ME 273 Spring Semester 2018
Project 1: Air Resistance in 2D Projectile

Mission:

Your mission, should you choose to accept it, is to provide thorough answers/solutions for each of the exercises, and produce a detailed report. Also provide any code that you used to produce the results shown in your report. It is imperative that the code you include be commented in detail with your own, original content. Detailed comments are the main way to uniquely identify your individual effort.

Exercise 1: 2D projecile with Air Resistance

Build a computational model of a 2-dimensional projectile subject to a resistive force that is proportional to the square of its instantaneous velocity. Assume that the drag coefficient and density of air remain constant throughout the duration of the projectile's motion. Build the model so that the initial projectile velocity and angle above the horizontal direction can be specified for each run. Run the simulation for a set of parameters of your choosing. Investigate the accuracy of your model, and determine a value of Δt that you deem produces tolerably accurate results.

Exercise 2: Shape of the 2D Projectile Trajectory

You may remember from your introductory physics course that, without air resistance, a projectile follows a parabolic trajectory. Are the trajectories produced by the model you built in Exercise 1 parabolic? Show in detail how you determined if the curve(s) were parabolic or not.

Exercise 3: Angle for Maximum Projectile Range

You may further recall from your introductory physics experience that when air resistance is neglected, the maximum range of a projectile is attained for a launch angle of 45 degrees. Use your model with air resistance to determine the new angle (with 0.1 degree accuracy) at which the maximum range occurs, for three different values of the drag coefficient (use a wide range), keeping all the other parameters constant. Make a graph of the max-range trajectory for these three drag coefficients (on the same figure), making sure to include labels indicating the value of the launch angle that produces the maximum range, and the drag coefficient for each one.

Exercise 4: 2D Model with Air Resistance-Naval Gun

Model a shell fired from one of the 16 inch guns on a World War II era battleship. Start with your realistic (and accurate!) 2D projectile model with air resistance, and then use real parameter values for a round fired from the naval gun. Since a range table exists (http://www.navweaps.com/Weapons/WNUS_16-50_mk7.php) for the Mark 7, let's model this gun firing an AP Mark 8 shell, so we have some data to which we can compare the results of our model. Of particular relevance for your projectile model are the Mark 7 muzzle velocity (magnitude of the initial velocity vector), the mass of the shell, and the radius of the shell (important for calculating cross-sectional area A).

There is one addition that must be made in order to make your battleship salvo model realistic. Because your projectile, as you will soon discover, is capable of attaining a relatively high altitude you will have take into account the fact that the atmospheric density varies with the vertical position of the fired shell. To model a realistic projectile that has been fired with a WWII era naval gun, we need to build into our model an altitude-dependent air density. The following equation accurately describes how the atmospheric density decreases with altitude:

$$\varrho(y) = \varrho_0 \left(1 - \frac{cy}{T_0}\right)^{\alpha}, \tag{1}$$

where $\varrho_0 = 1.2$ kg/m^3, $c = 6.5 \times 10^{-3}$ K/m, $T_0 = 300$ K (sea level temperature in Kelvins), and $\alpha = 2.5$. This equation is derived from thermodynamic considerations, and is a good approximation for the behavior of the earth's atmosphere up to about 10 kilometers. Note that the density depends only on the vertical distance above the surface of the Earth.

The most difficult parameter to determine is the drag coefficient D. The typical shells fired from the big naval guns were cylindrical with a point at the front end. Furthermore, the naval guns were rifled, so that the cylindrical projectiles were spinning during their flight. Thus, the drag coefficient D should be expected to be a very complicated function of velocity. However, we can determine an *effectively constant* value via the following procedure. The drag coefficient in your working program can be varied until the range produced in your model for different angles closely agrees with the actual measured values found in the Range Table (for "new gun" muzzle velocities) at the aforementioned web site. Reproduce the table of Range values, add another column for Drag Coefficient, and then for each angle specified, vary D in your model until the range is matched and record the value of D in the table. Include a column in your table for the maximum altitude achieved for each of the elevation angles.The Range Table is reproduced below for your convenience.

Ranges of projectiles fired at new gun muzzle velocities

Elevation	AP Mark 8	HC Mark 13
10 degrees	17,650 yards (16,139 m)	18,200 yards (16,642 m)
15 degrees	23,900 yards (21,854 m)	24,100 yards (22,037 m)
20 degrees	29,000 yards (26,518 m)	28,800 yards (26,335 m)
25 degrees	33,300 yards (30,450 m)	32,700 yards (29,901 m)
30 degrees	36,700 yards (33,558 m)	36,000 yards (32,918 m)
35 degrees	39,500 yards (36,119 m)	38,650 yards (35,342 m)
40 degrees	41,430 yards (37,884 m)	40,600 yards (37,163 m)
45 degrees	42,345 yards (38,720 m)	41,622 yards (38,059 m)

Exercise 5: Angle for Maximum Range

Use your naval gun model (with air resistance, the upgraded density dependence on altitude, and a relatively accurate constant value for D) to determine the angle (with 0.1 degree accuracy) at which the maximum range occurs. Make a graph of the trajectory for this angle, and be sure to include a label indicating the value of the launch angle that produces the maximum range. Is it greater or smaller than 45 degrees?