GA_1: Falling Sphere

ME 273 | Spring Semester 2018

Mission:

Your mission, should you choose to accept it, is to provide thorough answers/solutions to each of the Exercises, including numerical values and graphs where appropriate. Also provide any code that you used to produce the results shown in your solutions to the Exercises. It is imperative that the code you include be commented in detail with your own, original content. Detailed comments are the main way to uniquely identify your individual effort.

Exercises

Exercise 1: Computational Model of a Falling Sphere with Air resistance Produce a working computational model in MATLAB (or OCTAVE, or in another programming language of your choice) of a sphere that has been dropped from rest from a very tall building using the simple Euler method. Assume that the sphere will move entirely in one dimension, and that it is subject to the constant gravitational force near Earth's surface and to a drag force proportional to the square of the sphere's instantaneous speed.

Exercise 2: Accuracy of Computational Model: Velocity vs. Time Since the computational approach is based on an approximation, it is important to determine just how small Δt should be for the approximation to accurately solve the 1D air resistance problem. Make a comparison between the time dependence of the velocity predicted by the computational model, and that predicted by the exact result,

$$v_y(t) = \sqrt{\frac{2mg}{D\varrho A}} \tanh\left(\sqrt{\frac{D\varrho Ag}{2m}}t\right) .$$

Use parameters that describe a 16-pound bowling ball (you should look up the diameter, and convert to meters), and let it fall a distance equivalent to the height of the Sears, oops - Willis, tower (440 m). Assume the ball is initially at rest. Use a value of 0.5 for the drag coefficient, and the density of air near sea level. What value of Δt do you deem to be sufficiently small for the computational model to be accurate? Explain how you arrived at this value of Δt .

Exercise 3: Accuracy of Computational Model: Position vs. Time Carry out the same comparison (computational vs. exact analytical solution) for the bowling ball's position as a function of time. The exact result for the ball's position is given by

$$y(t) = \frac{2m}{D\varrho A} \ln \left[\cosh \left(\sqrt{\frac{D\varrho A g}{2m}} t \right) \right] .$$

Assume the bowling ball is falling the same distance of 440 m. Do you find the same value of Δt, as found for the velocity comparison of Exercise 2, to be acceptable for the position comparison?

Exercise 4: Position and Velocity of Dropped Bowling Ball Produce plots of the bowling ball's velocity and vertical position as functions of time from the results of the computational model, using the parameters from the previous exercises and the value of Δt (determined in Exercises 2 and 3) that produces a tolerably accurate computational solution. Has the bowling ball reached its terminal velocity by the time it hits the ground? Use your model to predict the time required for the bowling ball to fall the full 440 meters to the ground.

Exercise 5: Position and Velocity of a Dropped Mystery Sphere Repeat Exercise 4 for a different sphere (of your choice). How long does it take to travel the 440 meters to the ground, and has it reached its terminal velocity upon impact?