Problem Set 2-Wednesday

Group Number:	
Members:	Solution

Problem 3

If $\int_{-5}^{7} f(x)dx = -17$, $\int_{-5}^{11} f(x)dx = 32$, and $\int_{8}^{7} f(x)dx = 5$, then $\int_{11}^{8} f(x)dx =$.

$$\int_{11}^{8} f(x)\,dx = \int_{11}^{7} f(x)dx + \int_{7}^{8} f(x)dx$$

$$= \int_{11}^{-5} f(x)dx + \int_{-5}^{7} f(x)dx - \int_{8}^{7} f(x)dx$$

$$= -32 - 17 - 5 = -54$$

Problem 4

If $f(x) = \begin{cases} -x; & -4 \le x < 0 \\ \sqrt{4-x^2}; & 0 \le x \le 2 \end{cases}$, then the value of the integral $\int_{-4}^{2} f(x)dx$ by interpreting in terms of area(s) is.

$$\therefore \int_{-4}^{2} f(x)dx = A_1 + A_2 = \frac{1}{2}4 \cdot 4 + \frac{4\pi}{4}$$
$$= 16 + \pi$$

Problem 5

Write the limit as an integral (do not evaluate)

$$L = 2 \lim_{n\to\infty} \sum_{i=1}^{n} \left[1 + \sin\left(1 + \frac{i}{n}\right)\right] \frac{2}{n} =$$

$$= \int_a^b f(x)\,dx$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

Soln: $\Delta x = \frac{1}{n}$, $a = 0$, $b = 1$

$x_i = a + i\Delta x = \frac{i}{n}$

$\therefore f(x) = 2 + 2\sin(1+x)$

$\therefore \int_0^1 (2 + 2\sin(1+x))\,dx$

Soln 2: $\Delta x = \frac{2}{n}$, $a = 0$, $b = 2$

$x_i = a + i\Delta x = \frac{2i}{n}$.

$2\left(\frac{i}{n}\right) = x_i \Rightarrow \frac{i}{n} = \frac{x_i}{2}$

$\therefore f(x) = 1 + \sin\left(1 + \frac{x}{2}\right)$

$\therefore L = \int_0^2 \left(1 + \sin\left(1 + \frac{x}{2}\right)\right) dx$

Problem 6

$$\lim_{n\to\infty} \frac{2}{n^4}(1+8+27+\cdots+n^3) =$$

$$= \lim_{n\to\infty} \frac{2}{n^4} \left[\sum_{i=1}^{n} i^3 \right]$$

$$= \lim_{n\to\infty} \frac{2}{n^4} \left[\frac{n(n+1)}{2} \right]^2$$

$$= \lim_{n\to\infty} \frac{2n^2(n+1)^2}{2n^4} = \frac{1}{2}$$

Problem 7

If f is continuous function and

$$2 \le f(x) \le 5 \quad \text{for} \quad 3 \le x \le 9,$$

then ONE of the following statements is **FALSE**

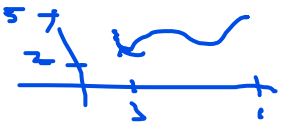
(A) $\int_3^9 |f(x)|\,dx \ge 12$

(B) $\int_3^9 (3 - f(x))\,dx \ge -12$

(C) $\int_3^9 (1 - |f(x)|)\,dx \ge -10$

(D) $\int_3^9 -2f(x)dx \le -24$

(E) $\int_3^9 (f(x))^2\,dx \ge 24$



$f(x) \ge 2$

$|f(x)| = f(x)$

$12 \le \int_3^9 f(x)dx \le 30$

(A) $\int_3^9 |f(x)|\,dx = \int_3^9 f(x)dx \ge 12$

(B) $2 \le f(x) \le 5$

$-5 \le -f(x) \le -2$

$-2 \le 3 - f(x) \le 1$

$-12 \le \int_3^9 (3 - f(x))dx \le 6$

(C) $-5 \le -|f(x)| \le -2$

$-4 \le 1 - |f(x)| \le -1$

$-24 \le \int_3^9 (1 - |f(x)|)dx \le -6$

X

(D) $-2f(x) \le -4$

$\int_3^9 -2f(x)dx \le -24$

(E) $2 \le f(x) \le 5$

$4 \le f^2(x) \le 25$

$24 \le \int_3^9 f^2(x)dx$