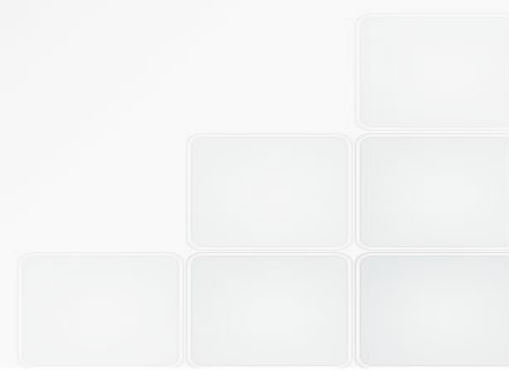


## Functions for the complex refractive index modeling

**Alberto Mittiga**

Energy Technologies and Renewable Sources Department,  
Photovoltaics and Smart Network Division  
ENEA – C.R. Casaccia,  
Via Anguillarese 301, 00123 Rome, Italy  
[alberto.mittiga@enea.it](mailto:alberto.mittiga@enea.it)



# Basic relations among $\tilde{n}$ , $\tilde{\chi}$ , $\tilde{\epsilon}$

The theories for the light-matter interaction give the simplest expression for the complex dielectric susceptibility  $\tilde{\chi} = \chi_1 - i \chi_2$ .

Refractive index, dielectric susceptibility and permittivity  $\tilde{\epsilon} = \epsilon_1 - i \epsilon_2$  are connected by the following simple equations:

$$\tilde{\epsilon} = (n - ik)^2 = 1 + \tilde{\chi}.$$

Therefore:

$$\epsilon_1 = 1 + \chi_1, \quad \epsilon_2 = \chi_2.$$

and:

$$n = \frac{1}{\sqrt{2}} \sqrt{\sqrt{\epsilon_1^2 + \epsilon_2^2} + \epsilon_1}, \quad k = \frac{1}{\sqrt{2}} \sqrt{\sqrt{\epsilon_1^2 + \epsilon_2^2} - \epsilon_1}.$$

Inside kSEMAWc  $\tilde{\epsilon}$  is calculated as a sum of "oscillators"  $\tilde{\chi}_i$  corresponding to different groups of electronic transitions:

$$\tilde{\epsilon} = 1 + \sum_i \tilde{\chi}_i = 1 + \sum_i C_i \tilde{f}_i$$

Sum rule on  $\chi_2$ :

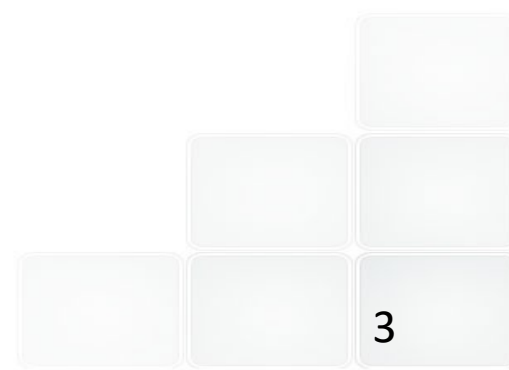
$$\int_0^\infty \chi_2(E) E dE = \frac{\pi(\hbar q)^2 N}{2\epsilon_0 m_e} = \frac{N(cm^{-3})}{4.61706 \cdot 10^{20}} eV^2$$

where  $N$  is electron number per unit volume of the material (all of them, including the core electrons),  $q$  is the electron charge and  $m_e$  is the electron mass.

As far as possible, the oscillator functions  $\tilde{f}_i$  in kSEMAWc are normalized according to:

$$\int_0^\infty \text{Im}(\tilde{f}_i(E)) E dE = 1$$

"ideal global fit ( $0 < E < \infty$ )"  $\implies \sum_i C_i = \frac{\pi(\hbar q)^2 N}{2\epsilon_0 m_e}$



Generally the fit is limited to energies lower than a few eV.

Transitions at higher energies, not explicitly included in the oscillator sum, give anyway a substantial contribute to  $\epsilon_1$ .

To simplify the fit, the 1 is replaced with a user-selectable parameter  $C_{flat}^2$ :

$$\tilde{\epsilon} = C_{flat}^2 + \sum_i \tilde{\chi}_i = C_{flat}^2 + \sum_i C_i \tilde{f}_i$$

$C_{flat}$  is the only parameter of a special “flat” oscillator and, since it is squared to obtain  $\epsilon_1$ ,  $C_{flat}$  corresponds fairly well to the refractive index offset generated by this “Flat” oscillator:  
 $n \simeq C_{flat} + \dots$

**A “flat” oscillator with  $C_{flat} > 1$  must be always included in the fit**



# Oscillators available in ksemawc v2.6

$\tilde{\epsilon}(E)$  is calculated summing up to 20 oscillators belonging to the following 18 classes:

Flat

Lorentz  
Quantum-homo  
Lorentz-Dirac  
Quantum-inhomo

Single peak in  $\epsilon_2$

Drude  
Drude-ionized

Free carrier effects

Direct Gap Cody-Lorentzian tail  
Direct Gap Tauc-Lorentzian tail  
Indirect Gap Cody-Lorentzian tail  
Indirect Gap Tauc-Lorentzian tail

Direct Gap Tauc-Exciton

Direct Gap Cody- $M_1$ - $M_2$   
Direct Gap Tauc- $M_1$ - $M_2$

Direct Gap Cody-Urbach (exponential) tail  
Direct Gap Tauc-Urbach (exponential) tail  
Indirect Gap Cody-Urbach (exponential) tail  
Indirect Gap Tauc-Urbach (exponential) tail

All the oscillators have less than four free parameters.

	$C \text{ (eV}^2\text{)}$	$E_0 \text{ (eV)}$	D	$W \text{ (eV)}$
Flat	$C > 1$	-	-	-
Single peak	intensity	Peak energy	Line width	-
Lorentz-Dirac	intensity	Peak energy	Line width	Line asymmetry
Drude	intensity	-	Line width	-
Dir/Indir. Gap Lorentz	intensity	$E_g$	Line width	Band width
Dir/Indir. Gap Urbach	intensity	$E_g$	Urbach tail	Band width
Dir. Gap Exciton	intensity	$E_g$	Line width	Binding energy
Dir. Gap M1-M2	intensity	$E_g$	$(E_1 - E_2)/2$	Band width

# Single peak oscillators formulas

“Lorentz”

$$\tilde{\chi} = \frac{2C}{\pi} \left[ \frac{1}{E_0^2 - E^2 + 2iED} \right]$$

“Quantum-homo”

$$\tilde{\chi} = \frac{C}{\pi E_0} \left[ \frac{1}{E_0 - E + iD} \right]$$

“Lorentz-Dirac”

$$\tilde{\chi} = \frac{2C}{\pi} \left[ \frac{1}{E_0^2 - E^2 + 2iE(D + E^2/W)} \right]$$

“Quantum-inhomo”

$$\tilde{\chi} = \frac{C}{K} \left\{ \frac{2}{\sqrt{\pi}} D_+ \left( \frac{E_0 - E}{D/\sqrt{\ln 2}} \right) - i \cdot \exp \left[ - \left( \frac{E - E_0}{D/\sqrt{\ln 2}} \right)^2 \right] \right\}$$

$C = (\pi \hbar^2 q^2 N) / (2m\epsilon_0)$  is the oscillator amplitude with dimensions [eV]<sup>2</sup>

$E_0$  is the resonance energy.

$D$  is the line width.

$W$  is the asymmetry parameter for the Lorentz-Dirac oscillator

$D_+$  is the Dawson function

$K$  is a normalization constant for the Quantum-inhomo oscillator



**Lorentz:** standard function for the fit of peak shaped absorption spectra. Its long tails may cause problems if used for materials with a high transparency in adjacent energy range.

**Quantum-homo:** obtained neglecting a not-resonant term in the Lorentz oscillator. Its simpler expression allows to obtain analytical results in convolution calculations.

