

EJERCICIOS 7.4

(A) $\mathcal{F}\{e^{-j\omega_1 t} \cos(\omega_c t)\}$, $\omega_1, \omega_c \in \mathbb{R}$

usamos la identidad $\cos(\omega_c t) = \frac{1}{2} (e^{j\omega_c t} + e^{-j\omega_c t})$

$$\Rightarrow e^{-j\omega_1 t} \cos(\omega_c t) = \frac{1}{2} (2\pi \delta(\omega - (\omega_c - \omega_1)) + 2\pi \delta(\omega - (-\omega_c - \omega_1)))$$

simplifico: $\mathcal{F}\{e^{-j\omega_1 t} \cos(\omega_c t)\} =$

$$= \pi [\delta(\omega - (\omega_c - \omega_1)) + \delta(\omega + (\omega_c + \omega_1))]$$

Esta es la forma de distribuciones: dos picos impulsivos en $\omega = \omega_c - \omega_1$ y $\omega = -\omega_c - \omega_1$

(B) $\mathcal{F}\{u(t) \cos^2(\omega_c t)\}$, $\omega_c \in \mathbb{R}$

primero usamos la identidad trigonométrica:

$$\cos^2(\omega_c t) = \frac{1}{2} (1 + \cos(2\omega_c t))$$

$$\Rightarrow u(t) \cos^2(\omega_c t) = \frac{1}{2} u(t) + \frac{1}{2} u(t) \cos(2\omega_c t)$$

la FT de $u(t)$ es (distribución) $= U(\omega) = \pi \delta(\omega) + \frac{1}{j\omega}$

para $u(t) \cos(2\omega_c t) = \frac{1}{4} u(t) (e^{j2\omega_c t} + e^{-j2\omega_c t})$ usamos desplazamiento en frecuencia:

$$\Rightarrow \mathcal{F}\{u(t) e^{j2\omega_c t}\} = U(\omega - 2\omega_c)$$

$$\mathcal{F}\{u(t) e^{-j2\omega_c t}\} = U(\omega + 2\omega_c)$$

$$\Rightarrow \mathcal{F}\{u(t) \cos^2(\omega_c t)\} = \frac{1}{2} U(\omega) + \frac{1}{2} \cdot \frac{1}{2} (U(\omega - 2\omega_c) + U(\omega + 2\omega_c))$$

$$= \frac{1}{2} U(\omega) + \frac{1}{4} (U(\omega - 2\omega_c) + U(\omega + 2\omega_c))$$

sustituyendo $U(\omega) = \pi \delta(\omega) + \frac{1}{j\omega}$

$$\Rightarrow \mathcal{F}\{u(t) \cos^2(\omega_c t)\} = \frac{7}{2} \left(\pi \delta(\omega) + \frac{1}{j\omega} \right) + \frac{7}{4} \left(\pi \delta(\omega - 2\omega_c) + \frac{1}{j(\omega - 2\omega_c)} + \pi \delta(\omega + 2\omega_c) + \frac{1}{j(\omega + 2\omega_c)} \right)$$

$$\textcircled{c} \mathcal{F}^{-1} \left\{ \frac{7}{\omega^2 + 6\omega + 45} * \frac{10}{(8 + j\frac{\omega}{3})^2} \right\}$$

$$\Rightarrow x(t) = x_1(t) \cdot x_2(t)$$

$$\text{donde } x_1(t) = \mathcal{F}^{-1} \left\{ \frac{7}{\omega^2 + 6\omega + 45} \right\}$$

$$x_2(t) = \frac{70}{(8 + j\frac{\omega}{3})^2}$$

$$\text{completo el cuadrado } \Rightarrow \frac{7}{\omega^2 + 6\omega + 45} = \frac{7}{12} \cdot \frac{12}{(\omega + 3)^2 + 6^2}$$

por base =

$$\mathcal{F}\{e^{-a|t|}\} = \int_{-\infty}^{\infty} e^{-a|t|} e^{-j\omega t} dt = \frac{2a}{a^2 + \omega^2} \quad (a > 0)$$

