

Problem Set Sections 11.5

Alternating Series and Absolute Convergence

If $s = \sum (-1)^{n-1} b_n$ where $b_n > 0$ is the sum of an alternating series that satisfies

$$(i) \ b_{n+1} \leq b_n \quad \text{and} \quad (ii) \ \lim_{n \rightarrow \infty} b_n = 0$$

then

$$|R_n| = |s - s_n| \leq b_{n+1}$$

A series $\sum a_n$ is called *absolutely convergent* if and only if the series of the absolute values $\sum |a_n|$ is convergent.

A series $\sum a_n$ is called *conditionally convergent* if $\sum a_n$ converges but $\sum |a_n|$ diverges.

Problem 1

Test for convergence $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3+5n}$.

Problem 2

Test for convergence $\sum_{n=1}^{\infty} (-1)^{n-1} \arctan n$.

Problem 3

Test for convergence $\sum_{n=1}^{\infty} \frac{n \cos n\pi}{2^n}$.

Problem 4

Test for convergence $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n})$.

Problem 5

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$$

Problem 6

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{1 + 2 \sin n}{n^3}$$

Problem 7

Show that the series is convergent. How many terms of the series do we need to add in order to find the sum such that $|\text{error}| < 0.0005$

$$\sum_{n=1}^{\infty} \frac{\left(-\frac{1}{3}\right)^n}{n}$$

