Sec1.1

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1 1.1 Differential Equations and Mathematical Models

1.0.1 Goals:

1. To discover a model.
2. To solve
3. To interpret.

1.1 Example

The **time rate of change** of a Population $P(t)$ with constant birth and death rates is proportional to the **size** of the population.

$$\frac{dP}{dt} = kP$$

1.2 Example

Suppose that * $P(t) = Ce^{kt}$ is a the population of a colony of bacteria at time t * $P(0hours) = 1000$ * the population doubled after 1 h. $P(1) = 2000$

1.3 Mathemtical Models

Real-world situation

Formulation Interpretation

Mathematical model Mathemtical analysis Mathemtical results

A *mathematica model* consists of a list of variables (P, t) that describe the given situation, together with one or more equations realating these variables ($\frac{dP}{dt} = kP, P(0) = P_0$) that are known or are assumed to hold.

1.4 Terminology

- The **order** of a differential equation is the order of the highest derivative that appears in it.
- The most general form of an ***n*th-order** differential equation with independent variable x and unknown function or dependent variable $y = y(t)$ is

$$F(x, y, y', y'', \cdots, y^{(n)}) = 0$$

- We say that the continuous function $u = u(x)$ is a **solution** of the DE above **on the interval** I provided that $u', u'', \cdots, u^{(n)}$ exist on I and

$$F(x, u, u', u'', \cdots, u^{(n)}) = 0$$

- **ordinary differential equations** vs **partial differential equations**
- **initial value problem** is a differential equation with an **an initial condition**

1.5 Example

Verify that $y = \frac{1}{1+x^2}$ is a solution to

$$y' + 2xy^2 = 0$$