Problem Set
Sections 11.2

Series

The geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots \text{ converges if } |r| < 1 \text{ and its sum is } \sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}, |r| < 1$$

if $|r| \geq 1$, the geometric series is divergent.

Divergence Test If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

The harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

Problem 1

The sequence $s_n = \dfrac{n^2 - 8}{5n^2 + 3}$ is the sequence of partial sums of the series $\sum_{n=1}^{\infty} a_n$. Does this series converge or diverge? If it is convergent, find its sum.

Problem 2

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\frac{1}{3} + \frac{2}{9} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} + \frac{2}{729} + \cdots$$

Problem 3

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{2+n}{1-2n}$$

Problem 4

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=2}^{\infty} \frac{1}{n^3 - n}$$

Problem 5

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty}\left(e^{\frac{1}{n}}-e^{\frac{1}{n+1}}\right)$$

Problem 6

Express the number as a ratio of integers $0.\bar{8} = 0.8888....$

Problem 7

If the nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is

$$s_n = \frac{n-1}{n+2}$$

find a_n and $\sum_{n=1}^{\infty} a_n$.