**Lorentz-Dirac:** asymmetric line shape. Useful for the fitting of noble metals optical constants.

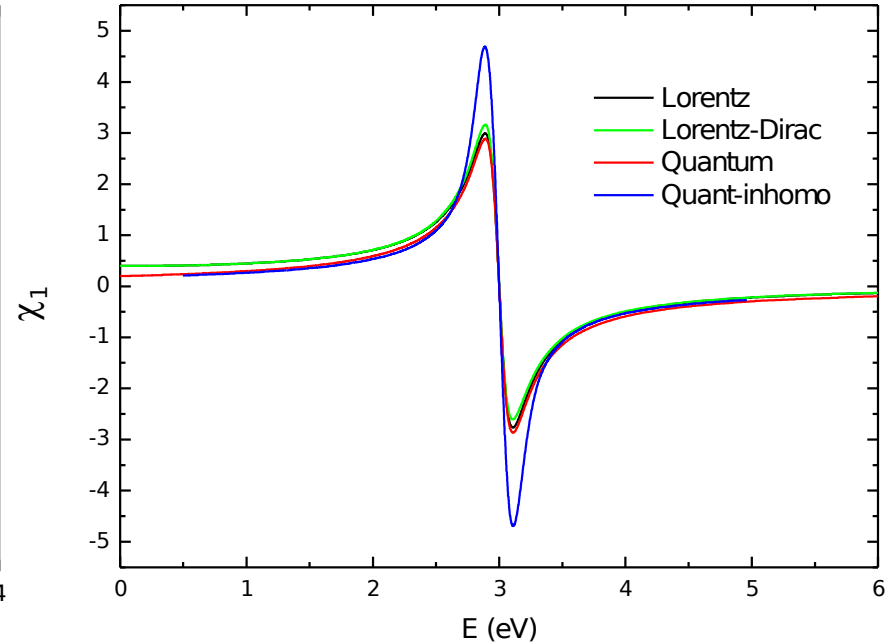
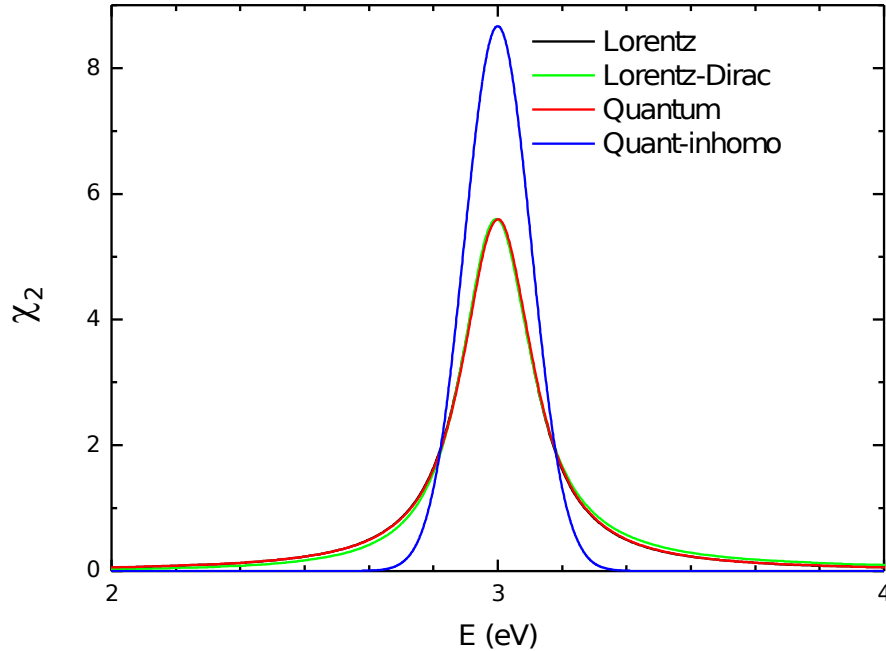
**Quantum-inhomo:** obtained as the convolution between a quantum oscillator and a Gaussian distribution centered in  $E_0$  and with HWHM equal to  $D$ ; the line width of the quantum oscillator is assumed to be much lower than the Gaussian width.

Useful to model a set of absorbing centres which cannot be considered as identical replicas due, for example, to proximity to other randomly distributed defects or to random fluctuations of strain (inhomogeneous broadening).

The fast decay of its  $\chi_2$  away from the resonance energy avoid some problems encountered using the Lorentz oscillator.



# Comparison of the 4 single oscillators



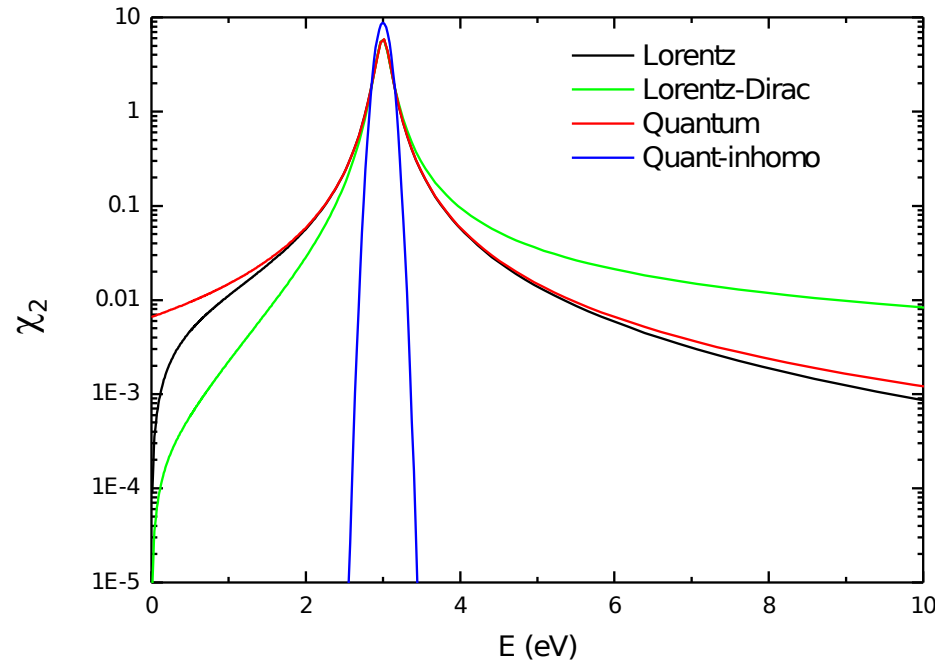
The oscillator parameter used for this figure are:

$C=5.6 \text{ eV}^2$  and  $E_0 = 3 \text{ eV}$ .

$D=0.1 \text{ eV}$  for Lorentz, Quantum and Quantum-inhomo oscillators.

$D=0.01 \text{ eV}$  and  $W=50 \text{ eV}$  for Lorentz-Dirac oscillator.

# Comparison of $\chi_2$ for the 4 single oscillators



$\chi_2$  for the 4 single peak oscillators (Lorentz, Lorentz-Dirac, Quantum and Quantum-inhomo) with  $C=5.6$  eV<sup>2</sup> and  $E_0 = 3$  eV.

$D=0.1$  eV for Lorentz, Quantum and Quantum-inhomo oscillators.

$D=0.01$  eV and  $W=50$  eV for Lorentz-Dirac oscillator.

The “Drude” oscillator describes fairly well the response of a free electron gas. It can be obtained simply setting  $E_0 = 0$  in the Lorentz oscillator :

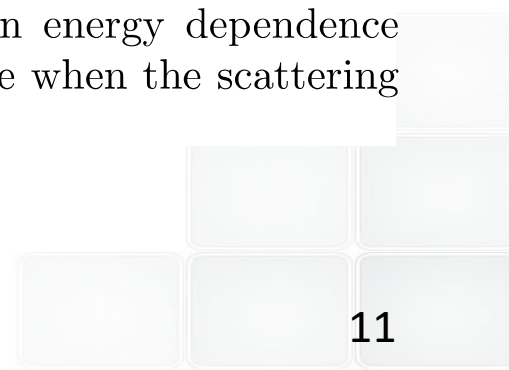
$$\chi_1 = -\frac{2C}{\pi} \frac{1}{D^2 + E^2}, \quad \chi_2 = \frac{2C}{\pi} \frac{D}{E(D^2 + E^2)}.$$

$D$  is related to the carrier scattering time  $\tau$  by  $D = \hbar/\tau$ .

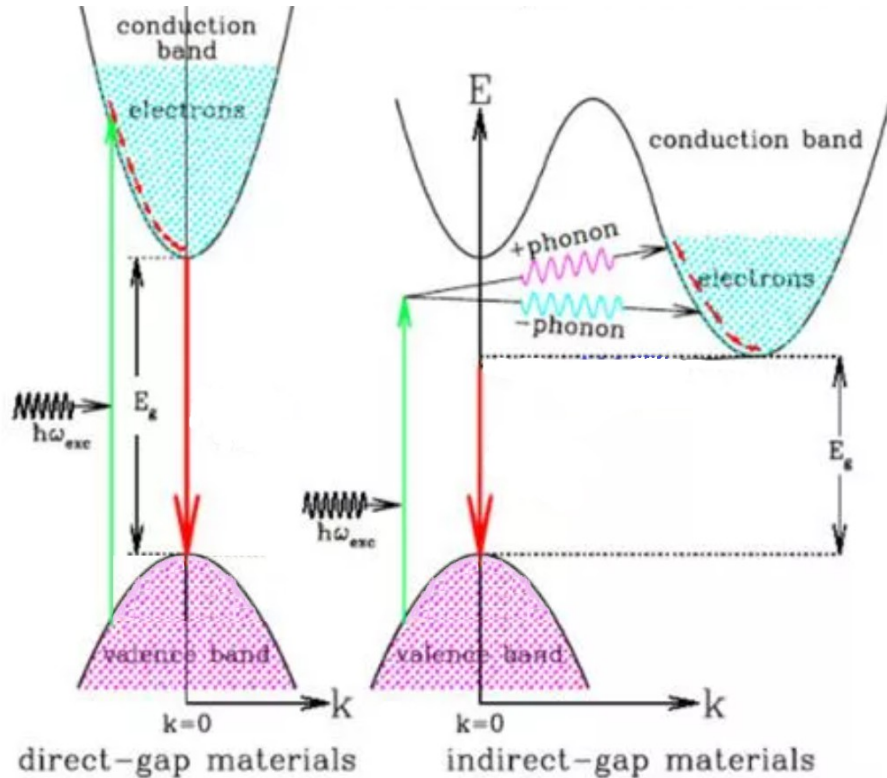
The optical mobility can be deduced from  $D$  using the equation  $\mu = q\tau/m^* = (q\hbar)/(m^*D)$ .

$C = (\pi\hbar^2 q^2 N)/(2m^* \epsilon_0)$  where  $N$  is the carrier density and  $m^*$  is their effective mass.

A slightly modified version (“Drude-ionized”) is available where  $D$  has an energy dependence ( $D(E)$ ) to approximate the energy dependence of the carrier scattering time when the scattering is caused by ionized impurities.



# Direct and Indirect gap oscillators



The oscillators introduced up to now are unsuitable to describe the optical constants of systems with a continuous distribution of density of states such as semiconductors.

The theories reported in textbooks on semiconductors usually derive  $\chi_2$  only and exclusively at energies near the fundamental gap.

The calculation is made by hypothesizing that the excited states have infinite lifetime, i.e. that every transition between two states may take place only by absorbing/emitting photons and phonons with a total energy exactly equal to the difference of their energetic levels.

