\ln a}{\ln b}\right)$

$=I$

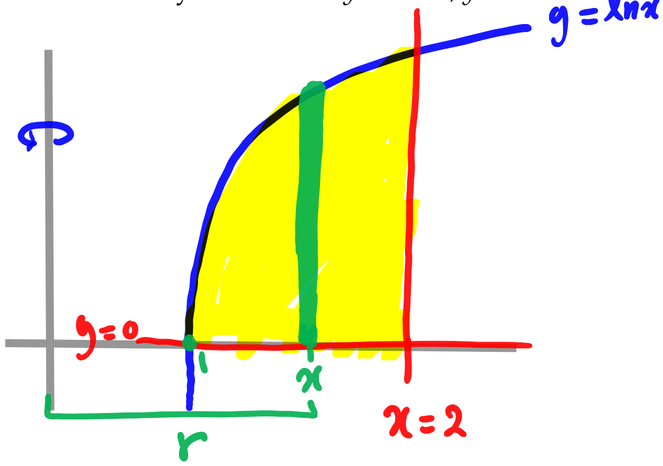
$$I = 2\log_3(x)\ln|x| - \boxed{\int \frac{\log_3 x^2}{x}\,dx}$$

$$2I = 2\log_3(x)\ln|x|$$

$$I = \log_3(x)\ln|x| + C$$

Problem 6

Find the volume generated by the rotating the region bounded by the curves $y = \ln x, y = 0$ and $x = 2$ about y-axis.



$h = \ln x$

$r = x$

$$V = 2\pi \int_1^2 x \ln x \, dx$$

by parts

$u = \ln x \qquad dv = x\, dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^2}{2}$

$$= 2\pi \left[\left[\frac{x^2}{2} \ln x \right]_1^2 - \int_1^2 \frac{1}{2} x \, dx \right]$$

$$= 2\pi \left[2 \ln 2 - \left[\frac{1}{4} x^2 \right]_1^2 \right]$$

$$= 2\pi \left[2 \ln 2 - \left(1 - \frac{1}{4} \right) \right]$$

$$= 4\pi \ln 2 - \frac{3}{2}\pi$$