1. $\lim_{n\to\infty}\left[\frac{10}{n}\sum_{i=1}^{n}\sqrt{\frac{10\,n+4i}{2n}}\right] =$

(a) $\frac{10}{3}(7\sqrt{7}-5\sqrt{5})$

(b) $\frac{10}{3}(\sqrt{7}-\sqrt{5})$

(c) $\frac{5}{3}(7\sqrt{7}-5\sqrt{5})$

(d) $\frac{5}{3}(\sqrt{7}-\sqrt{5})$

(e) $\frac{2}{3}(\sqrt{7}-\sqrt{5})$

2. Using 3 approximating rectangles and midpoints, the approximation of the area under $f(x)=\frac{x+1}{x}$, $1\le x\le 7$, is

(a) $\frac{47}{6}$

(b) $\frac{47}{12}$

(c) $\frac{37}{6}$

(d) $\frac{37}{12}$

(e) $\frac{43}{7}$

3. $\displaystyle\int_{\pi}^{3\pi/2} \sqrt{1-\cos^2 t}\, dt =$

(a) $1$

(b) $-1$

(c) $0$

(d) $\sqrt{2}$

(e) $-\sqrt{2}$

4. $\displaystyle\int_{4}^{10} \frac{t}{t^2-4}\, dt =$

(a) $\dfrac{3}{2}\ln 2$

(b) $3\ln 2$

(c) $\dfrac{3}{4}\ln 2$

(d) $\dfrac{1}{2}\ln 2$

(e) $3\ln 4$

5. The value of the definite integral $\int_0^{3/2} \sqrt{9-4x^2}\,dx$ is
(Hint: You may use areas)

(a) $\dfrac{9\,\pi}{8}$

(b) $\dfrac{3\,\pi}{8}$

(c) $\dfrac{9\,\pi}{16}$

(d) $\dfrac{\pi}{9}$

(e) $\dfrac{\pi}{8}$

6. $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \dfrac{n}{3n^2+k^2} =$

(a) $\dfrac{\pi}{6\sqrt{3}}$

(b) $\dfrac{\pi}{3\sqrt{3}}$

(c) $\dfrac{\pi}{4\sqrt{3}}$

(d) $\dfrac{1}{3\sqrt{3}}$

(e) $\dfrac{1}{6\sqrt{3}}$

7. Let $f$ be a continuous function on $[-1,4]$ such that:

$$\int_{-1}^{2} f(t)\,dt = 5 \text{ and } \int_{1}^{2} f(2t)\,dt = 2,$$

then $\int_{-1/3}^{4/3} f(3t)\,dt =$

(a) $3$

(b) $\frac{1}{3}$

(c) $-3$

(d) $\frac{-1}{3}$

(e) $9$

8. The volume of the solid generated by revolving the region bounded by the curves $y = \cos x$ and $y = 0$, for $0 \le x \le \frac{\pi}{2}$, about the line $y = -1$ is given by

(a) $\int_0^{\pi/2} \pi[(1+\cos x)^2 - 1]\,dx$

(b) $\int_0^{\pi/2} \pi[\cos^2 x - 1]\,dx$

(c) $\int_0^{\pi/2} \pi \cos^2 x\,dx$

(d) $\int_0^{\pi/2} \pi[1-(1-\cos x)^2]\,dx$

(e) $\int_0^{\pi/2} \pi(1+\cos^2 x)\,dx$

9. Let $f$ be the function defined on $R$ by

$$f(x) = \int_0^x (4 - t^2)\, e^{t^3}\, dt.$$

Which of the following statement is **FALSE**?

(a) $f$ is decreasing on $[-2, 2]$

(b) $f(2) > 0$

(c) $f(-2) < 0$

(d) $f(0) = 0$

(e) $f(1) > 0$

10. The area of the region bounded by the curves $y^2 = 2x + 1$ and $x + y = 1$ is equal to

(a) $\dfrac{16}{3}$

(b) $\dfrac{17}{3}$

(c) $\dfrac{15}{4}$

(d) $4$

(e) $\dfrac{13}{2}$

11. If $y = \int_{\cos x}^{\sin(3x)} \sqrt{t^2+3}\,dt$ then $\dfrac{dy}{dx}$ at $x = 0$ is equal to:

(a) $3\sqrt{3}$

(b) $\sqrt{3}$

(c) $\sqrt{3} - \sqrt{1+3}$

(d) $-\sqrt{1+3}$

(e) $3\sqrt{3} - \sqrt{1+3}$

12. A particle moves along a line so that its velocity at time $t$ is $v(t) = t^3 - 4t\ m/s$. Then the distance traveled during the time period $0 \le t \le 4$ is

