

Warming induced changes to body size stabilize consumer-resource dynamics

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Made with *Mathematica* 9

Preliminaries

Plot settings

```
LabelSize = 35; (*size of axis label text*)
FigureSize = 650; (*size of figure*)
TickSize = 20; (*size of tick text*)
Pad = {{90, 25}, {70, 10}}; (*whitespace to leave around figures,
{{left,right},{bottom,top}}*)
letpos = {.05, .93}; (*relative location of letter, eg A, in figure*)
ylabpos = {-0.1, 0.5}; (*relative location of y axis label position*)
LetterSize = 20; (*size of text in stability plots*)
```

Directories

```
SetDirectory[NotebookDirectory[]];
(*set current directory to be location of this file*)
imagedir = "IMAGES/"; (*directory to save figures in;
make this directory before running*)
```

The underlying consumer-resource dynamics

Equations 1 and 2 from Gilbert et al 2014 (with potential temperature dependencies added)

$$\frac{dR}{dt}[R, C, T] := r[T] R \left(1 - \frac{R}{K[T]} \right) - f[R, T] R C;$$

$$\frac{dC}{dt}[R, C, T] := e[T] f[R, T] R C - m[C, T] C;$$

where R is biomass of resource, C is biomass of consumer, T is temperature, K is resource carrying capacity, f is the functional response, e is the conversion efficiency of resources into new consumers, and m is consumer mortality.

BCR at a given T, as defined by Gilbert (Eqn 5),

$$\text{BCR}[T_] := \frac{e[T] a[T] K[T]}{m[T]}$$

Equilibrium biomasses at given temperature assuming a type I functional response and density-independent consumer mortality (as in most of Gilbert)

$$\text{Eq}[T_] := \text{Solve}[\{0 == \text{dRdt}[R, C, T], 0 == \text{dCdT}[R, C, T]\} /. f[R, T] \rightarrow a[T] /. m[C, T] \rightarrow m[T], \{R, C\}]$$

Equilibrium consumer to resource biomass ratio at a given temperature (at the equilibrium where both populations persist)

$$\text{CR}[T_] := \frac{C}{R} /. \text{Eq}[T] [[3]]$$

The Jacobian evaluated at equilibrium (linear stability analysis)

$$\text{Jac} = \{ \{ D[\text{dRdt}[R, C, T] /. f[R, T] \rightarrow a[T] /. m[C, T] \rightarrow m[T], R], \\ D[\text{dRdt}[R, C, T] /. f[R, T] \rightarrow a[T] /. m[C, T] \rightarrow m[T], C] \}, \\ \{ D[\text{dCdT}[R, C, T] /. f[R, T] \rightarrow a[T] /. m[C, T] \rightarrow m[T], R], \\ D[\text{dCdT}[R, C, T] /. f[R, T] \rightarrow a[T] /. m[C, T] \rightarrow m[T], C] \} \} /. \text{Eq}[T] [[3]];$$

The eigenvalues of the Jacobian are

$$\text{lambda} = \text{Eigenvalues}[\text{Jac}];$$

Re-creating Gilbert et al.'s Figure 3

Using equation for K in Table 1 of Gilbert. Want K = 100 at 15C (as in Figure 3 of Gilbert), so we need rate-constant K0 to be

$$\text{K15} = \text{Solve}\left[100 == K0 \text{Exp}\left[\frac{EB}{k T[R]} - \frac{ES}{k T[S]}\right] /. T[i_] \rightarrow T /. T \rightarrow 273.15 + 15, K0\right];$$

Reproducing Figure 3A of Gilbert (same shape but numbers too large; can't tell why from their figure legend)

```
Show[
Plot[
  BCR[T] /. K[T] → K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ] /. T[i_] → T + 273.15 /. K15 /. k → 8.62 * 10-5 /.
    a[T] → 0.1 /. e[T] → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.32 /.
    ES → 0.9, {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]}],
Plot[BCR[T] /. K[T] → K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ] /. T[i_] → T + 273.15 /. K15 /. k →
  8.62 * 10-5 /. a[T] → 0.1 /. e[T] → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.9 /.
  ES → 0.32, {T, 5, 30}, PlotStyle → {Black, Thickness[0.01], Dashing[Large]}],
Plot[BCR[T] /. K[T] → K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ] /. T[i_] → T + 273.15 /. K15 /.
  k → 8.62 * 10-5 /. a[T] → 0.1 /. e[T] → 0.15 /. m[T] → 0.6 /. r[T] → 2 /.
  EB → 0.9 /. ES → 0.9, {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]}],
Frame → True,
FrameLabel →
  {Style[(*"Temperature (Celcius)"*)"", LabelSize], Style["BCR", LabelSize], ,},
FrameStyle → Directive[FontSize → TickSize],
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
(*PlotRangeClipping→False,*)
Epilog → {
  Text[Style["A", LabelSize, Bold], Scaled@letpos],
  (*Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree] *)
}
]

(*Export[imagedir<>"BCRGilbert.pdf",%];*)
```

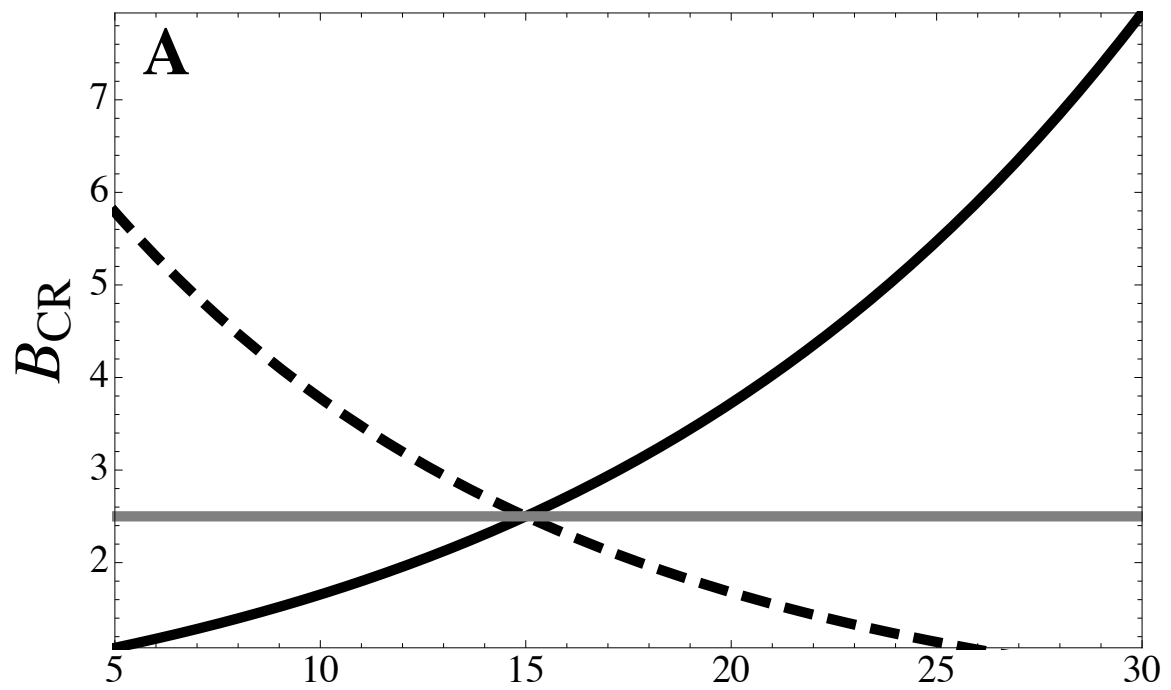


Figure 3b of Gilbert (off by factor of ~ 3 , again not sure why)

```
Show[Plot[
  CR[T] /. K[T] → K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ] /. T[i_] → T + 273.15 /. K15 /. k → 8.62 * 10-5 /.
    a[T] → 0.1 /. e[T] → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.32 /.
    ES → 0.9, {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]}],
  (*Plot[ CR[T] /. K[T] → K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ] /. T[i_] → T + 273.15 /. K15 /. k → 8.62 * 10-5 /.
    a[T] → 0.1 /. e[T] → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.9 /. ES → 0.32,
    {T, 5, 30}, PlotStyle → {Black, Thickness[0.01], Dashing[Large]}],
  Plot[ CR[T] /. K[T] → K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ] /. T[i_] → T + 273.15 /. K15 /. k → 8.62 * 10-5 /.
    a[T] → 0.1 /. e[T] → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.9 /. ES → 0.9,
    {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]}], *)
  Frame → True,
  FrameLabel →
    {Style[(*"Temperature (Celcius)"*)"", LabelSize], Style[" $\hat{C}:\hat{R}$ ", LabelSize], , },
  FrameStyle → Directive[FontSize → TickSize],
  ImagePadding → Pad,
  ImageSize → FigureSize,
  PlotRangePadding → None,
  Epilog → {
    Text[Style["B", LabelSize, Bold], Scaled@letpos],
    Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree]
  }
]
```

(*Export[imagedir<>"CtoRGilbert.pdf",%];*)

