Problem Set 17
8.1 and 8.2: Arc Length and Surface Area

Please indicate the members who are present. Also indicate the group coordinator.

Group Number:	
Members:	KEY

Problem 1

Find the length of the curve $y = \ln \sqrt{\sec 2x}, \quad 0 \leq x \leq \frac{\pi}{6}$.

$$\frac{dy}{dx} = \frac{1}{\cancel{\sqrt{\sec 2x}}} \cdot \frac{1}{\cancel{2\sqrt{\sec 2x}}} \cdot \cancel{\sec(2x)} \cdot \tan(2x) \cdot \cancel{2} = \tan(2x)$$

$$L = \int_0^{\frac{\pi}{6}} \sqrt{1 + \tan^2(2x)}\, dx = \int_0^{\frac{\pi}{6}} \sqrt{\sec^2(2x)}\, dx$$

$$= \int_0^{\frac{\pi}{6}} \sec(2x)\, dx = \frac{1}{2} \ln\left|\sec(2x) + \tan(2x)\right| \Big]_0^{\frac{\pi}{6}}$$

$$= \frac{1}{2}\left[\ln(2+\sqrt{3}) - \ln(1+0)\right]$$

$$= \frac{1}{2}\ln(2+\sqrt{3})$$

Problem 2

Find the length of the curve $y = \frac{1}{3}\left(x^2+2\right)^{3/2}$ from $x = 0$ to $x = 3$.

$$\frac{dy}{dx} = \frac{1}{2}\left(x^2+2\right)^{1/2} \cdot 2x = x\sqrt{x^2+2}$$

$$L = \int_0^3 \sqrt{1+\left(x\sqrt{x^2+2}\right)^2}\, dx = \int_0^3 \sqrt{1+x^4+2x^2}\, dx$$

$$= \int_0^3 \sqrt{\left(x^2+1\right)^2}\, dx = \int_0^3 \left(x^2+1\right) dx$$

$$= \left.\frac{x^3}{3} + x\right]_0^3$$

$$= 9+3 = 12$$

Problem 3

Find the length of the curve $y = \ln x$ from $x = 1$ to $x = e$.

$$\frac{dy}{dx} = \frac{1}{x}$$

$$L = \int_1^e \sqrt{1 + \frac{1}{x^2}}\, dx = \int_1^e \sqrt{\frac{x^2+1}{x^2}}\, dx$$

$$= \int_1^e \frac{\sqrt{x^2+1}}{x}\, dx$$

$x = \tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$dx = \sec^2\theta\, d\theta$

$\sqrt{\tan^2\theta + 1} = \sqrt{\sec^2\theta} = \sec\theta$

$x = e \rightarrow \theta = \tan^{-1} e = \alpha$

$x = 1 \rightarrow \theta = \frac{\pi}{4}$

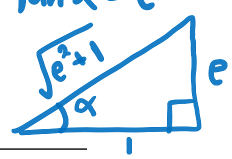
$$= \int_{\pi/4}^{\alpha} \frac{\sec\theta}{\tan\theta} \sec^2\theta\, d\theta = \int_{\frac{\pi}{4}}^{\alpha} \frac{\sec\theta\,(1+\tan^2\theta)}{\tan\theta}\, d\theta = \int_{\frac{\pi}{4}}^{\alpha} \frac{\sec\theta + \sec\theta\tan^2\theta}{\tan\theta}\, d\theta$$

$$= \int_{\pi/4}^{\alpha} \left[\csc\theta + \sec\theta\tan\theta\right] d\theta = \Big[\ln|\csc\theta - \cot\theta| + \sec\theta\Big]_{\frac{\pi}{4}}^{\alpha}$$

$$= \left[\ln\left|\csc(\alpha) - \frac{1}{\tan(\alpha)}\right| + \sec(\alpha)\right] - \left[\ln|\sqrt{2}-1| + \sqrt{2}\right]$$

$\tan^{-1}(e) = \alpha$

$\tan\alpha = e$

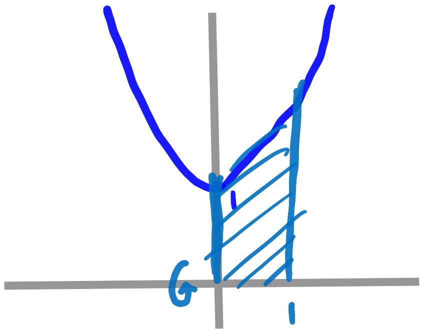


$$= \ln\left(\frac{\sqrt{e^2+1}}{e} - \frac{1}{e}\right) + \sqrt{e^2+1} - \ln(1-\sqrt{2}) - \sqrt{2}$$

Problem 4

Find the area of the surface generated by revolving the curve of $y = \cosh x, \quad 0 \leq x \leq 1$, about the x-axis.

$$\frac{dy}{dx} = \sinh x$$



$$S = \int_0^1 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = 2\pi \int_0^1 \cosh x \sqrt{1+\sinh^2 x}\, dx$$

$$= 2\pi \int_0^1 \cosh^2 x\, dx = 2\pi \int_0^1 \left[\frac{e^{x}+e^{-x}}{2}\right]^2 dx$$

$$= 2\pi \int_0^1 \left[\frac{e^{2x}+2+e^{-2x}}{4}\right] dx$$

$$= \frac{\pi}{2} \int_0^1 \left[e^{2x}+2+e^{-2x}\right] dx$$

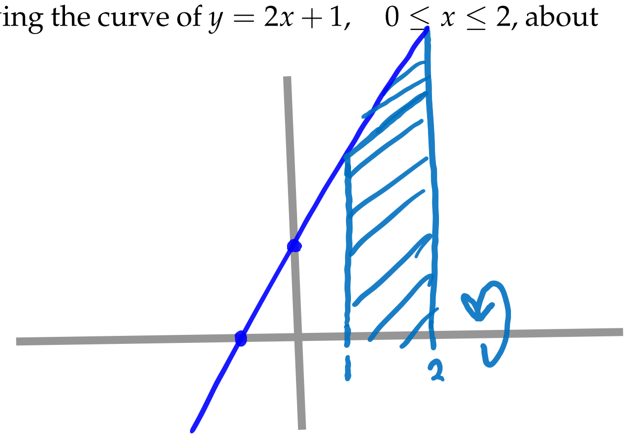
$$= \frac{\pi}{2} \left[\frac{1}{2}e^{2x}+2x-\frac{1}{2}e^{-2x}\right]_0^1$$

$$= \frac{\pi}{2}\left[\left(\frac{e^2}{2}+2-\frac{e^{-2}}{2}\right)-\left(\frac{1}{2}-\frac{1}{2}\right)\right] = \frac{\pi}{2}\left(\underbrace{\frac{e^2-e^{-2}}{2}}_{\sinh(2)}+2\right) = \frac{\pi}{2}\sinh 2 + \pi$$

Problem 5

Find the area of the surface generated by revolving the curve of $y = 2x + 1, \quad 0 \le x \le 2$, about the x-axis.

$$\frac{dy}{dx} = 2$$



$$S = \int_0^2 2\pi \cdot y \cdot \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = 2\pi \int_0^2 (2x+1) \cdot \sqrt{5}\, dx$$

$$= 2\pi\sqrt{5}\left[x^2 + x\right]_0^2$$

$$= 2\pi\sqrt{5}\,[4+2] = 12\pi\sqrt{5}$$