Noise in Elementary Transistor Circuits

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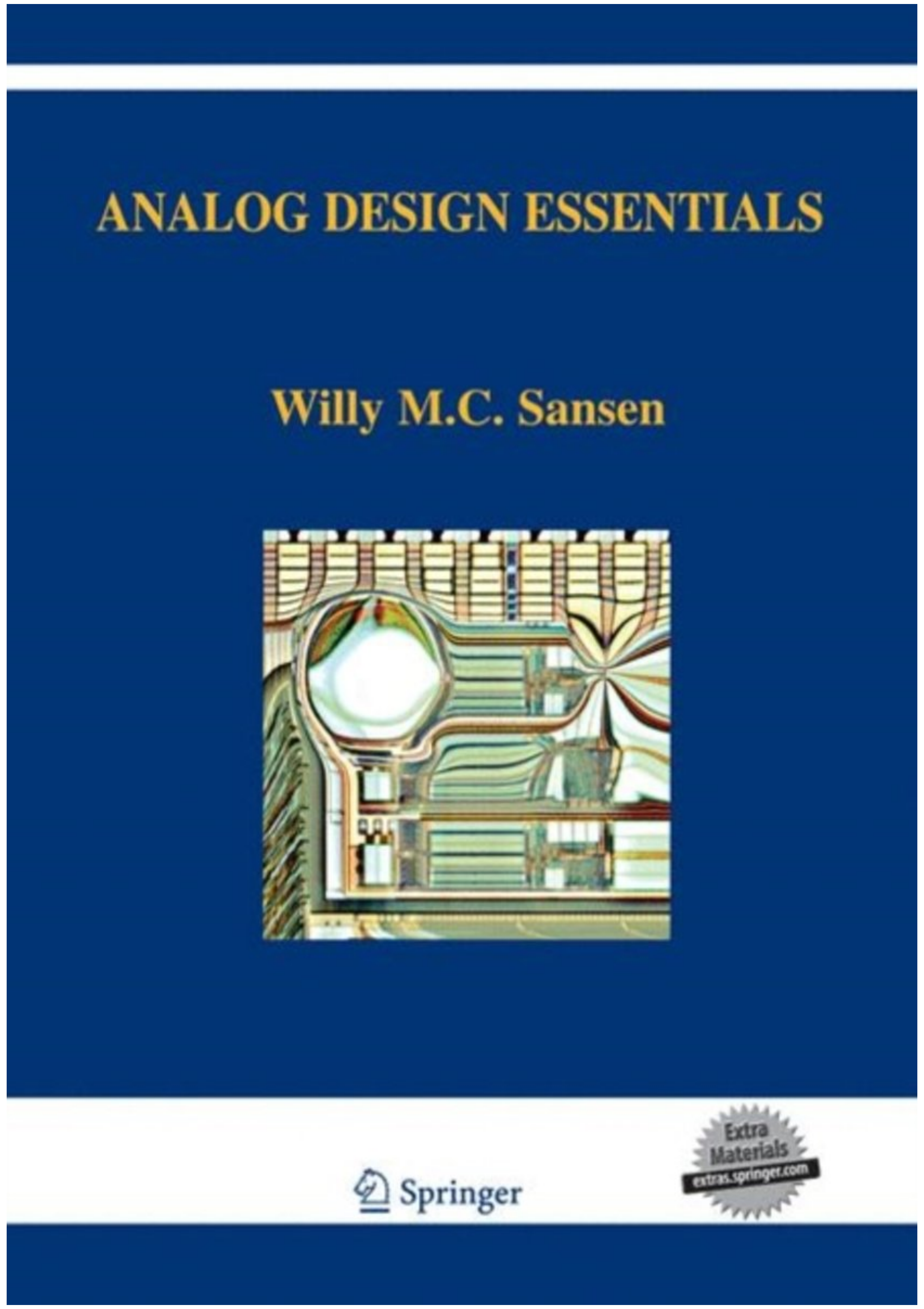
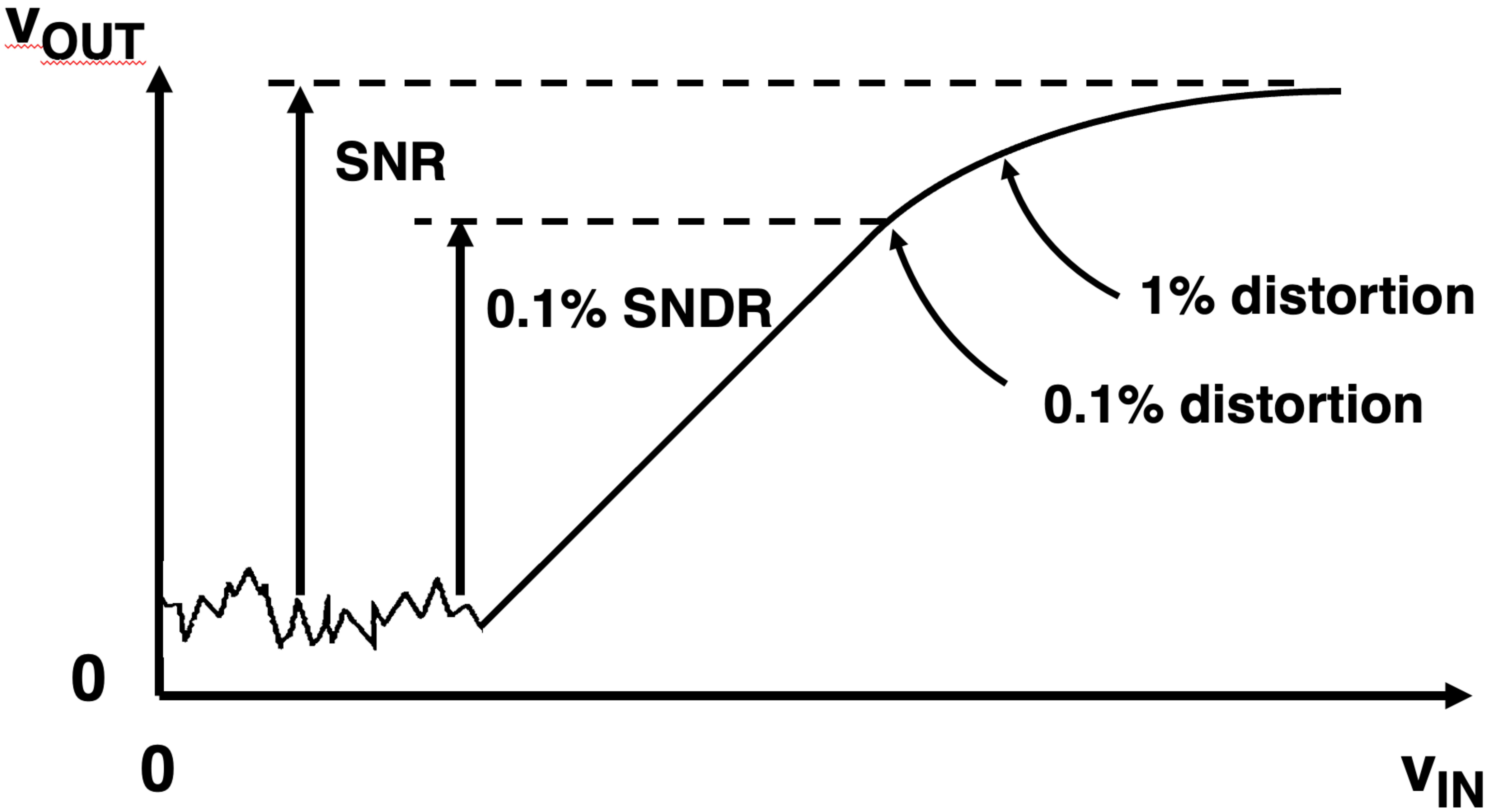
Outline

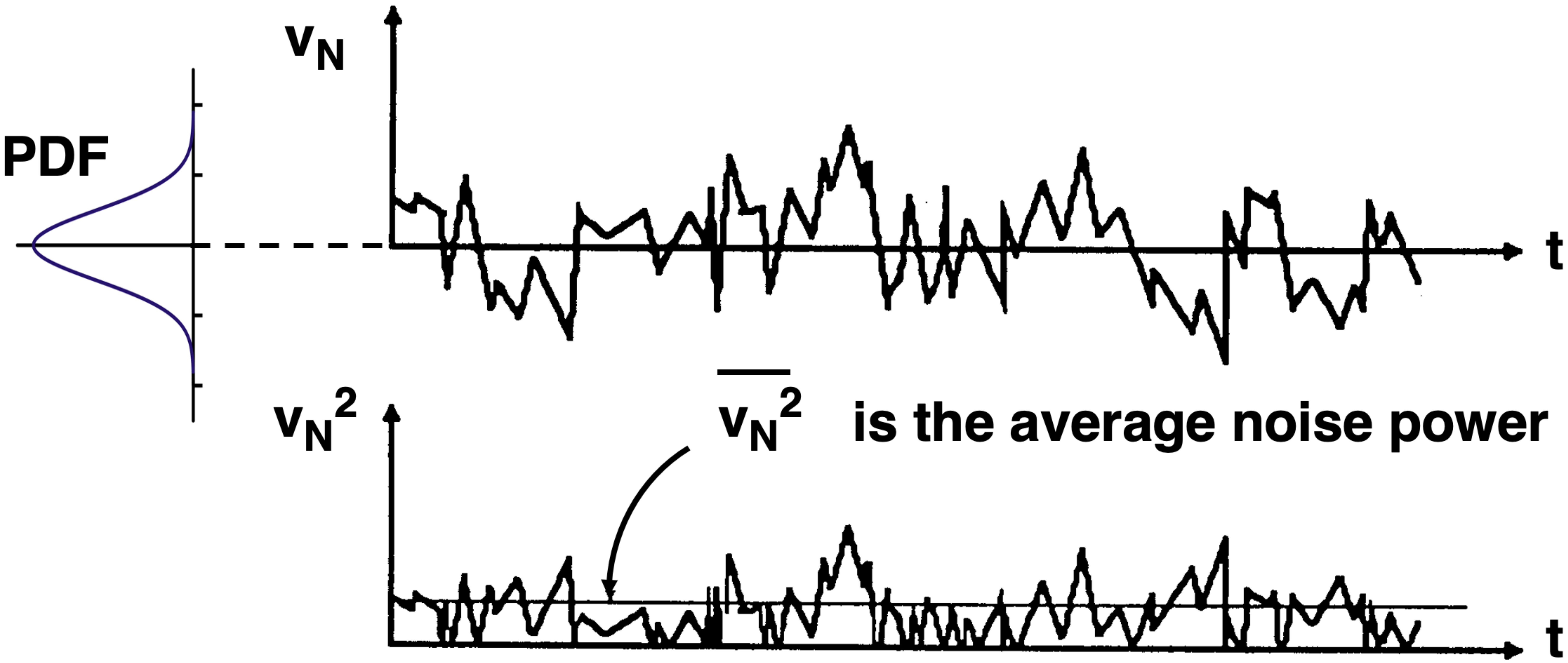
- **Definitions of noise**
- Noise of an amplifier
- Noise of a follower
- Noise of a cascode
- Noise of a current mirror
- Noise of a differential pair
- Capacitive noise matching



SNR and SNDR

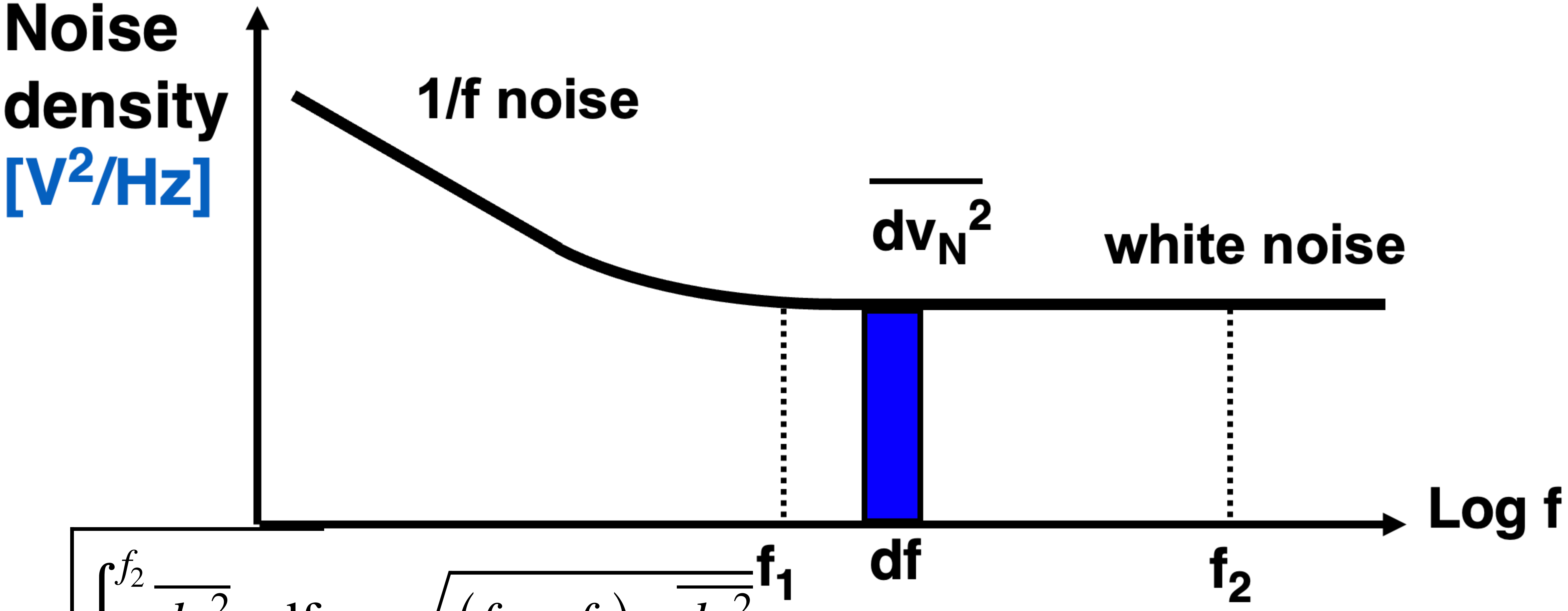


Noise Versus Time



Ref. Van der Ziel (Prentice Hall 1954, Wiley 1986), Ott (Wiley 1988)

Noise Versus Frequency

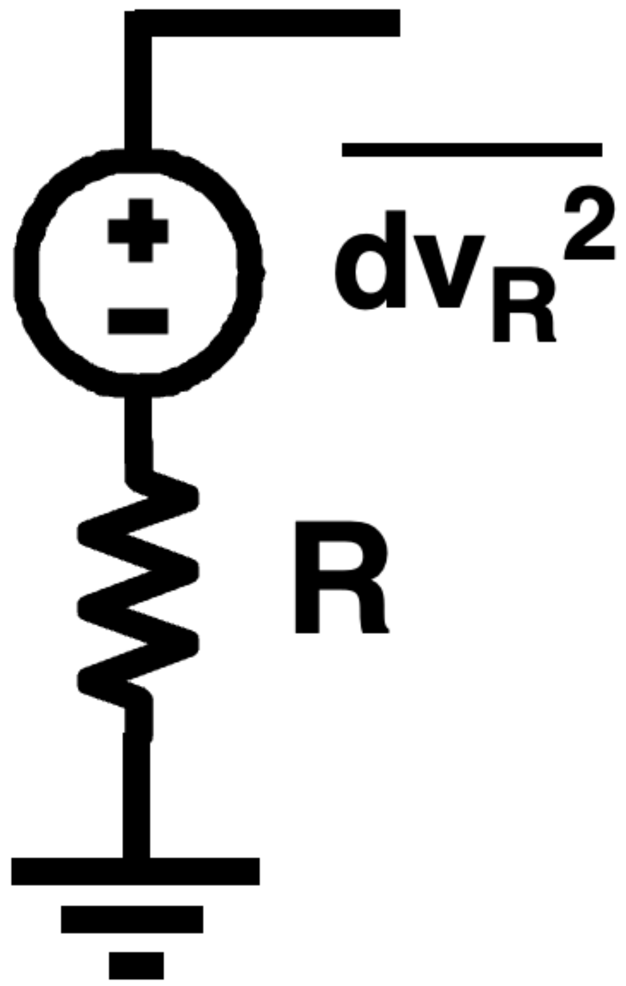


$$\overline{v_{12}} = \sqrt{\overline{v_n^2}} = \sqrt{\int_{f_1}^{f_2} \overline{dv_n^2} \cdot \mathrm{df}} = \sqrt{(f_2 - f_1) \cdot \overline{dv_n^2}}$$

Integrated noise: $\overline{v_{12}} = \sqrt{\overline{v_N^2}} = \sqrt{\int_{f_1}^{f_2} \overline{dv_N^2}\, df} = \sqrt{(f_2 - f_1)\, \overline{dv_N^2}}$

[V_{RMS}]

Resistor Thermal Noise

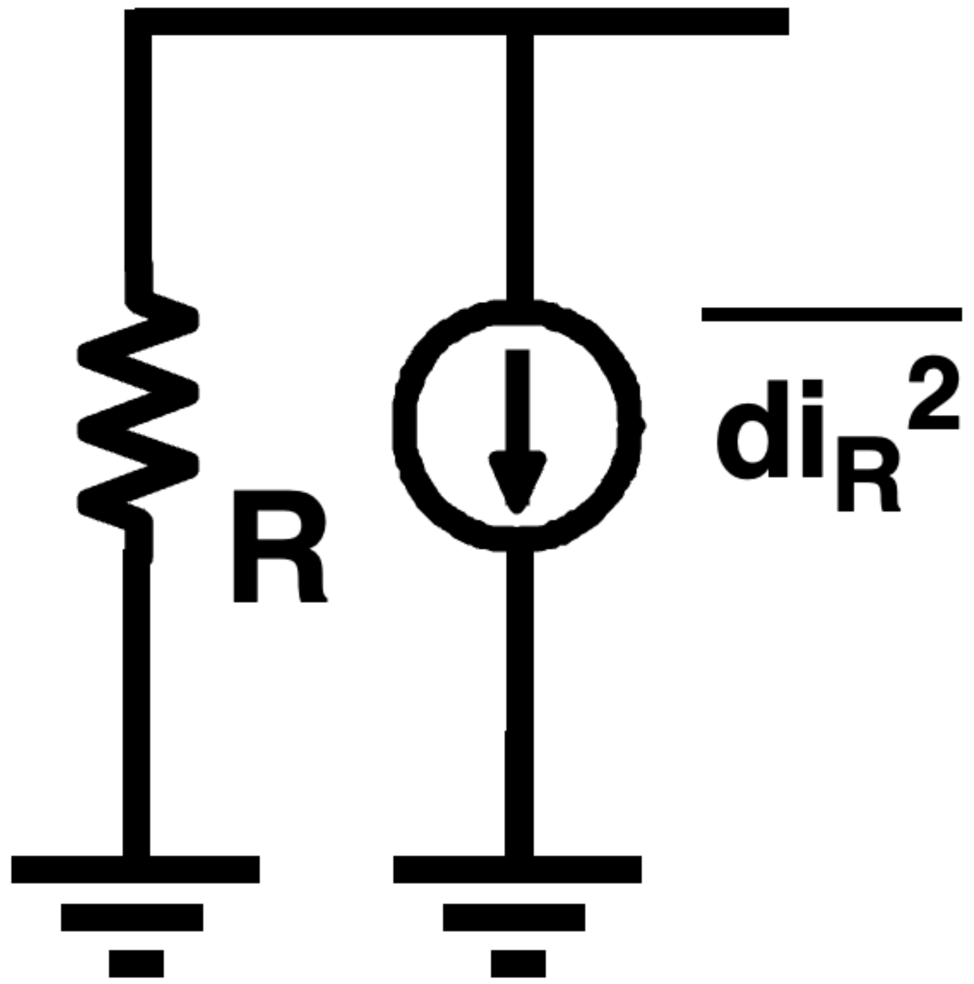


$\overline{dv_R^2} = 4kT\,R\,df$ is white

depends on T, not on I_R

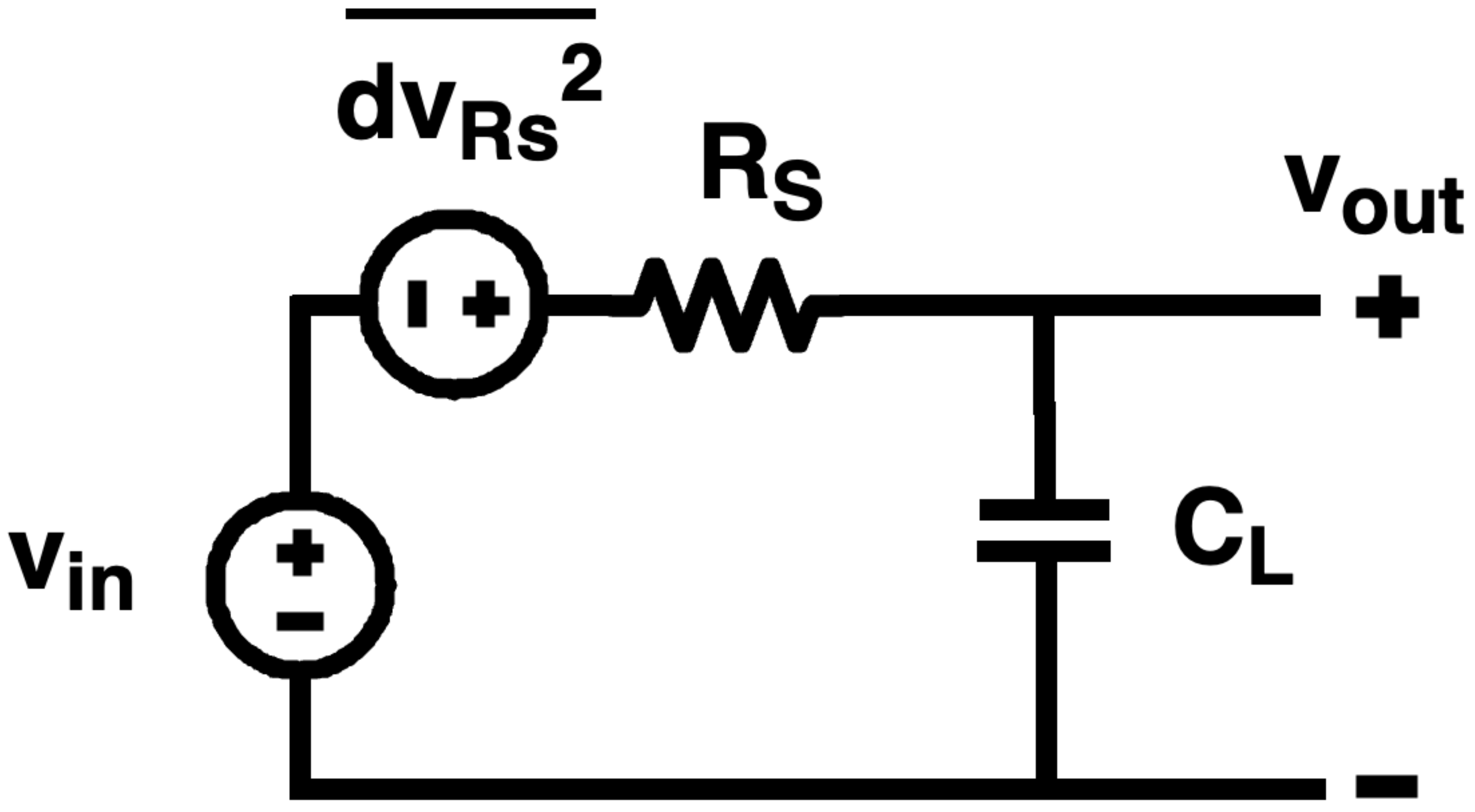
for $R = 1\ k\Omega$ $\sqrt{\overline{dv_R^2}} = 4\ nV_{RMS} / \sqrt{Hz}$

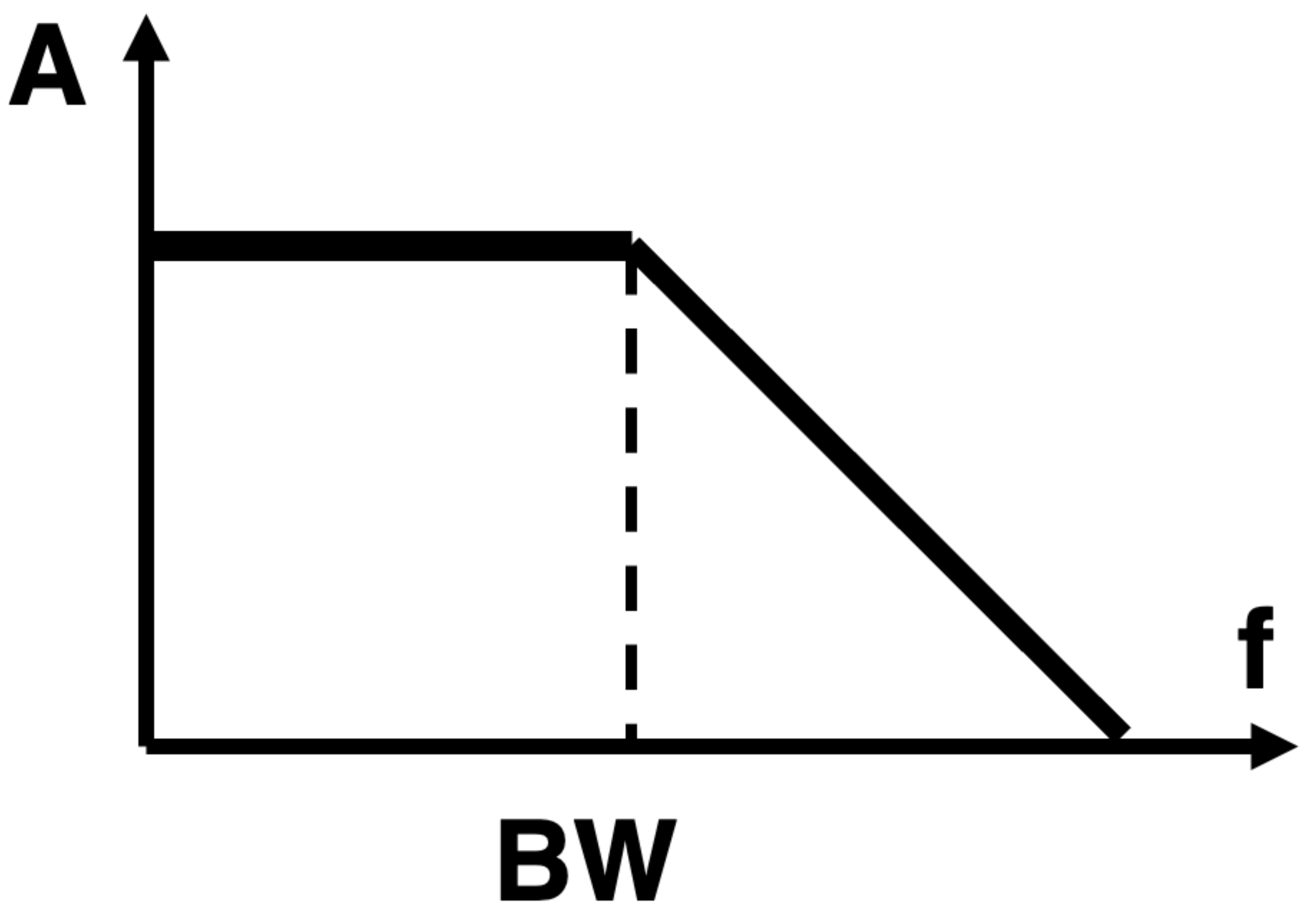
at $T = 300$ K or 27^oC



$$\overline{di_R^2} = \frac{\overline{dv_R^2}}{R^2} = \frac{4kT}{R} df$$ is white

Integrated Noise of a Resistor



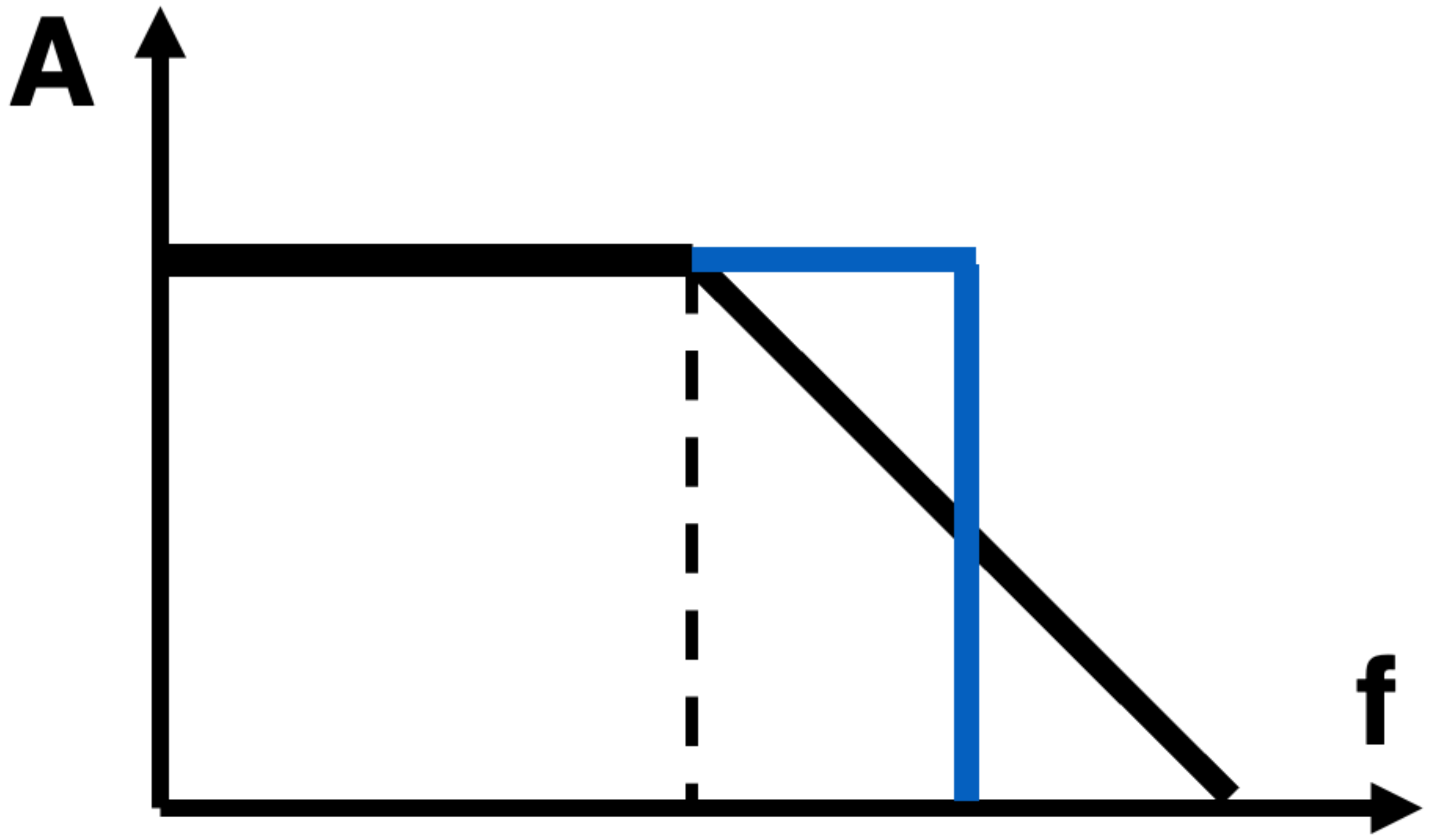


$$\overline{dv_{Rs}^2} = 4kT\,R_S\,df$$

$$BW = \frac{1}{2\pi R_S C_L}$$

$$\overline{v_{Rs}^2} = \int_0^{\infty} \frac{\overline{dv_{Rs}^2}}{1 + (f/BW)^2}$$

Integrated Noise and Noise Bandwidth



BW $BW_n = \frac{\pi}{2} BW$

$$\overline{v_{Rs}^2} = \int_0^\infty \frac{\overline{dv_{Rs}^2}}{1 + (f/BW)^2}$$

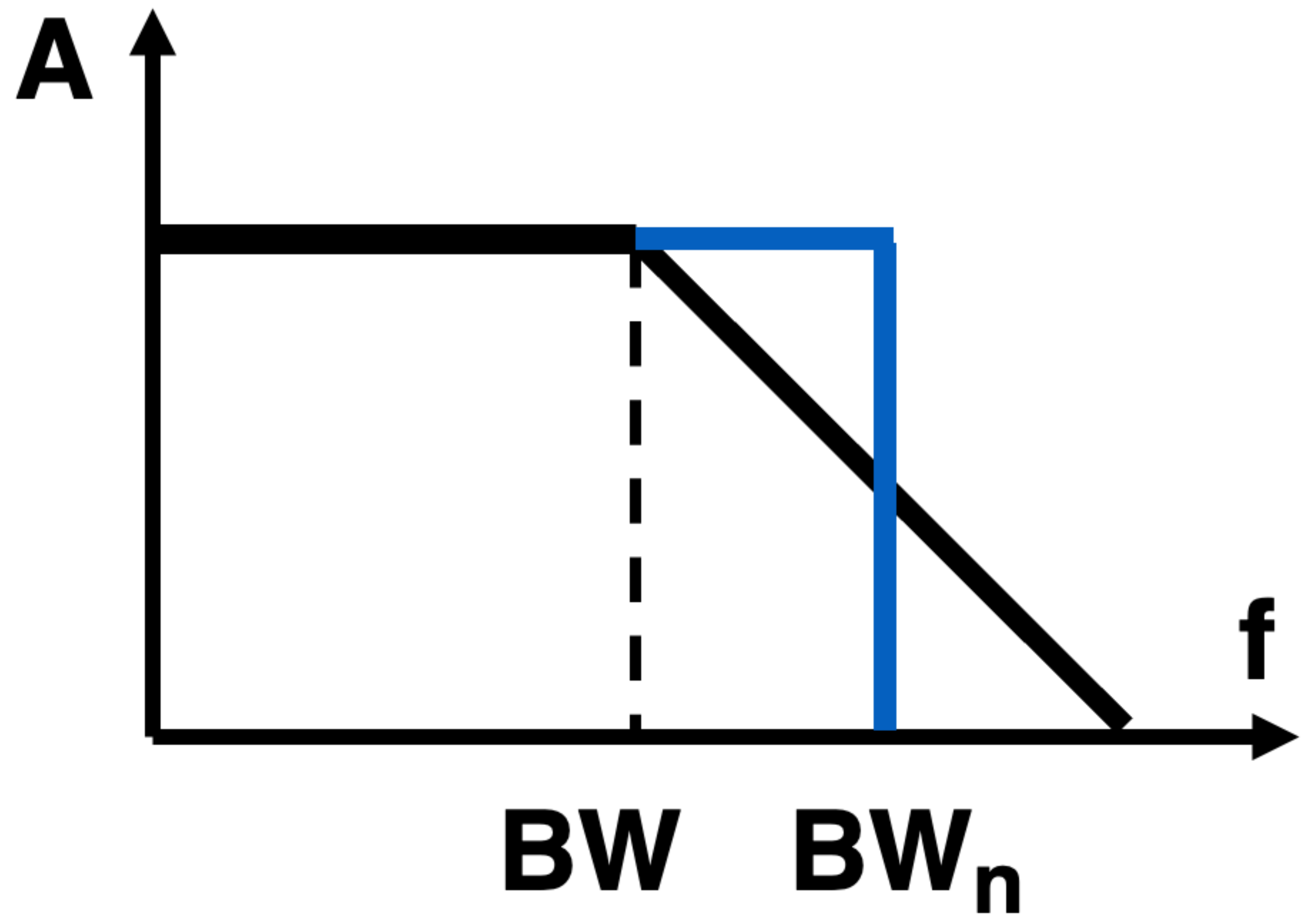
$$\boxed{\int_0^\infty \frac{dx}{1 + x^2} = \frac{\pi}{2}}$$

$$\overline{v_{Rs}^2} = 4kT\, R_S BW \frac{\pi}{2}$$

$$\overline{v_{Rs}^2} = \frac{kT}{C_L}$$

$$\boxed{C_L = 1\ \mathrm{pF} \quad v_{Rs} = 65\ \mu V_{RMS}}$$

Integrated Noise Versus Noise Density



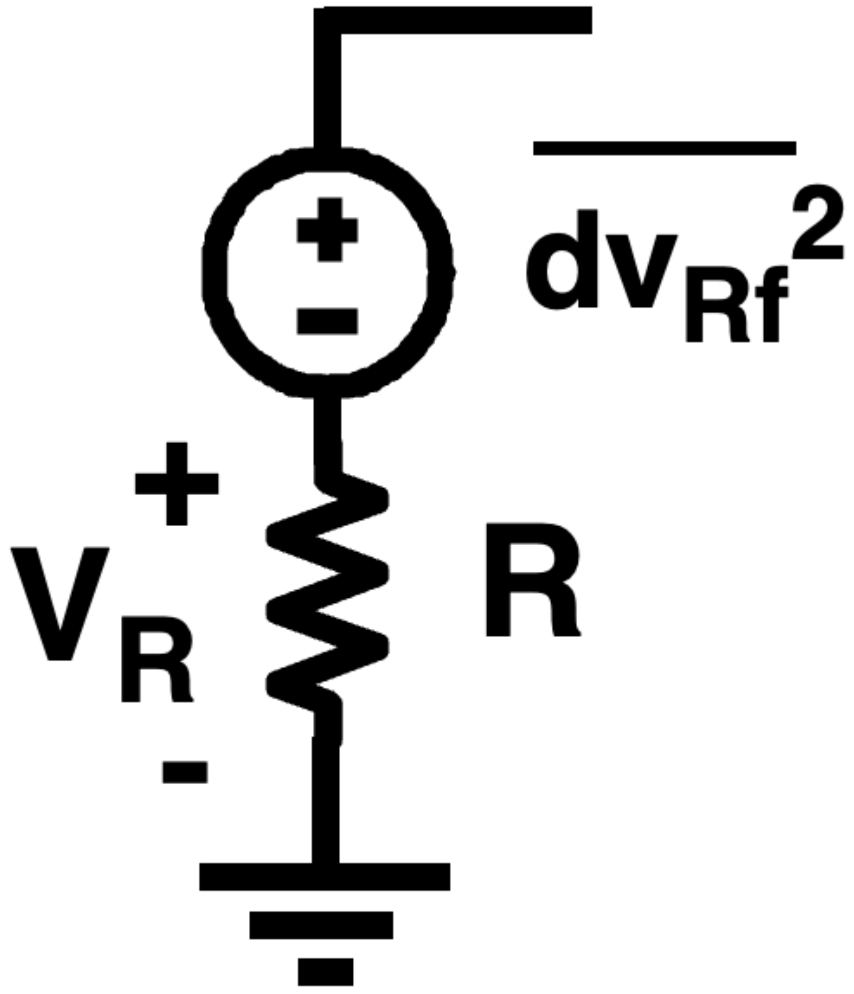
$$\overline{dv_{Rs}^2} = 4kT\, R_S\, df$$

$$\overline{v_{Rs}^2} = \int_0^{\infty} \frac{\overline{dv_{Rs}^2}}{1 + (f/BW)^2} = \frac{kT}{C_L}$$

Noise density [V^2/Hz] ~ R_S (or $1/g_m$)

Integrated noise [V_{RMS}] ~ $1/C_L$

Resistor 1/f Noise



$$\overline{dv_{Rf}^2} = V_R^2 \, \frac{KF_R R_\square}{A_R} \, \frac{df}{f} \qquad \text{is } 1/f$$

$$KF_{RSi} \approx 2\ 10^{-21}\ \text{Scm}^2$$

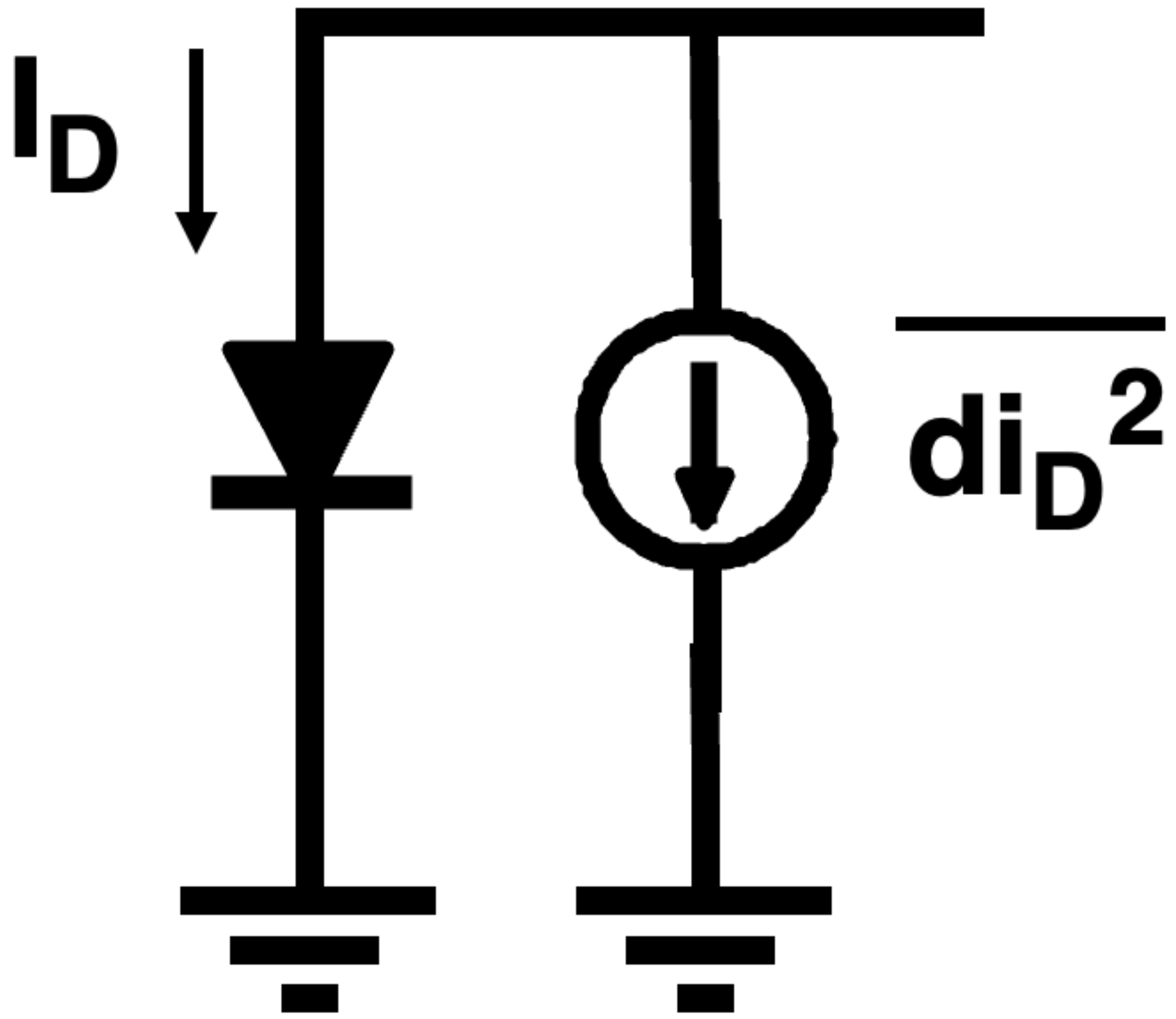
$$KF_{Rpoly} \approx 10\ KF_{RSi}$$

for R = 1 kΩ with 20 □'s of 50 Ω/□ and 1 μm wide and $V_R = 0.1$ V

$$\sqrt{\overline{dv_{Rf}^2}} = 16\ nV_{RMS}/\sqrt{Hz} \text{ at 1 Hz}$$

Ref. Vandamme, ESSDERC '04

Diode Shot Noise



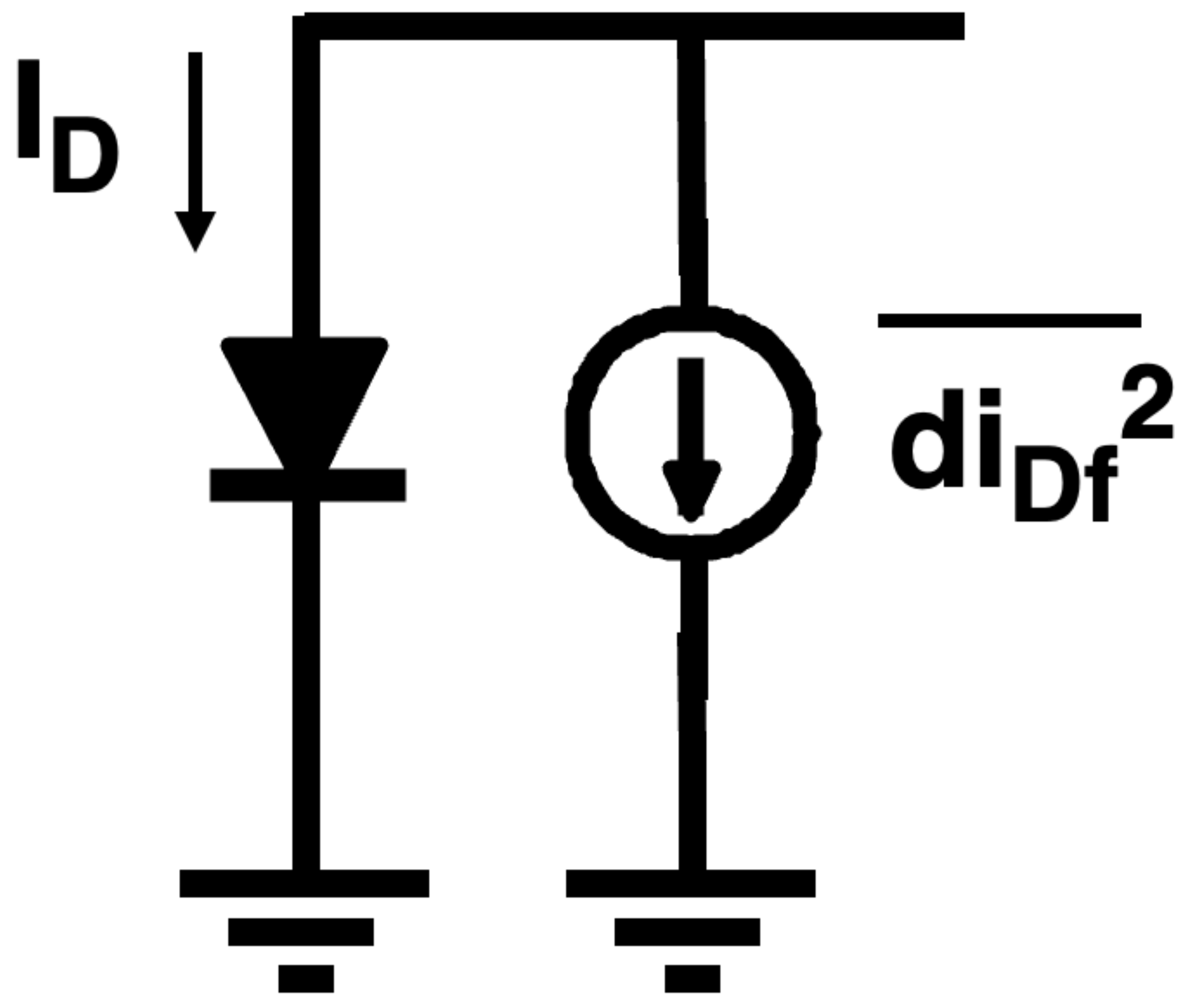
$\overline{di_D^2} = 2q\, I_D\, df$ is white

$q = 1.6\ 10^{-19}$ C

depends on I_D, not on T

for $I_D = 50\ \mu A$ $\sqrt{\overline{di_D^2}} = 4\ pA_{RMS} / \sqrt{Hz}$

Diode 1/f Noise



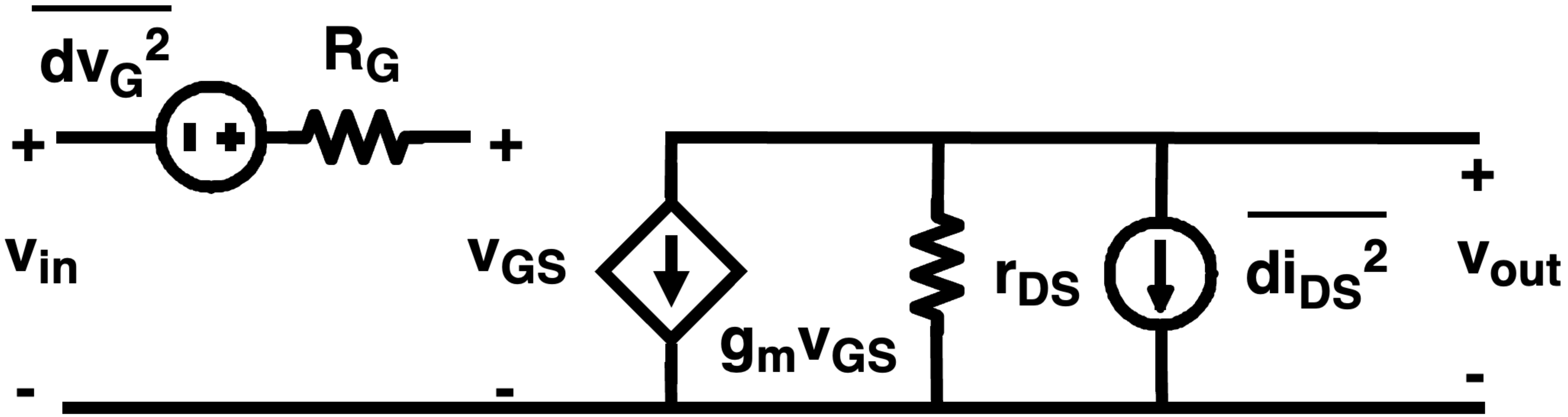
$$\overline{di_{Df}^2} = I_D \frac{KF_D}{A_D} \frac{df}{f} \qquad \text{is } 1/f$$

$$KF_D \approx 10^{-21}\ \mathrm{Acm}^2$$

For a diode of $A_D = 5 \times 2\ \mu m = 10\ \mu m^2$ and $I_D = 0.1$ mA

$$\sqrt{\overline{di_{Df}^2}} = 1\ nA_{RMS} / \sqrt{Hz} \text{ at } 1 \text{ Hz}$$

MOST Thermal Noise

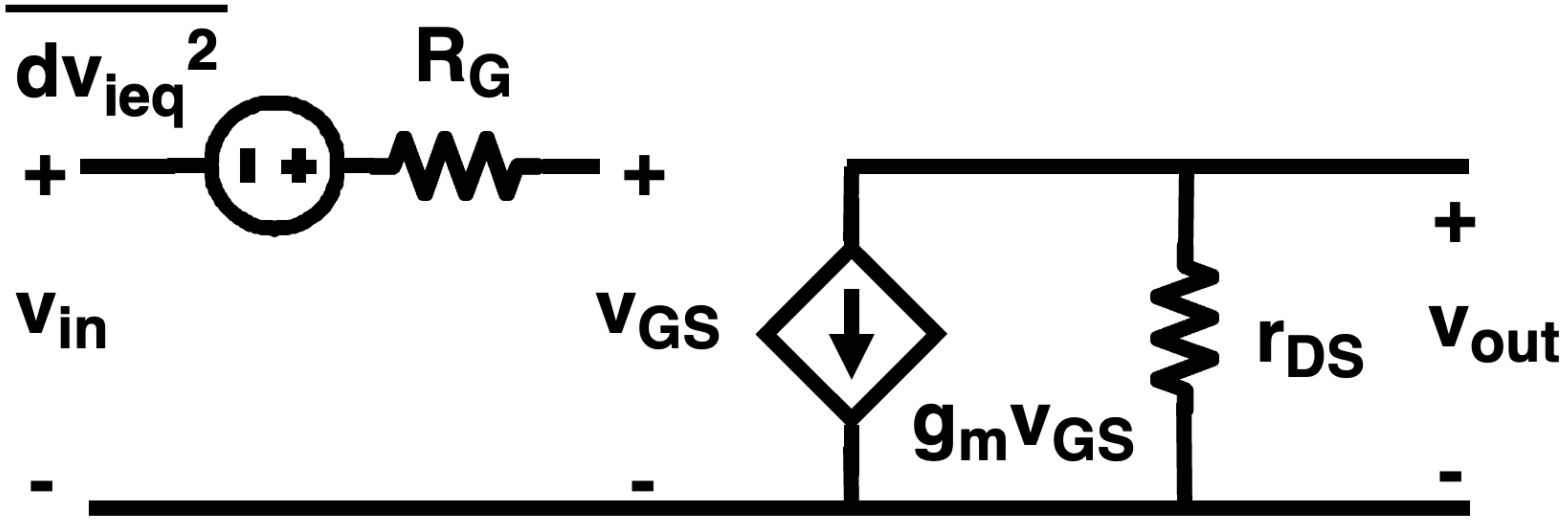


$$\overline{dv_G^2} = 4kT\, R_G\, df$$

$$\overline{di_{DS}^2} = \frac{4kT}{R_{CH}}\, df = 4kT \frac{2}{3}\, g_m\, df$$

Ref. Van der Ziel, Prentice Hall 1954, Wiley 1986.

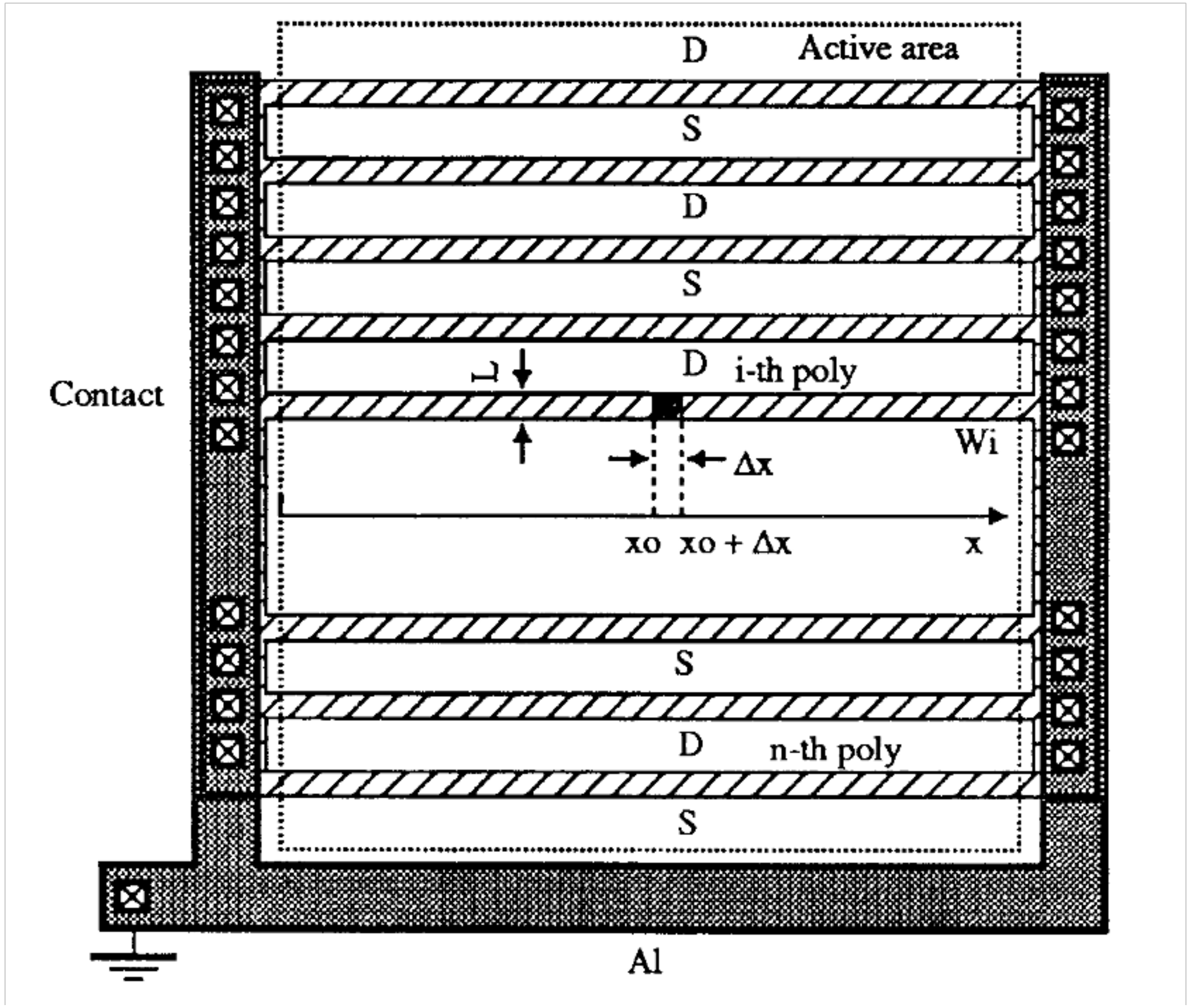
MOST Equivalent Input Noise



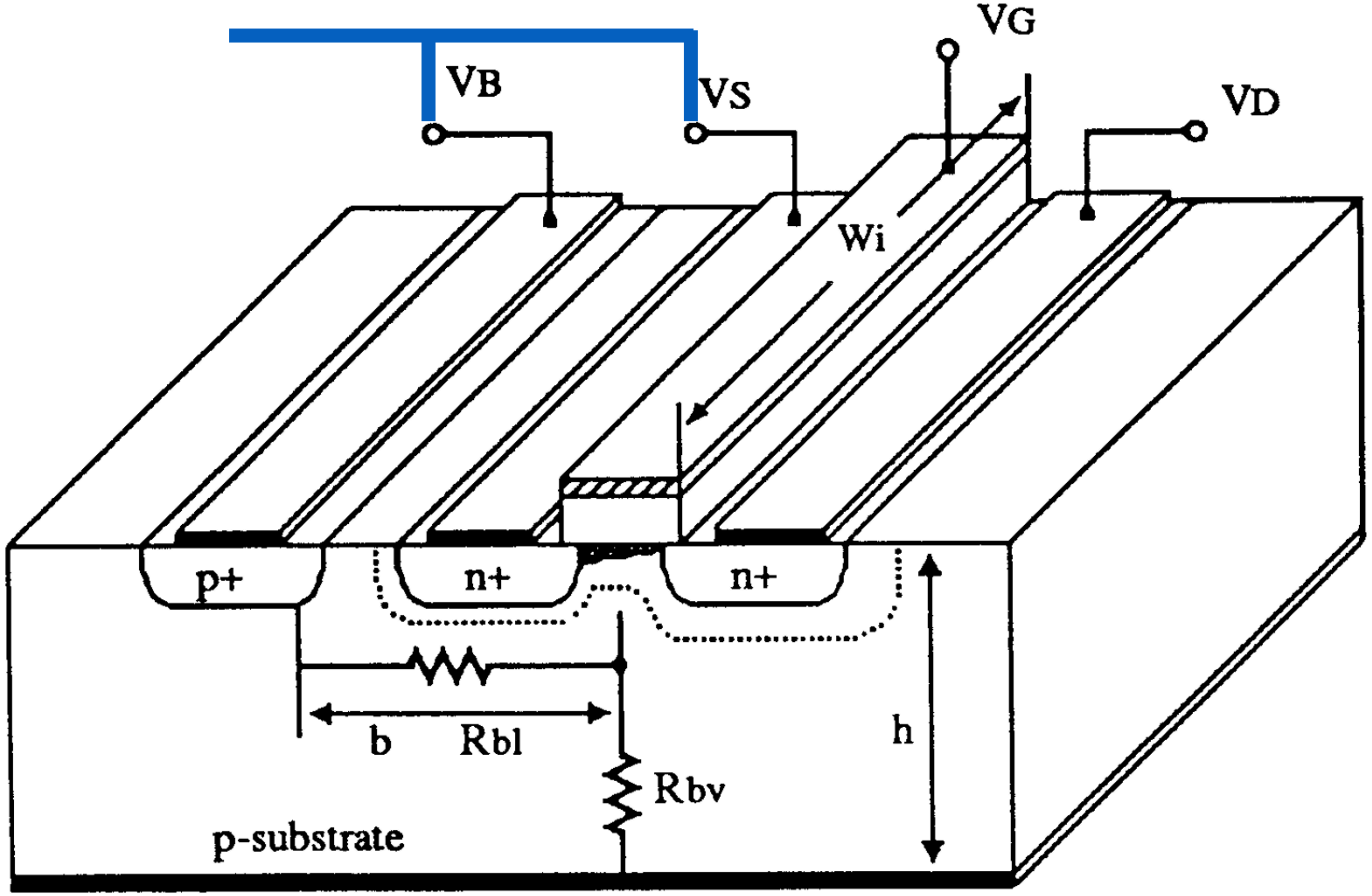
$$\overline{dv_{ieq}^2} = 4kT\,(R_{eff})\,df \qquad R_{eff} = \frac{2/3}{g_m} + R_G$$

High Freq.: $\overline{di_{ieq}^2} = (C_{GS}\,\omega)^2\,\overline{dv_{ieq}^2}$ **is correlated**