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

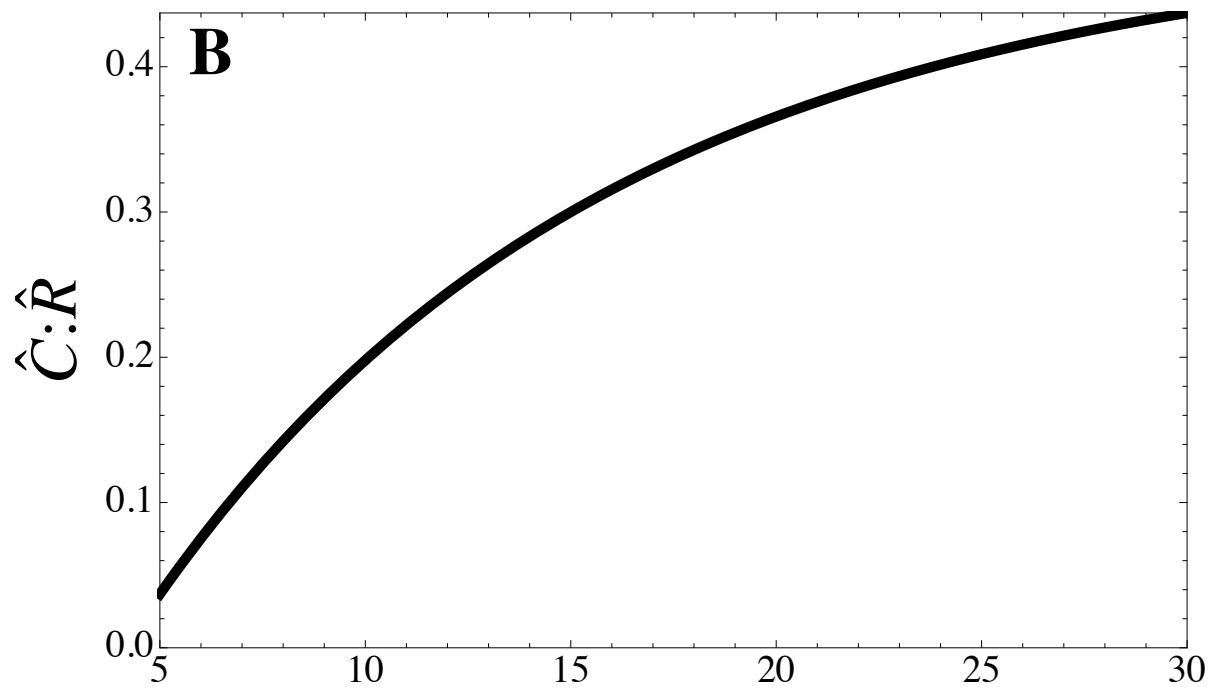
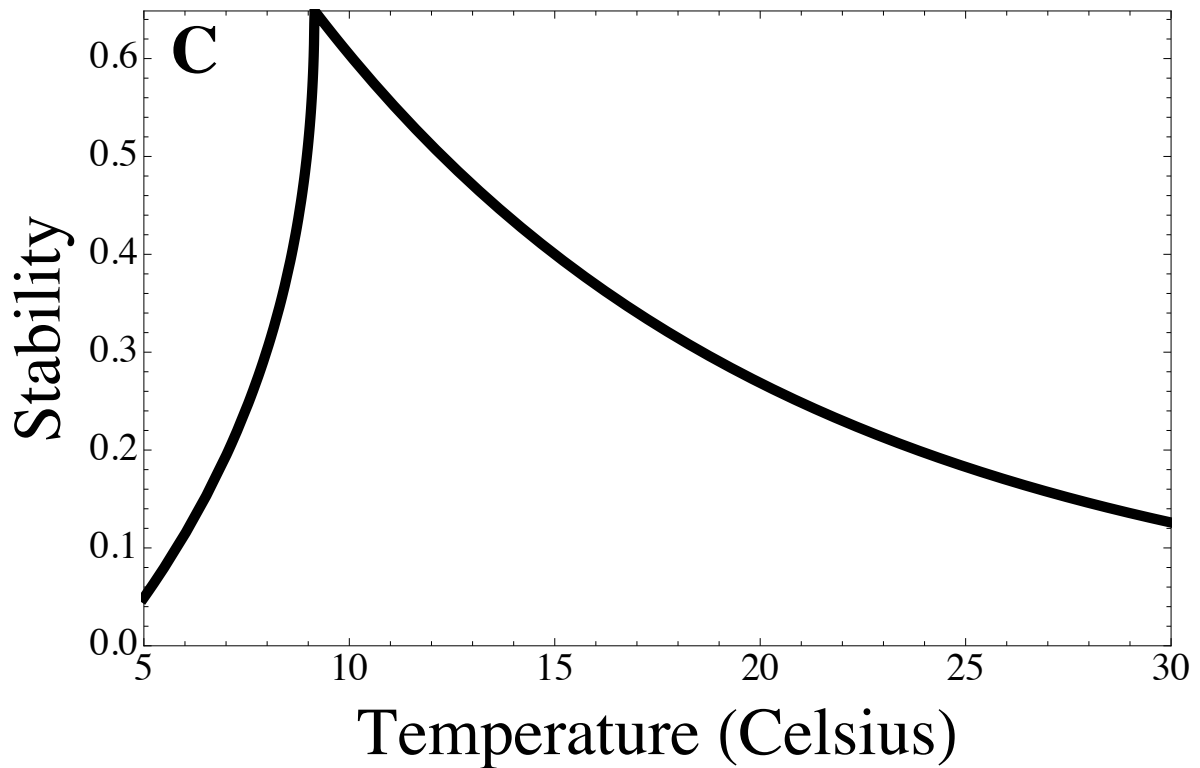


Figure 3c of Gilbert

```
Show[
  Plot[-Max[Re[lambda /. K[T] -> K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ] /. T[i_] -> T + 273.15 /. K15 /.
    k -> 8.62 * 10-5 /. a[T] -> 0.1 /. e[T] -> 0.15 /. m[T] -> 0.6 /. r[T] -> 2 /.
    EB -> 0.32 /. ES -> 0.9]], {T, 5, 30}, PlotStyle -> {Black, Thickness[0.01]}],
  (*Plot[-Max[Re[lambda /. K[T] -> K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ] /. T[i_] -> T + 273.15 /. K15 /. k -> 8.62 *
    10-5 /. a[T] -> 0.1 /. e[T] -> 0.15 /. m[T] -> 0.6 /. r[T] -> 2 /. EB -> 0.9 /. ES -> 0.32]],
    {T, 5, 30}, PlotStyle -> {Black, Thickness[0.01], Dashing[Large]}],
  Plot[-Max[Re[lambda /. K[T] -> K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ] /. T[i_] -> T + 273.15 /. K15 /. k -> 8.62 * 10-5 /.
    a[T] -> 0.1 /. e[T] -> 0.15 /. m[T] -> 0.6 /. r[T] -> 2 /. EB -> 0.9 /. ES -> 0.9]],
    {T, 5, 30}, PlotStyle -> {Gray, Thickness[0.01]}], *)
  Frame -> True,
  FrameLabel ->
    {Style["Temperature (Celsius)", LabelSize], Style["Stability", LabelSize], },
  FrameStyle -> Directive[FontSize -> TickSize],
  ImagePadding -> Pad,
  ImageSize -> FigureSize,
  PlotRangePadding -> None,
  Epilog -> {
    Text[Style["C", LabelSize, Bold], Scaled@letpos]
  }
]

(*Export[imagedir<>"StabilityGilbert.pdf", %];*)
```



Allowing all temperature dependencies

Here we add all the temperature dependencies, according to Table 1 of Gilbert et al.

```

cr = {C, R};
GilbertTable1 = {
  r[T] → r[M] Exp[ $\frac{-EB}{k T[R]}$ ],
  K[T] → K[M] Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ],
  m[T] → m[M] Exp[ $\frac{-Em}{k T[C]}$ ],
  a[T] → a[M] Sqrt[Sum[ $\left(v0[cr[[i]]] \text{Exp}\left[-\frac{Ev[cr[[i]]]}{k T[cr[[i]]}\right]\right)^2, \{i, 1, \text{Length}[cr]\}$ ]],
  e[T] → e[M]
};

```

To have the same population dynamics parameter values at 15 degrees as we did above with temperature dependence only in K, we need


```
aM15 = Solve[0.1 == a[T] /. GilbertTable1 /. T[i_] -> 273.15 + 15, a[M]] // Flatten
```

$$\left\{ a[M] \rightarrow \frac{0.1}{\sqrt{e^{-\frac{0.00694083 \text{ Ev}[C]}{k}} \nu_0[C]^2 + e^{-\frac{0.00694083 \text{ Ev}[R]}{k}} \nu_0[R]^2}} \right\}$$

```
eM15 = Solve[0.15 == e[T] /. GilbertTable1 /. T[i_] -> 273.15 + 15, e[M]] // Flatten
```

```
{e[M] -> 0.15}
```

```
KM15 = Solve[100 == K[T] /. GilbertTable1 /. T[i_] -> 273.15 + 15, K[M]] // Flatten
```

$$\left\{ K[M] \rightarrow 100. e^{-\frac{0.00347041 \text{ EB}}{k} + \frac{0.00347041 \text{ ES}}{k}} \right\}$$

```
rM15 = Solve[2 == r[T] /. GilbertTable1 /. T[i_] -> 273.15 + 15, r[M]] // Flatten
```

$$\left\{ r[M] \rightarrow 2. e^{\frac{0.00347041 \text{ EB}}{k}} \right\}$$

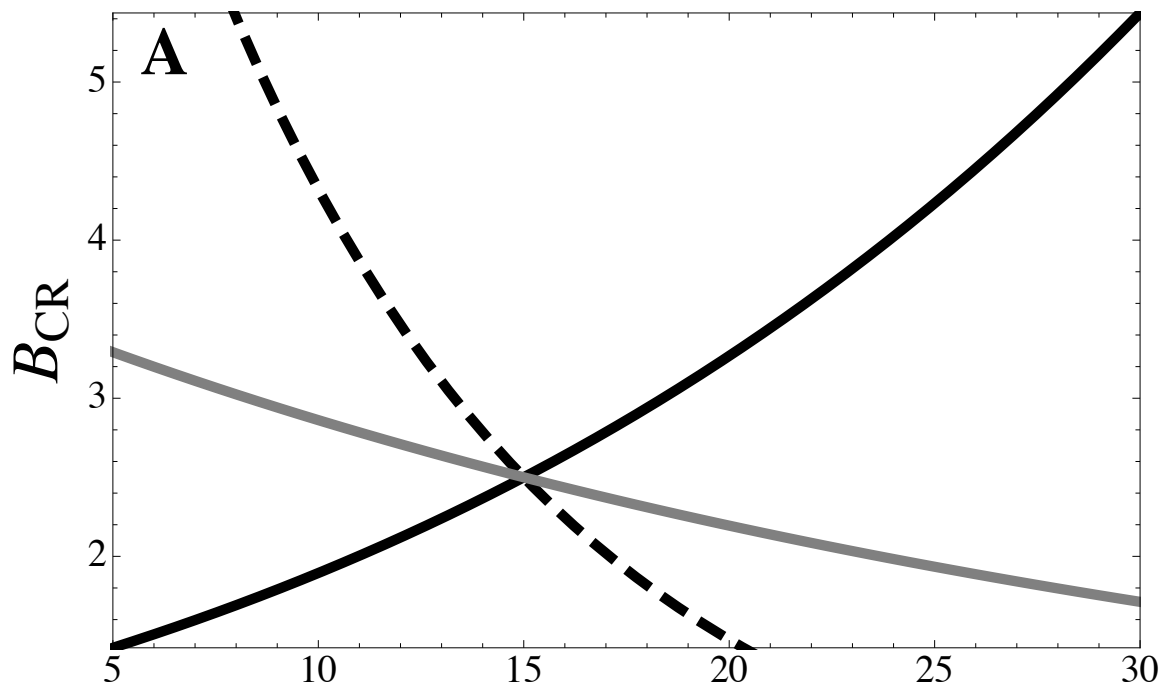
```
mM15 = Solve[0.6 == m[T] /. GilbertTable1 /. T[i_] -> 273.15 + 15, m[M]] // Flatten
```

$$\left\{ m[M] \rightarrow 0.6 e^{\frac{0.00347041 \text{ Em}}{k}} \right\}$$

Now, BCR as a function of temperature is

```
Show[
  Plot[BCR[T] /. GilbertTable1 /. aM15 /. eM15 /. KM15 /. rM15 /. mM15 /. EB -> 0.32 /.
    ES -> 0.9 /. k -> 8.62 * 10^-5 /. Em -> 0.65 /. Ev[i_] -> 0.46 /. v0[i_] -> 1 /.
    T[i_] -> T + 273.15, {T, 5, 30}, PlotStyle -> {Thickness[0.01], Black}],
  Plot[BCR[T] /. GilbertTable1 /. aM15 /. eM15 /. KM15 /. rM15 /. mM15 /. EB -> 0.9 /.
    ES -> 0.32 /. k -> 8.62 * 10^-5 /. Em -> 0.65 /.
    Ev[i_] -> 0.46 /. v0[i_] -> 1 /. T[i_] -> T + 273.15, {T, 5, 30},
    PlotStyle -> {Thickness[0.01], Black, Dashing[Large]}],
  Plot[BCR[T] /. GilbertTable1 /. aM15 /. eM15 /. KM15 /. rM15 /. mM15 /. EB -> 0.9 /.
    ES -> 0.9 /. k -> 8.62 * 10^-5 /. Em -> 0.65 /. Ev[i_] -> 0.46 /. v0[i_] -> 1 /.
    T[i_] -> T + 273.15, {T, 5, 30}, PlotStyle -> {Thickness[0.01], Gray}],
  Frame -> True,
  FrameLabel ->
    {Style[("Temperature (Celcius)"), LabelSize], Style["BCR", LabelSize], },
  FrameStyle -> Directive[FontSize -> TickSize],
  ImagePadding -> Pad,
  ImageSize -> FigureSize,
  PlotRangePadding -> None,
  (*PlotRangeClipping -> False, *)
  Epilog -> {
    Text[Style["A", LabelSize, Bold], Scaled@letpos],
    (*Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree] *)
  }
]
```

```
(*Export[imagedir<>"BCRAllTempDep.pdf", %]; *)
```



