Cache: Direct Mapping Address Structure

Direct Mapping Fundamentals

- Main Memory Structure:
  - Memory is divided into blocks (eg. 64 blocks)
  - Cache is made of lines

- Cache Mapping Formula:
  - A memory block maps to a specific cache line
    cache line index = (block #) mod (# of lines)

Example:
- Main Memory has 32 blocks (0-31), cache has 4 lines (0-3) * Unrealistic Example
- Block 25 maps to line 1 (25 mod 4 = 1)
- Block 4 and block 16 both map to line 0
- Consequence: If block 4 is in line 0, and block 16 is loaded, then block 4 gets evicted

Tag and Line Number: Address Breakdown

- When we store to cache, we want to store the block # alongside the data data, to better be able to check the cache for hits/misses
- But we don't want to store the entire block # since that would take up too much space

If we have a 4-line cache, any block # that is a multiple of 4 will map to line 0
Likewise, any mem block # that is a (multiple of 4) + 1 will map to cache line 1

line 0:

Mem block #	block # binary
block 0	0 0 0 0 0
block 4	0 0 1 0 0
block 8	0 1 0 0 0
block 12	0 1 1 0 0
⋮	

line 1:

Mem block #	block # binary
block 0	0 0 0 0 1
block 4	0 0 1 0 1
block 8	0 1 0 0 1
block 12	0 1 1 0 1
⋮	

We notice that the last 2 digits are the same, so we can simply store the first 3 digits and add back the last 2 digits on the fly when we want to access the cache. This "truncated" block # is called the tag

When a block is loaded into a cache line, its identity must be memoized using a tag
- Tag = top bits of the address (unique identifier)
- Line Number = lower bits (used to index into the cache)

General Formula

- Address size: S bits (memory size = 2^S)
- Cache size: 2^R lines (line index size = R bits)
- Tag size = S - R

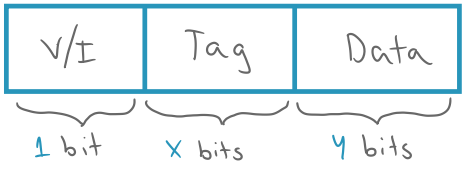
Example:
- Memory has 32 blocks = 5 bit address (S = 5)
- Cache has 4 lines = 2 bit line # (R = 2)
- Tag = 5 - 2 = 3 bits

Cache Structure

Each cache line contains:

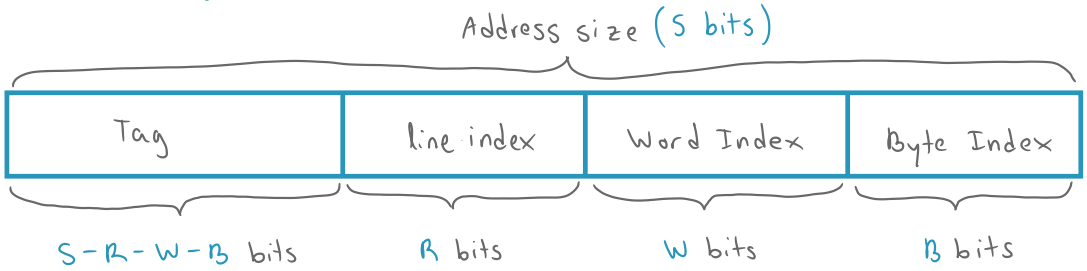
- Valid/invalid bit (V/I): Indicates if the data is valid
- Tag: Used for identifying the block stored
- Data: The contents of the block

Cache Line Format



Tag and data bit width will depend on the system being described

Direct Mapping Address Structure



- S = Total address bits
- R = Bits for line index
- W = Bits for word index (If multiple words per block. If only 1 word per block, we don't include a partition for the word index)
- B = Bits for byte index (if the system uses byte addressing, otherwise we don't include this partition for the byte index)

