2.9 ⓘ $e^{-2t} u(t) + e^{-3t} u(t)$

- Usamos la definición.

$$X(s) = \mathcal{L}\{e^{-2t} \cdot u(t)\} + \mathcal{L}\{e^{-3t} \cdot u(t)\}$$

con la transformada unilateral.

por el escalón unitario $u(t) = e^{-2t} u(t) = \begin{cases} 0 & , \ t < 0 \\ e^{-2t} & , \ t \geq 0 \end{cases}$

- $\mathcal{L}\{e^{-2t} \cdot u(t)\} = \int_0^\infty e^{-2t} \cdot e^{-st} \, dt = \int_0^\infty e^{-(2+s)t} \, dt$

$$= \left[\frac{e^{-(2+s)t}}{-(s+2)} \right]_0^\infty = \frac{1}{s+2}$$

Condición de convergencia $ROC = R\{s+2\} > 0$

$$\Rightarrow R\{s\} = -2$$

- $\mathcal{L}\{e^{-3t} \cdot u(t)\} = \int_0^\infty e^{-3t} \cdot e^{-st} \, dt = \int_0^\infty e^{-(3+s)t} \, dt$

$$= \left[\frac{e^{-(3+s)t}}{-(s+3)} \right]_0^\infty = \frac{1}{s+3}$$

Condición de convergencia $ROC = R\{s+3\} > 0$

$$\Rightarrow R\{s\} = -3$$

$$X(s) = \frac{1}{s+2} + \frac{1}{s+3} = \frac{2s+5}{(s+2)(s+3)} \quad \text{Transformada.}$$

• Polos = -2 , -3
Ceros = $2s+5$ = $s = \dfrac{5}{2}$ = $\delta = 2,5$

$Im\{s\}$

X = polos
O = ceros

$-3 \quad -2,5 \quad -2$

$Re\{s\}$

(ii) $e^{2t} u(t) + e^{-3t} u(-t)$

• $\mathcal{L}\{e^{2t} \cdot u(t)\} = \dfrac{1}{s-2}$, $k = s-2$

$R\{k\} > 0 \Rightarrow R\{s\} > 2$

• $\mathcal{L}\{e^{-3t} u(-t)\} = \displaystyle\int_{-\infty}^{0} e^{-3t} e^{-st} dt = \int_{-\infty}^{0} e^{-(3+s)t} dt$

$\rightarrow$ cambio de variable: $T = -T$

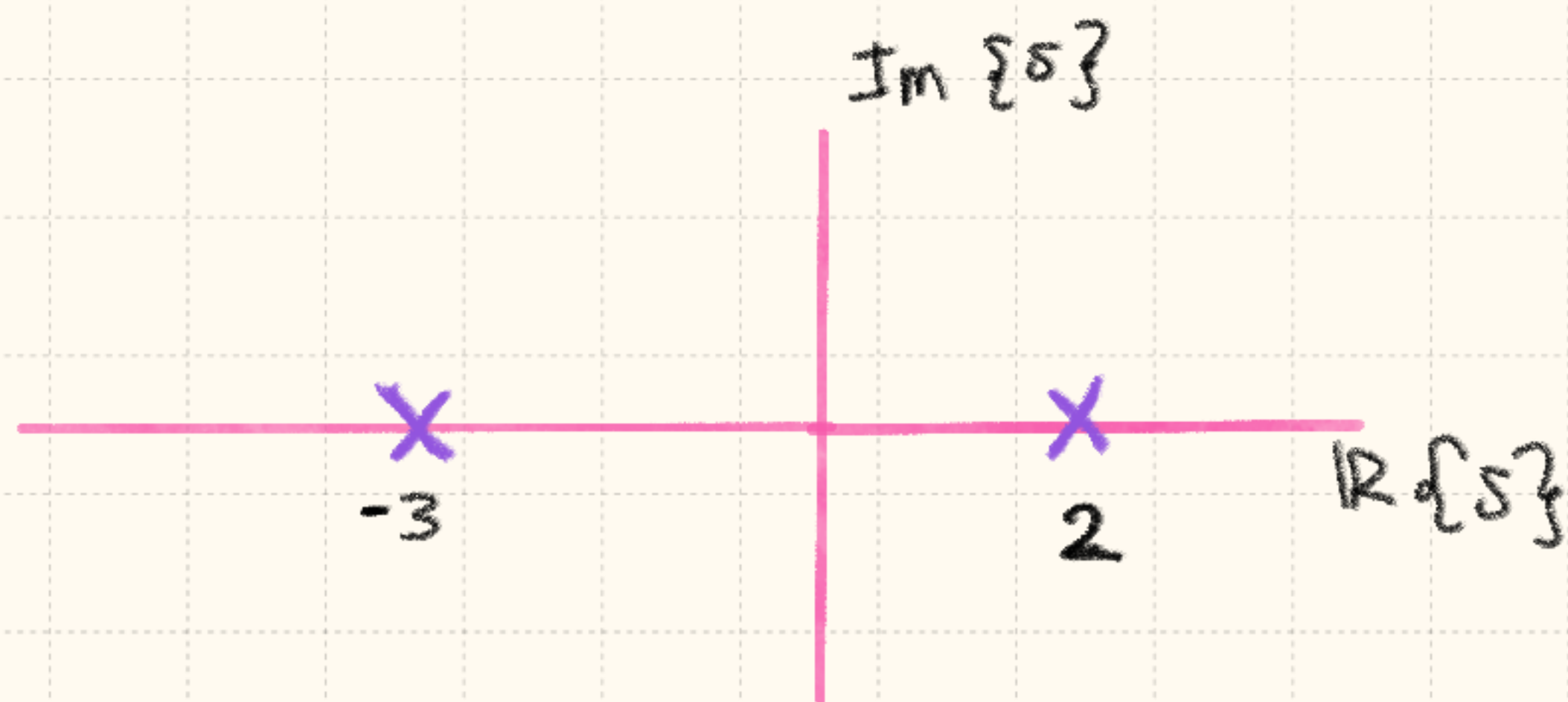$\displaystyle\int_{0}^{\infty} e^{(s+3)T} dT = \left[\dfrac{e^{(s+3)T}}{s+3} \right]_{0}^{\infty} = -\dfrac{1}{s+3}$