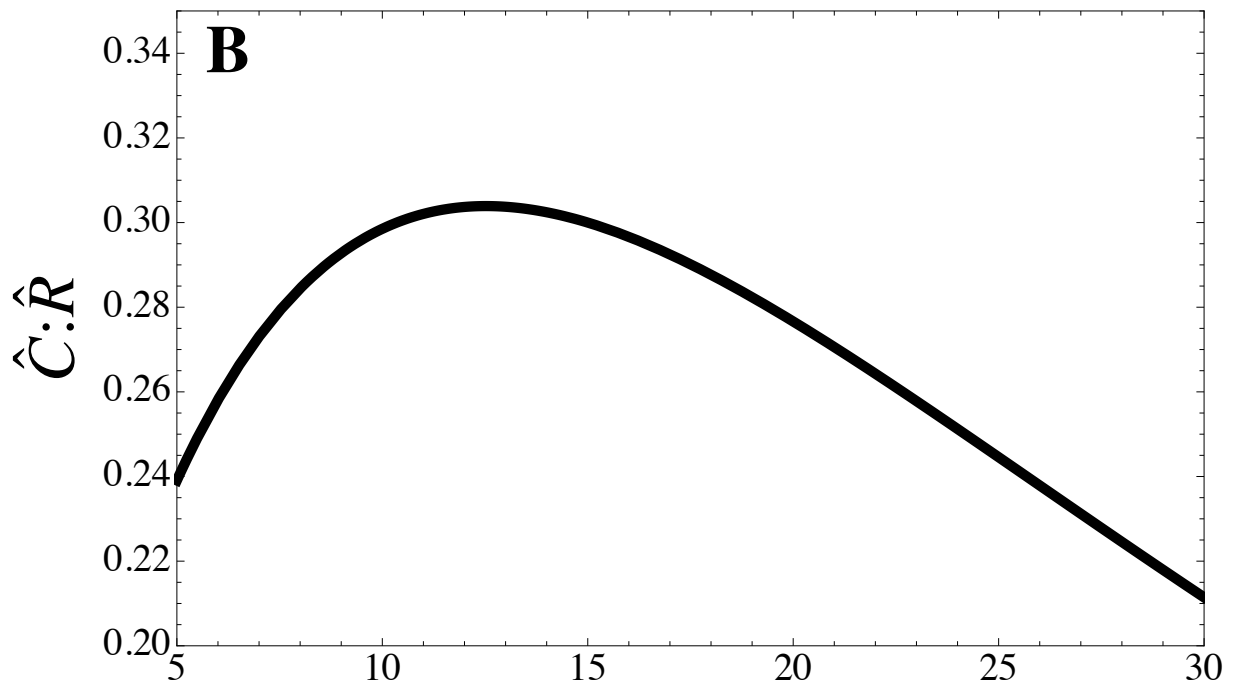
This is the same qualitative result as Gilbert et al Figure 3A, with the gray curve now with a slightly non-zero slope (because, although K doesn't vary with T , other parameters now do).

The biomass ratio is

```
Show[
Plot[CR[T] /. GilbertTable1 /. aM15 /. eM15 /. KM15 /. rM15 /. mM15 /. EB → 0.32 /.
    ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.
    T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]}, Axes → False],
(*Plot[CR[T] /. GilbertTable1 /. aM15 /. eM15 /. KM15 /. rM15 /. mM15 /. EB → 0.9 /. ES → 0.32 /.
    k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Black, Thickness[0.01], Dashing[Large]}, Axes → False],
Plot[CR[T] /. GilbertTable1 /. aM15 /. eM15 /. KM15 /. rM15 /. mM15 /. EB → 0.9 /. ES → 0.9 /.
    k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]}, Axes → False], *)
PlotRange → {{5, 30}, {0.2, 0.35}},
Frame → True,
FrameLabel →
    {Style[(*"Temperature (Celcius)"*)"", LabelSize], Style["C:R", LabelSize], , },
FrameStyle → Directive[FontSize → TickSize],
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
Epilog → {
    Text[Style["B", LabelSize, Bold], Scaled@letpos],
    Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree]
}
]
```

```
(*Export[imagedir<>"CtoRAllTempDep.pdf", %]; *)
```

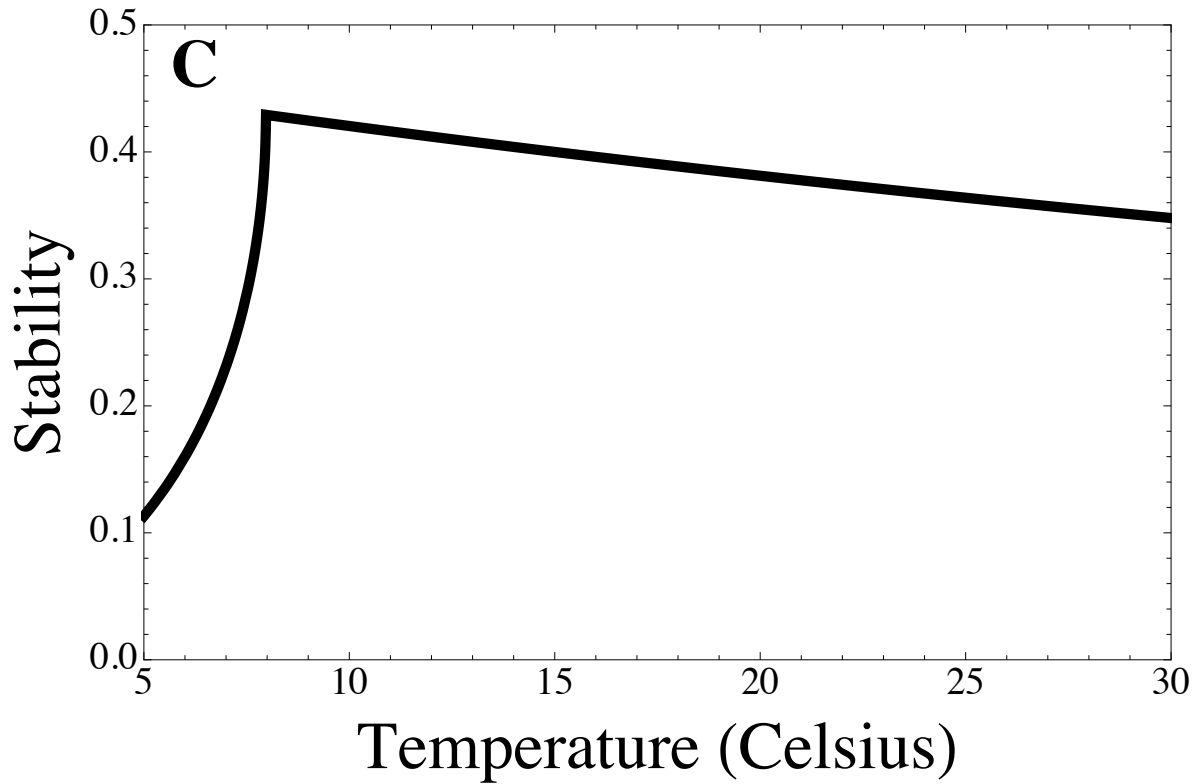
Solve::ratnz: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>



The ratio now decreases at high temperatures (as opposed to the increase in Gilbert et al Figure 3B).
And stability:

```
Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. aM15 /. eM15 /. KM15 /. rM15 /. mM15 /. EB → 0.32 /.
    ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /.
    Ev[i_] → 0.46 /. v0[i_] → 1 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]}, Axes → False,
  PlotRange → {0, All}],
(*Plot[
  -Max[Re[lambda /. GilbertTable1 /. aM15 /. eM15 /. KM15 /. rM15 /. mM15 /. EB → 0.9 /. ES → 0.32 /.
    k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thickness[0.01], Dashing[Large]},
  Axes → False,
  PlotRange → {0, All}],
Plot[-Max[Re[lambda /. GilbertTable1 /. aM15 /. eM15 /. KM15 /. rM15 /. mM15 /. EB → 0.9 /. ES →
  0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]}, Axes → False,
  PlotRange → {0, All}], *)
PlotRange → {0, 0.5},
Frame → True,
FrameLabel →
  {Style["Temperature (Celsius)", LabelSize], Style["Stability", LabelSize], , },
FrameStyle → Directive[FontSize → TickSize],
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
Epilog → {
  Text[Style["C", LabelSize, Bold], Scaled@letpos]
}
]

(*Export[imagedir<>"StabilityAllTempDep.pdf",%];*)
```



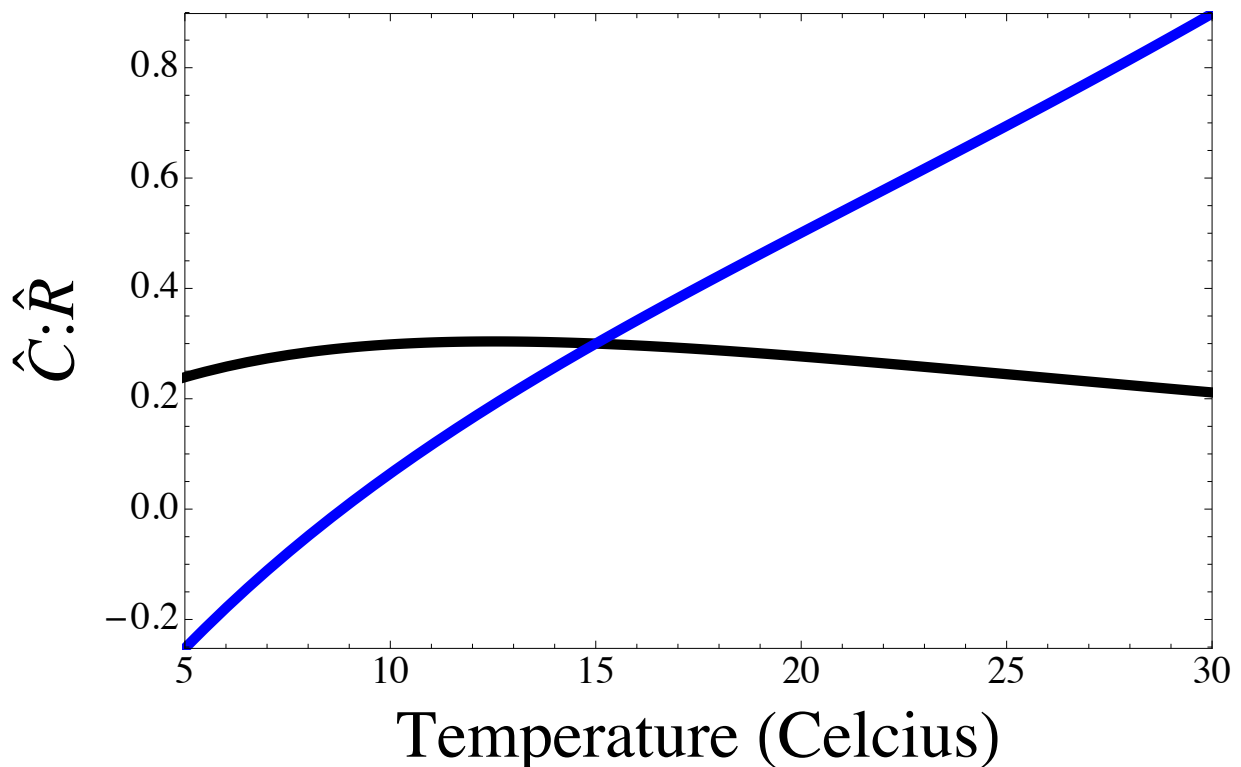
With all the dependencies, stability still decreases with temperature, albeit slower than when only K depended on temperature (Gilbert Figure 3c).