# $\chi_2^{inf}$ for excited states with infinite lifetime

Joint density of states

Calling  $\chi_2^{inf}$  the  $\chi_2$  calculated for excited states with infinite lifetime, we have for **direct transitions**:

$$\chi_2^{inf}(\hbar\omega) \propto C^{abs} \frac{|M_{cv}|^2}{\omega^2} \int_{E_C(k)-E_V(k)-\hbar\omega} \frac{dS}{\left| \vec{\nabla}(E_C(k) - E_V(k)) \right|}$$

For **indirect transitions** :

$$\chi_2^{inf}(\hbar\omega) \propto C^{abs} \frac{|M_{cv}|^2}{\omega^2} \int_{E_g}^{\hbar\omega + \hbar\omega_k} G_C(E_f - E_g) G_V(\hbar\omega + \hbar\omega_k - E_f) dE_f$$

Conduction and valence band convolution

Tauc approximation  $\iff$  constant momentum matrix element  $|M_{cv}|^2$

A constant  $|\langle r \rangle|^2$  assumption can be also used as proposed by Cody for the amorphous semiconductor case.

A relation between the matrix elements of these two operators can be obtained by using commutator relations:

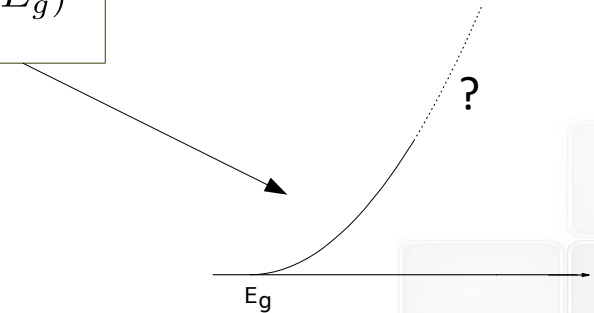
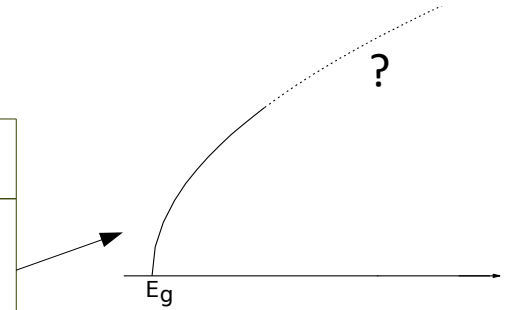
$$|\langle r \rangle|^2 = |M_{cv}|^2 \left( \frac{\hbar}{mE} \right)^2$$



# Energy dependence near $E_g$

If the bands are parabolic:

	Tauc	Cody
Direct gap	$\chi_2^{inf}(E) \propto \frac{ M_{cv} ^2}{E^2} \sqrt{E - E_g}$	$\chi_2^{inf}(E) \propto  \langle r \rangle ^2 \sqrt{E - E_g}$
Indirect gap	$\chi_2^{inf}(E) \propto \frac{ M_{cv} ^2}{E^2} (E - E_g)^2$	$\chi_2^{inf}(E) \propto  \langle r \rangle ^2 (E - E_g)^2$

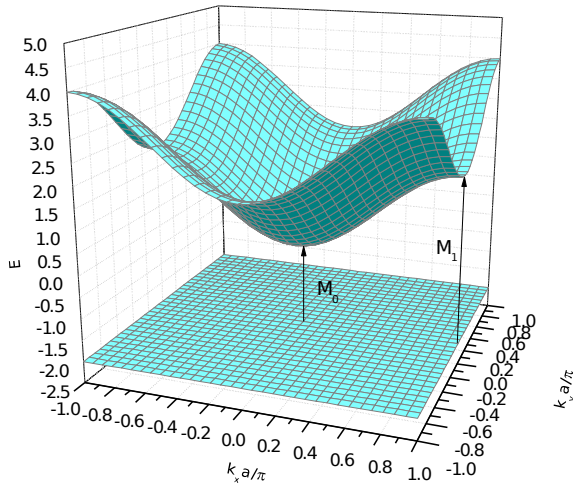


# 3D critical points

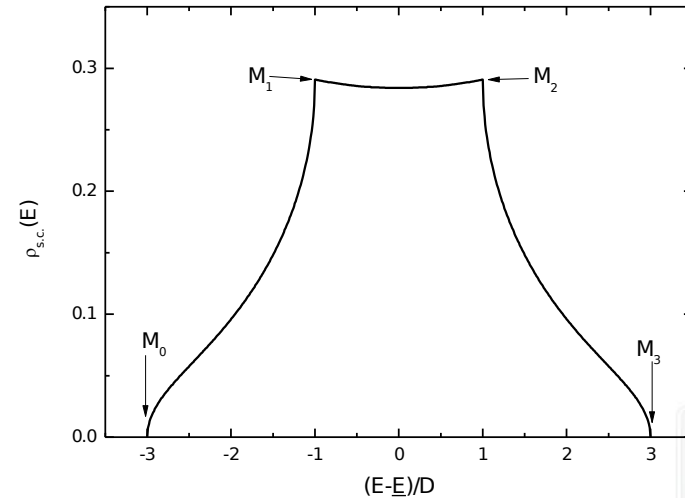
In 3 dimensions the Joint Density of States can show four types of critical points where  $\vec{\nabla}(E_C(k) - E_V(k)) = 0$  :  $M_0$ (minimum),  $M_1$ (saddle),  $M_2$  (saddle),  $M_3$  (maximum).

Band for a simple cubic lattice in the Tight-Binding s-level nearest-neighbor approximation with an overlap integral equal to  $D$ :

$$E(\vec{k}) = E_{lev} - 2D[\cos(k_x a) + \cos(k_y a) + \cos(k_z a)]$$



Plot for the 2-D case

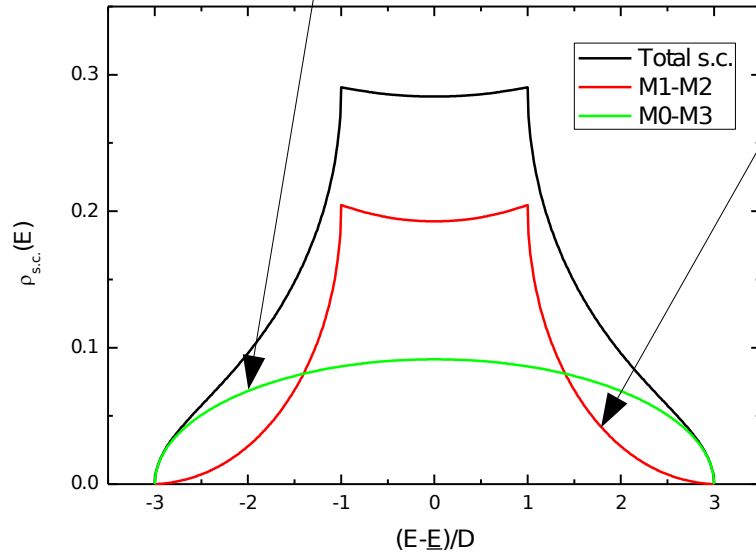


Density of states for the 3-D case



# 3D critical points : analytical approximation

$$\rho_{s.c.}(E) = A\sqrt{(E - E_0)(E_3 - E)} + B \left[ \sqrt{\frac{W}{2} - D} - \Theta(|E - \bar{E}| - D) \sqrt{|E - \bar{E}| - D} \right] + C(E - E_0)(E - E_3)$$

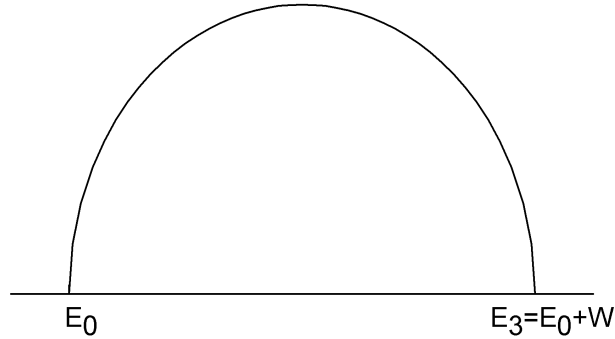


$$\bar{E} = (E_3 + E_0)/2, \quad W = E_3 - E_0$$

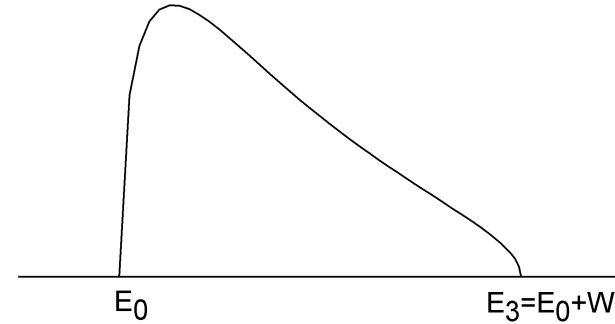
$$B = C \left[ 2(E_3 - E_0) \sqrt{\frac{E_3 - E_0}{2} - D} \right]$$

To keep the oscillators simple and with 4 parameters only,  $M_0$ - $M_3$  and  $M_1$ - $M_2$  contributions to direct transitions are simulated by separate oscillators.

