

EJERCICIOS 7.4

A) $\mathcal{F}\{e^{-j\omega_1 t} \cos(\omega_c t)\}$, $\omega_1, \omega_c \in \mathbb{R}$

usamos la identidad $\cos(\omega_c t) = \frac{1}{2}(e^{j\omega_c t} + e^{-j\omega_c t})$

$$\Rightarrow e^{-j\omega_1 t} \cos(\omega_c t) = \frac{1}{2} (2\pi\delta(\omega - (\omega_c - \omega_1)) + 2\pi\delta(\omega - (-\omega_c - \omega_1)))$$

Simplifico: $\mathcal{F}\{e^{-j\omega_1 t} \cos(\omega_c t)\} =$

$$= \pi [\delta(\omega - (\omega_c - \omega_1)) + \delta(\omega + (\omega_c + \omega_1))]$$

esta es la forma de distribuciones: dos picos impulsivos en $\omega = \omega_c - \omega_1$ y $\omega = -\omega_c - \omega_1$

B) $\mathcal{F}\{u(t) \cos^2(\omega_c t)\}$, $\omega_c \in \mathbb{R}$

primero usamos la identidad trigonométrica:

$$\cos^2(\omega_c t) = \frac{1}{2}(1 + \cos(2\omega_c t))$$

$$\Rightarrow u(t) \cos^2(\omega_c t) = \frac{1}{2} u(t) + \frac{1}{2} u(t) \cos(2\omega_c t)$$

$$(a) \text{ FT de } u(t) \text{ es (distribución)} = U(\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

para $u(t) \cos(2\omega_c t) = \frac{1}{4} u(t) (e^{j2\omega_c t})$ usamos desplazamiento en frecuencia:

$$\Rightarrow \mathcal{F}\{u(t)e^{j2\omega_c t}\} = U(\omega - 2\omega_c)$$

$$\mathcal{F}\{u(t)e^{-j2\omega_c t}\} = U(\omega + 2\omega_c)$$

$$\Rightarrow \mathcal{F}\{u(t) \cos^2(\omega_c t)\} = \frac{1}{2} U(\omega) + \frac{1}{2} \cdot \frac{1}{2} (U(\omega - 2\omega_c) + U(\omega + 2\omega_c))$$

$$= \frac{1}{2} U(\omega) + \frac{1}{4} (U(\omega - 2\omega_c) + U(\omega + 2\omega_c))$$

$$\text{sustituyendo } U(\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$\Rightarrow \Im(u(t) \cos^2(\omega t)) = \frac{7}{2} (\pi \delta(\omega) + \frac{1}{j\omega}) + \frac{7}{4} (\pi \delta(\omega - 2\omega_c) +$$

$$\frac{7}{j(\omega - 2\omega_c)} + \pi \delta(\omega + 2\omega_c) + \frac{7}{j(\omega + 2\omega_c)})$$

$$\textcircled{c} \quad F^{-1} \left\{ \frac{7}{\omega^2 + 6\omega + 45} * \frac{10}{(8 + j\frac{\omega}{3})^2} \right\}$$

$$\Rightarrow x(t) = x_1(t) \cdot x_2(t)$$

$$\text{donde } x_1(t) = F^{-1} \left\{ \frac{7}{\omega^2 + 6\omega + 45} \right\}$$

$$x_2(t) = \frac{70}{(8 + j\frac{\omega}{3})^2}$$

$$\text{completo el cuadrado} \Rightarrow \frac{7}{\omega^2 + 6\omega + 45} = \frac{7}{12} \cdot \frac{12}{(\omega + 3)^2 + 6^2}$$

por base

$$F\{e^{-at}\} = \sum_{n=0}^{\infty} e^{-at} e^{-jn\omega t} dt = \frac{2a}{a^2 + \omega^2} \quad (a > 0)$$

desplazamiento en frecuencia

$$\text{Si } X(\omega) = F(x(t)) \Rightarrow X(\omega - \omega_0) \xrightarrow{e^{j\omega_0 t}} x(t).$$

$$\Rightarrow a = 6, \omega_0 = -3$$

$$\frac{12}{(\omega + 3)^2 + 6^2} = \underbrace{\frac{2 \cdot 6}{6^2 + (\omega + 3)^2}}_{e^{-j3t}} = e^{j(-3)t} e^{-6|t|} = e^{-j3t} e^{-6|t|}$$

por base $\omega \rightarrow \omega + 3$

$$\Rightarrow x_1(t) = \frac{7}{12} e^{-6|t|} e^{j\beta t}$$

Ahora para:

$$\frac{10}{\left(\frac{8+jw}{3}\right)^2} = 10 \cdot \frac{9}{(24+jw)^2} = \frac{90}{(24+jw)^2}$$

por lo tanto con derivada

$$F\{e^{-at} u(t)\} = \frac{1}{a+jw}, \quad a > 0$$

Usando la propiedad $F\{t^n x(t)\} = j \frac{d}{dw} X(w)$

$$F\{t e^{-at} u(t)\} = j \frac{d}{dw} \frac{1}{a+jw} = \frac{j}{(a+jw)^2}$$

$$\Rightarrow X_2(t) = 90 t e^{-24t} u(t)$$

$$\Rightarrow X(t) = \left(\frac{1}{12} e^{-6t} e^{-j3t} \right) * (90 t e^{-24t} u(t))$$

(D) $F\{3t^3\}$

Propiedad $= X(t) = 1$ cuyo FT es $2\pi \delta(w)$

$$0 F\{t^n x(t)\} = j^n \frac{d^n}{dw^n} X(w)$$

$$j^3 = j^2 \cdot j = (-j) \cdot j = -j \Rightarrow F\{t^3\} = -j 2\pi \delta^{(3)}(w)$$

en donde $\delta^{(3)}$ es la tercera derivada de la delta de dirac (en el sentido de distribuciones).

(E) $\frac{B}{T} \sum_{n=-\infty}^{\infty} \left(\frac{1}{a^2 + (w-nw_0)^2} + \frac{1}{a+j(w-nw_0)} \right),$

donde $n \in \{0, \pm 1, \pm 2, \dots\}$. $w_0 = \frac{2\pi}{T}$ y $B, T \in \mathbb{R}^+$

$$\Rightarrow X_p(w) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(w-nw_0) \quad w_0 = \frac{2\pi}{T} \text{ frecuencia fundamental}$$

$$\Rightarrow \frac{1}{a^2 + (w - nw_0)^2}$$

$$F\{e^{-at}\} = \frac{2a}{a^2 + w^2}$$

$$\frac{1}{a + j(w - nw_0)}$$

$$F\{e^{-at} u(t)\} = \frac{1}{a + jw}$$

$$\frac{B}{T} \rightarrow \sum_{n=-\infty}^{\infty} f(w - nw_0)$$

$$\Rightarrow e^{\frac{Bt}{T}} \sum_{n=-\infty}^{\infty} \left(\frac{1}{a^2 + (w - nw_0)^2} + \frac{1}{a + j(w - nw_0)} \right)$$

$$\Rightarrow X_p(w) = \frac{B}{T} \sum_{n=-\infty}^{\infty} \left[\frac{1}{a^2 + (w - nw_0)^2} + \frac{1}{a + j(w - nw_0)} \right]$$

$$w_0 = \frac{2\pi}{T}, n \in \mathbb{Z}, B, T > 0$$