The decrease in consumer:resource biomass at high temperatures, relative to Gilbert et al Figure 3B, is largely driven by the temperature dependence of consumer mortality (here we drop that dependence and see consumer:resource biomass increase with temperature, as it did before):

```
Show[
  Plot[CR[T] /. GilbertTable1 /. aM15 /. eM15 /. KM15 /. rM15 /. mM15 /. EB → 0.32 /.
    ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.
    T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]}, Axes → False],
  Plot[CR[T] /. GilbertTable1 /. aM15 /. eM15 /. KM15 /. rM15 /. mM15 /. EB → 0.32 /.
    ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.
    T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Blue, Thickness[0.01]}, Axes → False],
  PlotRange → All,
  Frame → True,
  FrameLabel →
    {Style["Temperature (Celcius)", LabelSize], Style[" $\hat{C}:\hat{R}$ ", LabelSize], , },
  FrameStyle → Directive[FontSize → TickSize],
  ImagePadding → Pad,
  ImageSize → FigureSize,
  PlotRangePadding → None
]
```

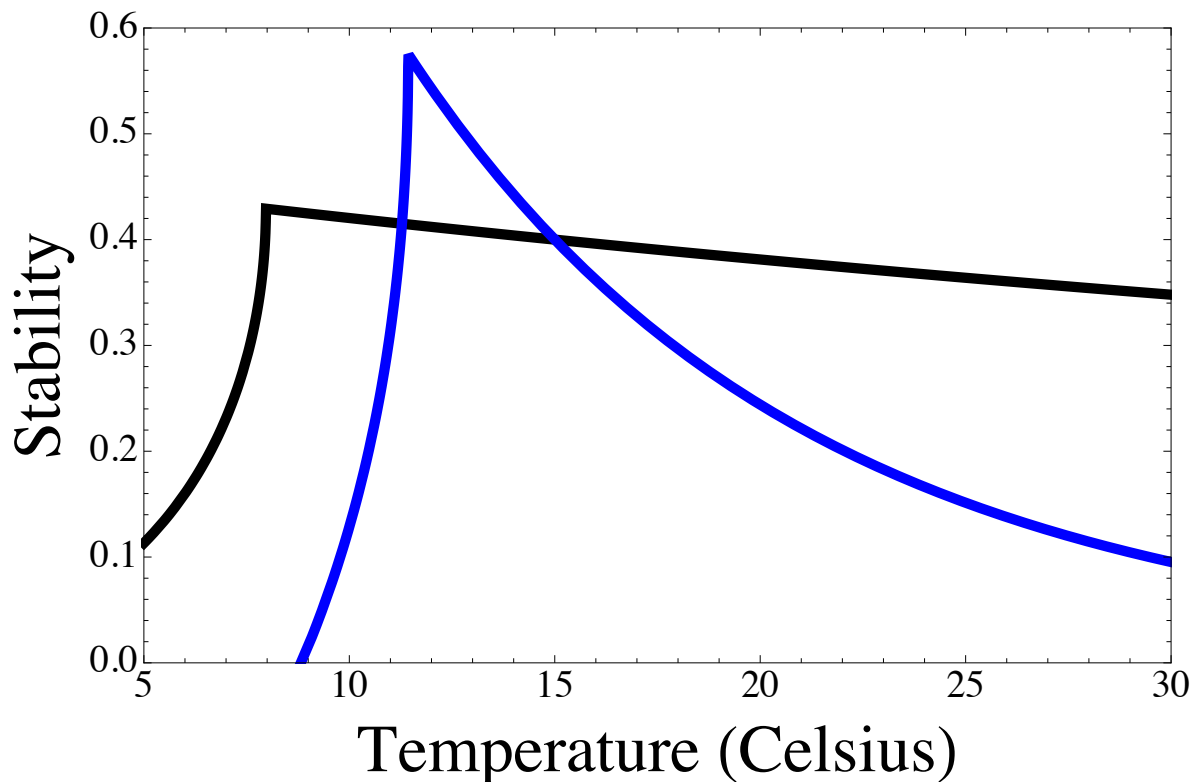
Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>



The increase in stability at high temperatures, relative to Gilbert et al Figure 3C, is largely driven by the temperature dependence of consumer mortality (here we drop that dependence and see stability decline, as it did in Gilbert):

```
Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. aM15 /. eM15 /. KM15 /. rM15 /. mM15 /. EB → 0.32 /.
    ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /.
    Ev[i_] → 0.46 /. v0[i_] → 1 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]}, Axes → False,
  PlotRange → {0, All}],
Plot[-Max[Re[lambda /. GilbertTable1 /. aM15 /. eM15 /. KM15 /. rM15 /. mM15 /.
  EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0 /.
  Ev[i_] → 0.46 /. v0[i_] → 1 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Blue, Thickness[0.01]},
  Axes →
  False, PlotRange →
  {0, All}],
PlotRange → {0, 0.6},
Frame → True,
FrameLabel →
  {Style["Temperature (Celsius)", LabelSize], Style["Stability", LabelSize], , },
FrameStyle → Directive[FontSize → TickSize],
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None
]
```



Adding mass-dependence and the temperature-size rule

Now we add dependence on body mass using the relations given in Table 1 of DeLong et al. 2015:

```
DeLongTable1 = {
  r[M] → r0 M[R]ρ,
  K[M] → K0 M[R]κ,
  a[M] → a0 M[C]α,
  e[M] → e0 M[C]ε,
  m[M] → m0 M[C]μ
};
```

We also want to add the temperature-size rule (TSR). From Forster et al. 2012 Figure 2 this is, for smaller organisms (where the TSR is linear):

```
TSR = M[i_] → M15[i] (1 - β[i] (T[i] - (273.15 + 15)));
```

where M15[i], i={R,C}, is the mass of the resource or consumer at 15 degrees Celsius, β[i] is the percent decline in body size with a degree increase in temperature, and T[i] is the current temperature (in Kelvins).

To have the same population dynamics parameter values at 15C as in Gilbert et al 2014 Figure 3 we need

```
a15 =
  Solve[0.1 == a[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, a0] //
  Flatten
```

$$\left\{ a0 \rightarrow \frac{0.1 M15[C]^{-1. \alpha}}{\sqrt{e^{-\frac{0.00694083 \text{ Ev}[C]}{k}} \sqrt{0[C]^2} + e^{-\frac{0.00694083 \text{ Ev}[R]}{k}} \sqrt{0[R]^2}}} \right\}$$

```
e15 = Solve[0.15 == e[M] /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, e0] // Flatten
```

$$\{e0 \rightarrow 0.15 M15[C]^{-1. \epsilon}\}$$

```
k15 =
  Solve[100 == K[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, K0] //
  Flatten
```

$$\left\{ K0 \rightarrow 100. e^{-\frac{0.00347041 \text{ EB}}{k} + \frac{0.00347041 \text{ ES}}{k}} M15[R]^{-1. \kappa} \right\}$$

```
r15 =
  Solve[2 == r[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, r0] //
  Flatten
```

$$\left\{ r0 \rightarrow 2. e^{\frac{0.00347041 \text{ EB}}{k}} M15[R]^{-1. \rho} \right\}$$

```
m15 =
  Solve[0.6 == m[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, m0] //
  Flatten
```

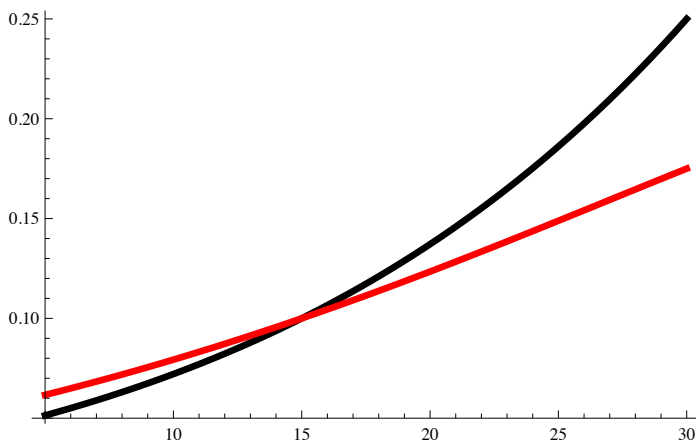
$$\left\{ m0 \rightarrow 0.6 e^{\frac{0.00347041 \text{ Em}}{k}} M15[C]^{-1. \mu} \right\}$$

Effects on each parameter

Lets see how each parameter changes with temperature, with (red) and without (black) the TSR.

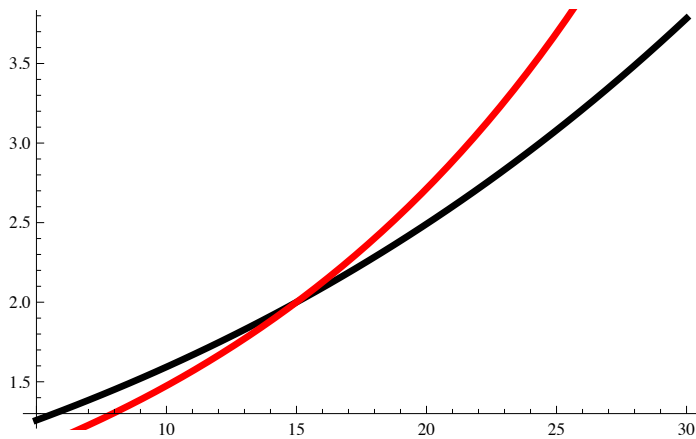
Attack rate: the TSR reduces the response to temperature

```
Show[
  Plot[
    a[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.32 /.
      ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.
      κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
      β[i_] → 0 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle →
      {Thickness[0.01], Black}],
  Plot[a[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
      {T, 5, 30}, PlotStyle → {Thickness[0.01], Red}]
]
```



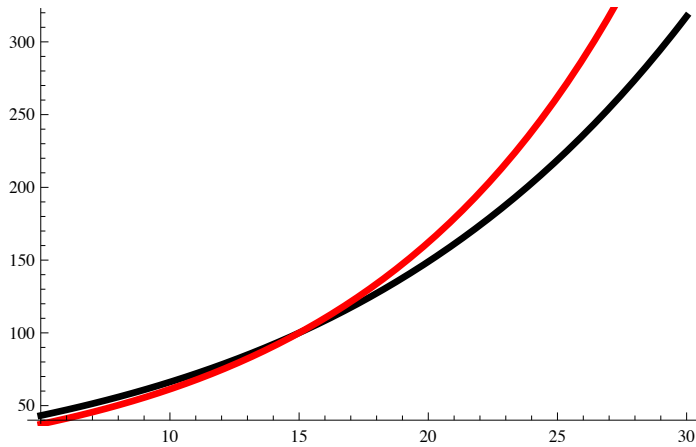
Resource intrinsic growth rate: the TSR increases the response to temperature

```
Show[
Plot[
  r[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.32 /.
    ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.
    κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
    β[i_] → 0 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle →
    {Thickness[0.01], Black}],
Plot[r[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
  EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
  v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
  ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Thickness[0.01], Red}]
]
```



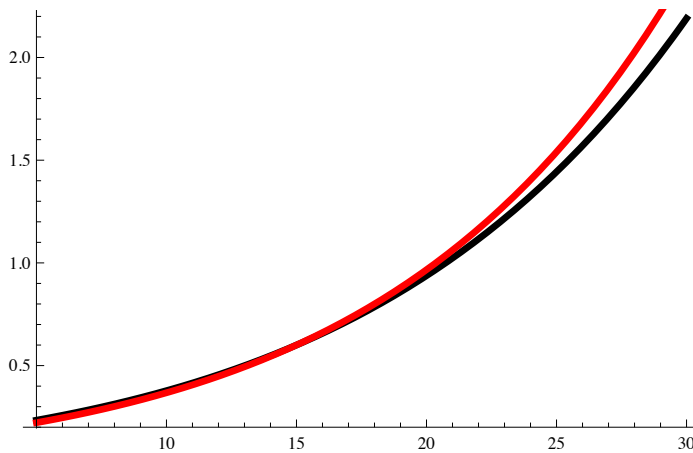
Resource carrying capacity: the TSR increases the response to temperature

```
Show[
Plot[
K[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.32 /.
      ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.
      κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
      β[i_] → 0 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle →
      {Thickness[0.01], Black}],
Plot[K[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
      {T, 5, 30}, PlotStyle → {Thickness[0.01], Red}]
]
```



Consumer mortality rate: the TSR increases the response to temperature

```
Show[
Plot[
m[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.32 /.
ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.
κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
β[i_] → 0 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle →
{Thickness[0.01], Black}],
Plot[m[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Thickness[0.01], Red}]
]
```



Note that for all temperature dependent parameters, the TSR increases the sensitivity to temperature in all except attack rate. In other words, the indirect effect of temperature on the parameters, through body mass, acts in the same direction as the direct effect of temperature on the parameters, except in the case of attack rate, where direct and indirect effects oppose one another and reduce the sensitivity of attack rate to temperature.

