Quiz3

March 21, 2018

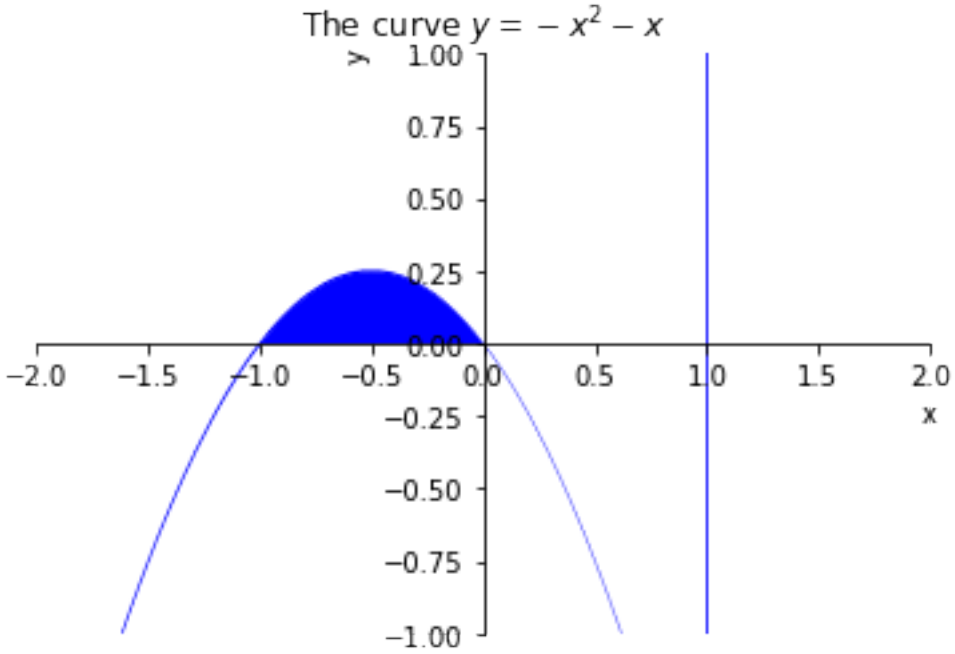
Quiz 3 Solution

Q1) Use the method of cylindrical shells to write (*DO NOT EVALUATE*) an integral for the volume of the solid generated by rotating the region bounded by the curves

$$y = -x^2 - x \quad \text{and } y = 0$$

about the line $x = 1$.

```
In [48]: from sympy import plot_implicit, symbols,init_printing, Eq, And, expand, latex
         init_printing()
         x, y =symbols('x y')
         p1 = plot_implicit(Eq(y,-x**2-x),  (x, -2, 2), (y, -1, 1),title="The curve $y=-x^2-x$"
         p2 = plot_implicit(y<-x**2-x,  (x, -1, 0), (y, 0, 1), show=False)
         p3 = plot_implicit(Eq(x,1), show=False)
         p1.append(p3[0])
         p1.append(p2[0])
         p1.show()
```



Q1 `solution`

$$r = 1 + x, \quad h = -x^2 - x$$

So the volume of the solid is found by evaluation the integral

$$V = \int_{-1}^{0} 2\pi(1+x)(-x^2-x)dx = \int_{-1}^{0} 2\pi(-x^3-2x^2-x)dx$$

Q2) Find the average value of the function

$$f(x) = \sin^{-1} x$$

on the interval $[0, 1]$.

Q2 `solution`

$$\begin{aligned} f_{\text{avg}} &= \tfrac{1}{1-0}\int_0^1 \sin^{-1} x dx \\ &= \int_0^1 \sin^{-1} x dx \end{aligned}$$

Now, use the substitution $u = \sin^{-1} x$ and therefore $x = \sin(u)$ and

$$dx = \cos u du$$

So,

$$\begin{aligned} f_{\text{avg}} &= \int_0^1 \sin^{-1} x dx \\ &= \int_0^{\pi/2} u \cos u du \end{aligned}$$

now, by parts, we get

$$\begin{aligned} y = u, &\quad dz = \cos u du \\ dy = du, &\quad z = \sin u \end{aligned}$$

$$\begin{aligned} \int_0^{\pi/2} u\cos u du &= u\sin u]_0^{\pi/2} - \int_0^{\pi/2} \sin u du \\ &= u\sin u]_0^{\pi/2} + \cos u]_0^{\pi/2} \\ &= \tfrac{\pi}{2} - 1 \end{aligned}$$

Therefore,

$$f_{\text{avg}} = \frac{\pi}{2} - 1$$