3) $n = 100$

$p = 2/3$

$E[X] = 2/3 \cdot 100 = n \cdot p = \frac{200}{3}$

$Var[X] = np(1-p) = 2/3 \cdot 100 \cdot 1/3 = 2/9 \cdot 100 = \frac{200}{9}$

S_{100} = # of heads in 100 trials

looking for $P(S_{100} \leq 50)$

$E[X_i] = 2/3$

$Var[X_i] = 2/9$

use CLT

$$P(0 \leq S_{100} \leq 50) = P\left(0 \leq \sum_{i=1}^{100} X_i \leq 50\right)$$

$$= P\left(\frac{0 - 100 \cdot 2/3}{\sqrt{2/9 \cdot 100}} \leq \frac{\sum_{i=1}^{100} X_i - 100 \cdot 2/3}{\sqrt{2/9 \cdot 100}} \leq \frac{50 - 100 \cdot 2/3}{\sqrt{2/9 \cdot 100}}\right)$$

$$\sqrt{\frac{200}{9}} = \sqrt{\frac{2 \cdot 4 \cdot 25}{9}} = 10\sqrt{2/9} = 10/3\sqrt{2}$$

$$= P\left(\frac{-200/3}{\sqrt{200/9}} \leq Z \leq \frac{-50/3}{\sqrt{200/9}}\right)$$

$$= \Phi\left(\frac{-5}{\sqrt{2}}\right) - \Phi\left(\frac{-20}{\sqrt{2}}\right)$$

$$\approx 2.035E-4 - 1.044E-45$$

$$\approx \boxed{2.035E-4}$$

2) $\text{proj}_S v = \text{proj}_{v_1} v + \text{proj}_{v_2} v$

$$= \frac{v \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{v \cdot v_2}{v_2 \cdot v_2} v_2$$

$v_1 = [1, 1, 1]$

$v_2 = [1, 0, 0]$

$P1 = [3, 3, 3]$

$\text{proj}_S P1 = \frac{9}{3} v_1 + \frac{3}{1} v_2 = 3v_1 + 3v_2$

$= [6, 3, 3]$

$P2 = [1, 2, 3]$

$\text{proj}_S P_2 = \frac{6}{3} v_1 + \frac{1}{1} v_2 = 2v_1 + v_2$

$= [3, 2, 2]$

$P3 = [0, 0, 1]$

$\text{proj}_S P_3 = \frac{1}{3} v_1 + \frac{0}{1} v_2 = \frac{1}{3} v_1$

$= [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$