Numerical solutions of temporal dynamics

Stability conditions for coexistence equilibrium (for stable node or focus need trace negative and determinant positive):

```
FullSimplify[Tr[Jac], {a[T] > 0, e[T] > 0, K[T] > 0, m[T] > 0, r[T] > 0}]
```

$$-\frac{m[T] r[T]}{a[T] e[T] K[T]}$$

So the trace is always negative, meaning we can't have an unstable node or focus. But we could have a saddle if the determinant can be negative:

```
FullSimplify[Det[Jac], {a[T] > 0, e[T] > 0, K[T] > 0, m[T] > 0, r[T] > 0}]
```

$$m[T] \left(1 - \frac{m[T]}{a[T] e[T] K[T]} \right) r[T]$$

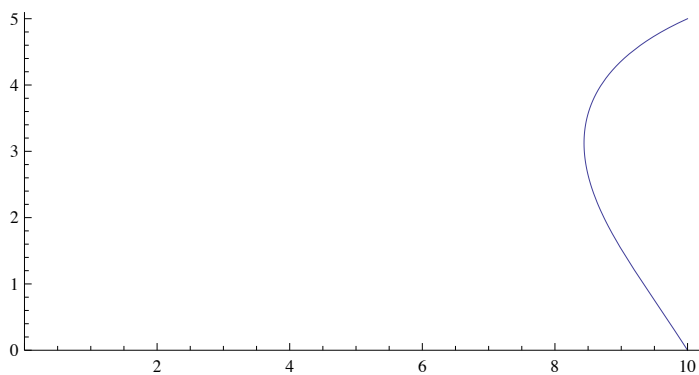
which is can be if $a \in K < m$.

Simplified dynamics (saddle)

```
deqns = {
    dRdt[R, C, T],
    dCdt[R, C, T]
} /. f[R, T] -> a[T] /. m[C, T] -> m[T] /. R -> R[t] /. C -> CO[t] /.
a[T] -> 0.1 /. e[T] -> 0.15 /. K[T] -> 10 /. r[T] -> 2 /. m[T] -> 0.6;

dsol = NDSolve[{
    D[R[t], t] == deqns[[1]],
    D[CO[t], t] == deqns[[2]],
    R[0] == 10,
    CO[0] == 5
},
{R, CO},
{t, 0, 100}
];

ParametricPlot[Evaluate[{R[t], CO[t]} /. dsol],
{t, 0, 100}, PlotRange -> {{0, All}, {0, All}}]
```



Simplified dynamics (stable)

```

deqns = {
    dRdt[R, C, T],
    dCdt[R, C, T]
} /. f[R, T] → a[T] /. m[C, T] → m[T] /. R → R[t] /. C → CO[t] /.
a[T] → 0.1 /. e[T] → 0.15 /. K[T] → 100 /. r[T] → 2 /. m[T] → 0.6;

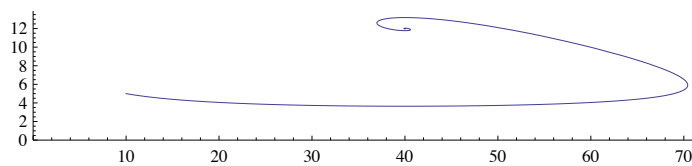
dsol = NDSolve[{
    D[R[t], t] == deqns[[1]],
    D[CO[t], t] == deqns[[2]],
    R[0] == 10,
    CO[0] == 5
},
{R, CO},
{t, 0, 100}
];

```

```

ParametricPlot[Evaluate[{R[t], CO[t]} /. dsol],
{t, 0, 100}, PlotRange → {{0, All}, {0, All}}]

```



The dynamics, with (black) and without (gray) the TSR

```

R0 = 10;
C0 = 10;
tmax = 100;
T = 30;

deqnsNoTSR = {
    dRdt[R, C, T],
    dCdt[R, C, T]
} /. f[R, T] → a[T] /. m[C, T] → m[T] /. R → R[t] /.
C → CC[t] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /.
k15 /. r15 /. m15 /. EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /.
Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15;

dsolNoTSR = NDSolve[{
    D[R[t], t] == deqnsNoTSR[[1]],
    D[CC[t], t] == deqnsNoTSR[[2]],
    R[0] == R0,
    CC[0] == C0
},
{R, CC},
{t, 0, tmax}
];

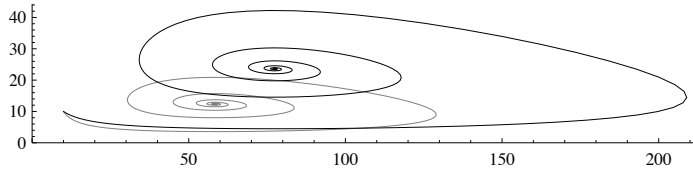
deqnsWithTSR = {
    dRdt[R, C, T],
    dCdt[R, C, T]
} /. f[R, T] → a[T] /. m[C, T] → m[T] /. R → R[t] /.
C → CC[t] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /.
k15 /. r15 /. m15 /. EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /.
Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15;

dsolWithTSR = NDSolve[{
    D[R[t], t] == deqnsWithTSR[[1]],
    D[CC[t], t] == deqnsWithTSR[[2]],
    R[0] == R0,
    CC[0] == C0
},
{R, CC},
{t, 0, tmax}
];

Show[
    ParametricPlot[Evaluate[{R[t], CC[t]} /. dsolNoTSR],
        {t, 0, tmax}, PlotRange → {{0, All}, {0, All}}, PlotStyle → Gray],
    ParametricPlot[Evaluate[{R[t], CC[t]} /. dsolWithTSR], {t, 0, tmax},
        PlotRange → {{0, All}, {0, All}}, PlotStyle → Black]
]

Clear[R0, C0, tmax, T, deqnsNoTSR, dsolNoTSR, deqnsWithTSR, dsolWithTSR]

```

BCR, C:R, and stability with and without the TSR

Now, with the TSR the BCR as a function of temperature is

```

Legended[
  Show[
    (*Plot[
      BCR[T]/.K[T]→K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ]/.T[i_]→T+273.15/.K15/.k→8.62*10-5/.a[T]→0.1/.
      e[T]→0.15/.m[T]→0.6/.r[T]→2/.EB→0.32/.ES→0.9,
      {T,5,30},PlotStyle→{Gray,Thickness[0.01]}],*)
    Plot[
      BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.
      EB→0.32/.ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.
      v0[i_]→1/.κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.
      ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→T+273.15,{T,5,30},
      PlotStyle→{Thickness[0.01],Black,Dashing[Large]}],
    (*Plot[BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.
      ES→0.32/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.
      α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→T+273.15,
      {T,5,30},PlotStyle→{Thickness[0.01],Black,Dashing[Large]}],
    Plot[BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.
      ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.
      κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.
      T[i_]→T+273.15,{T,5,30},PlotStyle→{Thick,Gray}],*)
    Plot[BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.
      EB→0.32/.ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.
      v0[i_]→1/.κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.
      ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
      {T,5,30},PlotStyle→{Thickness[0.01],Black}],
    (*Plot[BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.
      ES→0.32/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.
      α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
      {T,5,30},PlotStyle→{Thickness[0.01],Red,Dashing[Large]}],Plot[
      BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.9/.
      k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
      ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
      {T,5,30},PlotStyle→{Thickness[0.01],Pink}],*)
    Frame→True,
    FrameLabel→{Style[(*"Temperature (Celcius)"*)"",LabelSize],
      Style["BCR",LabelSize],,},

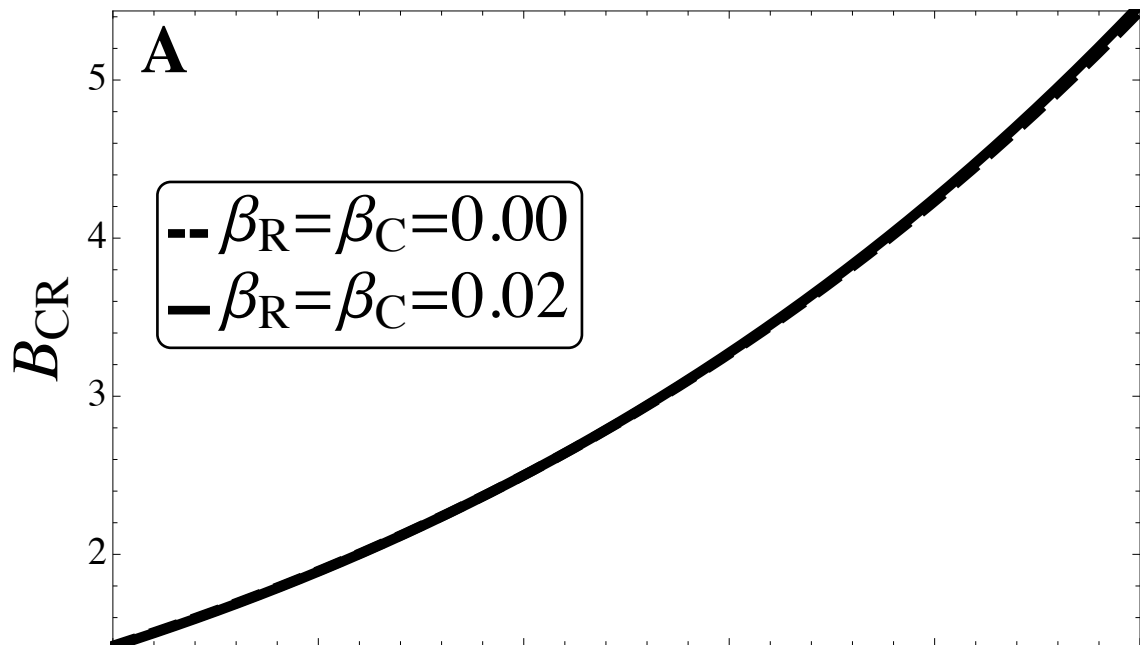
```

```

FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle → {{Black, Black}, {Directive[FontColor → White], Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
(*PlotRangeClipping→False,*)
Epilog → {
  Text[Style["A", LabelSize, Bold], Scaled@letpos],
  (*Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree]*)
},
],
Placed[
  LineLegend[{
    Directive[Black, Dashing[Medium], Thickness[0.25]],
    Directive[Black, Thickness[0.25]]
  },
  {
    Style[" $\beta_R = \beta_C = 0.00$ ", LabelSize],
    Style[" $\beta_R = \beta_C = 0.02$ ", LabelSize]
  },
  LegendFunction → "Frame",
  LegendLayout → "Column"
],
{0.25, 0.6}
]
]

(*Export[imagedir<>"BCRAllTempMassDep.pdf",%];*)

```



As the curves overlap almost perfectly, the added mass dependence and TSR do not affect the response of BCR to temperature.

