Sec1.4

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1 1.4 Separable Equations and Applications

The equation
$$\frac{dy}{dx} = H(x, y)$$
is called **separable** if $H(x, y)$ can be wrotten as the product of a function of x and a function of y:
$$\frac{dy}{dx} = g(x)h(y) = \frac{g(x)}{f(y)}$$

In this case the variables can be isolated *separated*.

1.1 Example

Solve the initial value problem
$$\frac{dy}{dx} = -6xy, \qquad y(0) = 7$$

1.1.1 Remark:

- If $y(0) = -4$ what is the solution?

1.2 Implicit Solutions

Solve differential equation
$$\frac{dy}{dx} = \frac{4 - 2x}{3y^2 - 5}.$$

See graph

[1]:
```
# Remark 1:
using SymPy, Plots;
f(x,y)=y^3-5y+x^2-4x;
x, y = symbols("x, y", real=true);
sol=solve(y^3-5y-9);
sol.n().
```

```
# plot(y->y^3-5y-9, -6:0.1:6)
```

[1]:

$$\begin{bmatrix} -1.42759826966035 - 1.05551430999854i \\ -1.42759826966035 + 1.05551430999854i \\ 2.8551965393207 \end{bmatrix}$$

1.2.1 Remark 2:

- solve with $y(1) = 0$ and $y(1) = -2$

[55]:
```
c2=f(1,0)
c3=f(1,-2)
c2,c3
```

[55]: (-3, -1)

1.3 Implicit, General, Singular Solutions

The equation

$$K(x, y) = 0$$

is called an **implicit solution** of DE if it is satisfied (on some interval) by some solution $y = y(x)$ of the DE.

1.3.1 Note

- A particular solution $y = y(x)$ of $K(x, y) = 0$ may or may not satisfy the givent IVP. For example $x^2 + y^2 = 4$ gives

$$x + y\frac{dy}{dx} = 0$$

- Do not assume that every possible algebraic solution $y = y(x)$ of an implicit solution satisfies the same DE. For example $(y - 2x)(x^2 + y^2 - 4) = 0$
- Also, solutions of a given DE can be either gained or lost when it is multiplied or divided by an algebraic factor.

$$(y - 2x)y\frac{dy}{dx} = -x(y - 2x)$$

extraneous solutions

1.3.2 General and Particular Solution

A solution that contains an ``arbitrary constant'' is called a **general solution**. A solution with a specific value of the the constant is called a **particular solution**.

1.3.3 Singular Solution

A particular solution that cannot be obtained from the general solution. (ex. $(y')^2 = 4y$ has general solution $y = (x - C)^2$, but $y = 0$ is a singular solution)

1.4 Example

Solve the DE

$$\frac{dy}{dx} = 6x(y-1)^{\frac{2}{3}}$$