# Direct gap-Cody and Direct gap-Tauc ( $M_0$ - $M_3$ )



$$\chi_{2-dir-Cody}^{inf}(E) = \frac{C}{K_{Cd}} \sqrt{(E - E_0)(E_3 - E)}$$



$$\chi_{2-dir-Tauc}^{inf}(E) = \frac{C}{K_{Td}} \frac{\sqrt{(E - E_0)(E_3 - E)}}{E^2}$$

$$K_{Cd} = \frac{\pi}{16} W^2 (2E_0 + W)$$

$$K_{Td} = \frac{\pi}{2} (2E_0 + W - 2\sqrt{E_0(E_0 + W)})$$

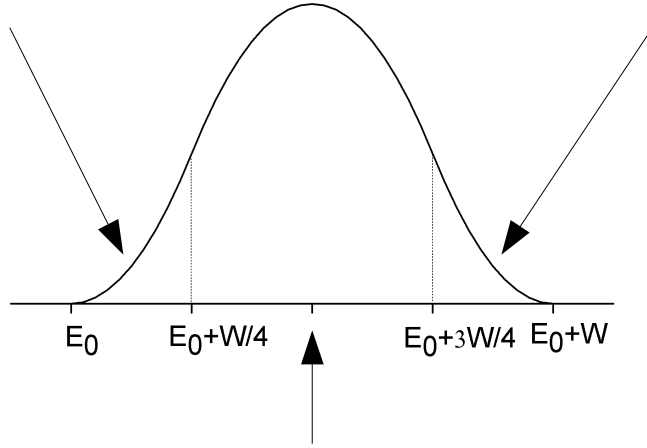
$$\int_0^\infty \epsilon_2^{osc}(E) E dE = C$$

# Indirect gap-Cody and Indirect gap-Tauc

$$\chi_{2-ind-Cody}^{inf}(E) = \frac{16C}{K_{Ci}W^2}(E - E_0)^2$$

$$\chi_{2-ind-Cody}^{inf}(E) = \frac{16C}{K_{Ci}W^2}[E - (E_0 + W)]^2$$

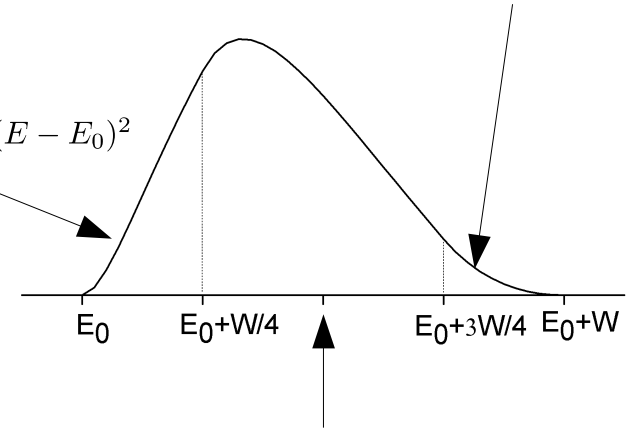
$$\chi_{2-ind-Tauc}^{inf}(E) = \frac{16C}{K_{Ci}W^2E^2}[E - (E_0 + W)]^2$$



$$\chi_{2-ind-Cody}^{inf}(E) = \frac{C}{K_{Ci}} \left\{ 2 - \frac{16}{W^2} [E - (E_0 + W/2)]^2 \right\}$$

$$K_{Ci} = E_0W + \frac{W^2}{2}$$

$$\chi_{2-ind-Tauc}^{inf}(E) = \frac{16C}{K_{Ci}W^2E^2}(E - E_0)^2$$



$$\chi_{2-ind-Tauc}^{inf}(E) = \frac{C}{K_{Ti}E^2} \left\{ 2 - \frac{16}{W^2} [E - (E_0 + W/2)]^2 \right\}$$

$$K_{Ti} = f(E_0/W)$$

$$f(r) = 2 \left[ 8r^2 \ln \left( \frac{4r+1}{4r} \right) + 8(r+1)^2 \ln \left( \frac{4r+4}{4r+3} \right) - (1 + 8(r^2 + r)) \ln \left( \frac{4r+3}{4r+1} \right) \right]$$

$E_0$  is the gap energy ,  $W$  is the absorption band width

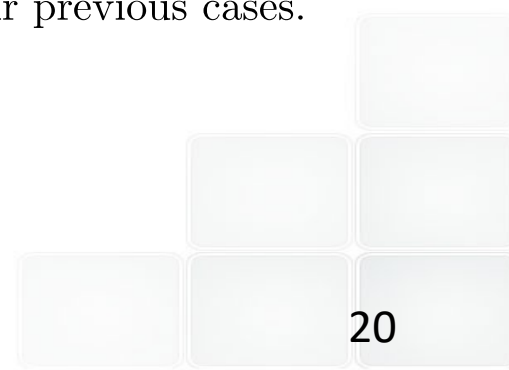
The analytical expressions for  $\tilde{\chi}(E)$  are obtained by performing the convolution between a suitable  $\chi_2^{inf}$  calculated for infinite lifetime and the normalized complex  $\tilde{\chi}_{qo}$  describing the response of a quantum oscillator:

$$\tilde{\chi}(E) = \int \chi_2^{inf}(E_r) \tilde{\chi}_{qo}(E_r - E) dE_r$$

The function  $\tilde{\chi}_{qo}$  is given by the expression given before, normalized so that the integral of its imaginary part is equal to 1:

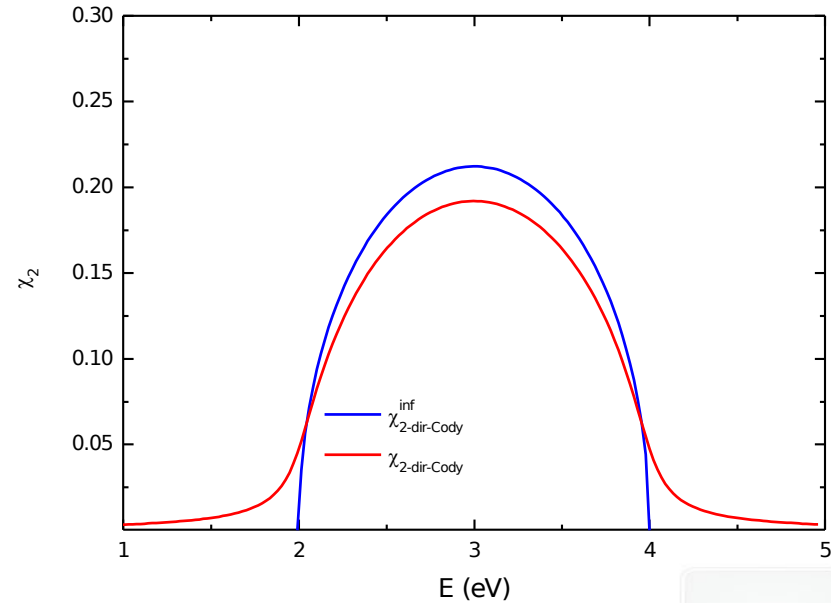
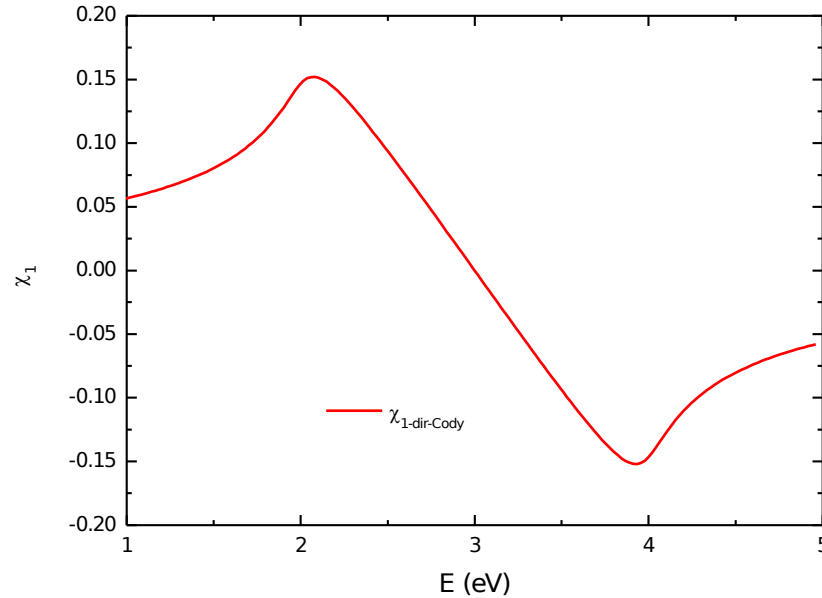
$$\tilde{\chi}_{qo}(E, E_r) = \frac{1}{\pi} \left[ \frac{1}{E_r - E + iD} \right]$$

The result of the convolution can be expressed as analytical functions in the four previous cases.



# Direct gap Cody - Lorentzian tail

$$\chi_{2-dir-Cody}^{inf}(E) = \frac{C}{K_{Cd}} \sqrt{(E - E_0)(E_3 - E)} \xrightarrow{\text{convolution with } \tilde{\chi}_{qo}} \tilde{\chi}_{dir-Cody}(E) = \frac{C}{K_{Cd}} \left[ \frac{E_0 + E_3}{2} - E + iD - \sqrt{E_0 - E + iD} \sqrt{E_3 - E + iD} \right]$$



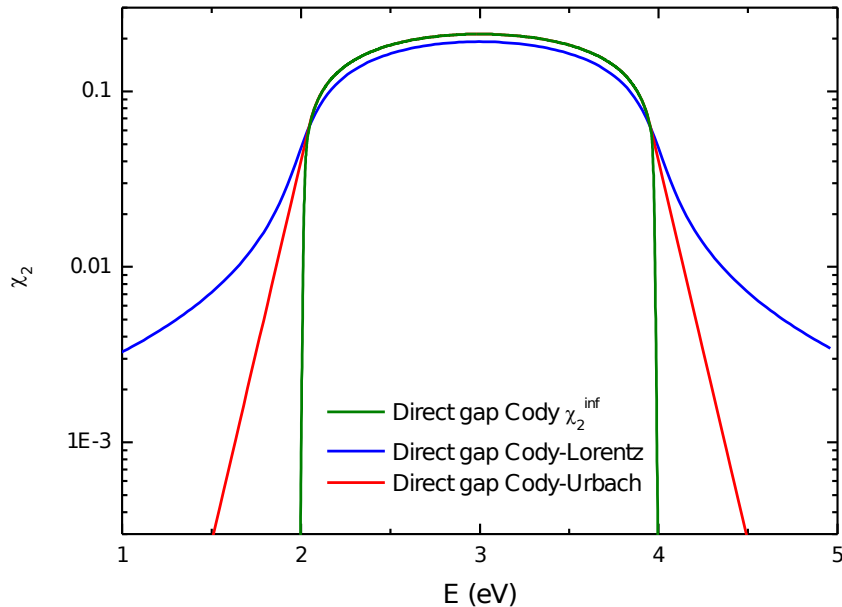
The example in this figure has:  $C=1$ ,  $E_0 = 2$  eV,  $W = E_3 - E_0 = 2$  eV,  $D = 0.1$  eV.