MOST Gate Resistance

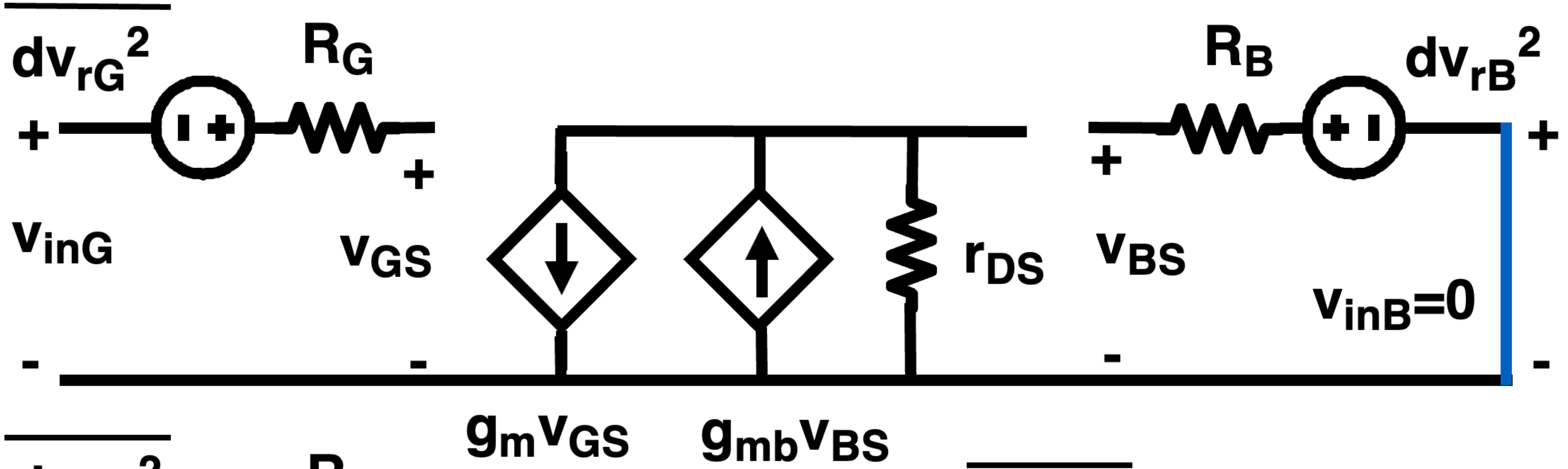


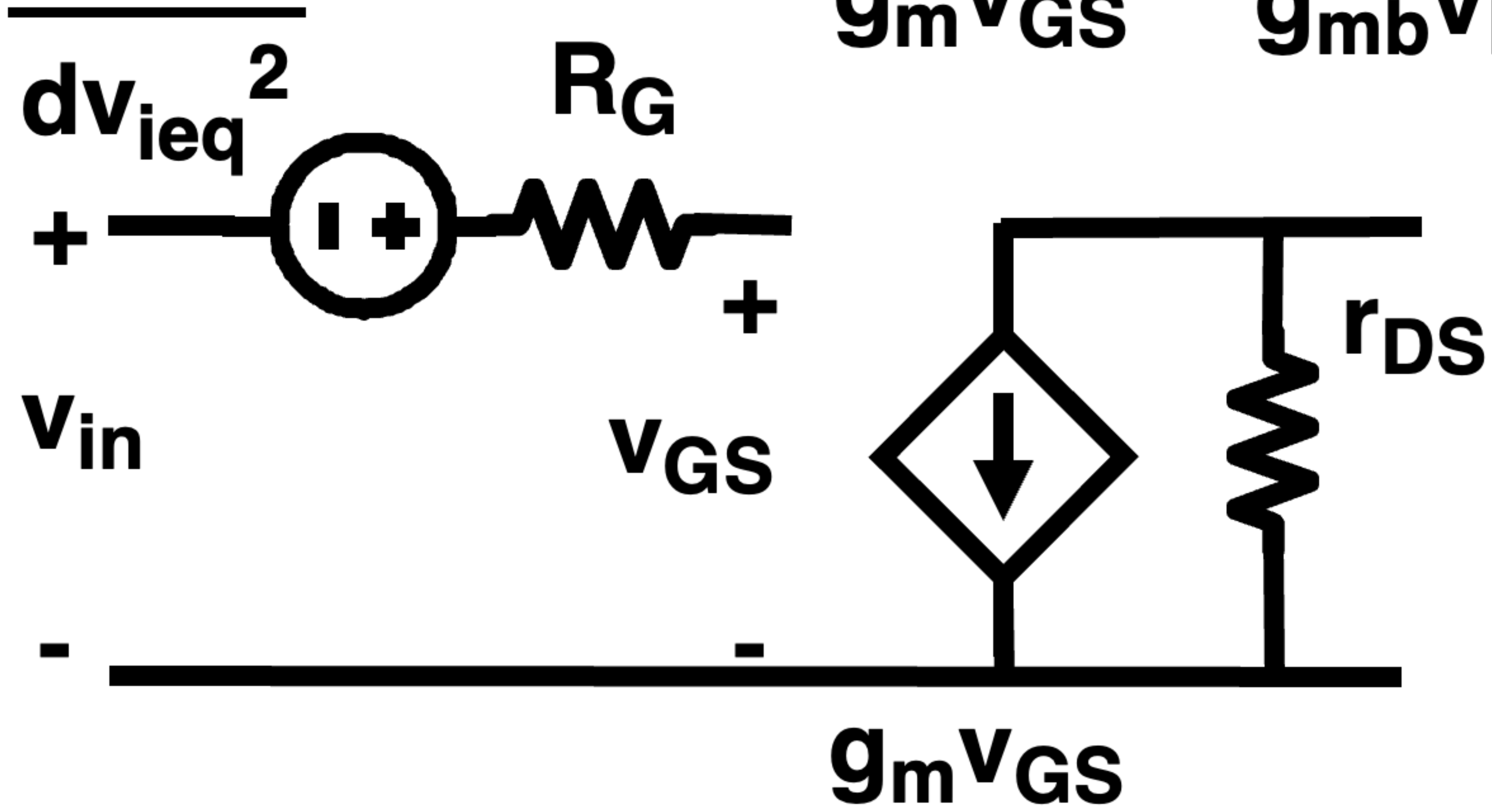
MOST Substrate Resistance



Ref. Chang, Kluwer 1991

MOST Total Thermal Noise



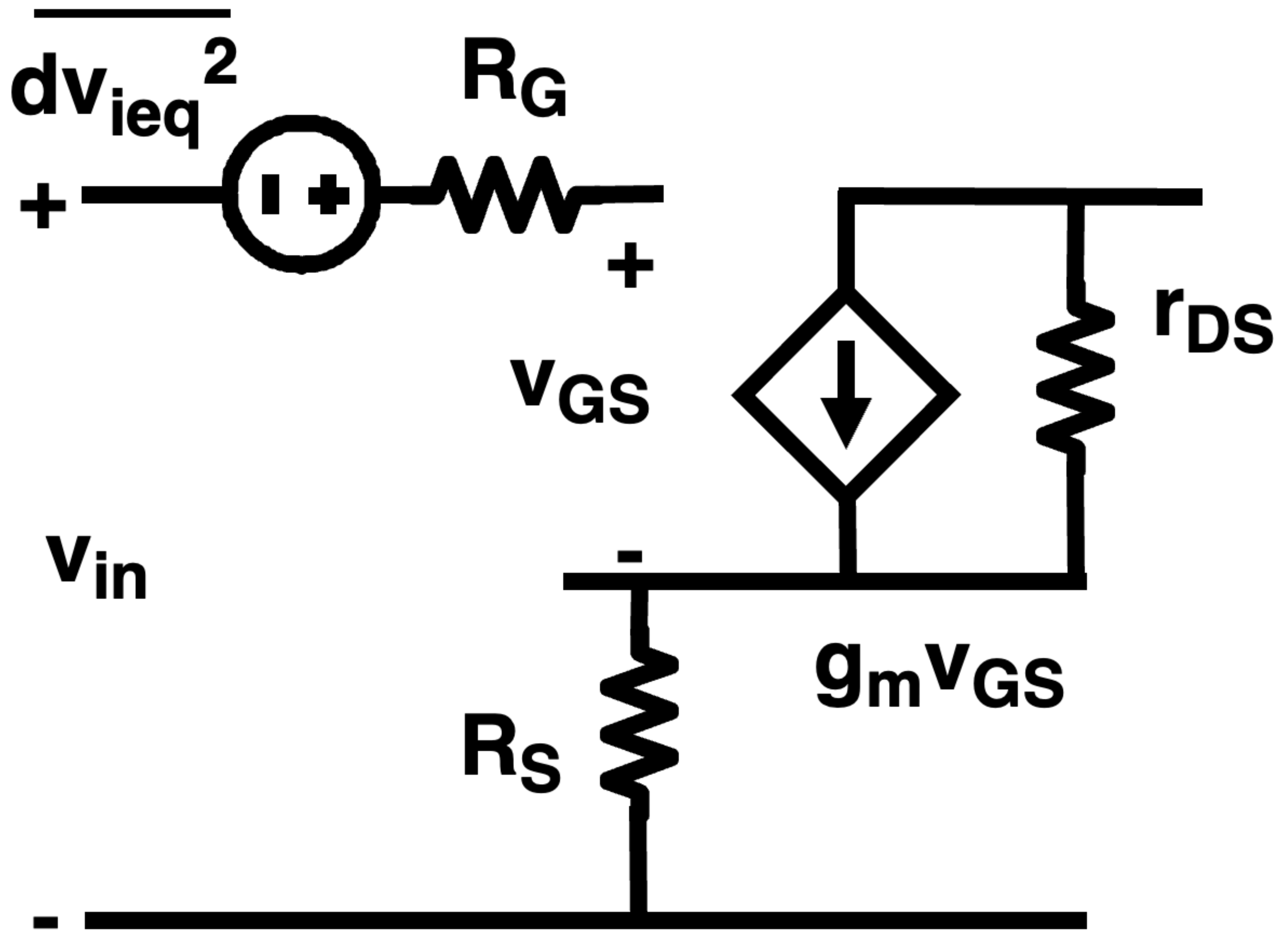


$$\overline{dv_{ieq}^2} = 4kT\,(R_{eff})\,df$$

$$R_{eff} = \frac{2/3}{g_m} + R_G + R_B\,(n-1)^2$$

$$(n-1) = C_D/C_{ox} = g_{mb}/g_m$$

MOST Source Resistance

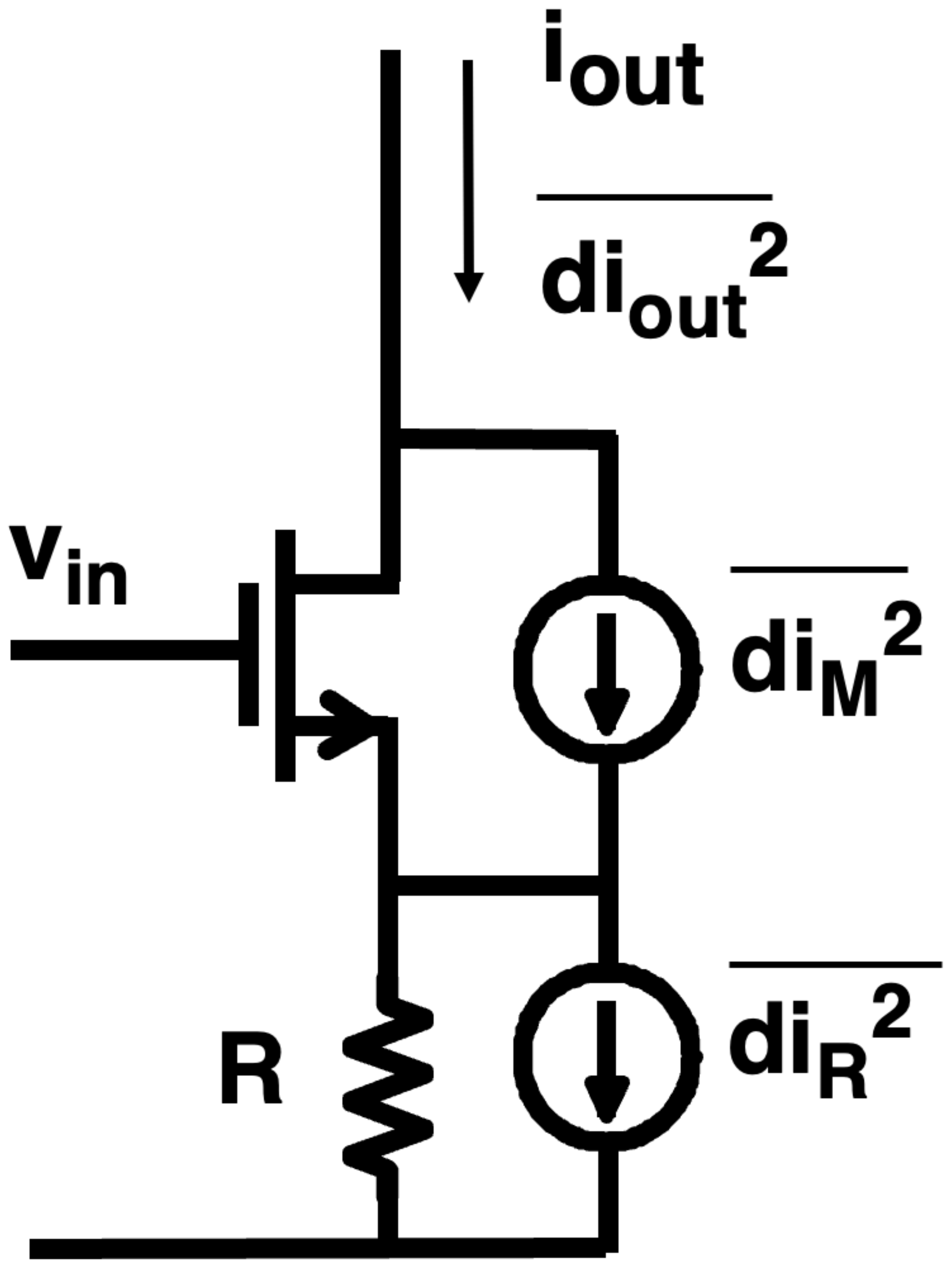


Noise of R_S
= noise R_G

$$\overline{dv_{ieq}^2} = 4kT\,(R_{eff})\,df$$

$$R_{eff} = \frac{2/3}{g_m} + R_G + R_S + R_B\,(n-1)^2$$

Noise by Source Resistance



$g_m R >> 1$

$$i_{out} = \frac{v_{in}}{R}$$

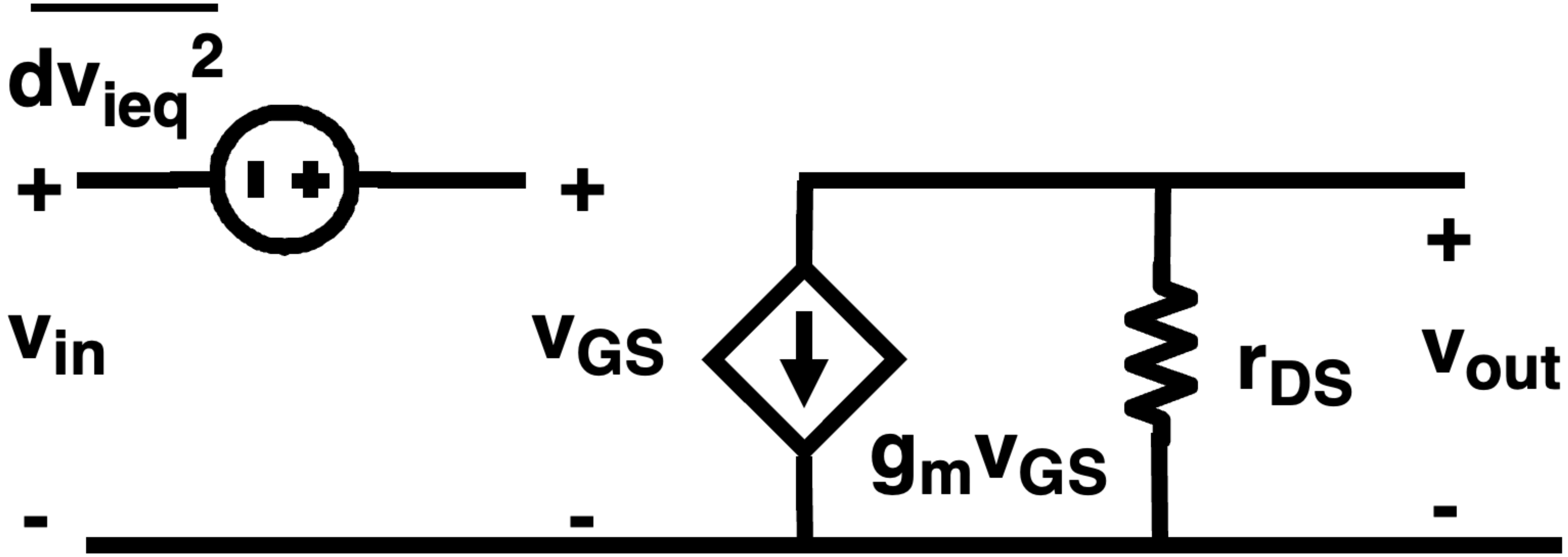
$$\overline{di_M^2} = 4kT\ 2/3\ g_m\ df \qquad \overline{di_{outM}^2} = \frac{\overline{di_M^2}}{(g_m R)^2}$$

$$\overline{di_R^2} = \frac{4kT}{R} df \qquad \overline{di_{outR}^2} = \overline{di_R^2}$$

$$\overline{di_{out}^2} = \frac{4kT}{R} \left(\frac{2/3}{g_m R} + 1 \right) df \approx \frac{4kT}{R} df$$

$$\overline{dv_{in}^2} = 4kT\ R\ df$$

Example: MOST Equivalent Input Noise

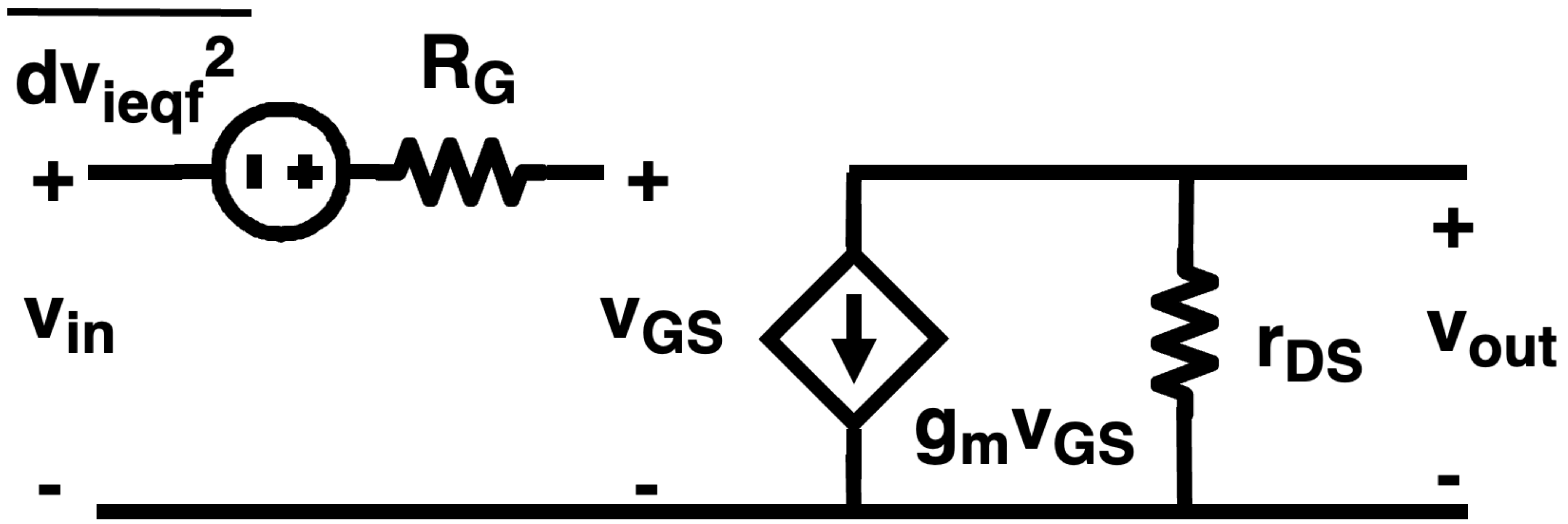


$$\overline{dv_{ieq}^2} \approx 4kT\left(\frac{2/3}{g_m}\right) df$$

$$\overline{dv_{ieq}^2} \approx ?$$

for $I_{DS} = 65\ \mu A$

MOST 1/f Noise



$$\overline{dv_{ieqf}^2} = \frac{KF_F}{WL\ C_{ox}^2}\ \frac{df}{f}$$

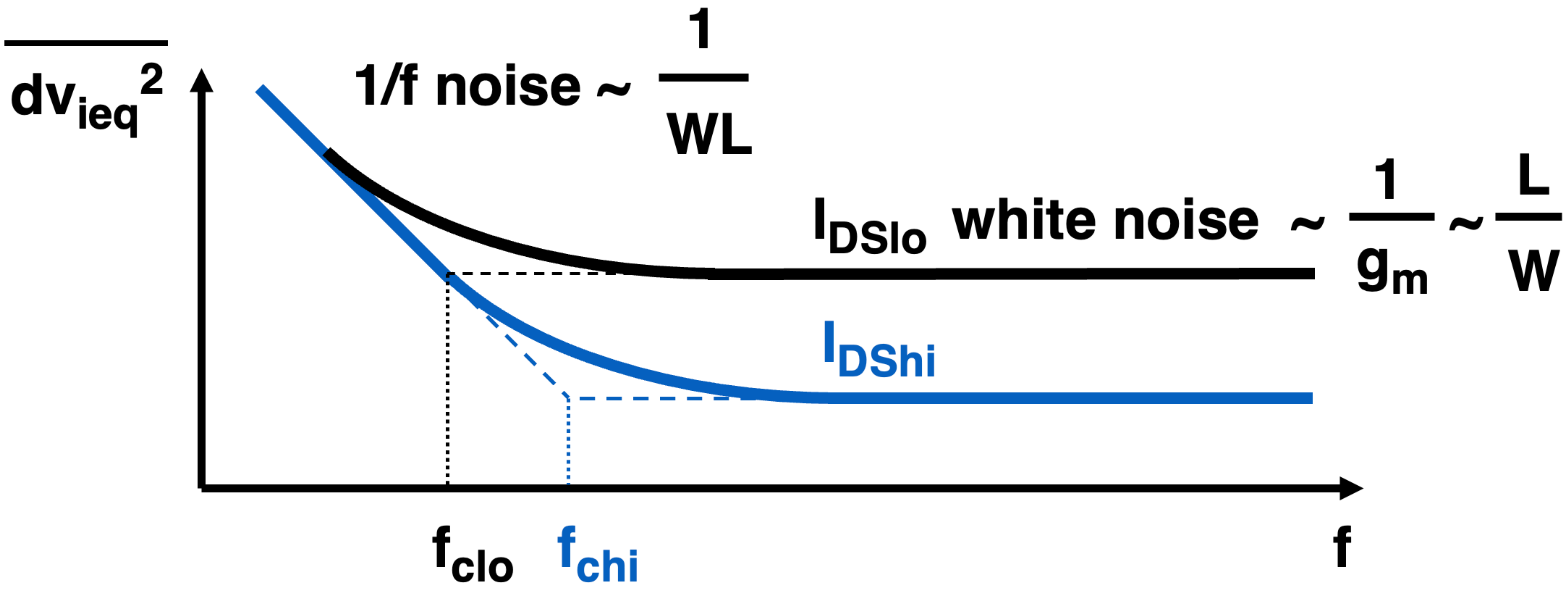
pMOST $KF_F \approx 10^{-32}\ C^2/cm^2$

nMOST $KF_F \approx 4\ 10^{-31}\ C^2/cm^2$

pJFET $KF_F \approx 10^{-33}\ C^2/cm^2$

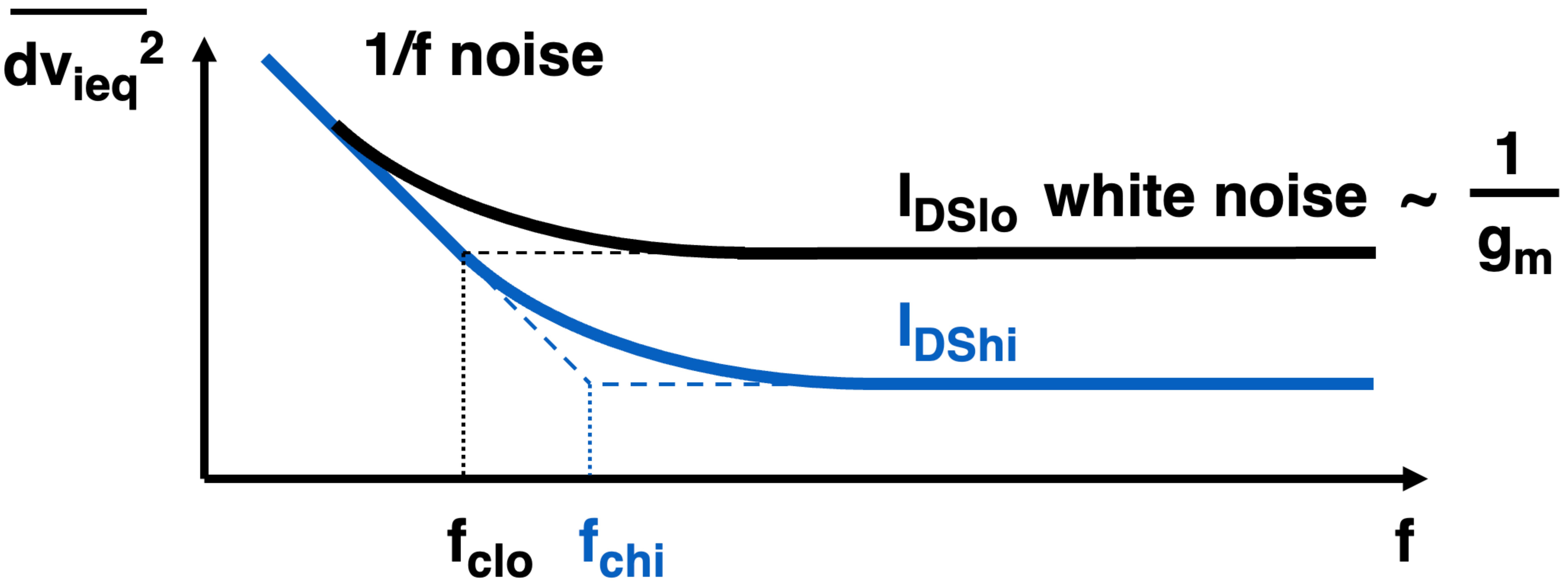
W & L in cm; C_{ox} in F/cm^2

MOST Noise Versus Frequency



Corner frequency ~ g_m

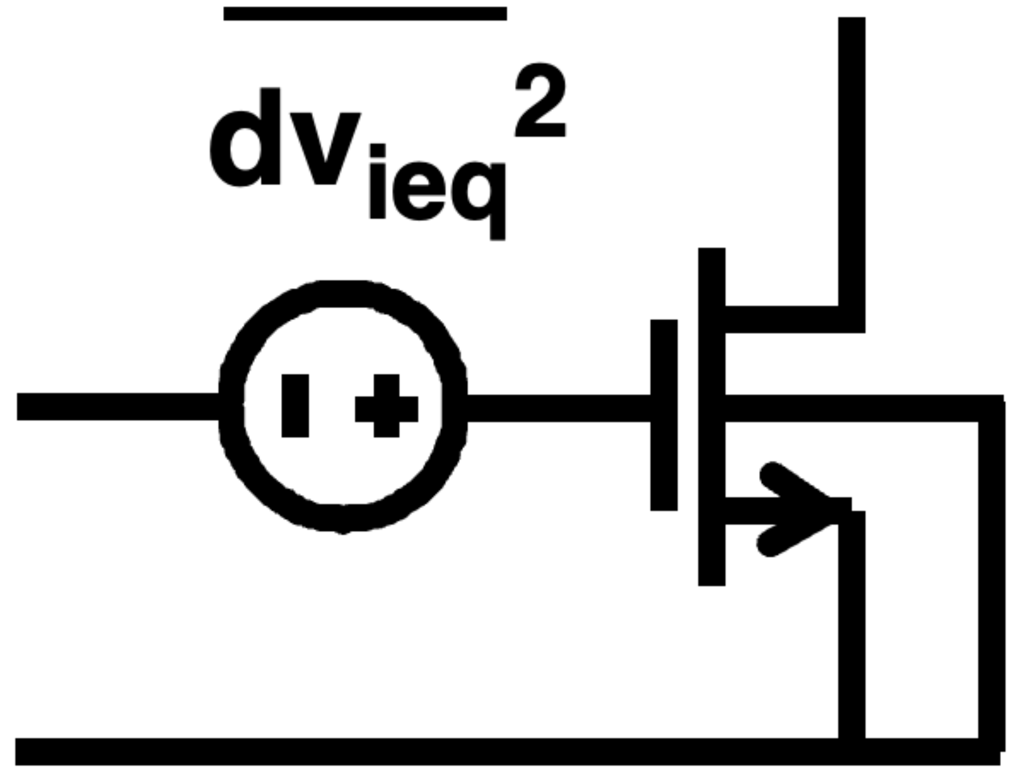
Example: MOST Noise Corner Frequency

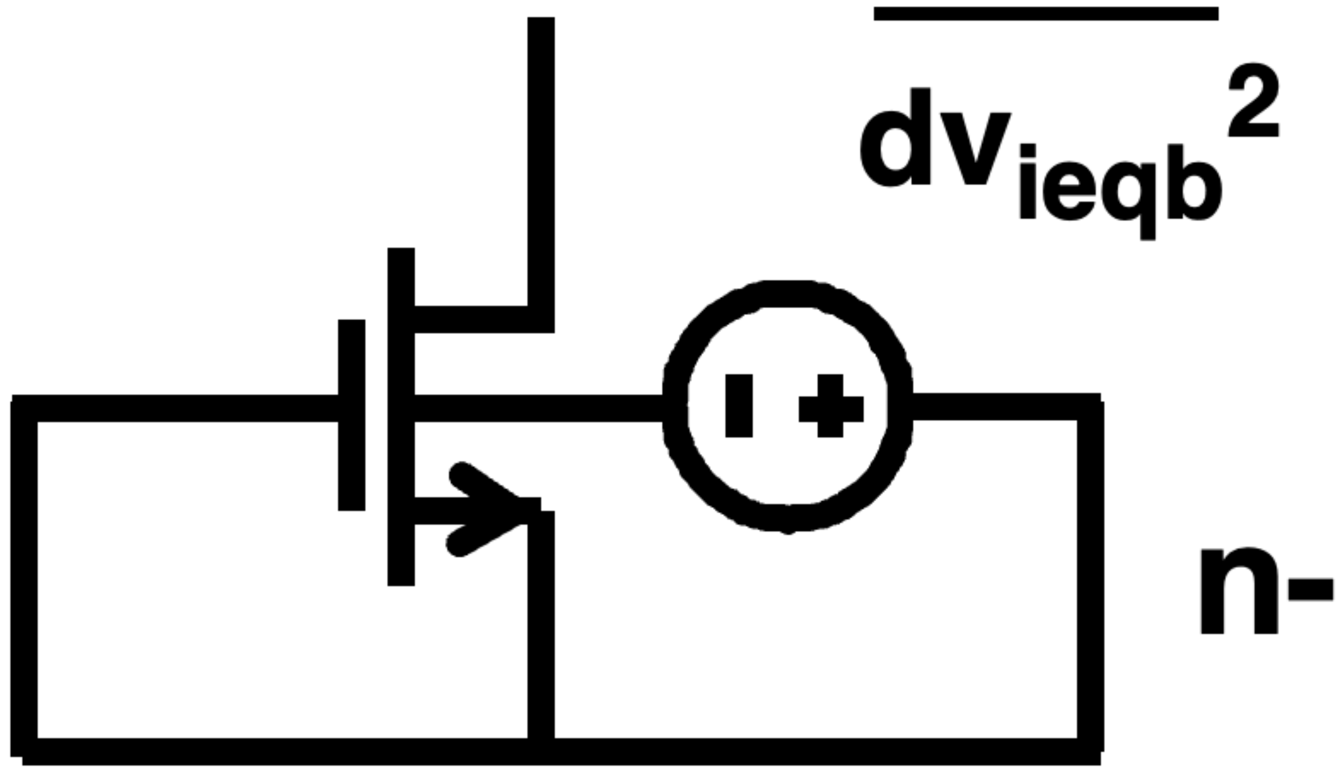


Ex. : f_c ? For $I_{DS} = 65\ \mu A$;

$K'_n = 60\ \mu A/V^2$ and $L = 1\ \mu m$ (0.35 μm process)

MOST Equivalent Noise at Bulk



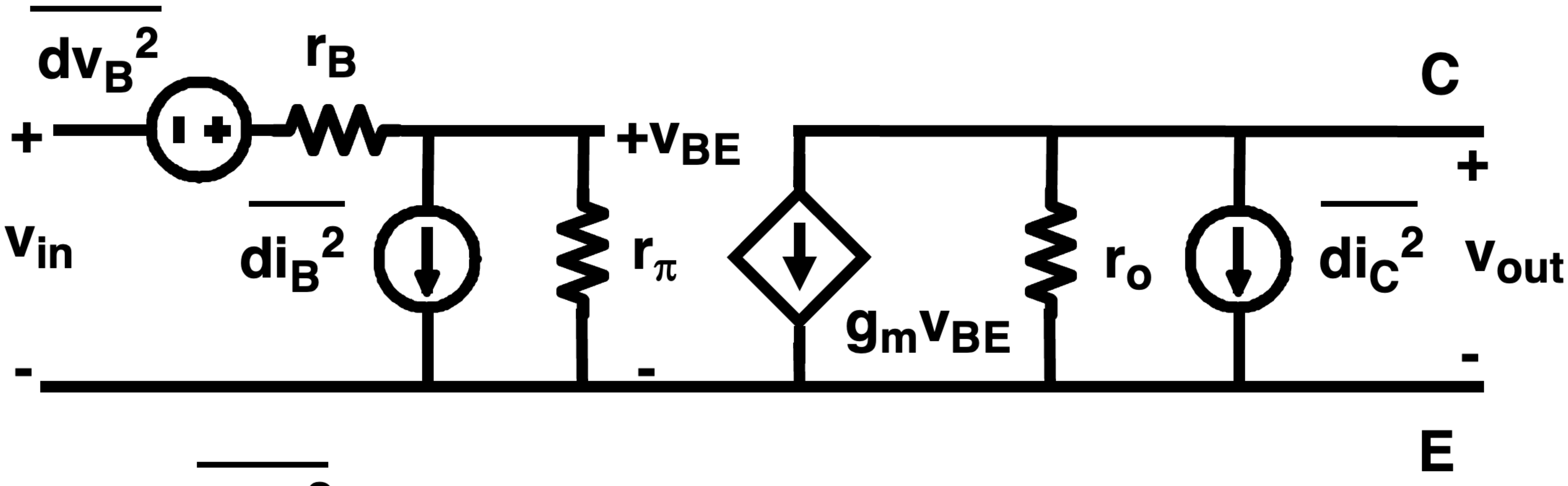


$$n-1 = \frac{g_{mb}}{g_m}$$

$$\overline{dv_{ieq}^2} = 4kT\left(\frac{2/3}{g_m}\right) df \qquad \overline{dv_{ieqb}^2} = 4kT\left(\frac{2/3\ g_m}{g_{mb}^2}\right) df$$

$$\overline{dv_{ieqf}^2} = \frac{KF_F}{WL\ C_{ox}^2}\frac{df}{f} \qquad \overline{dv_{ieqfb}^2} = \frac{KF_F}{WL\ C_{ox}^2}\frac{g_m^2}{g_{mb}^2}\frac{df}{f}$$

BJT Noise



$$\overline{dv_B^2} = 4kT\, r_B\, df$$

$$\overline{di_B^2} = 2q\, I_B\, df$$

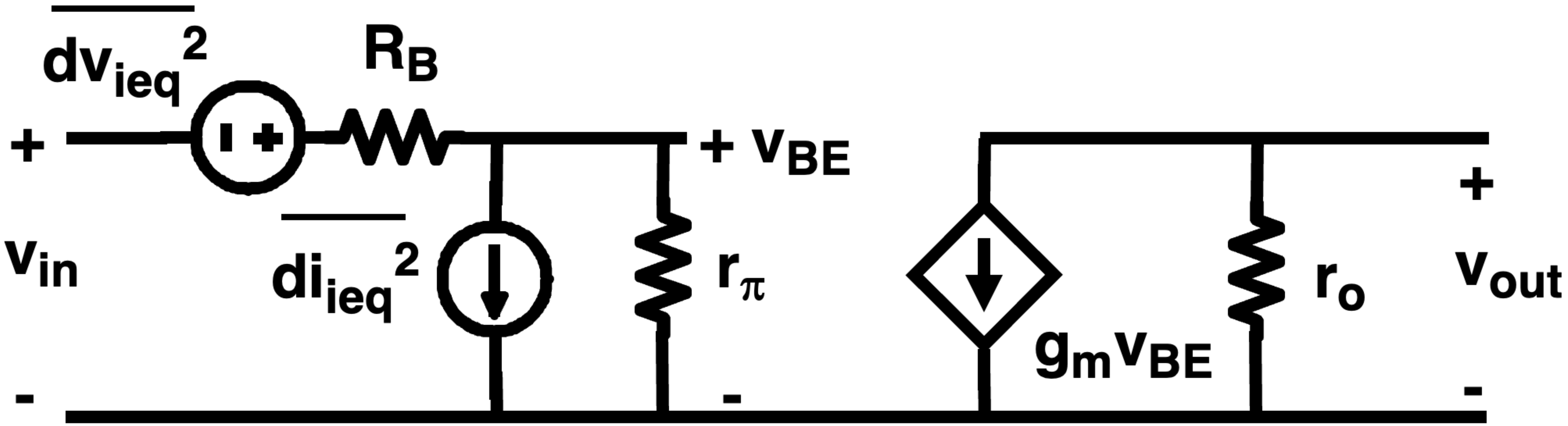
$$\overline{di_C^2} = 2q\, I_C\, df$$

$$\overline{di_{Bf}^2} = \frac{KF_B\, I_B}{A_{EB}} \frac{df}{f}$$

$$KF_B \approx 10^{-21}\ \mathrm{Acm^2}$$

Ref. Van der Ziel (Prentice Hall 1954)

BJT Equivalent Input Noise



$$\overline{dv_{ieq}^2} = 4kT\,(R_{eff})\,df \qquad R_{eff} = \frac{1/2}{g_m} + R_B + R_E$$

$$\overline{di_{ieq}^2} = \overline{di_B^2} = 2q\,I_B\,df$$

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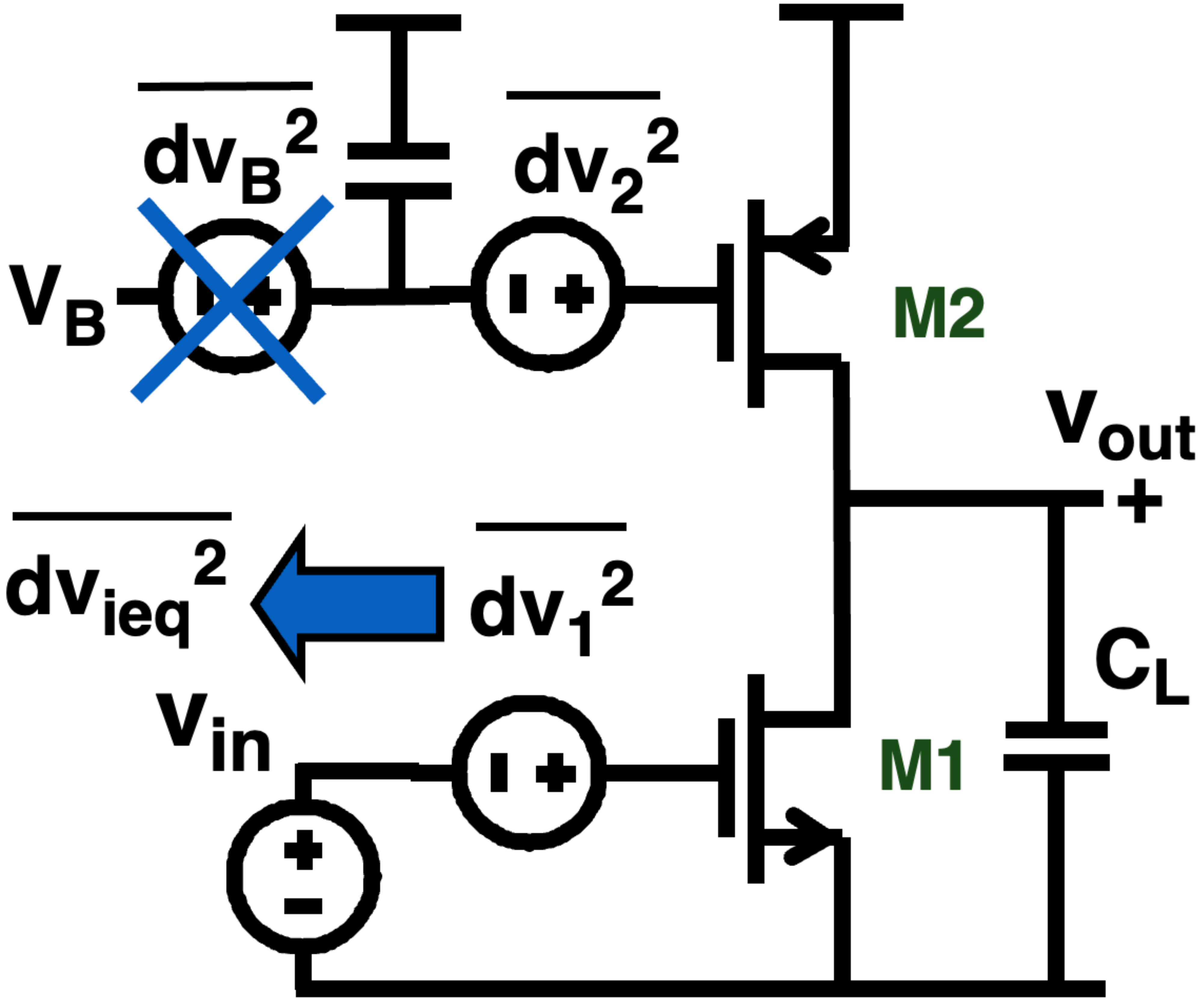
Amplifier Thermal Noise



If $\overline{dv_B^2}$ is negligible:

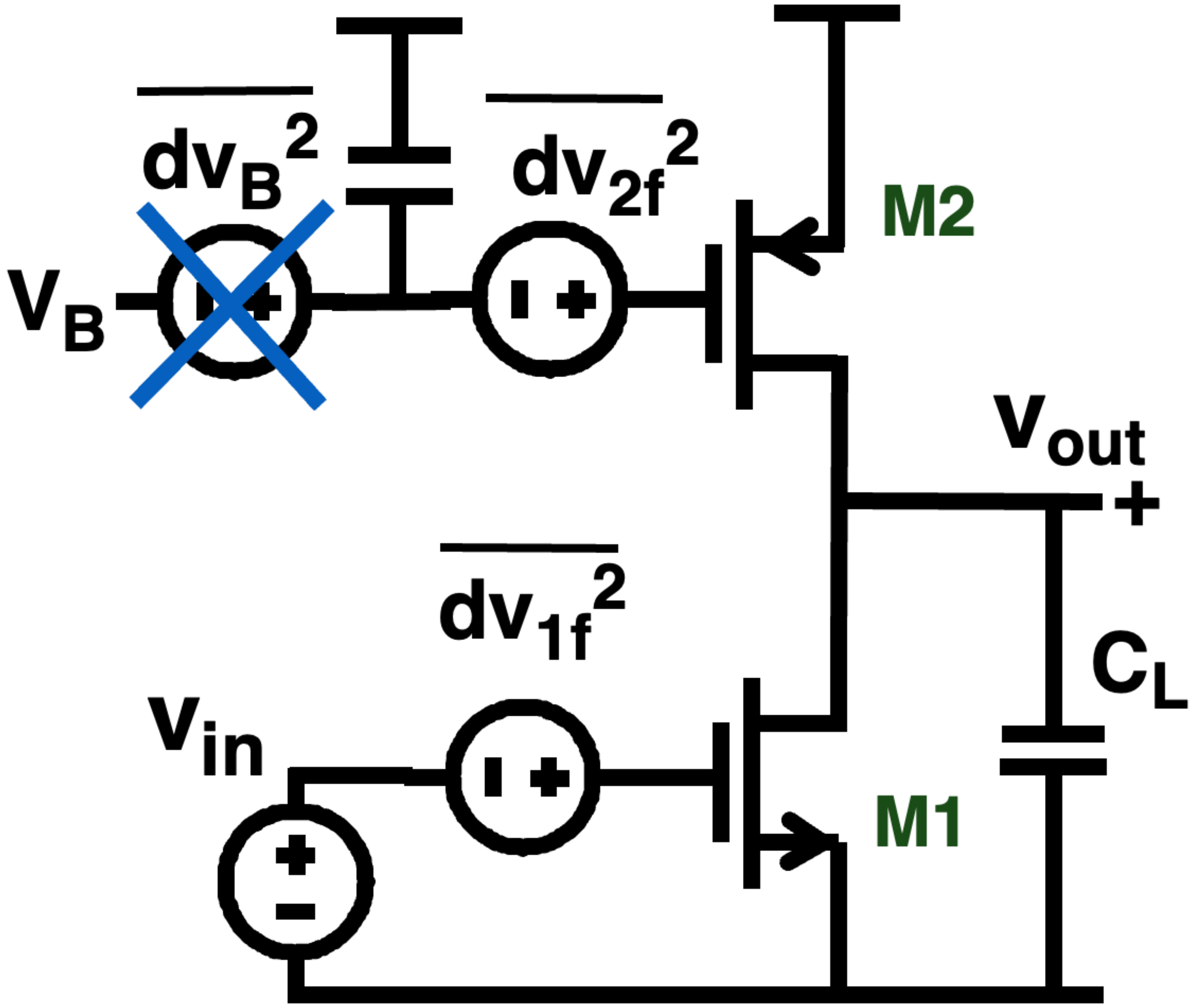
$$\overline{di_{out}^2} = g_{m1}^2\,\overline{dv_1^2} + g_{m2}^2\,\overline{dv_2^2}$$

$$\overline{dv_{ieq}^2} = \overline{dv_1^2} + \overline{dv_2^2}\left(\frac{g_{m2}}{g_{m1}}\right)^2$$

$$\overline{dv_{ieq}^2} = \overline{dv_1^2}\left(1 + \frac{g_{m2}}{g_{m1}}\right)$$

Small g_{m2}: small $(W/L)_2$ or large $(V_{GS} - V_T)_2$

Amplifier 1/f Noise



If $\overline{dv_B^2}$ is negligible :

$$\overline{dv_{if}^2} = \overline{dv_{1f}^2} + \overline{dv_{2f}^2}\left(\frac{g_{m2}}{g_{m1}}\right)^2$$

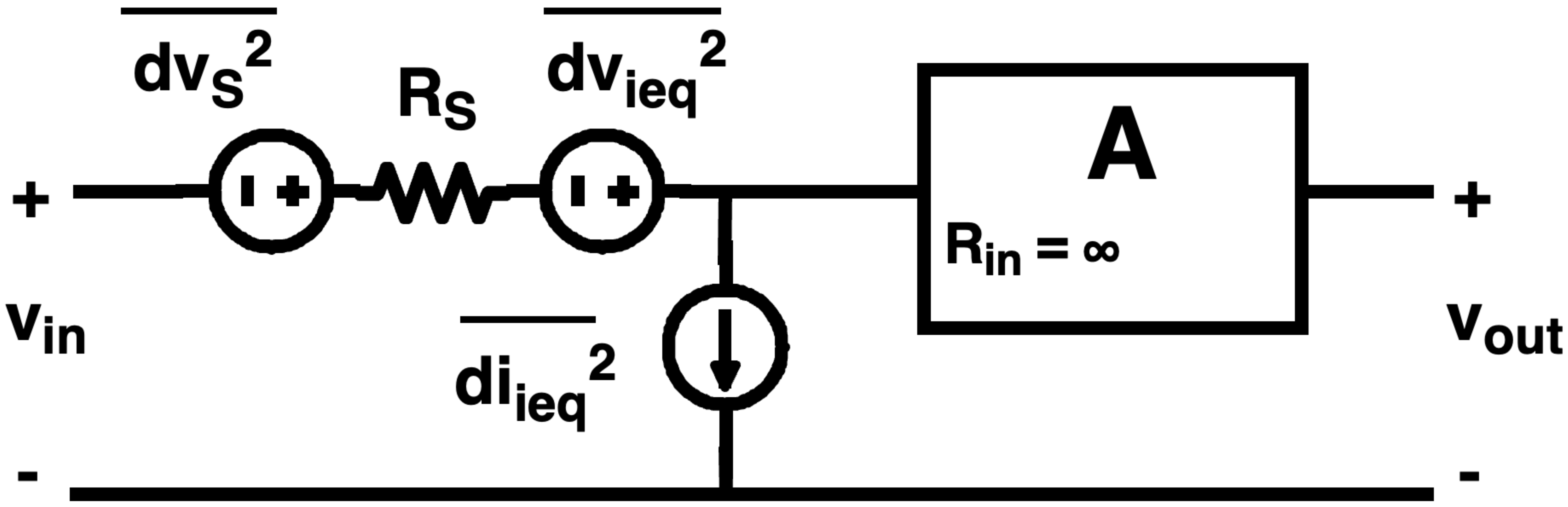
$$\overline{dv_{if}^2} = \overline{dv_{1f}^2}\left[1 + \left(\frac{g_{m2}}{g_{m1}}\right)^2 \left(\frac{\overline{dv_{2f}}}{\overline{dv_{1f}}}\right)^2\right]$$

$$\overline{dv_{if}^2} = \overline{dv_{1f}^2}\left[1 + \frac{KF_2}{KF_1}\,\frac{K'_2}{K'_1}\left(\frac{L_1}{L_2}\right)^2\right]$$

$\overline{dv_{if}^2}$ has minimum at:

$$L_{1opt} = L_2\sqrt{\frac{KF_1}{KF_2}\,\frac{K'_1}{K'_2}} \approx 10\ L_2\text{, then } \overline{dv_{if}^2} = 2\ \overline{dv_{1f}^2}$$

Amplifier Noise Figure



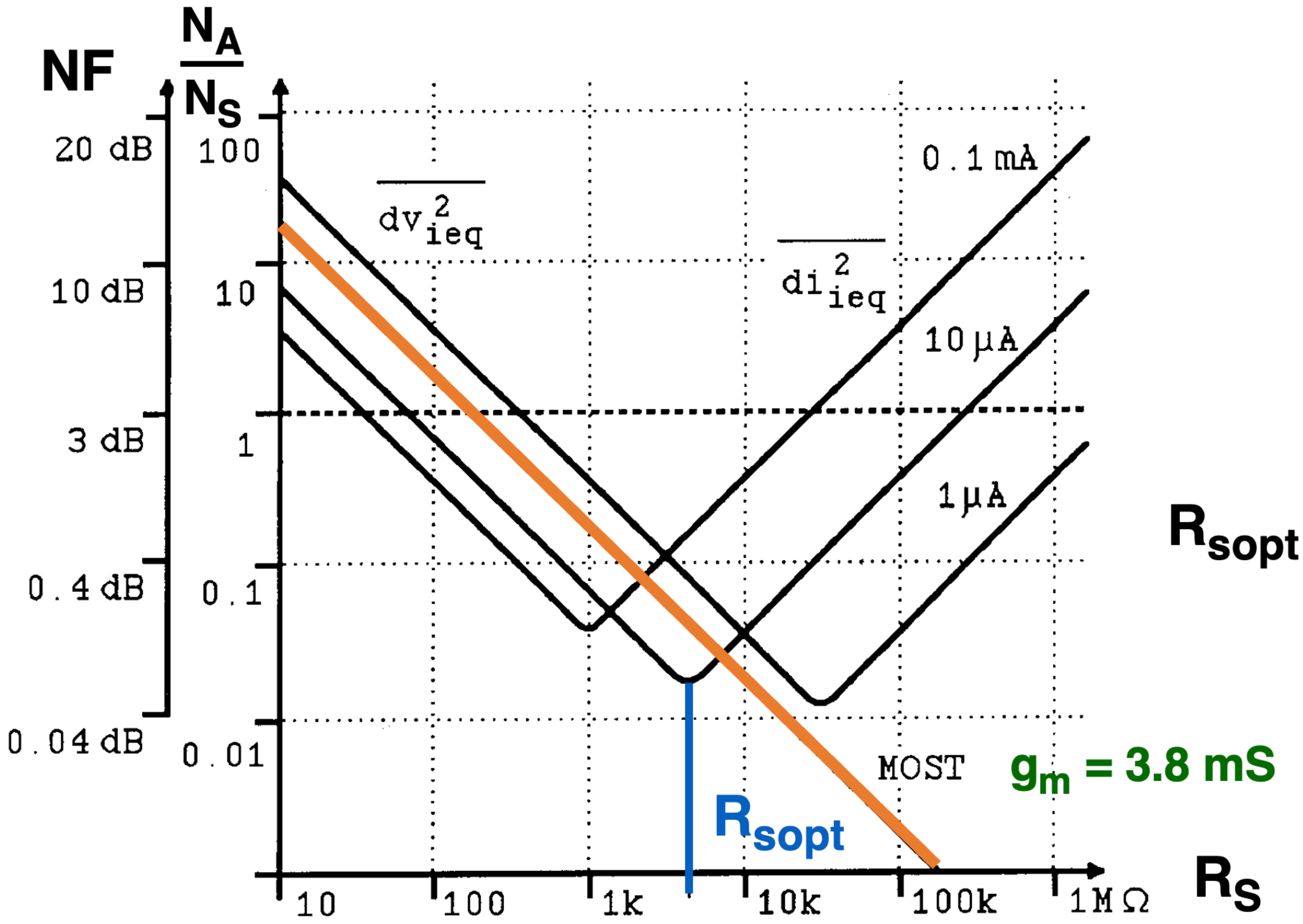
$$NF = \frac{N_S + N_A}{N_S} = 1 + \frac{N_A}{N_S}$$

$$NF = 1 + \frac{\overline{dv_{ieq}^2} + R_S^2\,\overline{di_{ieq}^2}}{4kT\,R_S\,df}$$

Voltage drive $NF \sim \frac{1}{R_S}$

Current drive $NF \sim R_S$

Resistive Noise Matching



$\beta = 100$

$r_B = 100\ \Omega$

$$R_{sopt} = \sqrt{\frac{\overline{dv_{ieq}^2}}{\overline{di_{ieq}^2}}}$$

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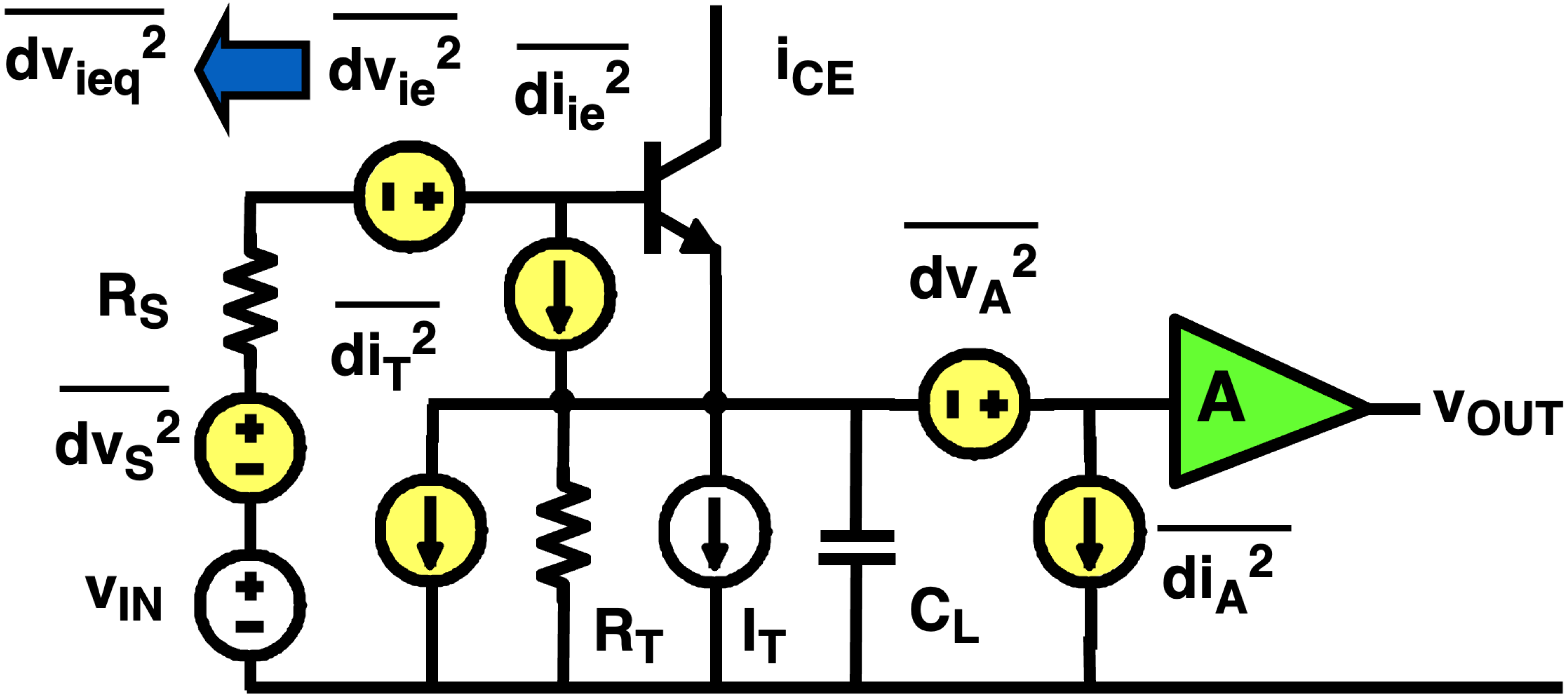
Emitter Follower Noise



$$\overline{dv_{ieq}^2} = \overline{dv_{ie}^2} + \overline{dv_A^2} + \left(R_S - \frac{1}{g_m}\right)^2 \overline{di_{ie}^2} + \frac{\overline{di_T^2} + \overline{di_A^2}}{g_m^2}$$

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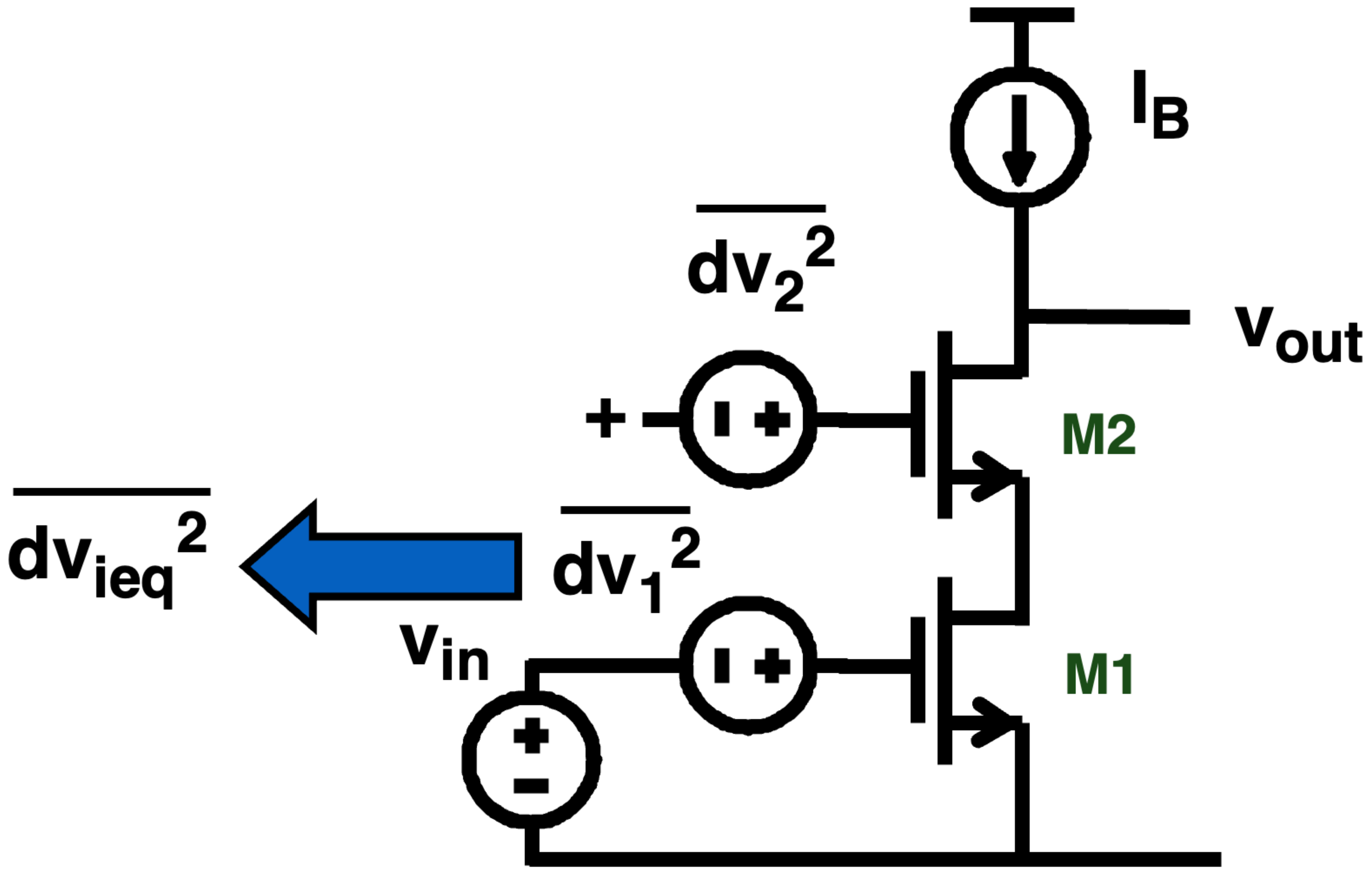
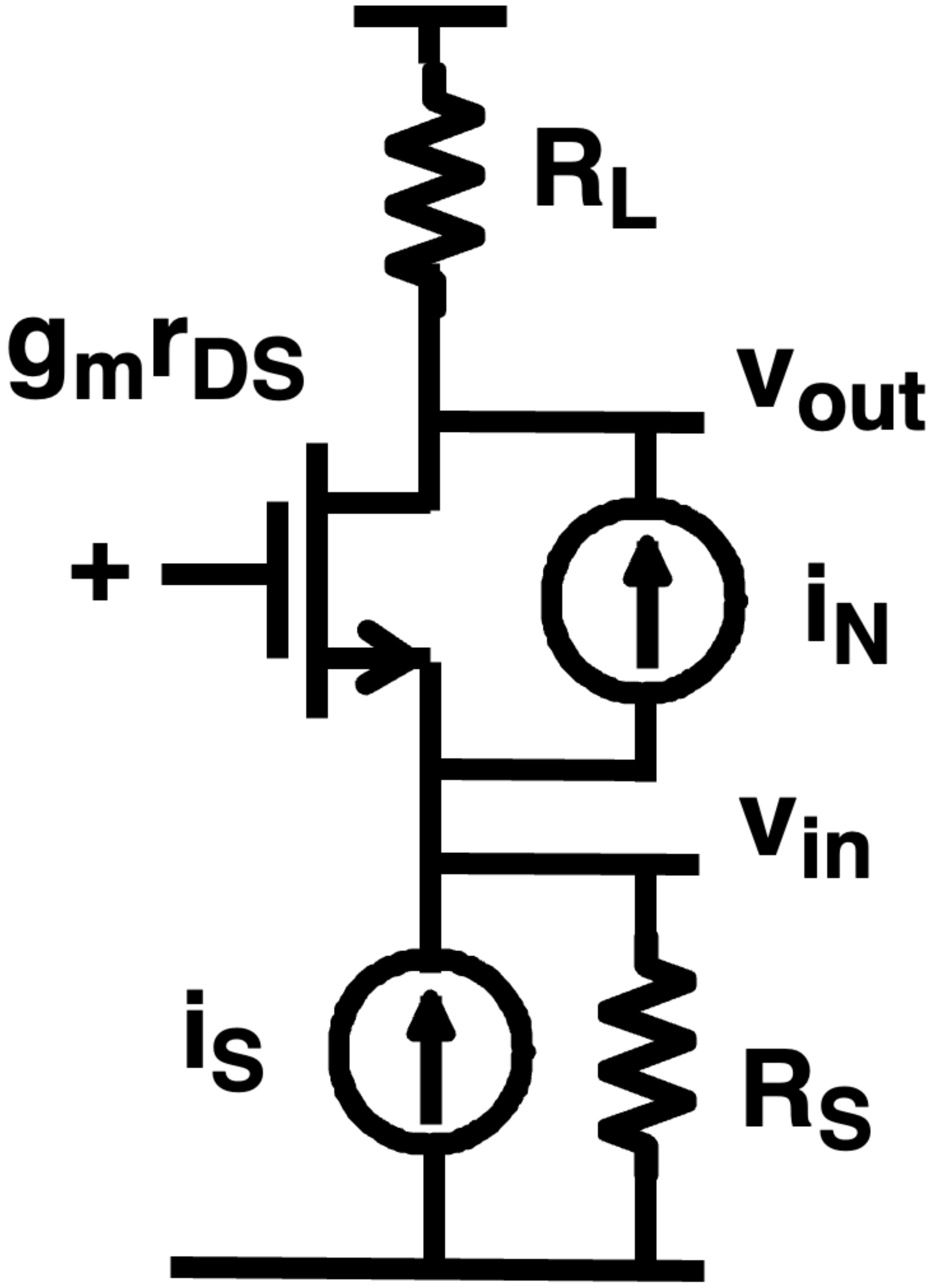
Cascode Noise

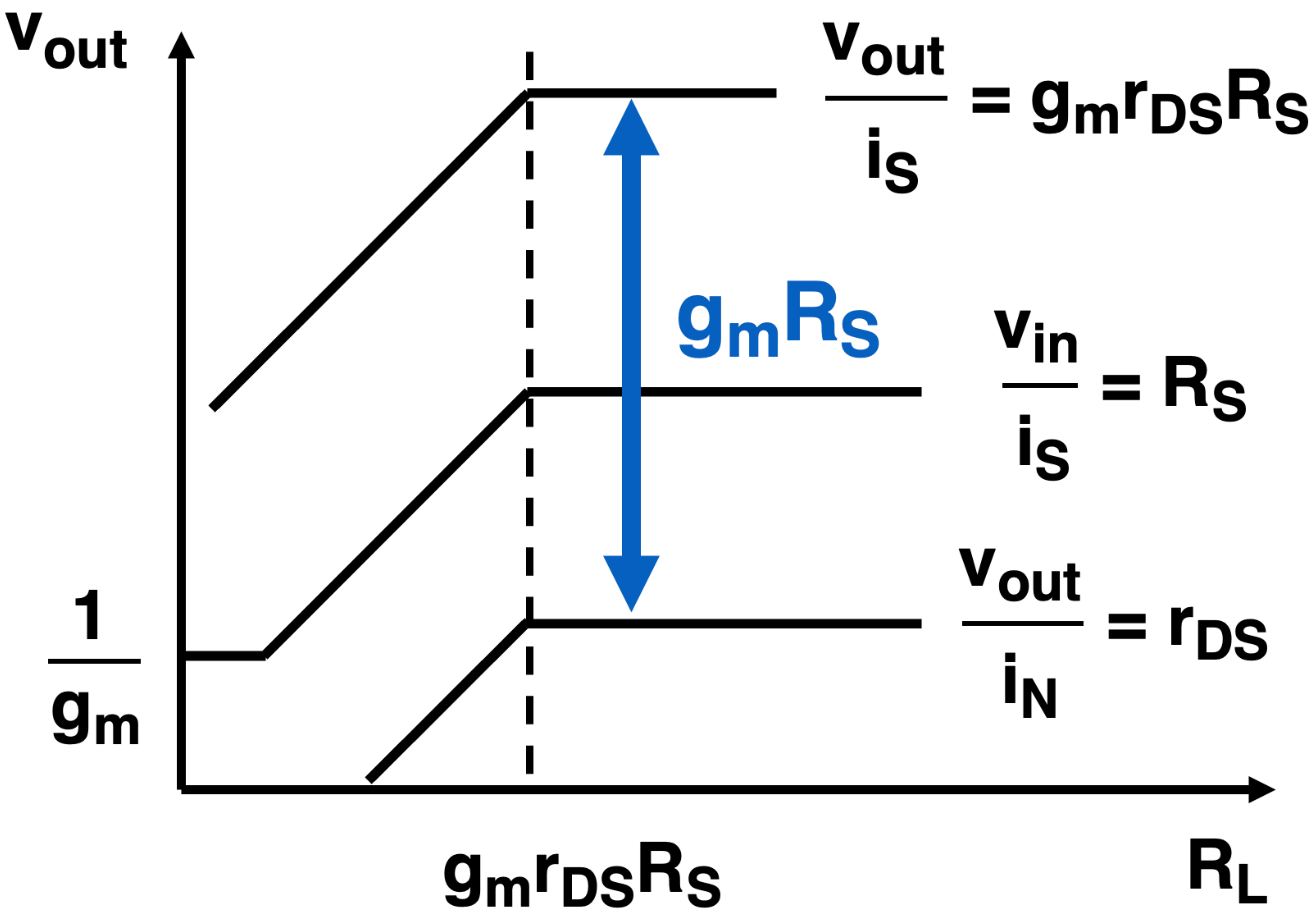


$$\overline{dv_{ieq}^2} = \overline{dv_1^2} + \overline{dv_2^2}\,\frac{1}{(g_{m1}\,r_{o1})^2} \approx \overline{dv_1^2}$$

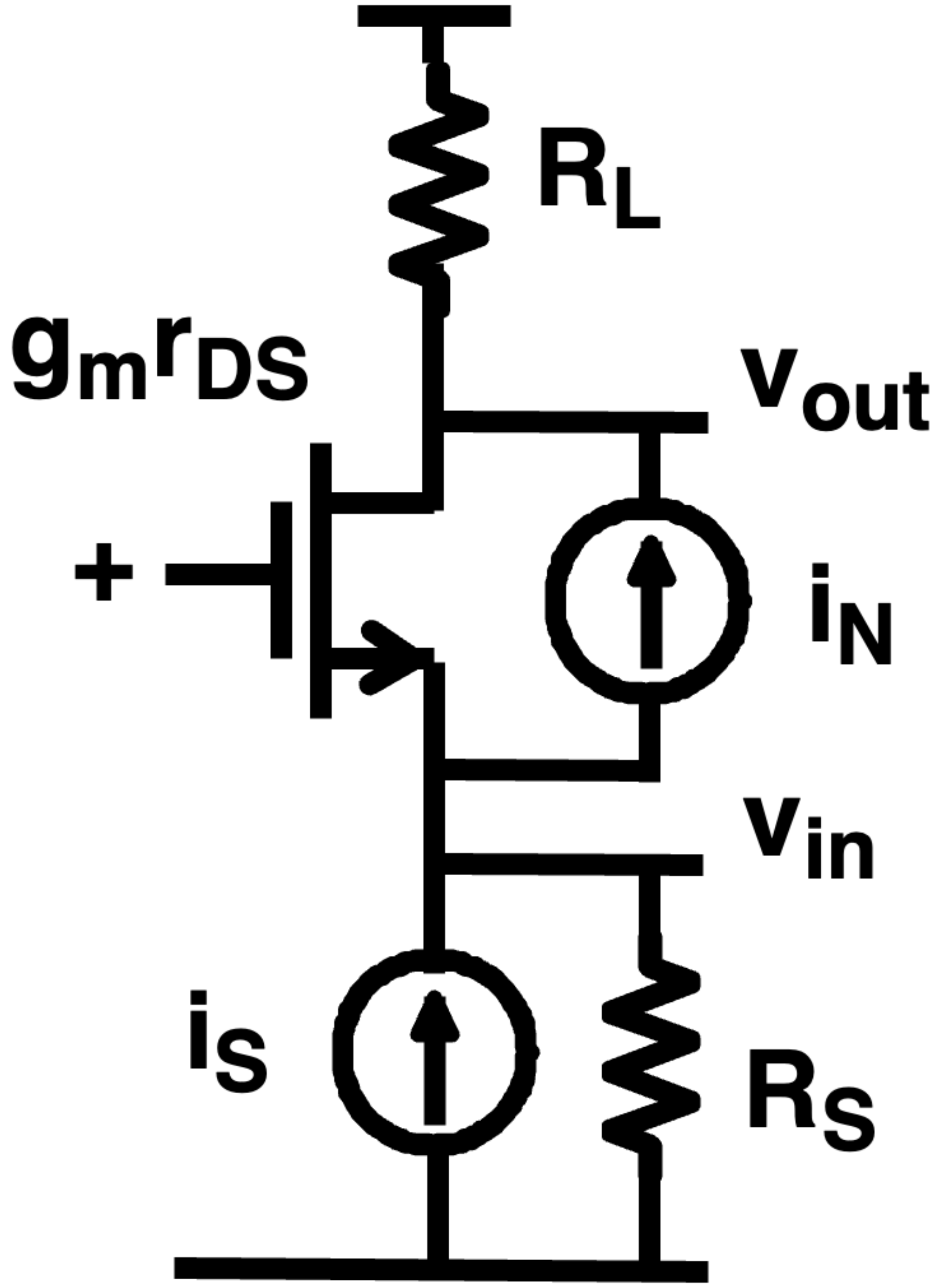
Cascode Input-Referred Noise

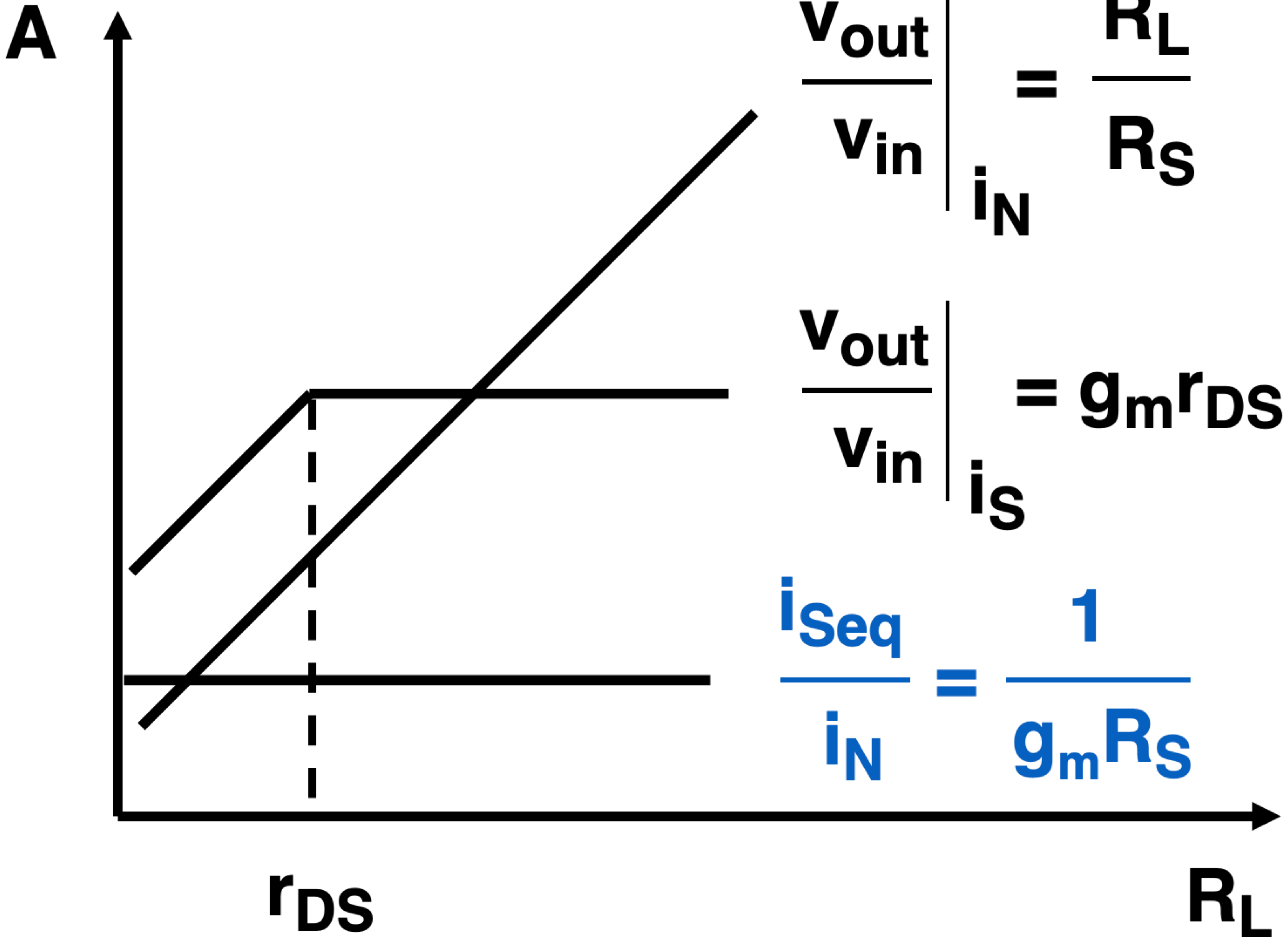


$g_m r_{DS} >> 1$



Cascode Noise Transfer Functions





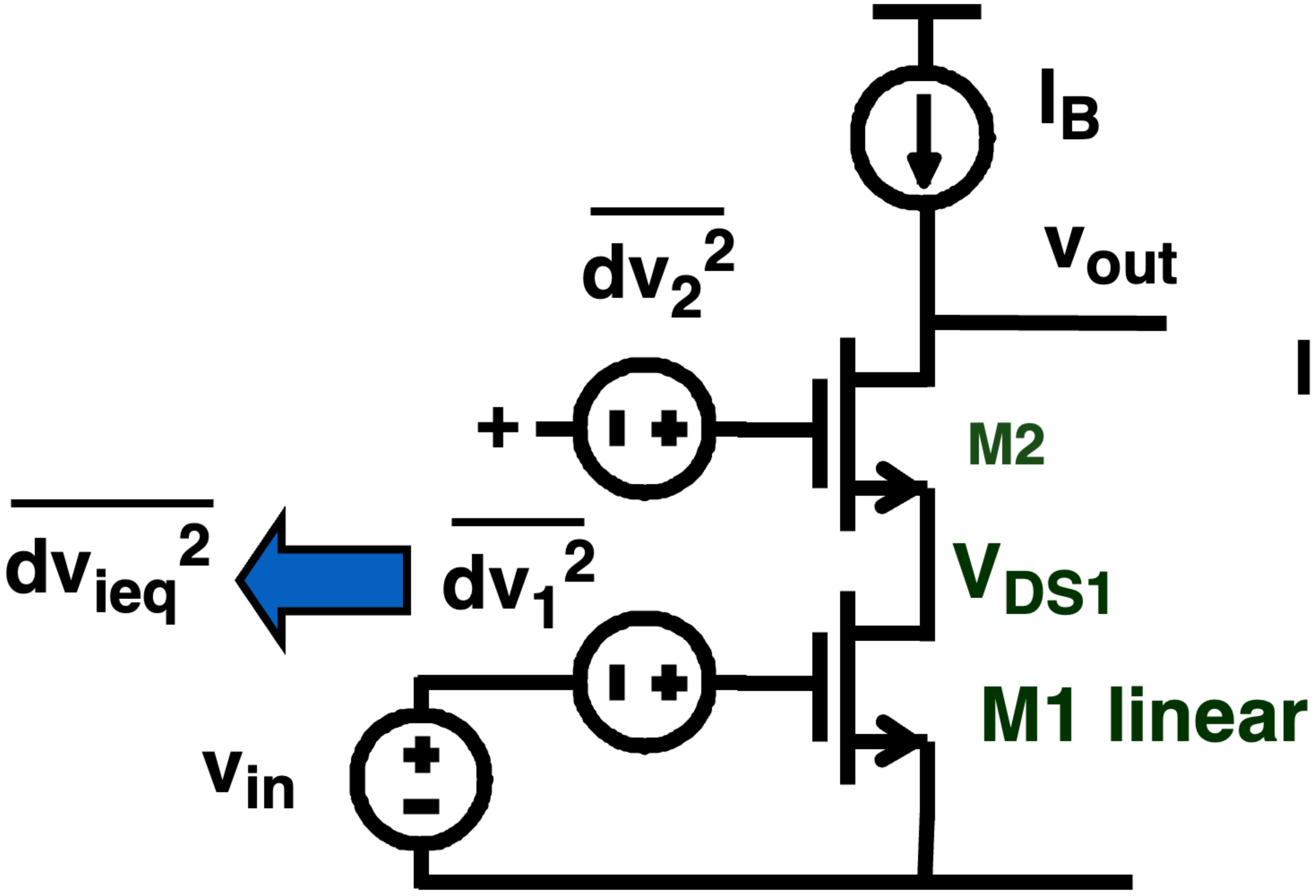
$$\left.\frac{v_{out}}{v_{in}}\right|_{i_N} = \frac{R_L}{R_S}$$

$$\left.\frac{v_{out}}{v_{in}}\right|_{i_S} = g_m r_{DS}$$

$$\frac{i_{Seq}}{i_N} = \frac{1}{g_m R_S}$$

Cascode noise i_N is only negligible if R_S is large!!!

