Problem Set 14
7.5: Strategy of Integration

Please indicate the members who are present. Also indicate the group coordinator.

Group Number:	
Members:	KEY

Problem 1

Find the integral $\int \frac{x+\arcsin(x)}{\sqrt{1-x^2}}\,dx$.

$$I = \underbrace{\int \frac{x}{\sqrt{1-x^2}}\,dx}_{I_1} + \underbrace{\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx}_{I_2}$$

$$I_1 = \frac{-1}{2}\int \frac{du}{\sqrt{u}}$$

let $u = 1-x^2$

$du = -2x\,dx$

$$= -\frac{1}{2}\cdot 2u^{\frac{1}{2}} + C_1 = -(1-x^2)^{\frac{1}{2}} + C_1$$

$$I_2 = \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$$

$$\boxed{\begin{aligned} v &= \sin^{-1}x \\ dv &= \frac{1}{\sqrt{1-x^2}}\,dx \end{aligned}}$$

$$= \int v\,dv = \frac{v^2}{2} + C_2 = \frac{(\sin^{-1}x)^2}{2} + C_2$$

$$I = -\sqrt{1-x^2} + \frac{(\sin^{-1}x)^2}{2} + C$$

$$\therefore \boxed{C = C_1 + C_2}$$

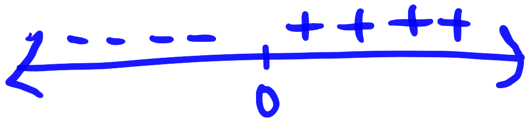
Problem 2

Evaluate $\int_{-1}^{2} |e^x - 1| dx$.

$e^x - 1 = 0$

$e^x = 1$

$x = 0$



$$I = \int_{-1}^{0} -(e^x - 1)\,dx + \int_{0}^{2} (e^x - 1)\,dx$$

$$= -\left[e^x - x\right]_{-1}^{0} + \left[e^x - x\right]_{0}^{2}$$

$$= -\left[1 - \left(\frac{1}{e} + 1\right)\right] + \left[e^2 - 2 - (1)\right]$$

$$= \frac{1}{e} + e^2 - 3$$

Problem 3

Evaluate $\int \frac{\tan^{-1} x}{x^2} dx$.

$$u = \tan^{-1} x \qquad dv = \frac{1}{x^2}\,dx$$
$$du = \frac{1}{x^2+1} \qquad v = -\frac{1}{x}$$

$$I = \frac{-\tan^{-1} x}{x} - \int -\frac{1}{x(x^2+1)}\,dx$$

$$= \frac{-\tan^{-1}}{x} + \int \frac{1}{\underline{x(x^2+1)}}\,dx$$

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

Partial fraction decomposition

$$1 = A(x^2+1) + (Bx+C)x$$

$x=0$: $\boxed{A=1}$ $\qquad 1 = Ax^2 + A + Bx^2 + Cx$

$\boxed{C=0}$ $\qquad 0 = A + B$

$\boxed{B=-1}$

$$\therefore I = \frac{-\tan^{-1} x}{x} + \int \frac{1}{x}\,dx - \int \frac{x}{x^2+1}\,dx$$

$$= \frac{-\tan^{-1} x}{x} + \ln|x| - \frac{1}{2}\ln(x^2+1) + C$$

$$= \frac{-\tan^{-1} x}{x} + \ln\left|\frac{x}{\sqrt{x^2+1}}\right| + C$$

Problem 4

(a) Evaluate $\int \sqrt{1-\sin(x)}dx.$ $= I = \int \sqrt{1-\sin x}\cdot\frac{\sqrt{1+\sin x}}{\sqrt{1+\sin x}}\,dx$

(b) Evaluate $\int_{-\pi/2}^{\pi/2} \sqrt{1-\sin(x)}dx.$

$$= \int \frac{\sqrt{1-\sin^2 x}}{\sqrt{1+\sin x}}\,dx = \int \frac{\sqrt{\cos^2 x}}{\sqrt{1+\sin x}}\,dx = \int \frac{|\cos x|}{\sqrt{1+\sin x}}\,dx$$

$$= \int \frac{du}{(u)^{\frac{1}{2}}}$$

$$\boxed{\begin{aligned} &\text{Let } u = 1+\sin x \\ &du = \cos x\,dx \end{aligned}}$$

$$I = 2u^{\frac{1}{2}} + C = 2\sqrt{1+\sin x} + C$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin x}\,dx = 2\left[\sqrt{1+\sin x}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$\boxed{= 2\sqrt{2}}$$

Problem 5

Find the integral $\int x \sin^2 x \cos x \, dx$.

$$u = x \qquad dv = \sin^2 x \cos x \, dx$$
$$du = dx \qquad v = \frac{1}{3}\sin^3 x$$

$$I = \frac{x}{3}\sin^3 x - \frac{1}{3}\underbrace{\int \sin^3 x \, dx}_{I_1}$$

$$w = \cos x$$
$$dw = -\sin x \, dx$$

$$I_1 = \int \sin^2 x \quad \sin x \, dx$$

$$= \int (1 - \cos^2 x) \sin x \, dx = -\int (1 - w^2) \, dw$$

$$= -w + \frac{1}{3} w^3 + C_1 = -\cos x + \frac{1}{3}\cos^3 x + C_1$$

$$I = \frac{x}{3}\sin^3 x + \frac{1}{3}\cos x - \frac{1}{9}\cos^3 x - \frac{C_1}{3}$$

$$= \frac{x}{3}\sin^3 x + \frac{1}{3}\cos x - \frac{1}{9}\cos^3 x + C$$

$$\boxed{C = \frac{-C_1}{3}}$$

Problem 6

Find the integral $\int \frac{dx}{\sqrt{x}+x\sqrt{x}}$.

$$u = x^{\frac{1}{2}}$$

$$du = \frac{1}{2} x^{-\frac{1}{2}} dx$$

$$2du = \frac{1}{x^{1/2}} dx$$

$$= \int \frac{dx}{x^{\frac{1}{2}} + x^{3/2}}$$

$$= \int \frac{dx}{x^{\frac{1}{2}}(1+x)} dx$$

$$= \int \frac{2}{1+u^2} du$$

$$= 2\tan^{-1} u + C$$

$$= 2\tan^{-1}(\sqrt{x}) + C$$