$k = s+3$
$R\{k\} < 0 \Rightarrow R\{s\} < -3$

$$\Rightarrow \frac{1}{S-2} - \frac{1}{S+3} = \boxed{\frac{5}{(S-2)(S+3)}} \; \text{Transformada}$$

$$\text{Polos} = S = 2 \quad X$$
$$S = -3 \quad X$$

ceros = No hay (numerador constante)

 $e^{-a|t|}$

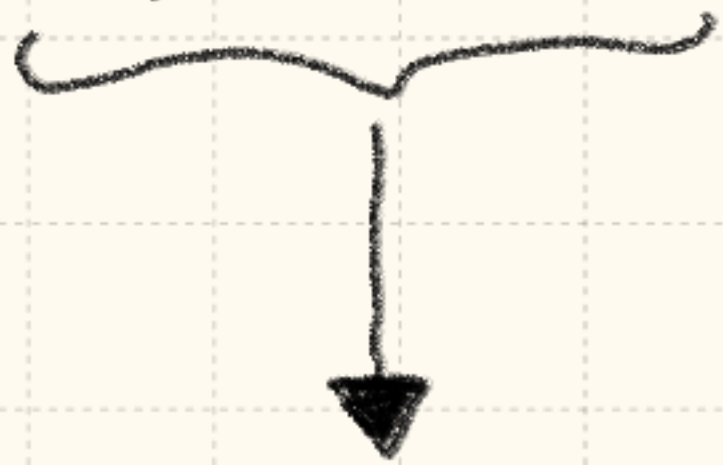utilizando la transformada bilateral : $X(s) = \int_{-\infty}^{\infty} e^{-a|t|} e^{-st} dt$

$$\Rightarrow |t| = \begin{cases} -t, & t<0 \\ t, & t \geq 0 \end{cases}$$

$\Rightarrow$ para $t \geq 0 : |t| = t \Rightarrow e^{-a|t|} = e^{-at}$

para $t \leq 0 : |t| = t \Rightarrow e^{-a|t|} = e^{at}$

$$\Rightarrow \int_{-\infty}^{0} e^{at} e^{-st} dt + \int_{0}^{\infty} e^{-at} e^{-st} dt$$

$$= \underbrace{\int_{-\infty}^{0} e^{(a-s)t} dt} + \int_{0}^{\infty} e^{-(a+s)t} dt$$

$$= \left[\frac{e^{(a-s)t}}{a-s} \right]_{-\infty}^{0} = \frac{1}{a-s} \left(e^{(a-s)0} - \lim_{t \to -\infty} e^{(a-s)t} \right)$$