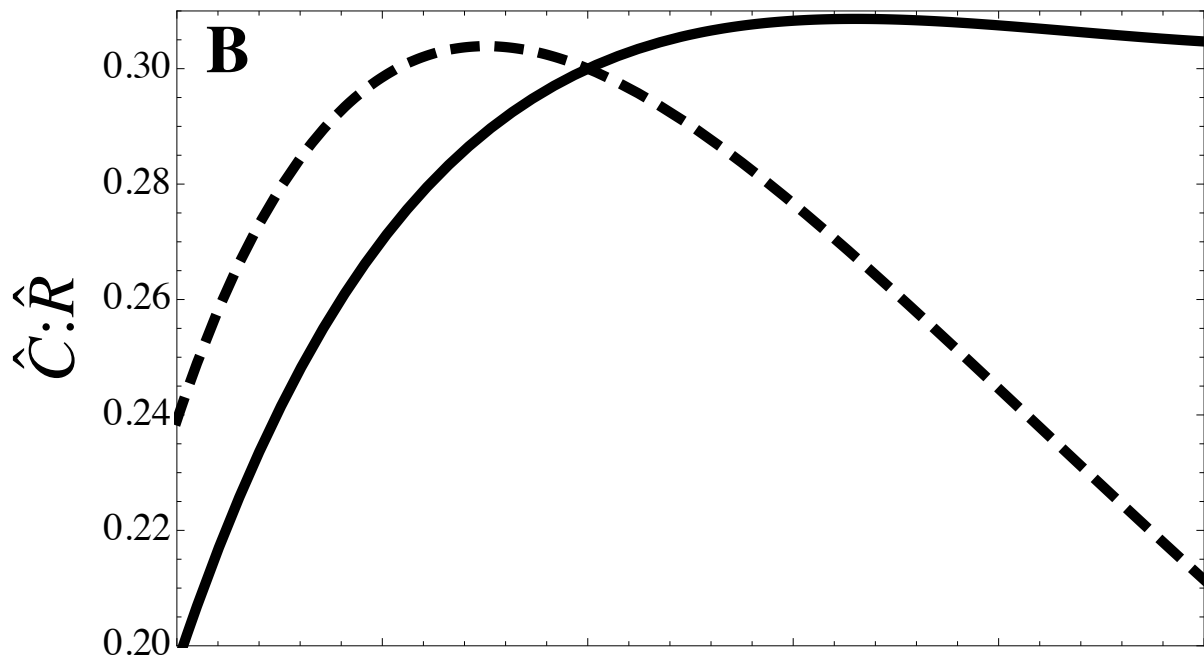
Biomass ratios:

```
Show[
  (*Plot[
    CR[T]/.K[T]→K0 Exp[ $\frac{EB}{k_{T[R]}} - \frac{ES}{k_{T[S]}}$ ]/.T[i_]→T+273.15/.K15/.k→8.62*10-5/.a[T]→0.1/.
    e[T]→0.15/.m[T]→0.6/.r[T]→2/.EB→0.32/.ES→0.9,
    {T,5,30},PlotStyle→{Gray,Thickness[0.01]}],*)
  Plot[CR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.
    EB→0.32/.ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.
    v0[i_]→1/.κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.
    ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→T+273.15,{T,5,30},
    PlotStyle→{Black,Dashing[Large],Thickness[0.01]},
    Axes→
    False],
  Plot[CR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.
    EB→0.32/.ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.
    v0[i_]→1/.κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.
    ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
    {T,5,30},PlotStyle→{Black,Thickness[0.01]},
    Axes→
    False],
  (*Plot[CR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.
    EB→0.32/.ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.
    κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.04/.
    T[i_]→T+273.15,{T,5,30},PlotStyle→{Blue,Thickness[0.01]},Axes→
    False],*)
  PlotRange→{0.2,0.31},
  Frame→True,
  FrameLabel→
    {Style[(*"Temperature (Celcius)"*)"",LabelSize],Style["C:R",LabelSize],,},
  FrameStyle→Directive[FontSize→TickSize],
  FrameTicksStyle→{{Black,Black},{Directive[FontColor→White],Black}},
  ImagePadding→Pad,
  ImageSize→FigureSize,
  PlotRangePadding→None,
  Epilog→{
    Text[Style["B",LabelSize,Bold],Scaled@letpos],
    Rotate[Text[Style["BCR",LabelSize],Scaled@ylabpos],90 Degree]
  }
]
```

```
(*Export[imagedir<"CtoRAllTempMassDep.pdf",%];*)
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

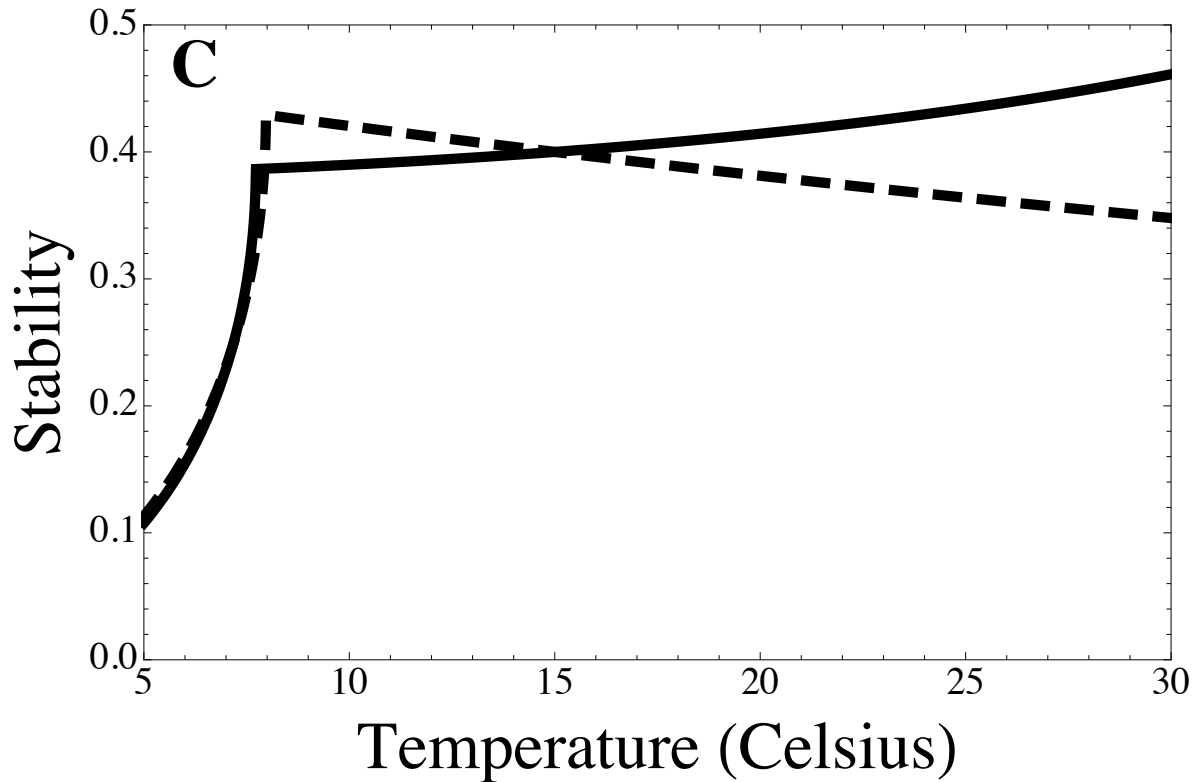


Here we see that when we add the TSR the ratio no longer decreases over reasonable temperatures. I.e., temperature directly decreases consumer:resource biomass (through its affect on cosumer mortality; see previous section), but it also decreases body sizes, and decreased body size increases consumer:resource biomass (by increasing conversion efficiency and intrinsic growth rate; see below). In other words, the direct and indirect effects of temperature are opposing.

Stability:

```
Show[(*Plot[
  -Max[Re[lambda/.K[T]→K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ]/.T[i_]→T+273.15/.K15/.k→8.62*10-5/.
    a[T]→0.1/.e[T]→0.15/.m[T]→0.6/.r[T]→2/.EB→0.32/.ES→0.9]],
  {T,5,30},PlotStyle→{Gray,Thickness[0.01]}],*)Plot[
  -Max[Re[lambda/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.
    EB→0.32/.ES→0.9/.k→8.62*10-5/.Em→0.65/.
    Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.ε→-0.5/.
    μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→T+273.15]],
  {T,5,30},PlotStyle→{Black,Dashing[Large],Thickness[0.01]},
  Axes→
    False,PlotRange→
    {0,All}],Plot[
  -Max[Re[lambda/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.
    EB→0.32/.ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→
    0.46/.v0[i_]→1/.κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.
    ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15]],
  {T,5,30},PlotStyle→{Black,Thickness[0.01]},
  PlotRange→
    {0,All}],
  (*Plot[-Max[Re[
    lambda/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.
    ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.
    α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.04/.T[i_]→T+273.15]],
  {T,5,30},PlotStyle→{Blue,Thickness[0.01]},PlotRange→
    {0,
    All}],*)
  PlotRange→{0,0.5},
  Frame→True,
  FrameLabel→
    {Style["Temperature (Celsius)",LabelSize],Style["Stability",LabelSize],,},
  FrameStyle→Directive[FontSize→TickSize],
  ImagePadding→Pad,
  ImageSize→FigureSize,
  PlotRangePadding→None,
  Epilog→{
    Text[Style["C",LabelSize,Bold],Scaled@letpos]
  }
]
```

(*Export[imagedir<>"StabilityAllTempMassDep.pdf",%];*)



With the temperature-size rule, stability increases with increasing temperature (as opposed to the decrease without the TSR). Again, temperature directly destabilizes, but indirectly, through its effect on body size (which decreases attack rate and increases intrinsic growth rate), increasing temperature stabilizes the dynamics.

The lack of decrease in C:R at high temperatures, relative to the case without the TSR, is driven by the indirect effect of temperature on conversion efficiency (purple; increases with decreasing body size and hence with temperature) and resource intrinsic growth rate (blue; increases with decreasing body size and hence with temperature). When we drop these effects the curve goes back to the without TSR case:

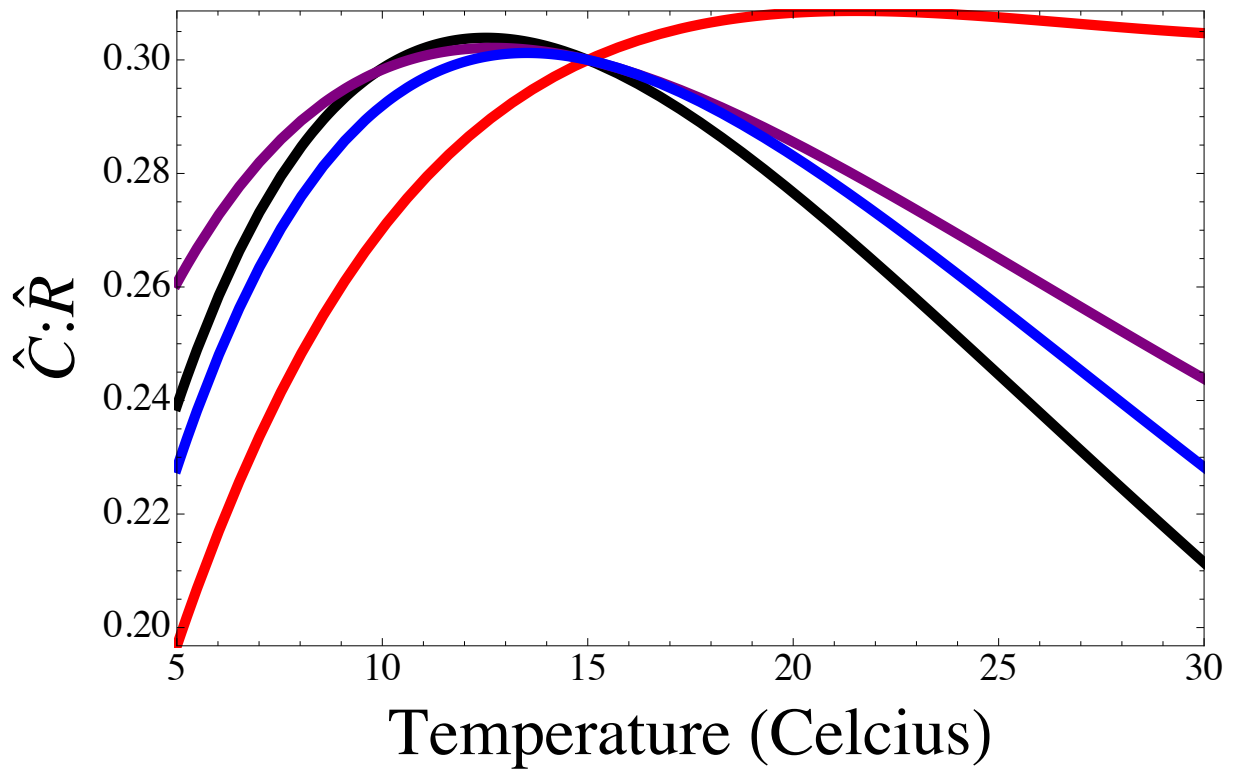
```
Show[
  Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
    v0[i_] → 1 /. x → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
    Axes →
      False],
  Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
    v0[i_] → 1 /. x → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Red, Thickness[0.01]},
    Axes →
      False],
  Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
    v0[i_] → 1 /. x → -0.81 /. α → 1 /. ε → 0 /. μ → -0.29 /. ρ → -0.81 /.
    TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15, {T, 5, 30},
    PlotStyle → {Purple, Thickness[0.01]}, Axes → False],
  Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
    v0[i_] → 1 /. x → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → 0 /.
    TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15, {T, 5, 30},
    PlotStyle → {Blue, Thickness[0.01]}, Axes → False],
  PlotRange → All,
  Frame → True,
  FrameLabel →
    {Style["Temperature (Celcius)", LabelSize], Style["C:R", LabelSize], , },
  FrameStyle → Directive[FontSize → TickSize],
  ImagePadding → Pad,
  ImageSize → FigureSize,
  PlotRangePadding → None,
  Epilog → {
    Text[Style["", LabelSize, Bold], Scaled@letpos],
    Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree]
  }
]
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