# Direct gap Tauc - Lorentzian tail

The convolution between the  $\chi_{2-dirTauc}^{inf}$  and the normalized complex  $\tilde{\chi}_{qo}$  gives:

$$\begin{aligned} \chi_{1-dir-Tauc}(E) = & \frac{C}{2DK_{Td}} \Re \left\{ \frac{1}{2(D-iE)} \left( 2iD + 2E - 2\sqrt{E_0E_3} \right. \right. \\ & \left. \left. - (i-1)\sqrt{2}\sqrt{D-i(E-E_0)}\sqrt{-iD-E+E_3} \right) \right. \\ & + \frac{1}{2(D+iE)} \left( -2iD + 2E - 2\sqrt{E_0E_3} + (i+1)\sqrt{2}\sqrt{D+i(E-E_0)}\sqrt{iD-E+E_3} \right) \\ & + E(D-iE)^2\sqrt{D+i(E-E_0)}\sqrt{D+i(E-E_3)} / (D^2+E^2)^2 \\ & + E(D+iE)^2\sqrt{D-i(E-E_0)}\sqrt{D-i(E-E_3)} / (D^2+E^2)^2 \\ & \left. \left. - ED(D^2(E_0+E_3) + E(-4E_0E_3 + E(E_0+E_3))) / ((D^2+E^2)^2\sqrt{E_0E_3}) \right\} \end{aligned}$$

$$\begin{aligned} \chi_{2-dir-Tauc}(E) = & \frac{CD}{K_{Td}} \\ & \times \Re \left\{ - \left[ (D-iE)^2\sqrt{D+i(E-E_0)}\sqrt{D+i(E-E_3)} \right] / (2D(D^2+E^2)^2) \right. \\ & - \left[ (D+iE)^2\sqrt{D-i(E-E_0)}\sqrt{D-i(E-E_3)} \right] / (2D(D^2+E^2)^2) \\ & \left. + D[D^2(E_0+E_3) + E(-4E_0E_3 + E(E_0+E_3))] / (2D(D^2+E^2)^2\sqrt{E_0E_3}) \right\} \end{aligned}$$



The parameters are:  $C=1$ ,  $E_0 = 2$  eV,  $W = 2$  eV,  $D = 0.1$  eV.  $D$  corresponds to the quantum oscillator line width in one case and to the characteristic energy  $D$  of the Urbach tail ( $\chi_2(E) \propto \exp(\pm E/D)$ ) in the other one.

$\chi_2^{inf}$  is modified adding an Urbach tail on both band edges. The real part  $\chi_1$  is then obtained by means of the numerical integration of the Kramers-Kronig relation:

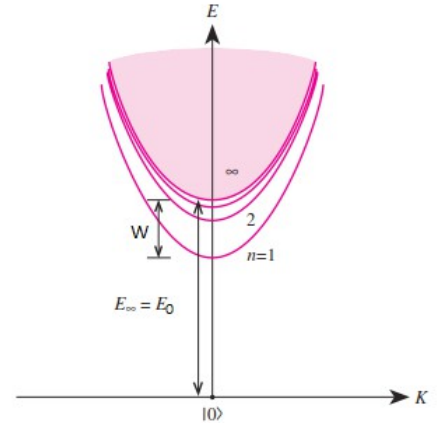
$$\chi_1(E) = \frac{2}{\pi} \mathcal{P} \int_0^\infty \frac{\chi_2(x)x}{x^2 - E^2} dx$$

The oscillator included in kSEMAWc is based on an improved version of the Elliott's theory presented by Tanguy in 1995 [1] which gives a very compact expression for the susceptibility of a semiconductor with a direct gap in the Tauc approximation taking into account the excitons and a Lorentzian broadening of the states :

$$\tilde{\chi}(E) = \frac{C\sqrt{W}}{(E + iD)^2} [g_a(\xi(E + iD)) + g_a(\xi(-E - iD)) - 2g_a(\xi(0))]$$

$$g_a(\xi) = 2 \ln \xi - 2\pi \cot(\pi\xi) - 2\psi(\xi) - 1/\xi$$

$$\xi(z) = \sqrt{\frac{W}{E_0 - z}}$$

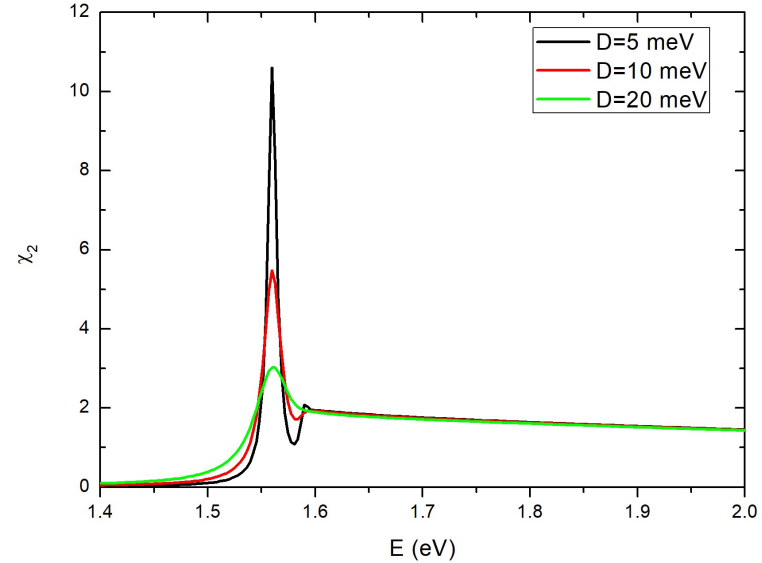
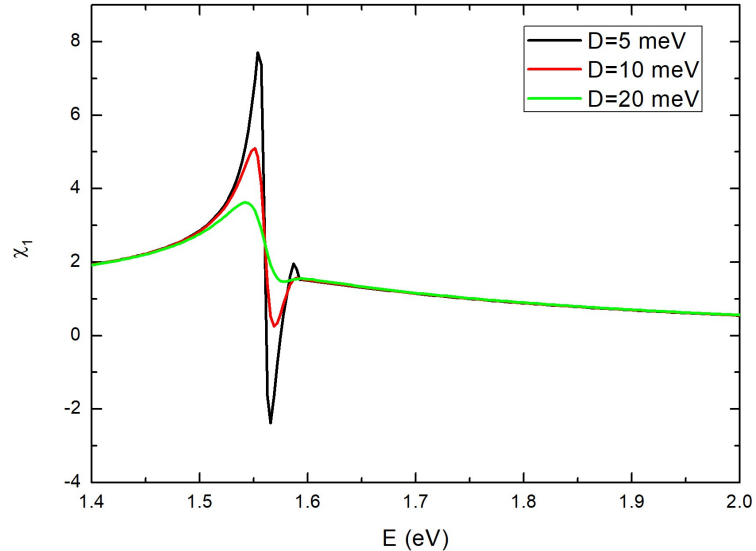


where  $\psi(z) = d \ln \Gamma(z)/dz$  is the complex digamma function,  $D$  is the width of the Lorentzian broadening function,  $E_0$  is the energy gap and  $W$  is the exciton binding energy.

[1] C. Tanguy, Optical dispersion by Wannier excitons, Phys. Rev. Lett. 75, 4090–4093 (1995).

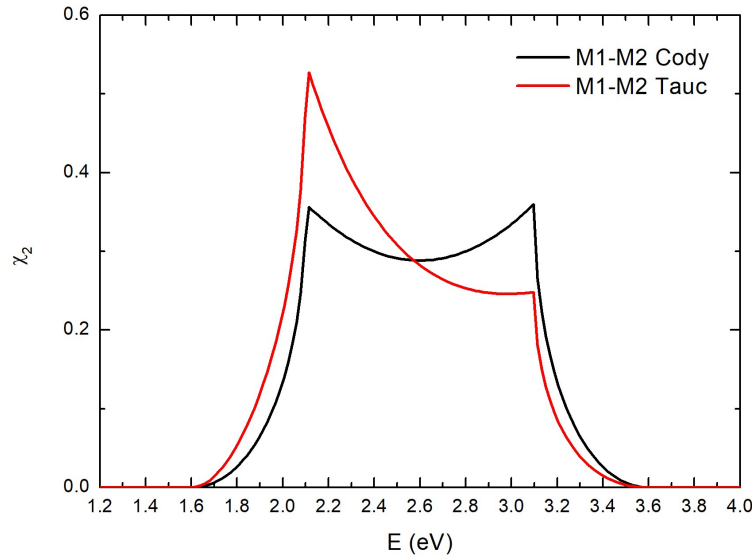


# Direct gap Tauc-exciton: broadening effect



The other parameters are:  $C=4$ ,  $E_0 = 1.6$  eV,  $W = 40$  meV .

