Problem Set 6-~~Wednesday~~ Monday

Please indicate the members who are present. Also indicate the group coordinator.

Group Number:	
Members:	

Solution Key

Problem 1

$$A = \int_a^b \left| x^2 - 2x - 2 + x \right| dx$$

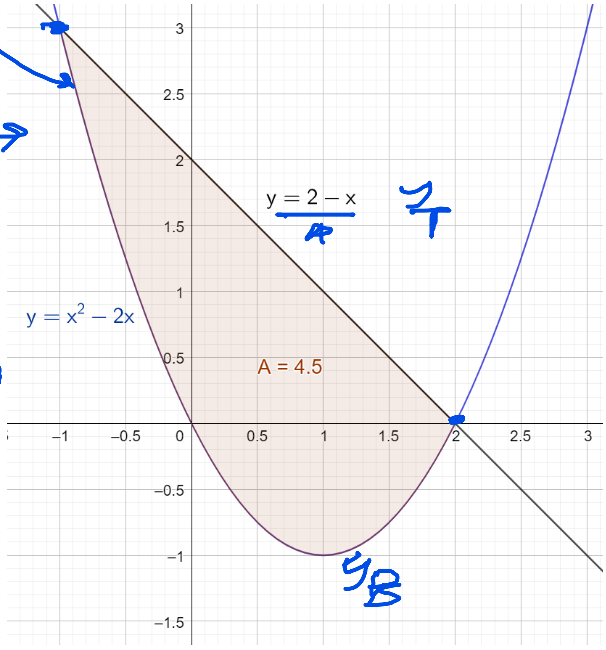
Find the area of the region enclosed by the curves $y = x^2 - 2x$ and $y = 2 - x$.

POI: $x^2 - 2x = 2 - x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = -1, 2$



Clearly from the figure shown $y = 2 - x$ is above $y = x^2 - 2x$

$$\therefore A = \int_{-1}^{2} \left[(2 - x) - (x^2 - 2x) \right] dx$$

$$= \int_{-1}^{2} (2 + x - x^2)\, dx$$

$$= \left[2x + \frac{x^2}{2} - \frac{x^3}{3} \right]_{-1}^{2}$$

$$= \left(4 + 2 - \frac{8}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right)$$

$$= 6 - \frac{8}{3} + 2 - \frac{1}{2} - \frac{1}{3}$$

$$= 8 - 3 - \frac{1}{2} = 5 - \frac{1}{2} = \frac{9}{2} = 4.5$$

Problem 2

$(x-h) = (y-k)^2$ $\quad x + 4 = (y^2 - 4y + 4) \Longleftrightarrow$

$x + 4 = (y-2)^2$

$(-4, 2)$

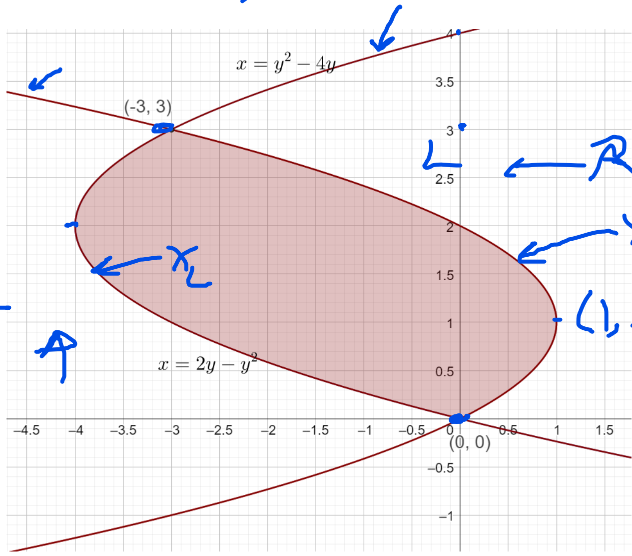
Find the area of the region enclosed by the curves $x = y^2 - 4y$ and $x = 2y - y^2$.