desplazamiento en frecuencia

$$\text{Si } X(\omega) = \mathcal{F}\{x(t)\} \Rightarrow X(\omega - \omega_0) \longleftrightarrow e^{j\omega_0 t} x(t)$$

$$\Rightarrow a = 6, \quad \omega_0 = -3$$

$$\frac{12}{(\omega + 3)^2 + 6^2} = \frac{2 \cdot 6}{6^2 + (\omega + 3)^2} = e^{j(-3)t} e^{-6|t|} = e^{-j3t} e^{-6|t|}$$

por base $\omega \mapsto \omega + 3$

$$\Rightarrow x_1(t) = \frac{7}{12} e^{-6|t|} e^{-j3t}$$

Ahora para:

$$\frac{10}{\left(\frac{8+j\omega}{3}\right)^2} = 10 \cdot \frac{9}{(24+j\omega)^2} = \frac{90}{(24+j\omega)^2}$$

por base con derivada.

$$\mathcal{F}\{e^{-at} u(t)\} = \frac{1}{a+j\omega}, \quad a > 0$$

Usando la propiedad $\mathcal{F}\{t x(t)\} = j \frac{d}{d\omega} X(\omega)$

$$\mathcal{F}\{t e^{-at} u(t)\} = j \frac{d}{d\omega} \frac{1}{a+j\omega} = \frac{-j}{(a+j\omega)^2}$$

$$\Rightarrow x_2(t) = 90 t e^{-24t} u(t)$$

$$\Rightarrow X(\omega) = \left(\frac{7}{12} e^{-6|t|} e^{-j3t} \right) * \left(90 t e^{-24t} u(t) \right)$$

Ⓓ $\mathcal{F}\{t^3\}$

Propiedad = $X(t) = 1$ cuyo FT es $2\pi \delta(\omega)$

$$\circ \mathcal{F}\{t^n X(t)\} = j^n \frac{d^n}{d\omega^n} X(\omega)$$

$$j^3 = j^2 \cdot j = (-1) \cdot j = -j \Rightarrow \mathcal{F}\{t^3\} = -j 2\pi \delta^{(3)}(\omega)$$

en donde $\delta^{(3)}$ es la tercera derivada de la delta de Dirac (en el sentido de distribuciones)

Ⓔ $\frac{B}{T} \sum_{n=-\infty}^{\infty} \left(\frac{1}{a^2 + (\omega - n\omega_0)^2} + \frac{1}{a + j(\omega - n\omega_0)} \right)$

donde $n \in \{0, \pm 1, \pm 2, \dots\}$. $\omega_0 = \frac{2\pi}{T}$ y $B, T \in \mathbb{R}^+$

$$\Rightarrow X_p(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_0) \quad \omega_0 = \frac{2\pi}{T} \text{ frecuencia fundamental}$$

$$\Rightarrow \frac{1}{a^2 + (\omega - n\omega_0)^2} \quad \left| \quad \frac{1}{a + j(\omega - n\omega_0)} \right.$$

$$F\{e^{-at}u(t)\} = \frac{2a}{a^2 + \omega^2} \quad F\{e^{-at}u(t)\} = \frac{1}{a + j\omega}$$

$$\frac{B}{T} \Rightarrow \sum_{n=-\infty}^{\infty} f(\omega - n\omega_0)$$

$$\Rightarrow e^{B/T} \sum_{n=-\infty}^{\infty} \left(\frac{1}{a^2 + (\omega - n\omega_0)^2} + \frac{1}{a + j(\omega - n\omega_0)} \right)$$

$$\Rightarrow X_p(\omega) = \frac{B}{T} \sum_{n=-\infty}^{\infty} \left[\frac{1}{a^2 + (\omega - n\omega_0)^2} + \frac{1}{a + j(\omega - n\omega_0)} \right]$$

$$\omega_0 = \frac{2\pi}{T} \quad n \in \mathbb{Z}, \quad B, T > 0$$