Problem Set 8
6.3: Volumes by Cylindrical Shells

Please indicate the members who are present. Also indicate the group coordinator.

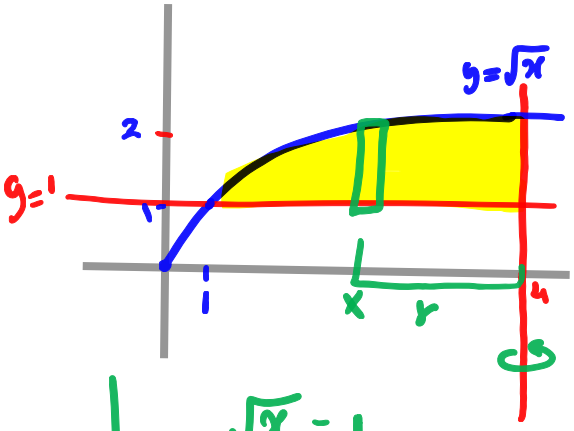
Group Number:	Solution key, many thanks to Majed Bamardouf and his team
Members:	

Problem 1

Using the **method of cylindrical shells**, find the volume of the solid generated by rotating the region enclosed by the curves

$$y = \sqrt{x}, \quad y = 1, \quad , x = 4$$

about the line $x = 4$.



$h = \sqrt{x} - 1$

$r = 4 - x$

$\sqrt{x} = 1$

$x = 1$

$$V = 2\pi \int_1^4 \left[(4-x)(\sqrt{x}-1)\right] dx$$

$$= 2\pi \int_1^4 \left(4x^{\frac{1}{2}} - 4 - x^{\frac{3}{2}} + x\right) dx$$

$$= 2\pi \left[\frac{8}{3}x^{\frac{3}{2}} - 4x - \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{2}x^2\right]_1^4$$

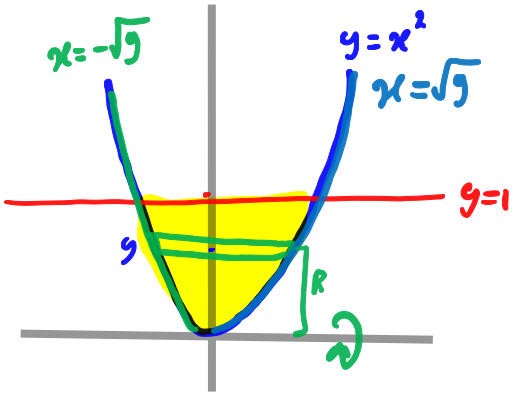
$$= 2\pi \left[\left(\frac{64}{3} - 16 - \frac{64}{5} + 8\right) - \left(\frac{8}{3} - 4 - \frac{2}{5} + \frac{1}{2}\right)\right]$$

$$= 2\pi \left[\frac{56}{3} - \frac{9}{2} - \frac{62}{5}\right] = 2\pi \left[\frac{53}{30}\right]$$

$$= \frac{53\pi}{15}$$

Problem 2

Using the **method of cylindrical shells**, find the volume of the solid generated by rotating the region bounded by the curves $y = x^2$ and $y = 1$ about the x-axis.



$$h = \sqrt{y} - (-\sqrt{y}) = 2\sqrt{y}$$

$$r = y - 0$$

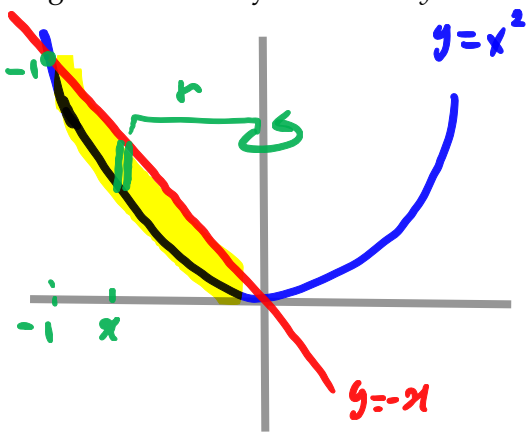
$$V = 2\pi \int_0^1 \left[(y)(2\sqrt{y})\right] dy$$

$$= 2\pi \int_0^1 \left[2y^{\frac{3}{2}}\right] dy$$

$$= 2\pi \left[\frac{4}{5} y^{\frac{5}{2}}\right]_0^1 = \frac{8\pi}{5}$$

Problem 3

Find the volume of the solid generated by rotating the region enclosed by the curves $y = x^2$ and $y = -x$ about the y-axis.



$r = -x$

$h = -x - x^2$

$x^2 = -x$

$x^2 + x = 0$

$x(x+1) = 0$

$x = 0, -1$

$$V = 2\pi \int_{-1}^{0} [(-x)(-x-x^2)]\,dx$$

$$= 2\pi \int_{-1}^{0} (x^2 + x^3)\,dx = 2\pi \left[\frac{1}{3}x^3 + \frac{1}{4}x^4\right]_{-1}^{0}$$

$$= 2\pi \left[-\left(-\frac{1}{3} + \frac{1}{4}\right)\right]$$

$$= \frac{\pi}{6}$$

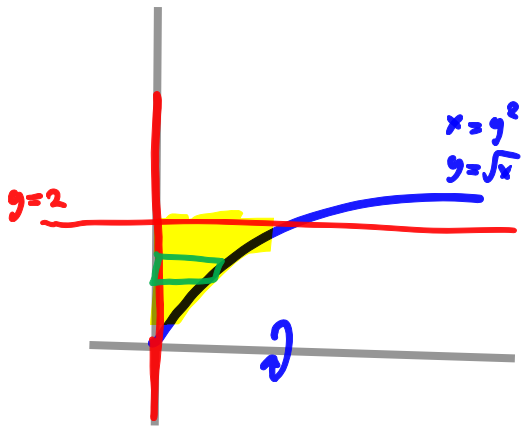
Problem 4

Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}, x = 0,$ and $y = 2$ about the x-axis. **(Use both methods)**

Cylindrical Shell

$h = y^2$

$r = y$

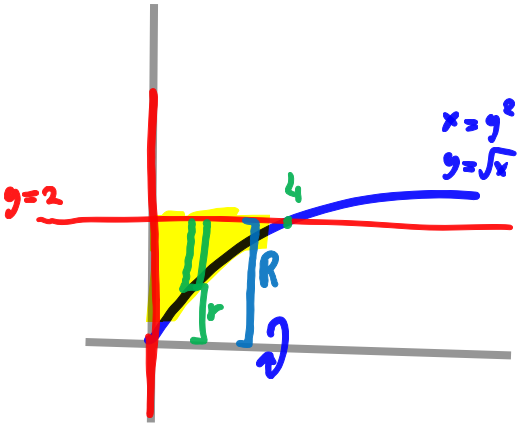


$$V = 2\pi \int_0^2 [(y)(y^2)]\,dy = 2\pi \left[\frac{1}{4}y^4\right]_0^2 = \boxed{8\pi}$$

Washer

$R = 2 - 0$

$r = \sqrt{x} - 0$



$$V = \pi \int_0^4 \left[(2)^2 - (\sqrt{x})^2\right] dx$$

$\sqrt{x} = 2$

$x = 4$

$$= \pi \int_0^4 (4 - x)\,dx = \pi \left[4x - \frac{1}{2}x^2\right]_0^4 = \pi[16 - 8]$$

$$= \boxed{8\pi}$$