# 3D $M_1$ - $M_2$ critical point oscillator

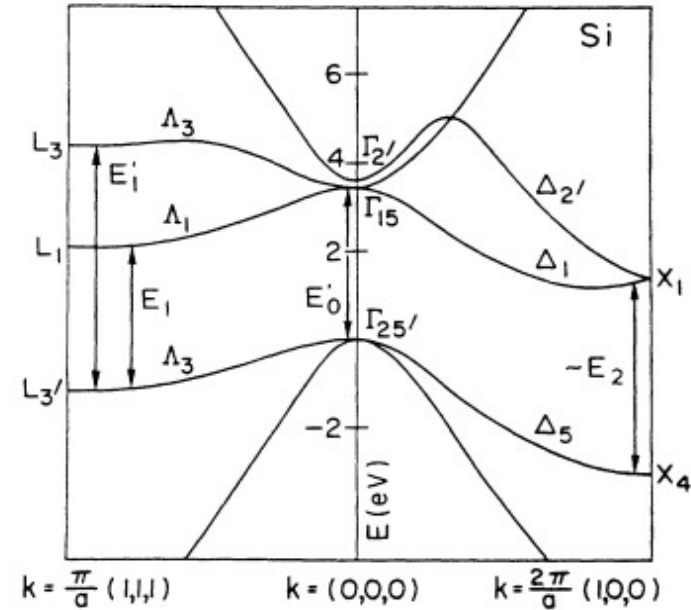
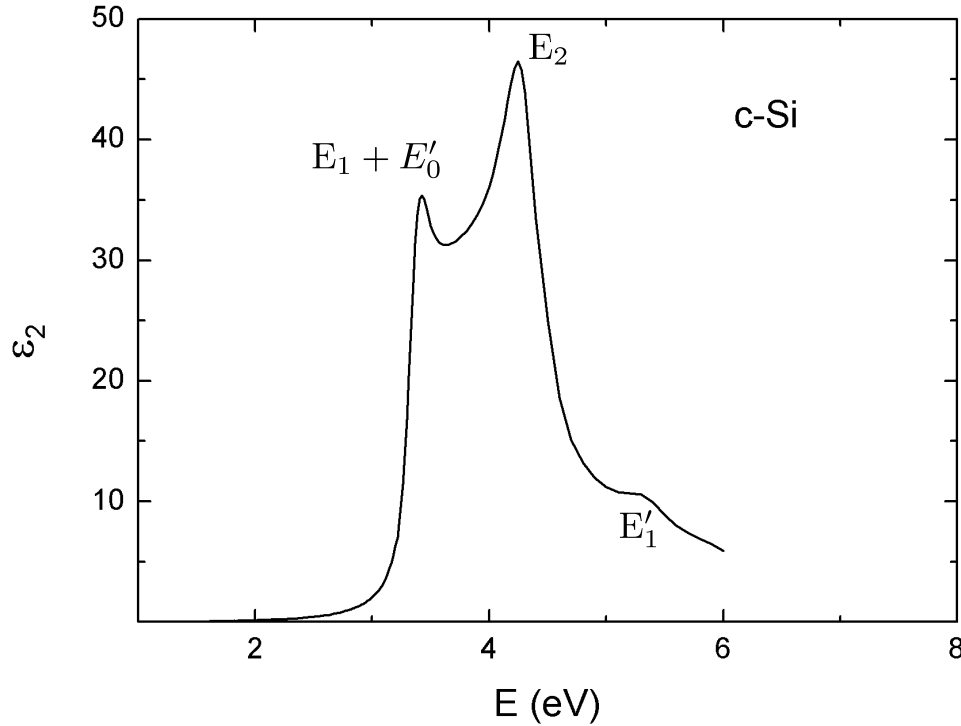


Plot of  $\chi_2$  for a  $M_1$ - $M_2$  oscillators with  $C=1 \text{ eV}^2$ ,  $E_0 = 1.6 \text{ eV}$ ,  $W = 2 \text{ eV}$  and  $D=0.5 \text{ eV}$  using the Cody (black curve) and Tauc (red curve) approximations.

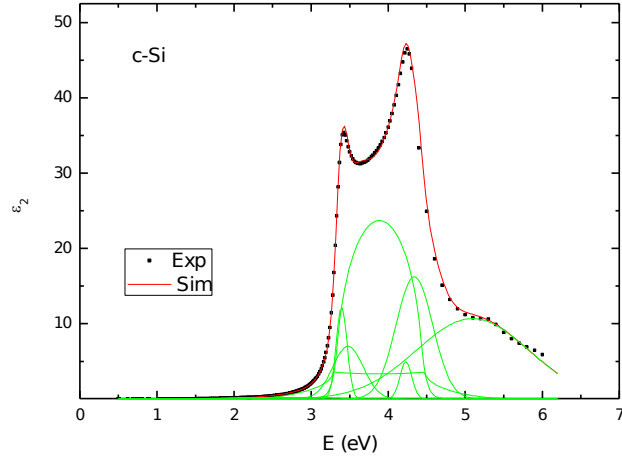
The real part  $\chi_1$  is obtained by means of the numerical integration of the Kramers-Kronig relation.

$$\chi_{2-CM12}(E) = \frac{C}{K_{CM12}} \left\{ K_1 \left[ \sqrt{\frac{W}{2}} - D - \Theta(|E - \bar{E}| - D) \sqrt{|E - \bar{E}| - D} \right] + (E - E_0)(E - E_3) \right\}$$

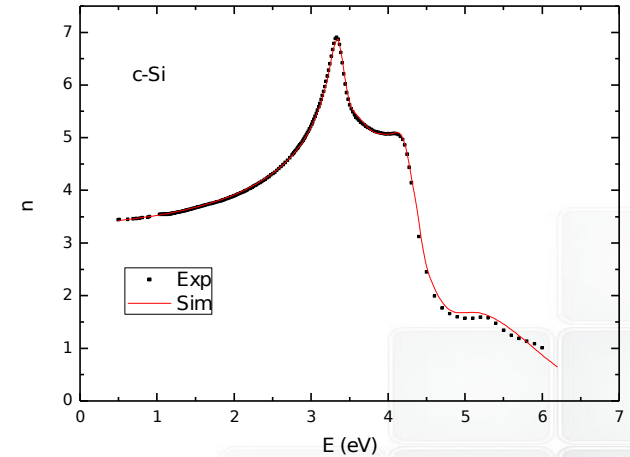
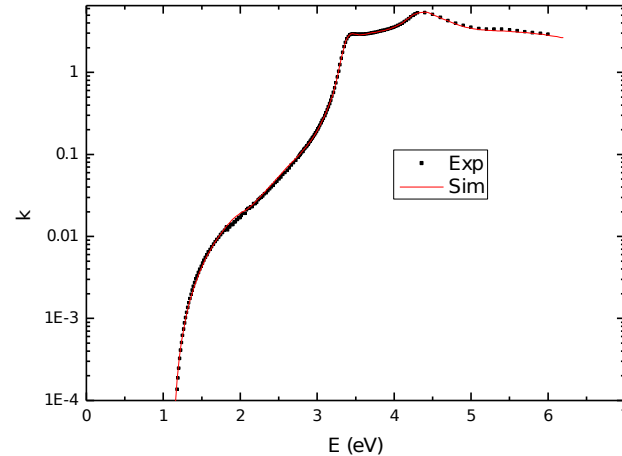
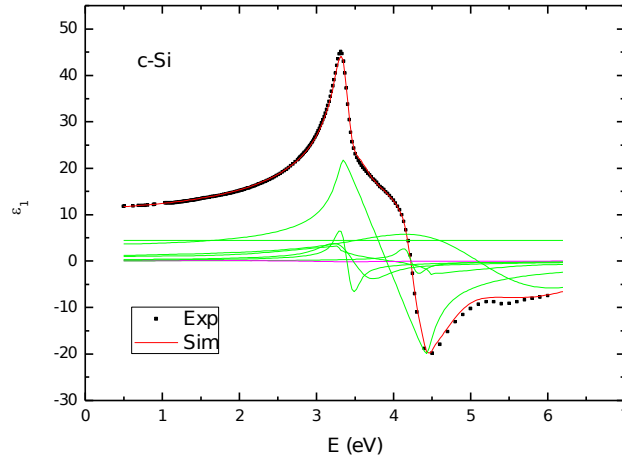
# A familiar case: crystalline silicon