Cascode With Linear M1



$$\alpha_1 = \frac{V_{DS1}}{V_{GS1} - V_T} \qquad \alpha_1 < 0.5$$

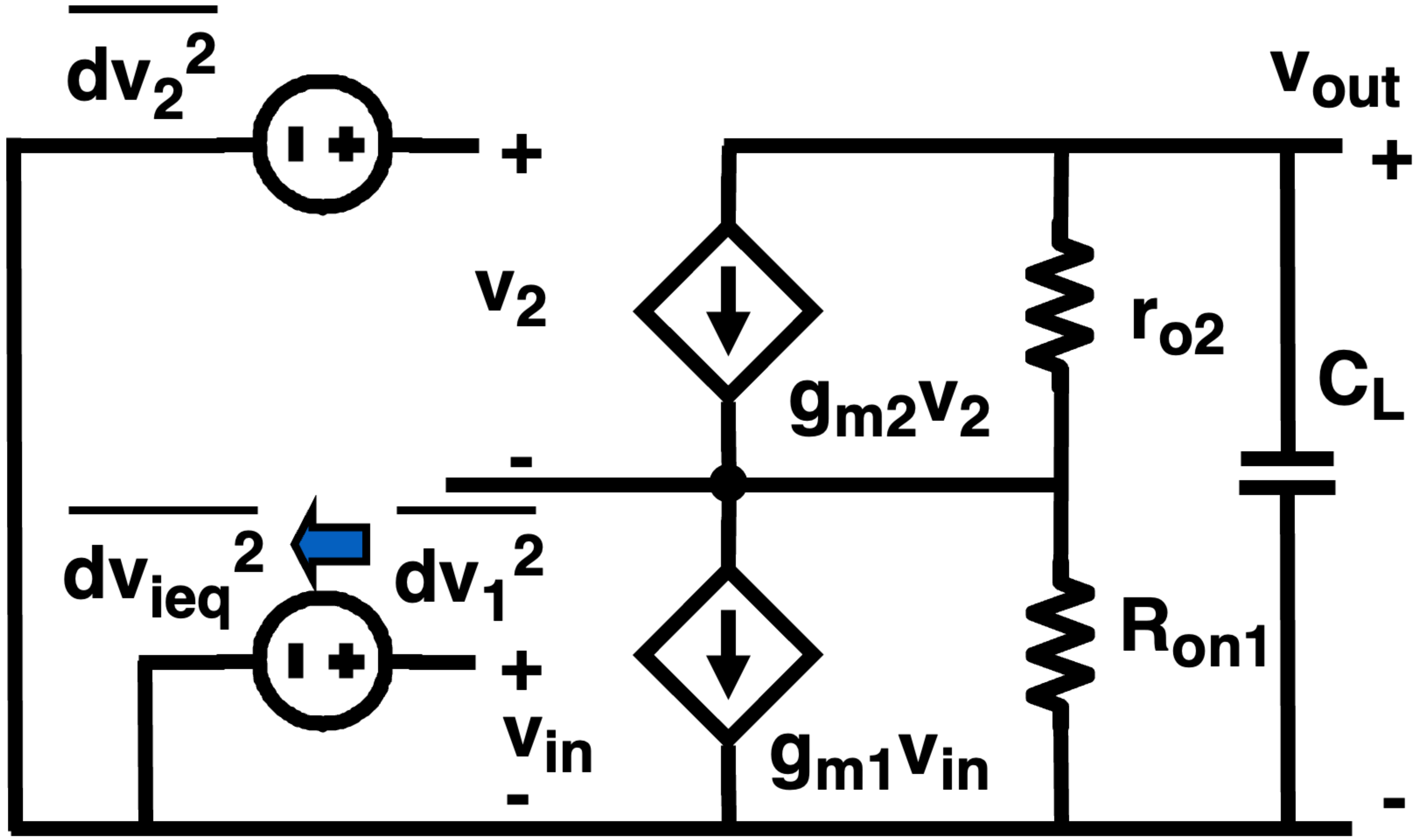
$$I_{DS1} = \beta_1 V_{DS1}(V_{GS1} - V_T)$$

$$R_{on1} = \frac{1}{\beta_1 (V_{GS1} - V_T)}$$

$$A_v = \alpha_1 g_{m2} r_{o2}$$

$$\overline{dv_{ieq}^2} = \frac{4kT}{\alpha_1^2}\left(R_{on1} + \frac{2/3}{g_{m2}}\right) df$$

Cascode With Linear M1 Noise



$$I_{DS1} = \beta_1 V_{DS1}(V_{GS1} - V_T)$$

$$R_{on1} = \frac{1}{\beta_1 (V_{GS1} - V_T)}$$

$$\overline{dv_1^2} = \frac{4kT\, R_{on1}}{\alpha_1^2}$$

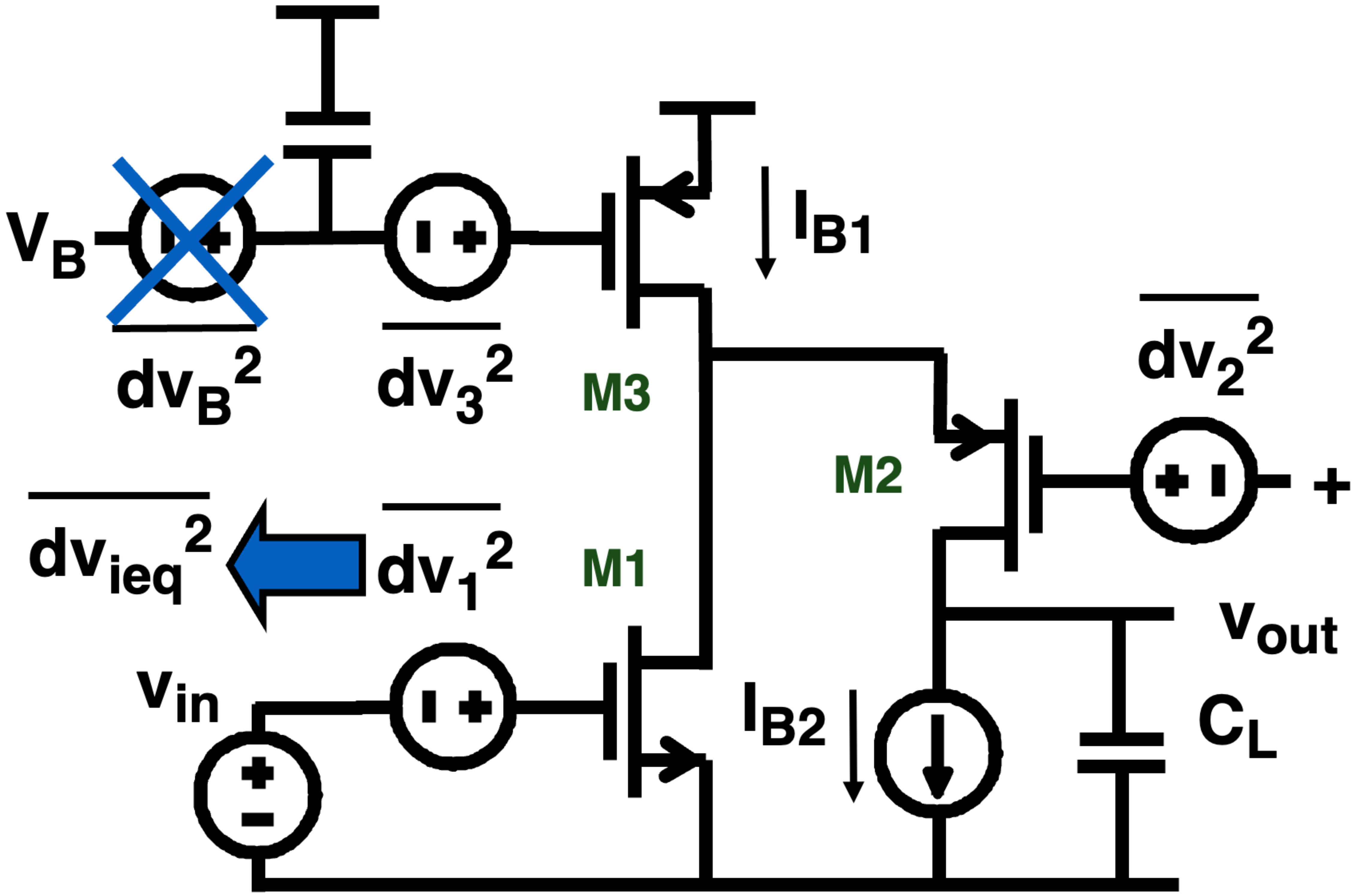
$$\overline{dv_2^2} = \frac{4kT\, 2/3}{g_{m2}}$$

$$v_{out} / dv_1 = \alpha_1\, g_{m2}\, r_{o2}$$

$$v_{out} / dv_2 = g_{m2}\, r_{o2}$$

$$\overline{dv_{ieq}^2} = \frac{4kT}{\alpha_1^2} \left(R_{on1} + \frac{2/3}{g_{m2}}\right) df$$

Folded Cascode Noise



If $\overline{dv_B^2}$ is negligible:

$$\overline{dv_{ieq}^2} = \overline{dv_1^2} + \overline{dv_2^2}\,\frac{1}{(g_{m1}\,r_{o1})^2} + \overline{dv_3^2}\,\frac{(g_{m3})^2}{(g_{m1})^2}$$

Small g_{m3}:

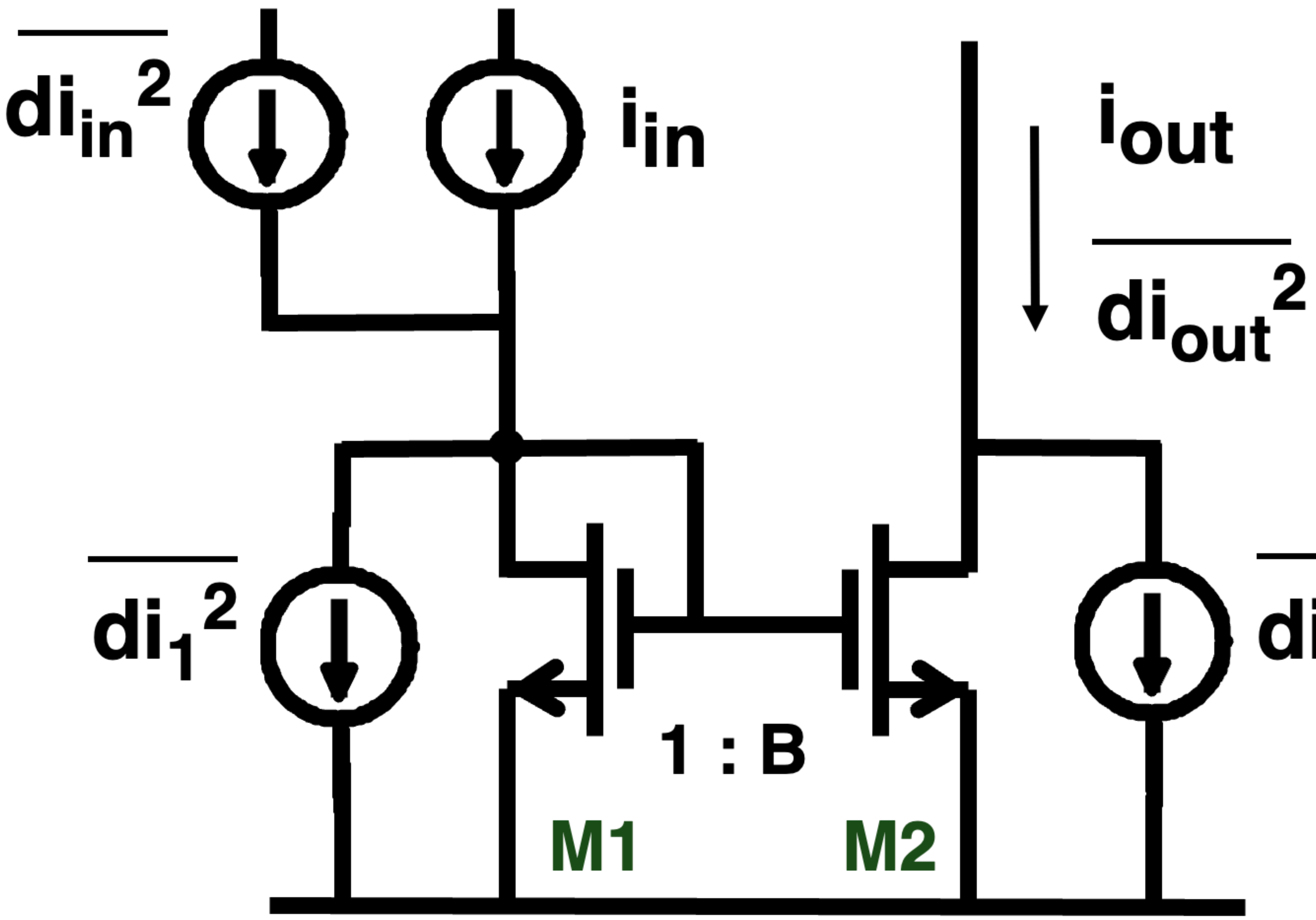
$(W/L)_3 \downarrow$

$(V_{GS}\text{-}V_T)_3 \uparrow$

Outline

- Definitions of noise
- Noise of an amplifier
- Noise of a follower
- Noise of a cascode
- **Noise of a current mirror**
- Noise of a differential pair
- Capacitive noise matching

Current Mirror Noise



$$\overline{di_2^2} = \frac{2}{3} 4kT\, g_{m2}\, df$$

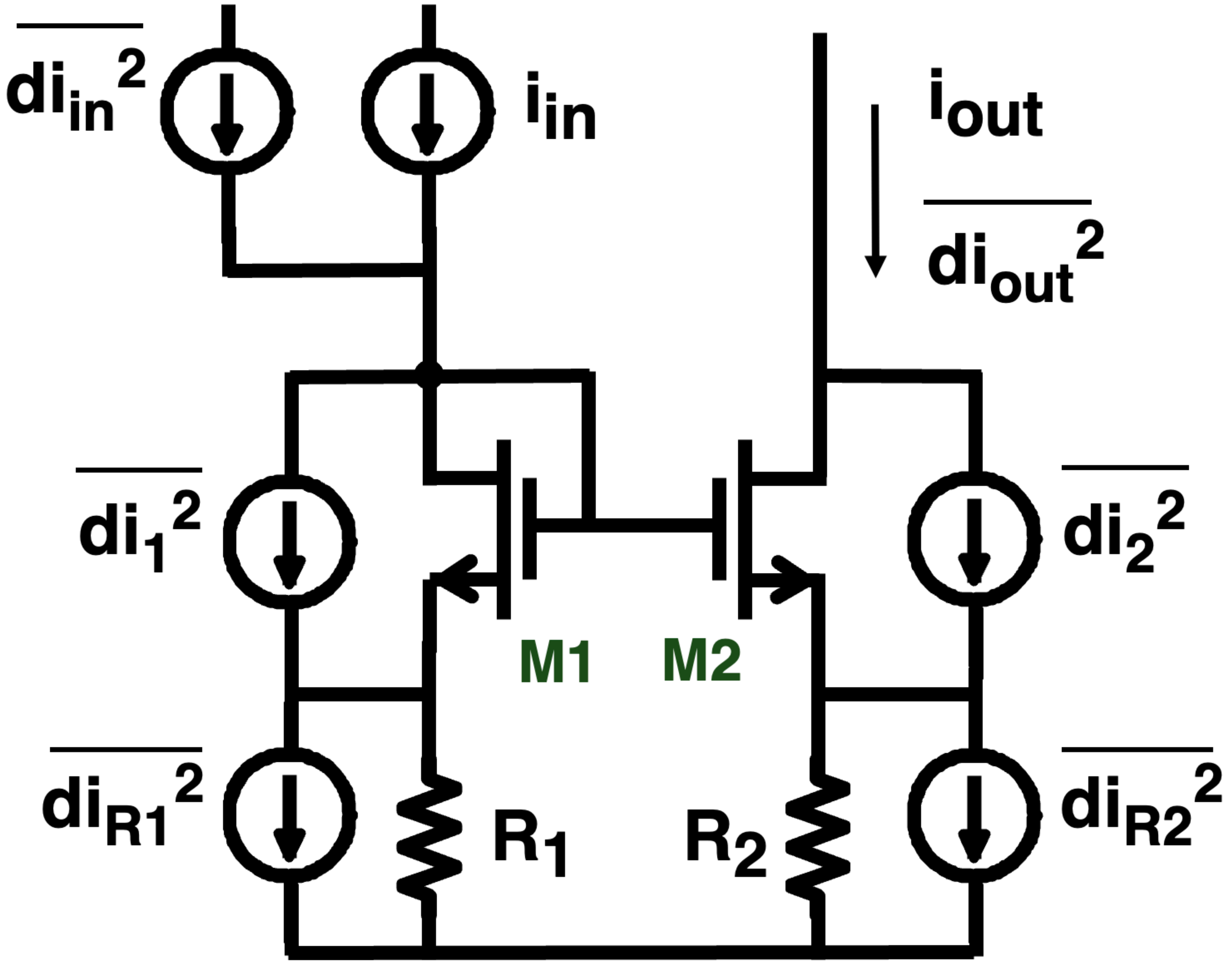
$$\overline{di_{out}^2} = \overline{di_2^2} + B^2 \left(\overline{di_{in}^2} + \overline{di_1^2}\right)$$

Small g_m :

$(W/L) \downarrow$

$(V_{GS}-V_T) \uparrow$

Source-Degenerated Current Mirror Noise



If $\overline{di_{in}^2}$ can be neglected:

$$\overline{di_{out}^2} = \frac{\overline{di_2^2}}{(1 + g_{m2}R_2)^2} + \overline{di_{R2}^2} + \frac{(R_1)^2}{(R_2)^2}\left(\frac{\overline{di_1^2}}{(1 + g_{m1}R_1)^2} + \overline{di_{R1}^2}\right)$$

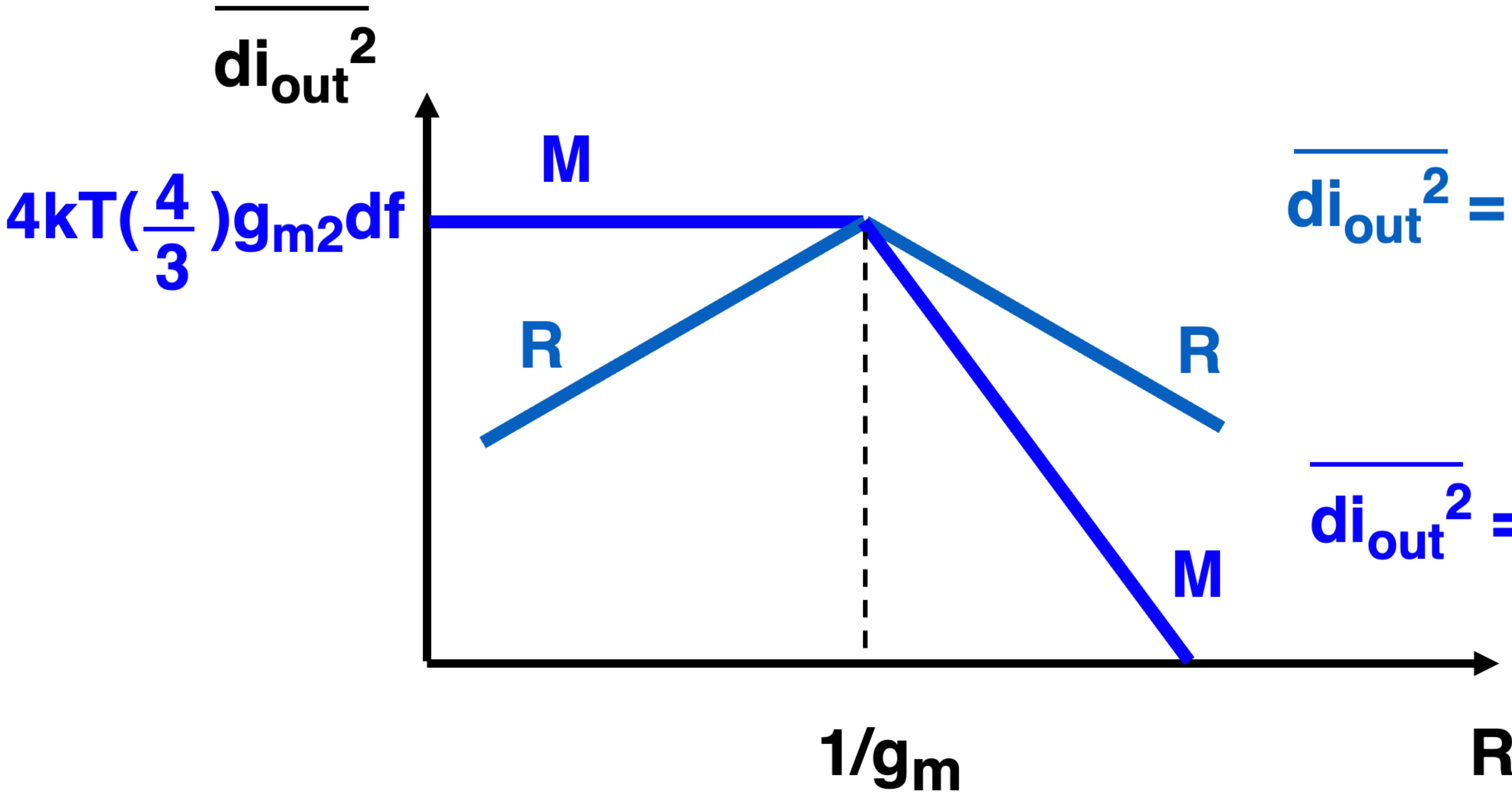
Small g_m :

$(W/L) \downarrow$

$(V_{GS}-V_T) \uparrow$

$R \uparrow$

Source-Degenerated Current Mirror Noise

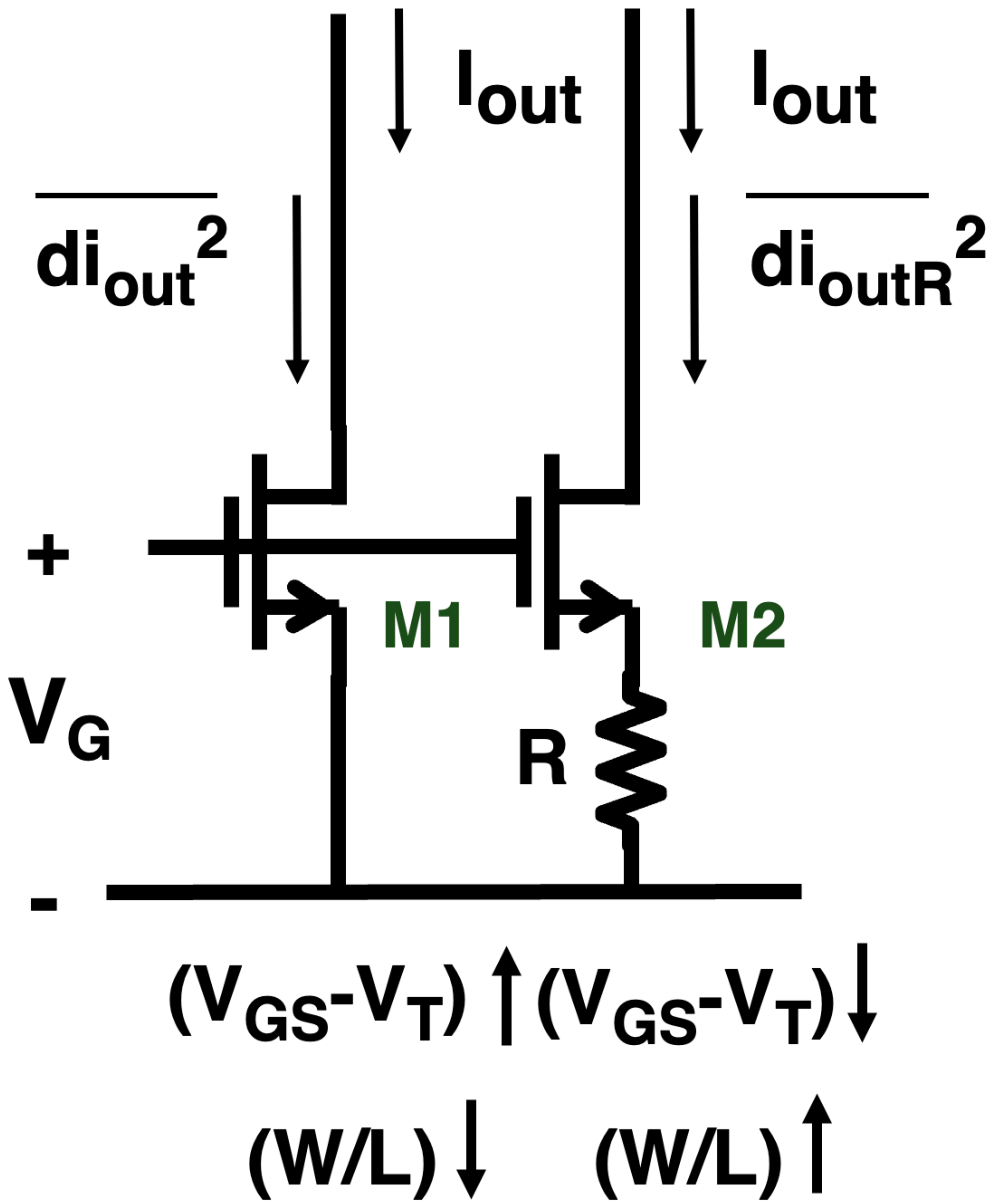


$$\overline{di_{out}^2} = \frac{8kT}{R_2} df$$

$$\overline{di_{out}^2} = \frac{\frac{4}{3} 4kTg_{m2}}{(g_{m2}R_2)^2} df$$

Bilotti, JSSC Dec 75, 516-524

Current Mirror Comparison



Same I_{out} & same V_G :

$$\boxed{\overline{di_{outR}}^2 = \overline{di_{out}}^2}$$

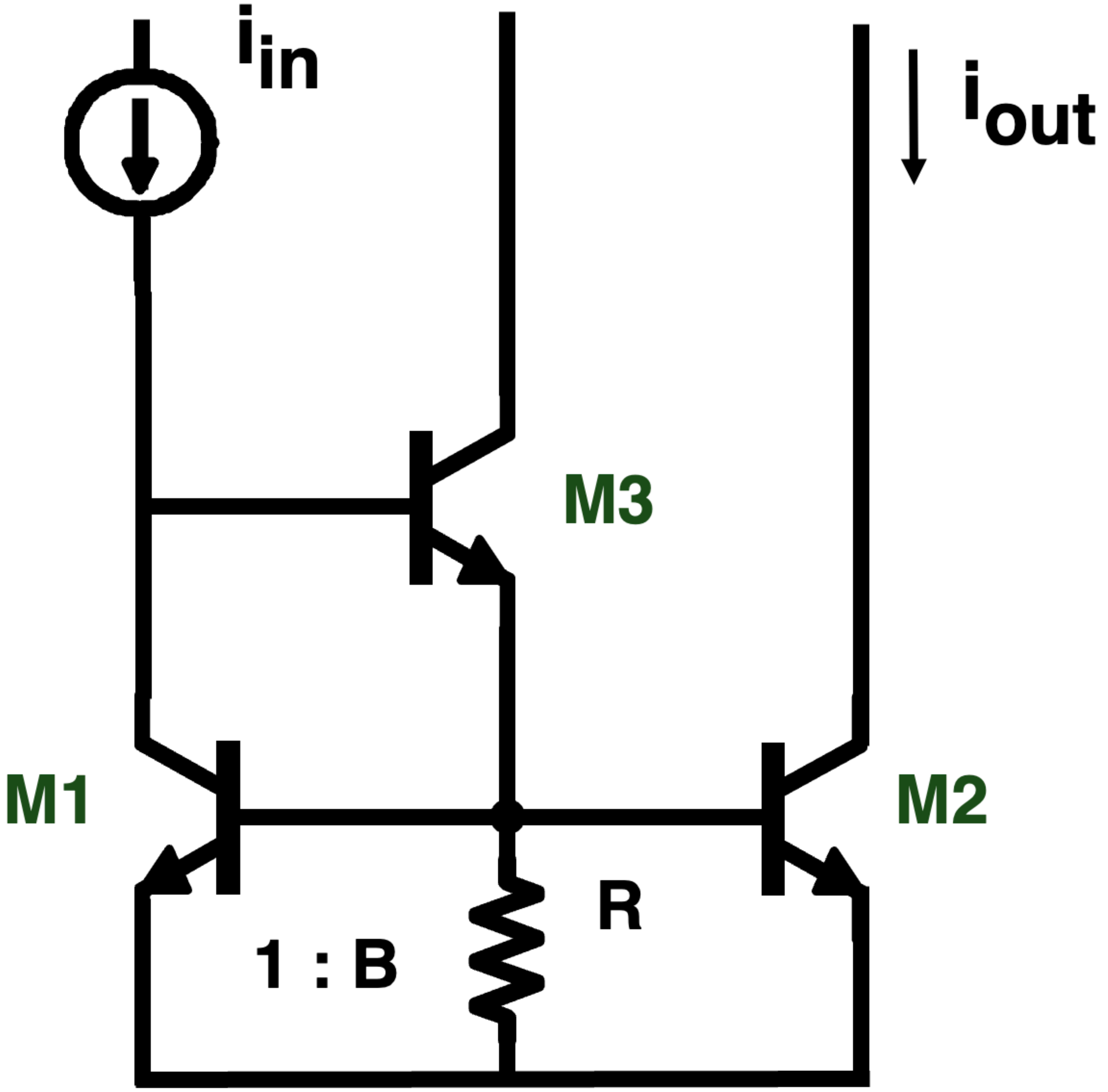
Small g_m :

$(W/L)\downarrow$

$(V_{GS}-V_T)\uparrow$

$V_G\uparrow$

BJT Current Mirror Noise



Noise added by M3 :

$$\overline{di_{M3}^2} = 2qI_{C3}\, df$$

Noise added by R :

$$\overline{di_R^2} = 4kT/R \;\; df$$

Both are divided by β_3^2 to be added to the output and are thus negligible!

Outline

- Definitions of noise
- Noise of an amplifier
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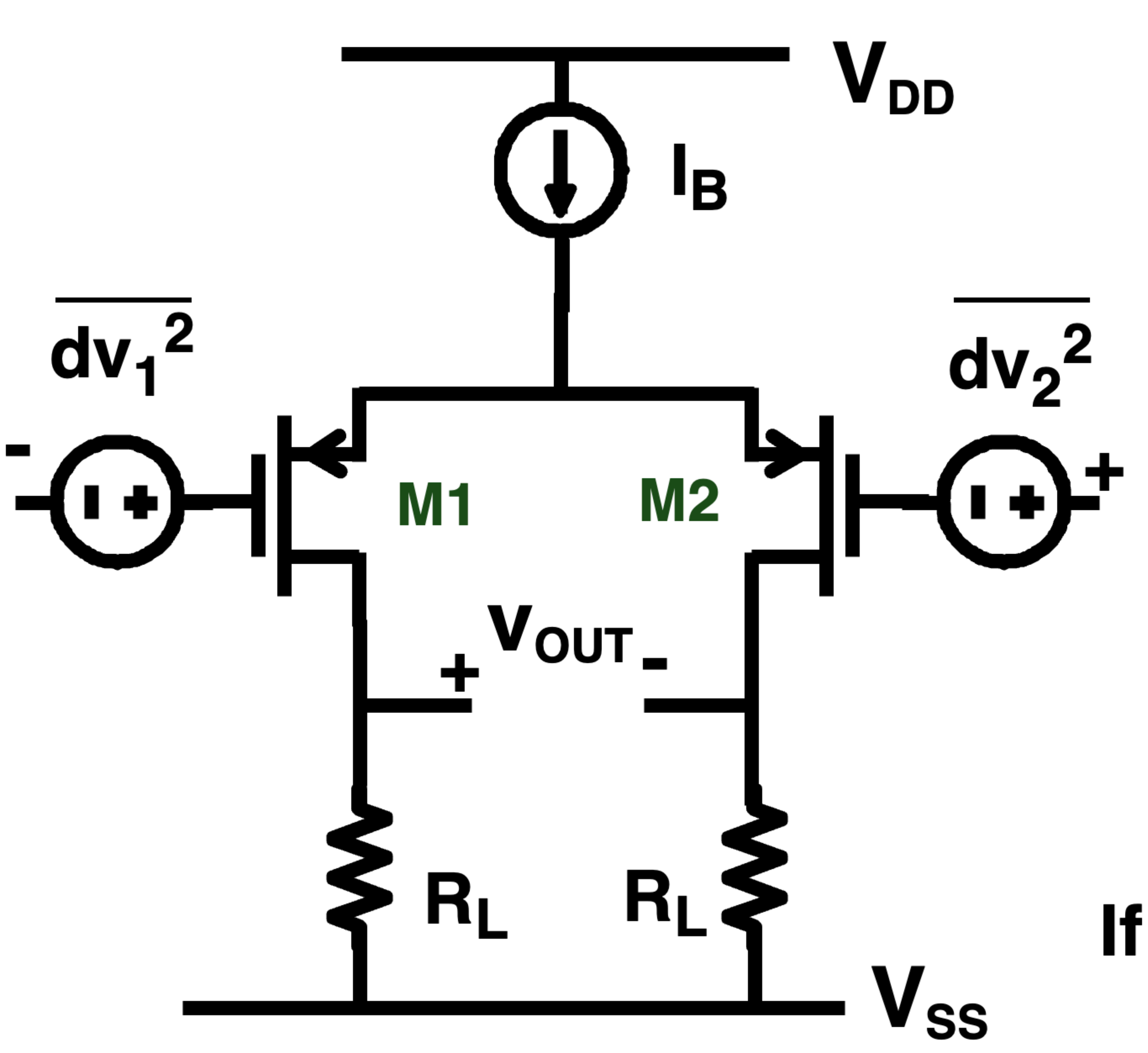
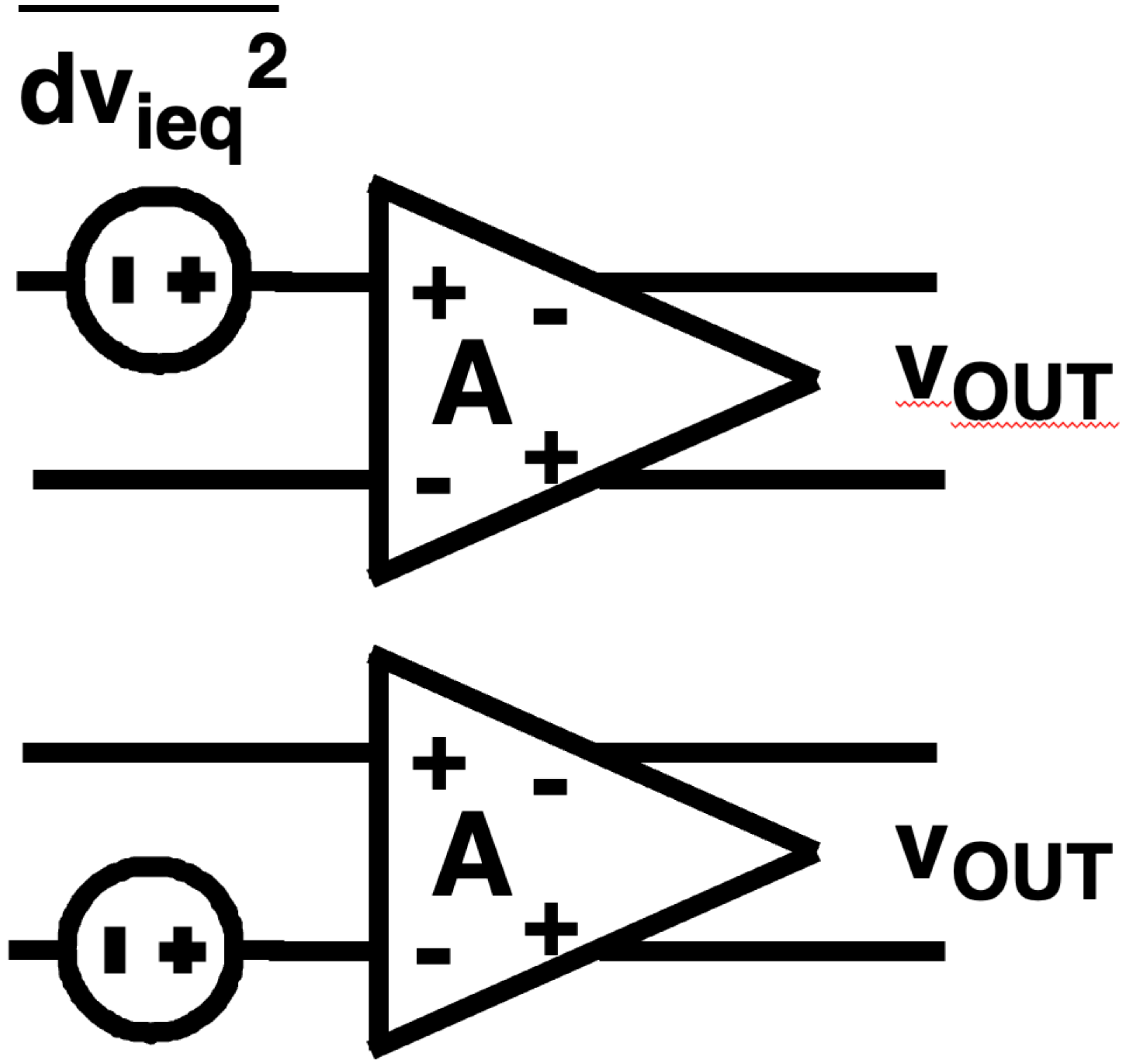
- Noise of a current mirror
- **Noise of a differential pair**
- Capacitive noise matching

Differential Pair Noise

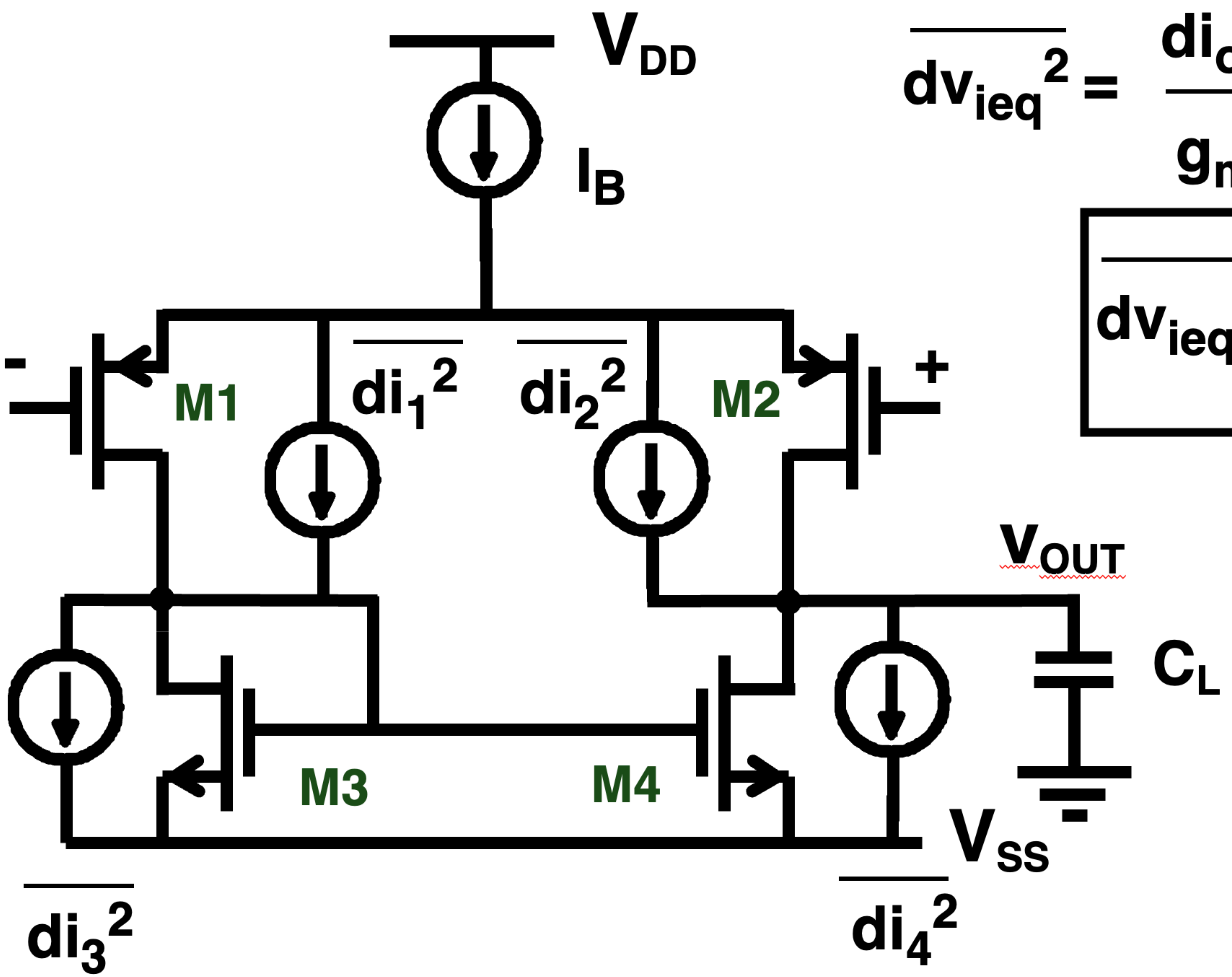




If R_L is considered noiseless:

$$\overline{dv_{ieq}^2} = 2\,\overline{dv_1^2}$$

Differential Amplifier Noise



$$\overline{dv_{ieq}^2} = \frac{\overline{di_{out}^2}}{g_{m1}^2} = \frac{2\,\overline{di_1^2} + 2\,\overline{di_3^2}}{g_{m1}^2}$$

$$\boxed{\overline{dv_{ieq}^2} = 2\,\overline{dv_1^2}\left(1 + \frac{g_{m3}}{g_{m1}}\right)}$$

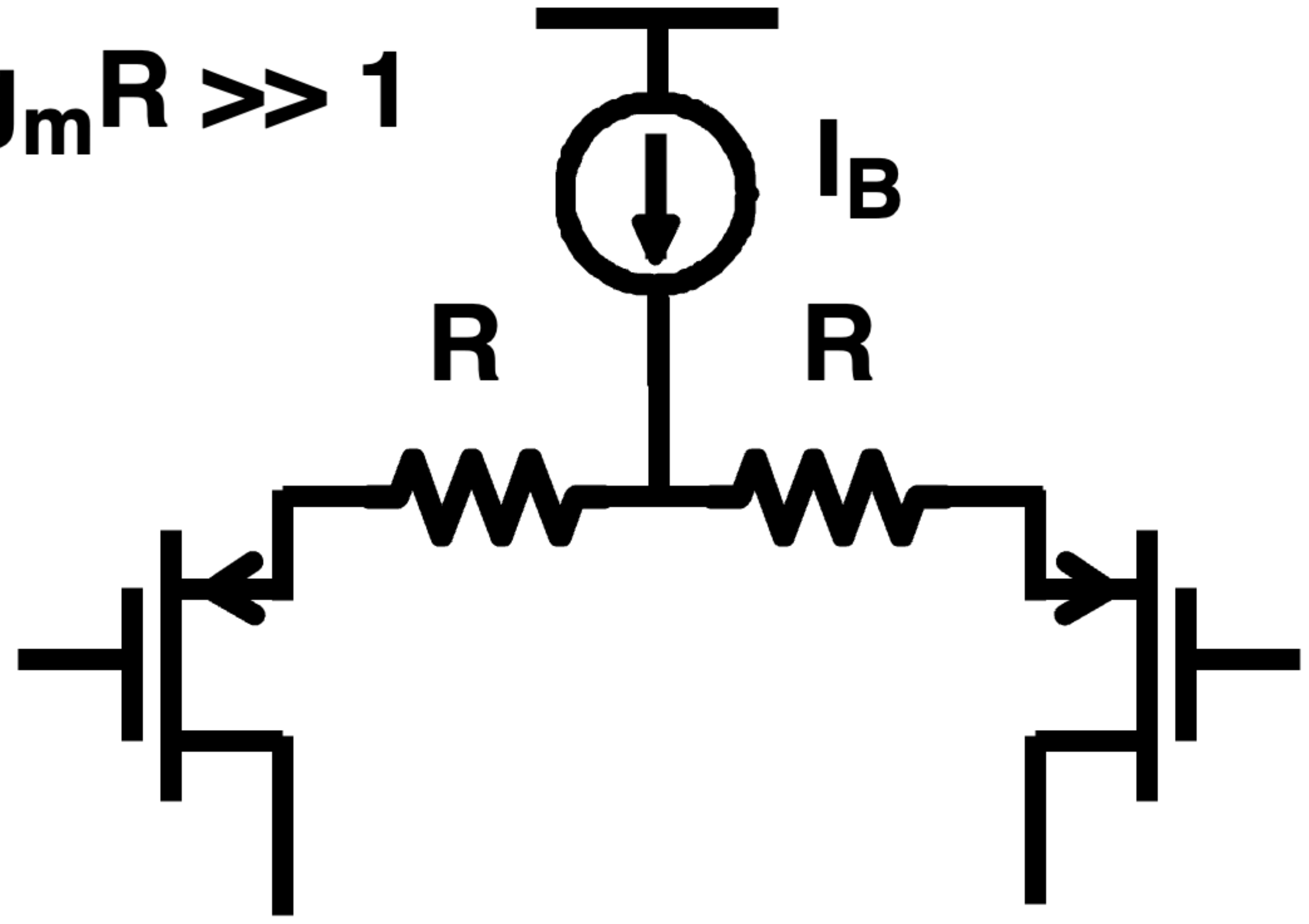
Small g_{m3} :

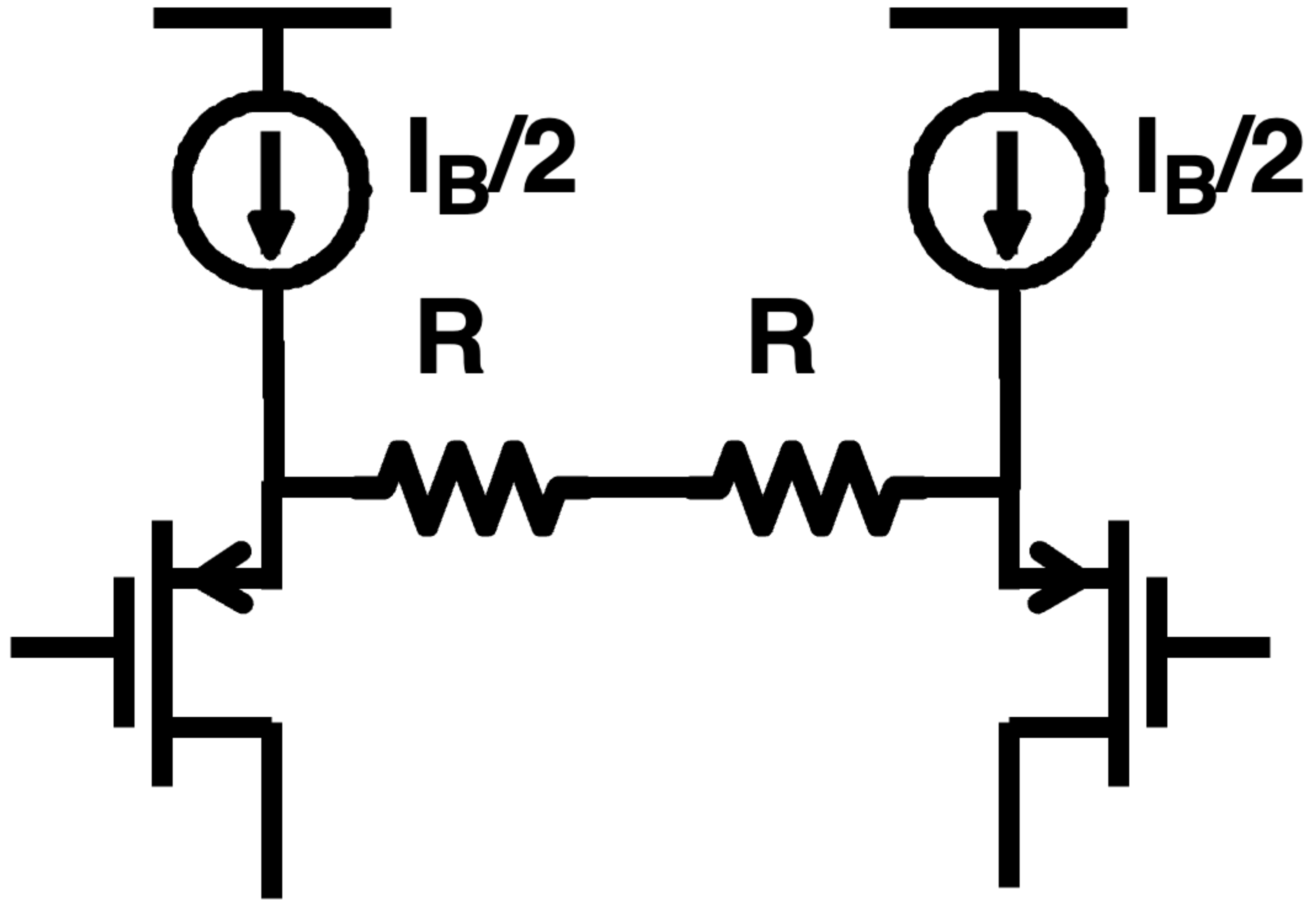
$(W/L)_3 \downarrow$

$(V_{GS}-V_T)_3 \uparrow$

Source-Degenerated Differential Pair Noise

$g_m R >> 1$





$$\overline{di_{out}^2} = 2 \frac{4kT}{R} df$$

$$\overline{dv_{in}^2} = 2\,(4kT\,R\,df)$$

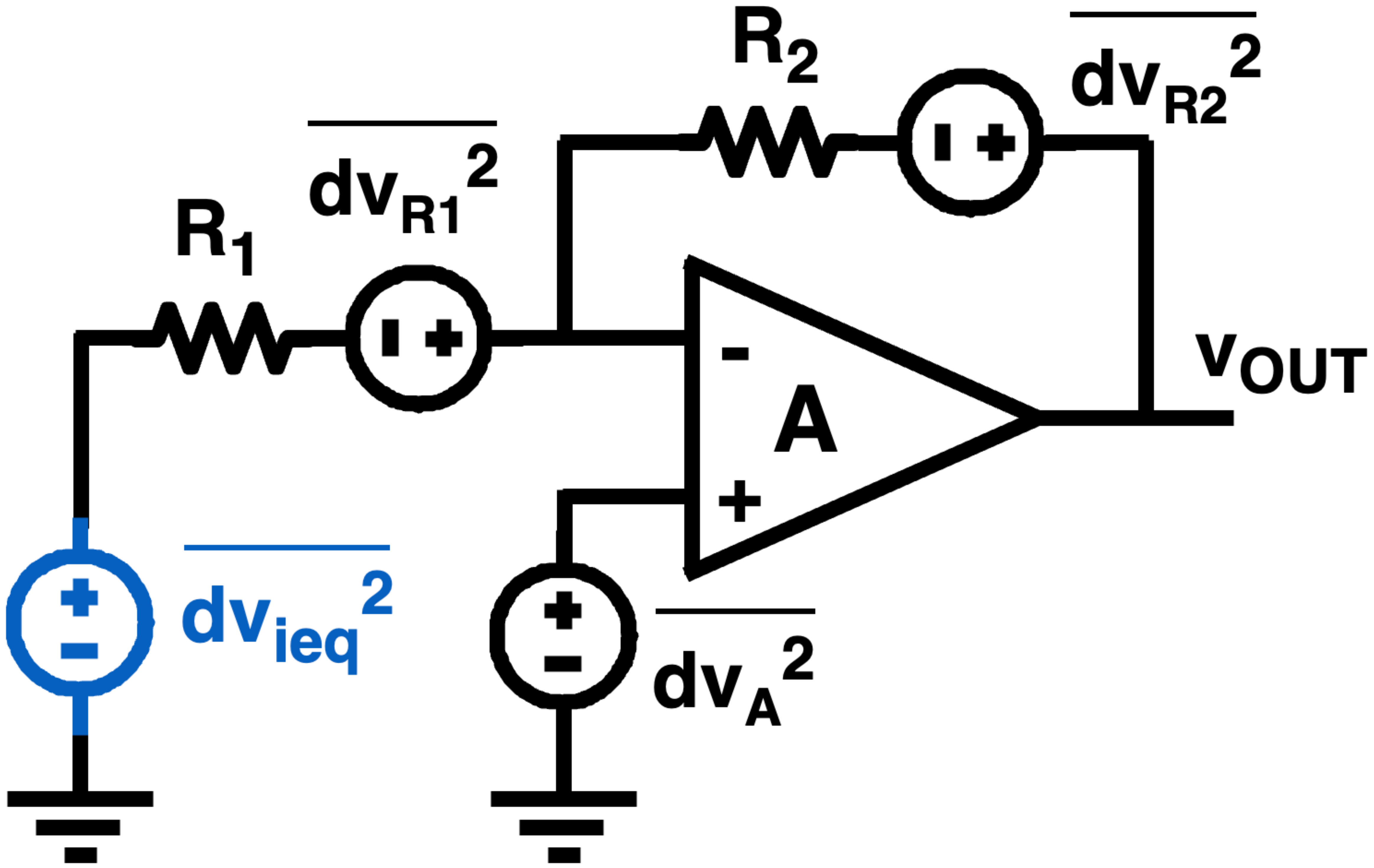
$\overline{di_B^2}$ is negligible

$$\overline{di_{out}^2} = 2 \left(\frac{4kT}{R} df + \overline{di_B^2}\right)$$

$$\overline{di_B^2} = 4kT\,2/3\,g_{mB}\,df$$

$$\overline{dv_{in}^2} = 2\,(4kT\,R\,df)\,(1 + 2/3\,g_{mB}R)$$

Opamp With Feedback Noise



$$\overline{dv_{ieq}^2} = \Sigma\, \overline{dv_{out}^2} \left(\frac{R_1}{R_2}\right)^2$$

$$\overline{dv_{out}^2} = \overline{dv_{R1}^2} \left(\frac{R_2}{R_1}\right)^2$$

$$\overline{dv_{out}^2} = \overline{dv_{R2}^2}$$

$$\overline{dv_{out}^2} = \overline{dv_A^2} \left(1 + \frac{R_2}{R_1}\right)^2$$

$$\overline{dv_{ieq}^2} = \overline{dv_{R1}^2} + \overline{dv_{R2}^2} \left(\frac{R_1}{R_2}\right)^2 + \overline{dv_A^2} \left(1 + \frac{R_1}{R_2}\right)^2 \approx \overline{dv_{R1}^2} + \overline{dv_A^2}$$

Outline

- Definitions of noise
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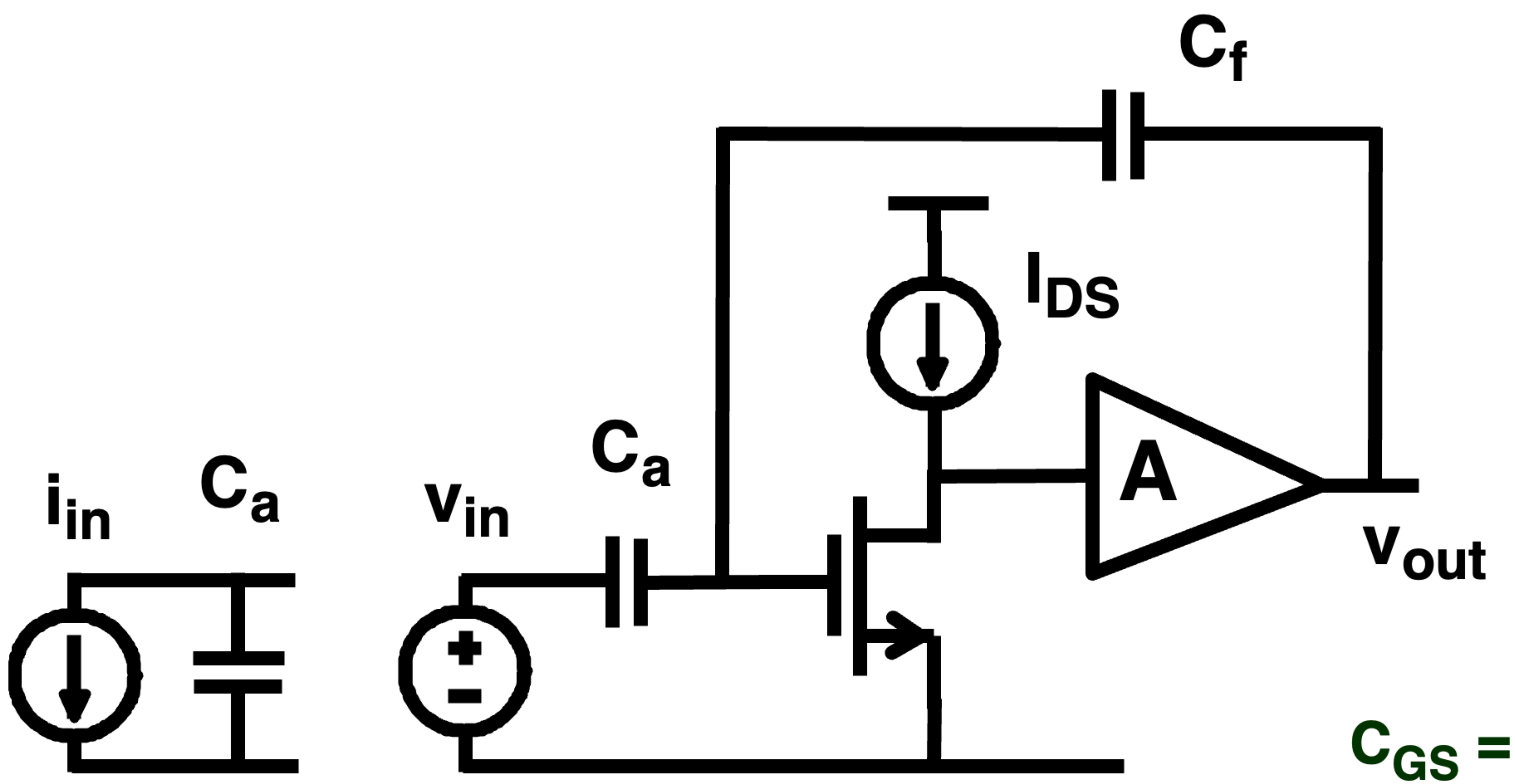
- **Capacitive noise matching**

Amplifier With Capacitive Source



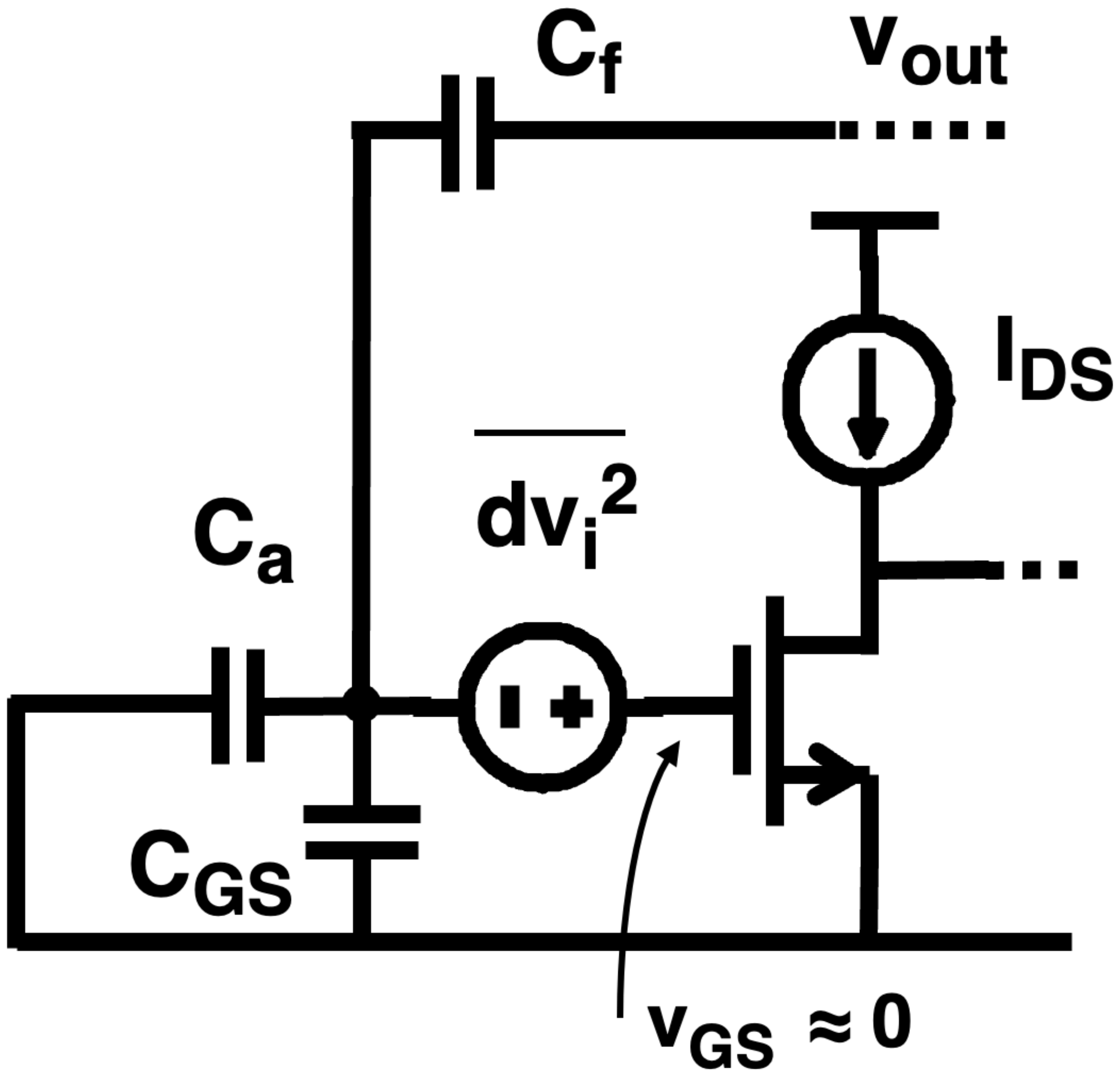
$$A_v = \frac{C_a}{C_f}$$

$C_a = 5$ pF
$C_f = 1$ pF

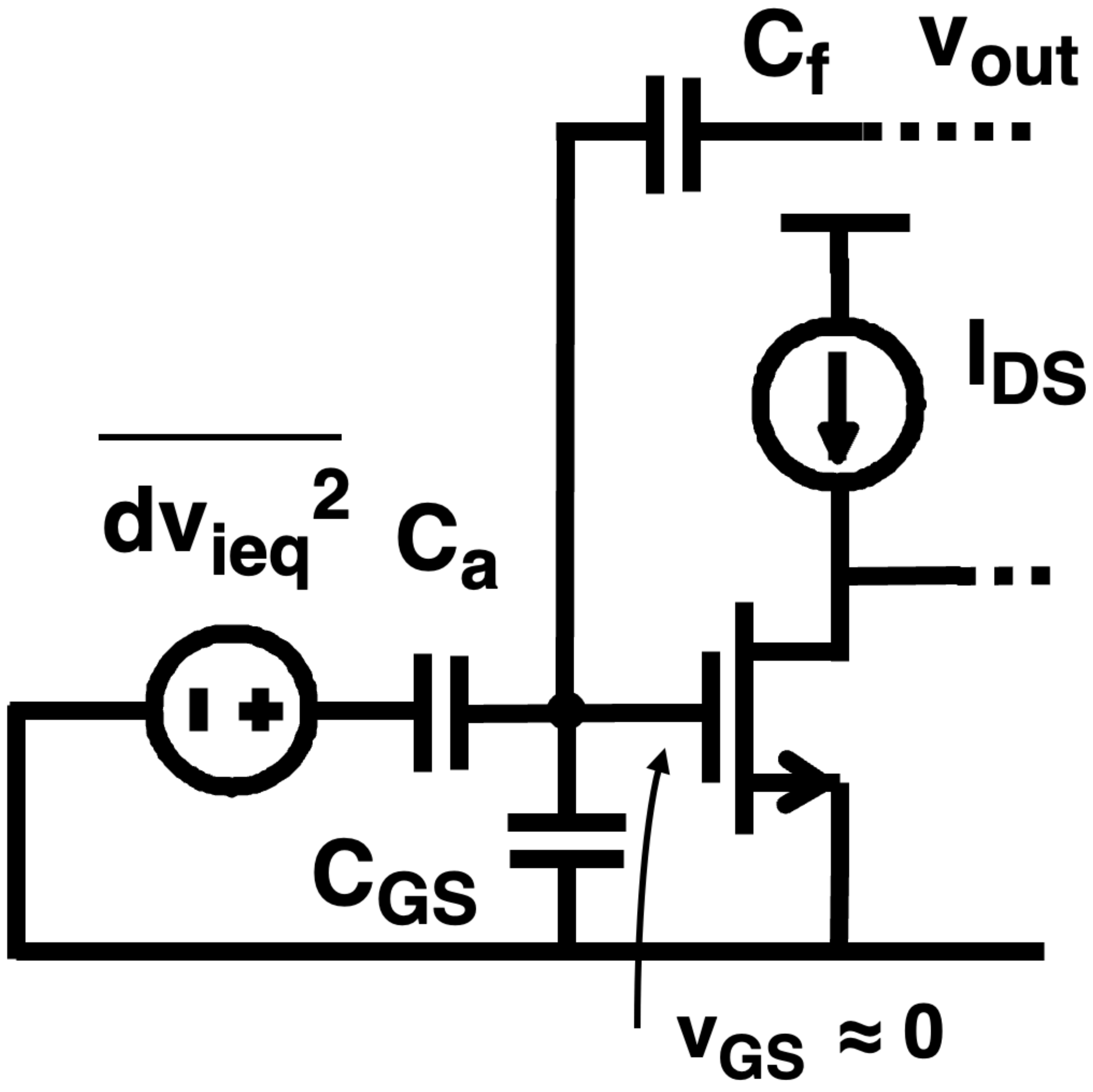
$C_{GS} = kW \quad k \approx 2$ fF/μm

W_{opt}? I_{DSopt}? S/N_{opt} for $V_{in} = 10\ mV_{RMS}$?

