M. Molter
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1.19 Obtain a linear approximation of the function $f(h) = \sqrt{h}$, valid near $h = 25$.

We can use a Taylor Series to obtain a polynomial approximation of a non–linear function near some reference point. The generalized form is:

$$y = (\theta_R + \Delta\theta) = y\theta_R + \frac{\Delta\theta}{1!}\frac{dy}{d\theta}(\theta_R) + \frac{(\Delta\theta)^2}{2!}\frac{d^2y}{d\theta^2}(\theta_R) + \frac{(\Delta\theta)^3}{3!}\frac{d^3y}{d\theta^3}(\theta_R) + \dots$$

The problem asks for the linear approximation, so we can neglect the higher order terms and the approximation becomes:

$$y = (\theta_R + \Delta\theta) = y\theta_R + \frac{\Delta\theta}{1!}\frac{dy}{d\theta}(\theta_R)$$

We will need the derivative of $\sqrt{h}$ about $h = 25$.

$$\frac{d}{d\theta}\left[\sqrt{\theta}\right] = \frac{1}{2}\frac{1}{\sqrt{\theta}}$$

Therefore, the linear approximation becomes:

$$y = (25 + \Delta\theta) = (5)(25) + \Delta\theta\frac{1}{2 \times \sqrt{25}} = 125 + \frac{1}{10}\Delta\theta$$