General::stop : Further output of Solve::ratnz will be suppressed during this calculation. >>

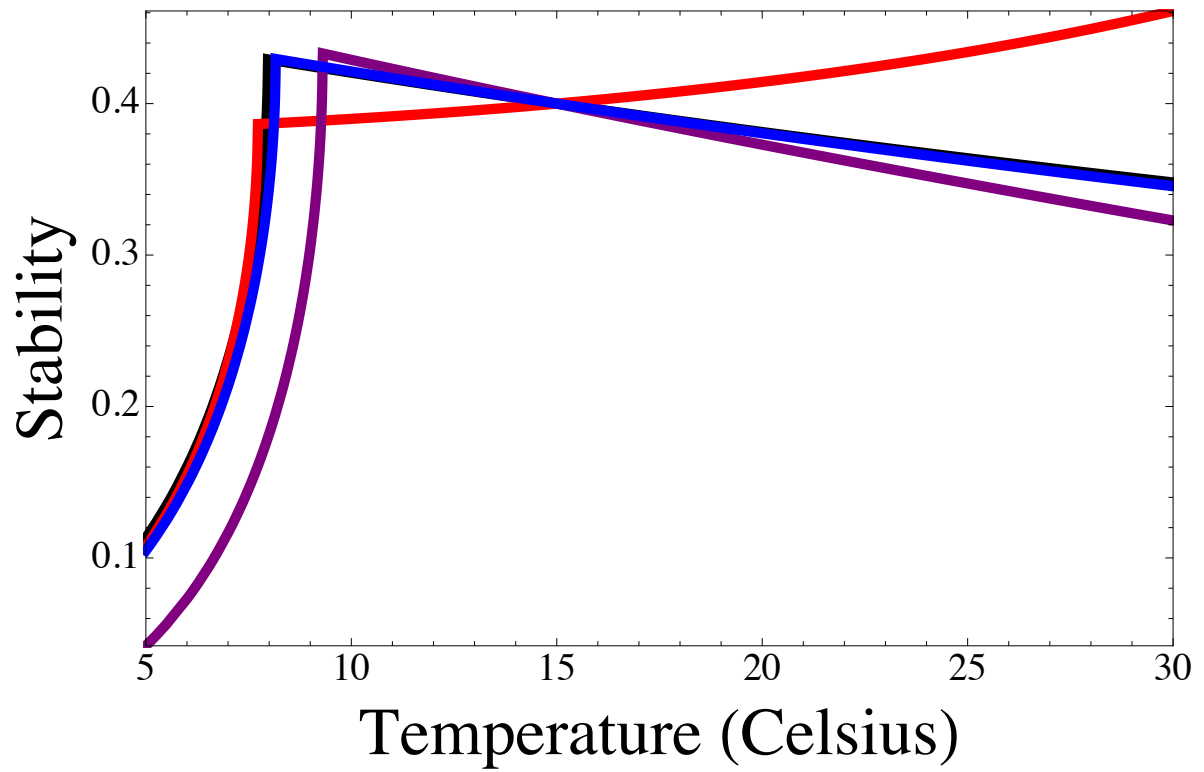


The increase in stability at high temperatures, relative to the case without the TSR, is driven by the indirect effect of temperature on attack rate (which decreases with decreasing body size) and resource intrinsic growth rate (which increases with increasing body size). When we drop these effects the response looks similar to the response without the TSR:

```

Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /.
    Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
    μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
  Axes →
    False, PlotRange →
      {0, All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] →
    0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Red, Thickness[0.01]},
  PlotRange →
    {0, All}],
  Plot[
    -Max[
      Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
        0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
        v0[i_] → 1 /. κ → -0.81 /. α → 0 /. ε → -0.5 /. μ → -0.29 /.
        ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15]],
      {T, 5, 30}, PlotStyle → {Purple, Thickness[0.01]},
      PlotRange →
        {0,
          All}],
    Plot[-Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /.
      m15 /. EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /.
      Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
      μ → -0.29 /. ρ → 0 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15]],
      {T, 5, 30}, PlotStyle → {Blue, Thickness[0.01]},
      PlotRange →
        {0,
          All}],
    PlotRange → All,
    Frame → True,
    FrameLabel →
      {Style["Temperature (Celsius)", LabelSize], Style["Stability", LabelSize], },
    FrameStyle → Directive[FontSize → TickSize],
    ImagePadding → Pad,
    ImageSize → FigureSize,
    PlotRangePadding → None,
    Epilog → {
      Text[Style["", LabelSize, Bold], Scaled@letpos]
    }
  ]
]

```



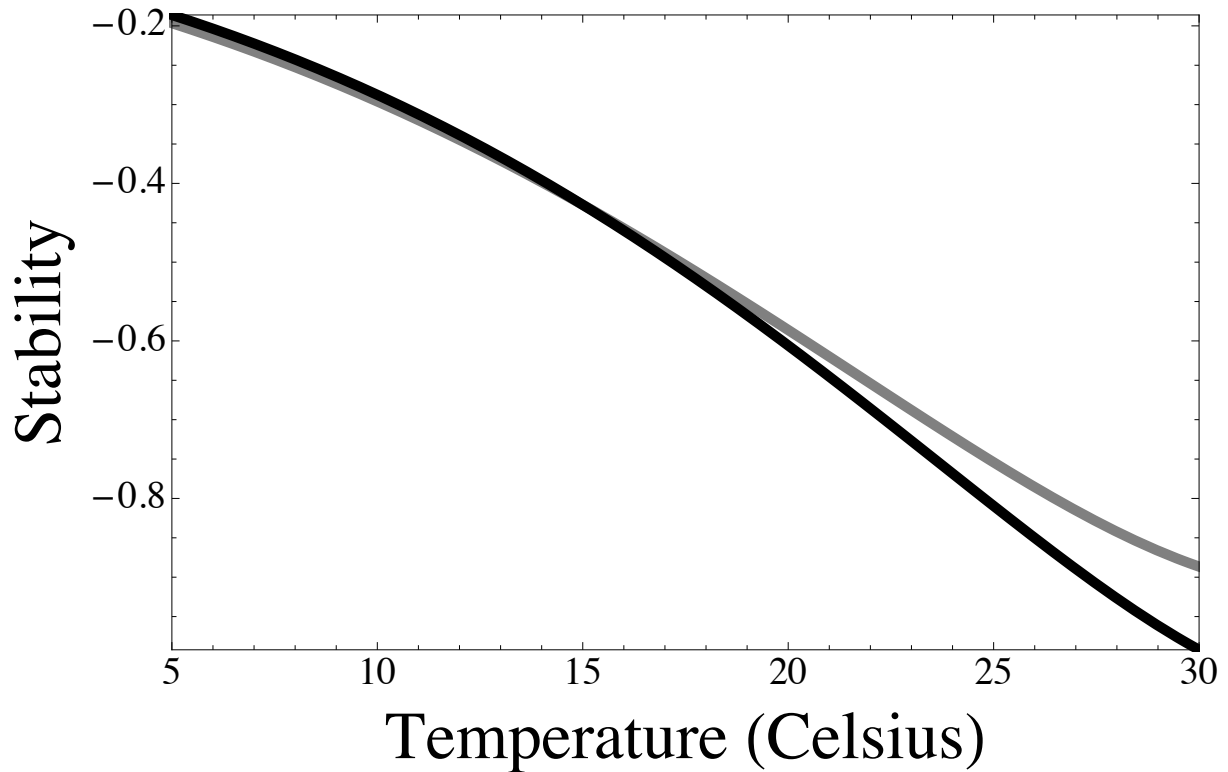
Note that when things are already not stable (saddle node created by lowering K , meaning there aren't enough resources to support consumer), then the TSR causes further instability.

```

k15Saddle =
  Solve[10 == K[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, K0] //
  Flatten;

Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15Saddle /. r15 /.
    m15 /. EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /.
    Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
    μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]},
  Axes →
    False,
  PlotRange →
    All], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15Saddle /. r15 /.
    m15 /. EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /.
    Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
    μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
  PlotRange →
    All],
  PlotRange → All,
  Frame → True,
  FrameLabel →
    {Style["Temperature (Celsius)", LabelSize], Style["Stability", LabelSize], , },
  FrameStyle → Directive[FontSize → TickSize],
  ImagePadding → Pad,
  ImageSize → FigureSize,
  PlotRangePadding → None,
  Epilog → {
    Text[Style["", LabelSize, Bold], Scaled@letpos]
  }
]

```



Parameter perturbation [$r(T_{\text{ref}}) = 0.2$, $K(T_{\text{ref}}) = 1000$]

```

r15b =
  Solve[0.2 == r[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] -> 273.15 + 15, r0] //
  Flatten

{r0 -> 0.2 e $\frac{0.00347041 \text{ EB}}{k}$  M15[R]-1. ρ}

k15b = Solve[1000 == K[T] /. GilbertTable1 /. DeLongTable1 /. TSR /.
  T[i_] -> 273.15 + 15, K0] // Flatten

{K0 -> 1000. e $-\frac{0.00347041 \text{ EB}}{k} + \frac{0.00347041 \text{ ES}}{k}$  M15[R]-1. κ}

Legended[
  Show[
    (*Plot[
      BCR[T] /. K[T] -> K0 Exp $\left[\frac{\text{EB}}{k \text{ T[R]}} - \frac{\text{ES}}{k \text{ T[S]}}\right]$  /. T[i_] -> T + 273.15 /. K15 /. k -> 8.62 * 10-5 /. a[T] -> 0.1 /.
      e[T] -> 0.15 /. m[T] -> 0.6 /. r[T] -> 2 /. EB -> 0.32 /. ES -> 0.9,
      {T, 5, 30}, PlotStyle -> {Gray, Thickness[0.01]}], *)
    Plot[
      BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15b /. r15b /. m15 /.
      EB -> 0.32 /. ES -> 0.9 /. k -> 8.62 * 10-5 /. Em -> 0.65 /. Ev[i_] -> 0.46 /.