Calculating Equivalent Input Noise



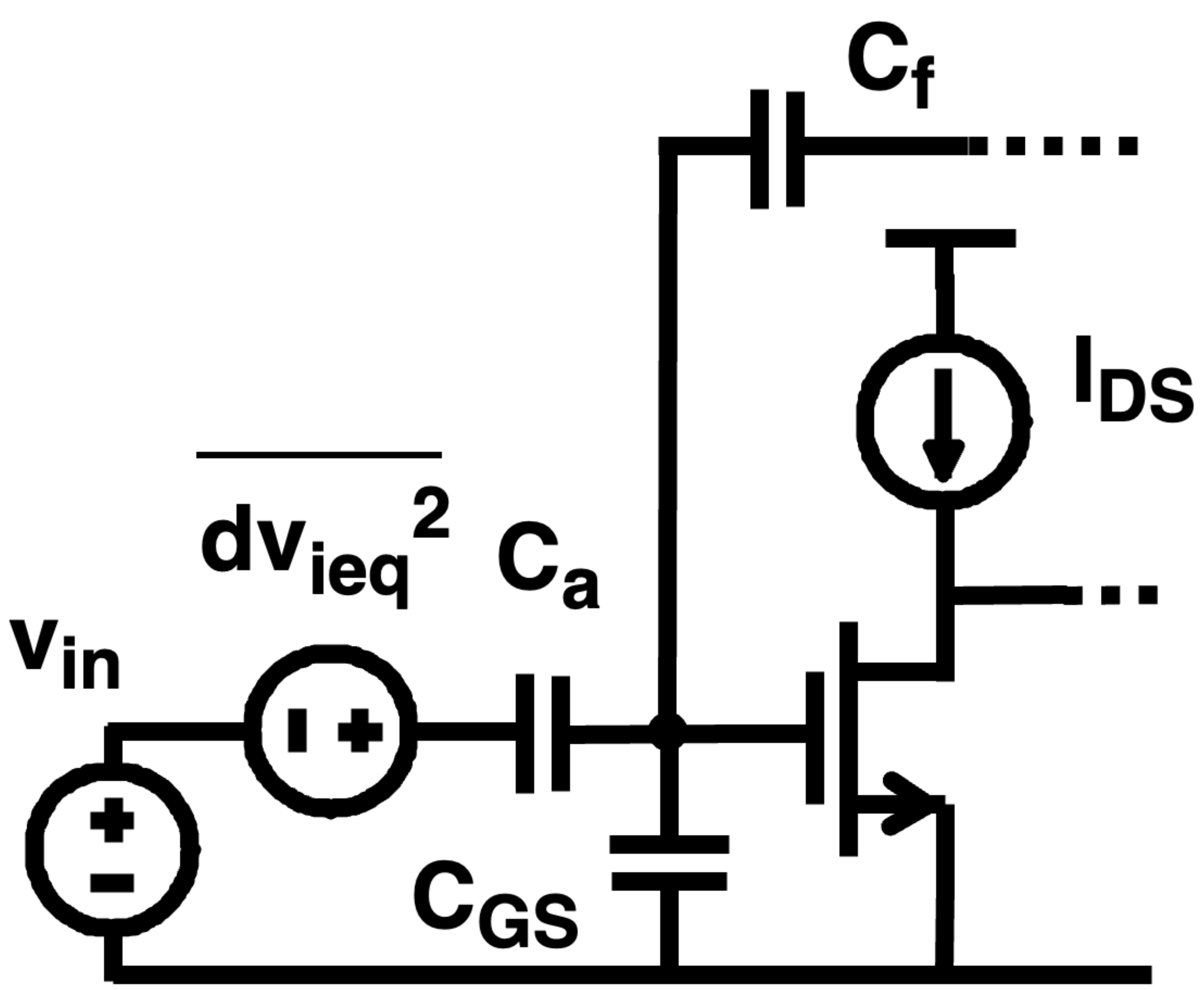
$$\frac{v_{out}}{v_i} = \frac{C_f + C_a + C_{GS}}{C_f}$$



$$\frac{v_{out}}{v_{ieq}} = \frac{C_a}{C_f}$$

No Miller with C_{DG} !!!

Calculating Equivalent Input Noise



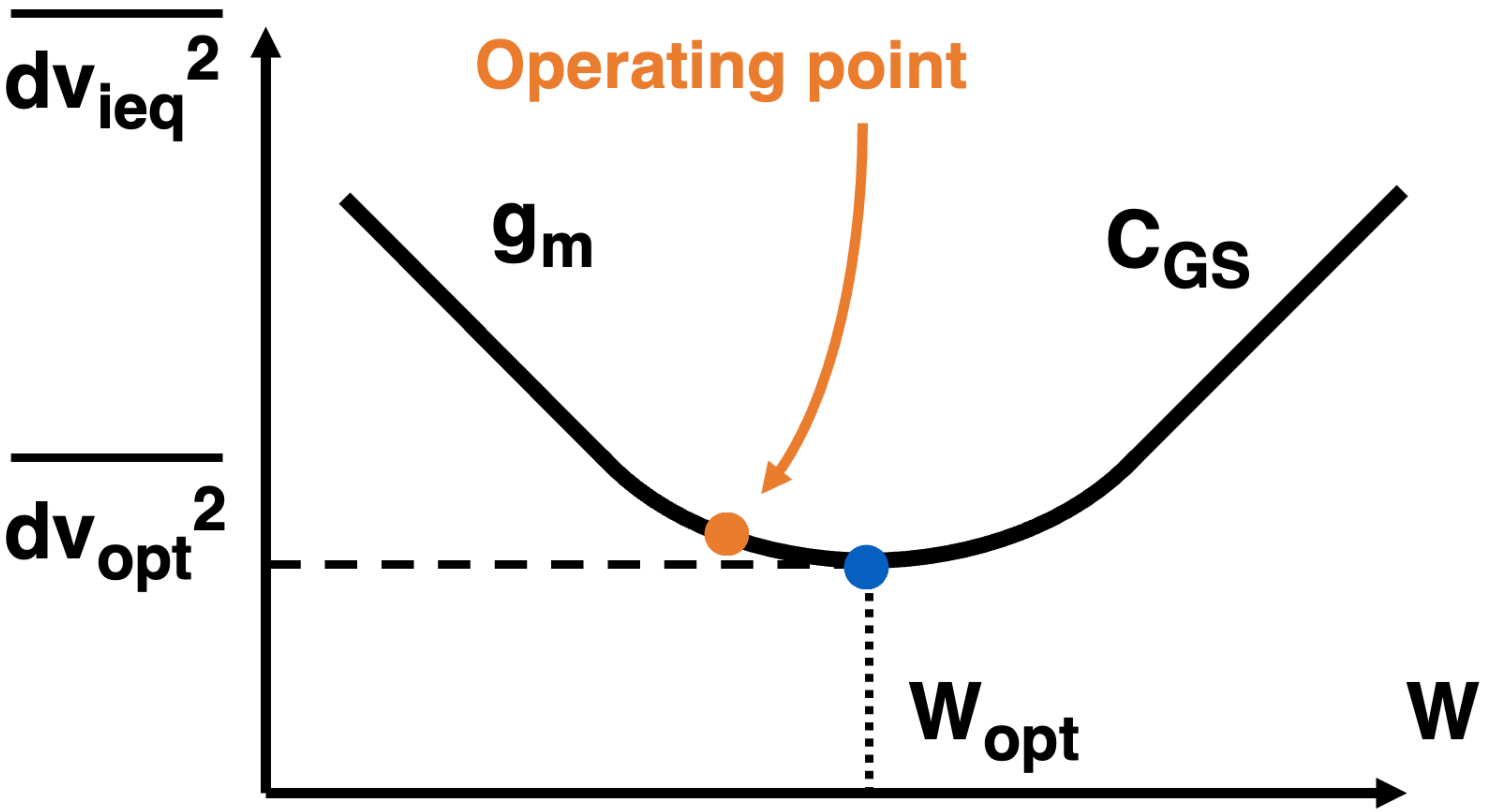
$$\overline{dv_{ieq}^2} = \frac{(C_f + C_a + C_{GS})^2}{C_a^2}\,\overline{dv_i^2}$$

$$\overline{dv_i^2} = \frac{8kT}{3}\frac{1}{g_m}\,df$$

$$g_m = 2\,K'_n\,\frac{W}{L}\,(V_{GS} - V_T)$$

$$\overline{dv_{ieq}^2} = \frac{(C_f + C_a + kW)^2}{C_a^2}\,\frac{L}{W}\,\frac{8kT}{3}\,\frac{1}{2\,K'_n\,(V_{GS} - V_T)}$$

Capacitive Noise Matching



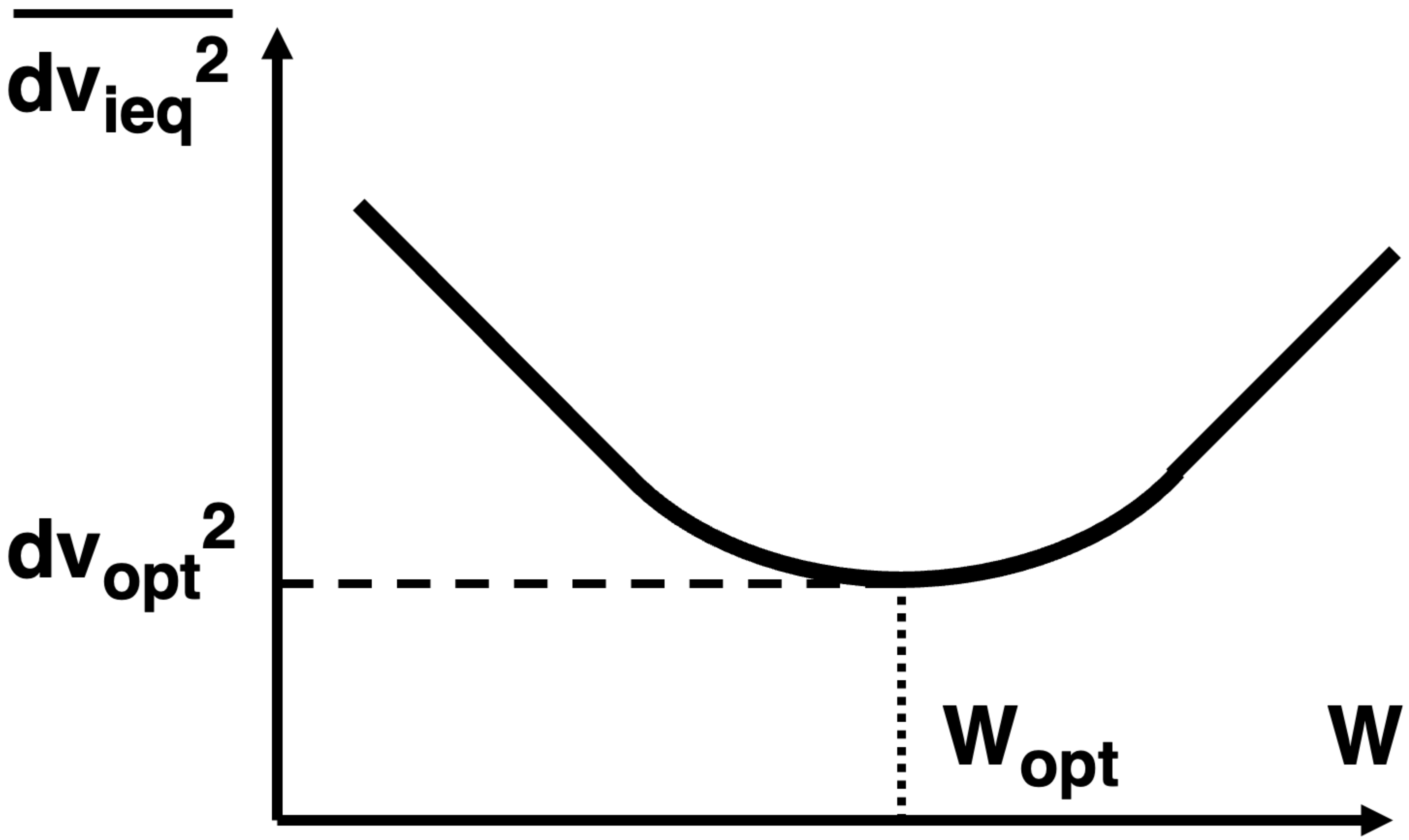
Noise matching where

$C_{GS} = C_f + C_a$

$$\boxed{W_{opt} = \frac{C_f + C_a}{k}}$$

$$\overline{dv_{ieq}^2} = \frac{(C_f + C_a + kW)^2}{C_a^2} \frac{L}{W} \frac{8kT}{3} \frac{1}{2K'_n(V_{GS} - V_T)}$$

SNR With Capacitive Noise Matching



$$W_{opt} = \frac{C_f + C_a}{k}$$

C_{GSopt}

I_{DSopt} , g_{mopt}

$$\overline{dv_{opt}^2} = 4\,\frac{8kT}{3}\,\frac{df}{g_{mopt}}$$

$$BW_n = \frac{\pi}{2} BW = \frac{\pi}{2}\frac{f_T}{A_v} = \frac{1}{4A_v}\frac{g_{mopt}}{C_{GSopt}}$$

$$\frac{S}{N_{opt}} = \frac{10\ mV_{RMS}}{\sqrt{\overline{dv_{opt}^2}\,BW_n}}$$

Outline

- Definitions of noise
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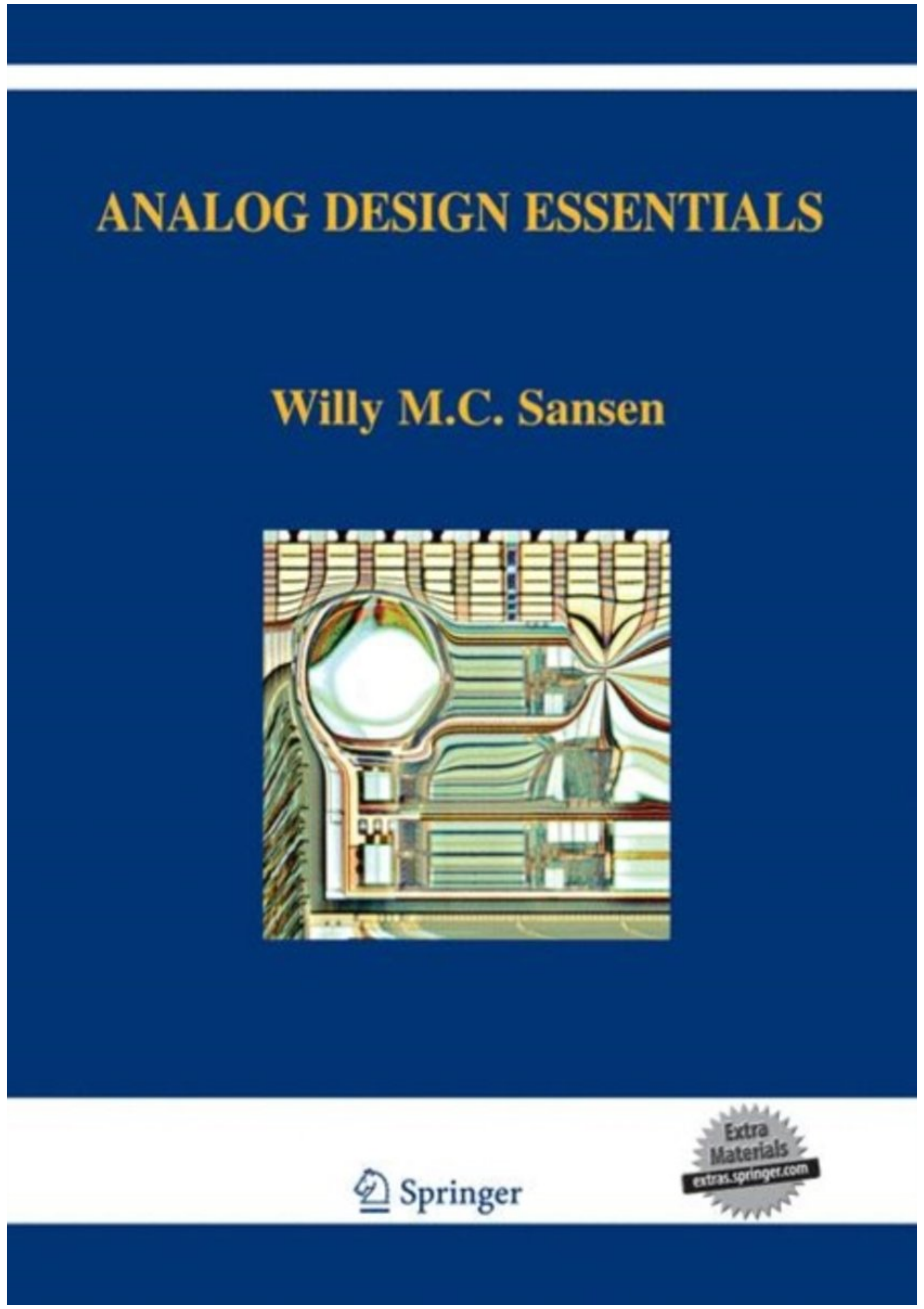
KU LEUVEN

Distortion in elementary transistor circuits

prof. dr. ir. Filip Tavernier

Outline

- **Definitions and metrics of distortion**
- Distortion in components
- Distortion reduction
- Distortion in OPAMPs
- Summary



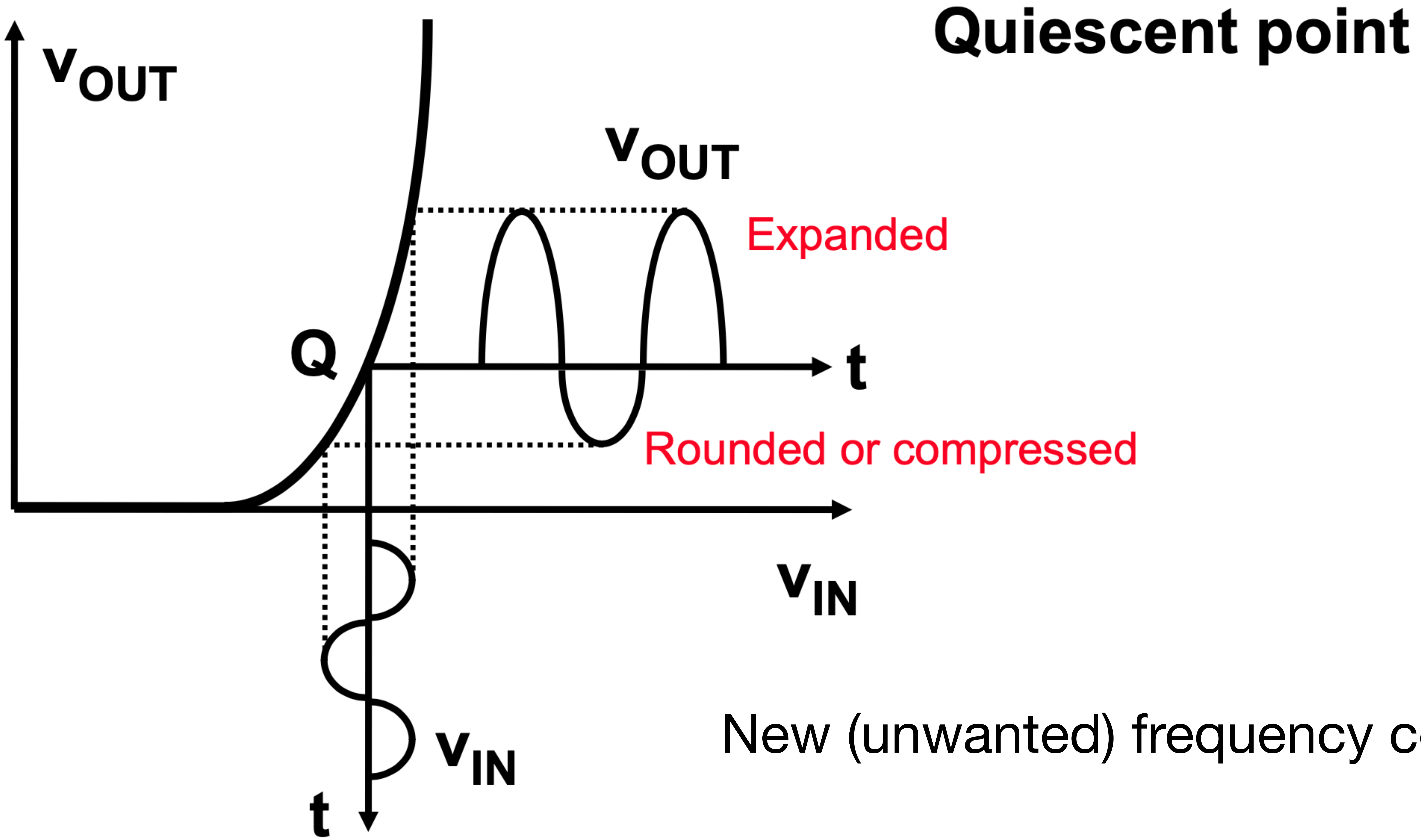
What it is NOT

Distortion is NOT the result of linear filtering!

No new frequency components

What it is

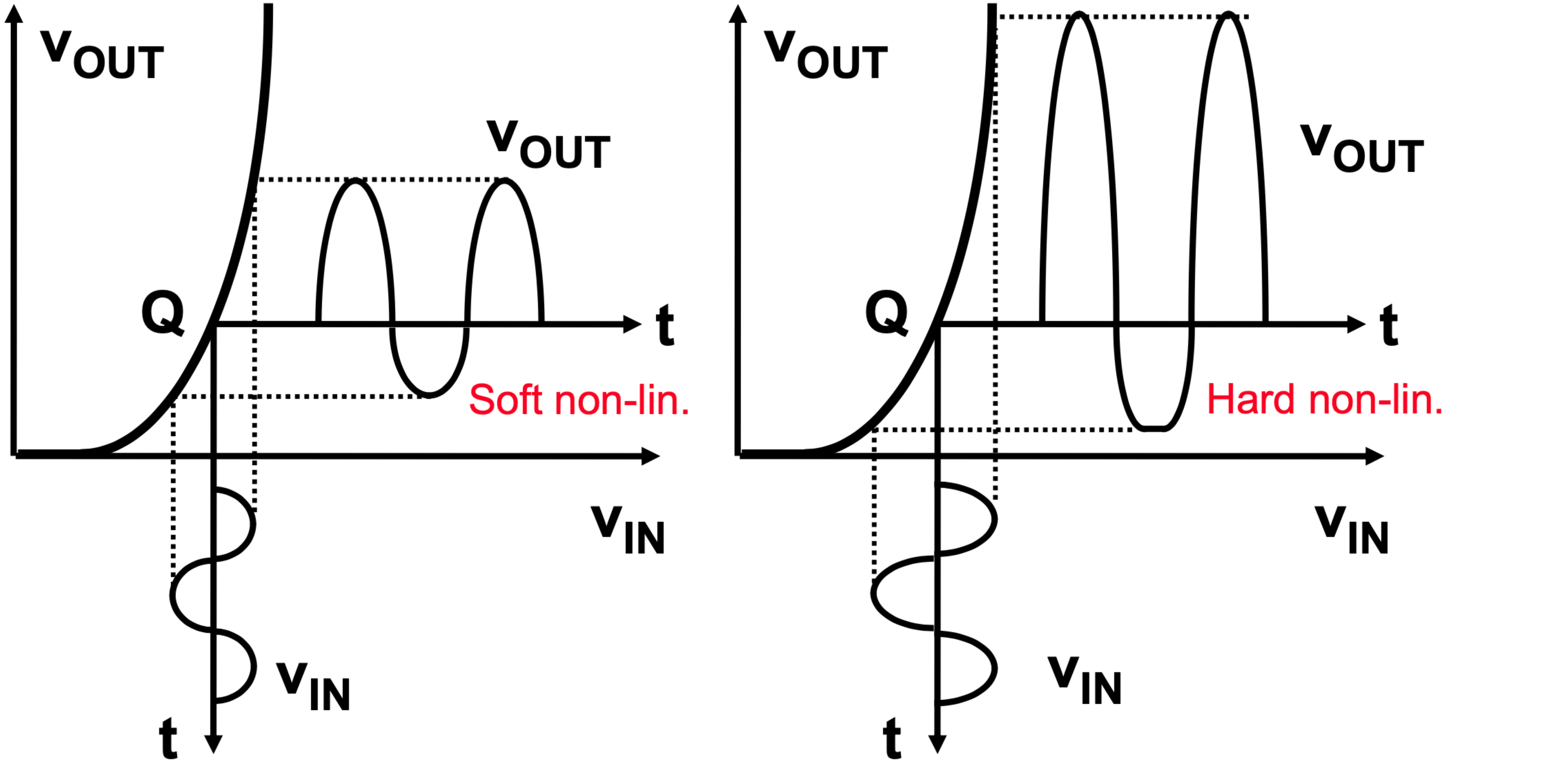
Distortion is the result of amplitude dependency



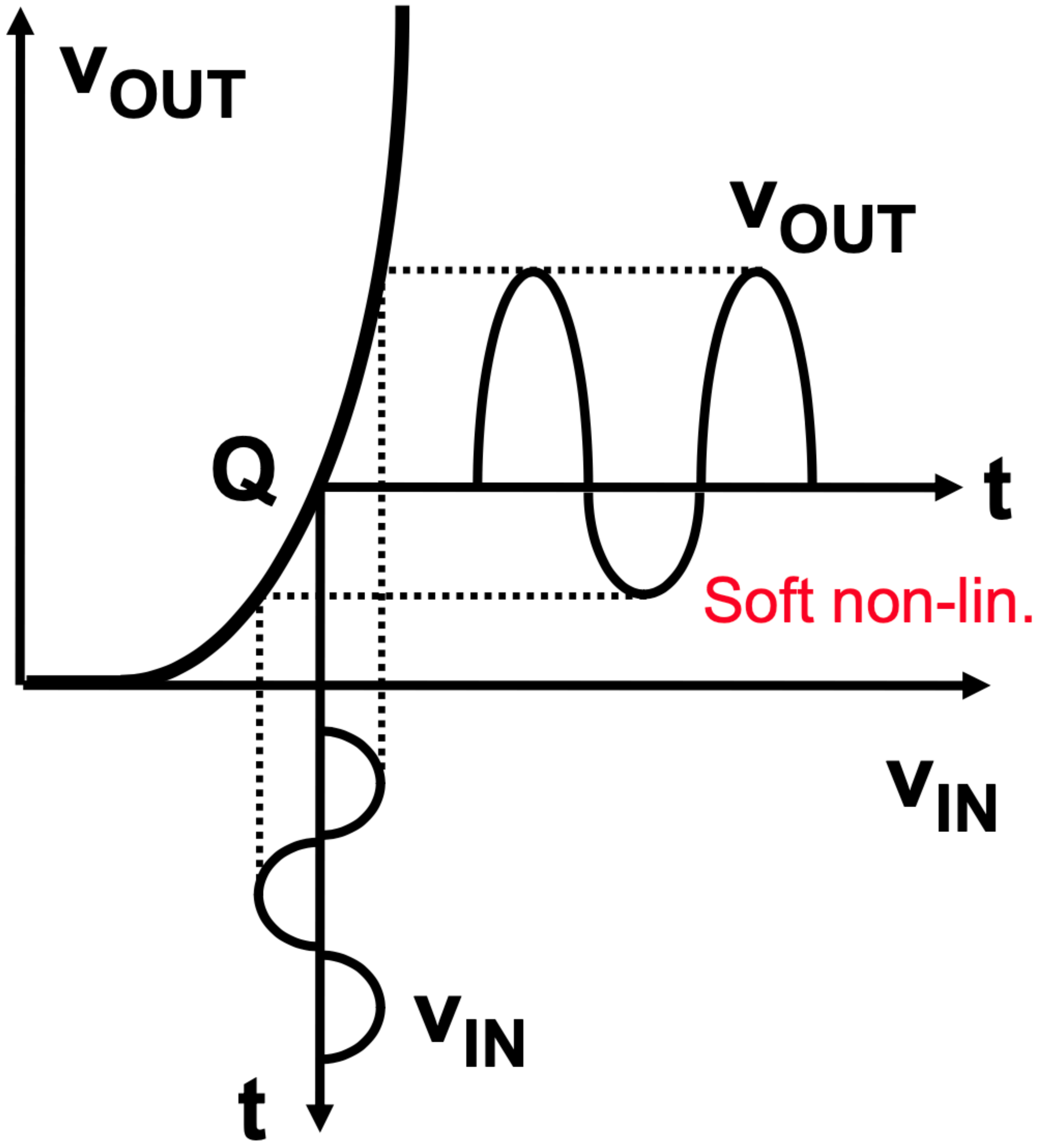
New (unwanted) frequency components

Soft and hard distortion

Hard distortion is difficult to model and hardly ever allowable



Modeling distortion with power series



$v_{IN}(t) \Longrightarrow v_{OUT}(t)$

$$v_{OUT} = a_0 + a_1 v_{IN} + a_2 v_{IN}^2 + a_3 v_{IN}^3 + \ldots$$

Finding the distortion coefficients

$$y = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + \ldots$$

$$a_0 = y\Big|_{u=0}$$

$$a_1 = \frac{dy}{du}\Big|_{u=0}$$

$$a_2 = \frac{1}{2}\,\frac{d^2y}{du^2}\Big|_{u=0}$$

$$a_3 = \frac{1}{6}\,\frac{d^3y}{du^3}\Big|_{u=0}$$

Harmonic distortion (HD)

$$y = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + \dots$$

With $u = U \cos \omega t$

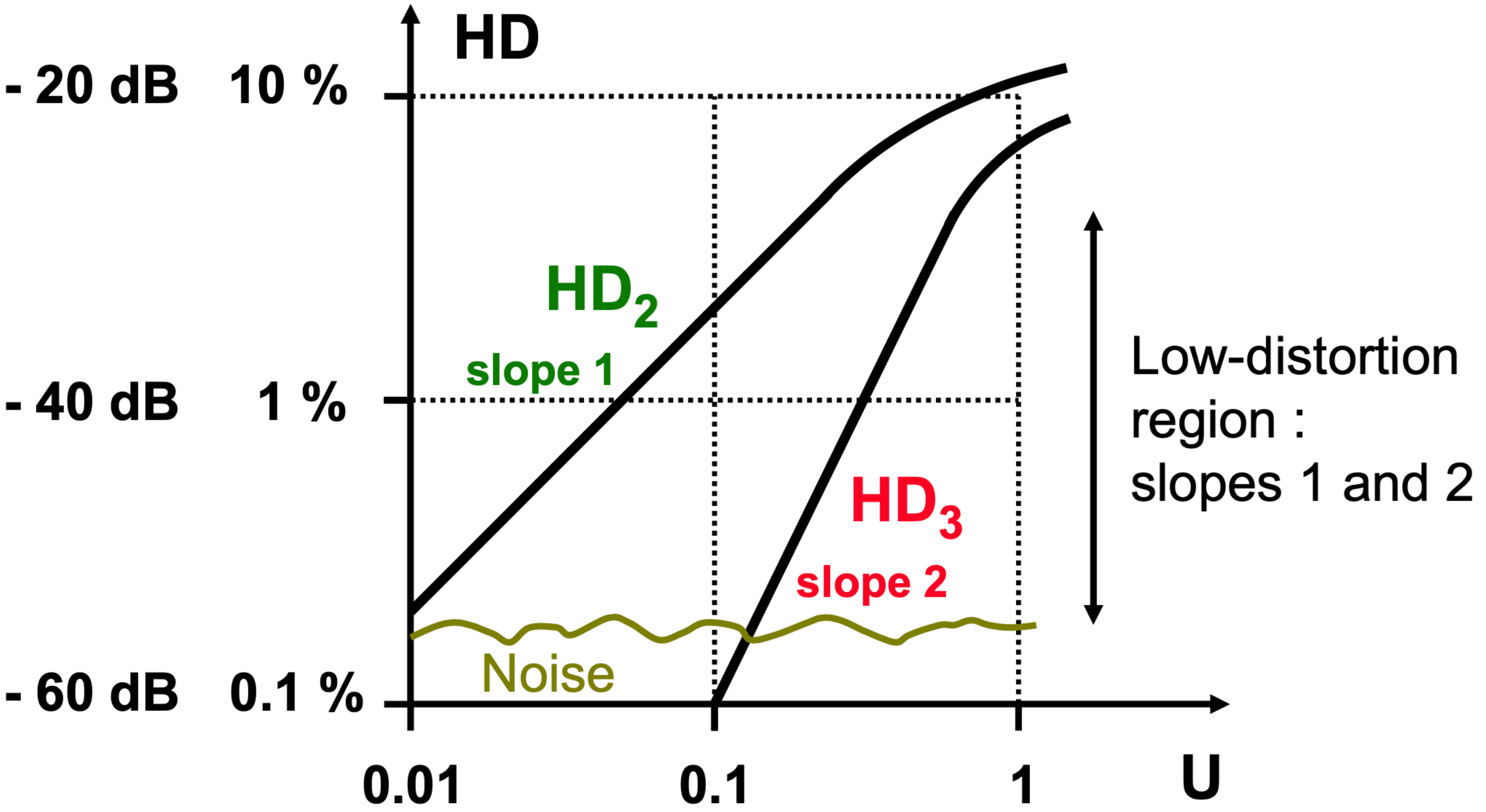
$$\cos^2 x = 1/2\,(1 + \cos 2x)$$
$$\cos^3 x = 1/4\,(3 \cos x + \cos 3x)$$

$$y = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + \dots \quad = a_0 +$$

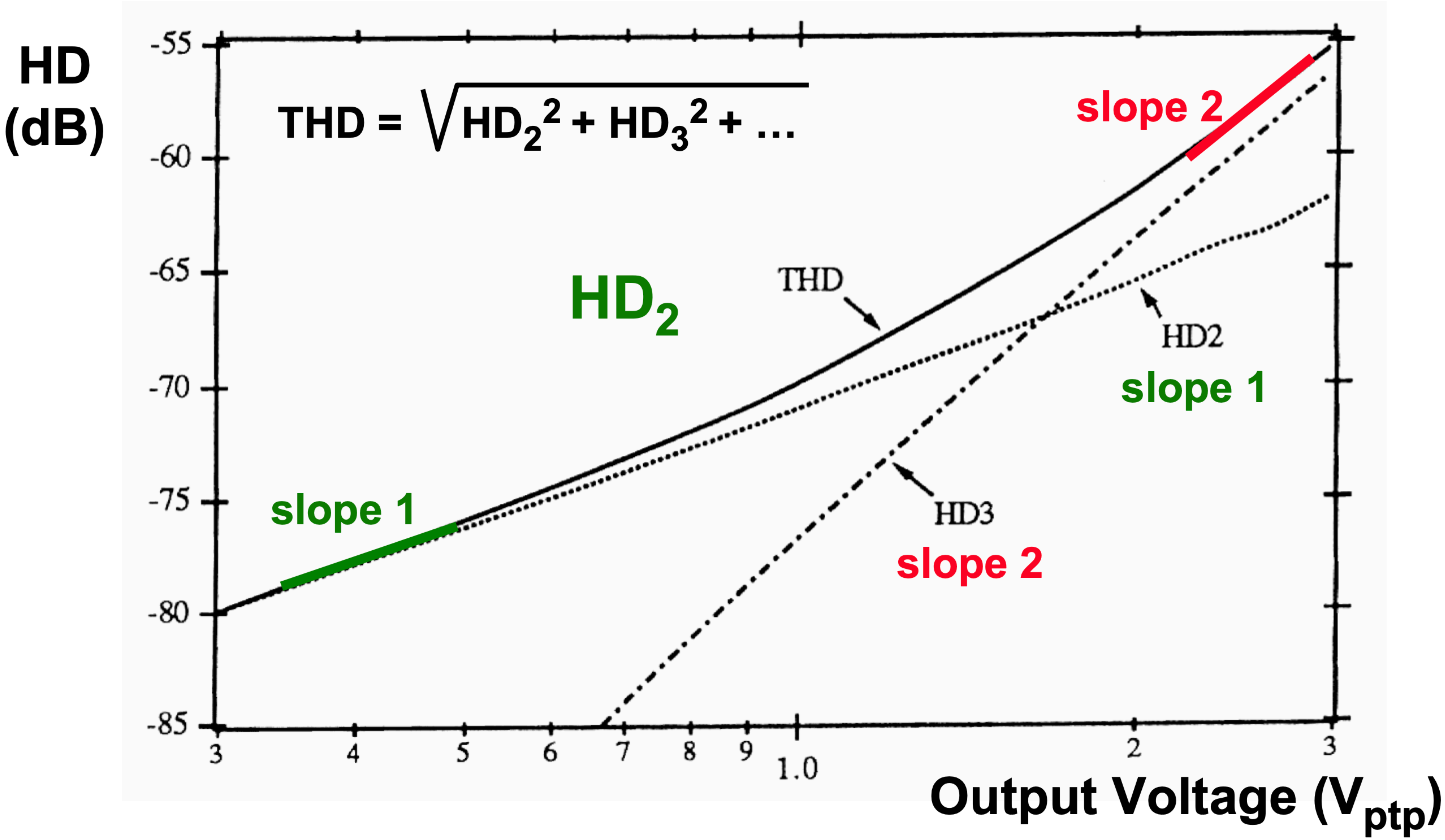
$$\left(a_1 + \frac{3}{4} a_3 U^2\right) U \cos \omega t + \frac{a_2}{2} U^2 \cos 2\omega t + \frac{a_3}{4} U^3 \cos 3\omega t$$

$$\boxed{HD_2 = \frac{1}{2} \frac{a_2}{a_1} U} \qquad \boxed{HD_3 = \frac{1}{4} \frac{a_3}{a_1} U^2}$$

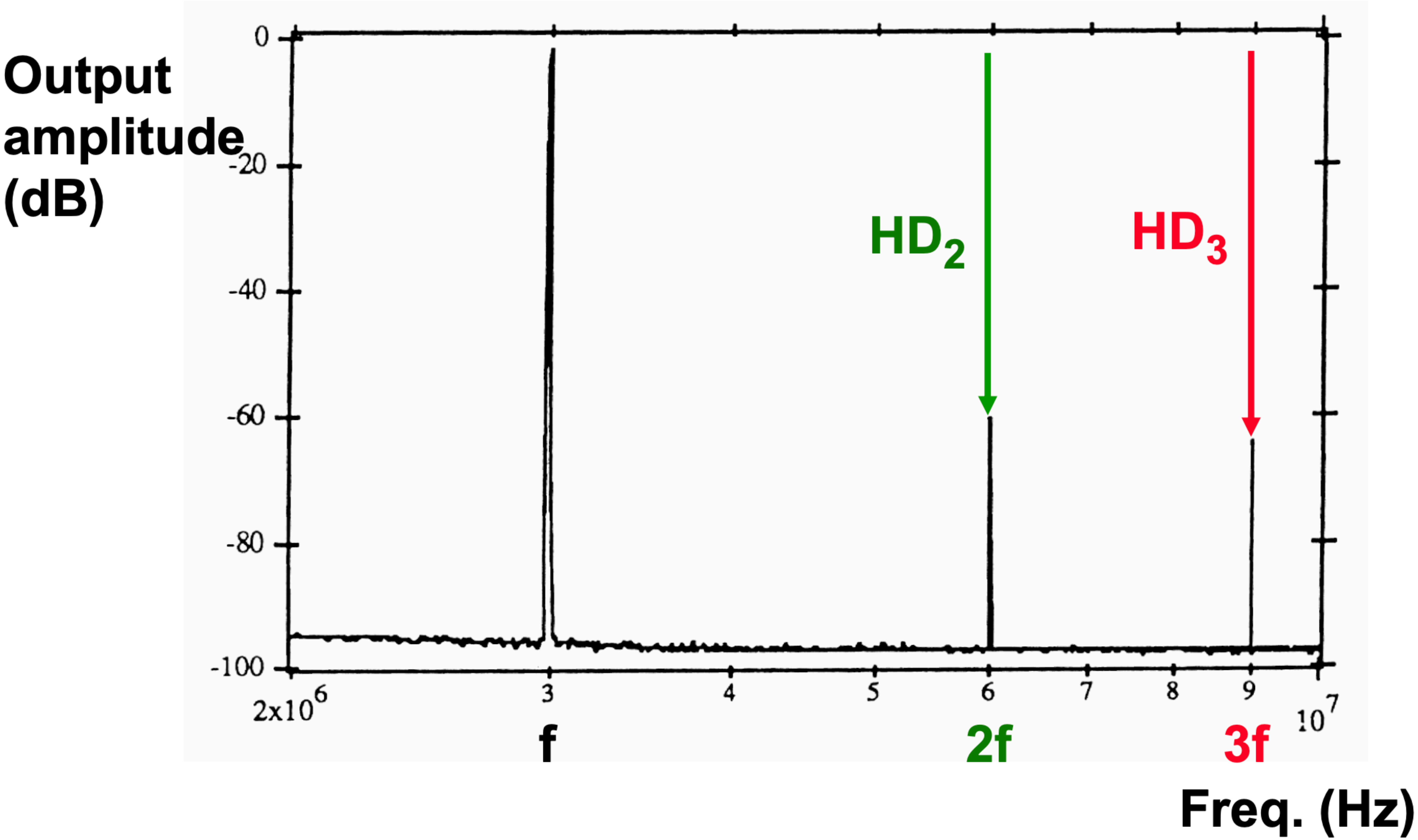
HD versus input amplitude



Example: HD of a resistor



HD in the output spectrum



Intermodulation distortion (IM)

$$y = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + \ldots$$

with $u = U\,(\cos \omega_1 t + \cos \omega_2 t)$

$$y = a_0 + \ldots$$

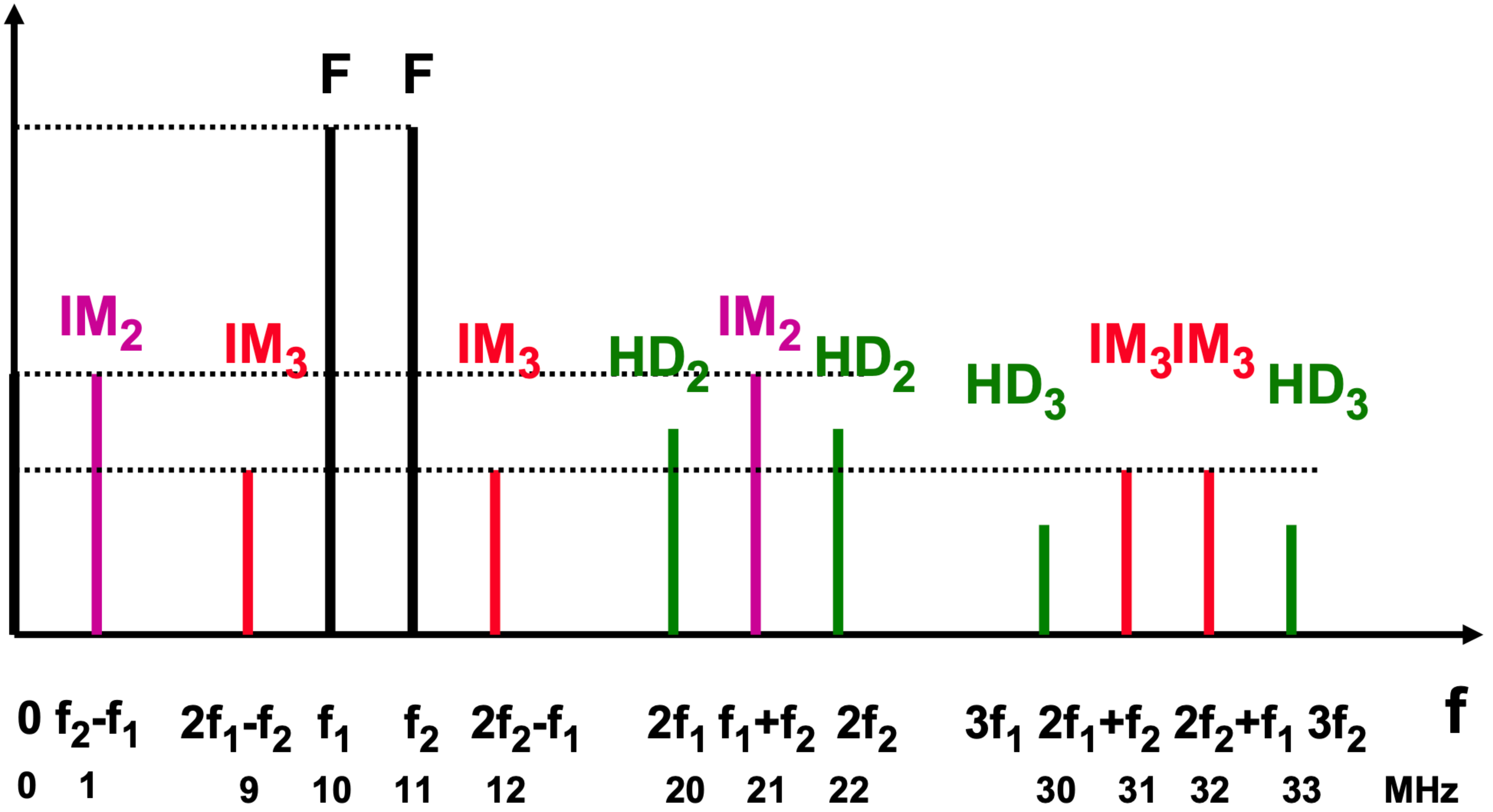
IM_2 at $\omega_1 \pm \omega_2$

IM_3 at $2\omega_1 \pm \omega_2$ and $\omega_1 \pm 2\omega_2$

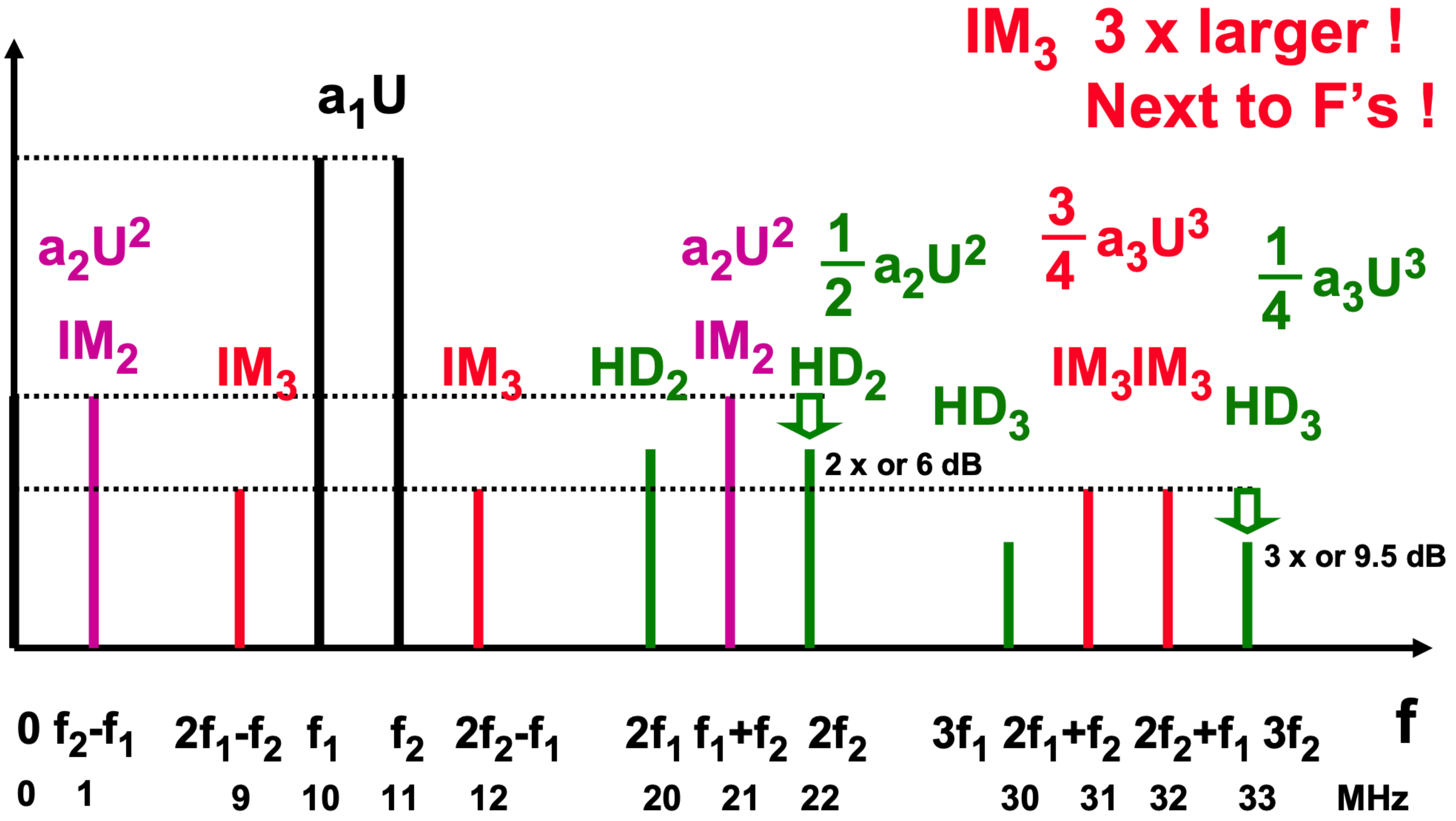
$$\boxed{IM_2 = 2\,HD_2 = \frac{a_2}{a_1}\,U}$$

$$\boxed{IM_3 = 3\,HD_3 = \frac{3}{4}\,\frac{a_3}{a_1}\,U^2}$$

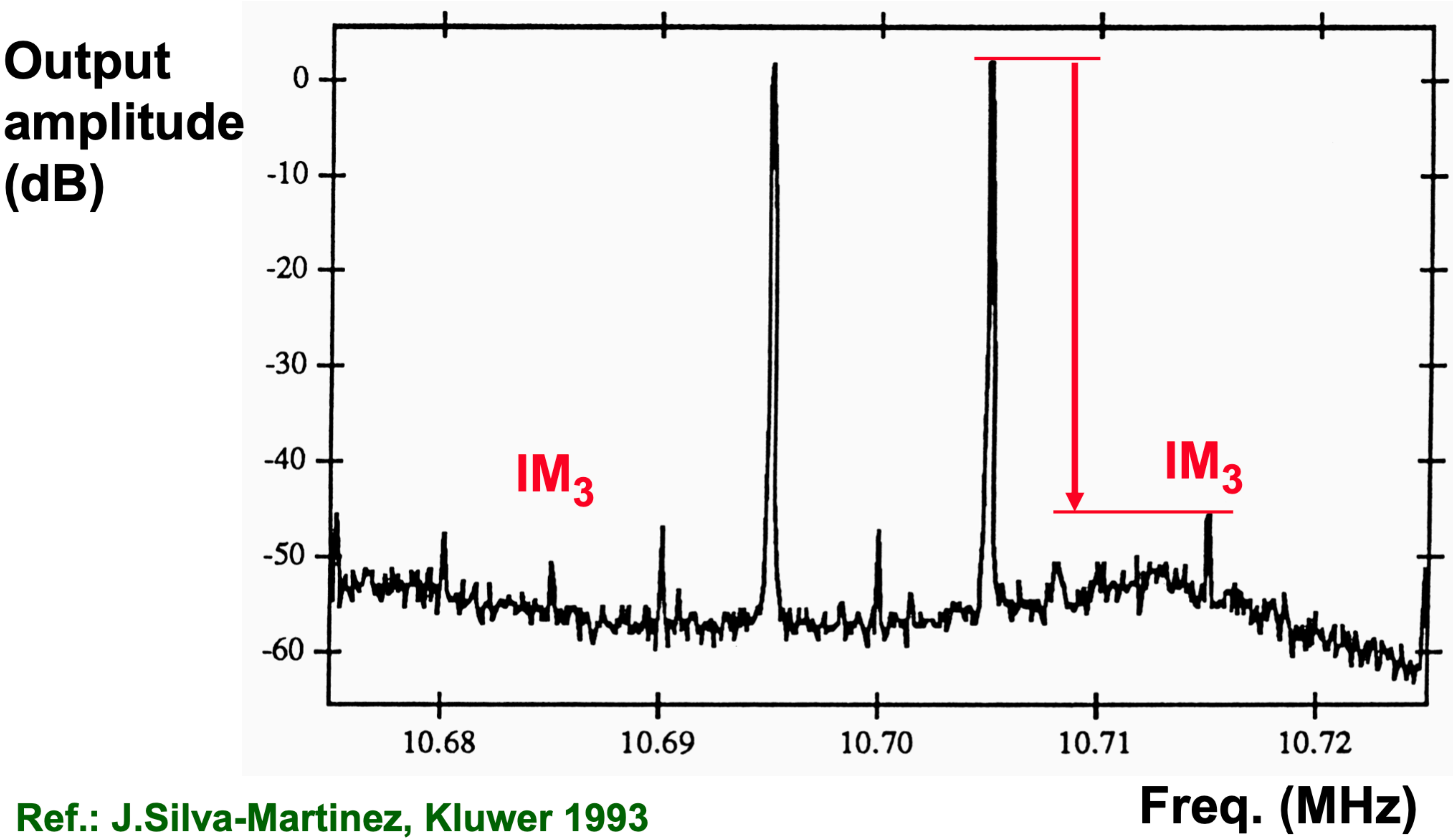
HD and IM in the spectrum



The relevance of IM_3



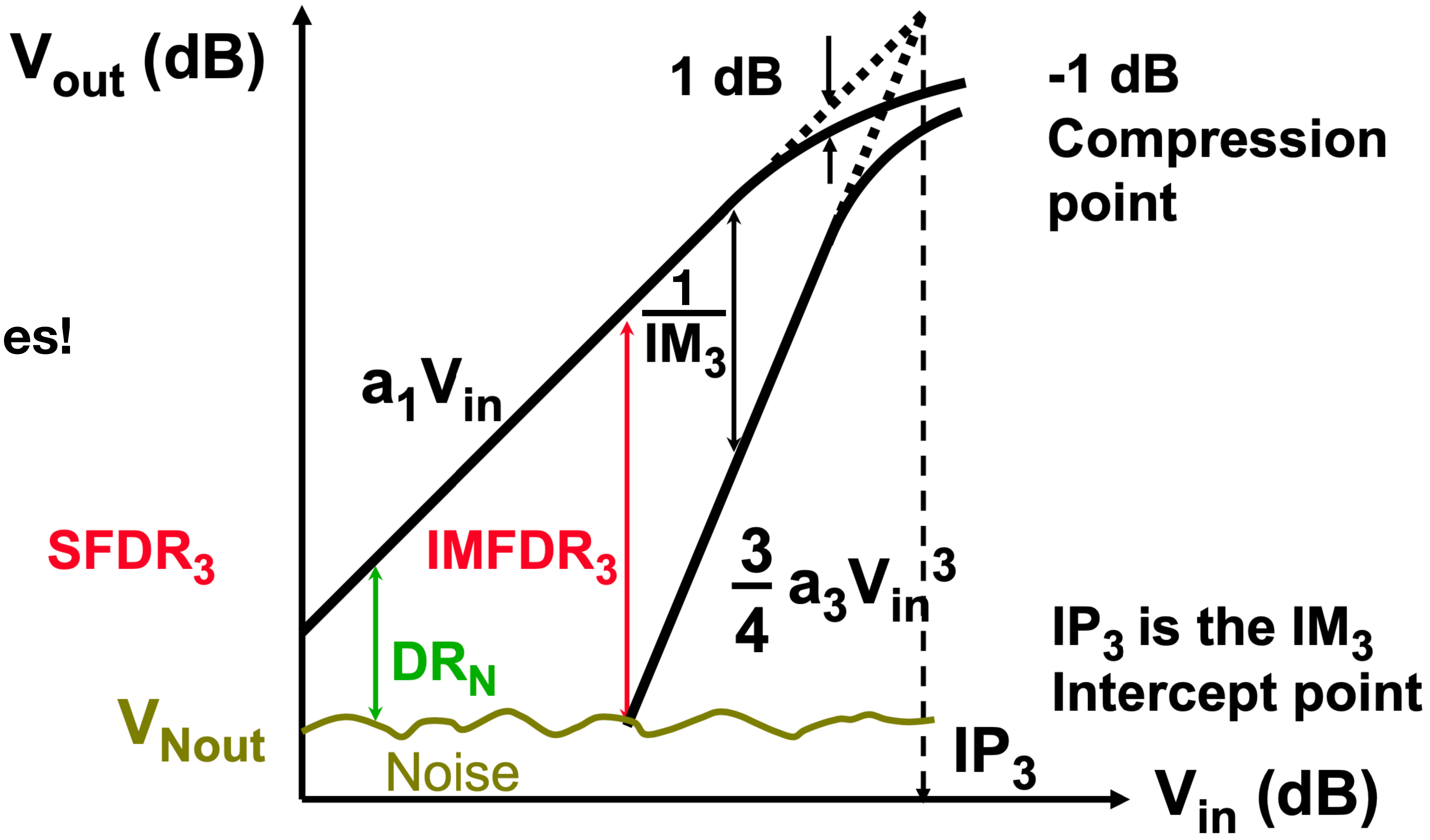
Example: amplifier output spectrum



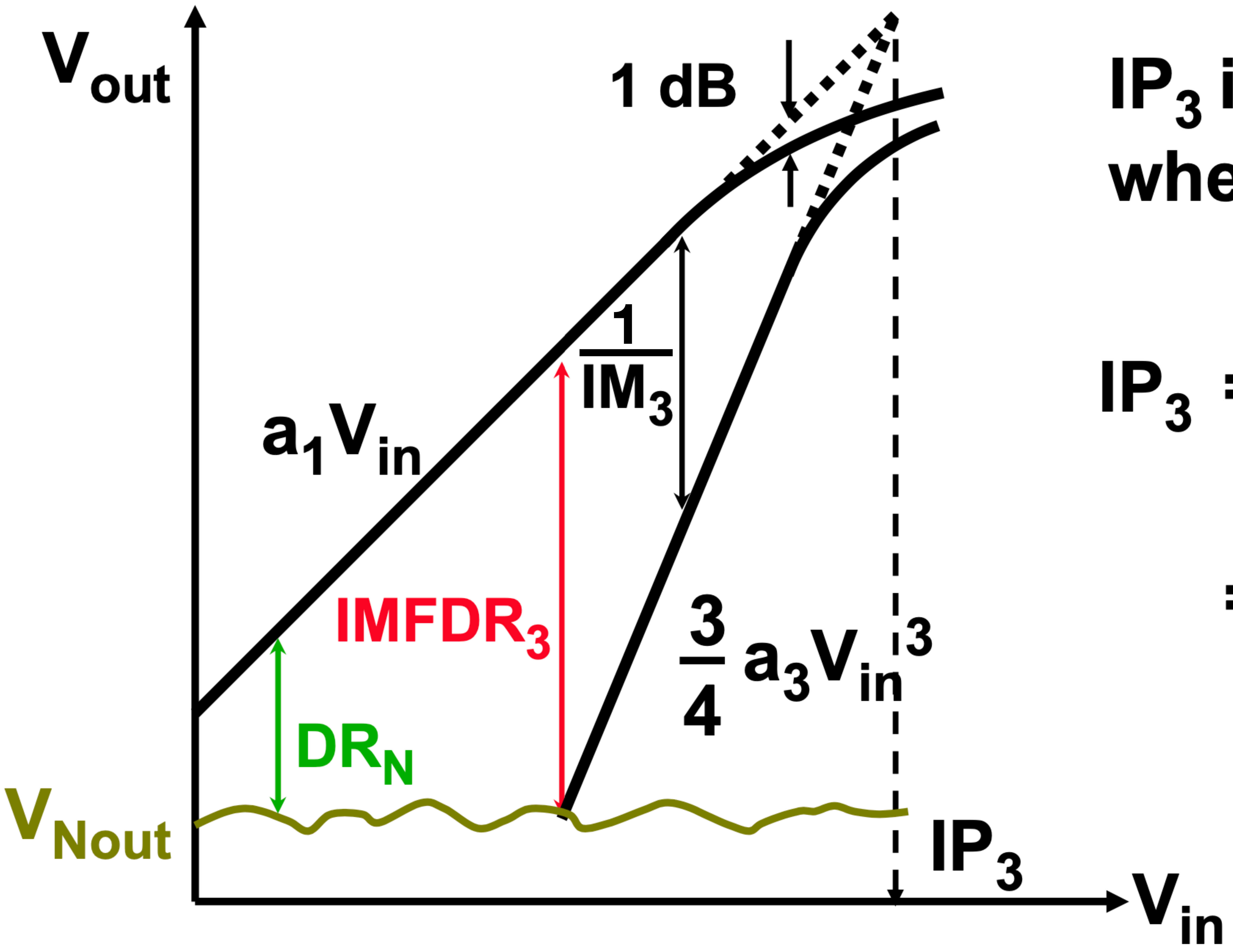
Ref.: J.Silva-Martinez, Kluwer 1993

IM_3 versus input amplitude

Logarithmic scales!