# A familiar case: crystalline silicon

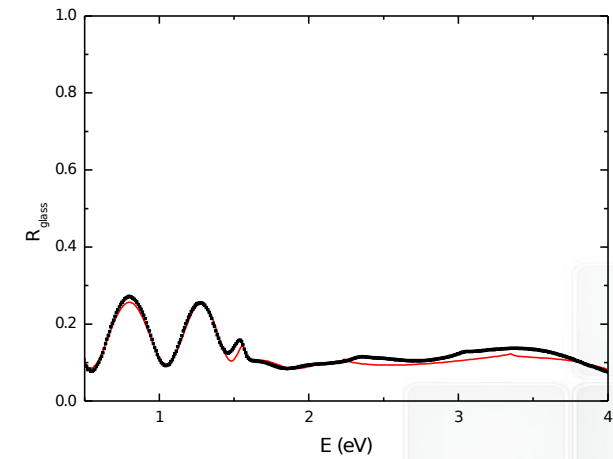
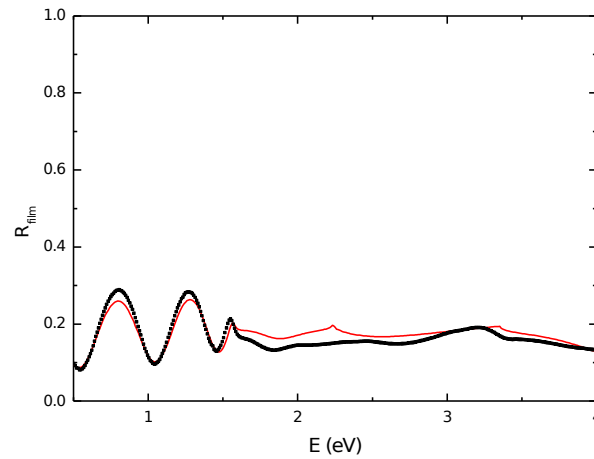
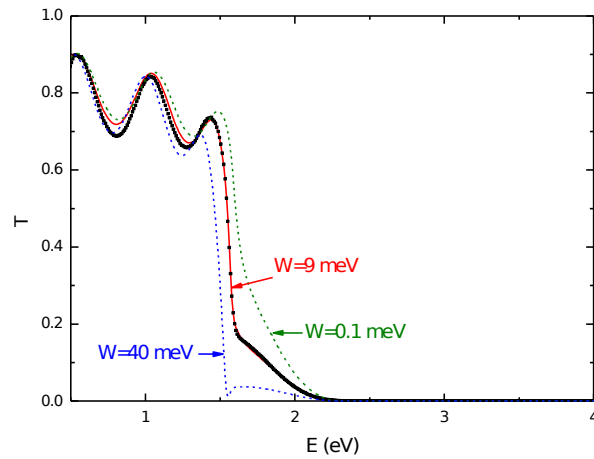


#Osc	type	C	E0 (eV)	D (eV)	W (eV)
1 <input checked="" type="checkbox"/>	Quant-inhomo	7.30972	3.39575	0.0831545	unused
2 <input checked="" type="checkbox"/>	Quant-inhomo	4	4.23	0.09	unused
3 <input checked="" type="checkbox"/>	Dir-Gap-Cody-M1M2	23	2.05	0.6	3.66
4 <input checked="" type="checkbox"/>	Ind-Gap-Cody-Urbach	0.85	1.1	0.03	3
5 <input checked="" type="checkbox"/>	Quant-inhomo	12.1081	3.47921	0.233499	unused
6 <input checked="" type="checkbox"/>	Quant-inhomo	42	4.34	0.28	unused
7 <input checked="" type="checkbox"/>	Flat	2.1	unused	unused	unused
8 <input checked="" type="checkbox"/>	Dir-Gap-Cody-Urbach	83.0221	3.31295	0.0600965	1.14763
9 <input checked="" type="checkbox"/>	Quant-inhomo	99.4053	5.09589	0.857884	unused



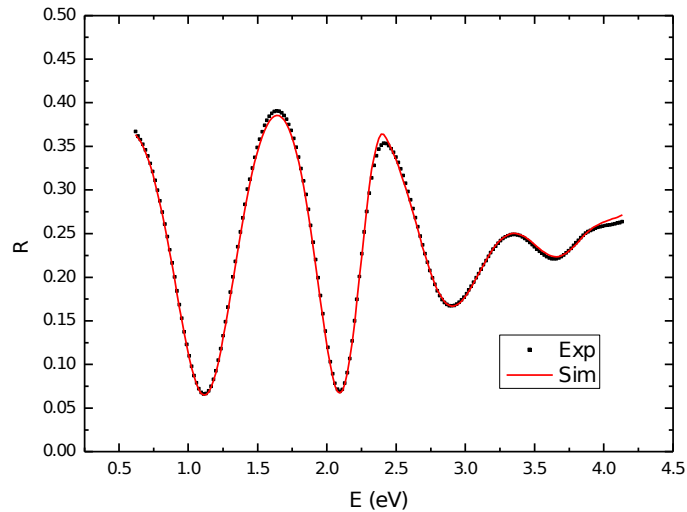
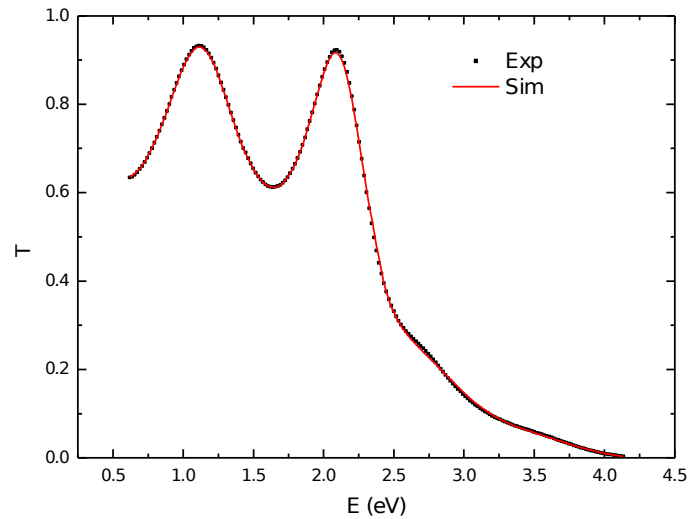
# Perovskite film on glass

#Osc	type	C	E0 (eV)	D (eV)	W (eV)
1 <input checked="" type="checkbox"/>	Flat	1	unused	unused	unused
2 <input checked="" type="checkbox"/>	Lorentz	1000	17.4	1.85	unused
3 <input checked="" type="checkbox"/>	Dir-Gap-Tauc-Exciton	3.42	1.59	0.022	0.009
4 <input checked="" type="checkbox"/>	Dir-Gap-Tauc-M1M2	3.6	1.68	0.56	2.23
5 <input checked="" type="checkbox"/>	Quant-inhomo	13.3	3.52	0.67	unused



# ZnSe film on glass

#Osc	type	C	E0 (eV)	D (eV)	W (eV)
1 <input checked="" type="checkbox"/>	Flat	1	unused	unused	unused
2 <input checked="" type="checkbox"/>	Dir-Gap-Cody-Urbach	122.799	2.37205	0.0948758	9.21083
3 <input checked="" type="checkbox"/>	Dir-Gap-Cody-Urbach	195.239	4.07019	0.238088	5.76904



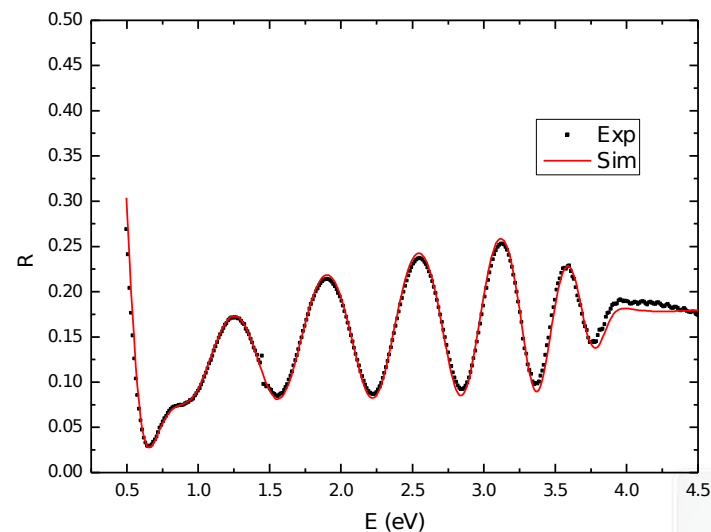
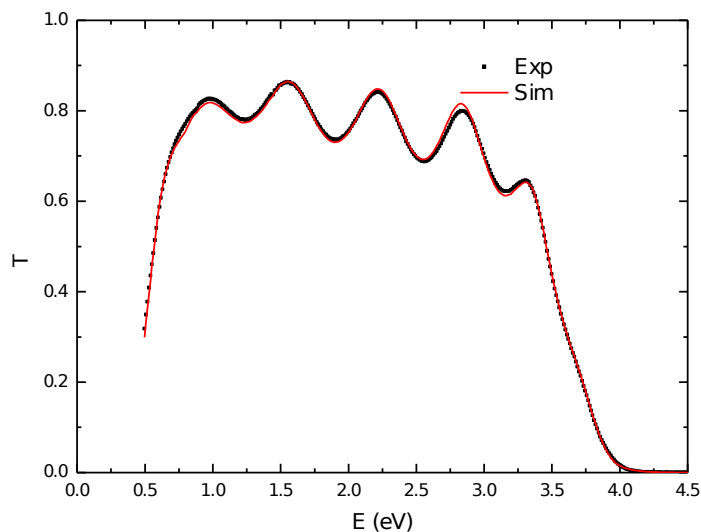
# IWO film on glass

From Drude parameters, using  $m^* = 0.35m_e$ , we get:

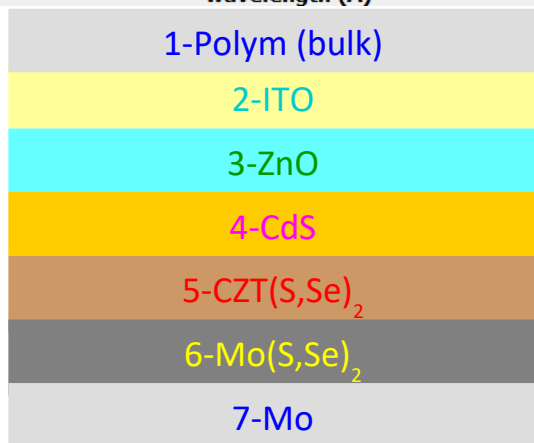
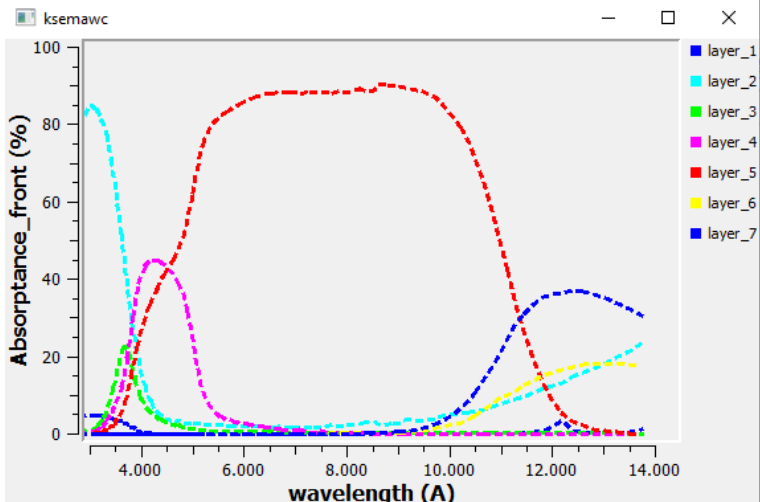
$$N(\text{cm}^{-3}) = C(\text{eV}^2) \frac{m^*}{m_e} 4.617 \cdot 10^{20} = 2.83 \cdot 10^{20} \text{cm}^{-3}$$

$$\mu\left(\frac{\text{cm}^2}{\text{Vs}}\right) = \frac{1.159}{D(\text{eV})m^*/m_e} = 38.5\left(\frac{\text{cm}^2}{\text{Vs}}\right)$$

#Osc	type	C	E0 (eV)	D (eV)	W (eV)
1 <input checked="" type="checkbox"/>	Flat	1	unused	unused	unused
2 <input checked="" type="checkbox"/>	Drude-ionized	1.75451	unused	0.0857026	unused
3 <input checked="" type="checkbox"/>	Dir-Gap-Cody-Urbach	489.375	3.80252	0.25282	13.97
4 <input checked="" type="checkbox"/>	Dir-Gap-Cody	0.139687	0.872585	2.8876e-07	2.9341



# CZTSSe solar cell



Valin Model Simulation Numerical Search Data Fit Graph Range

Model of the optical device (from Front to Back)

type	Move	d(mm-A)	rough (Å)	GRAD-n	dGRAD-n/dE	CURV-n	GRAD-k	CURV-k	material
bulk	-	dw	0.10	0.0	0.0000	0.000	0.0000	0.0000	Material #6
homo. film	up	dw	2400.00	500.0	0.0000	0.000	0.0000	0.0000	Material #3
homo. film	up	dw	500.00	0.0	0.0000	0.000	0.0000	0.0000	Material #5
homo. film	up	dw	800.00	0.0	0.0000	0.000	0.0000	0.0000	Material #4
homo. film	up	dw	10000.00	1000.0	0.0000	0.000	0.0000	0.0000	Material #8
homo. film	up	dw	2500.00	0.0	0.0000	0.000	0.0000	0.0000	Material #7
homo. film	up	dw	5000.00	0.0	0.0000	0.000	0.0000	0.0000	Material #2
bulk	up	dw	0.00	0.0	0.0000	0.000	0.0000	0.0000	Material #1
bulk	up	-	0.00	0.0	0.0000	0.000	0.0000	0.0000	Material #1

N. layers 7 ☐ Symmetric on 2nd face inhomogeneity as 23 subfilm computing <f(x)> with N= 5

Materials

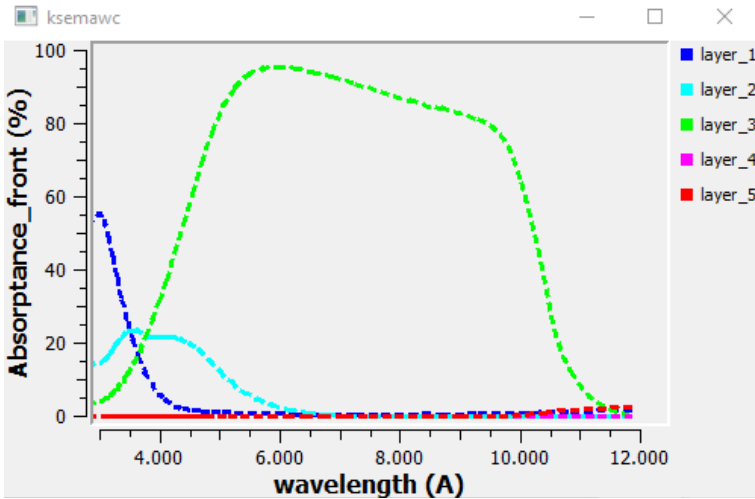
Material #	unknown	n	k	with	EMA	f
Material #1	unknown	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
Material #2	CZTSSe cells/Mo ENEA	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
Material #3	CZTSSe cells/ITO standard	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
Material #4	CZTSSe cells/CdS ENEA	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
Material #5	CZTSSe cells/ZnO ENEA	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
Material #6	CZTSSe cells/Polym IPC 1L 0	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
Material #7	CZTSSe cells/MoSeS2 Richter	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
Material #8	CZTSSe cells/CZTSSe Gokmen	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
output SF	constant nk	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
input SF	constant nk	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
input PDS	constant nk	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
input ELI-1	constant nk	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
input ELI-2	constant nk	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
input ELI-3	constant nk	1	0	<input type="checkbox"/>	air n=1 k=0	0.000
input ELI-4	constant nk	1	0	<input type="checkbox"/>	air n=1 k=0	0.000

Load Spj! Save Spj! CZTSSe\_cells/CZTSSe\_cell\_poly\_IPC.Spj info: CZTSSe cell with a 100 um front polymer encapsulant

Load nk! Clear nk! CZTSSe\_cells/CZTSSe\_Gokmen.nk info: CZTSSe Eg=1.13 eV k:Adachi\_2015 Book+ Gokmen\_2014 (DOI:10.1063/1.4890844) n: Adachi\_2015 Book



# HJ c-Si solar cell



Valin Model Simulation Numerical Search Data Fit Graph Range

Model of the optical device (from Front to Back)

type	Move	d(mm-A)	rough (Å)	GRAD-n	dGRAD-n/dE	CURV-n	GRAD-k	CURV-k	material
homo. film	-	dw	690.00	0.0	0.0000	0.000	0.0000	0.0000	Material #2
homo. film	up	dw	120.00	0.0	0.0000	0.000	0.0000	0.0000	Material #3
homo. film	up	dw	2500000.00	1000.0	0.0000	0.000	0.0000	0.0000	Material #4
homo. film	up	dw	150.00	0.0	0.0000	0.000	0.0000	0.0000	Material #3
homo. film	up	dw	690.00	0.0	0.0000	0.000	0.0000	0.0000	Material #2
bulk	up	dw	0.00	0.0	0.0000	0.000	0.0000	0.0000	Material #1
bulk	up	dw	0.00	0.0	0.0000	0.000	0.0000	0.0000	Material #1
bulk	up	dw	0.00	0.0	0.0000	0.000	0.0000	0.0000	Material #1
bulk	up	-	0.00	0.0	0.0000	0.000	0.0000	0.0000	Material #1

N. layers 5

☐ Symmetric on 2nd face

inhomogeneity as 23 subfilm

computing <f(x)> with N= 5

Plot Exp Meas! Simulate!

Materials

Material #	unknown	n	k	with
Material #1	ITO/ITO standard	1	0	<input type="checkbox"/>
Material #2	HJ stack/a-SiH CNR	1	0	<input type="checkbox"/>
Material #3	sil/cSioc. 1	1	0	<input type="checkbox"/>
Material #4	constant nk	1	0	<input type="checkbox"/>
Material #5	constant nk	1	0	<input type="checkbox"/>
Material #6	constant nk	1	0	<input type="checkbox"/>
Material #7	constant nk	1	0	<input type="checkbox"/>
Material #8	constant nk	1	0	<input type="checkbox"/>
output SF	constant nk	1	0	<input type="checkbox"/>
input SF	constant nk	1	0	<input type="checkbox"/>
input PDS	constant nk	1	0	<input type="checkbox"/>
input ELI-1	constant nk	1	0	<input type="checkbox"/>
input ELI-2	constant nk	1	0	<input type="checkbox"/>
input ELI-3	constant nk	1	0	<input type="checkbox"/>
input ELI-4	constant nk	1	0	<input type="checkbox"/>

EMA f

air n=1 k=0	0.000
air n=1 k=0	0.000
air n=1 k=0	0.000
air n=1 k=0	0.000
air n=1 k=0	0.000
air n=1 k=0	0.000
air n=1 k=0	0.000
air n=1 k=0	0.000
air n=1 k=0	0.000
air n=1 k=0	0.000
air n=1 k=0	0.000
air n=1 k=0	0.000
air n=1 k=0	0.000
air n=1 k=0	0.000
air n=1 k=0	0.000

Load Spj! Save Spj! HJ\_stack/Simulaz\_HJFlat\_amorfi.2.Spj info: TCO-aSi-cSi-aSi-TCO

Besides **Marco Montecchi** and **Alberto Mittiga** many peoples helped in the kSEMAWc development. We thank:

- **Claudia Malerba** and **Francesca Menchini** for their work in testing and improving kSEMAWc to characterize semiconductor materials
- **Enrico Nichelatti** for the wise search for analytical solutions of the integrals on the density of the states, as well as for the LATEX revision of the manual
- **Francesco Biccari** for his useful suggestions for simplifying the software distribution
- **Luca Serenelli** for his support in outlining the Linux installation procedure
- **Giacomo Mazzamuto** for his support in the compilation and linking of the GSL library under Windows

**And...Thank You for your attention !**

This research was financially supported by:

- EU H2020 research and innovation programme under the grant agreement 823802 (SFERA-III)
- EU H2020 research and innovation programme under the grant agreement 952982 (Custom-Art)
- the Italian Ministry of Economic Development in the framework of the Operating Agreement with ENEA for the Research on the Electric System.