Address Decomposition for Word and Byte Addressing

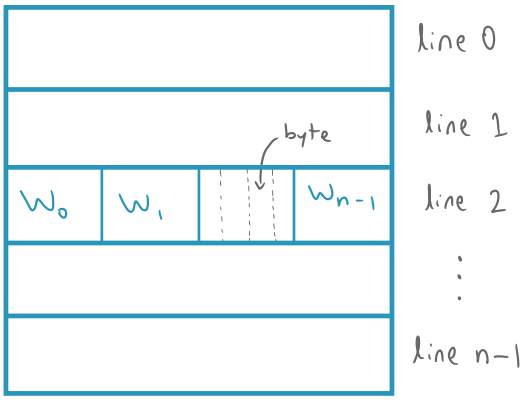
Scenarios:

- Word Addressing, 1 word per block

- Byte Addressing, 1 word per block, word size = 4 bytes

- Word Addressing, multiple words per block

- Byte Addressing, multiple words per block



Cache Initialization and Context Switching

- On program startup: all cache lines are marked invalid
- On context switch:
  - Previous process's cache data becomes irrelevant
  - All cache lines are again marked invalid
- Valid bit is set to 1 only when new block data is loaded

Total Cache Size Calculation

Formula:

$$\text{Total cache size} = \#\text{ of lines} \times (1 + \text{Tag Bits} + \text{Data bits})$$

- Find the total # of bytes for a direct mapping cache to store 64 KB in 1-word blocks assuming a word size of 32 bits and MIPS addressing

↳ Break down the specifications:
- Word size = 32 bits → 32 bit address
- Block size = 1 word
- Addressing mode: byte addressing (from what we know of MIPS)
- Data: 64 KB

* b = bits
* B = bytes

Block size = 1 word → 1 word × 32 bits/word = 32 b → 4 B = 2^2 B

↳ We don't need to allocate any bits for word select, since we have only 1

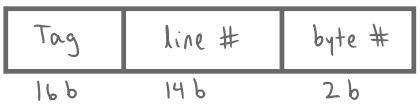
Word size = 32 bits → 4 B = 2^2 B → Allocate 2 bits for addressing byte # (byte addressing)

Data = 64 KB = 64×2^{10} B = $2^6 \times 2^{10}$ B = 2^{16} B

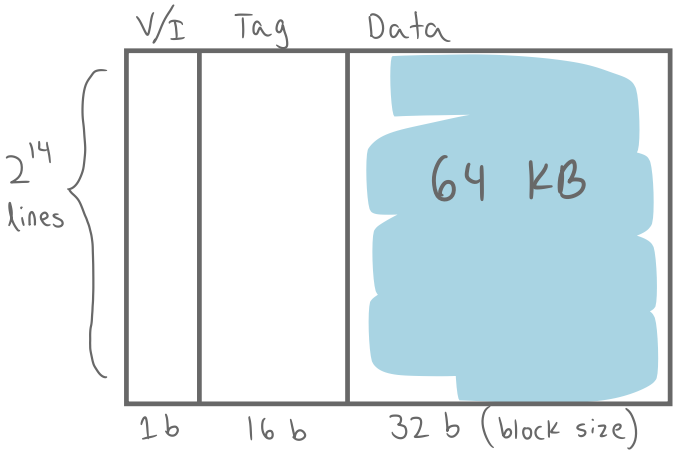
↳ # of lines = $\frac{\text{Data size}}{\text{block size}} = \frac{2^{16}\text{ B}}{2^2\text{ B}} = 2^{14}$ lines

Tag = 32 − (14 + 2) = 16 bits

↳ Address Structure



Cache Structure



→ 39 bits

Total cache size

2^{14} lines × 39 bits/line = $(39 \cdot 2^{14})$ bits

$= (39 \cdot 2^4)(2^{10})$ bits $= (39 \cdot 16)(1\text{ Kb}) = (39 \cdot 16)$ Kb

$= (39 \cdot 2)$ KB = **78 KB**