Relation between IP_3 and IM_3



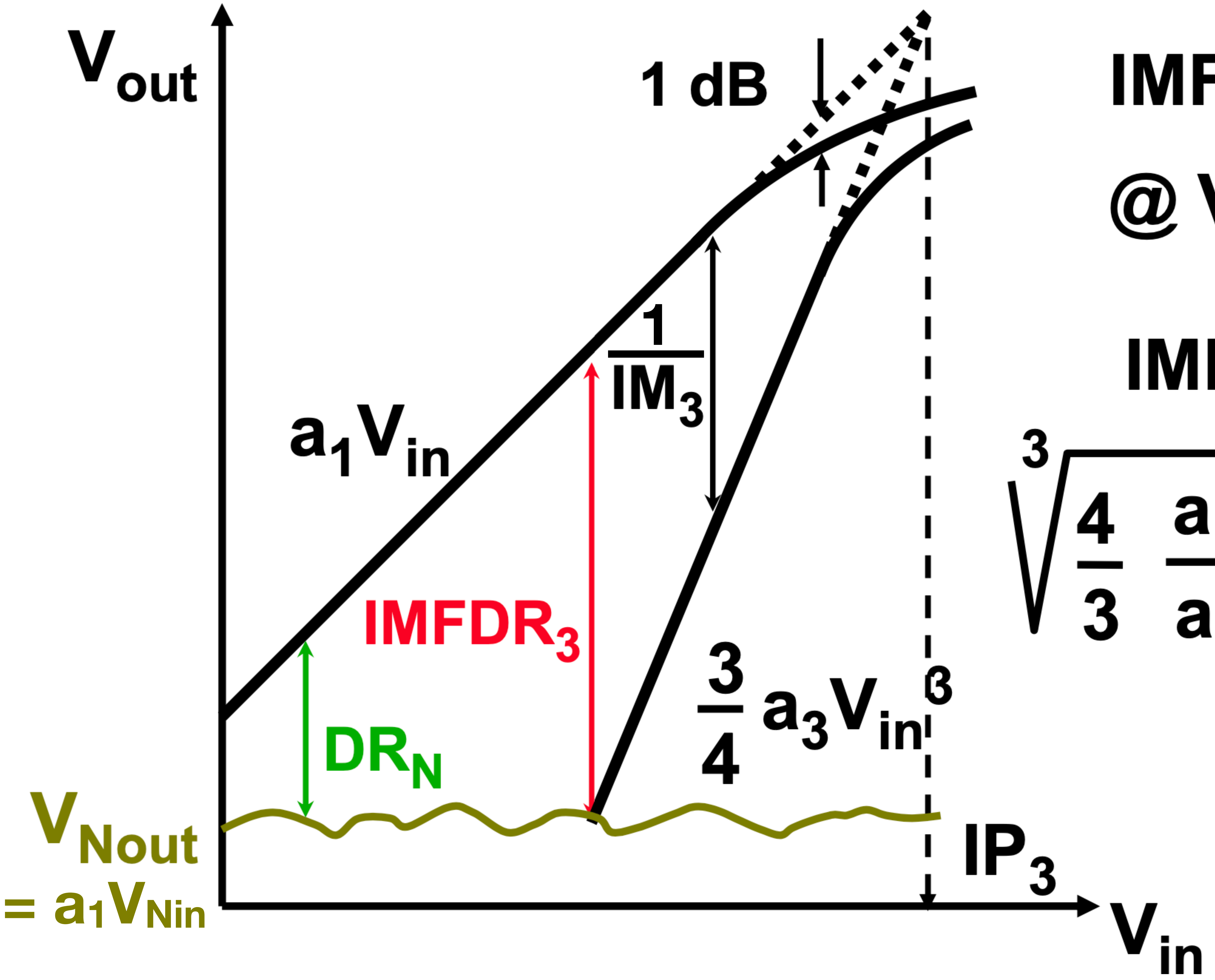
IP_3 is V_{in} where $IM_3 = F$

$$IP_3 = \sqrt{\frac{4}{3}\frac{a_1}{a_3}}$$

$$= V_{in}\frac{1}{\sqrt{IM_3}}$$

$$= V_{indB} - \frac{1}{2}IM_{3dB}$$

Relation between IMFDR$_3$ and IP$_3$



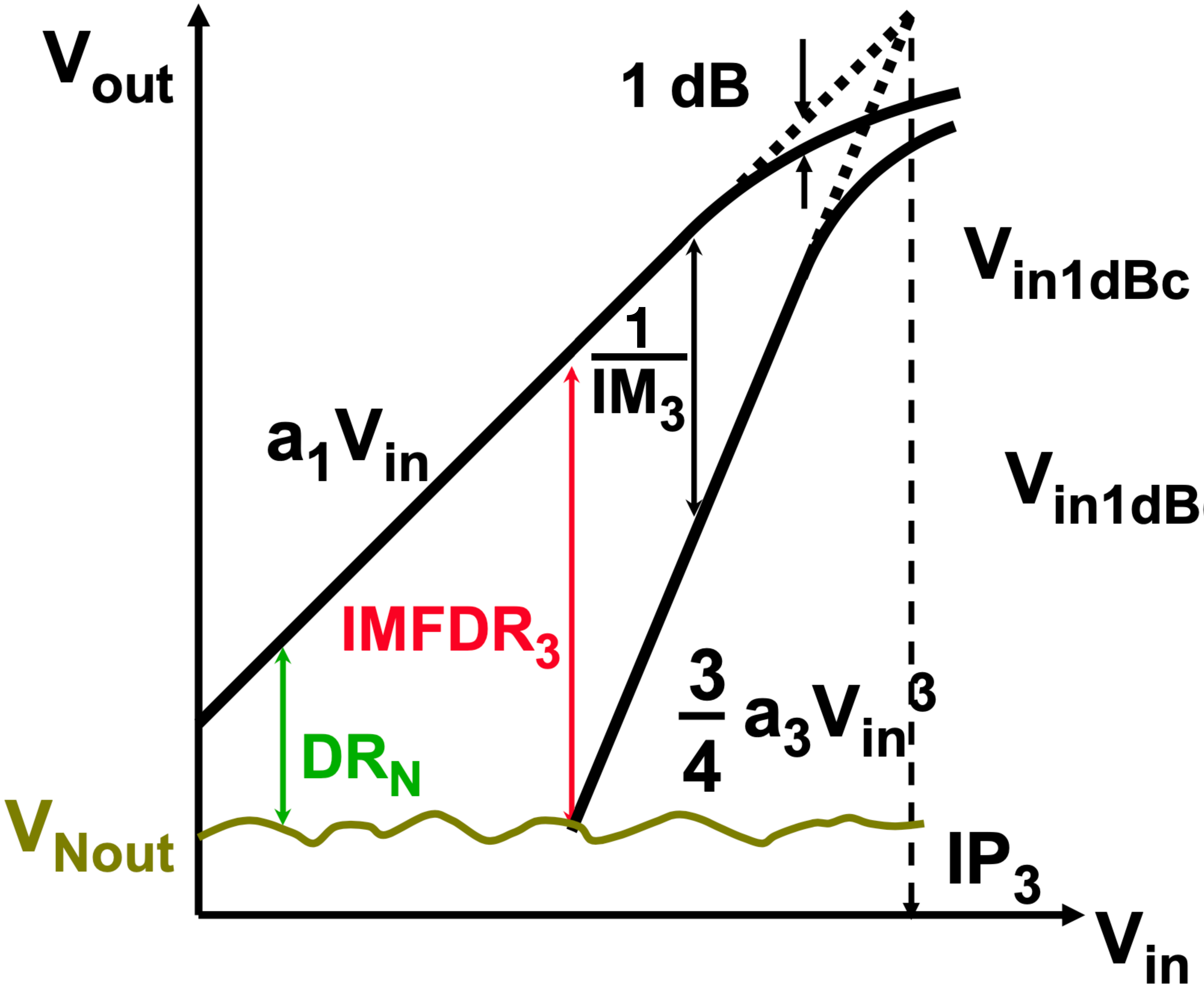
$$IMFDR_3 = \max DR$$

$$@\ V_{Nout} = \frac{3}{4} a_3 V_{in}^3$$

$$IMFDR_3 = \sqrt[3]{\frac{4}{3} \frac{a_1}{a_3} \frac{1}{V_{Nin}^2}} = \left(\frac{IP_3}{V_{Nin}}\right)^{2/3}$$

$$= \frac{2}{3} (IP_{3dB} - V_{NindB})$$

Relationship between V_{in1dBc} and IP_3



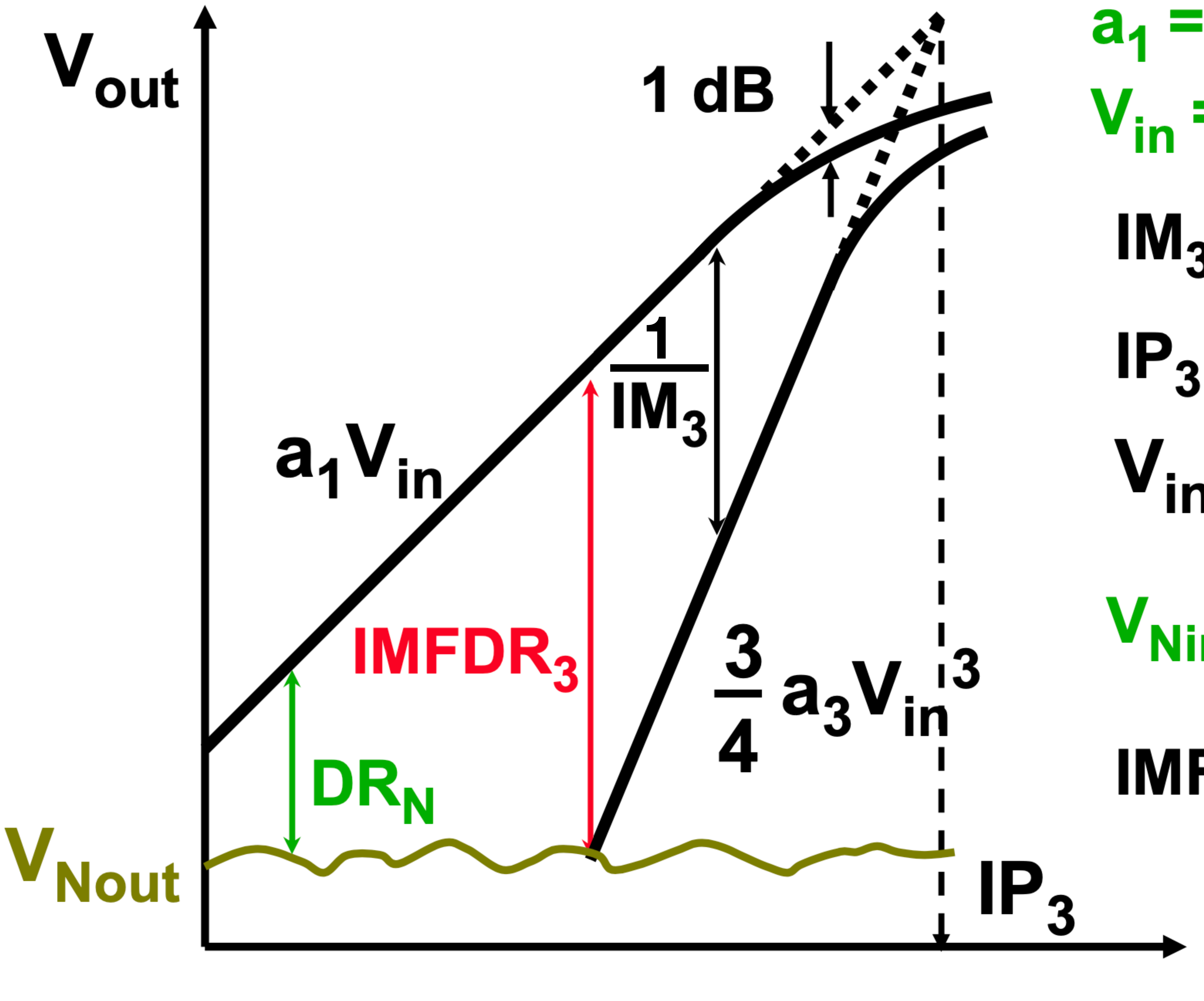
-1 dB is x 0.891

$$V_{in1dBc} = \sqrt{0.109\,\frac{4}{3}\,\frac{a_1}{a_3}}$$

$$V_{in1dBc} = \sqrt{0.109}\;IP_3$$

$$= IP_{3dB} - 9.6\ \text{dB}$$

Exercise



$a_1 = 20 \quad a_3 = 0.4$

$V_{in} = 0.45\ V_{RMS}$ or 6 dBm

$IM_3 = 0.3$ % or -50 dB

$IP_3 = 6 + 25 = 31$ dBm

$V_{in1dBc} = 21$ dBm

$V_{Nin} = 30\ \mu V_{RMS}$ (-78 dBm)

$$IMFDR_3 = \frac{2}{3}119 = 73 \text{ dB}$$

Outline

- Definitions and metrics of distortion
- **Distortion in components**
  - **MOST**
  - BJT
  - Passive components
- Distortion reduction
- Distortion in OPAMPs
- Summary

Definitions

$$i_{DS} = K\,(v_{GS} - V_T)^2 \qquad\qquad K = K' \frac{W}{L}$$

$$I_{DS} + i_{ds} = K\,(V_{GS} + v_{gs} - V_T)^2$$

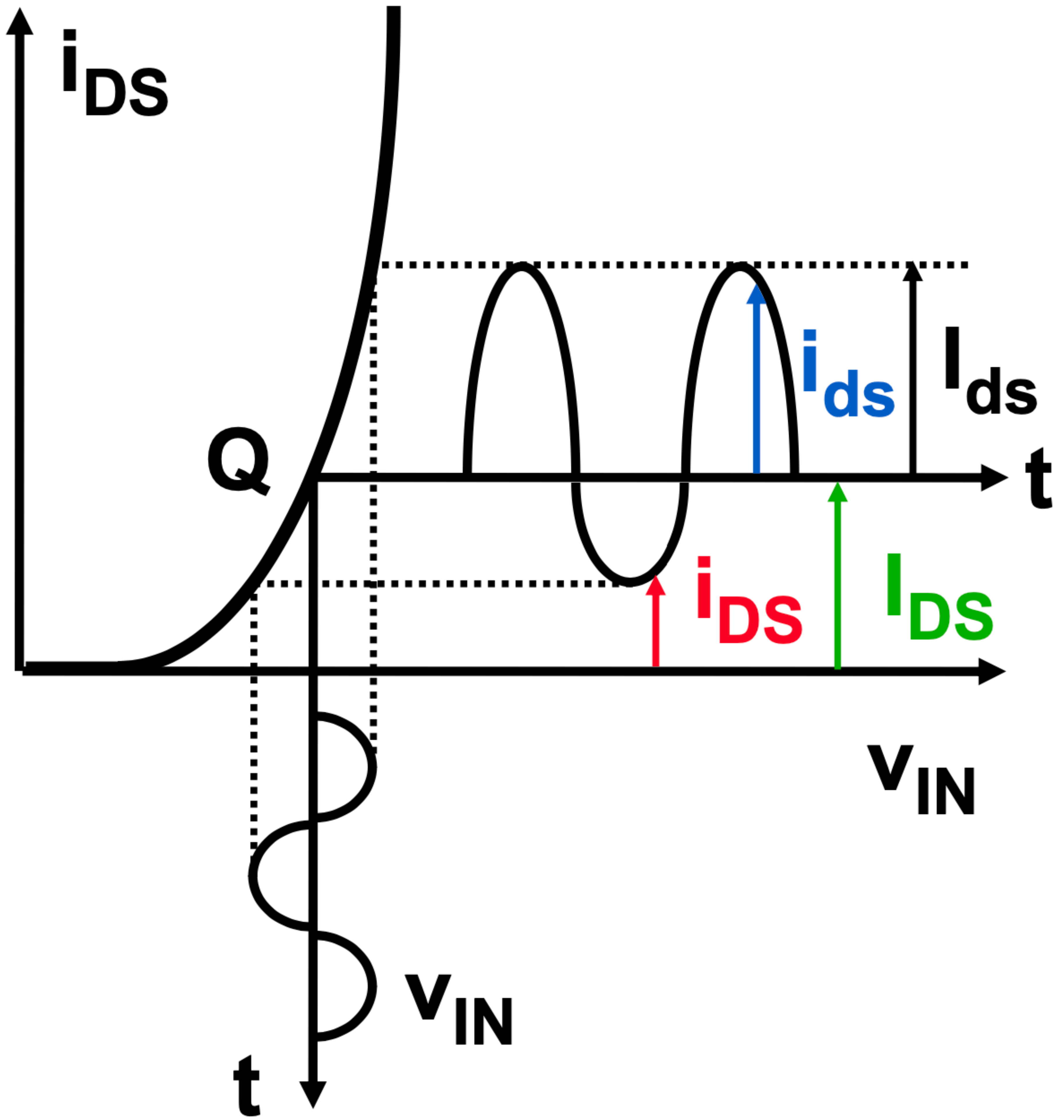
I_{DS} is the DC component

i_{DS} is the DC + ac component

i_{ds} is the ac component

I_{ds} is the amplitude of the ac component

Definitions - graphical



I_{DS} : DC component

i_{DS} : DC + ac component

i_{ds} : ac component

I_{ds} : amplitude of the ac component

Distortion of MOST current

$$I_{DS} = K\,(V_{GS} - V_T)^2 \qquad K = K'\,\frac{W}{L}$$

$$I_{DS} + i_{ds} = K\,(V_{GS} + v_{gs} - V_T)^2$$

$$i_{ds} = K\,(V_{GS} + v_{gs} - V_T)^2 - K\,(V_{GS} - V_T)^2$$

$$i_{ds} = 2K\,(V_{GS} - V_T)\,v_{gs} + K\,v_{gs}^{\;2} \quad \text{or} \quad i_{ds} = g_1 v_{gs} + g_2 v_{gs}^{\;2} + g_3 v_{gs}^{\;3} + \ldots$$

$$g_1 = 2K\,(V_{GS} - V_T)$$

$$g_2 = K$$

$$g_3 = 0$$

$$\boxed{IM_2 = \frac{g_2}{g_1} V_{gs} = \frac{V_{gs}}{2(V_{GS}-V_T)} \qquad \& \qquad IM_3 = 0}$$

Normalization of signals

$$i_{ds} = 2K\,(V_{GS} - V_T)\,v_{gs} + K\,v_{gs}^2 \qquad i_{DS} = K\,(v_{GS} - V_T)^2$$

or $y = a_1 u + a_2 u^2 + a_3 u^3 + ..$

$$y = \frac{i_{ds}}{I_{DS}} = \frac{2\,v_{gs}}{V_{GS} - V_T} + \frac{1}{4}\left(\frac{2\,v_{gs}}{V_{GS} - V_T}\right)^2$$

$$y = \frac{I_{ds}}{I_{DS}} = u + \frac{1}{4}u^2 \qquad U = \frac{V_{gs}}{(V_{GS} - V_T)/2}$$

y is the relative current swing !

Example

The peak value of V_{gs} is V_{gsp} = 100 mV

(then $V_{gsRMS} = 100/\sqrt{2} = 71\ mV_{RMS}$)

if $V_{GS}-V_T = 0.5$ V then $V_{gsp}/[\ 2(V_{GS}-V_T] = 0.1$

gives $IM_2 = 10\ \%$ ($HD_2 = 5\ \%$) & $IM_3 = 0$

The relative current swing $U = 0.1/0.25 = 0.4$!

More MOST distortion components

In general

$$i_{ds} = g_m v_{gs} + K_{2gm} v_{gs}^2 + K_{3gm} v_{gs}^3 +$$ non-linear transconductance

$$g_o v_{ds} + K_{2go} v_{ds}^2 + K_{3go} v_{ds}^3 +$$ non-linear output resistance

$$g_{mb} v_{bs} + K_{2gmb} v_{bs}^2 + K_{3gmb} v_{bs}^3 +$$ non-linear body transconductance

$$K_{2gm\&gmb} v_{gs} v_{bs} + K_{3,2gm\&gmb} v_{gs}^2 v_{bs}$$

$$+ K_{3,gm\&2gmb} v_{gs} v_{bs}^2 +$$

$$\ldots\ldots\ldots\ldots +$$

$$K_{3gm\&gmb\&go} v_{gs} v_{ds} v_{bs}$$

Distortion of a MOST diode

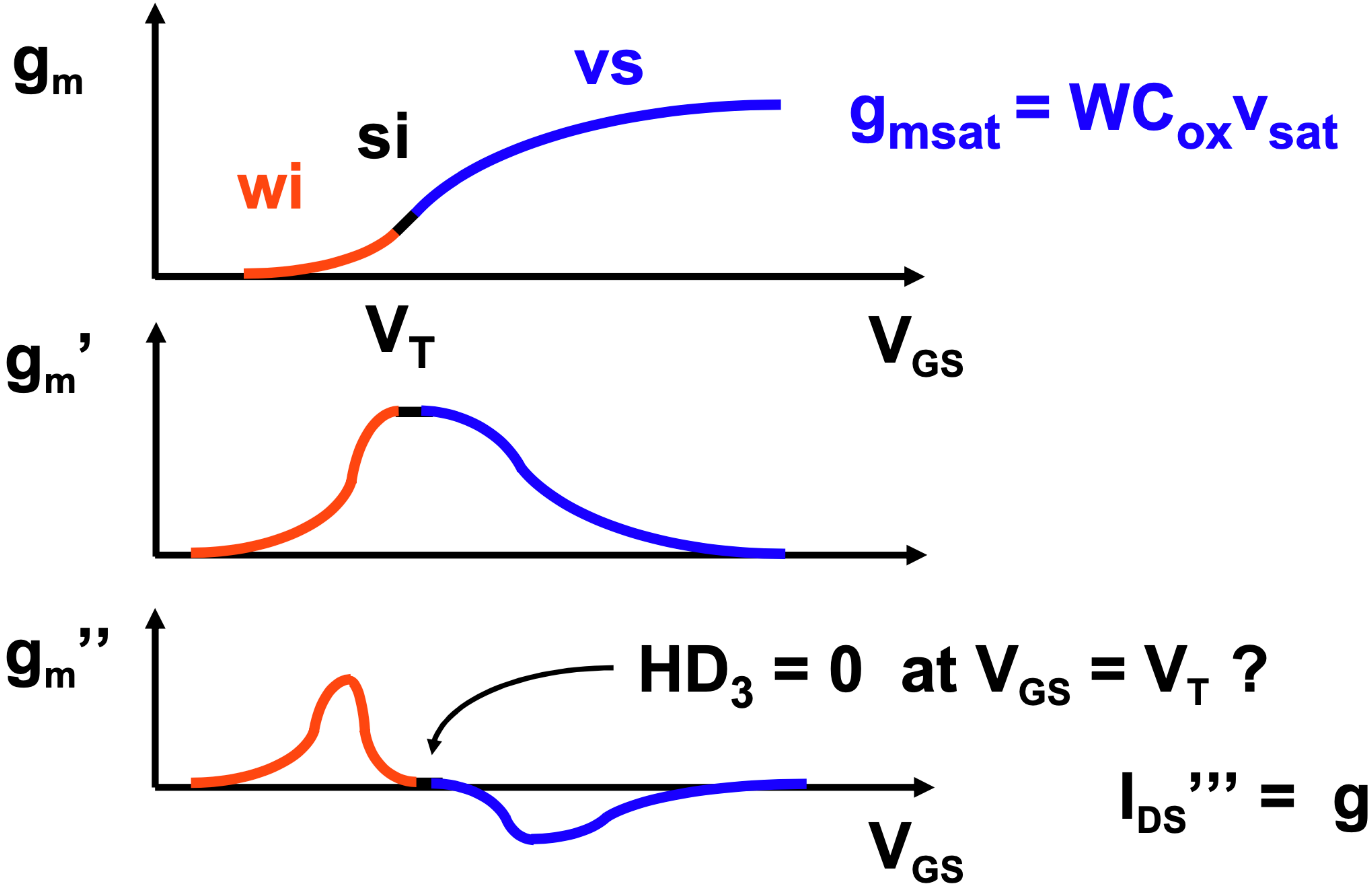
$$i_{DS} = K\,(v_{DS} - V_T)^2$$

$$y = \frac{i_{ds}}{I_{DS}} = \frac{2\,v_{ds}}{V_{DS} - V_T} + \frac{1}{4}\left(\frac{2\,v_{ds}}{V_{DS} - V_T}\right)^2$$

$$y = \frac{I_{ds}}{I_{DS}} = u + \frac{1}{4}u^2 \qquad U = \frac{V_{ds}}{(V_{DS} - V_T)/2}$$

Same as for a MOST transistor amplifier !

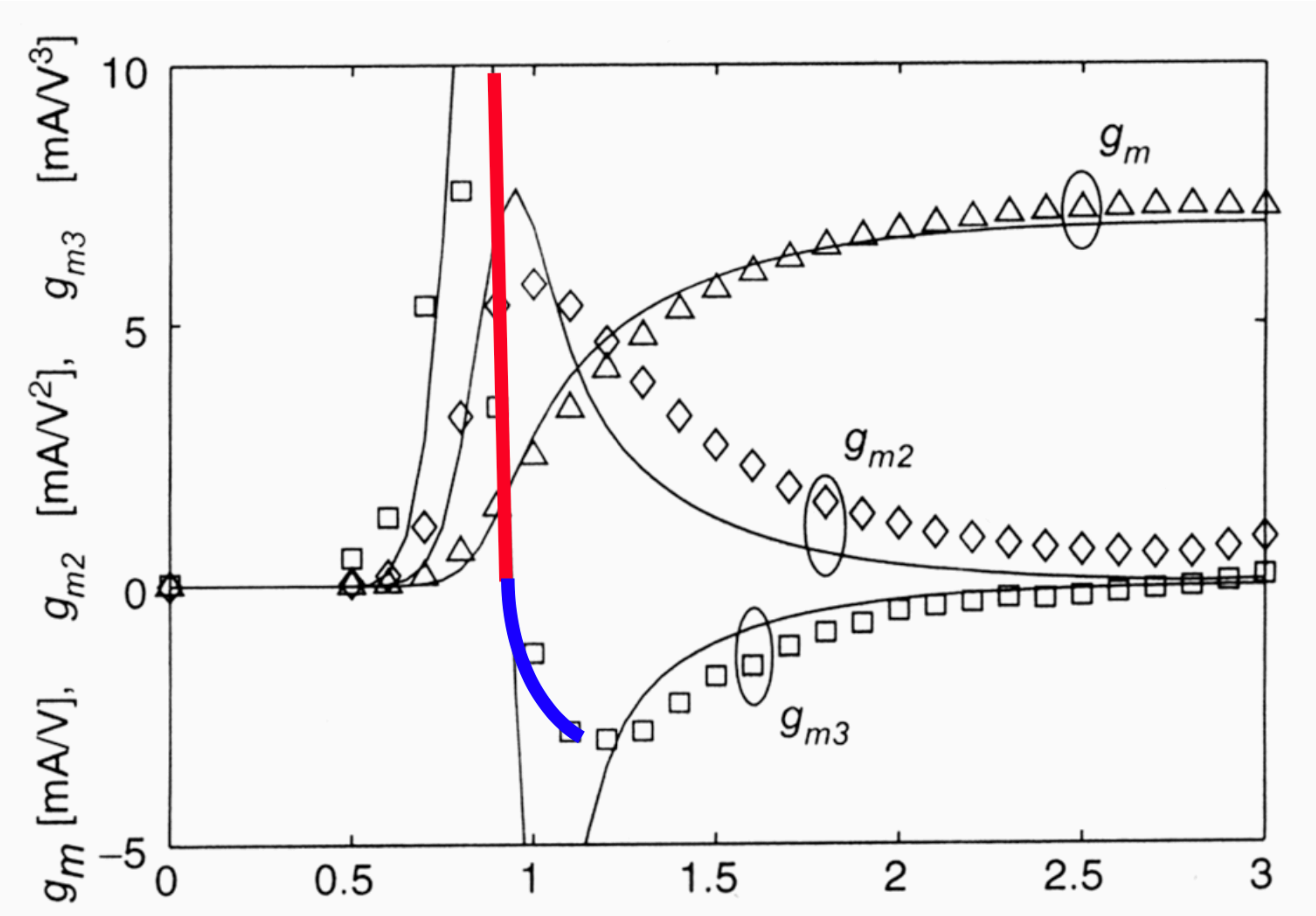
MOST biasing for no HD_3



$I_{DS}''' = g_m''$

MOST biasing for no HD3

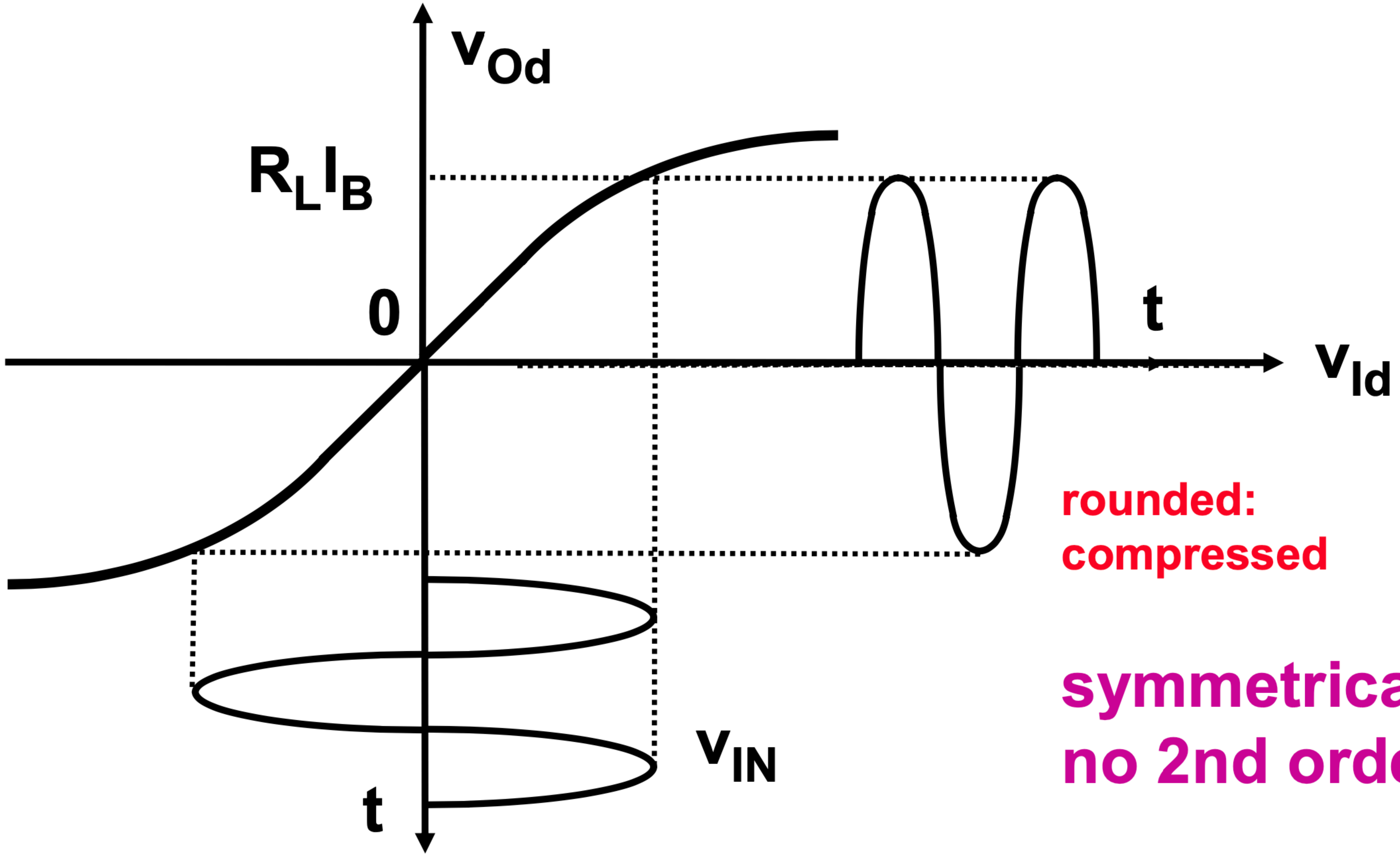
Very steep g_{m3} → very sensitive!



W = 60 μm
L = 0.6 μm
V_{DS} = 2 V

Ref. Fager JSSC Jan. 2004, 24-33

MOST differential pair



rounded:
compressed

symmetrical:
no 2nd order

even order distortion:
same amplitude and sign
in both transistors
→ removed in differential signal

uneven order distortion:
same amplitude but different sign
in both transistors
→ maintained in differential signal

Distortion in a MOST differential pair

$$y = \frac{i_{Od}}{I_B} = \frac{v_{Id}}{V_{GS}-V_T}\sqrt{1 - \frac{1}{4}\left(\frac{v_{Id}}{V_{GS}-V_T}\right)^2}$$

v_{Id} is the differential input voltage

i_{Od} is the differential output current ($g_m v_{Id}$) or twice the circular current $g_m v_{Id}/2$

I_B is the total DC current in the pair

Note that $g_m = \dfrac{I_B}{V_{GS} - V_T} = K' \dfrac{W}{L}(V_{GS} - V_T)$

Distortion in a MOST differential pair

$$y = \frac{i_{Od}}{I_B} = \frac{v_{Id}}{V_{GS}-V_T}\sqrt{1 - \frac{1}{4}\left(\frac{v_{Id}}{V_{GS}-V_T}\right)^2}$$

IM_3 is result of current limitation
→ compression

$$\sqrt{1-x} \approx 1 - \frac{x}{2}$$

$$y = \frac{I_{Od}}{I_B} = U\sqrt{1 - \frac{1}{4}U^2} \approx U - \frac{1}{8}U^3$$

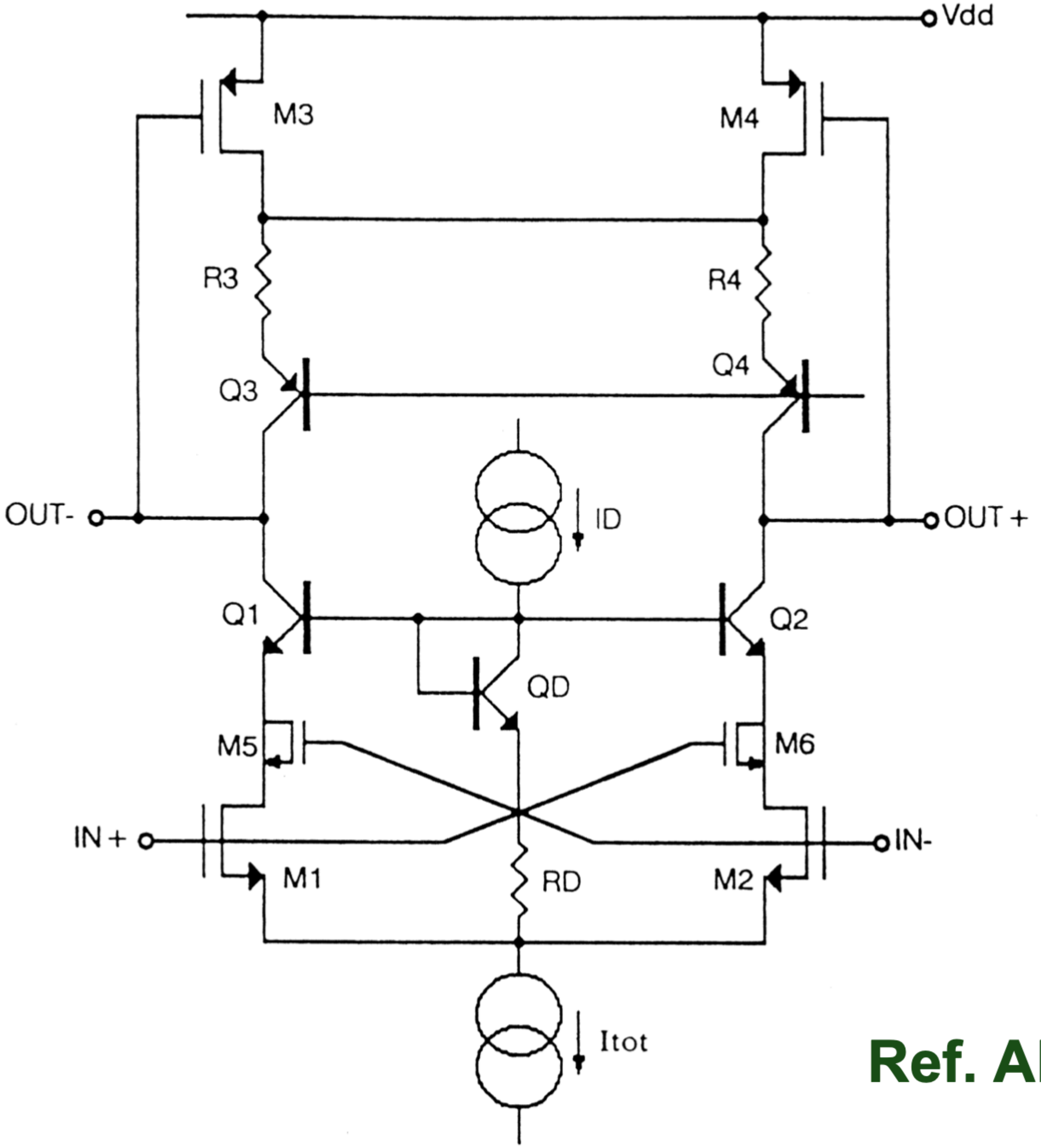
$$U = \frac{V_{Id}}{V_{GS} - V_T}$$

$$IM_2 = 0 \qquad \boxed{IM_3 = \frac{3}{32}U^2}$$

U is the relative current swing

$$IP_3 = 4\sqrt{\frac{2}{3}}(V_{GS} - V_T) \approx 3.3\,(V_{GS} - V_T)$$

MOST in linear region



$V_{DS1} = R_D I_D \approx 0.2$ V

$I_{DS1} = \beta_1 V_{DS1}(V_{GS1} - V_T)$

$g_{m1} = \beta_1 V_{DS1}$ is constant

Low distortion !

Ref. Alini,JSSC, Dec.92, pp.1905-1915

Outline

- Definitions and metrics of distortion
- **Distortion in components**
  - MOST
  - **BJT**
  - Passive components
- Distortion reduction
- Distortion in OPAMPs
- Summary

Distortion of BJT current

$$I_{CE} = I_S \exp\left(\frac{V_{BE}}{kT_e/q}\right)$$

$$I_{CE} + i_{ce} = I_S \exp\left(\frac{V_{BE} + v_{be}}{kT_e/q}\right)$$

$$1 + y = \exp\left(\frac{v_{be}}{kT_e/q}\right)$$

$$\approx \exp(u) = 1 + u + \frac{u^2}{2} + \frac{u^3}{6} + \ldots \quad \text{if } u \ll 1$$

I_{CE} DC component
i_{CE} DC + ac component
i_{ce} ac component
I_{ce} amplitude of the ac component

Distortion of BJT current

$$y \approx u + \frac{u^2}{2} + \frac{u^3}{6} + \ldots \qquad U = \frac{V_{be}}{kT_e/q}$$

is the non-linear equation

y is the relative current swing !

$a_1 = 1$

$a_2 = 1/2$

$a_3 = 1/6$

$$IM_2 = \frac{a_2}{a_1} U = \frac{1}{2} \frac{V_{be}}{kT_e/q}$$

$$IM_3 = \frac{3}{4} \frac{a_3}{a_1} U^2 = \frac{1}{8} \left(\frac{V_{be}}{kT_e/q}\right)^2$$

Example

1. Relative current swing is 10 %

$y_p = 0.1$ gives $IM_2 = 5$ % ($HD_2 = 2.5$%)

$IM_3 = 0.125$ % ($HD_3 = 0.04$ %)

As a result $V_{bep} = y_p(kT_e/q) = 2.6\ mV_p$ ($1.8\ mV_{RMS}$)

$IP_3 = \sqrt{8}\,(kT_e/q) = 74\ mV_p$ or $50\ mV_{RMS}$ or -13 dBm

2. $V_{bep} = 100$ mV

then $y_p = 0.1/0.026 \approx 4$ (must be << 1 !!)

gives $IM_2 =$?? Too high distortion !!

Distortion in a BJT diode

$$i_D = I_S \exp\left(\frac{v_D}{kT_e/q}\right)$$

$$y \approx u + \frac{u^2}{2} + \frac{u^3}{6} + \ldots$$

$$y = \frac{I_d}{I_D} = u + \frac{u^2}{2} + \frac{u^3}{6}$$

$$U = \frac{V_d}{kT_e/q}$$

Same as for a Bipolar transistor amplifier !

Distortion in a BJT differential pair

$$y = \frac{i_{Od}}{I_B} = \tanh \frac{V_{Id}}{2kT_e/q}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\approx x - \frac{1}{3} x^3$$

$$y = \frac{I_{Od}}{I_B} \approx U - \frac{1}{3} U^3$$

$$U = \frac{V_{Id}}{2kT_e/q}$$

U is the relative current swing

$$IM_2 = 0$$

$$\boxed{IM_3 = \frac{1}{4} U^2}$$

$$IP_3 = 4\ kT_e/q$$

$\boxed{\text{IM}_3 \text{ is result of current limitation AND intrinsic third order distortion}}$

Outline

- Definitions and metrics of distortion
- **Distortion in components**
    - MOST
    - BJT
    - **Passive components**
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Distortion in resistors and capacitors

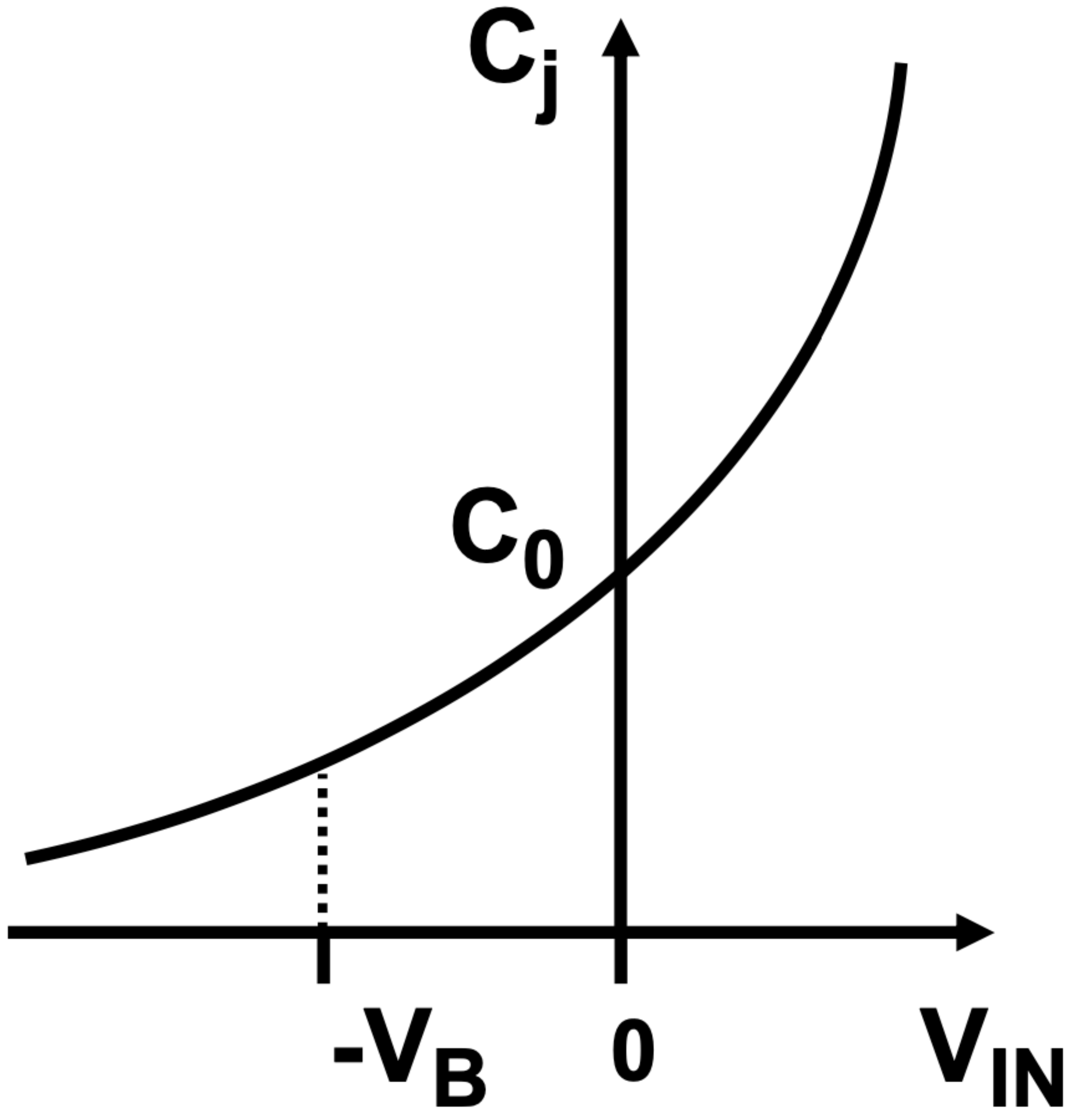
$R = R_0 (1 + a_1V + a_2V^2 + ...)$ [≈ JFET with large V_P]

For diffused resistors : $a_1 \approx 5$ ppm/V
$a_2 \approx 1$ ppm/V^2

$C = C_0 (1 + a_1V + a_2V^2 + ...)$

For poly-poly caps : $a_1 \approx 20$ ppm/V
$a_2 \approx 2$ ppm/V^2

Non-linear depletion capacitance



$$C_j = \frac{C_0}{\sqrt{1 - \frac{v_{IN}}{\Phi}}}$$

$$v_{IN} = V_B + v_{in}$$

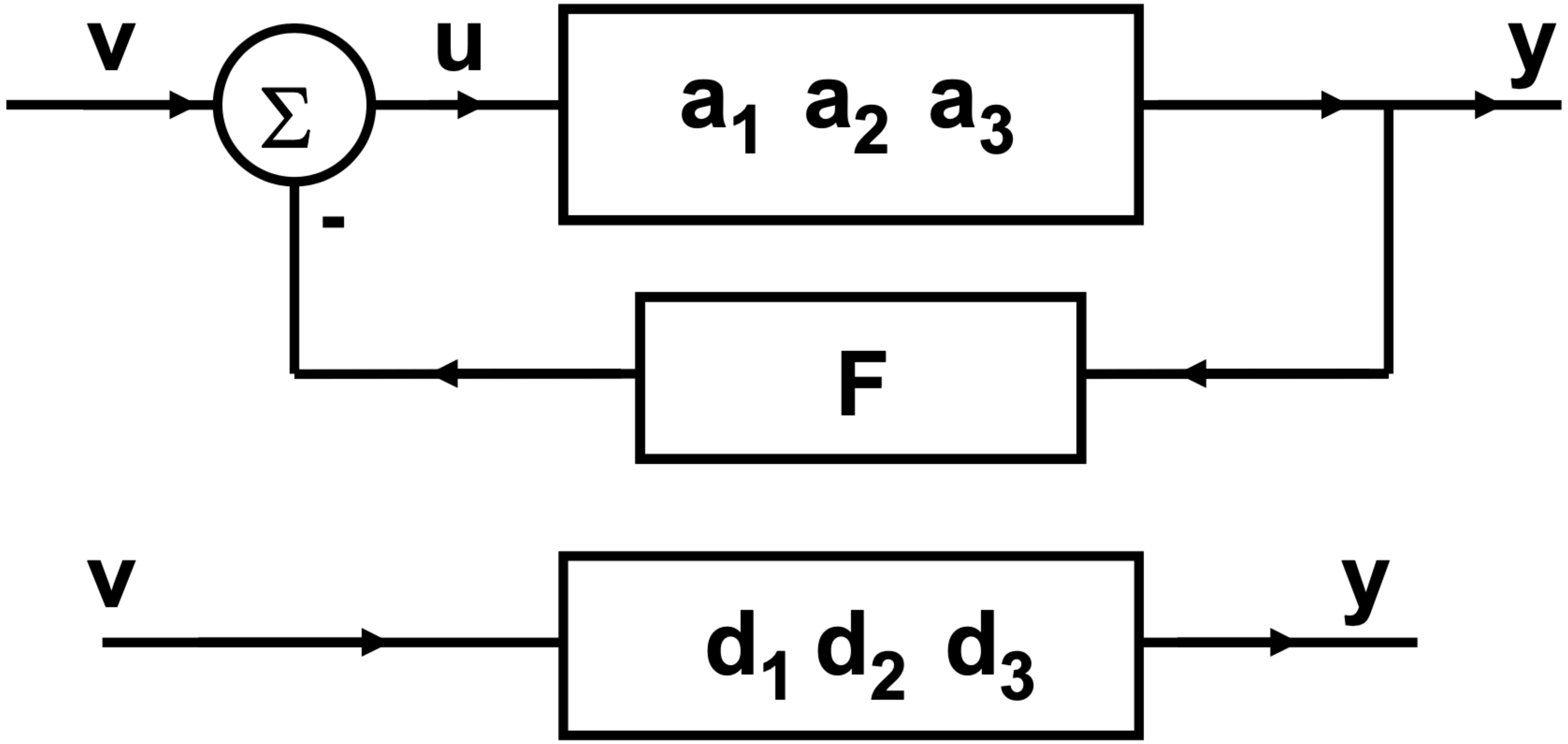
$$C_j = \frac{C_0}{\sqrt{1 + \frac{V_B}{\Phi}}} \frac{1}{\sqrt{1 + \underbrace{\frac{v_{in}}{V_B + \Phi}}_{x}}}$$

$$C_j = C_{0B}\,(1 + x)^{-1/2} = C_{0B}\,(1 - 1/2\,x + 3/8\,x^2 - 5/16\,x^3 + ..)$$

Outline

- Definitions and metrics of distortion
- Distortion in components
- **Distortion reduction**
    - **Feedback**
    - Cancellation
- Distortion in OPAMPs
- Summary

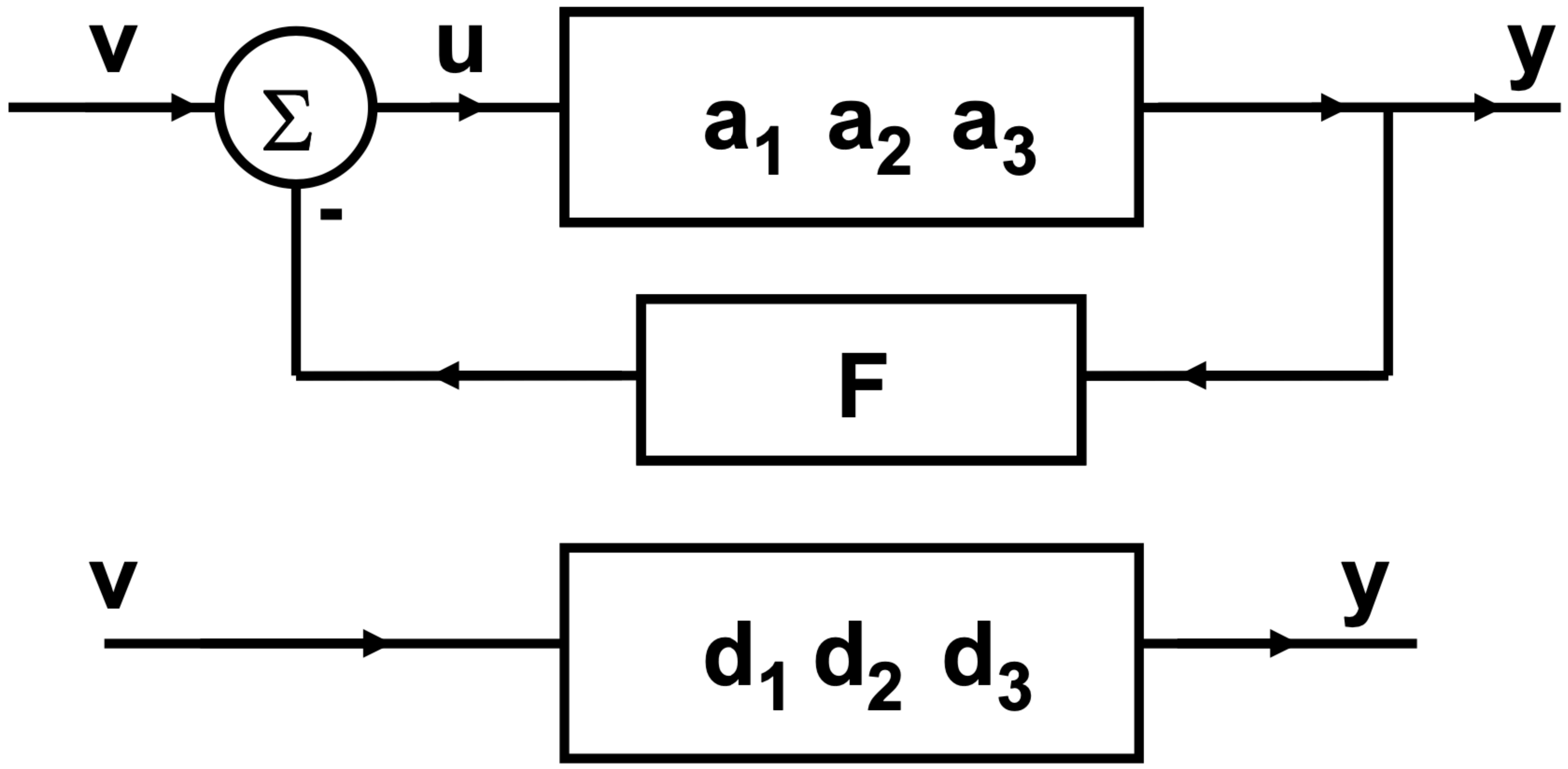
Feedback distortion coefficients



$$\left.\begin{array}{l} u = v - Fy \\ y = a_1 u + a_2 u^2 + a_3 u^3 \end{array}\right\} \text{elim. } u$$

$$\left.\begin{array}{l} \text{elim. } u \\ y = d_1 v + d_2 v^2 + d_3 v^3 \end{array}\right\} \text{elim. } y \left\{\begin{array}{l} \text{coeff } v : d_1 \\ \text{coeff } v^2 : d_2 \\ \text{coeff } v^3 : d_3 \end{array}\right.$$

Distortion reduction by feedback



$$u = v - Fy$$

$$d_1 = \frac{a_1}{1+T} \approx \frac{1}{F}$$

$$d_2 = \frac{a_2}{(1+T)^3}$$

$$d_3 = \frac{a_3(1+T) - 2F a_2^2}{(1+T)^5}$$

Loop gain $1+T = 1+a_1F$

u is (1+T) times smaller than v :

v is reduced by loop gain (1+T)

IM with feedback

$$IM_{2f} = \frac{d_2}{d_1}V = \frac{a_2}{a_1}\frac{V}{(1+T)^2} = \frac{a_2}{a_1}\underbrace{\frac{1}{(1+T)}}_{\text{reduction by loop gain}}\underbrace{\frac{V}{(1+T)}}_{\text{reduction in current swing}}$$

$$IM_{3f} = \frac{3}{4}\frac{d_3}{d_1}V^2 = \frac{3}{4}\left[\underbrace{\frac{a_3}{a_1}\frac{1}{(1+T)}}_{\text{compression}} - \underbrace{\left(\frac{a_2}{a_1}\right)^2\frac{2T}{(1+T)^2}}_{\text{expansion}}\right]\underbrace{\frac{V^2}{(1+T)^2}}_{\text{reduction in current swing}}$$

IM_{3f} for large T

$$IM_{3f} = \frac{3}{4}\frac{d_3}{d_1}V^2 = \frac{3}{4}\underbrace{\left[\frac{a_3}{a_1}\frac{1}{(1+T)} - \left(\frac{a_2}{a_1}\right)^2\frac{2T}{(1+T)^2}\right]}\frac{V^2}{(1+T)^2}$$

$$\text{For large T :}\quad \frac{a_3 a_1 - 2 a_2^2}{a_1^2}\frac{1}{T} = \frac{a_3}{a_1}\left(1 - \frac{2 a_2^2}{a_1 a_3}\right)\frac{1}{T}$$

MOST : $a_3 = 0$: a_2 dominant

Bipolar : $a_1 = 1$ $a_2 = 1/2$ $a_3 = 1/6$: a_2 dominant

Diff. pair : $a_2 = 0$: a_3 dominant

IM_{2f} with emitter degeneration

$$T = g_m R_E = \frac{V_{RE}}{kT_e/q} \qquad \frac{a_2}{a_1} = \frac{1}{2}$$

$$IM_{2f} = \frac{1}{2} \frac{1}{(1+T)^2} \frac{V_{in}}{kT_e/q} = \frac{1}{(1+T)} \frac{U}{2}$$

$$U = \frac{1}{(1+T)} \frac{V_{in}}{kT_e/q}$$ is the relative current swing

IM_{2f} decreases linearly with T for constant U !

IM_{3f} with emitter degeneration

$$IM_{3f} = \frac{1 - 2T}{(1 + T)^2} \frac{U^2}{8}$$

$$\frac{a_2}{a_1} = \frac{1}{2} \qquad \frac{a_3}{a_1} = \frac{1}{6}$$

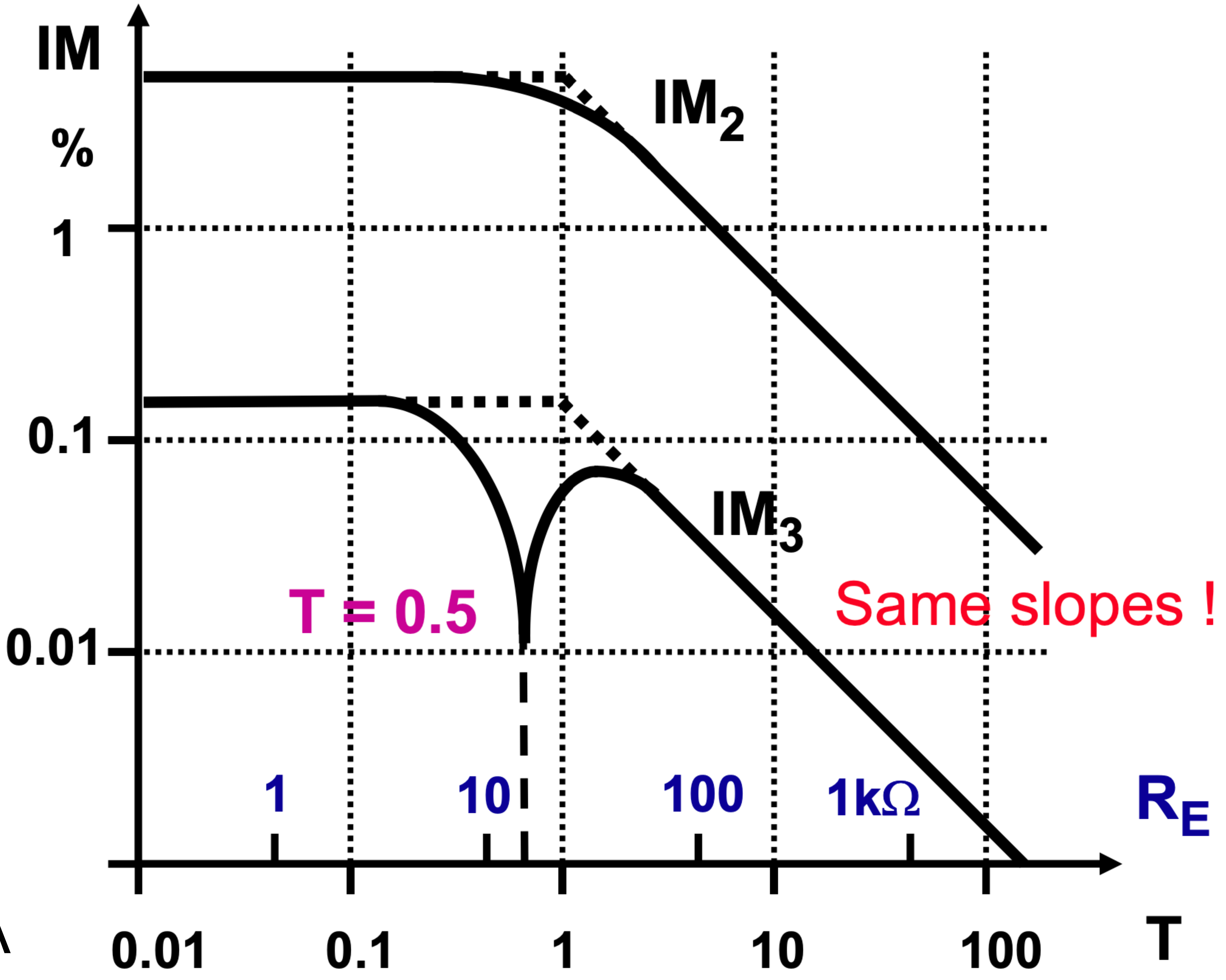
$$U = \frac{1}{(1 + T)} \frac{V_{in}}{kT_e/q}$$

is the relative current swing

Null for T = 0.5

IM_{3f} also decreases with T for constant U for large T !!

Emitter degeneration distortion versus T



I_{CE} = 1 mA

Null in IM_3 if

$$a_3 (1 + T) = 2f\, a_2^2$$

$$a_3 (1 + T) = 2T \frac{a_2^2}{a_1}$$

$$T = \frac{1}{\frac{2a_2^2}{a_1 a_3} - 1}$$

NO IM_{3f} for T = 0.5, but IM_{2f} not reduced!

Emitter degeneration for large T

$$U = \frac{1}{T}\,\frac{V_{in}}{kT_e/q} = \frac{V_{in}}{R_E I_{CE}}$$

$$IM_{2fT} = \frac{U}{2T} = \frac{V_{in}}{kT_e/q}\;\frac{1}{2\,T^2} = \frac{V_{in}\,kT_e/q}{2\,(R_E I_{CE})^2}$$

$$IM_{3fT} = \frac{U^2}{4T} = \left(\frac{V_{in}}{kT_e/q}\right)^2\frac{1}{4\,T^3} = \frac{V_{in}^2\;kT_e/q}{4\,(R_E I_{CE})^3}$$

Source degeneration reduces distortion

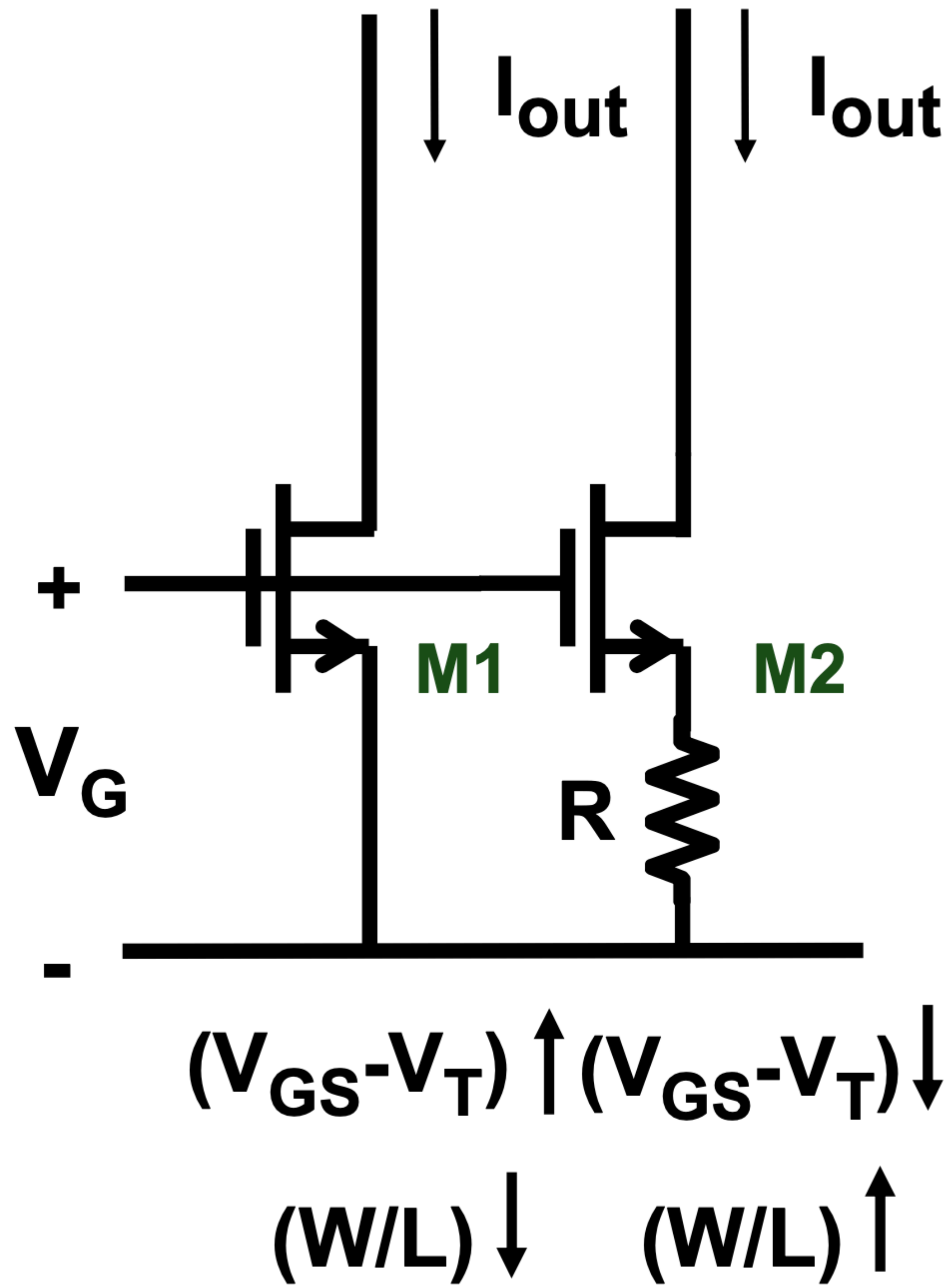
$$T = g_m R_S = \frac{V_{RS}}{(V_{GS}-V_T)/2} \qquad \frac{a_2}{a_1} = \frac{1}{4} \qquad a_3 = 0$$

$$U = \frac{1}{(1+T)} \frac{V_{in}}{(V_{GS}-V_T)/2}$$ is the relative current swing

$$IM_{2f} = \frac{1}{(1+T)} \frac{U}{4} \approx \frac{V_{in}}{(V_{GS}-V_T)/2} \frac{1}{4\,T^2} = \frac{V_{in}\,(V_{GS}-V_T)/2}{4\,(R_S I_{DS})^2}$$

$$IM_{3f} = \frac{T}{(1+T)^2} \frac{3U^2}{32} \approx \frac{V_{in}^{\,2}}{(V_{GS}-V_T)^2/4} \frac{3}{32T^3} = \frac{3V_{in}^{\,2}\,(V_{GS}-V_T)/2}{32\,(R_S I_{DS})^3}$$

High V_{GS} - V_T or source degeneration?



Same I_{out} & same V_G :

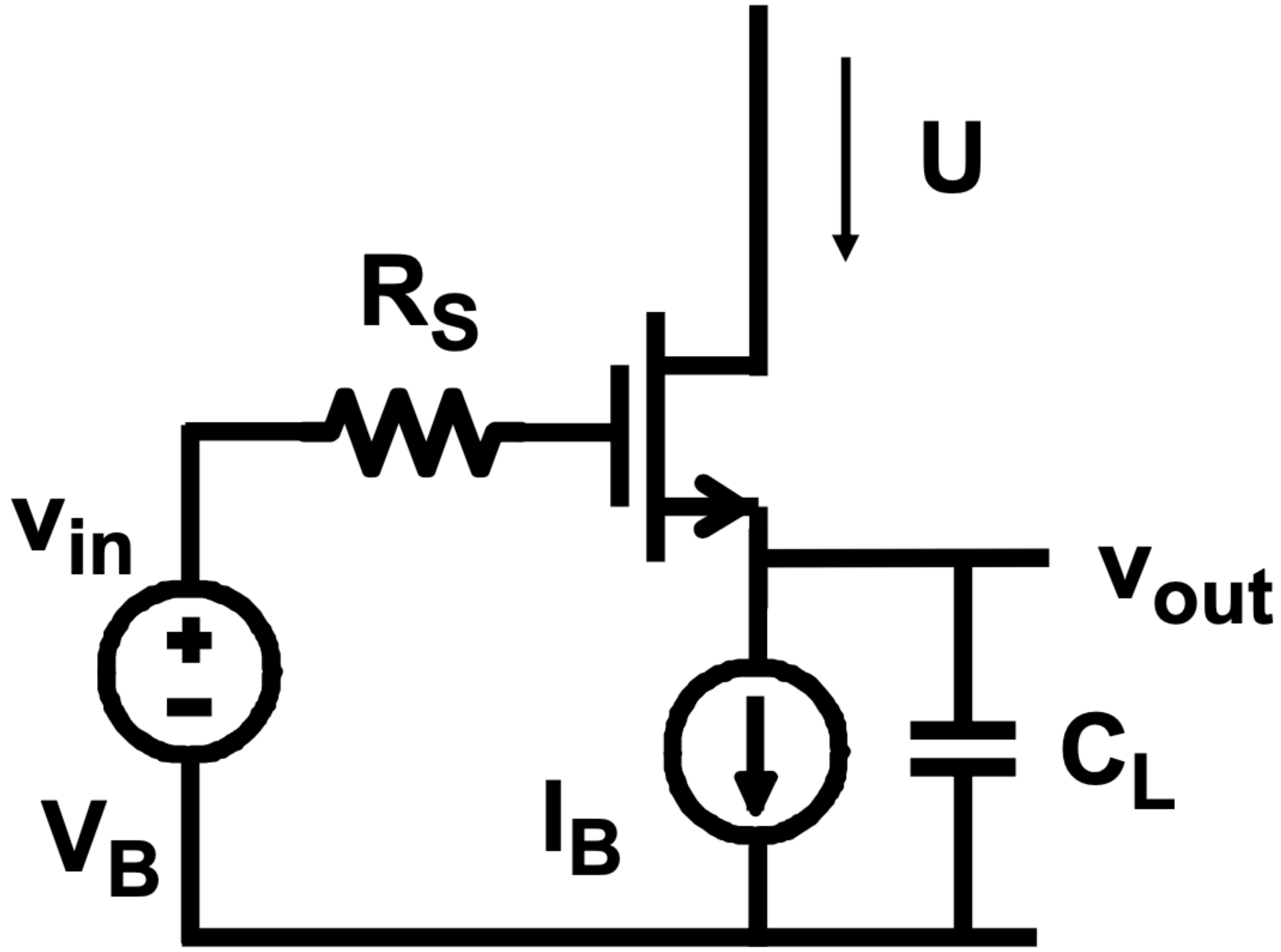
Same gain !
Same output noise !

