2,10 • Resolver la transformada inversa de $X(s)$

$\Rightarrow X(s) = \dfrac{2s^2+14s+124}{s^3+8s^2+46s+68}$

① Factorizar el denominador (Busco una raíz entera, $s=-1,-2,\ldots$)

$\Rightarrow$ con $s=-2$

$(-2)^3+8(-2)^2+46(-2)+68 = -8+32-92+68 =$

con esto $s=-2$ es una raíz $\Rightarrow (s+2)$ es un factor.

• División sintética $\dfrac{s^3+8s^2+46s+68}{s+2} = s^2+6s+34$

$$\frac{2s^2+14s+124}{(s+2)(s^2+6s+34)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+6s+34}$$

$= 2s^2+14s+124 = A(s^2+6s+34)+(Bs+C)(s+2)$

• $A(s^2+6s+34) = As^2+6As+34A$

• $(Bs+C)(s+2) = Bs(s+2)+C(s+2) = Bs^2+2Bs+Cs+2C$

agrupando: s^2 : Bs^2

s: $2Bs+Cs = (2B+C)s$

cte: $2C$

$\Rightarrow (Bs+C)(s+2) = Bs^2+(2B+C)+2C$

$\Rightarrow As^2 + 6As + 34A + Bs^2 + (2B+C)s + 2C$

agrupando: $s^2 = (A+B)s^2$

$s = (6A + 2B + C)s$

$cte = 34A + 2C$

$\Rightarrow 2s^2 + 14s + 124 = (A+B)s^2 + (6A+2B+C)s + (34A+2C)$

sistema de ecuaciones

$$\begin{array}{l} s^2 \\ s \\ ind. \end{array} \begin{cases} A+B = 2 \rightarrow \boxed{B = 2-A} \\ 6A+2B+C = 14 \rightarrow 6A + 2(2-A) + C = 6A+4-2A+C = 14 \\ 34A+2C = 124 \end{cases}$$

$\rightarrow 4A + C = 10$

$\boxed{C = 10 - 4A}$

$34A + 2(10-4A) = 124$

$= 34A + 20 - 8A = 124$

$26A = 104$

$A = \frac{104}{26} \rightarrow \boxed{A = 4}$

reemplazo $\boxed{A = 4}$

$\Rightarrow B = 2 - A \Rightarrow \boxed{B = -2}$

$C = 10 - 4A \Rightarrow \boxed{C = -6}$

sustituyo los valores de A, B, C en las Fracciones Parciales.

$$\Rightarrow \frac{4}{s+2} + \frac{-2s-6}{s^2+6s+34}$$

$$= \frac{4}{s+2} - \frac{2(s+3)}{s^2+6s+34}$$

$\rightarrow (s+3)^2 + 25 = (s+3)^2 + 5^2$

$$= \frac{4}{s+2} - \frac{2(s+3)}{(s+3)+5^2}$$

Transformada inversa de laplace. $x(t)$.

$\Rightarrow$ • $\mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}u(t)$

• $\mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2+b^2}\right\} = e^{-at}\cos(bt)u(t)$

$\Rightarrow$ • $\frac{4}{s+2} = 4e^{-2t}u(t)$

• $-\frac{2(s+3)}{(s+3)+5^2}$ $-2e^{-3t}\cos(5t)u(t)$

$\Rightarrow$ $\boxed{x(t) = 4e^{-2t}u(t) - 2e^{-3t}\cos(5t)u(t)}$

• $X(s) = \frac{1}{(s+1)(s+2)^2}$ para $\Re\{s\} \geq -1$

• Fracciones parciales: $\frac{1}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$

$\Rightarrow 1 = A(s+2)^2 + B(s+1)(s+2) + C(s+1)$

expando $\Rightarrow \bullet A(s+2)^2 = A(s^2+4s+4) = As^2+4As+4A$

- $(s+1)(s+2) = s^2+2s+s+2 \Rightarrow s^2+3s+2$

$\Rightarrow B(s^2+3s+2) = Bs^2+3Bs+2$

- $C(s+1) = Cs+C$

$\longrightarrow As^2+4As+4+Bs^2+3Bs+2+Cs+C$

$s^2 = (A+B)s^2$

$s = (4A+3B+C)s$

ind $= 4A+2B+C$

$\Rightarrow 1 = (A+B)^2 + (4A+3B+C)s + (4A+2B+C)$

sistema de ecuaciones

$$\begin{cases} A+B=0 \rightarrow B=-A \\ 4A+3B+C=0 \rightarrow 4A-3A+C=0 \rightarrow A+C=0 \rightarrow C=-A \\ 4A+2B+C=1 \rightarrow 4A-2A-A=1 \rightarrow A=1 \end{cases}$$

$\Rightarrow A=1,\ B=-1,\ C=-1$

$\Rightarrow \frac{1}{s+1} - \frac{1}{s+2} - \frac{1}{(s+2)^2}$

- Transformada inversa

$\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}u(t)$

- $\mathcal{L}^{-1}\left\{-\frac{1}{s+2}\right\} = -e^{-2t}u(t)$

- $\mathcal{L}^{-1}\left\{-\frac{1}{(s+2)^2}\right\} = -te^{-2t}u(t)$

$\rightarrow \boxed{x(t) = e^{-t} - (1+t)e^{-2t}}$