# Sec1.6

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## 1 1.6 Substitution Methods and Exact Equations

### 1.1 Example

Solve DE

$$\frac{dy}{dx} = (x + y + 3)^2.$$

**Solution In class**

### 1.2 Homogeneous Equations

A **homogeneous** first-order DE is one that can be written in the form

$$\frac{dy}{dx} = F(\frac{y}{x})$$

In this case we take the substitution $v = \frac{y}{x}$

### 1.3 Example

Solve the DE

$$2xy\frac{dy}{dx} = 4x^2 + 3y^2.$$

**Solution In class**

### 1.4 Example

Solve the DE

$$x\frac{dy}{dx} = y + \sqrt{x^2 - y^2}, \quad y(x_0) = 0.$$

where $x_0 > 0$.

**Solution In class**

## 1.5 Bernoulli Equations

It is A first-order DE of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n.$$

Use the substitution $v = y^{1-n}$

## 1.6 Example (Revisited)

Solve the DE

$$2xy\frac{dy}{dx} = 4x^2 + 3y^2.$$

**Solution In class**

## 1.7 Example

Solve the DE

$$x\frac{dy}{dx} + 6y = 3xy^{\frac{4}{3}}.$$

**Solution In class**

## 1.8 Example

Solve the DE

$$2xe^{2y}\frac{dy}{dx} = 3x^4 + e^{2y}.$$

**Solution In class**

## 1.9 Exact DE

The general solution of a first-order DE has the form

$$F(x, y(x)) = C$$

Differentiating, we get

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{dy}{dx} = 0$$

that is

$$M(x, y) + N(x, y)\frac{dy}{dx} = 0$$

This eqaution is called **exact differential equation** with

$$M(x, y) = \frac{\partial F}{\partial x}, \quad N(x, y) = \frac{\partial F}{\partial y}$$

### 1.10 Example

Solve DE

$$y^3dx + 3xy^2dy = 0$$

**THEOREM 1 Criterion for Exactness**

Suppose that the functions $M(x, y)$ and $N(x, y)$ are continuous and have continuous first-order partial derivatives in the open rectangle $R: a < x < b$, $c < y < d$. Then the differential equation

$$M(x, y)\,dx + N(x, y)\,dy = 0 \tag{23}$$

is exact in $R$ if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \tag{24}$$

at each point of $R$. That is, there exists a function $F(x, y)$ defined on $R$ with $\partial F/\partial x = M$ and $\partial F/\partial y = N$ if and only if Eq. (24) holds on $R$.

### 1.11 Example

Solve

$$(6xy - y^3)dx + (4y + 3x^2 - 3xy^2)dy = 0$$

### 1.12 Example

Solve

$$(\cos x + \ln y)dx + (\frac{x}{y} + e^y)dy = 0$$

### 1.13 Reducible Second-Order Equations

A second-order DE is

$$F(x, y, y', y'') = 0$$

- if either $x$ or $y$ are missing then we

#### 1.13.1 Case: $y$ is missing:

Take

$$p = y', \quad y'' = p'$$

### 1.13.2 Case: $x$ is missing:

Take

$$p = y', \quad y'' = p\frac{dp}{dy}$$

## 1.14 Example

Solve 1. $xy'' + 2y' = 6x$. 2. $yy'' = (y')^2$.