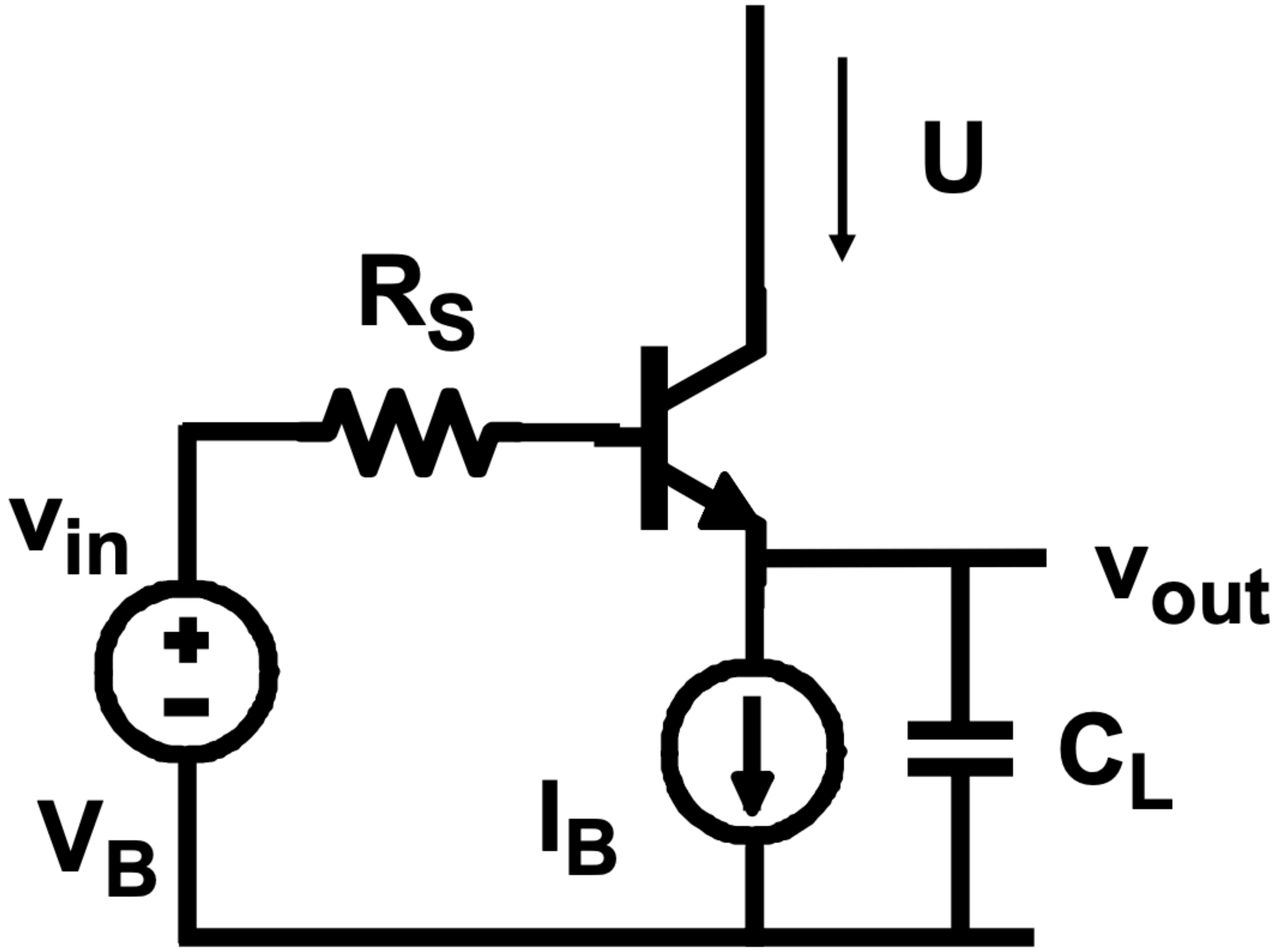
Same distortion ?

$$\frac{IM_{2f}}{IM_2} = \frac{1 - \frac{V_R}{V_{GST1}}}{\left(1 + \frac{V_R}{V_{GST1}}\right)^2}$$

IM_3? → high V_{GS} - V_T
IM_2? → high V_R

Distortion in followers

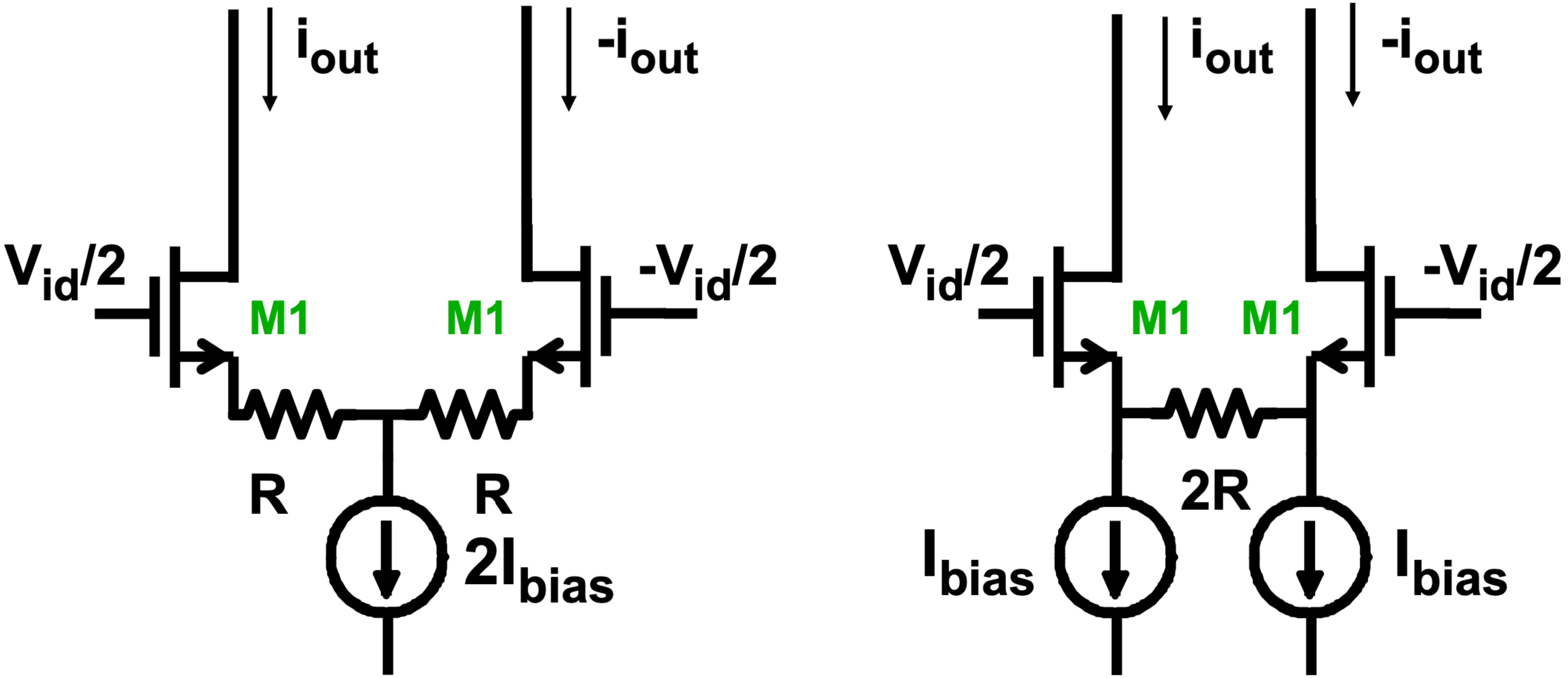




$$U = \frac{1}{g_m r_{DS}} \frac{V_{in}}{(V_{GS} - V_T)/2} = \frac{V_{in}}{V_{En}L} \qquad U = \frac{1}{g_m\, r_o} \frac{V_{in}}{kT_e/q} = \frac{V_{in}}{V_E}$$

If v_{BS} = 0 !!

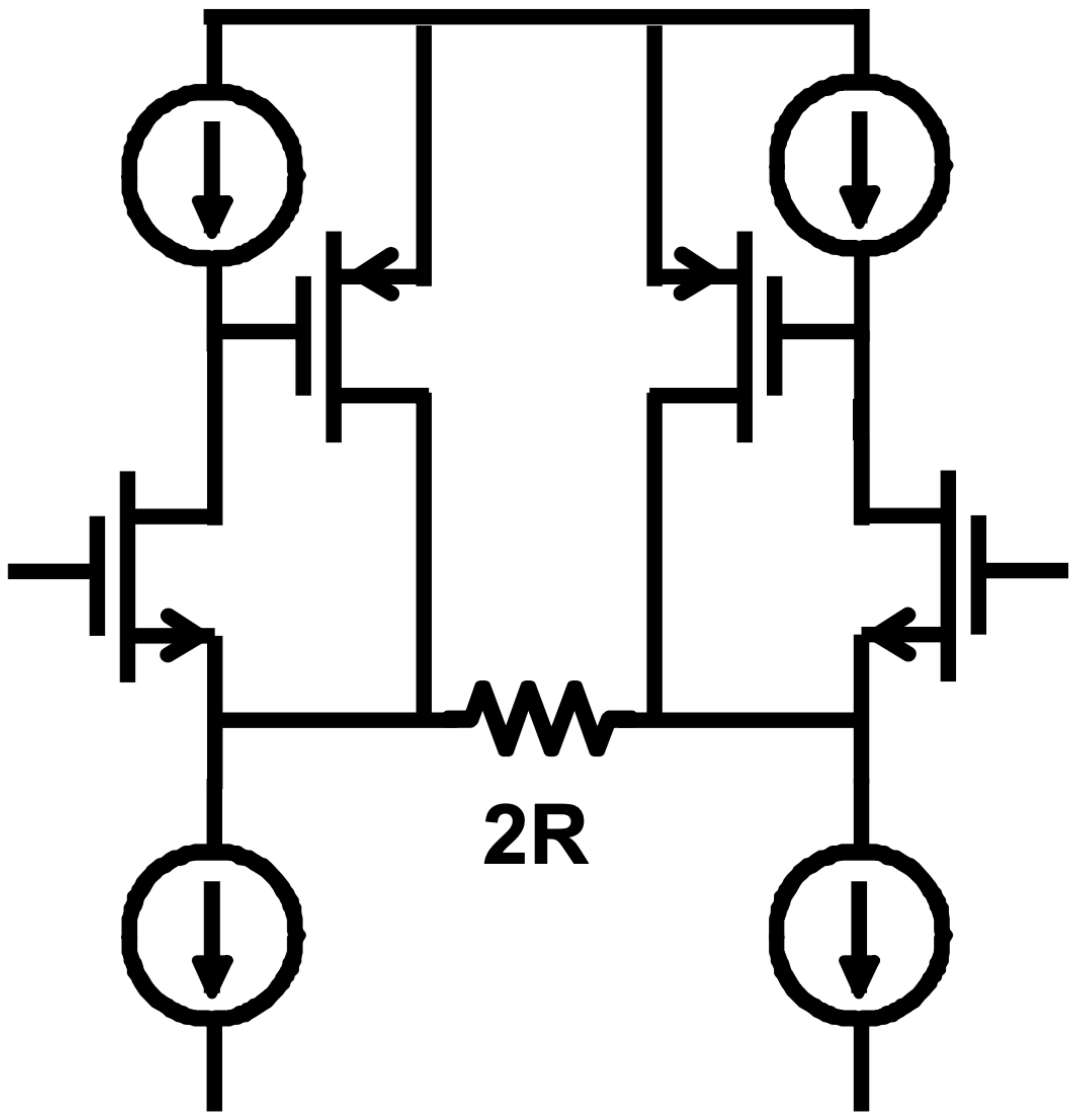
Degeneration of differential pair



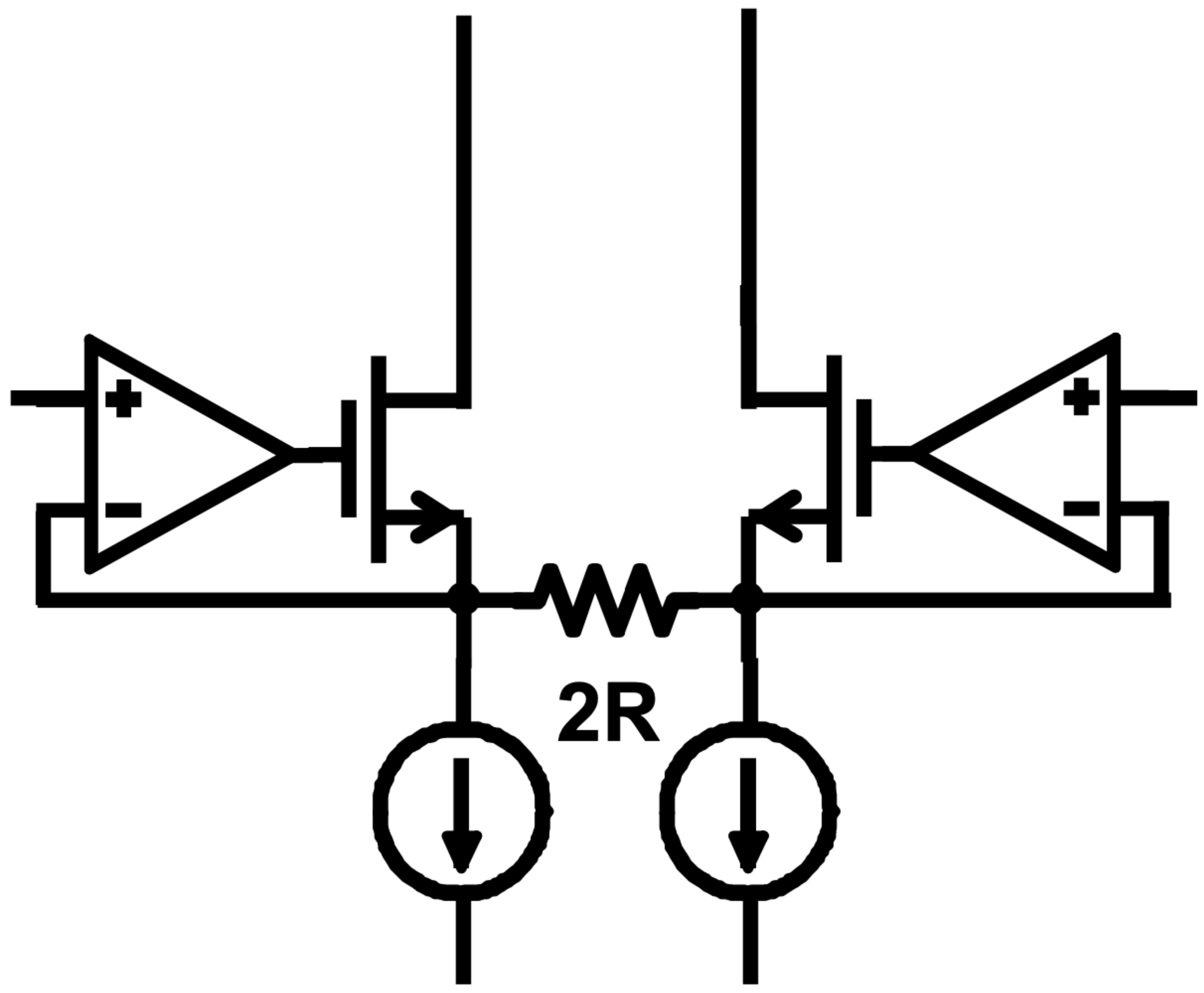
$IP_3 \approx 3.3\,(V_{GS}-V_T)(1+g_{m1}R)^2 \quad HD_3/n^2 \quad n= 1+g_{m1}R$

HD_3 = - 60 dB for V_{id} = 1 V requires V_{GS}-V_T = 0.38 V and $g_{m1}R$ = 3 !!!

More feedback reduces distortion



Additional local FB



More FB with opamps

Outline

- Definitions and metrics of distortion
- Distortion in components
- **Distortion reduction**
  - Feedback
  - **Cancellation**
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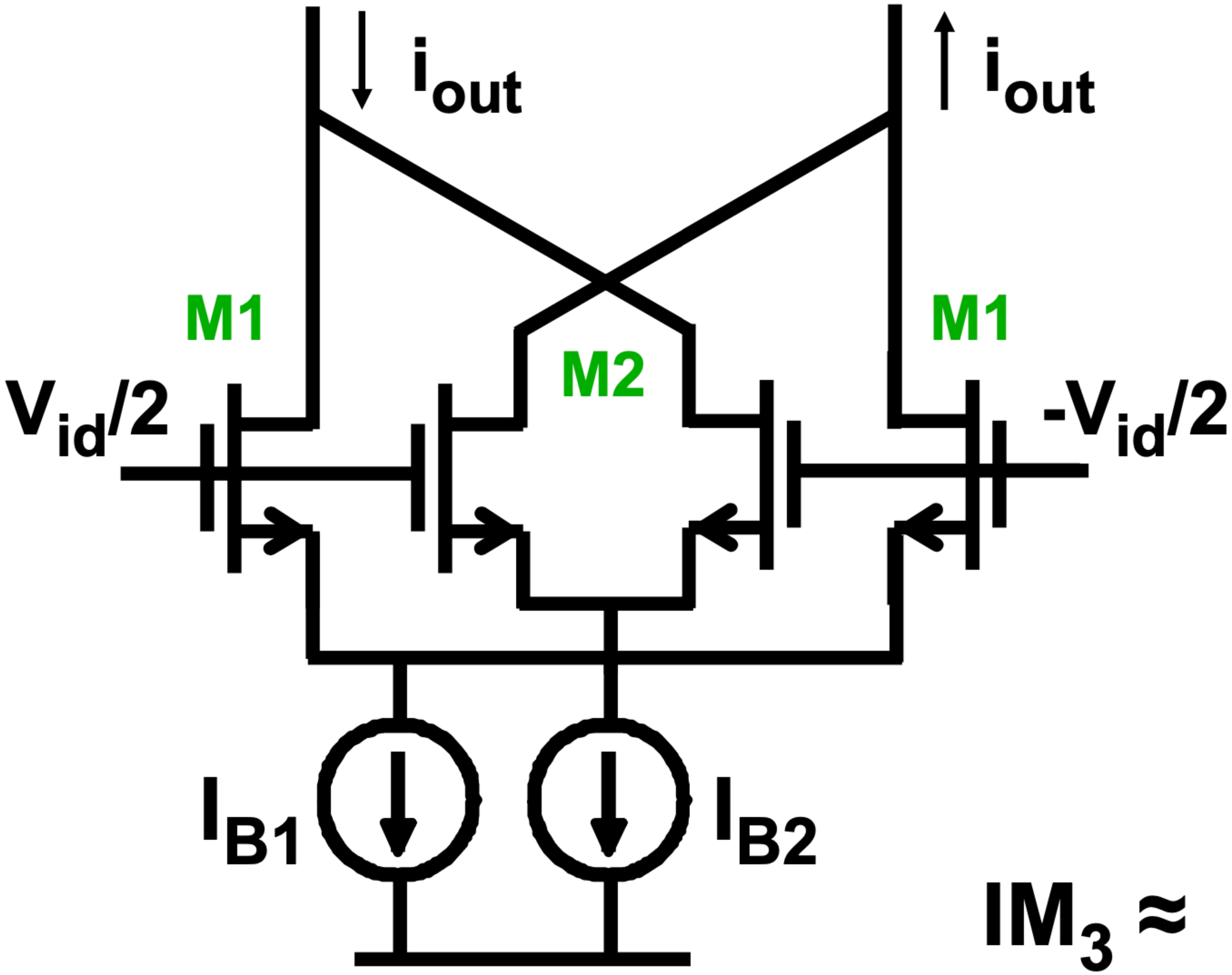
Cancelling distortion by subtraction



Parameters :

$\alpha = I_{B2} / I_{B1}$

≈ 0.25

$v = V_{GST1} / V_{GST2}$

≈ 1.6

$V_{GST} = V_{GS} - V_T$

$IM_3 \approx 0$ if $v = \alpha^{-1/3}$

then $i_{out} = g_{m1} V_{id} (1 - \alpha^{2/3})$

Condition for zero IM_3

$$i_{out} = 2\,(i_{DS1} - i_{DS2})$$

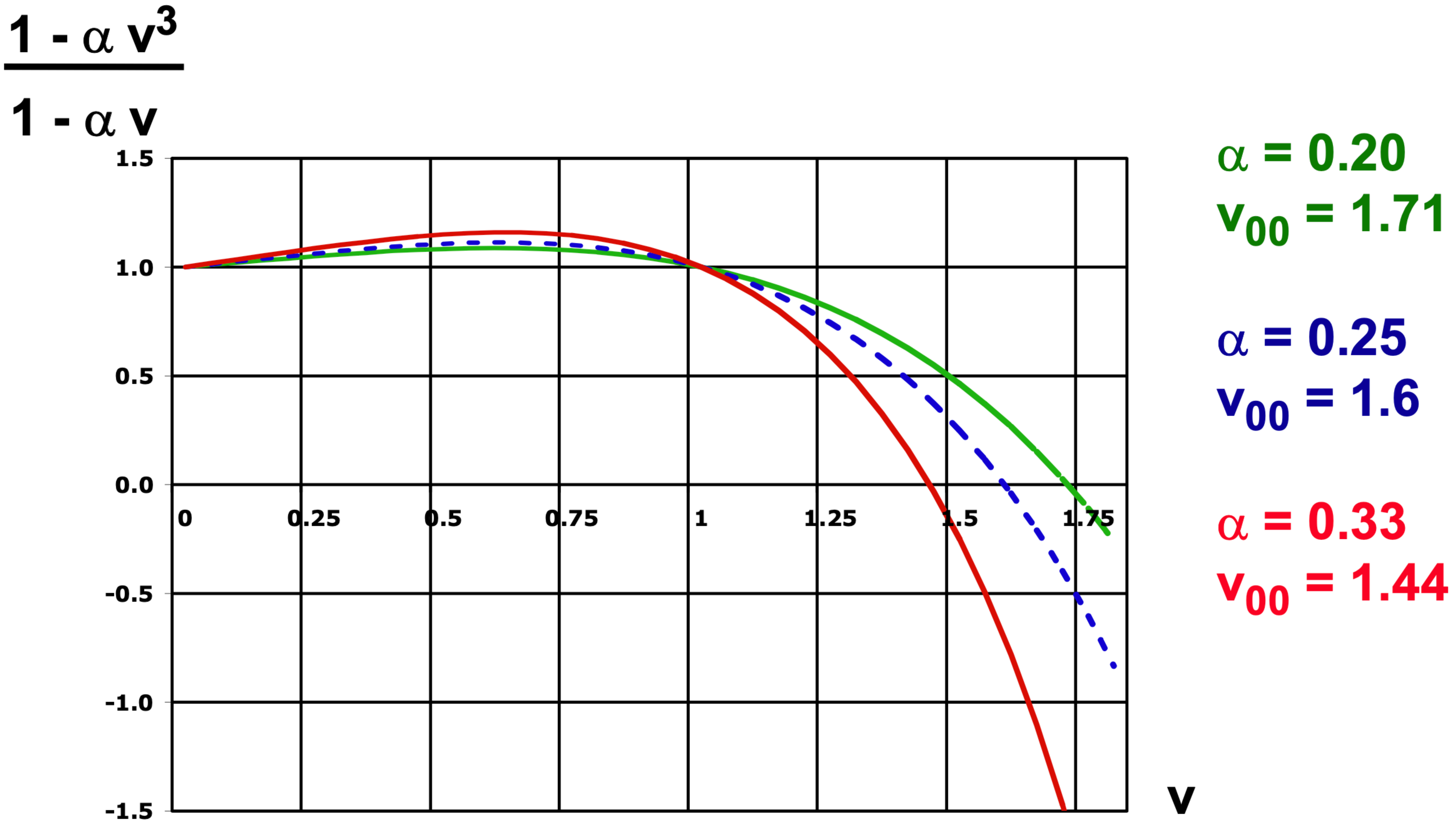
$$\frac{i_{DS}}{I_B} = U - \frac{1}{8}U^3 \qquad U = \frac{V_{id}}{V_{GS} - V_T} \qquad IM_3 = \frac{3}{32}U^2$$

$$IM_3 \approx \frac{3}{32}\left(\frac{V_{id}}{V_{GS1} - V_T}\right)^2 \frac{1 - \alpha\, v^3}{1 - \alpha\, v}$$

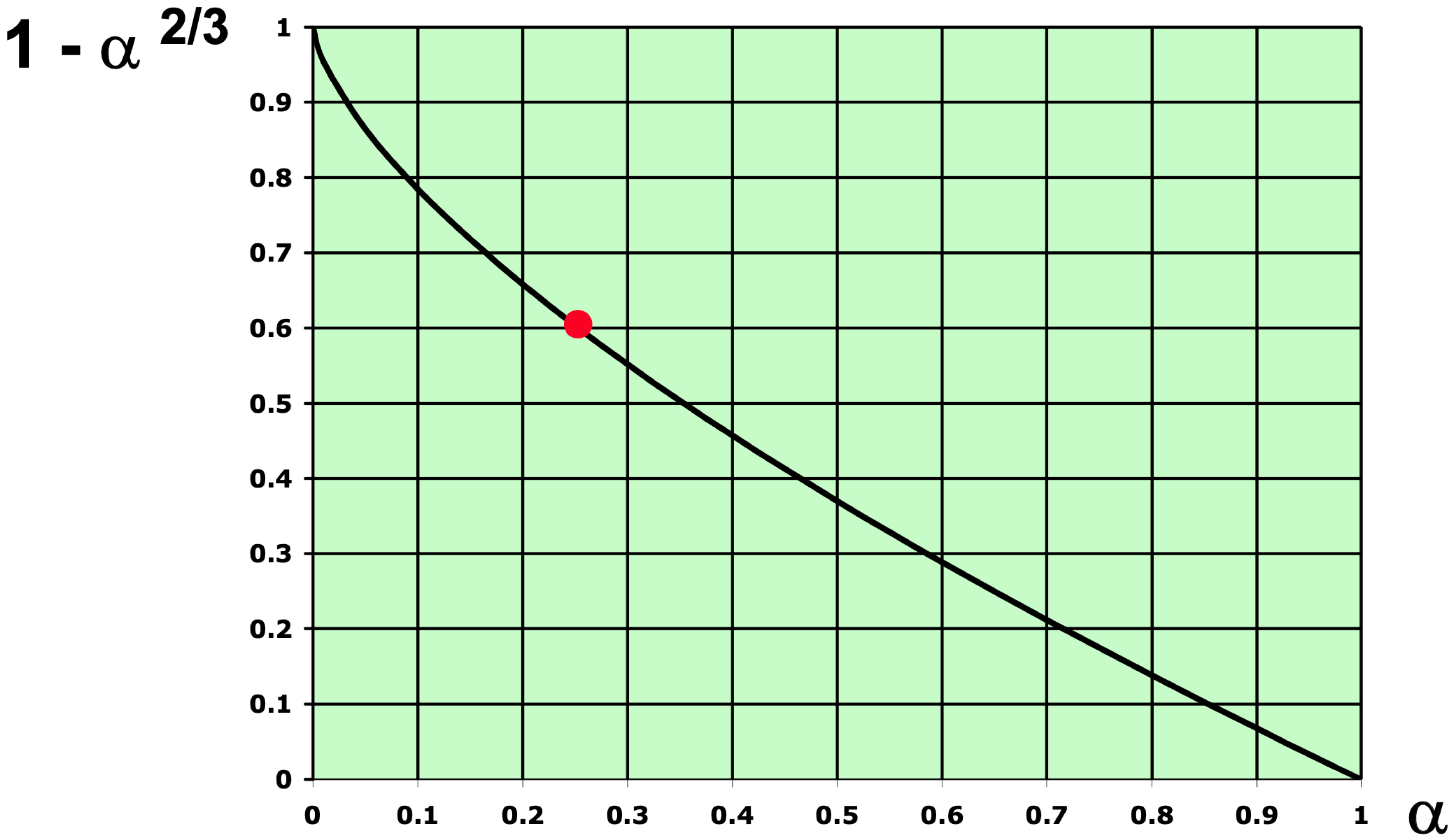
$$IM_3 \approx 0 \quad \text{if} \quad v_{00} = \alpha^{-1/3}$$

at which point $i_{out} = g_{m1}\, V_{id}\, (1 - \alpha^{2/3})$

Zero IM_3 for different α



Subtracting currents reduces signal



$\alpha \approx 0.25$

$v_{00} = 1.6$

x 0.6

Outline

- Definitions and metrics of distortion
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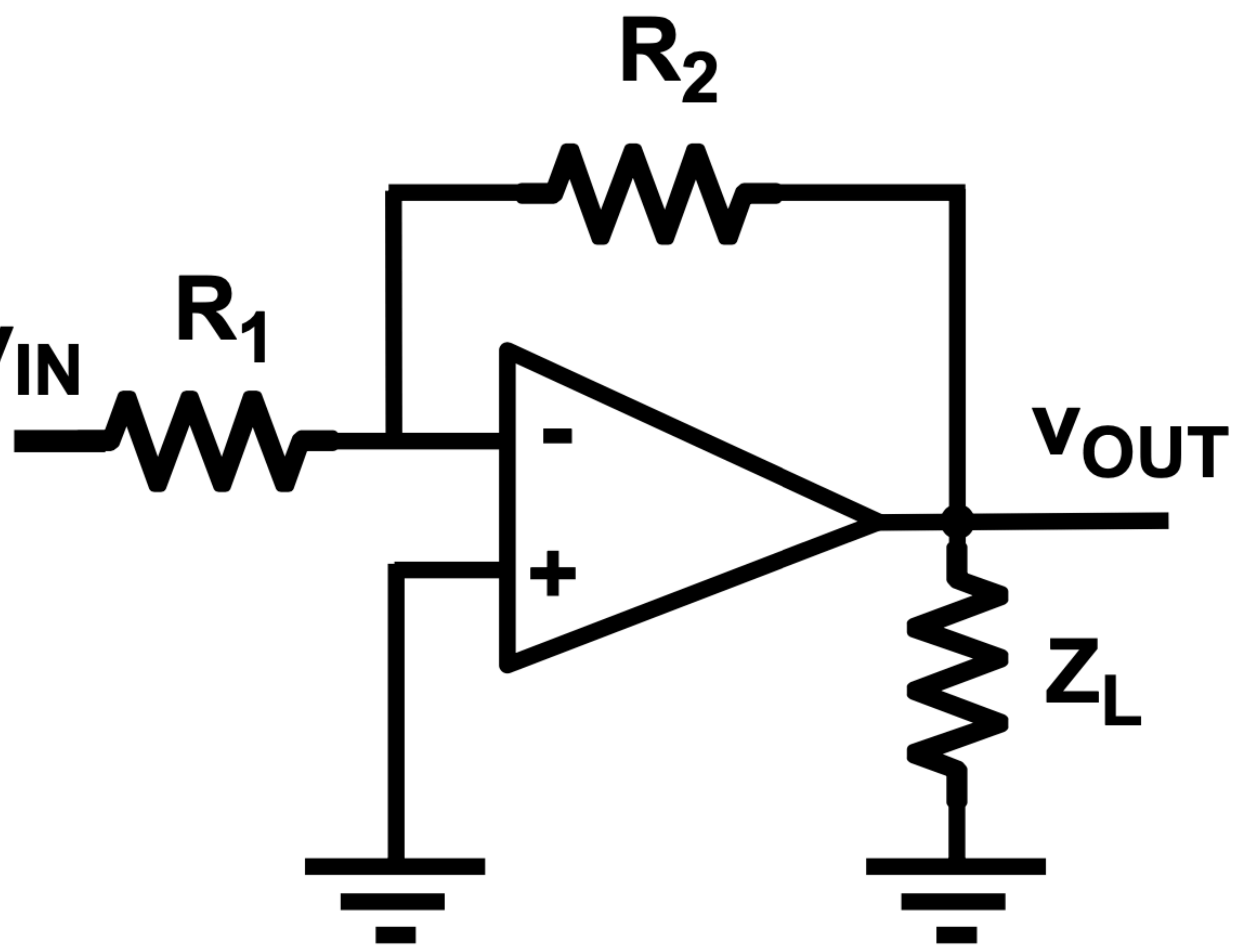
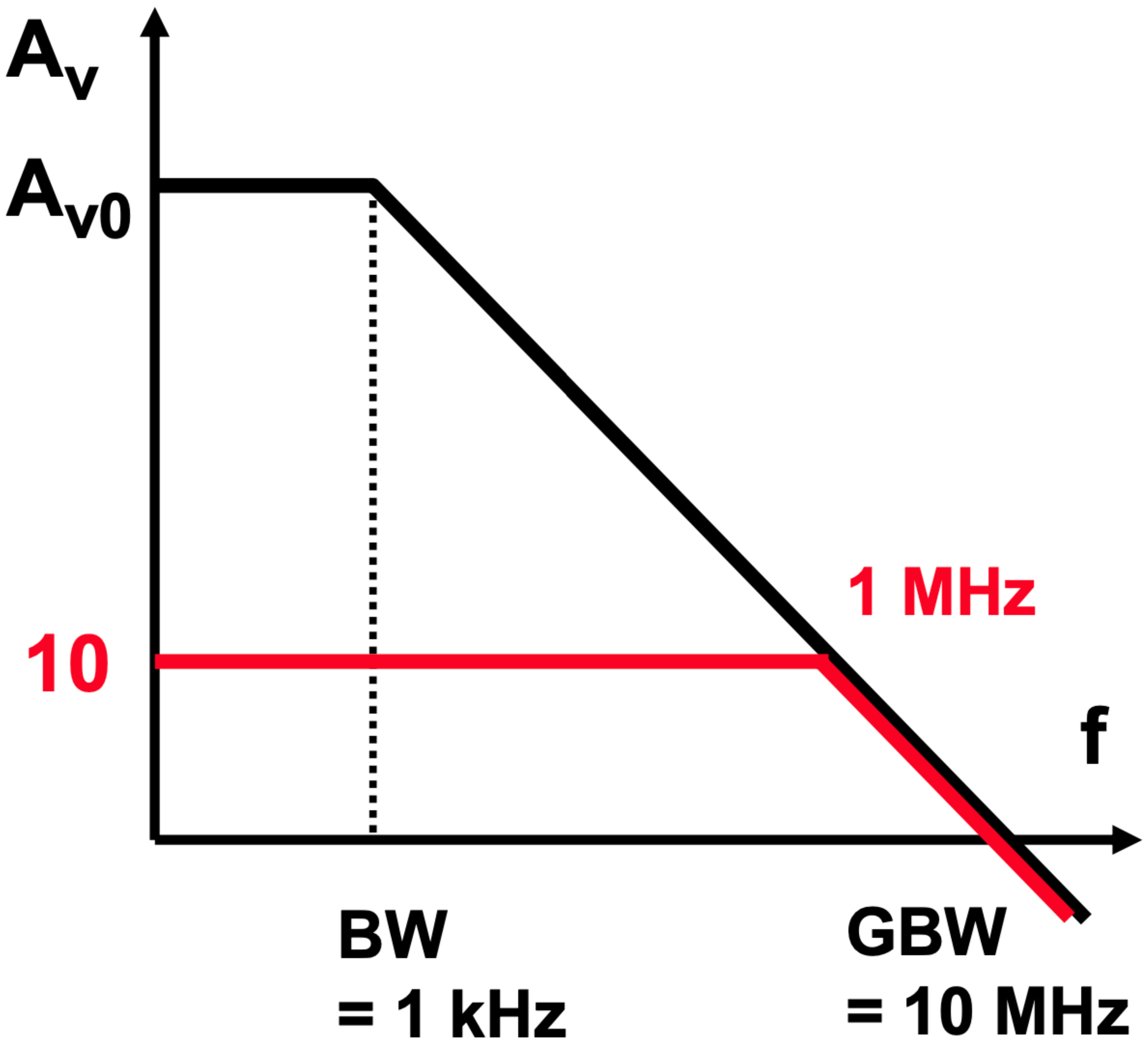
Miller CMOS OTA with feedback

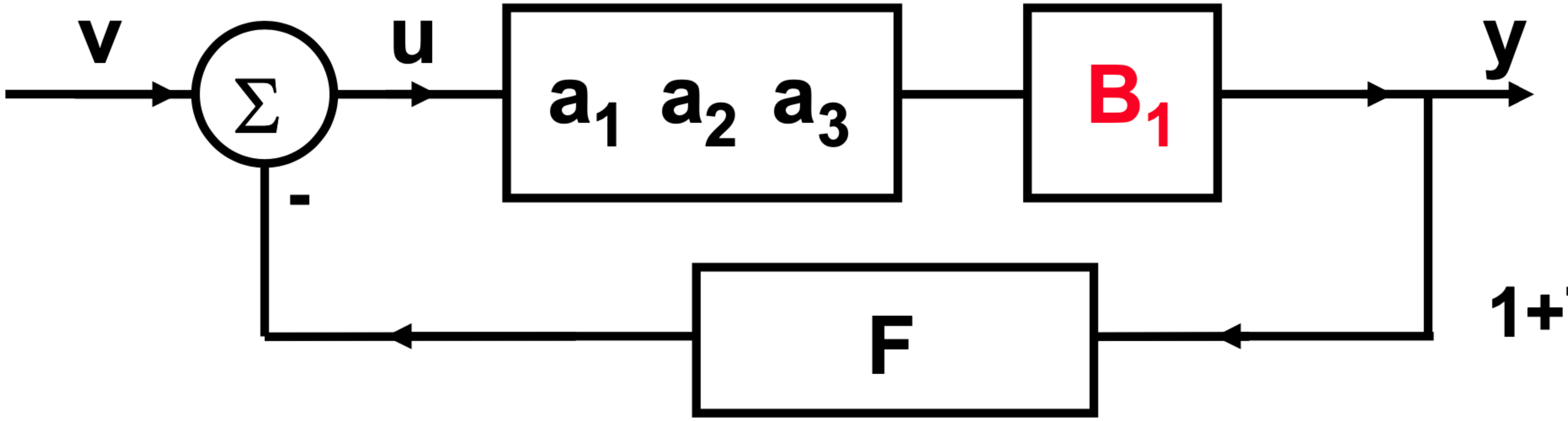




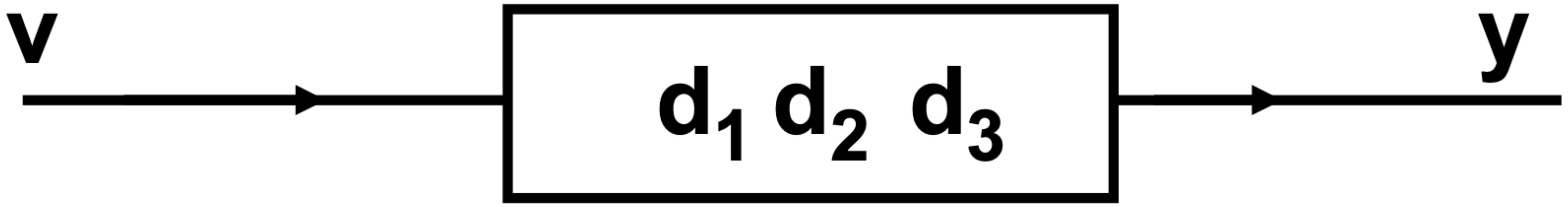
GBW = 10 MHz & A_{vc} = 10

Z_L = 100 kΩ//5pF

Distortion in first stage



$1+T = 1 + B_1 a_1 F$



$$\left.\begin{array}{l} \left.\begin{array}{l} u = v - Fy \\ y = B_1(a_1 u + a_2\, u^2 + a_3\, u^3) \end{array}\right\} \text{elim. } u \\ y = d_1 v + d_2\, v^2 + d_3\, v^3 \end{array}\right\} \text{elim. } y \left\{\begin{array}{l} \text{coeff } v \;: d_1 \\ \text{coeff } v^2 : d_2 \\ \text{coeff } v^3 : d_3 \end{array}\right.$$

IM_{2f} and IM_{3f} of first stage

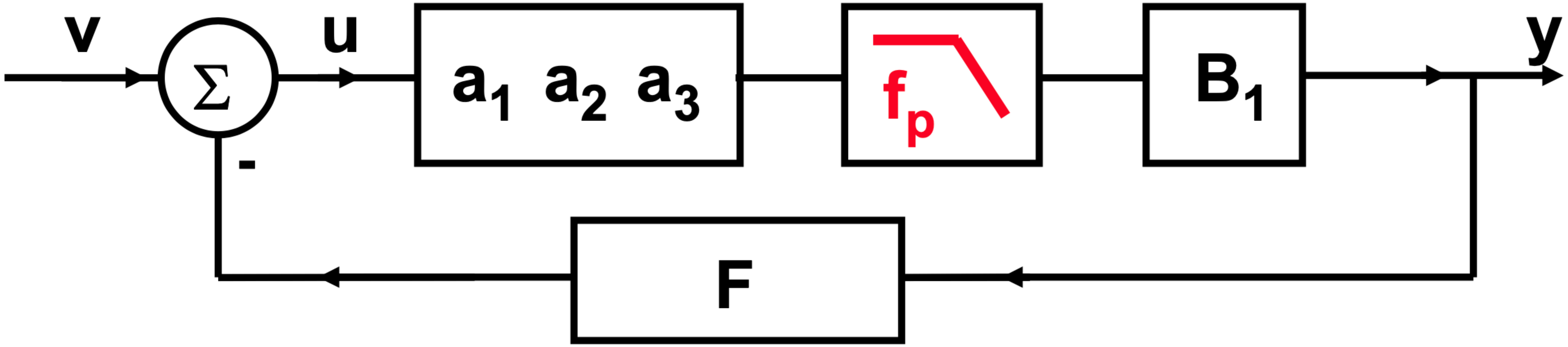
$$IM_{2f} = \frac{d_2}{d_1} V = \frac{a_2}{a_1} \frac{V}{(1+T)^2} = \frac{a_2}{a_1} \frac{1}{(1+T)} \frac{V}{(1+T)}$$

$$IM_{3f} = \frac{3}{4} \frac{d_3}{d_1} V^2 = \frac{3}{4} \left[\frac{a_3}{a_1} \frac{1}{(1+T)} - \left(\frac{a_2}{a_1} \right)^2 \frac{2T}{(1+T)^2} \right] \frac{V^2}{(1+T)^2}$$

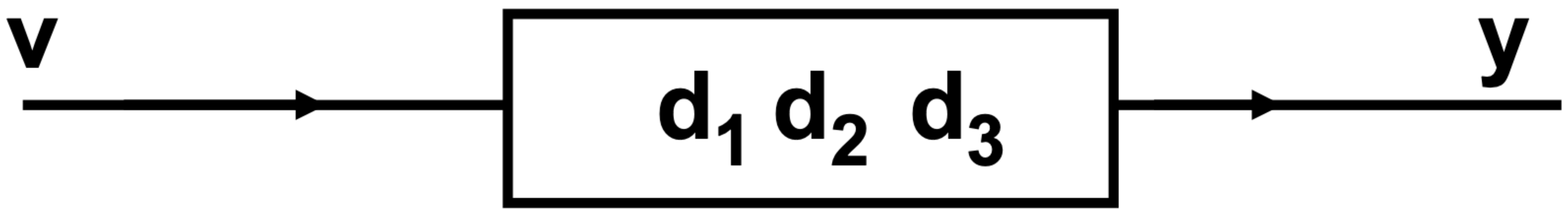
Same as before but with different Loop gain :

$$1+T = 1 + B_1 a_1 F$$

Distortion in first stage with f_p



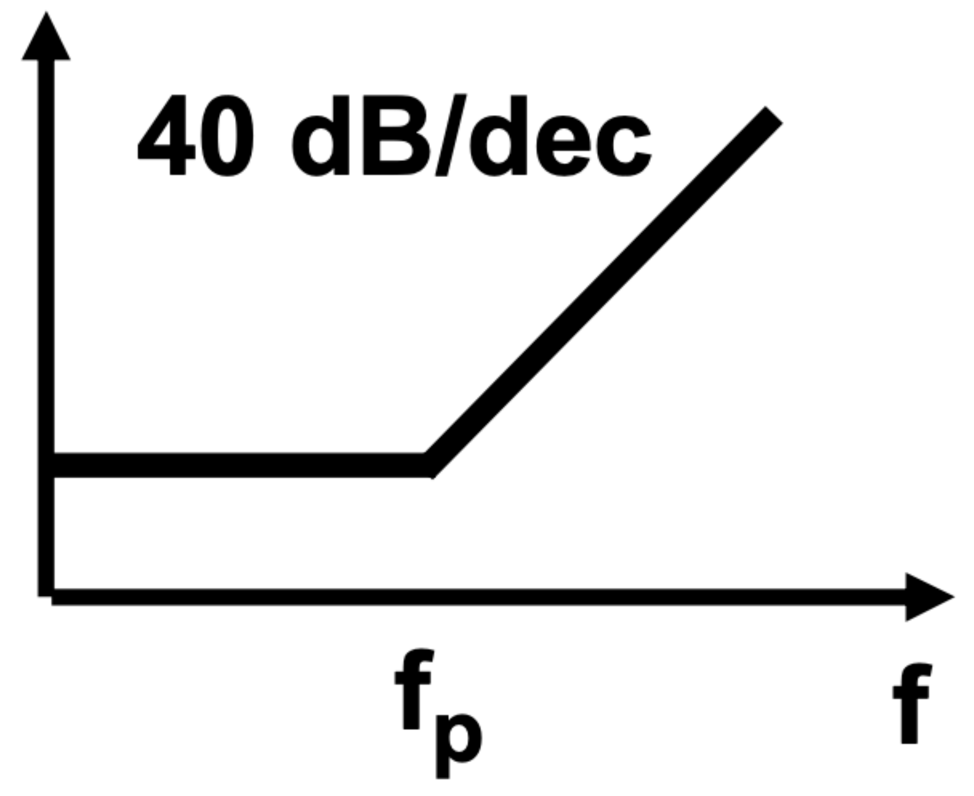
$$1+T = 1 + B_{1p}a_1F$$



$$\left.\begin{array}{l} \left.\begin{array}{l} u = v - Fy \\ y = B_{1p}(a_1u + a_2\,u^2 + a_3\,u^3) \end{array}\right\}\text{elim. } u \\ y = d_1v + d_2\,v^2 + d_3\,v^3 \end{array}\right\}\text{elim. } y \begin{cases} \text{coeff } v : d_1 \\ \text{coeff } v^2 : d_2 \\ \text{coeff } v^3 : d_3 \end{cases}$$

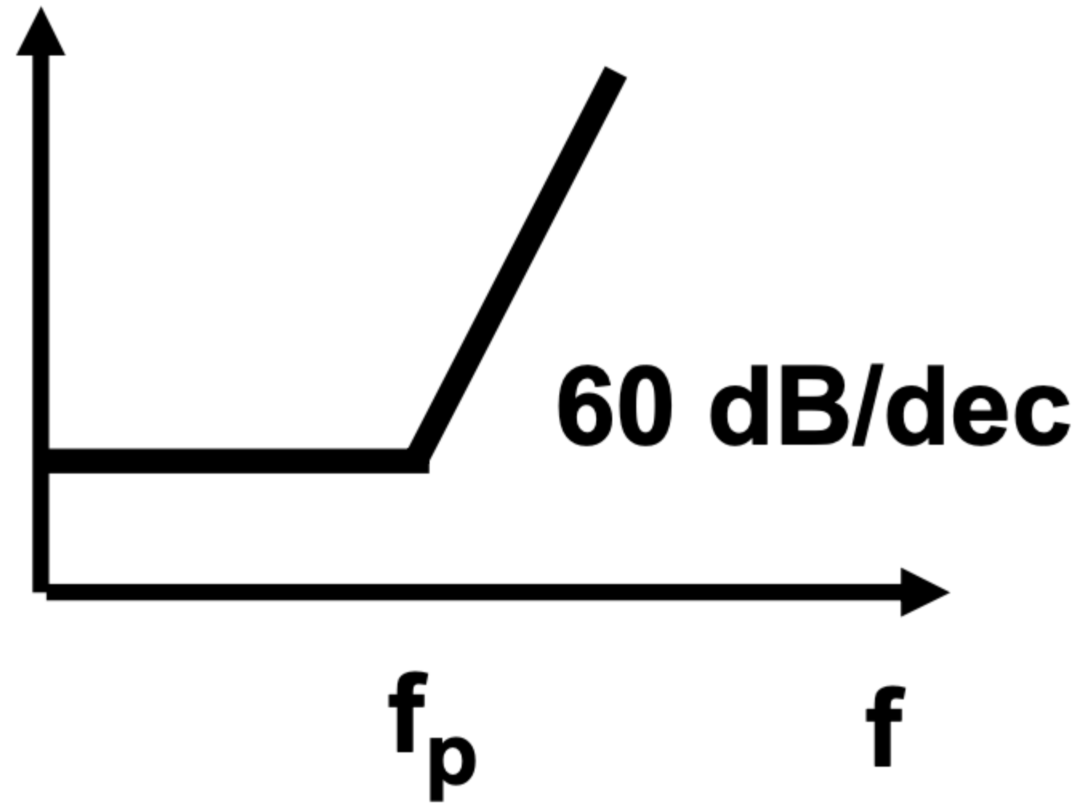
IM_{2f} and IM_{3f} of first stage with f_p

$$IM_{2f} = \frac{a_2}{a_1}\frac{V}{(1+T)^2} = \frac{a_2}{a_1}\frac{1}{(B_{1p}a_1F)^2}V$$

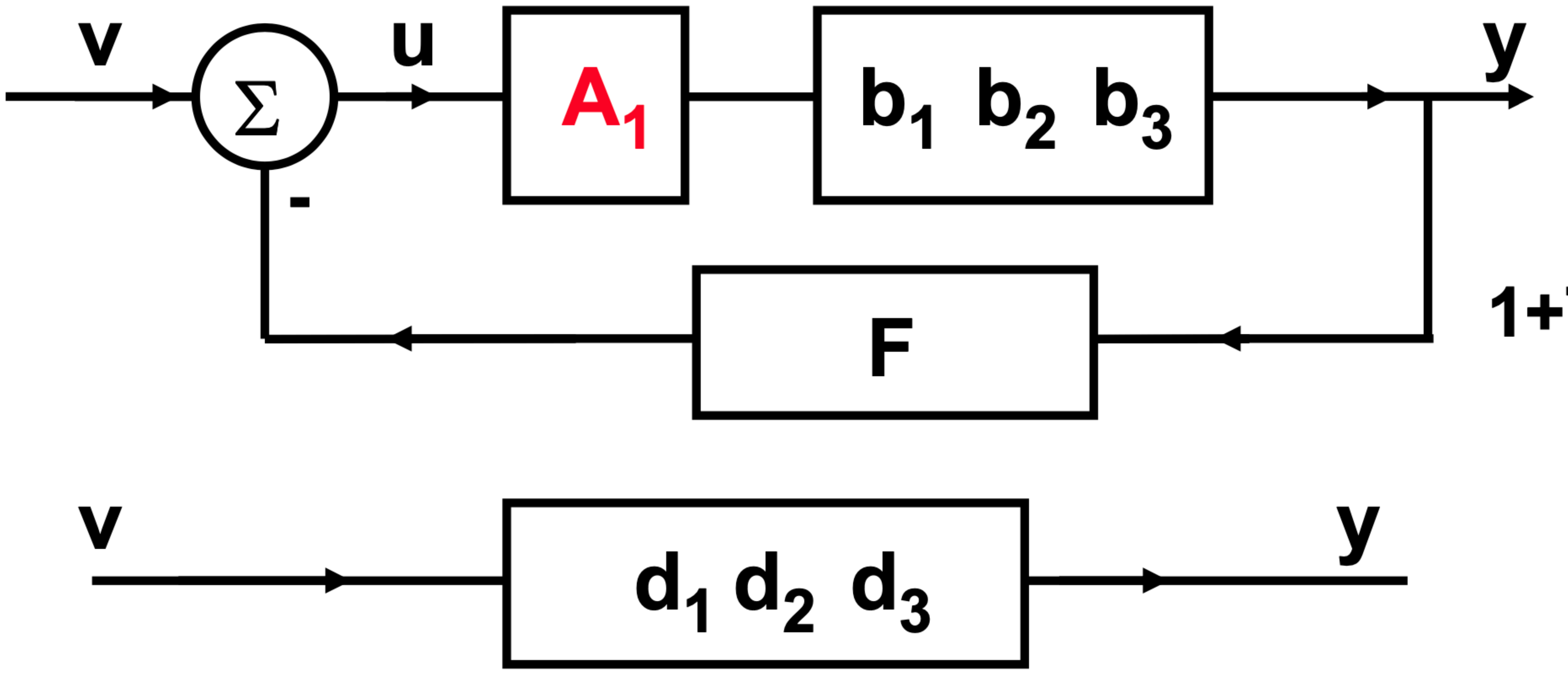


$$IM_{3f}\Big|_{\text{diff.pair}} = \frac{3}{4}\frac{a_3}{a_1}\frac{1}{(1+T)}\frac{V^2}{(1+T)^2} = \frac{3}{4}\frac{a_3}{a_1}\frac{1}{(B_{1p}a_1F)^3}V^2$$

$$IM_{3f}\Big|_{\text{Single trans.}} = \frac{3}{4}\frac{{a_2}^2}{{a_1}^2}\frac{2}{(B_{1p}a_1F)^3}V^2$$



Distortion in second stage



$1+T = 1 + A_1 b_1 F$

$$\left.\begin{array}{l} u = v - Fy \\ y = A_1 b_1 u + A_1^2 b_2 u^2 + A_1^3 b_3 u^3 \end{array}\right\} \text{elim. } u$$

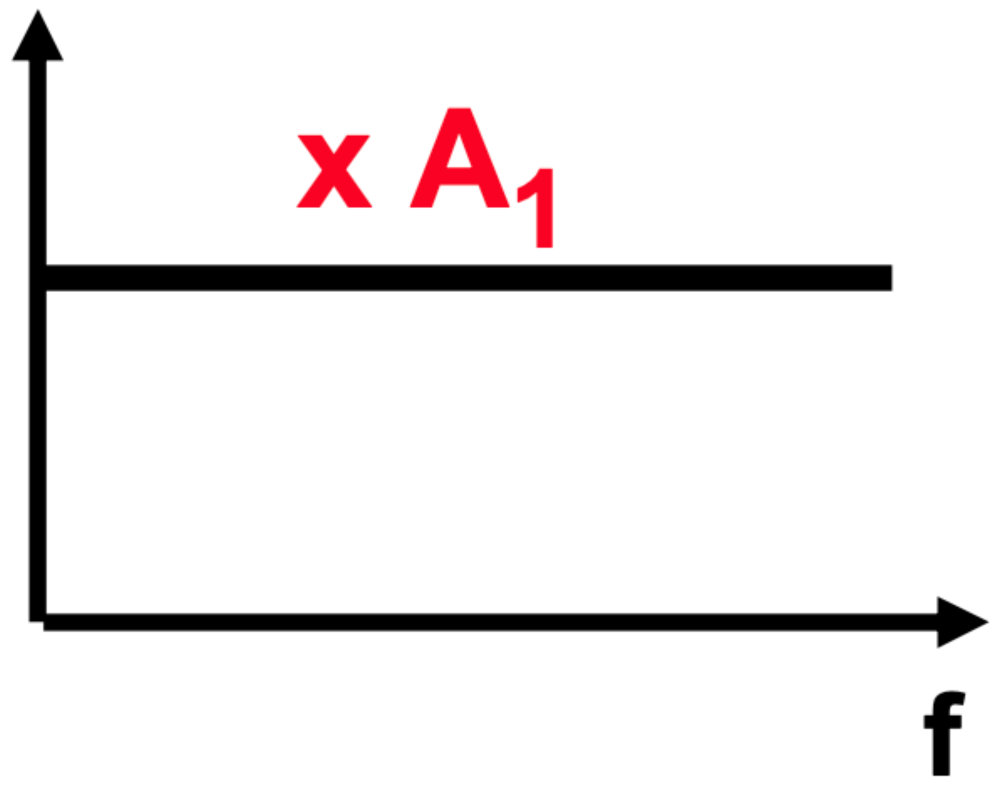
$$y = d_1 v + d_2 v^2 + d_3 v^3$$

elim. y

$$\begin{cases} \text{coeff } v : d_1 \\ \text{coeff } v^2 : d_2 \\ \text{coeff } v^3 : d_3 \end{cases}$$

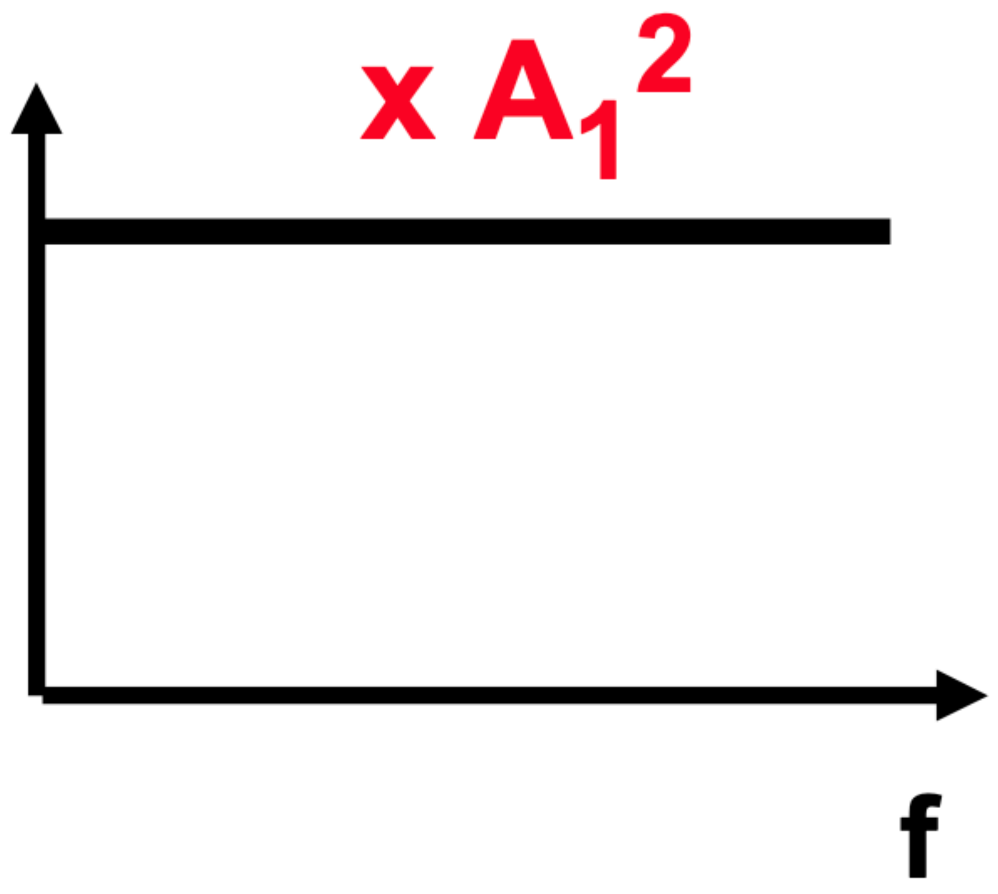
IM_{2f} and IM_{3f} of second stage

$$IM_{2f} = \frac{b_2}{b_1}\,\frac{V}{(1+T)^2} = \frac{b_2}{b_1}\,\frac{A_1}{(A_1 b_1 F)^2}\,V$$

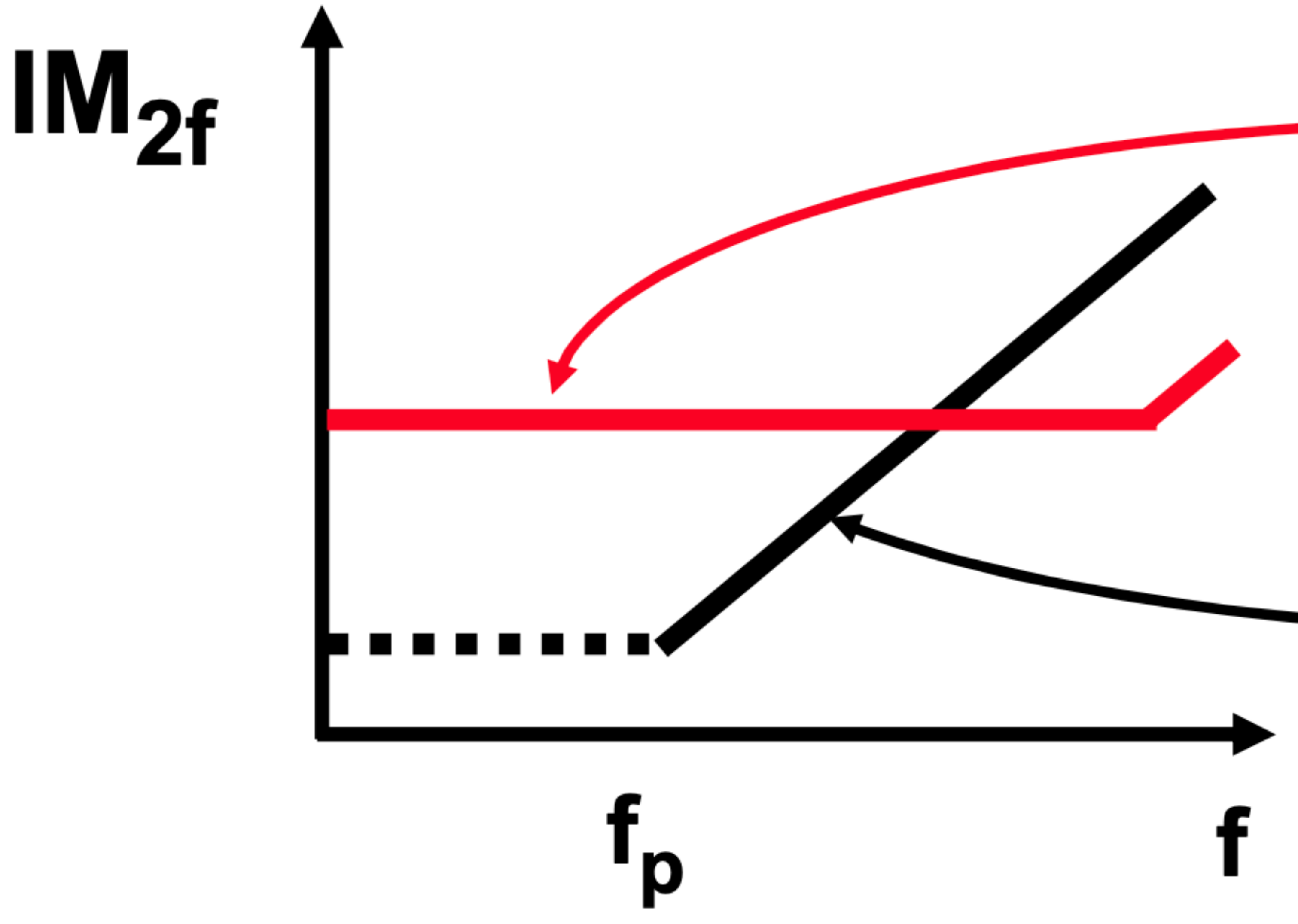


$$IM_{3f}\Big|_{\text{Single trans.}} = \frac{3}{4}\left(\frac{b_2}{b_1}\right)^2 \frac{2T}{(1+T)^2}\,\frac{V^2}{(1+T)^2}$$

$$= \frac{3}{4}\,\frac{b_2^{\,2}}{b_1^{\,2}}\,\frac{2\,A_1^{\,2}}{(A_1 b_1 F)^3}\,V^2$$



IM_{2f} and IM_{3f} of both stages



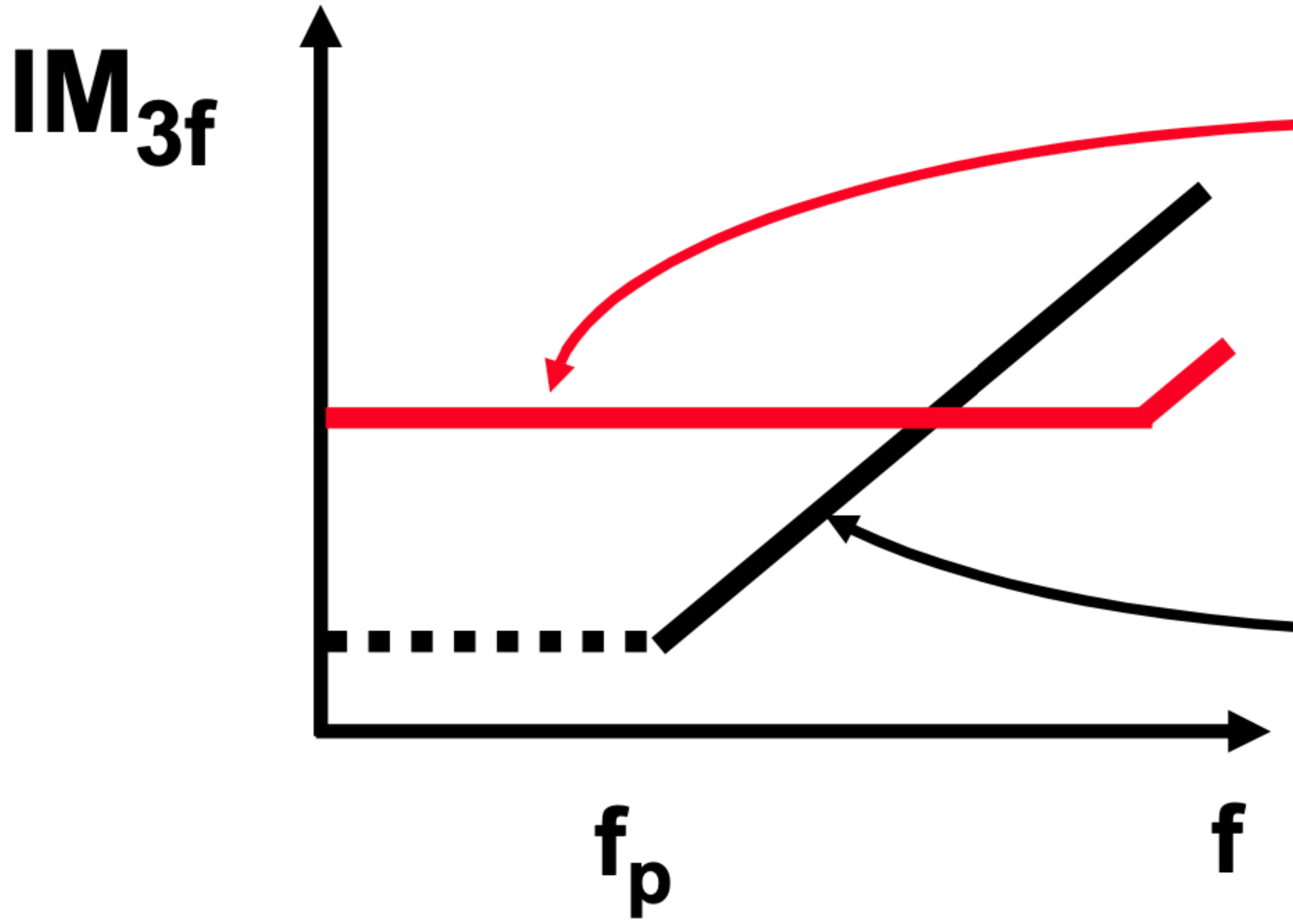
$$\frac{b_2}{b_1}\, a_1 \,\frac{V}{T^2}$$

$$T = a_1 b_1 F$$

$$\frac{a_2}{a_1}\,\frac{V}{T^2}\left(\frac{f}{f_p}\right)^2$$

Output stage dominates at low frequencies

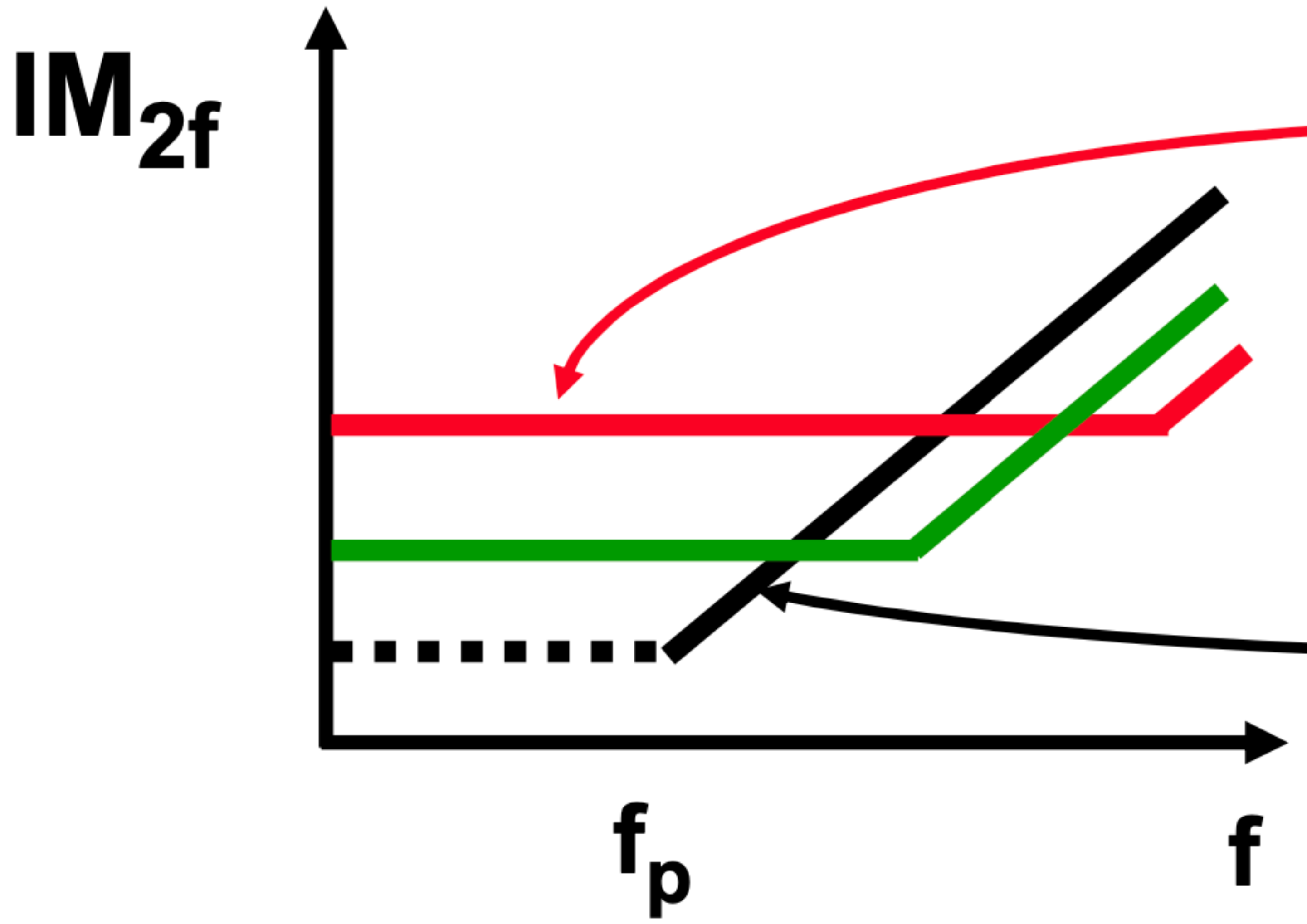
Input stage dominates at high frequencies



$$\frac{3}{4}\,\frac{b_3}{b_1}\, a_1^2 \,\frac{V^2}{T^3}\left(\frac{b_1 b_3 - 2\, b_2^2}{b_1 b_3}\right)$$

$$\frac{3}{4}\,\frac{a_3}{a_1}\,\frac{V^2}{T^3}\left(\frac{f}{f_p}\right)^3\left(\frac{a_1 a_3 - 2\, a_2^2}{a_1 a_3}\right)$$

