Problem Set 19
11.2: Series

Please indicate the members who are present. Also indicate the group coordinator.

Group Number:	
Members:	

The geometric series $\sum_{n=0}^{\infty} a r^n$

$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots$ converges if $|r| < 1$ and its sum is $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}, |r| < 1$

if $|r| \geq 1$, the geometric series is divergent.

Divergence Test If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

The harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots = \lim_{n\to\infty} \underbrace{\sum_{i=1}^{n} a_i}_{S_n} = S$$

$$\sum_{n=1}^{\infty} a_n = S$$

sequence of partial sums

Problem 1

$s_1 = a_1 = \frac{1-8}{5+3}$ / $s_2 = a_1 + a_2 =$

$s_7 = a -$

The sequence $s_n = \dfrac{n^2-8}{5n^2+3}$ is the sequence of partial sums of the series $\sum_{n=1}^{\infty} a_n$. Does this series converge or diverge? If it is convergent, find its sum.

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{n^2-8}{5n^2+3} = \frac{1}{5}$$

$$\sum_{n=1}^{\infty} a_n = \frac{1}{5}$$

Convergent

Problem 2

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\frac{1}{3}+\frac{2}{9}+\frac{1}{27}+\frac{2}{81}+\frac{1}{243}+\frac{2}{729}+\cdots$$

$$\cdot \sum_{n=0}^{\infty} a_n = \frac{1}{3}+\frac{1}{27}+\frac{1}{243}+\cdots = \sum_{n=0}^{\infty}\frac{1}{3^{2n+1}} = \sum_{n=0}^{\infty}\frac{1}{3}\left(\frac{1}{9}\right)^n$$

$$= \frac{\frac{1}{3}}{1-\frac{1}{9}} \qquad \text{[geometric with } r=\tfrac{1}{9},\ a=\tfrac{1}{3}]$$

$$= \frac{\frac{1}{3}}{\frac{8}{9}} = \frac{3}{8}$$

$$\cdot \sum_{n=0}^{\infty} b_n = \frac{2}{9}+\frac{2}{81}+\frac{2}{729}+\cdots = \sum_{n=0}^{\infty} 2\left(\frac{1}{9}\right)^{n+1}$$

$$= \sum_{n=0}^{\infty}\frac{2}{9}\left(\frac{1}{9}\right)^n$$

$$= \frac{\frac{2}{9}}{1-\frac{1}{9}} \qquad \text{[geometric with } r=\tfrac{1}{9},\ a=\tfrac{2}{9}]$$

$$= \frac{\frac{2}{9}}{\frac{8}{9}} = \frac{2}{8}$$

$$\therefore \frac{1}{3}+\frac{2}{9}+\frac{1}{27}+\frac{2}{81}+\cdots = \frac{3}{8}+\frac{2}{8} = \frac{5}{8}$$

Problem 3

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \underbrace{\frac{2+n}{1-2n}}_{a_n}$$

$$\lim_{n\to\infty} \frac{2+n}{1-2n} = -\frac{1}{2} \neq 0$$

$\therefore$ By the divergece test

the series diverges

Problem 4

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=2}^{\infty} \frac{1}{n^3 - n}$$

$\lim_{n\to\infty} \frac{1}{n^3-n} = 0$

$$\sum_{n=2}^{\infty} \frac{1}{n^3-n} = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \boxed{\sum_{i=2}^{n} \frac{1}{i^3-i}}$$

$$\frac{1}{i^3-i} = \frac{1}{i(i-1)(i+1)} = \frac{-1}{i} + \frac{\frac{1}{2}}{i-1} + \frac{\frac{1}{2}}{i+1}$$

$$= \frac{-1}{i} + \frac{1}{2(i-1)} + \frac{1}{2(i+1)}$$

$$s_n = \sum_{i=2}^{n} \left[\frac{-1}{i} + \frac{1}{2(i-1)} + \frac{1}{2(i+1)}\right]$$

$$= \left(\cancel{\frac{-1}{2}} + \cancel{\frac{1}{2}} + \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{-1}{3}} + \underbrace{\frac{1}{4}} + \cancel{\frac{1}{8}}\right)$$

$$+ \left(\cancel{\frac{-1}{4}} + \cancel{\frac{1}{6}} + \cancel{\frac{1}{10}}\right) + \left(\cancel{\frac{-1}{5}} + \cancel{\frac{1}{8}} + \cancel{\frac{1}{10}}\right) + \cdots$$

$$+ \cdots + \left(\cancel{\frac{-1}{n-1}} + \cancel{\frac{1}{2(n-2)}} + \cancel{\frac{1}{2(n)}}\right) + \left(\cancel{\frac{-1}{n}} + \cancel{\frac{1}{2n}} + \frac{1}{2(n+1)}\right)$$

$$= \underbrace{\frac{1}{4} + \frac{1}{2(n+1)}}$$

$$\sum_{n=2}^{\infty} \frac{1}{n^3-n} = \lim_{n\to\infty} s_n = \frac{1}{4}$$

Convergent

Problem 5

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty}\left(e^{\frac{1}{n}} - e^{\frac{1}{n+1}}\right) = \lim_{n\to\infty} s_n$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n}\left(e^{\frac{1}{i}} - e^{\frac{1}{i+1}}\right)$$

$$= \lim_{n\to\infty}\left[\left(e^{1} - \cancel{e^{\frac{1}{2}}}\right) + \left(\cancel{e^{\frac{1}{2}}} - \cancel{e^{\frac{1}{3}}}\right) + \left(\cancel{e^{\frac{1}{3}}} - \cancel{e^{\frac{1}{4}}}\right) + \dots + \left(\cancel{e^{\frac{1}{n}}} - e^{\frac{1}{n+1}}\right)\right]$$

$$= \lim_{n\to\infty}\left(e - e^{\frac{1}{n+1}}\right)$$

$$= \underline{\underline{e - 1}}$$

Convergent

Problem 6

Express the number as a ratio of integers $0.\bar{8} = 0.8888....$

$$0.\overline{8} = \frac{8}{10} + \frac{8}{100} + \frac{8}{1000} + \cdots$$

$$= \sum_{n=1}^{\infty} 8\left(\frac{1}{10}\right)^{n} = \sum_{n=1}^{\infty} \frac{8}{10}\left(\frac{1}{10}\right)^{n-1}$$

$$= \frac{8/10}{1 - 1/10} = \frac{8/10}{9/10} = \frac{8}{9}$$

Problem 7

If the nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is

$$s_n = \frac{n-1}{n+2}$$

find a_n and $\sum_{n=1}^{\infty} a_n$.

$$S_n = a_1 + a_2 + \cdots + a_{n-1} + a_n$$

$$S_{n-1} = a_1 + a_2 + \cdots + a_{n-1}$$

$$\boxed{S_n - S_{n-1} = a_n}$$

$$a_n = S_n - S_{n-1} = \frac{n-1}{n+2} - \frac{n-2}{n+1}$$

$$= \frac{n^2 - 1 - n^2 + 4}{(n+1)(n+2)} = \frac{3}{(n+1)(n+2)}$$

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{3}{(n+1)(n+2)} = \lim_{n \to \infty} S_n = 1$$