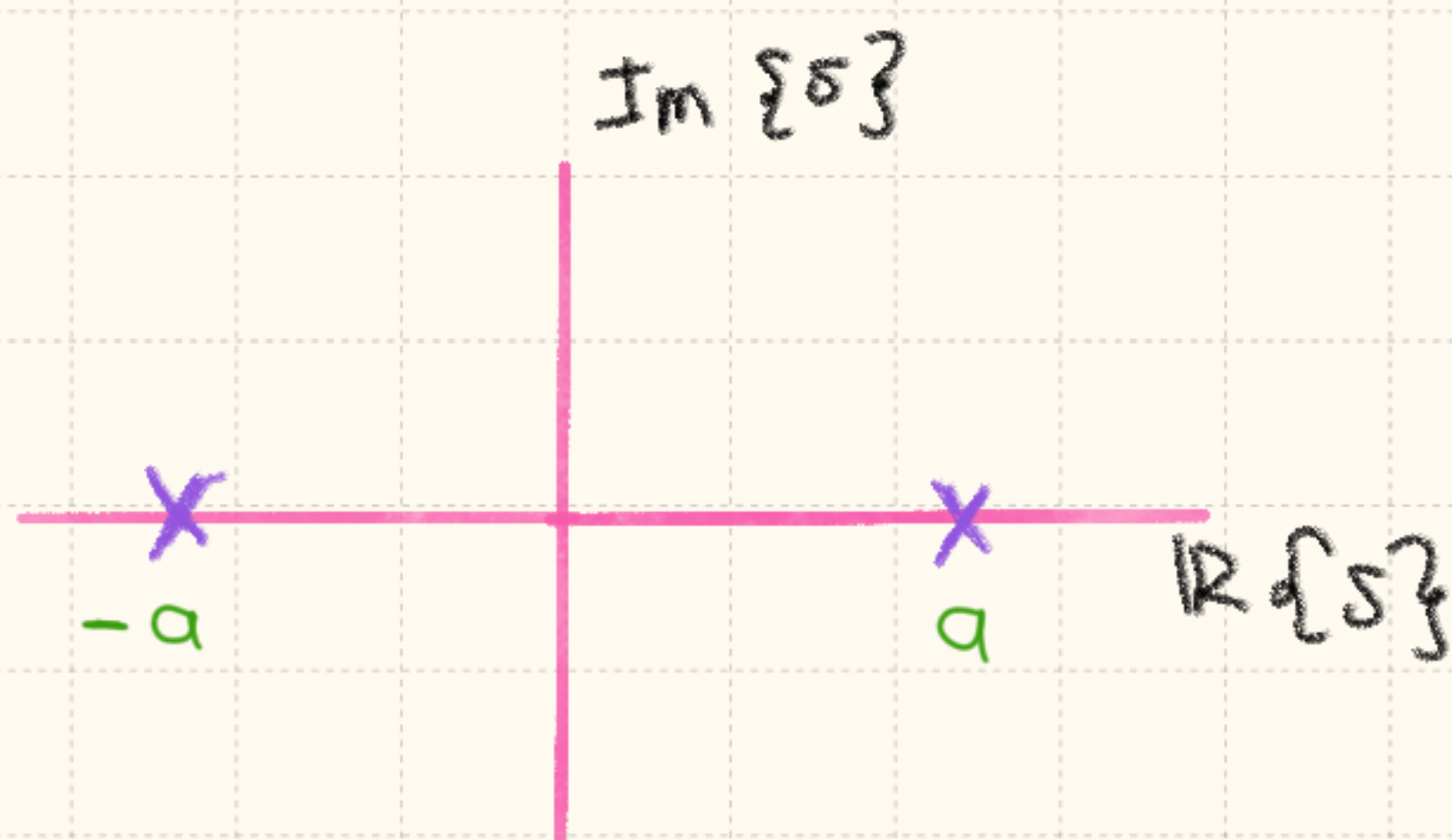$$\Rightarrow \mathbb{R}\{a-s\} > 0 \Rightarrow a - \mathbb{R}\{s\} > 0 \Rightarrow \mathbb{R}\{s\} < a$$

$$\bullet \int_0^\infty e^{-(a+s)t} dt = \frac{1}{a+s} \quad \cdot \quad \mathbb{R}\{a+s\} > 0 \Rightarrow \mathbb{R}\{s\} > -a$$

$$\Rightarrow X(s) = \frac{1}{a-s} + \frac{1}{a+s} = \boxed{\frac{2a}{(a-s)(a+s)}} \quad \text{Transformad}$$

Polos: $\pm a$ ✗
ceros: ninguno O
ROC: $-a < \mathbb{R}\{s\} < a$

$$\text{(iv)} \quad e^{-2t} [u(t) - u(t-5)]$$

$$x(t) = \begin{cases} e^{-2t} , & 0 < t < 5 \\ 0 , & \text{en otro caso.} \end{cases}$$

transformada bilateral

$$\int_0^5 e^{-2t} e^{-st} \, dt = \int_0^5 e^{-(2+s)t} \, dt.$$

$$X(s) = \frac{e^{-(2+s)t}}{-(2+s)} \Bigg]_0^5 = \boxed{\frac{1 - e^{-10} e^{-5s}}{s+2}} \quad \text{transformada}$$

ROC: $-\infty < \mathcal{R}\{s\} < 0$

Todo el plano S