(a) $40\,m$

(b) $32\,m$

(c) $36\,m$

(d) $30\,m$

(e) $46\,m$

13. $\displaystyle\int_0^1 \frac{1}{(2-\sqrt{x})^2}\,dx =$

(a) $2 - 2\ln 2$

(b) $-2 + 2\ \ln 2$

(c) $\ln 2 + 2$

(d) $2$

(e) $1 - 2\ \ln 2$

14. $\displaystyle\int \frac{\sec^2\left(\frac{\pi}{x}\right)}{x^2}\,dx =$

(a) $\displaystyle -\frac{1}{\pi}\ \tan\left(\frac{\pi}{x}\right) + C$

(b) $\displaystyle -\frac{1}{\pi}\ \tan\left(\frac{\pi}{x^2}\right) + C$

(c) $\displaystyle \pi\ \frac{\tan\left(\frac{\pi}{x}\right)}{x^3} + C$

(d) $\displaystyle \pi\ \cot\left(\frac{\pi}{x}\right) + C$

(e) $\displaystyle \frac{\pi\ \sec^3\left(\frac{\pi}{x}\right)}{3x^3} + C$

15. $\displaystyle\int \frac{(1+\sqrt{t})^{2/3}}{\sqrt{t}}\,dt =$

(a) $\frac{6}{5}(1+\sqrt{t})^{5/3}+C$

(b) $\frac{6}{5}(1+\sqrt{t})^{1/3}+C$

(c) $\frac{6}{5}(1+\sqrt{t})^{4/3}+C$

(d) $\frac{6}{5}(1+\sqrt{t})^{3/4}+C$

(e) $\frac{6}{5}(1+\sqrt{t})^{3/5}+C$

16. Let $g$ be a differentiable function and let

$$f(x)=\int_0^{g(x)} \frac{t}{\sqrt{4+5t^2}}\,dt.$$

If $f'(1)=1$, and $g'(1)=3$, then $f(1)=$

(a) $\frac{1}{5}$

(b) $\frac{1}{5}-\sqrt{2}$

(c) $\sqrt{3}-2$

(d) $2$

(e) $\sqrt{5}+3$

17. The base of a solid is a semi-circle of radius 1. Parallel cross-sections perpendicular to the base are squares with two of their vertices on the semi-circle. The volume of the solid is:

(a) $\frac{8}{3}$

(b) $\frac{4}{3}$

(c) $\frac{3}{4}$

(d) $\frac{3}{8}$

(e) $\frac{2}{3}$

18. The volume of the solid obtained by rotating the region enclosed by the curves $y = x^2$ and $y = \sin\left(\frac{\pi x}{2}\right)$ around the $x$-axis is

(Hint: Use the identity $\sin^2\theta = (1 - \cos 2\theta)/2$).

(a) $\frac{3\pi}{10}$

(b) $\frac{3\pi}{10} - \frac{1}{2}$

(c) $\frac{3\pi}{10} + \frac{1}{2}$

(d) $\frac{\pi}{10}$

(e) $\frac{\pi}{10} - \frac{1}{2}$

19. Let $m$ be an integer $\geq 2$ and $f$ be the function defined on $[2,\infty)$ by $f(x)=\int_2^x \frac{1}{t\,(\ln\, t)^m}\,dt$.
Which of the following statements is **FALSE**:

(a) $f(x)=\dfrac{2}{1-m}\left[\dfrac{1}{(\ln\,(x))^{m-1}}-\dfrac{1}{(\ln\, 2)^{m-1}}\right]$

(b) $f'(x)=\dfrac{1}{x(\ln\, x))^m}$

(c) $f(2)=0$

(d) $f(3)>f\left(\dfrac{5}{2}\right)$

(e) $f$ is continuous on $[2,\infty)$

20. Let $f$ be a continuous increasing function defined on the interval $[a,b]$ such that $f(x)\geq 0$, $\forall x\in[a,b]$. Let $R_n$ be the Riemann sum of $f$ corresponding to a subdivision of $[a,b]$ into $n$ subintervals using right end points, $L_n$ is the corresponding sum using left end points. Let $A$ be the area of the region under the graph $y=f(x)$. Which of the following statements is **FALSE**?

(a) $A<R_n-\left(\dfrac{b-a}{n}\right)[f(b)-f(a)]$

(b) $L_n\leq A\leq R_n$

(c) $R_n-L_n=\left(\dfrac{b-a}{n}\right)[f(b)-f(a)]$

(d) $R_n-A\geq 0$

(e) $A\geq R_n-\left(\dfrac{b-a}{n}\right)[f(b)-f(a)]$