IM_{2f} and IM_{3f} with three stages

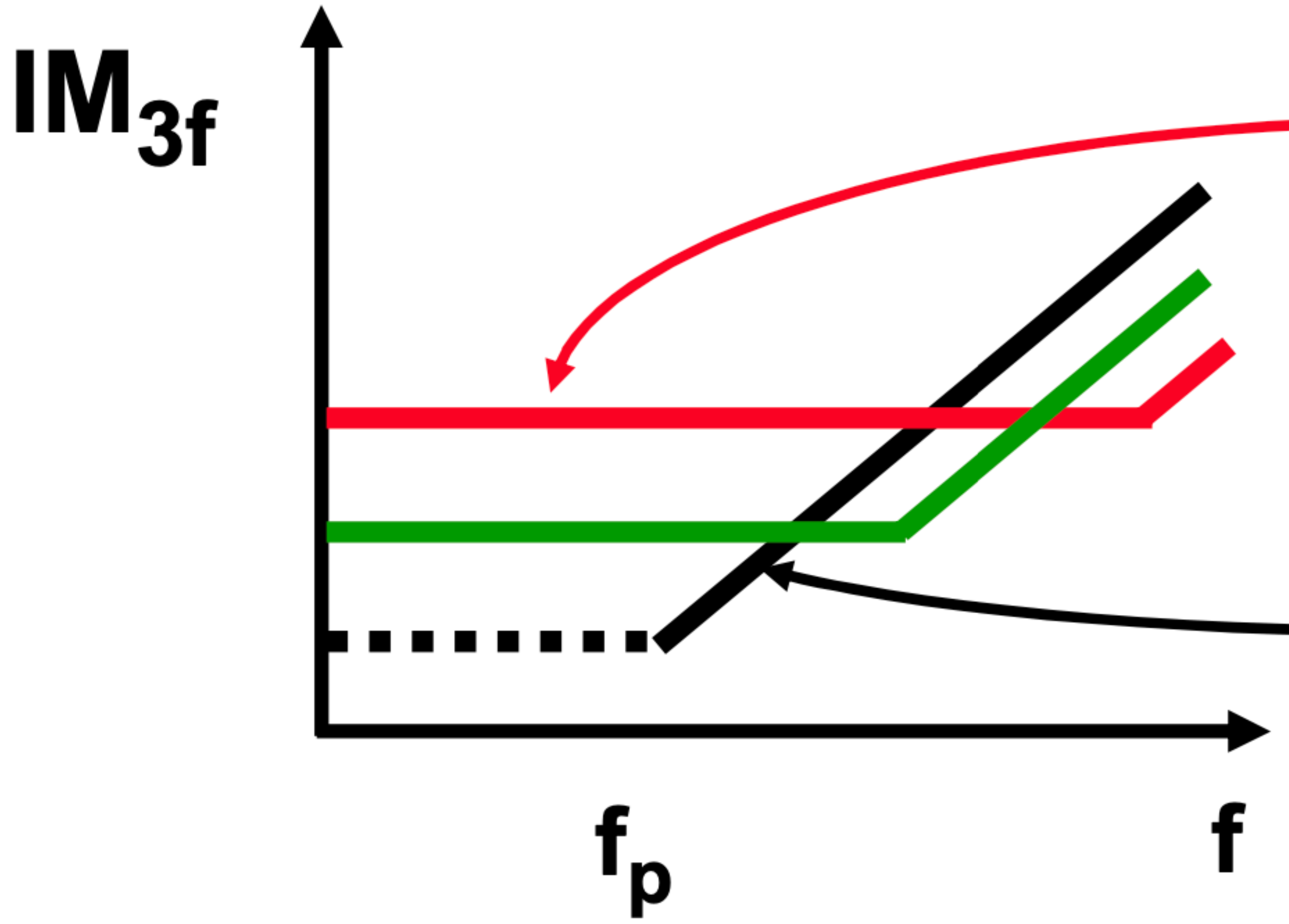


$$\frac{c_2}{c_1} a_1 b_1 \frac{V}{T^2}$$

$$T = a_1 b_1 c_1 F$$

$$\frac{a_2}{a_1} \frac{V}{T^2} \left(\frac{f}{f_p}\right)^2$$

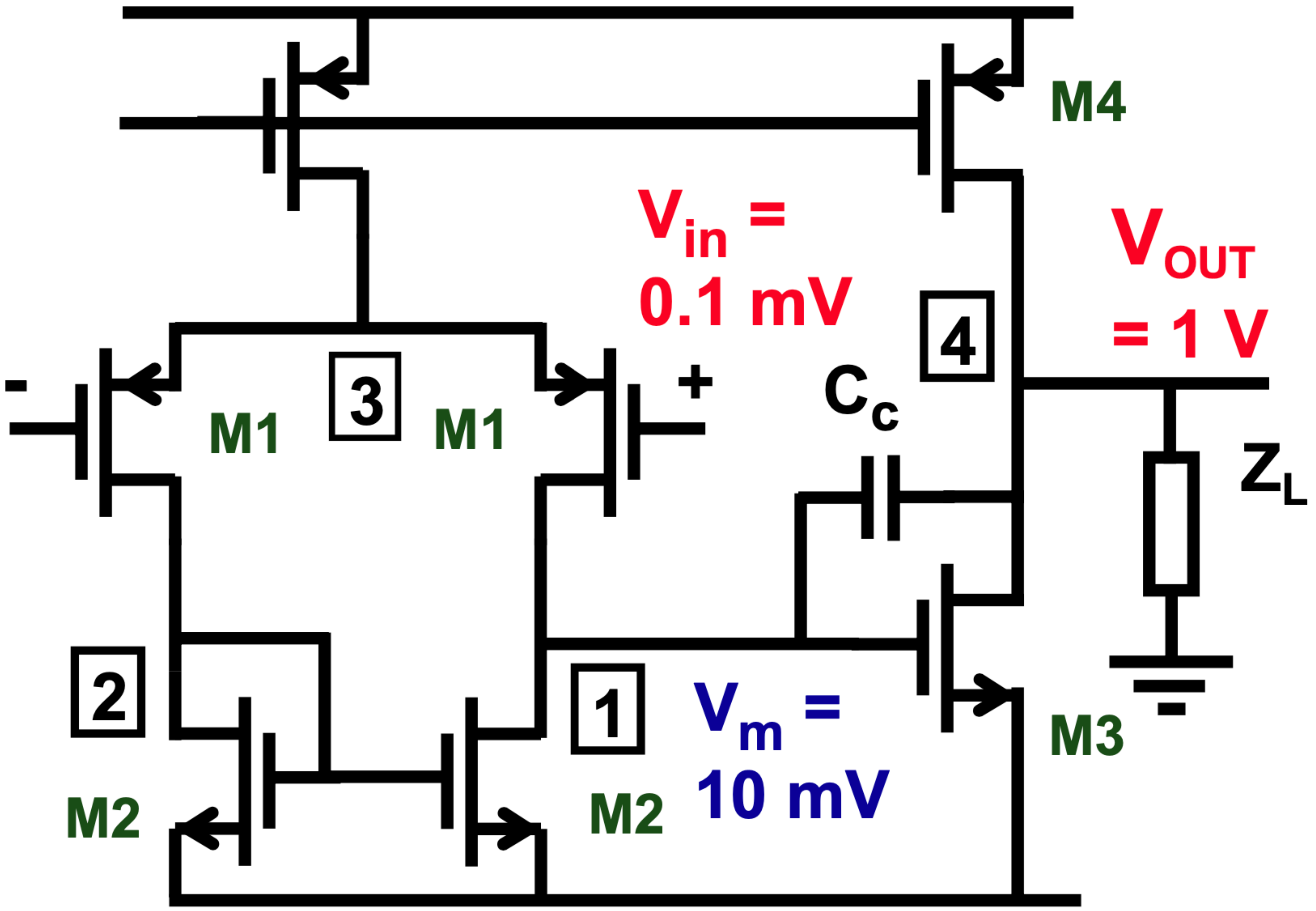
Intermediate stage never dominates



$$\frac{3}{4} \frac{c_3}{c_1} a_1^2 b_1^2 \frac{V^2}{T^3} \left(\frac{c_1 c_3 - 2\, c_2^2}{c_1 c_3}\right)$$

$$\frac{3}{4} \frac{a_3}{a_1} \frac{V^2}{T^3} \left(\frac{f}{f_p}\right)^3 \left(\frac{a_1 a_3 - 2\, a_2^2}{a_1 a_3}\right)$$

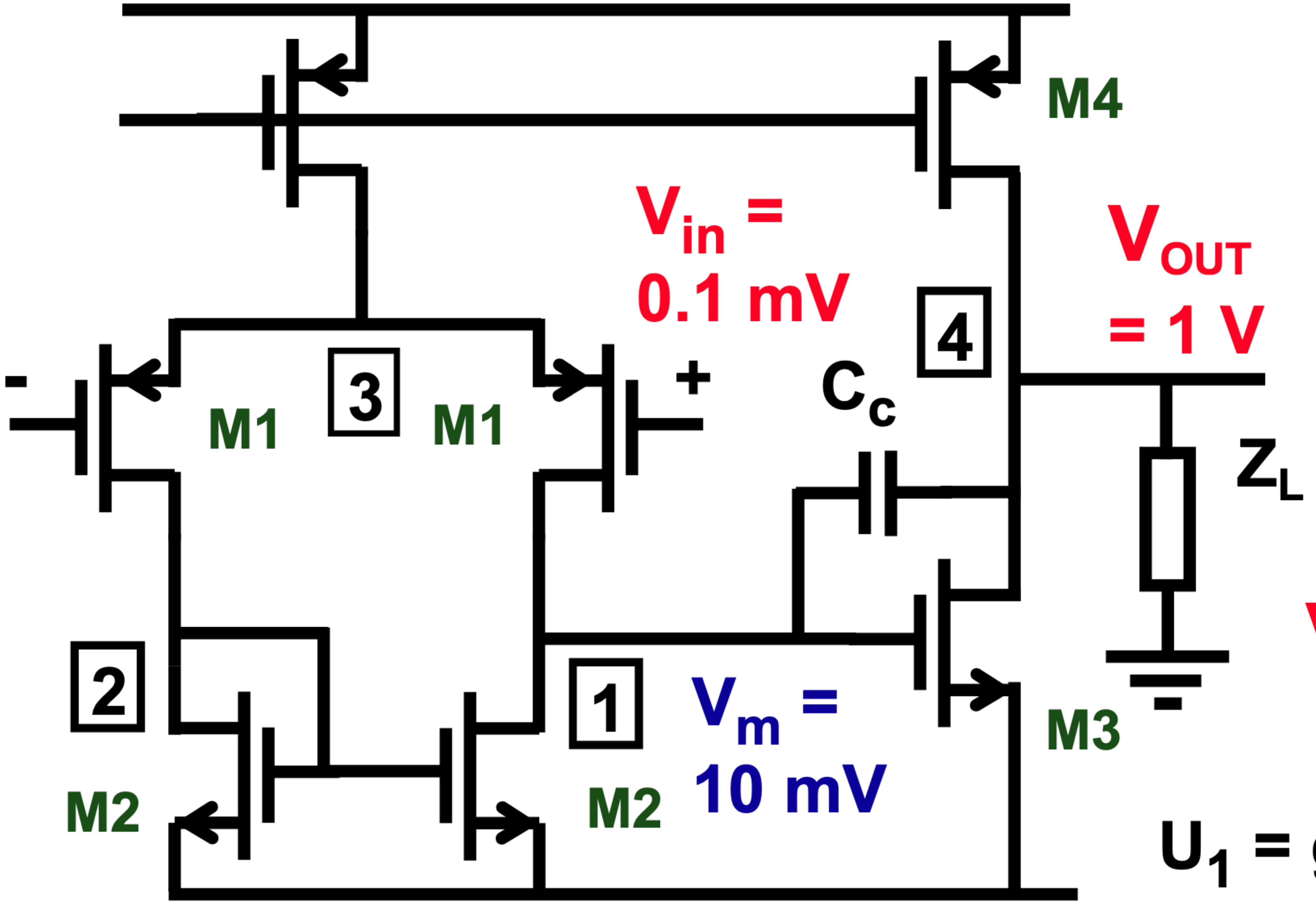
Example



GBW = 10 MHz
A_{v0} = 10.000
BW = 1 kHz
A_{vc} = 10

I_{DS1} = 6 μA
g_{m1} = 60 μS
I_{DS3} = 120 μA
g_{m3} = 1.2 mS
R_L = 100 k Ω
C_L = 5 pF
C_c = 1 pF

Signal swing at low frequencies



GBW = 10 MHz

A_{vc} = 10

I_{DS1} = 6 µA

I_{DS3} = 120 µA

V_{node} at 100 Hz ?

$U_1 = g_{m1}V_{in}/I_{DS1} = 5\ 10^{-4}$

$U_3 = g_{m3}V_m/I_{DS3} = 0.1$

IM_{2f} and IM_{3f} at low frequencies

Distortion generation by nonlinear output stage :

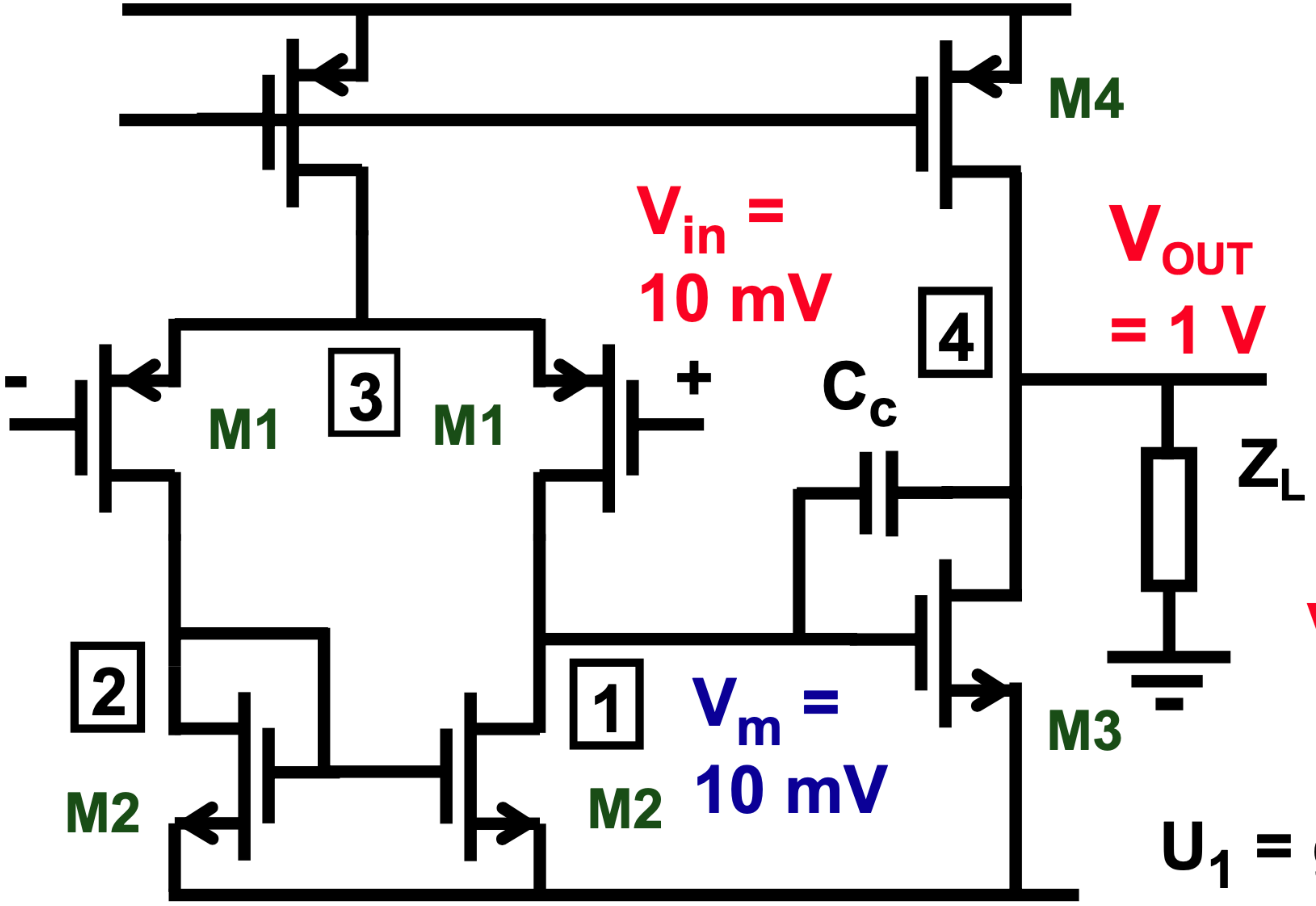
$$U_3 = g_{m3} V_m / I_{DS3} = 0.1$$

$$IM_2 = U_3/4 = 0.25\ 0.1 = 2.5\ \%$$

Distortion reduction by feedback :

$T = 1000$ $\qquad IM_{2f} = 2.5\ \%/1000 = 0.0025\ \%$ **Negligible !**

Signal swings at high frequencies



GBW = 10 MHz

A_{vc} = 10

I_{DS1} = 6 μA

I_{DS3} = 120 μA

V_{node} at 100 kHz ?

$U_1 = g_{m1}V_{in}/I_{DS1} = 5\ 10^{-2}$

$U_3 = g_{m3}V_m/I_{DS3} = 0.1$

IM_{2f} and IM_{3f} at high frequencies

Distortion generation by nonlinear output stage :

$$U_3 = g_{m3}V_m/I_{DS3} = 0.1$$

$$IM_2 = U_3/4 = 0.25\ 0.1 = 2.5\ \%$$

Distortion generation by nonlinear input stage :

$$U_1 = g_{m1}V_m/I_{DS1} = 0.05$$

$$IM_3 = U_1^2/10 = 0.0025/10 \ = 0.025\ \%$$ **Negligible !**

Distortion reduction by feedback :

$T = 10 \qquad IM_{2f} = 2.5\ \%/100 = 0.25\ \%$

Outline

- Definitions and metrics of distortion
- Distortion in components
- Distortion reduction
- Distortion in OPAMPs
- **Summary**

Guidelines for low distortion

- Reduce relative signal swings
- Use feedback
- Use distortion cancellation
- Use differential circuits

IM_2 and IM_3 without feedback

$U_p = V_{ip}/V_{ref}$	$IM_2 \times U_p$	$IM_3 \times U_p^2$	V_{ref}
BJT	1/2	1/8	kT_e/q
MOST	1/4	0	$(V_{GS} - V_T)/2$
BJT differential pair	0	1/4	$2kT_e/q$
MOST differential pair	0	3/32	$V_{GS} - V_T$

IM_2 and IM_3 with feedback (T > 5)

$U_p = V_{ip}/V_{ref}$	$IM_2 \times U_p$	$IM_3 \times U_p^2$	V_{ref}
BJT	1/2T	1/4T	$kT_e/q \times T$
MOST	1/4T	3/32T	$(V_{GS} - V_T)/2 \times T$
BJT differential pair	0	1/4T	$2kT_e/q \times T$
MOST differential pair	0	3/32T	$V_{GS} - V_T \times T$

References

P.Wambacq, W.Sansen : Distortion analysis of analog Integrated Circuits, Kluwer Ac. Publ. 1998

W.Sansen : “Distortion in elementary transistor circuits” IEEE Trans. CAS II Vol 46, No 3, March 1999, pp.315-324

J. Silva-Martinez, etal : High-performance CMOS continuous-time filters, Kluwer Ac. Publ. 1993

B. Hernes, T. Saether : Design criteria for low-distortion in feedback opamp circuits, Kluwer Ac. Publ. 2003

G. Palumbo, S. Pennisi : Feedback amplifiers, Kluwer Ac. Publ. 2002

Offset and CMRR

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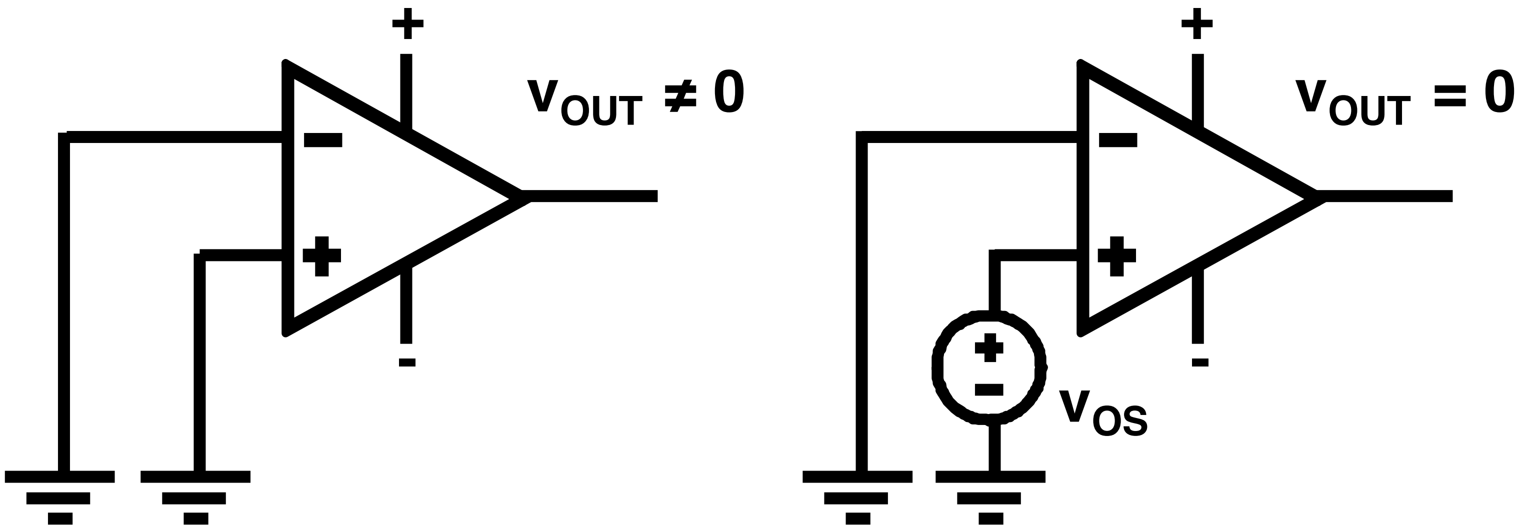
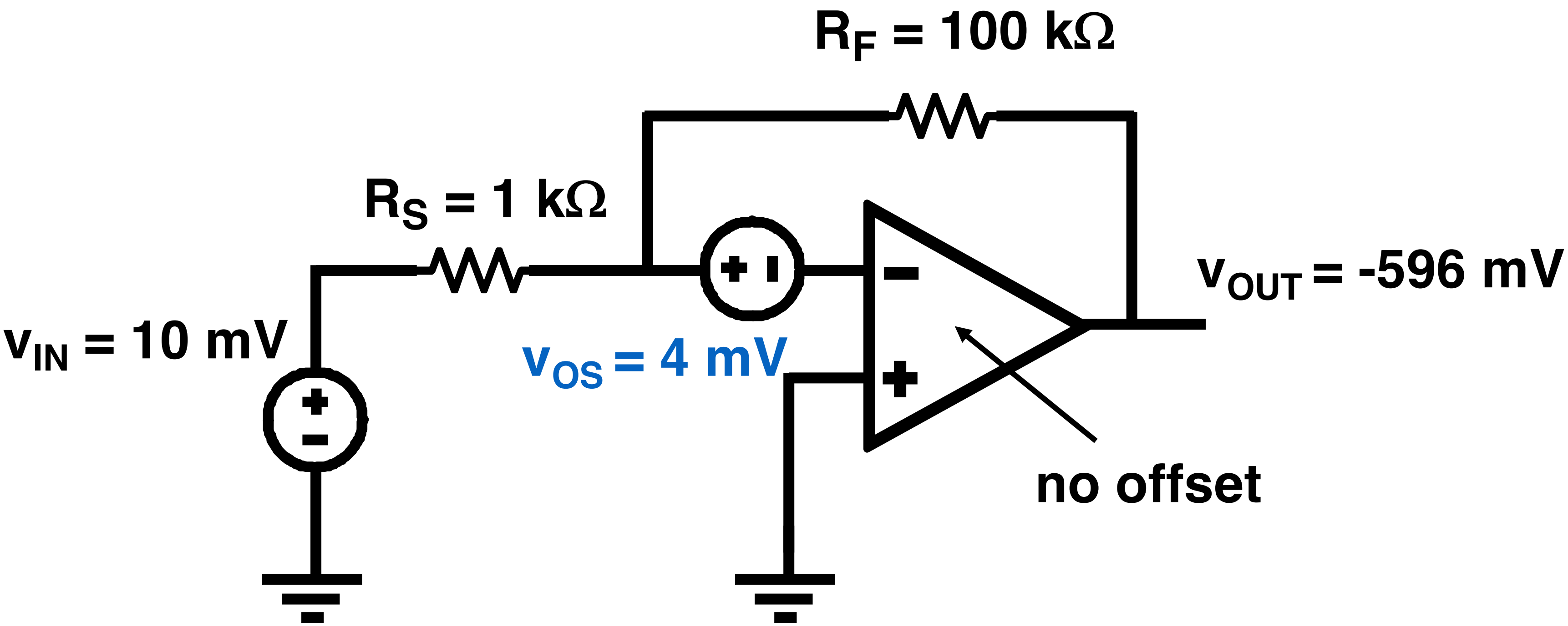
KU Leuven – Department of Electrical Engineering – MICAS

Definition of offset



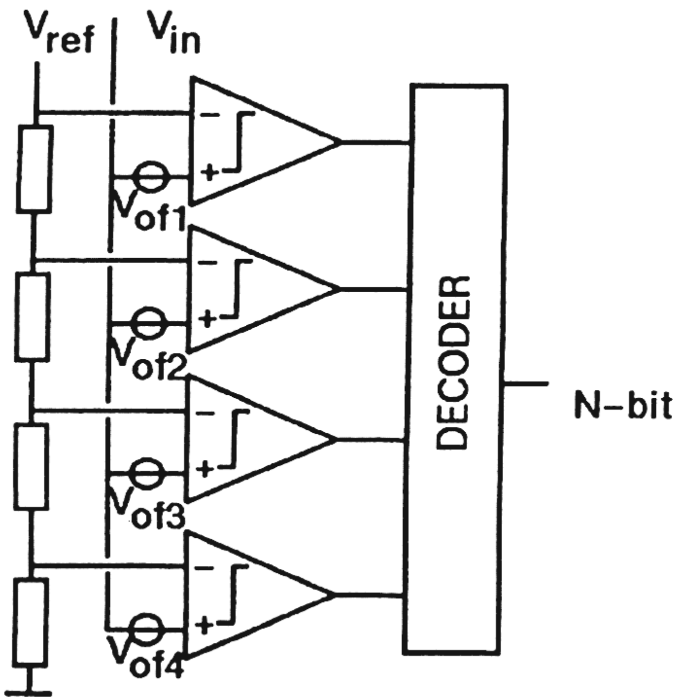
Offset voltage v_{os}

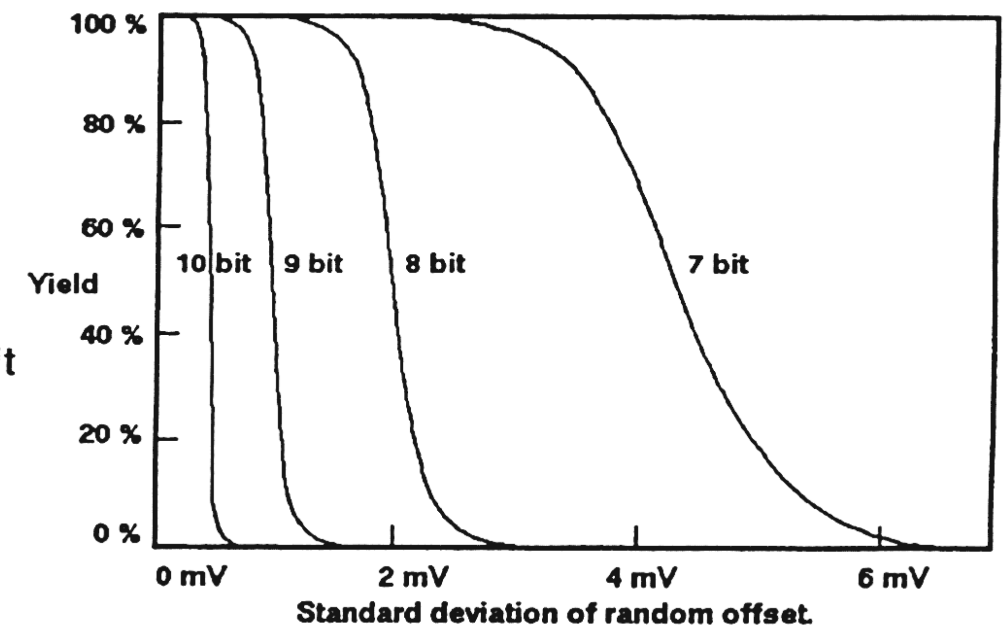
Offset introduces gain errors



The DC output voltage is -596 mV instead of -1 V!

Offset leads to reduced yield in ADCs





Ref: Pelgrom, IEDM 1998, pp.789.

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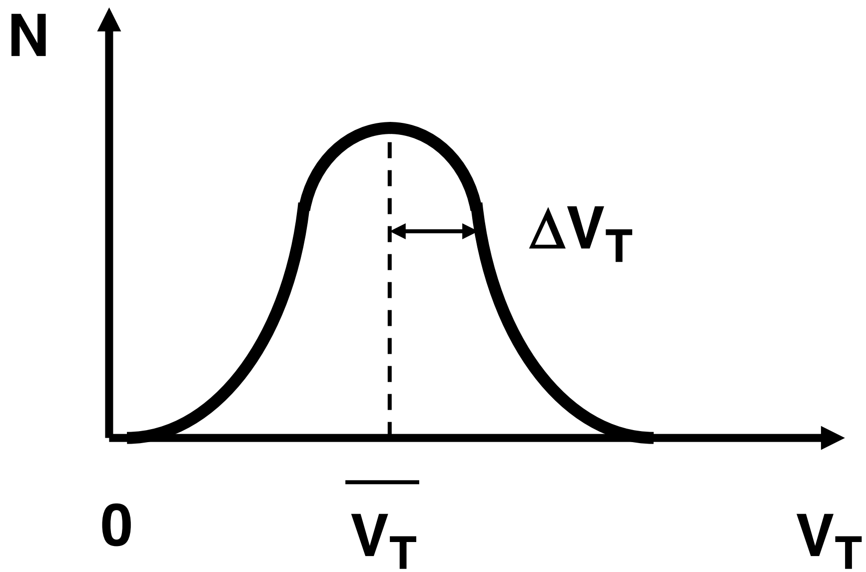
Offset is the result of mismatch



$$I_{DS} = K' \frac{W}{L} (V_{GS} - V_T)^2$$

$$\sigma_{\Delta VT} = \frac{A_{VT}}{\sqrt{WL}}$$

$$A_{VT} \sim t_{ox} \sqrt[4]{N_B}$$

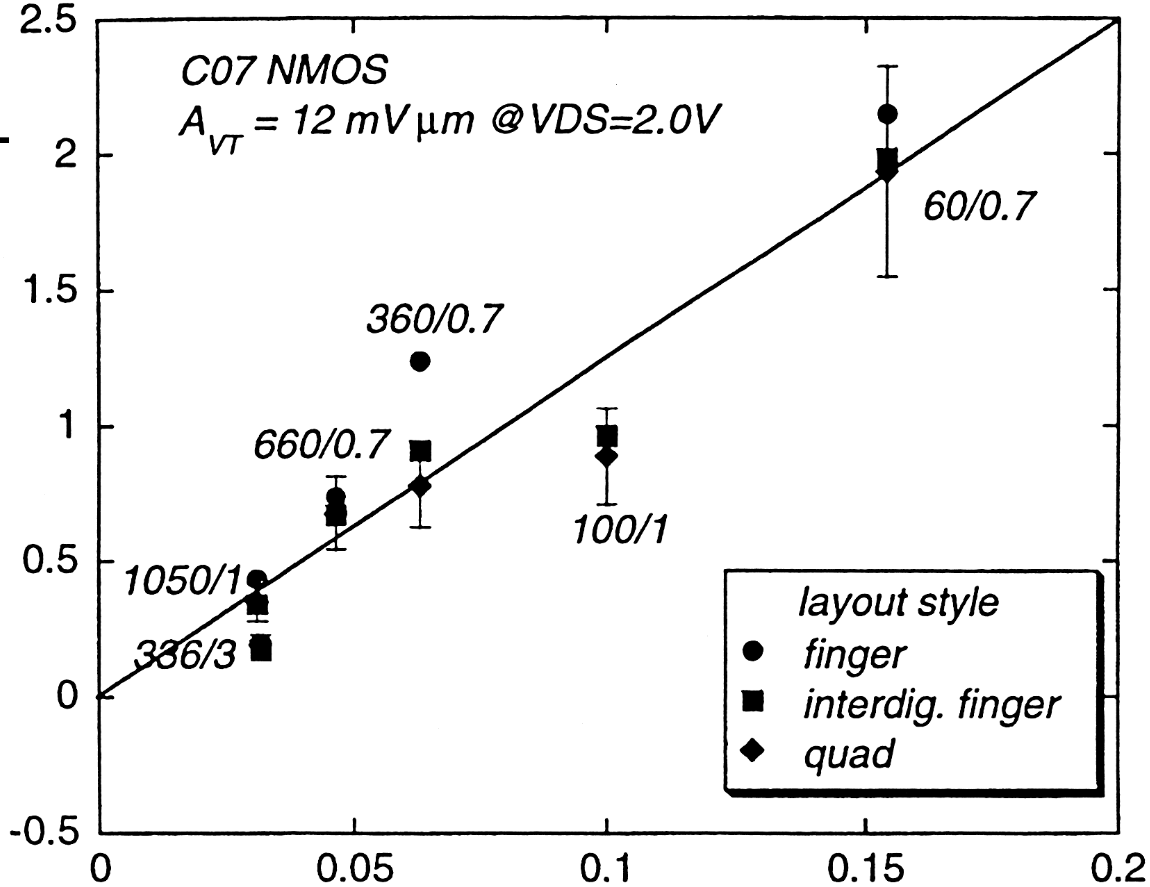
$A_{VT} \approx 5$ mVμm

for 0.25 μm nMOST

+50 % for pMOST

Ref: Keyes, JSSC Aug. 1975, 245-247
Shyu, JSSC Dec 1984, 948-955
Lakshmikumar, JSSC Dec 1986, 1057-1066
Pelgrom, JSSC Oct.1989, 1433-1439
Croon, JSSC Aug. 2002, 1056-1064

Standard deviation is a function of the area



$$\sigma_{\Delta VT} = \frac{A_{VT}}{\sqrt{WL}}$$

$\sigma(\Delta V_T) \approx 2$ mV

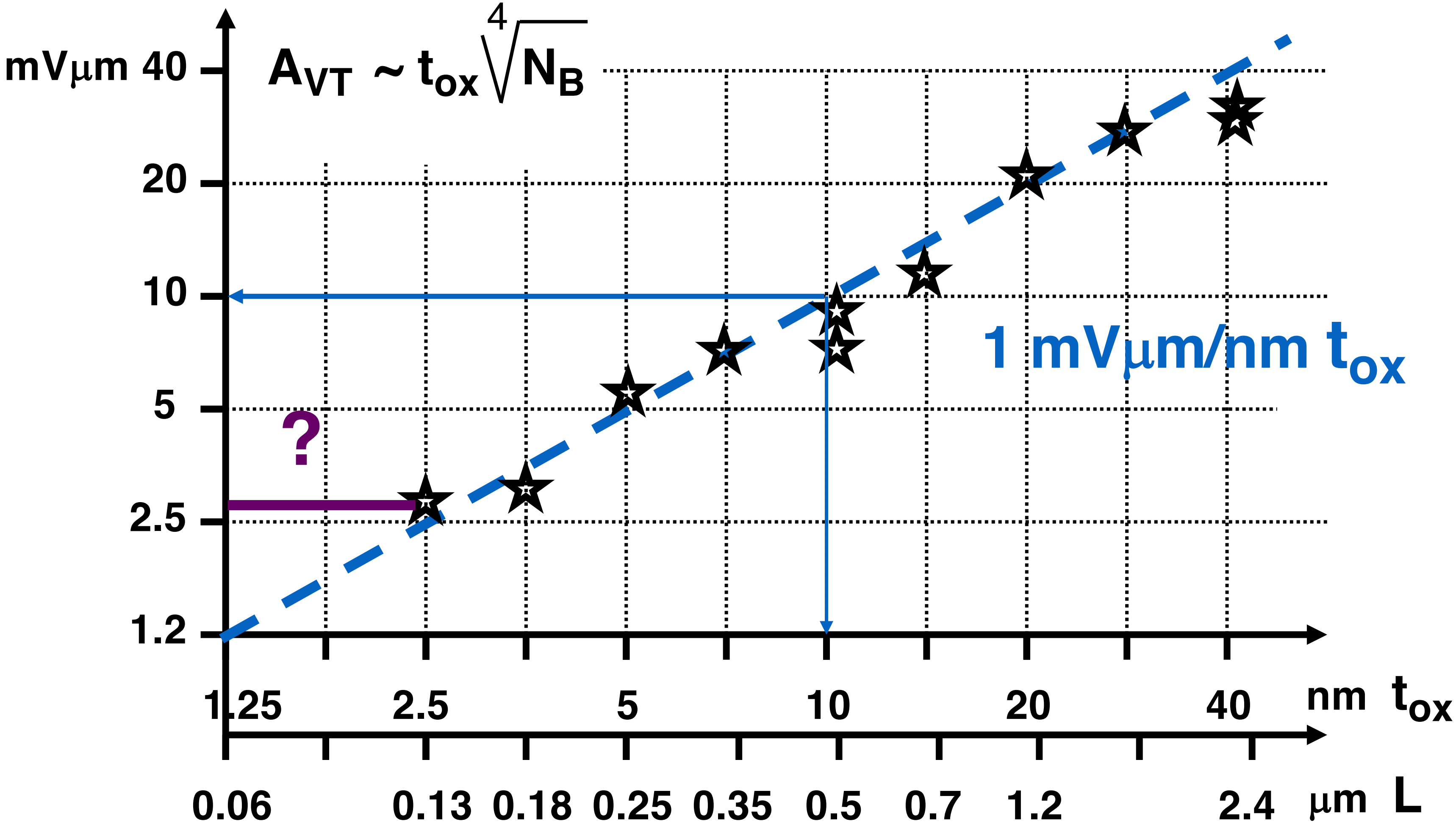
W/L = 60/0.7 µm

in 0.7 µm CMOS

$1/\sqrt{WL}$

1/µm

A_{VT} as a function of technology node



Other MOST mismatch parameters

$$\frac{\Delta K'}{K'} = \frac{A_{K'}}{\sqrt{WL}}$$

$A_{K'} \approx 0.0056\ \mu m$ +50% for pMOST

$$\frac{\Delta W/L}{W/L} = A_{WL}\sqrt{\frac{1}{W^2}+\frac{1}{L^2}}$$

$A_{WL} \approx 0.02\ \mu m$ +50% for pMOST

$$\frac{\Delta\gamma}{\gamma} = \frac{A_{\gamma}}{\sqrt{WL}}$$

$A_{\gamma} \approx 0.016\ \mu m$ -25% for pMOST

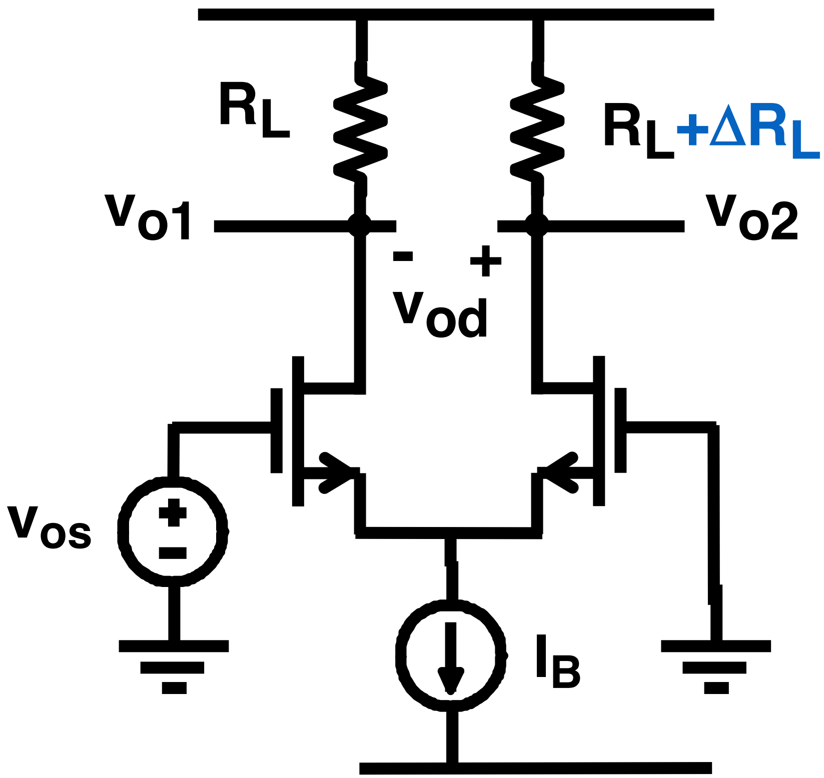
negligible if B = S

Ref.: Pelgrom : JSSC Oct.1989, pp.1430-1440

nMOST mismatch coefficients

Techno L (μm) / t_{ox} (nm)	2.5 / 50	1.2 / 25	0.7 / 15	0.5 / 11	0.35 / 8	0.25 / 6
A_{VT} (mV μm)	30	21	13	7.1	6	⇨ 0
A_{WL} (% μm)	2.5	1.8	2.5	1.3	2	⇨ 1.8
S_{VT} (mV/mm)	0.3	0.3	0.4	0.2		
S_{WL} (%/mm)	0.3		0.2	0.2		

Random offset in a differential pair



$$v_{od} = \Delta R_L \frac{I_B}{2}$$

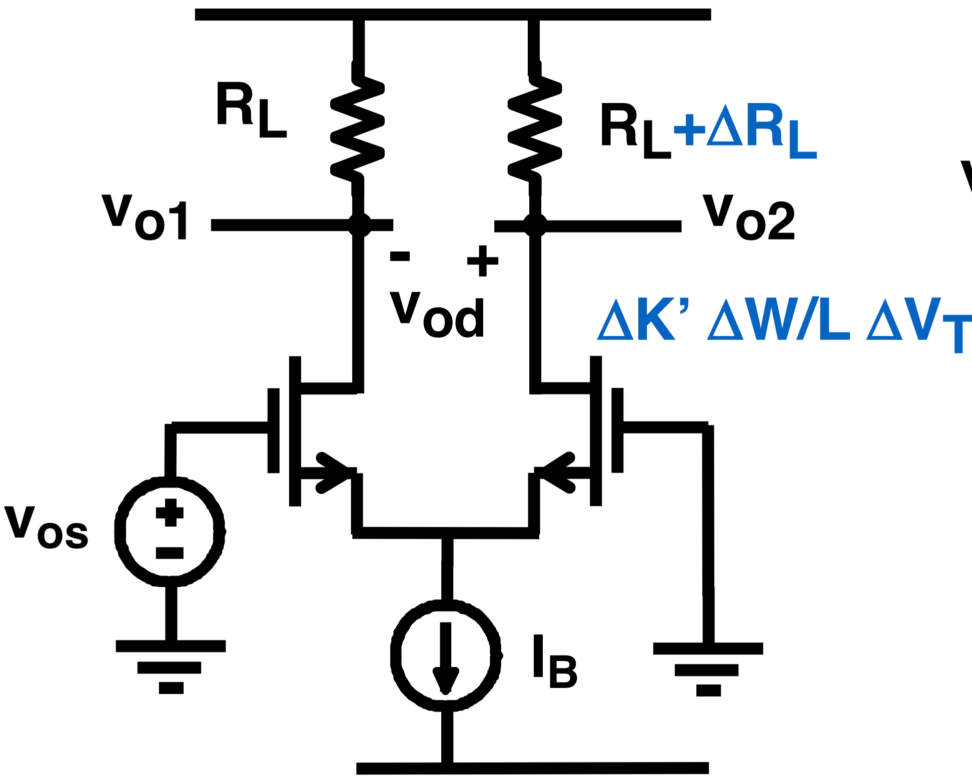
$$v_{os} = \frac{v_{od}}{g_m R_L}$$

$$v_{os} = \frac{\Delta R_L}{R_L} \frac{I_B}{2 g_m}$$

$$v_{os} = \frac{\Delta R_L}{R_L} \frac{V_{GS} - V_T}{2}$$

Ref.: Laker, Sansen : Design of analog …, MacGrawHill 1994

Random offset in a differential pair

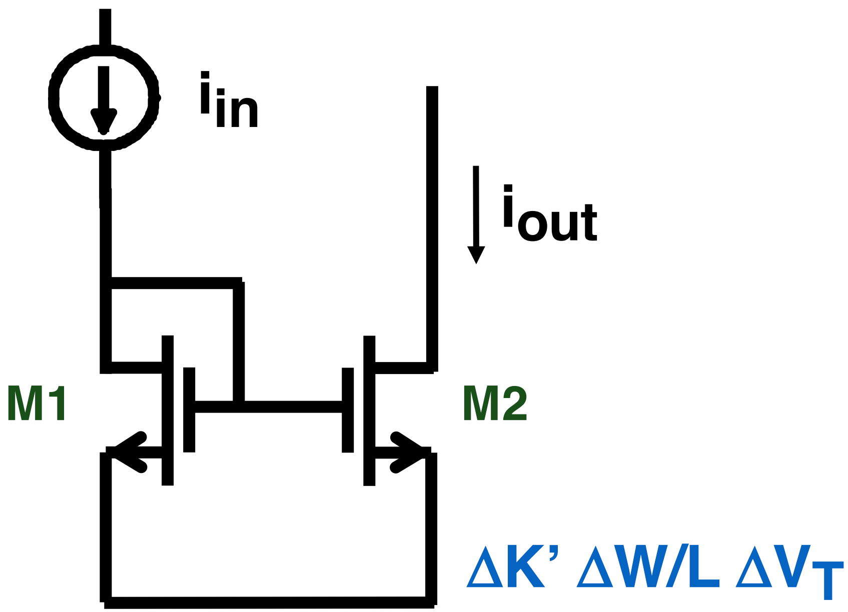


$$v_{OS} = \Delta V_T + \frac{V_{GS} - V_T}{2}\left(\frac{\Delta R_L}{R_L} + \frac{\Delta K'}{K'} + \frac{\Delta W/L}{W/L}\right)$$

$\Rightarrow$ **small $V_{GS} - V_T$**

Ref.: Laker, Sansen : Design of analog …, MacGrawHill 1994

Random offset in a current mirror



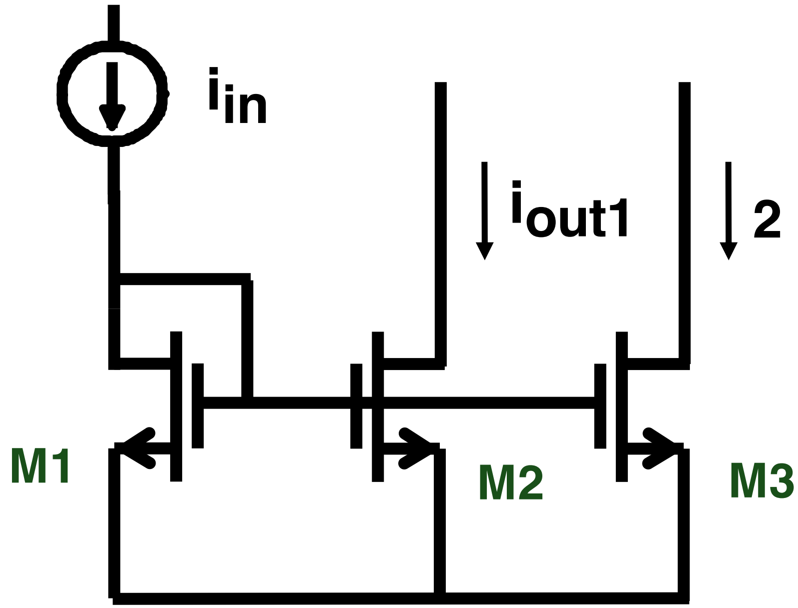
$$\frac{\Delta I_{out}}{I_{out}} =$$

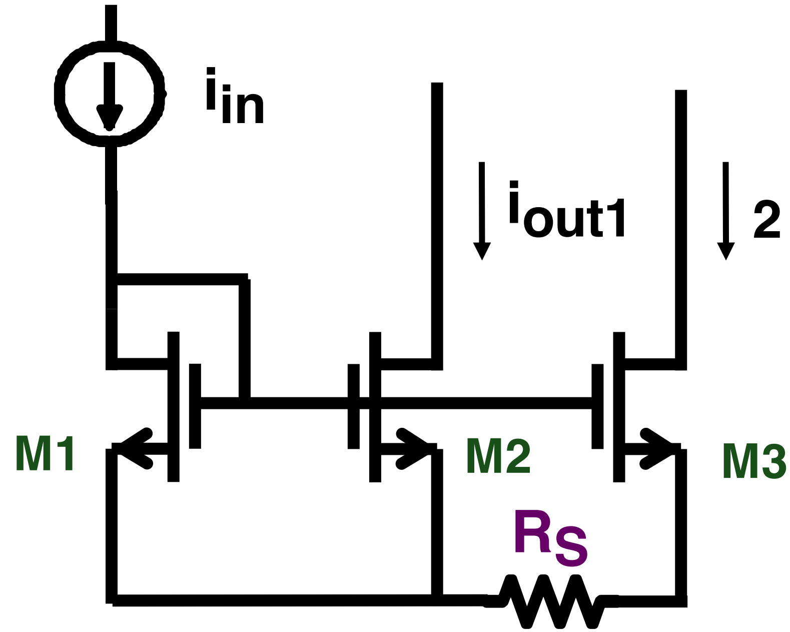
$$\frac{\Delta V_T}{(V_{GS} - V_T)/2} + \frac{\Delta K'}{K'} + \frac{\Delta W/L}{W/L}$$

$\Rightarrow$ large $V_{GS} - V_T$

Ref.: Laker, Sansen : Design of analog …, MacGrawHill 1994

Other sources of offset





$\Delta K'$ $\Delta W/L$ ΔV_T

$\Delta K'$ $\Delta W/L$ ΔV_T R_S

MOST drain current mismatch

$$I_{DS} = \frac{\beta}{2}(V_{GS} - V_T)^2 \qquad \beta = \frac{K'}{n}\frac{W}{L}$$

$$\frac{\Delta I_{DS}}{I_{DS}} = \frac{\Delta\beta}{\beta} - \Delta V_T \frac{2}{V_{GS} - V_T}$$

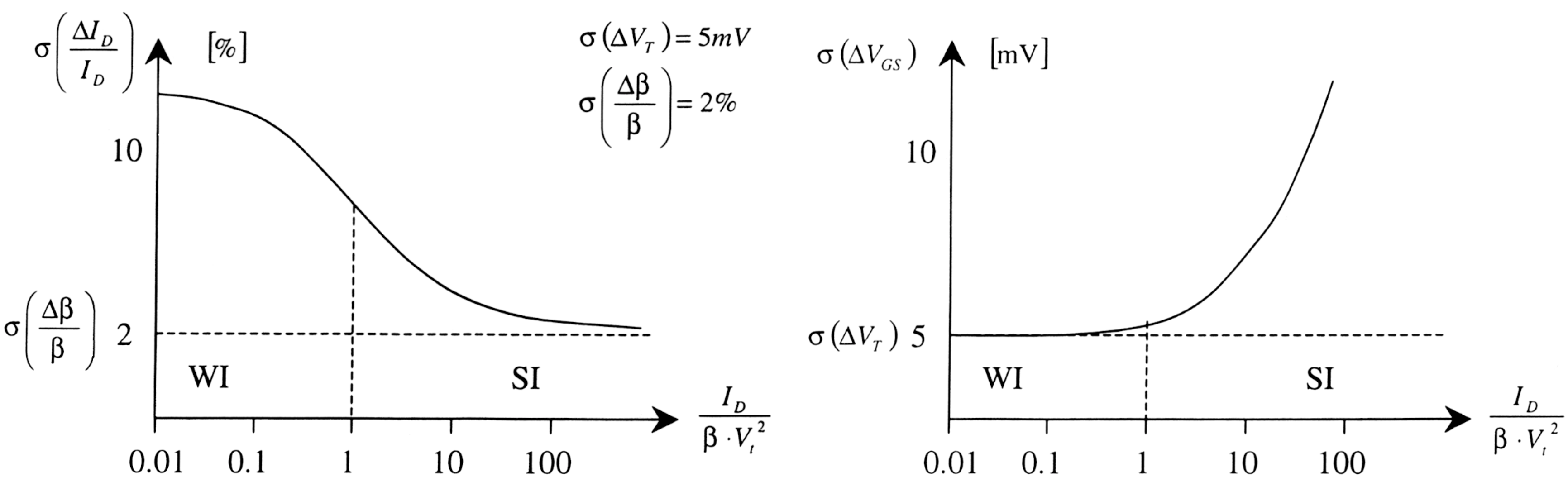
$$\sigma^2\left(\frac{\Delta I_{DS}}{I_{DS}}\right) = \sigma^2\left(\frac{\Delta\beta}{\beta}\right) + \sigma^2(\Delta V_T)\underbrace{\frac{4}{(V_{GS} - V_T)^2}}$$

$$\frac{1}{(nkT/q)^2} \quad \text{in wi}$$

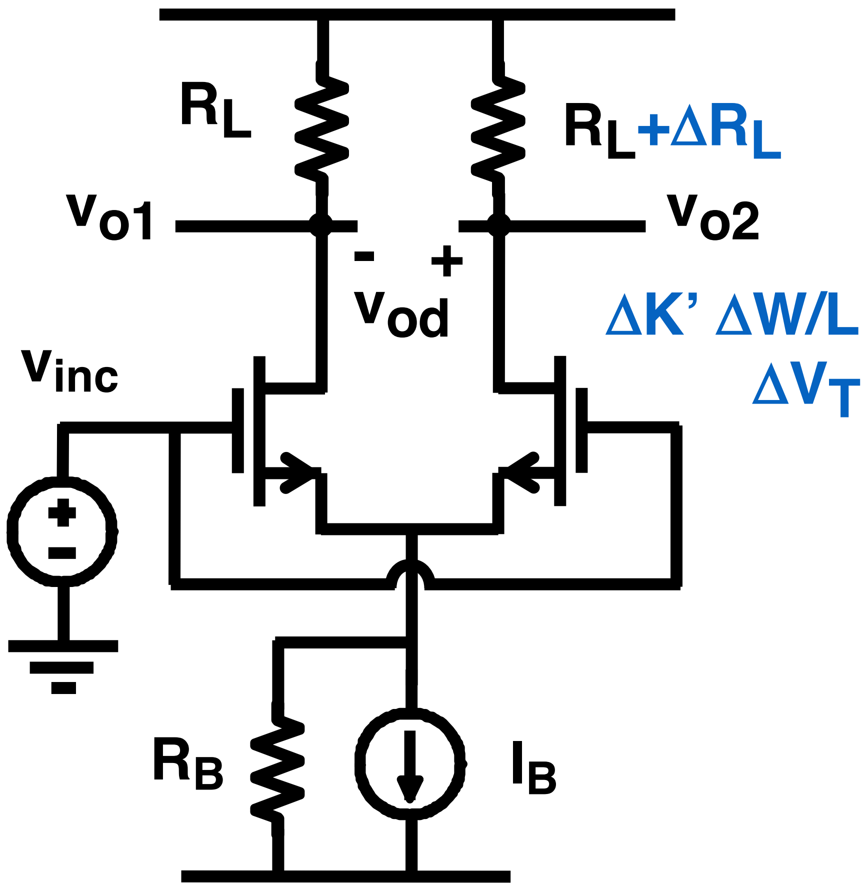
$$\left(\frac{g_m}{I_{DS}}\right)^2 \quad \text{in general}$$

MOST drain current mismatch in WI and SI

$$\sigma^2\left(\frac{\Delta I_{DS}}{I_{DS}}\right) = \sigma^2\left(\frac{\Delta\beta}{\beta}\right) + \sigma^2(\Delta V_T)\ \underbrace{\frac{4}{(V_{GS}-V_T)^2}}_{\text{in si}}\ \text{or}\ \underbrace{\frac{1}{(nkT/q)^2}}_{\text{in wi}}$$



CMRR$_r$ in a differential pair



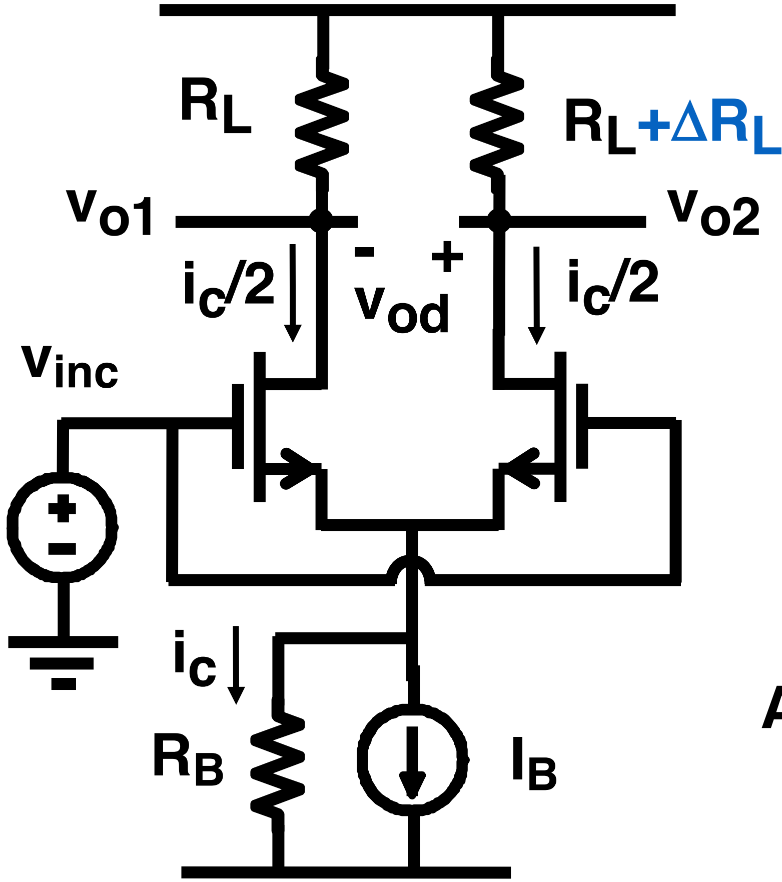
$$v_{od} = A_{dd}\, v_{id} + A_{dc}\, v_{ic}$$
$$v_{oc} = A_{cd}\, v_{id} + A_{cc}\, v_{ic}$$

$$A_{dd} = \left.\frac{v_{od}}{v_{id}}\right|_{v_{ic}=0} = g_m R_L$$

$$A_{dc} = \left.\frac{v_{od}}{v_{ic}}\right|_{v_{id}=0} \approx 0$$

$$\mathrm{CMRR} = \frac{A_{dd}}{A_{dc}} \approx \infty$$

$CMRR_r$ in a differential pair



$$A_{dc} = \left.\frac{v_{od}}{v_{ic}}\right|_{v_{id}=0} \neq 0$$

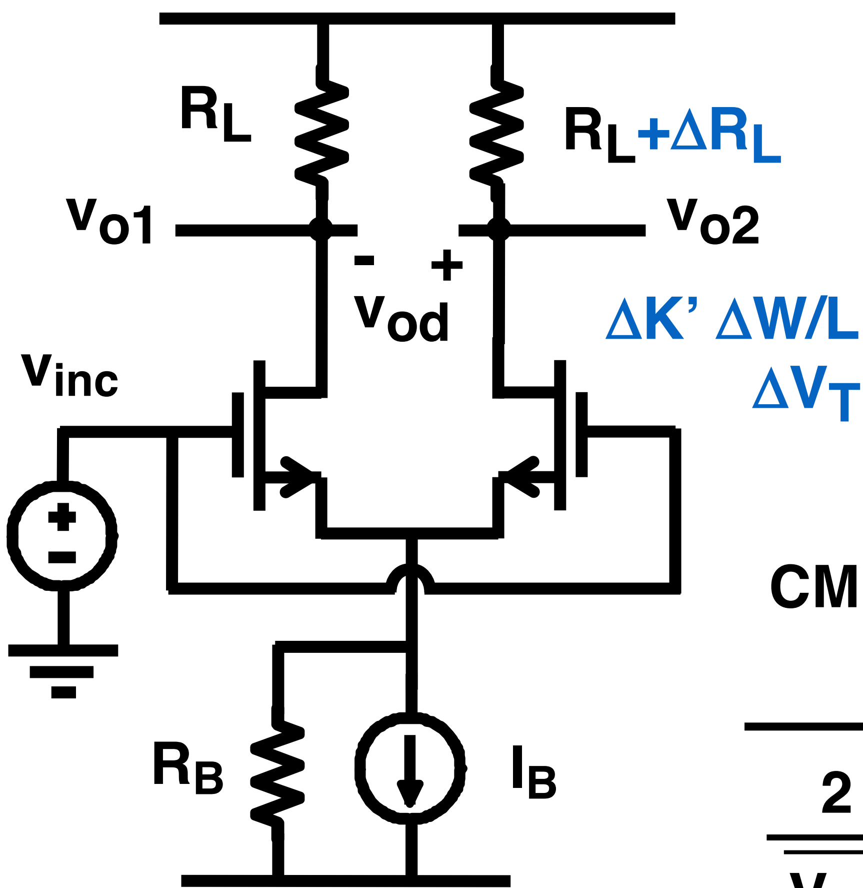
$$v_{ic} = v_{inc} \Rightarrow i_c = \frac{v_{inc}}{R_B}$$

$$v_{od} = \Delta R_L \, i_c/2$$

$$A_{dc} = \frac{\Delta R_L}{2 R_B}$$

$$\boxed{CMRR = \frac{2 g_m R_B}{\Delta R_L / R_L}}$$

CMRR$_r$ in a differential pair



$$\text{CMRR} = \frac{2\, g_m R_B}{\dfrac{2\, \Delta V_T}{V_{GS} - V_T} + \dfrac{\Delta R_L}{R_L} + \dfrac{\Delta K'}{K'} + \dfrac{\Delta W/L}{W/L}}$$

Relation between random offset and $CMRR_r$

$$v_{OSr} = \Delta V_T + \frac{V_{GS} - V_T}{2} \left(\frac{\Delta R_L}{\overline{R_L}} + \frac{\Delta K'}{\overline{K'}} + \frac{\Delta W/L}{\overline{W/L}} \right)$$

$$CMRR_r = \frac{2\, g_m R_B}{\dfrac{2\,\Delta V_T}{\overline{V_{GS} - V_T}} + \dfrac{\Delta R_L}{\overline{R_L}} + \dfrac{\Delta K'}{\overline{K'}} + \dfrac{\Delta W/L}{\overline{W/L}}}$$

$$v_{OSr}\, CMRR_r = \frac{V_{GS} - V_T}{2}\, 2\, g_m R_B = I_B R_B = V_E L_B = 5 \ldots 15 \text{ V}$$

$$v_{OSr}\, CMRR_r = 10 \text{ V}$$

Relation between random offset and $CMRR_r$

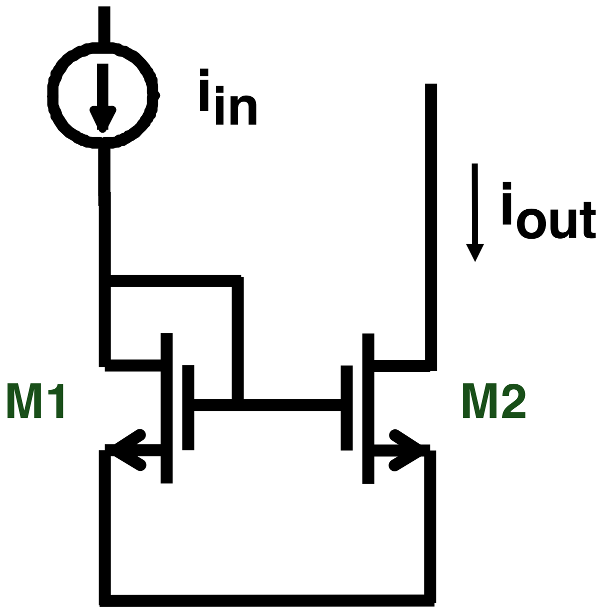
v_{OSr}	$CMRR_r \approx V_E L_B$	$\approx$ 10 V	(~ L_B)
10 mV	60 dB	$\approx$ 10 V	as for MOSTs
1 mV	80 dB	$\approx$ 10 V	as for bipolar transistors
10 μV	120 dB	$\approx$ 10 V	with trimming: with laser with Zener zap with fusable links

