Sec1.4

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1 1.4 Separable Equations and Applications

The equation

$$\frac{dy}{dx} = H(x, y)$$

is called **separable** if $H(x, y)$ can be written as the product of a function of x and a function of y:

$$\frac{dy}{dx} = g(x)h(y) = \frac{g(x)}{f(y)}$$

In this case the variables can be isolated *separated*.

1.1 Example

Solve the initial value problem

$$\frac{dy}{dx} = -6xy, \qquad y(0) = 7$$

Solution:

$$\frac{dy}{y} = -6x * dx$$

$$\int \frac{dy}{y} = \int -6x * dx$$

Since the initial value is positive, we can declare y positive inside the ln

$$\ln(y) = -3x^2 + C$$

$$y = e^{(-3x^2+C)}$$

Find the value of C through the initial value

$$7 = e^{(-3(0)^2+c)}$$

$$7 = e^c$$

$$\ln(7) = c$$

$$y = e^{(-3x^2+\ln(7))}$$

1.1.1 Remark:

- If $y(0) = -4$ what is the solution?

1.2 Implicit Solutions

Solve differential equation

$$\frac{dy}{dx} = \frac{4-2x}{3y^2-5}.$$

Solution:

$$(3y^2-5)dy = (4-2x)dx$$
$$\int 3y^2 - 5dy = \int 4 - 2xdx$$
$$y^3 - 5y = 4x - x^2 + C$$

Solution is set to be Implicit See graph

In [46]:
```
# Remark 1:
using SymPy, Plots;
f(x,y)=y^3-5y+x^2-4x;
x, y = symbols("x, y", real=true);
sol=solve(y^3-5y-9);
sol.n()

# plot(y->y^3-5y-9, -6:0.1:6)
```

Out[46]:

$$\left[\begin{array}{r} -1.42759826966035 - 1.05551430999854i \\ -1.42759826966035 + 1.05551430999854i \\ 2.8551965393207 \end{array}\right]$$

1.2.1 Remark 2:

- solve with $y(1) = 0$ and $y(1) = -2$

In [55]:
```
c2=f(1,0)
c3=f(1,-2)
c2,c3
```

Out[55]: (-3, -1)

1.3 Implicit, General, Singular Solutions

The equation

$$K(x,y) = 0$$

is called an **implicit solution** of DE if it is satisfied (on some interval) by some solution $y = y(x)$ of the DE.

1.3.1 Note

- A particular solution $y = y(x)$ of $K(x, y) = 0$ mau or may not satisfy the givent IVP. For example $x^2 + y^2 = 4$ gives
$$x + y\frac{dy}{dx} = 0$$
- Do not assume that every possible algebraic solution $y = y(x)$ of an implicit solution satisfies the same DE.
- Also, solutions of a given DE can be either gained or lost when it is multiplied or divided by an algebraic factor.
$$(y - 2x)y\frac{dy}{dx} = -x(y - 2x)$$
extraneous solutions

1.3.2 General and Particular Solution

A solution that contains an "arbitrary constant" is called a **general solution**. A solution with a specific value of the the constant is called a **particular solution**.

1.3.3 Singular Solution

A particular solution that cannot be obtained from the general solution. (ex. $(y')^2 = 4y$ has general solution $y = (x - C)^2$, but $y = 0$ is a singular solution)

1.4 Example

Solve the DE
$$\frac{dy}{dx} = 6x(y - 1)^{\frac{2}{3}}$$
Solution
$$\frac{(y-1)^{\frac{2}{3}}}{dy} = 6xdx$$
$$\int \frac{dy}{3(y-1)^{\frac{2}{3}}} = \int 2xdx$$
$$(y-1)^{\frac{1}{3}} = x^2 + C$$
$$y = (x^2 + C)^3 + 1$$

y=1 is a singular solution.