Problem Set 2

Group Number:	
Members:	
	Solution

Problem 1

If R_n is the Riemann sum for $f(x) = 4 + \frac{x^2}{8}, 0 \le x \le 4$ with n subintervals and taking sample points to be the right end points, then $R_n =$

$$R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x \quad , \quad a=0, \; b=4$$

$$\Delta x = \frac{4-0}{n} = \frac{4}{n} \quad , \quad x_i = a + i\Delta x = \frac{4i}{n}$$

$$R_n = \sum_{i=1}^{n} f\left(\frac{4i}{n}\right)\frac{4}{n} = \frac{4}{n}\sum_{i=1}^{n}\left[4 + \frac{16\,i^2}{8n^2}\right]$$

$$= \frac{4}{n}\left[\sum_{i=1}^{n} 4 + \frac{2}{n^2}\sum_{i=1}^{n} i^2\right]$$

$$= \frac{4}{n}\left[4n + \frac{\cancel{2}}{n^2}\,\frac{n(n+1)(2n+1)}{\cancel{6}\,3}\right]$$

$$= \underline{16 + \frac{4}{3}\,\frac{(n+1)(2n+1)}{n^2}}$$

Problem 2

$\lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\frac{1}{n}}_{\Delta x} \cos\left(1+\frac{i}{n}\right)^2 =$

(A) $\int_1^2 \cos(1+x^2)dx$.

(B) $\int_1^2 \cos(x^2)dx$.

(C) $\int_1^2 \cos^2(x)dx$.

(D) $\int_0^1 \cos(x^2)dx$.

(E) $\int_0^1 \cos(1+x^2)dx$.

First decide what is Δx

$\Delta x = \frac{1}{n} = \frac{b-a}{n}$

$\Rightarrow b-a=1$

According to the options

$a=1, \quad b=2$

or $a=0, \quad b=1$

If $a=1, b=2$

then $x_i = a + i\Delta x = 1 + \frac{i}{n}$

So $f(x) = \cos x^2$ So the integral is $\int_1^2 \cos x^2 \, dx$

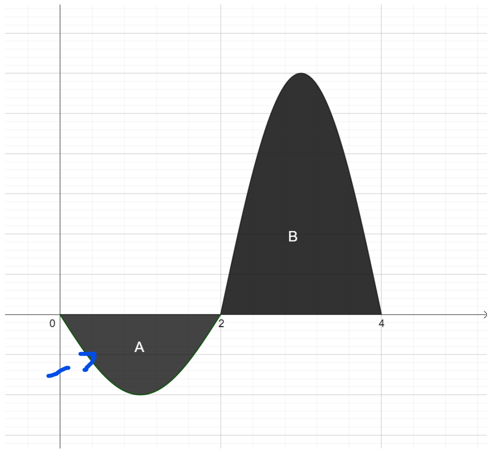
is it one of the option?

Yes.

Suppose it wasn't the we check the othe option

$a=0, b=1$ & So on...

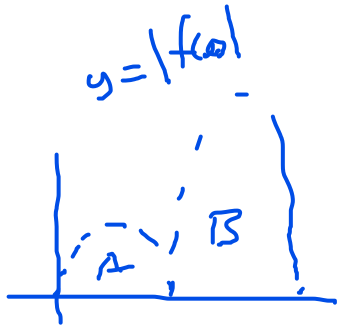
Problem 3



In the figure shown, regions A and B are bounded by the graph of a function f and the x-axis. If the area of region A is $\frac{1}{6}$ and the area of the region B is $\frac{3}{8}$, then

$$\int_0^4 f(x)dx + \int_0^4 |f(x)|dx =$$

$$\int_0^4 f(x)\,dx = -A + B$$



$$\int_0^4 f(x)\,dx = A + B$$

$$\therefore \int_0^4 f(x)\,dx + \int_0^4 |f(x)|\,dx = -A + B + A + B$$

$$= 2B = \frac{3}{4}$$