$x = 2y - y^2$

$x - 1 = -(y^2 - 2y + 1)$

$(x-1) = -(y-1)^2$

vertex: $(1,1)$



PoI: $y^2 - 4y = 2y - y^2$

$2y^2 - 6y = 0$

$y(y-3) = 0$

$y = 0, 3$

$$\therefore A = \int_0^3 (x_R - x_L)\,dy$$

$$= \int_0^3 \left[(2y - y^2) - (y^2 - 4y)\right] dy$$

$$= \int_0^3 (6y - 2y^2)\,dy$$

$$= \left[\frac{6y^2}{2} - \frac{2y^3}{3}\right]_0^3$$

$$= 3\cdot 9 - \frac{2}{3}\cdot 27 = \underline{\underline{9}}$$

Problem 3

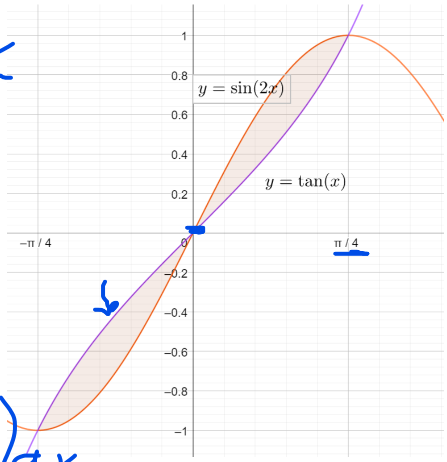
$0 \le 2x \le \pi$

$0 \le x \le \boxed{\frac{\pi}{2}}$

Find the area of the region enclosed by the curves

$$y = \sin 2x \quad \text{and} \quad y = \tan x, \quad \frac{-\pi}{4} \le x \le \frac{\pi}{4}.$$

$$A = \int_{-\pi/4}^{\pi/4} \underbrace{|\sin 2x - \tan x|}\, dx$$



$$A = \int_{-\pi/4}^{0} (\tan x - \sin 2x)\, dx + \int_{0}^{\pi/4} (\sin 2x - \tan x)\, dx$$

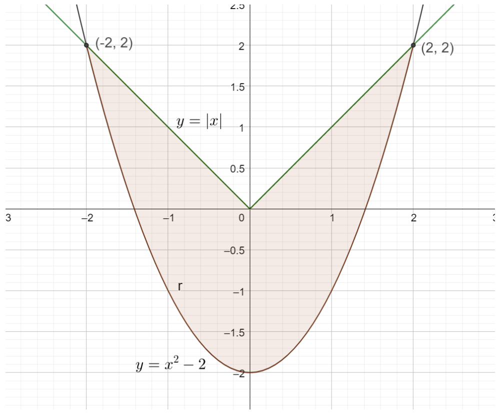
$$= \left[\ln|\sec x| + \frac{\cos 2x}{2}\right]_{-\pi/4}^{0} + \left[-\frac{\cos 2x}{2} - \ln|\sec x|\right]_{0}^{\pi/4}$$

$$= \left[\left(0 + \frac{1}{2}\right) - \left(\ln\sqrt{2} + 0\right)\right] + \left[0 - \ln\sqrt{2} + \frac{1}{2} + 0\right]$$

$$= 1 - 2\ln\sqrt{2} = \boxed{1 - \ln 2}$$

Problem 4

Find the area of the region enclosed by the curves $y = |x|$ and $y = x^2 - 2$.



$|x| = x^2 - 2$

P_I: $x \geq 0$:

$x = x^2 - 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = \cancel{-1}, \underline{\underline{2}}$

$x < 0$

$\underline{\underline{-x}} = x^2 - 2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, \cancel{1}$

$$\therefore A = 2\int_{-2}^{0} (-x - x^2 + 2)\,dx$$

$$= 2\left[-\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_{-2}^{0}$$

$$= 2\left[0 - \left(-\frac{4}{2} + \frac{8}{3} - 4\right)\right]$$

$$= 2\left(6 - \frac{8}{3}\right) = 2\left(\frac{18-8}{3}\right) = \frac{2 \cdot 10}{3} = \frac{20}{3} \ \#$$