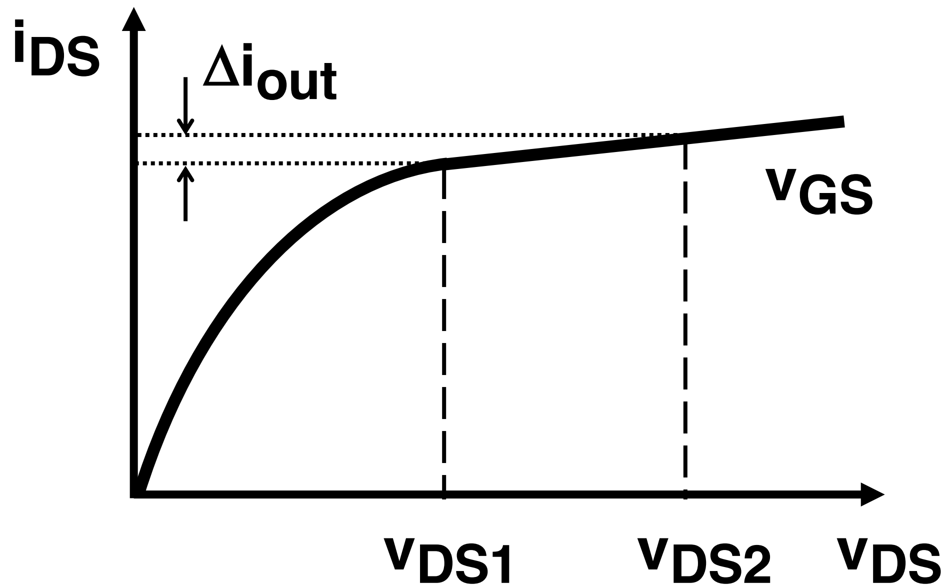
low offset voltage = high CMRR

Table of contents

- Random offset and $CMRR_r$
- **Systematic offset and $CMRR_s$**
- Total CMRR and frequency dependency
- Good layout practices
- Accuracy limit of analog circuits

Systematic offset in a current mirror

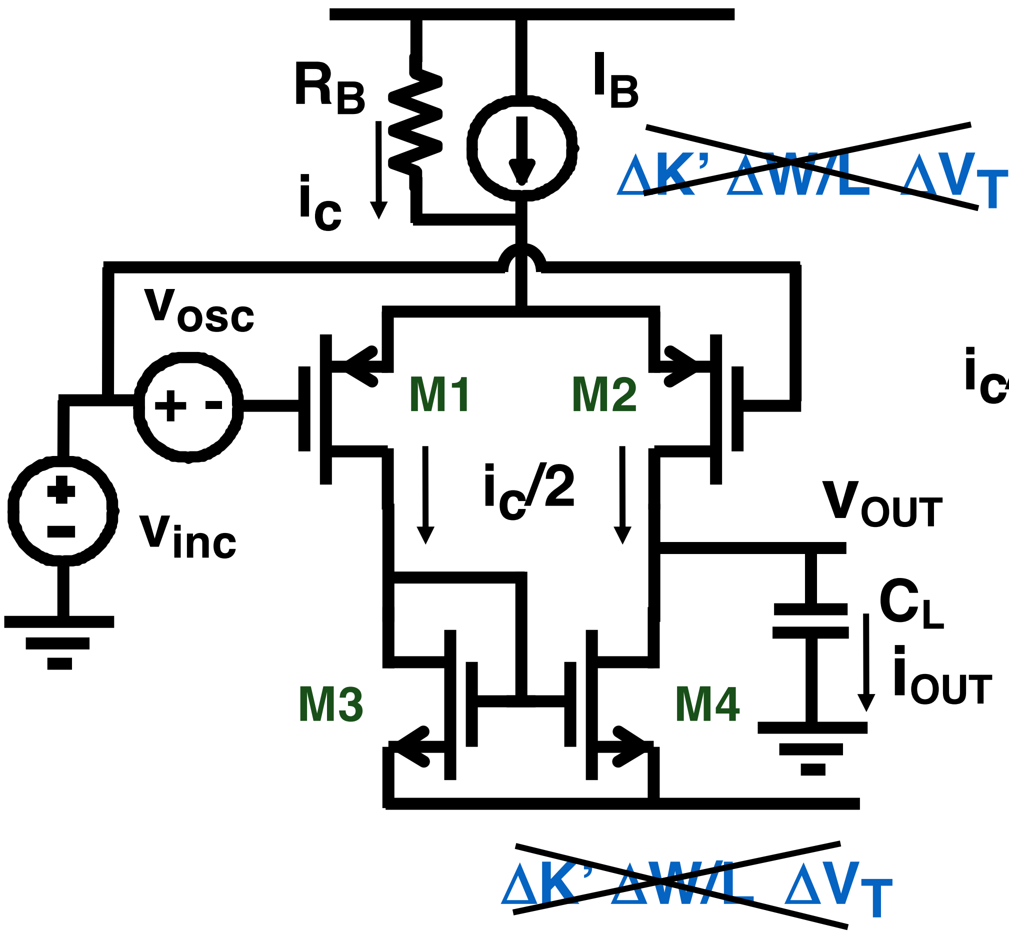




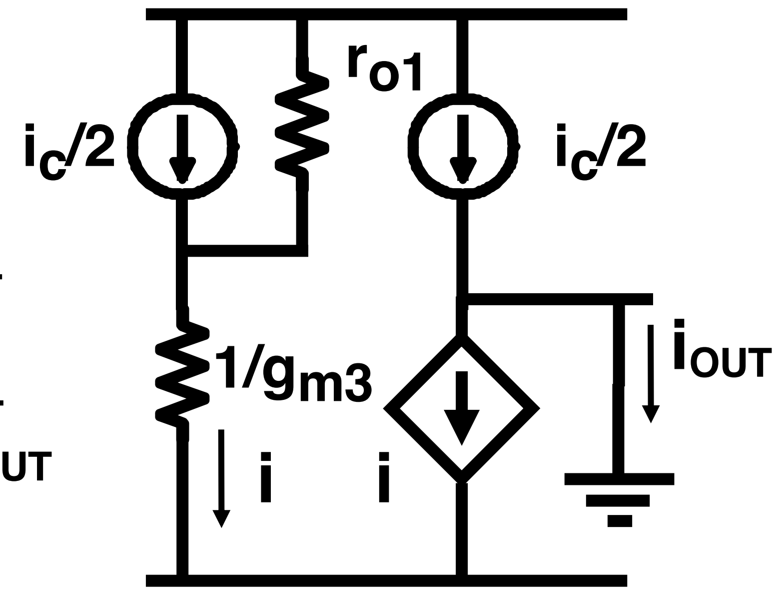
$\cancel{\Delta K' \;\; \Delta W/L \;\; \Delta V_T}$

$$\frac{\Delta i_{out}}{i_{out}} = \frac{v_{DS2} - v_{DS1}}{V_E L_2}$$

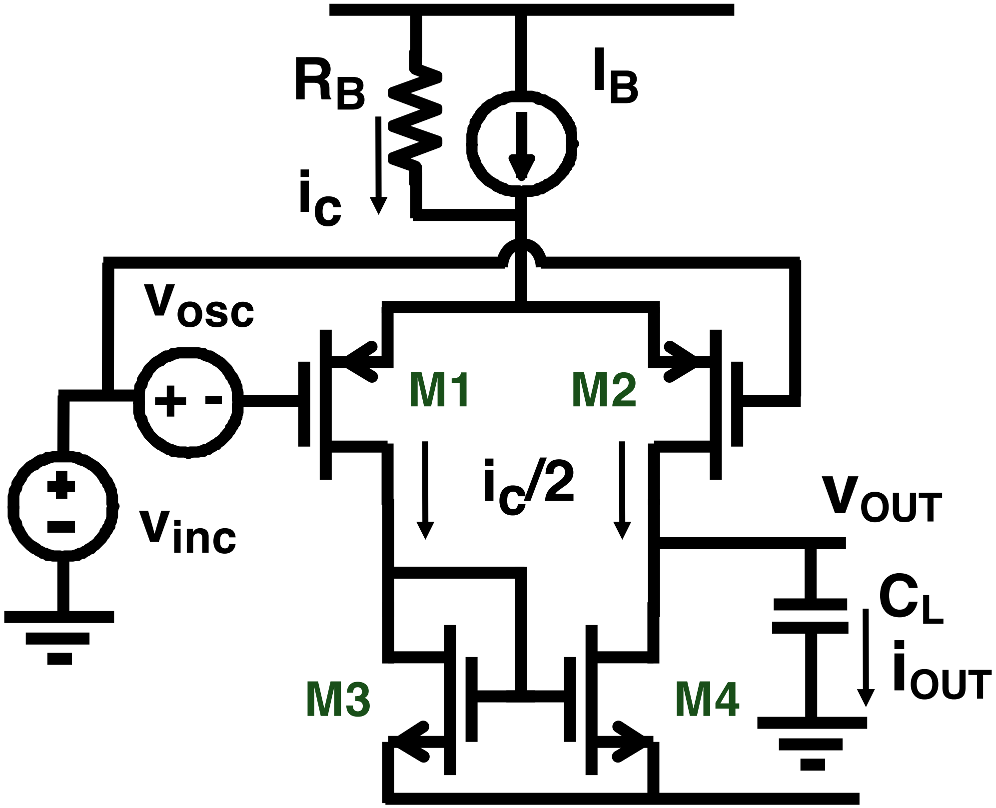
CMRR$_s$ in the basic OTA



$$i_c = \frac{v_{inc}}{R_B} \qquad \frac{i_{out}}{i_c} = \frac{1}{2g_{m3}r_{o1}}$$



$CMRR_s$ in the basic OTA



$$\frac{i_{out}}{v_{inc}} = \frac{1}{R_B}\,\frac{2}{g_{m3} r_{o1}}$$

$$\frac{i_{out}}{v_{osc}} = g_{m1}$$

v_{ind}

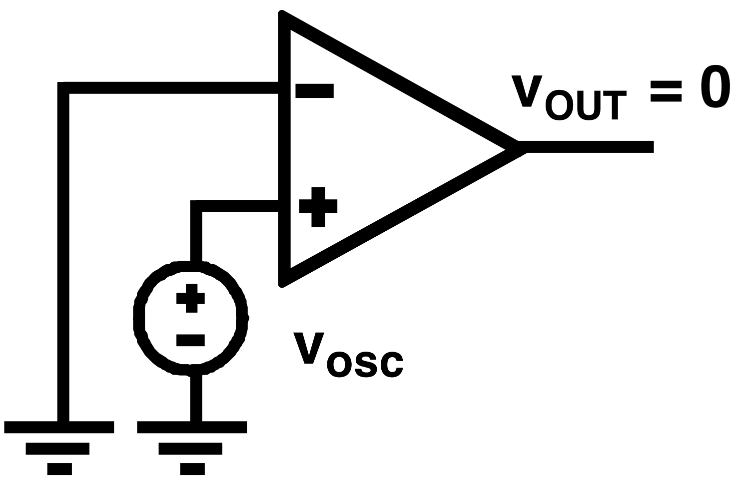
$$\frac{\frac{i_{out}}{v_{inc}}}{\frac{i_{out}}{v_{osc}}} = \frac{A_{dc}}{A_{dd}} = \frac{1}{CMRR_s}$$

$\mathrm{CMRR_s}$ in the basic CMOS OTA

$$\frac{\left(\dfrac{i_{out}}{v_{inc}}\right)}{\left(\dfrac{i_{out}}{v_{osc}}\right)} = \frac{v_{osc}}{v_{inc}} = \frac{A_{dc}}{A_{dd}} = \frac{1}{\mathrm{CMRR_s}}$$

$$\mathrm{CMRR_s} = \frac{1}{2} g_{m1} R_B\, g_{m3} r_{o1}$$

$$\mathrm{CMRR_s}\, v_{osc} = v_{inc}$$



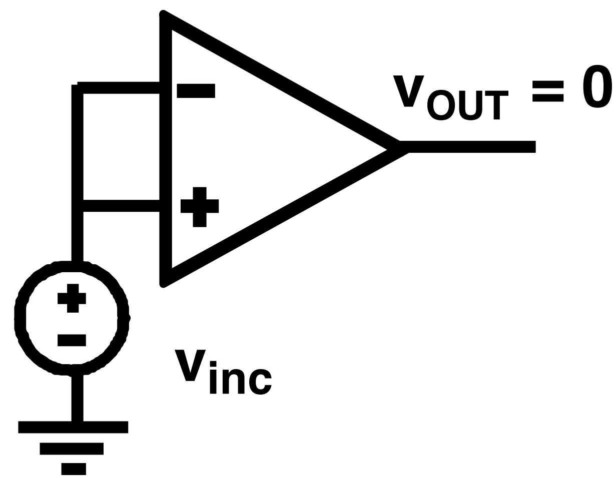
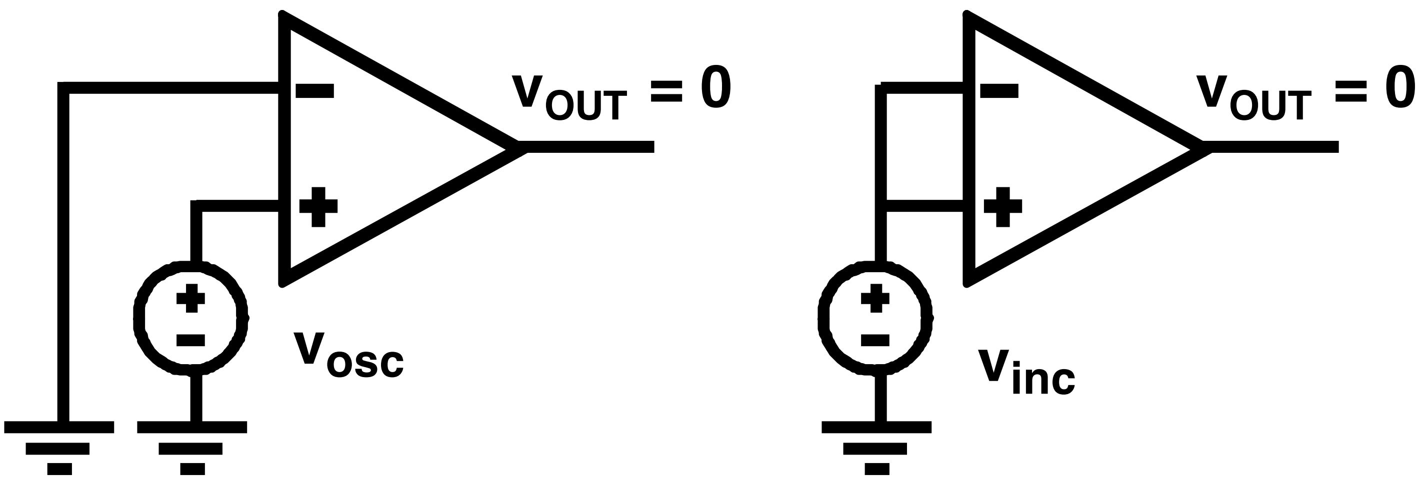


Table of contents

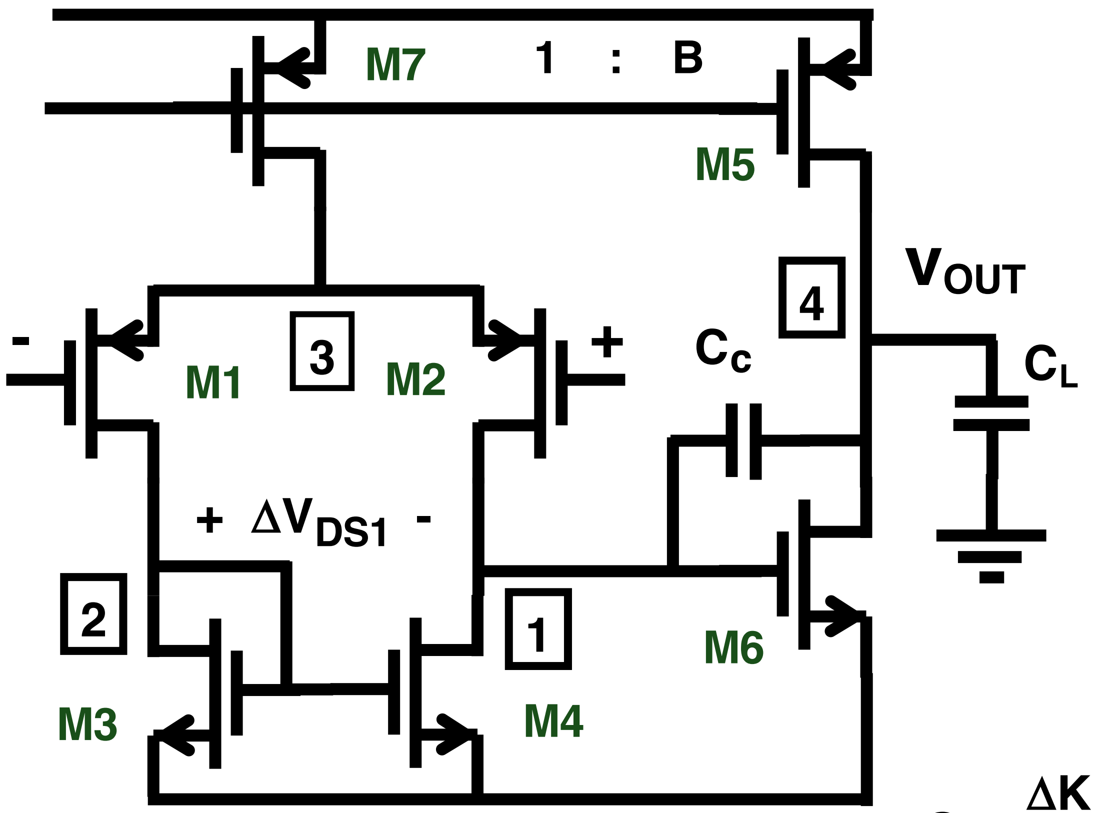
- Random offset and $CMRR_r$
- Systematic offset and $CMRR_s$
- **Total CMRR and frequency dependency**
- Good layout practices
- Accuracy limit of analog circuits

Total CMRR

$$\frac{1}{CMRR} = \frac{1}{CMRR_r} + \frac{1}{CMRR_s}$$



Total offset of the Miller OTA

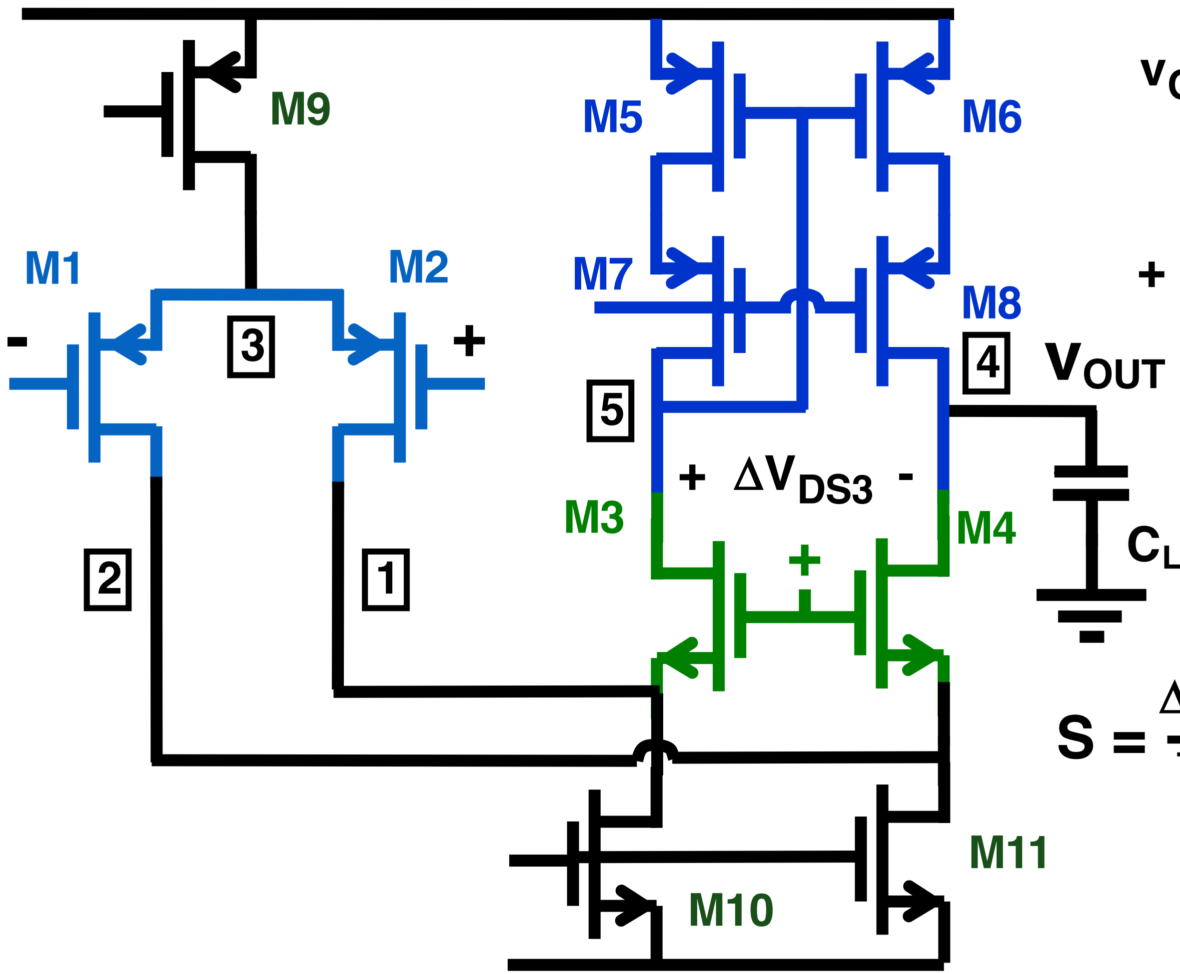


$$A_{v1} = g_{m1}\, r_{02} // r_{04}$$

$$v_{OS} = \frac{\Delta V_{DS1}}{A_{v1}} + \Delta V_{T1} + \frac{g_{m3}}{g_{m1}} \Delta V_{T3}^{*} + \frac{V_{GS1} - V_T}{2} S$$

$$S = \frac{\Delta K'_n}{K'_n} + \frac{\Delta K'_p}{K'_p} + \frac{\Delta W/L_1}{W/L_1} + \frac{\Delta W/L_3}{W/L_3}$$

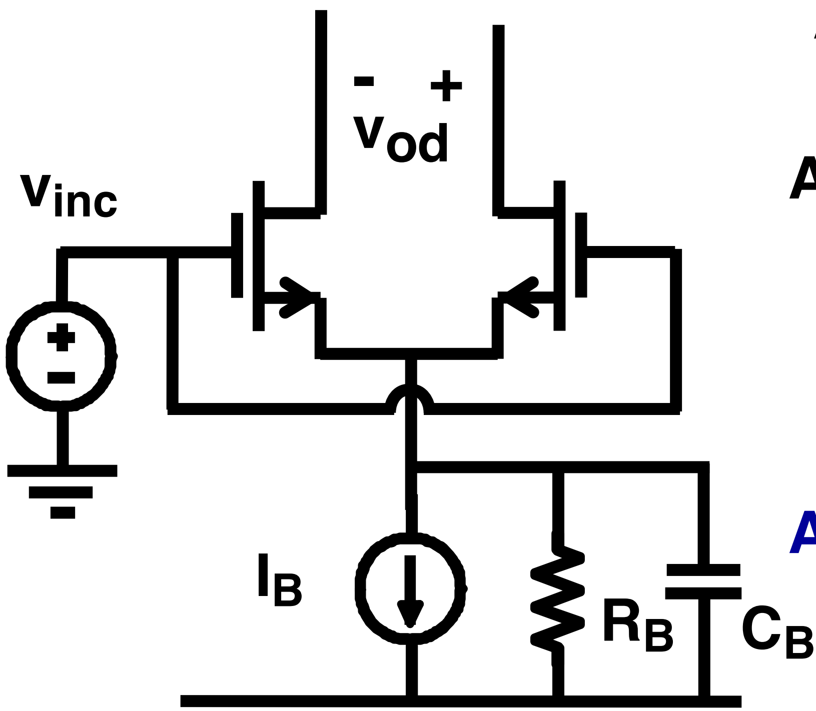
Total offset of the folded-cascode OTA

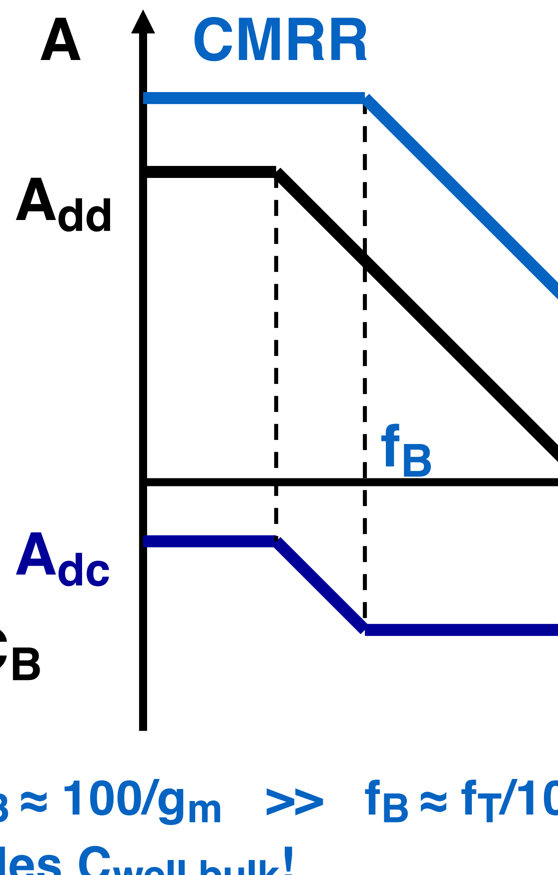


$$v_{OS} = \frac{\Delta V_{DS3}}{A_v} + \Delta V_{T1} + \frac{g_{m6}}{g_{m1}} \Delta V_{T5} + \frac{g_{m11}}{g_{m1}} \Delta V_{T11}^{*} + \frac{V_{GS1} - V_T}{2} S$$

$$S = \frac{\Delta K'_n}{\overline{K'_n}} + \frac{\Delta K'_p}{\overline{K'_p}} + \frac{\Delta W/L_{1,6,11}}{\overline{W/L_{1,6,11}}}$$

CMRR as a function of frequency





$$f_B = \frac{1}{2\pi R_B C_B}$$

$C_B \approx C_{GS}$ $R_B \approx 100/g_m$ >> $f_B \approx f_T/100$

BUT C_B includes $C_{well,bulk}$!

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Good layout practices for low offset

1. **Equal nature**
2. Equal temperature
3. Large area
4. Minimum distance
5. Same orientation
6. Same area/perimeter ratio
7. Round shape
8. Centroide layout
9. End dummies
10. BJT better than MOST

Hastings,
"The Art of Analog Layout"
Prentice Hall 2001
R. Soin, .."A-D Asics, .. "
Peregrinus, 1991

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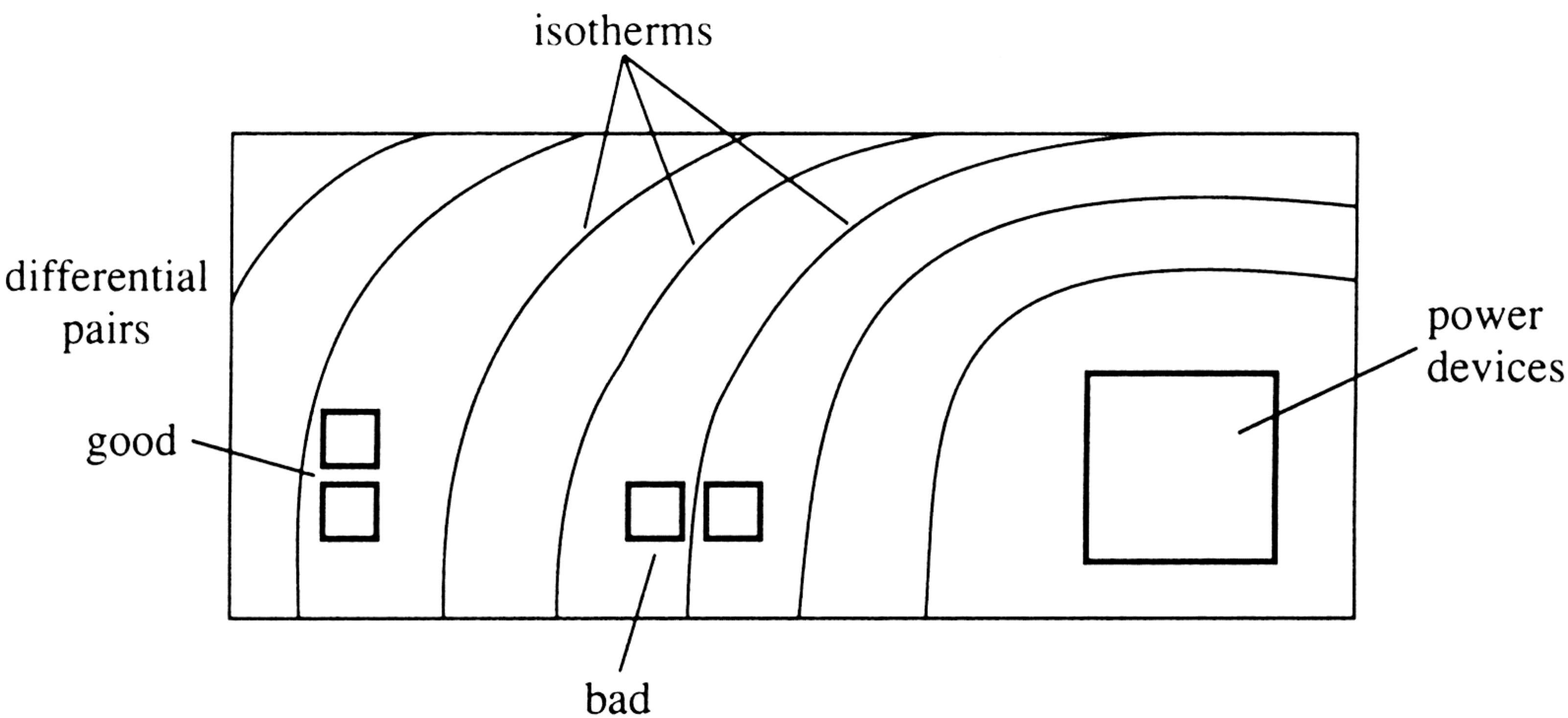
On same isotherm



Solomon, JSSC Dec 74, 314-332

Good layout practices for low offset

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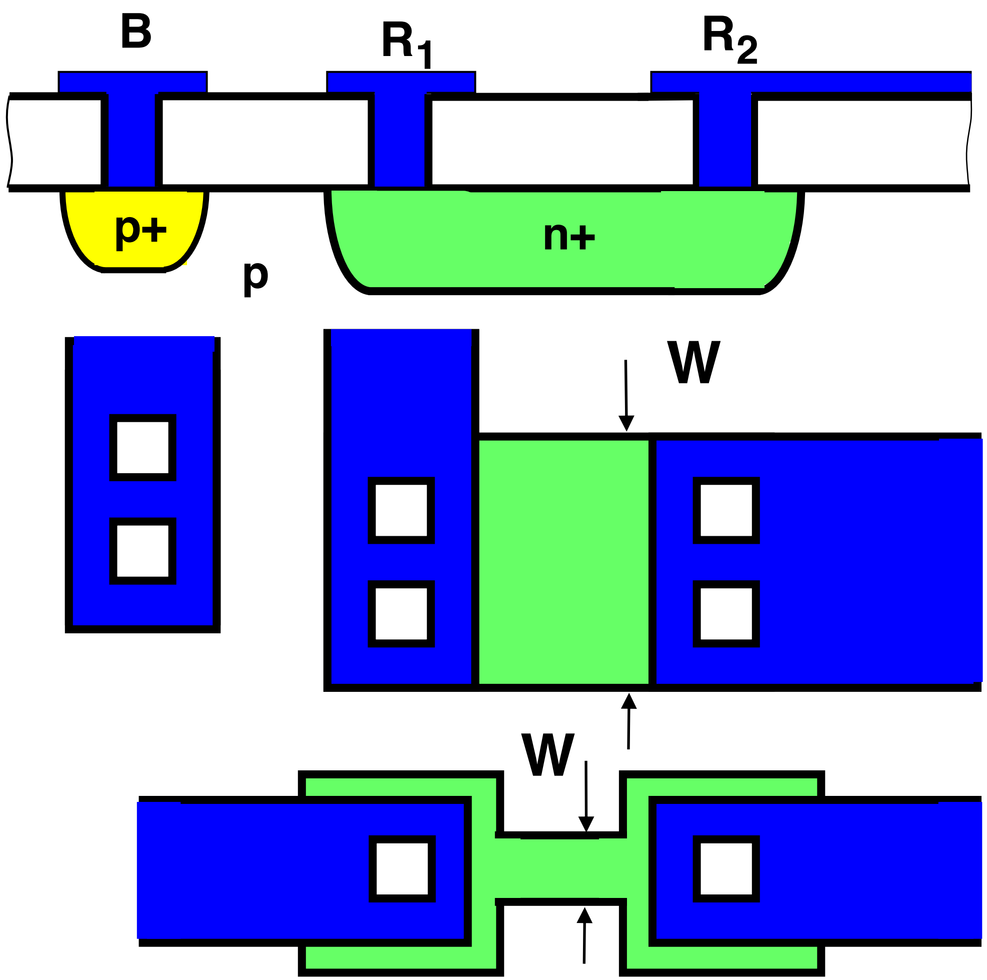
Layout of an integrated resistor



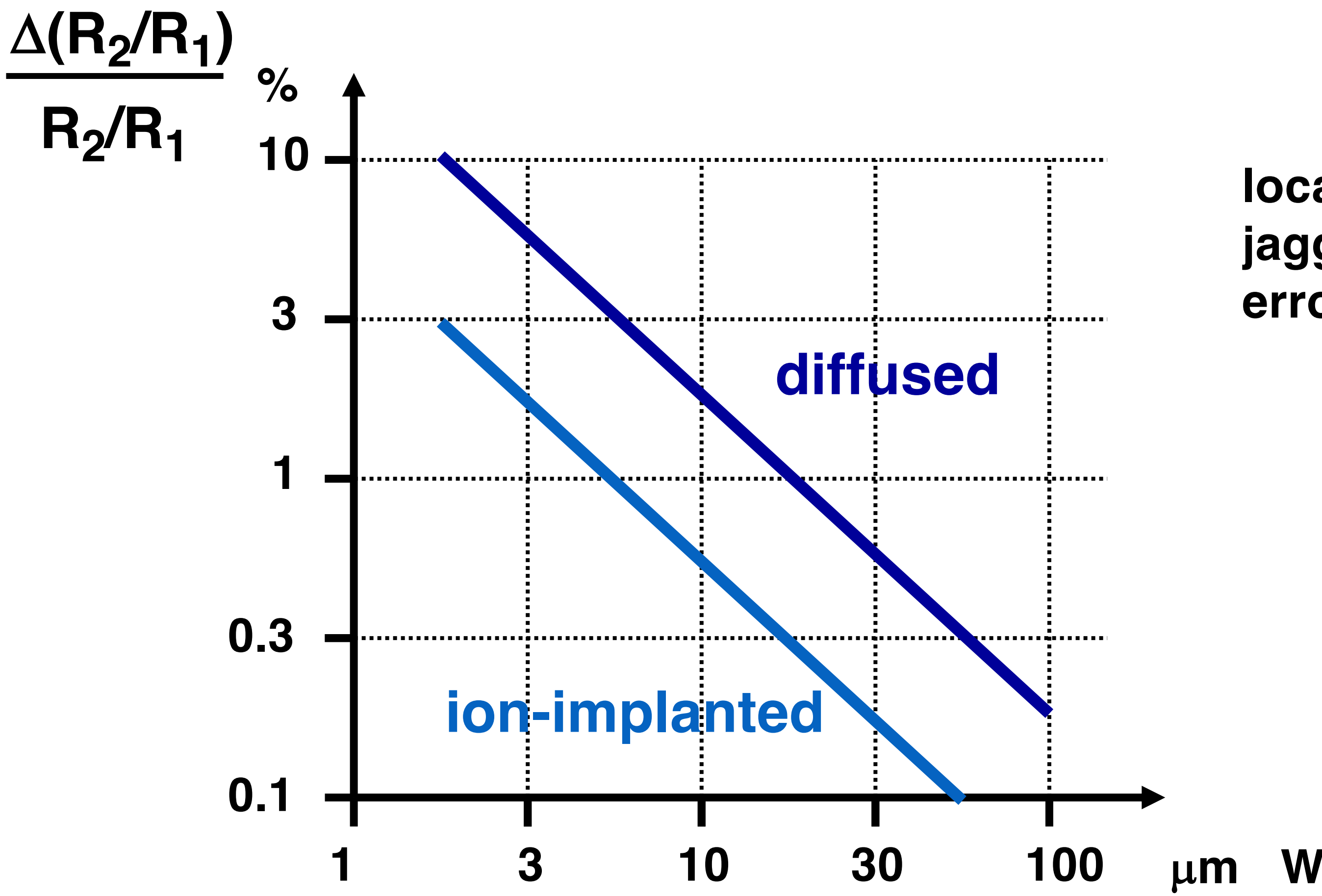
Source/drain diffusion resistor in CMOS

Ref.: Laker, Sansen : Design of analog …, MacGrawHill 1994 Table 2-6

Resistor technology parameters

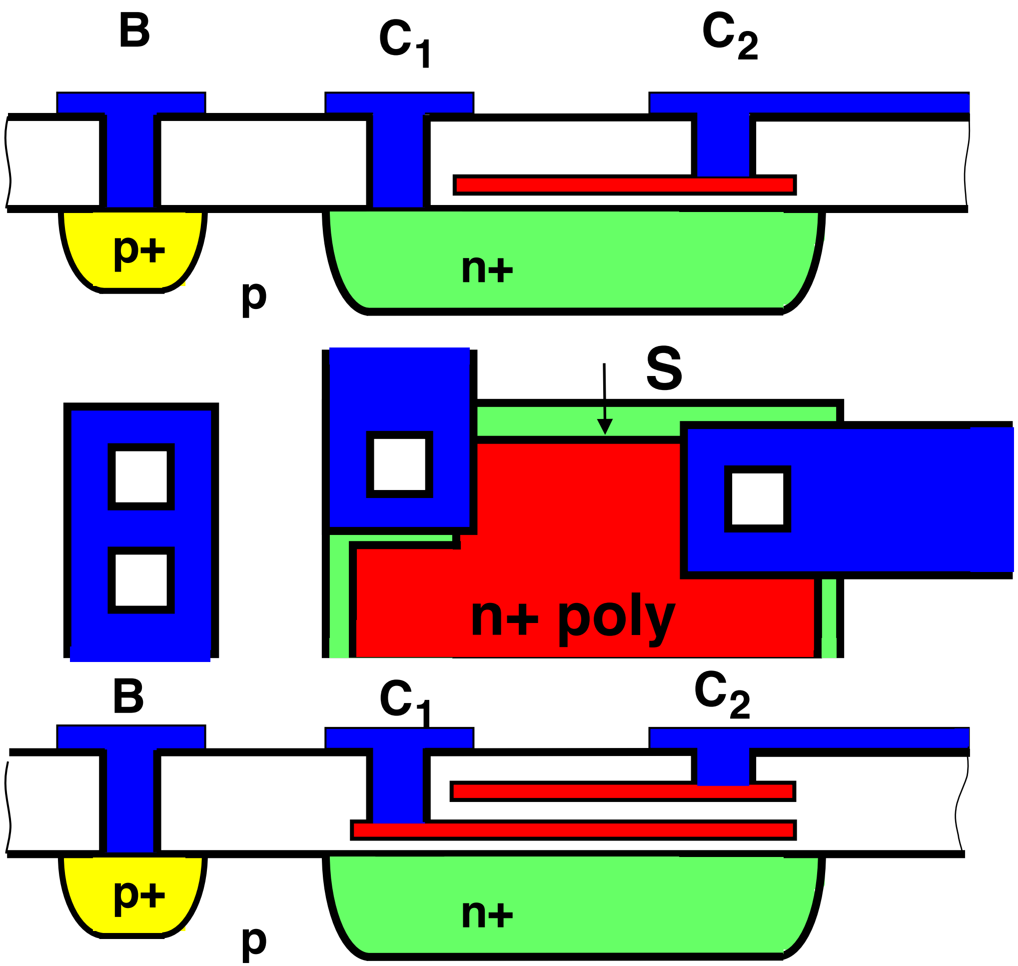
Process	Type	$\rho\square$ $\Omega/\square$	absolute accuracy percent	temperature coefficient percent/°C	voltage coefficient percent/V	breakdown voltage V
Bipolar	base diffusion	150	10	0.12	2	50
	emitter diffusion	10	20	0.02	0.5	7
	pinch resistance	5 k	40	0.33	5	7
	epi layer	1 k	10	0.3	1	60
	aluminum	50 m	20	0.01	0.02	90
	ion-implantation	2 k	1	0.02	0.2	20
	ion-implantation	200	0.3	0.02	0.05	20
CMOS	S/D diffusion	20-50	20	0.2	0.5	20
	well	2.5 k	10	0.3	1	20
	poly gate	50	20	0.2	0.05	40
	poly resistance	1.5 k	1	0.05	0.02	20
	aluminum	50 m	20	0.01	0.02	90
Thin film	NiCr(Ta)	200	1	0.005	0.005	90
	aluminum	50 m	20	0.01	0.02	90

Mismatch as a function of resistor area



local errors:
jagged edges, ...
error ~ 1/size

Layout of an integrated capacitor



Poly to S/D capacitor

Poly to poly capacitor

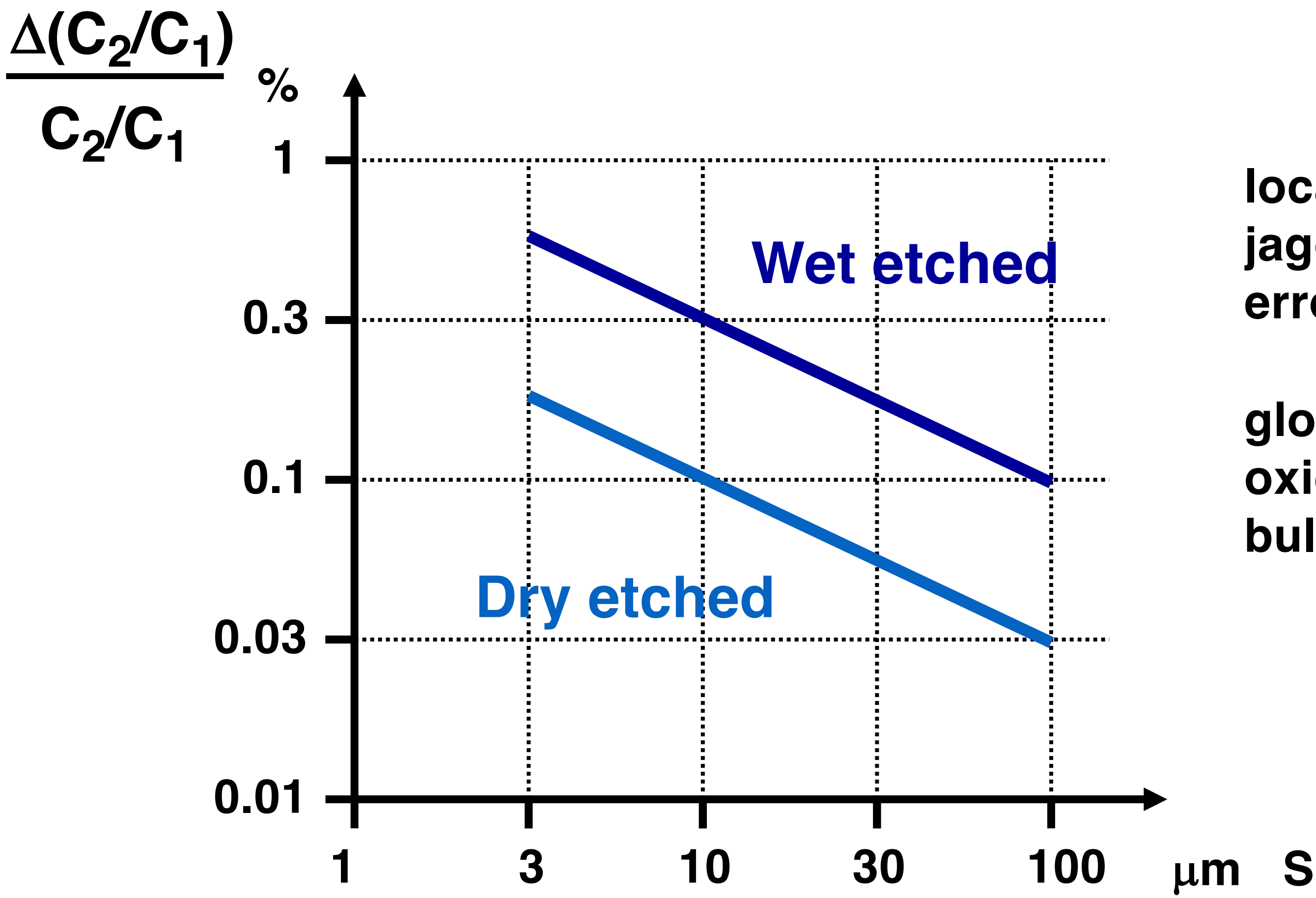
$$C_{par} \approx \frac{1}{6 \ldots 15} C_{pp}$$

Capacitor technology table

Process	Type	C nF/cm^2	absolute accuracy percent	temperature coefficient percent/°C	voltage coefficient percent V	breakdown voltage V
Bipolar	C_{CB}	16	10	0.02	2	50
	C_{EB}	50	10	0.02	1	7
	C_{CS}	8	20	0.01	0.5	60
CMOS	C_{ox}(50 nm)	70	5	0.002	0.005	40
	$C_{m,\text{poly}}$	12	10	0.002	0.005	40
	$C_{\text{poly,poly}}$	56	2	0.002	0.005	40
	$C_{\text{poly,substrate}}$	6.5	10	0.01	0.05	20
	$C_{m,\text{substrate}}$	5.2	10	0.01	0.05	20
	$C_{\text{poly,substrate}}$	6.5	10	0.01	0.05	20

Ref.: Laker, Sansen : Design of analog …, MacGrawHill 1994 Table 2-7

Mismatch as a function of capacitor area



local errors:
jagged edges, ...
error ~ 1/size

global errors:
oxide thickness,
bulk doping, ...

Good layout practices for low offset

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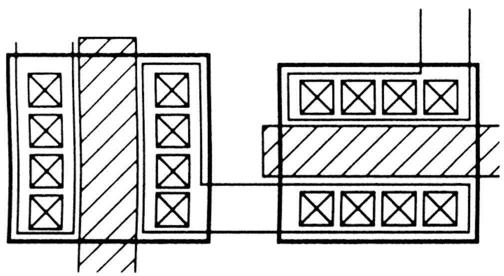
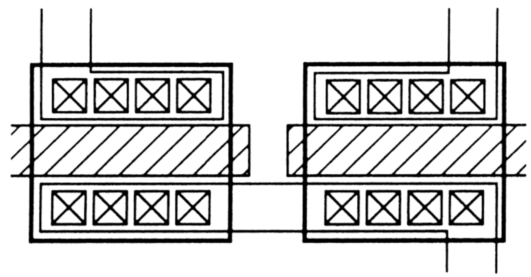
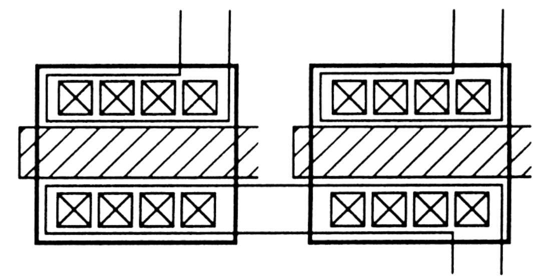
Matching of transistor pair



Bad



Better



Better

Good layout practices for low offset

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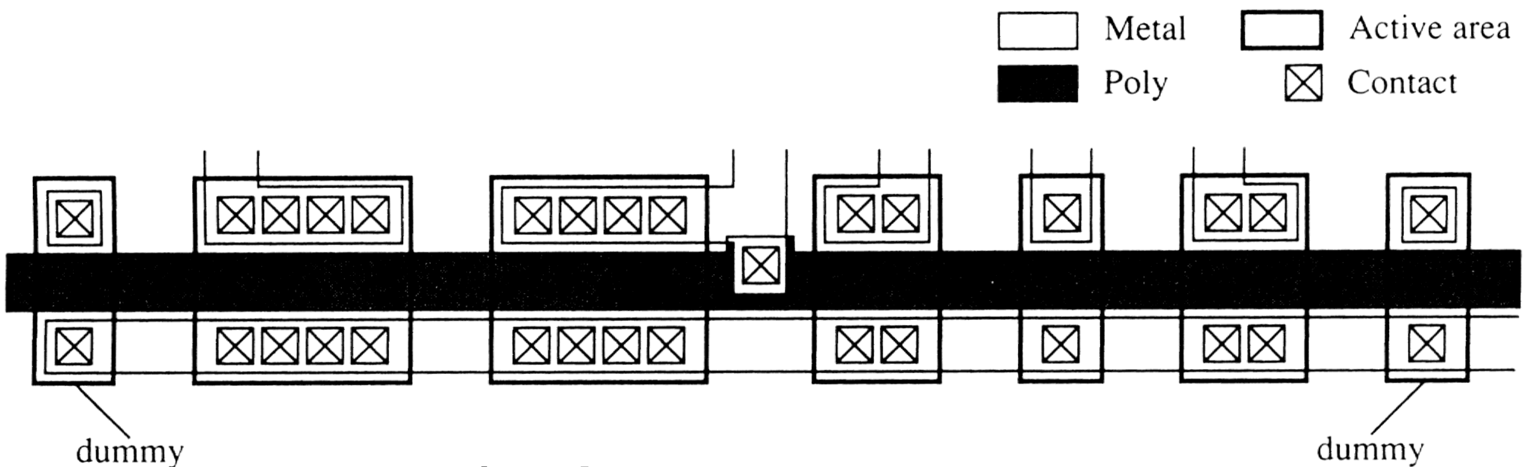
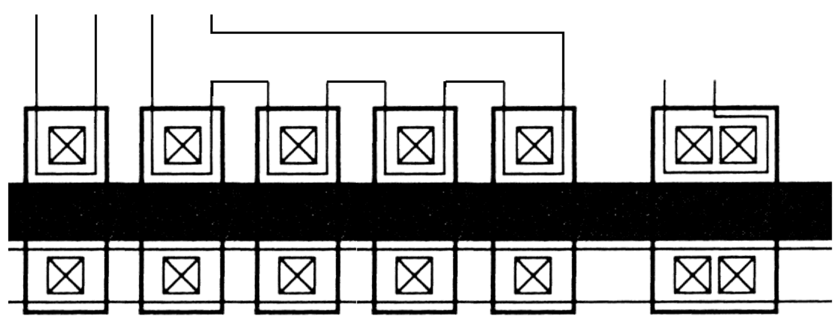
Matching of current mirrors

Current mirror 4:4:2:1:2 with end dummies.



1 : 4



Good layout practices for low offset

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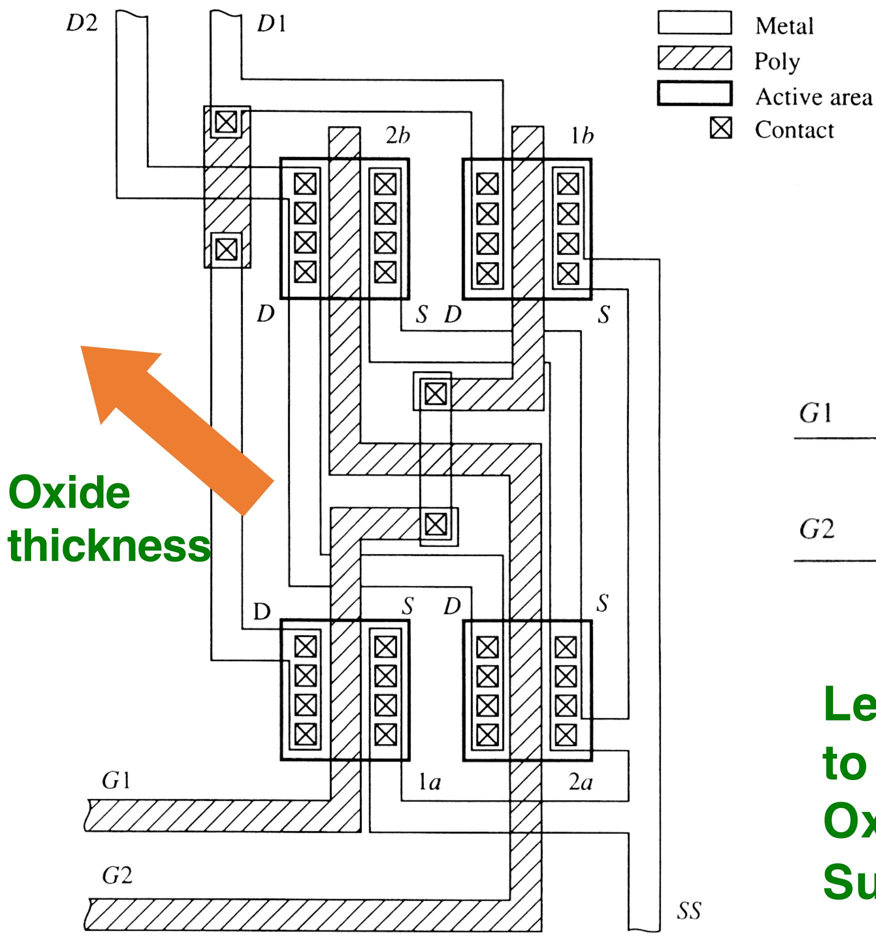
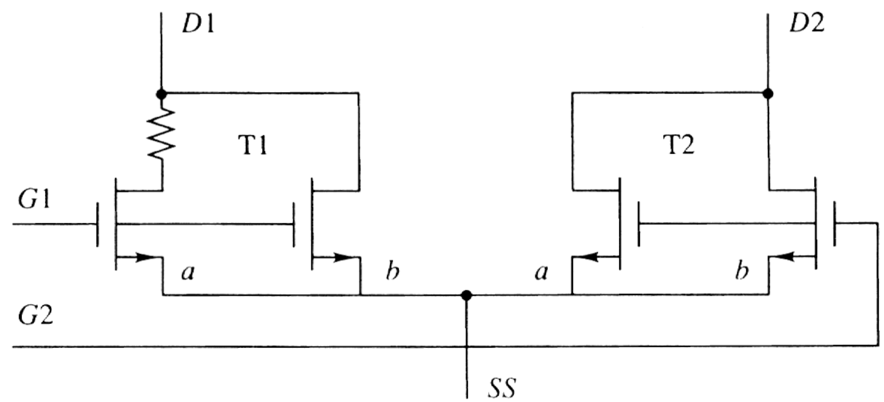
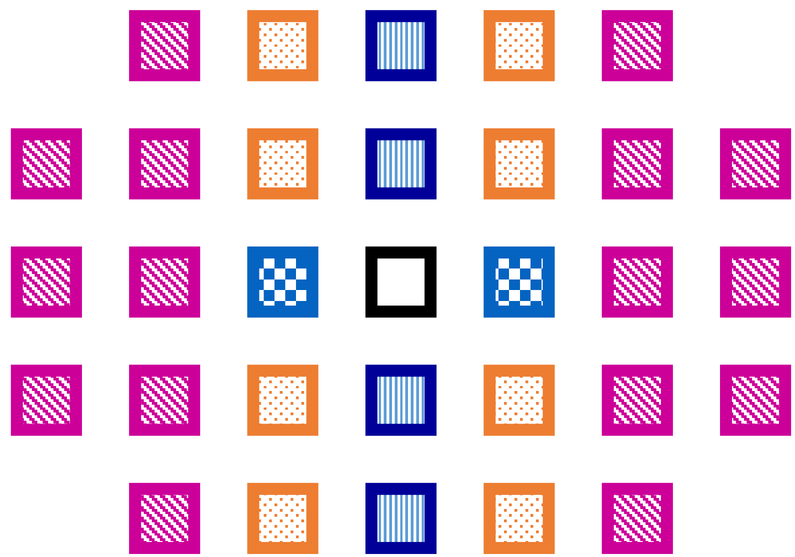
Centroide layout of a differential pair





Less sensitive
to global variations :
Oxide thickness
Substrate doping level
……..

Centroide layout of capacitor bank



Ratio:

1
2
4
8
16

Good layout practices for low offset

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2. Equal temperature
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4. Minimum distance
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6. Same area/perimeter ratio
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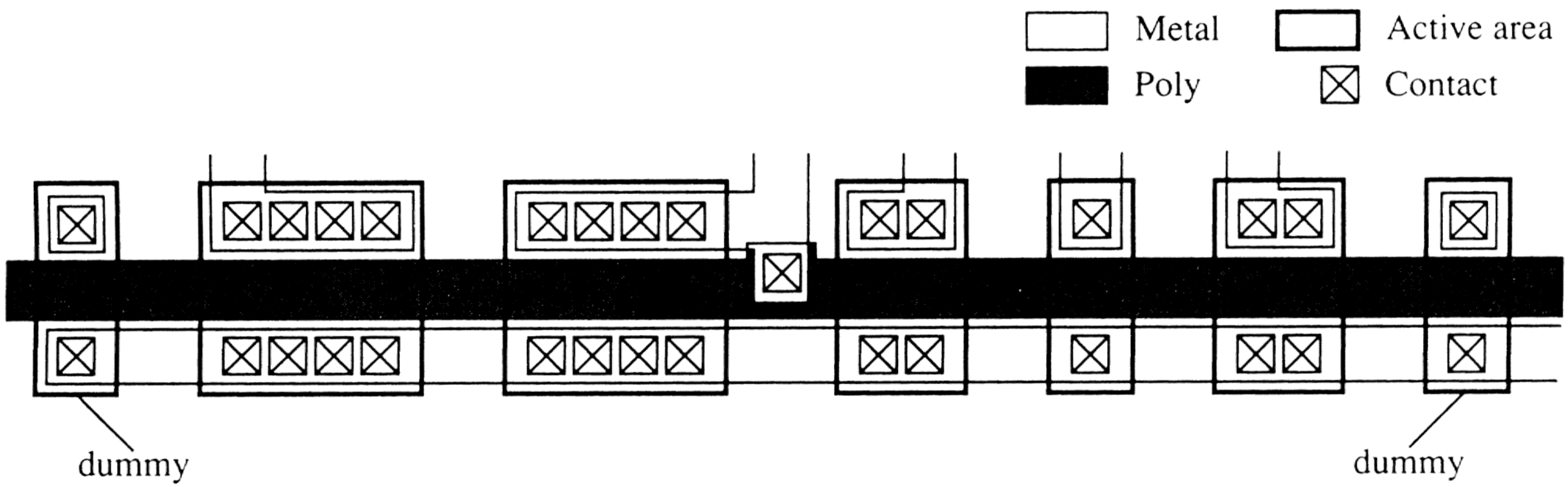
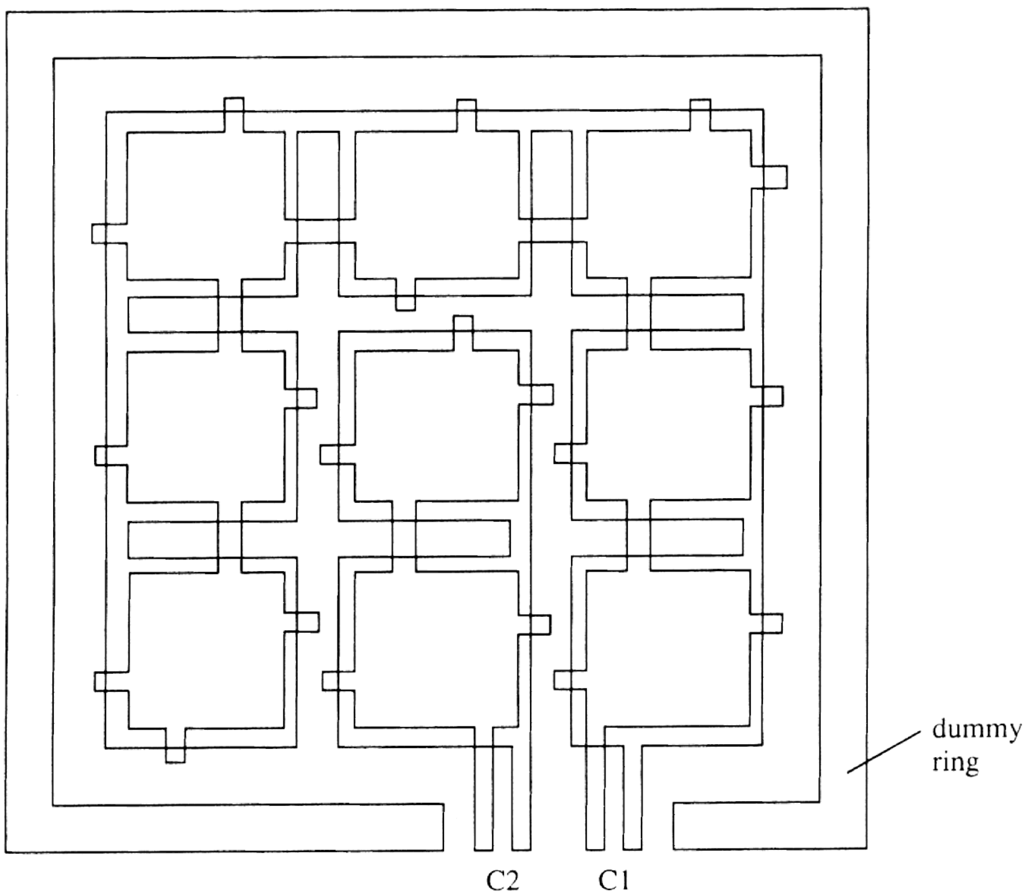
Matching of current mirrors

Current mirror 4:4:2:1:2 with end dummies.



Matching of capacitors



Ratio 7:2 = 3.5

Courtesy Vittoz

Good layout practices for low offset

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6. Same area/perimeter ratio
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10. **BJT better than MOST**

Offset for MOST and BJT

MOST: $$v_{OS} = \Delta V_T + \frac{V_{GS} - V_T}{2}\left(\frac{\Delta R_L}{R_L} + \frac{\Delta K'}{K'} + \frac{\Delta W/L}{W/L}\right)$$

Bipolar: $$v_{OS} = \frac{kT}{q}\left(\frac{\Delta R_L}{R_L} + \frac{\Delta I_S}{I_S}\right)$$ **is much smaller!**

1) no V_T

2) $kT/q \ll (V_{GS}-V_T)/2$

3) Drift decreases with v_{OS}: $$\frac{\Delta v_{OS}}{\Delta T} = \frac{v_{OS}}{T}$$

Bipolar: Base current!

Table of contents

- Random offset and $CMRR_r$
- Systematic offset and $CMRR_s$
- Total CMRR and frequency dependency
- Good layout practices
- **Accuracy limit of analog circuits**

Limits because of mismatch

$$\frac{1}{(\text{Accuracy})^2} \approx \sigma^2\left(\frac{\Delta I_{DS}}{I_{DS}}\right) \approx \frac{4\,A_{VT}^{\;2}}{WL\,\underbrace{(V_{GS} - V_T)}^2}$$

$$\text{Speed} \approx f_T = \frac{2\,I_{DS}}{2\pi\, WL\, 2/3\, C_{ox}\,\underbrace{(V_{GS} - V_T)}}$$

$$\frac{V_{DD}}{2}$$

$$\frac{\text{Speed x }(\text{Accuracy})^2}{\text{Power}} = \frac{1}{C_{ox}\,A_{VT}^2} \sim \frac{1}{t_{ox}}$$

= technological constant

Limits because of noise

$$S/N = \frac{V_{pp}^2 / 2}{4kT\,R\,BW}$$

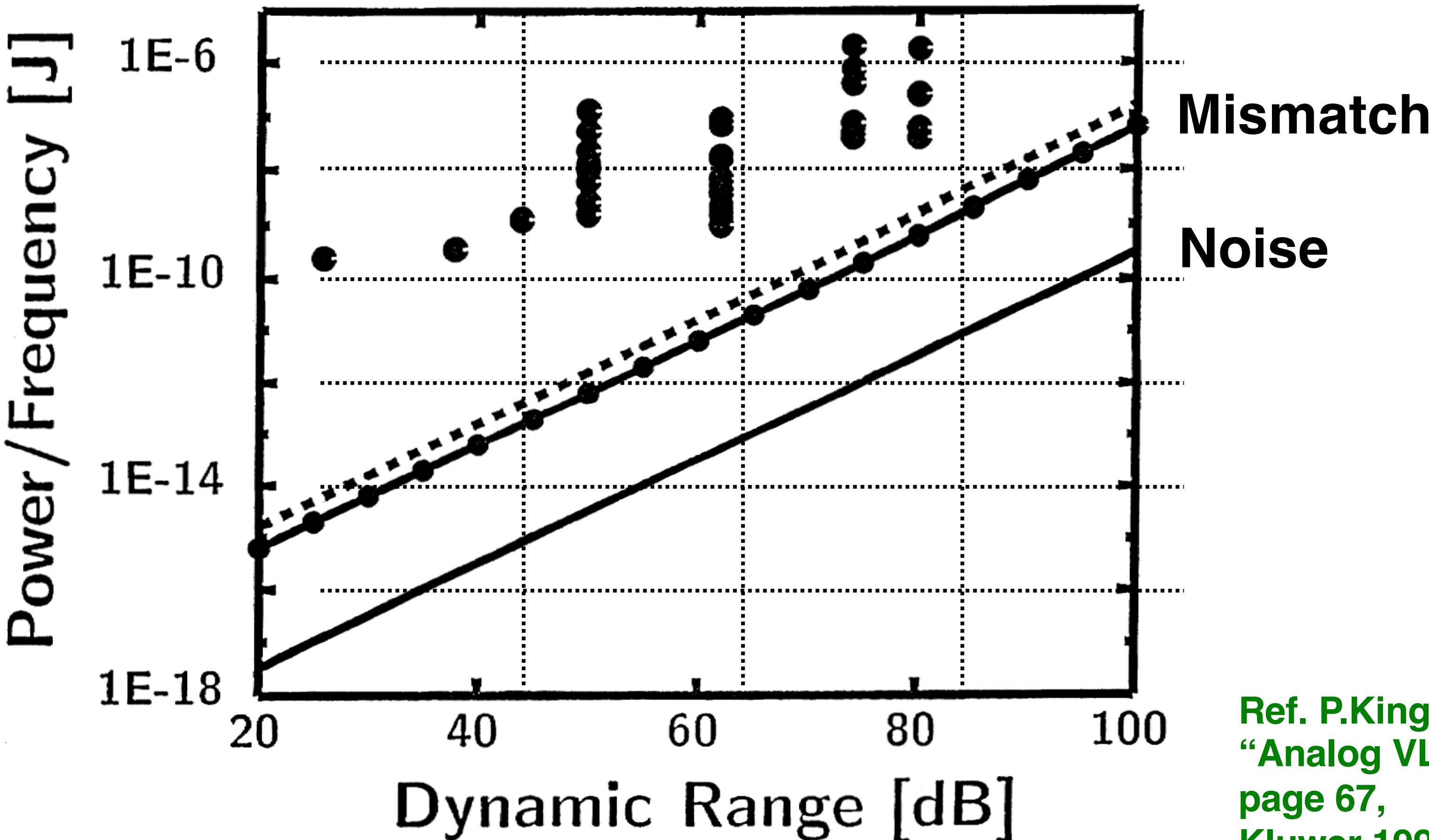
$$S/N = \frac{V_{pp}^2 / 8}{kT / C}$$

$$P_{min} = \frac{V_{pp}^2}{R}$$

$$P_{min} = V_{DD}\,BW\,V_{pp}\,C$$

$$P_{min} \approx 8kT\,BW\,S/N$$

Speed-accuracy-power trade-off



Ref. P.Kinget, ...
"Analog VLSI .."
page 67,
Kluwer 1997.

The challenge of technology scaling