```

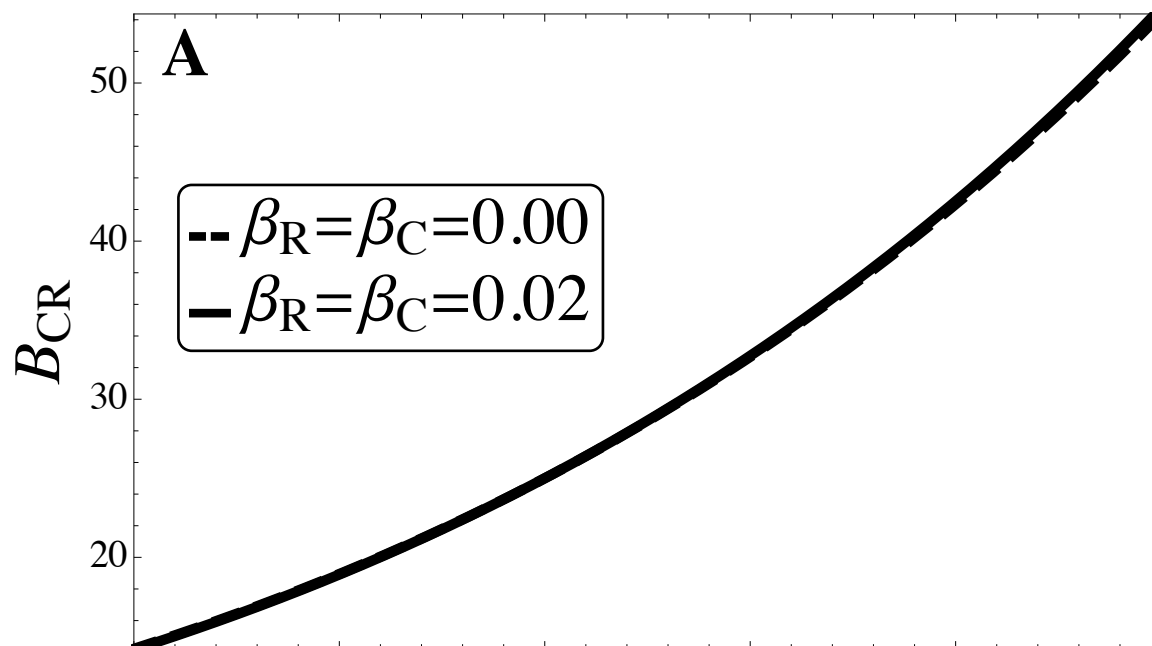
```

v0[i_] → 1 /.  $\kappa \rightarrow -0.81$  /.  $\alpha \rightarrow 1$  /.  $\epsilon \rightarrow -0.5$  /.  $\mu \rightarrow -0.29$  /.
 $\rho \rightarrow -0.81$  /. TSR /.  $\beta[i_] \rightarrow 0$  /.  $T[i_] \rightarrow T + 273.15$ , {T, 5, 30},
PlotStyle → {Thickness[0.01], Black, Dashing[Large] }],
(*Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /.
ES → 0.32 /.  $k \rightarrow 8.62 \cdot 10^{-5}$  /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.  $\kappa \rightarrow -0.81$  /.
 $\alpha \rightarrow 1$  /.  $\epsilon \rightarrow -0.5$  /.  $\mu \rightarrow -0.29$  /.  $\rho \rightarrow -0.81$  /. TSR /.  $\beta[i_] \rightarrow 0$  /.  $T[i_] \rightarrow T + 273.15$ ,
{T, 5, 30}, PlotStyle → {Thickness[0.01], Black, Dashing[Large] }],
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /.
ES → 0.9 /.  $k \rightarrow 8.62 \cdot 10^{-5}$  /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.
 $\kappa \rightarrow -0.81$  /.  $\alpha \rightarrow 1$  /.  $\epsilon \rightarrow -0.5$  /.  $\mu \rightarrow -0.29$  /.  $\rho \rightarrow -0.81$  /. TSR /.  $\beta[i_] \rightarrow 0$  /.
T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick, Gray} ], *)
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15b /. r15b /. m15 /.
EB → 0.32 /. ES → 0.9 /.  $k \rightarrow 8.62 \cdot 10^{-5}$  /. Em → 0.65 /. Ev[i_] → 0.46 /.
v0[i_] → 1 /.  $\kappa \rightarrow -0.81$  /.  $\alpha \rightarrow 1$  /.  $\epsilon \rightarrow -0.5$  /.  $\mu \rightarrow -0.29$  /.
 $\rho \rightarrow -0.81$  /. TSR /.  $\beta[i_] \rightarrow 0.02$  /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Thickness[0.01], Black} ],
(*Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /.
ES → 0.32 /.  $k \rightarrow 8.62 \cdot 10^{-5}$  /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.  $\kappa \rightarrow -0.81$  /.
 $\alpha \rightarrow 1$  /.  $\epsilon \rightarrow -0.5$  /.  $\mu \rightarrow -0.29$  /.  $\rho \rightarrow -0.81$  /. TSR /.  $\beta[i_] \rightarrow 0.02$  /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Thickness[0.01], Red, Dashing[Large] }], Plot[
BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /. ES → 0.9 /.
 $k \rightarrow 8.62 \cdot 10^{-5}$  /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.  $\kappa \rightarrow -0.81$  /.  $\alpha \rightarrow 1$  /.
 $\epsilon \rightarrow -0.5$  /.  $\mu \rightarrow -0.29$  /.  $\rho \rightarrow -0.81$  /. TSR /.  $\beta[i_] \rightarrow 0.02$  /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Thickness[0.01], Pink} ], *)
Frame → True,
FrameLabel → {Style[(*"Temperature (Celcius)"*)"", LabelSize],
Style["BCR", LabelSize], },
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle → {{Black, Black}, {Directive[FontColor → White], Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
(*PlotRangeClipping → False, *)
Epilog → {
Text[Style["A", LabelSize, Bold], Scaled@letpos],
(*Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree] *)
}
],
Placed[
LineLegend[{
Directive[Black, Dashing[Medium], Thickness[0.25]],
Directive[Black, Thickness[0.25]]
},
{
Style[" $\beta_R = \beta_C = 0.00$ ", LabelSize],
Style[" $\beta_R = \beta_C = 0.02$ ", LabelSize]
},
LegendFunction → "Frame",
LegendLayout → "Column"
],
{0.25, 0.6}
]

```

]

```
(*Export[imagedir<>"BCRAllTempMassDep.pdf",%];*)
```

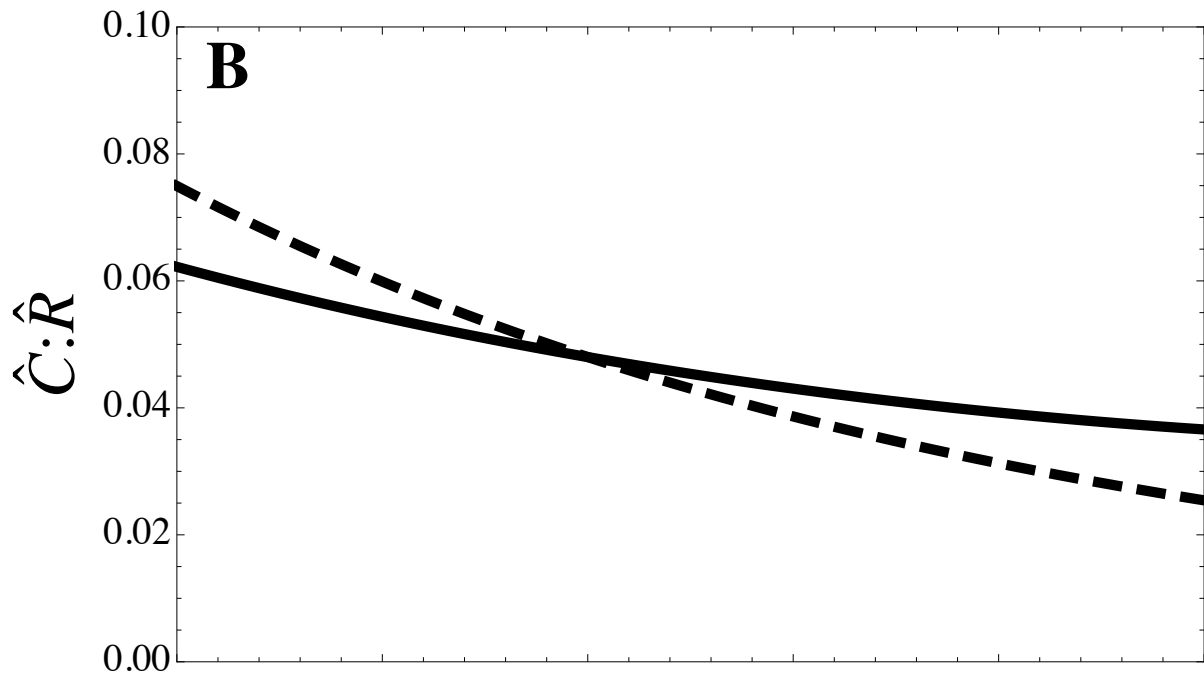


```
Show[
(*Plot[
  CR[T]/.K[T]→K0 Exp[ $\frac{EB}{k_{T[R]}} - \frac{ES}{k_{T[S]}}$ ]/.T[i_]→T+273.15/.K15/.k→8.62*10-5/.a[T]→0.1/.
    e[T]→0.15/.m[T]→0.6/.r[T]→2/.EB→0.32/.ES→0.9,
  {T,5,30},PlotStyle→{Gray,Thickness[0.01]}],*)
Plot[CR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15b/.r15b/.m15/.
  EB→0.32/.ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.
  v0[i_]→1/.κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.
  ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→T+273.15,{T,5,30},
  PlotStyle→{Black,Dashing[Large],Thickness[0.01]}],
  Axes→
  False],
Plot[CR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15b/.r15b/.m15/.
  EB→0.32/.ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.
  v0[i_]→1/.κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.
  ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
  {T,5,30},PlotStyle→{Black,Thickness[0.01]}],
  Axes→
  False],
(*Plot[CR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.
  EB→0.32/.ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.
  κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.04/.
  T[i_]→T+273.15,{T,5,30},PlotStyle→{Blue,Thickness[0.01]}],Axes→
  False],*)
PlotRange→{0,0.1},
Frame→True,
FrameLabel→
  {Style[(*"Temperature (Celcius)"*)"",LabelSize],Style["C:R",LabelSize],,},
FrameStyle→Directive[FontSize→TickSize],
FrameTicksStyle→{{Black,Black},{Directive[FontColor→White],Black}},
ImagePadding→Pad,
ImageSize→FigureSize,
PlotRangePadding→None,
Epilog→{
  Text[Style["B",LabelSize,Bold],Scaled@letpos],
  Rotate[Text[Style["BCR",LabelSize],Scaled@ylabpos],90 Degree]
}
]
```

```
(*Export[imagedir<"CtoRAllTempMassDep.pdf",%];*)
```

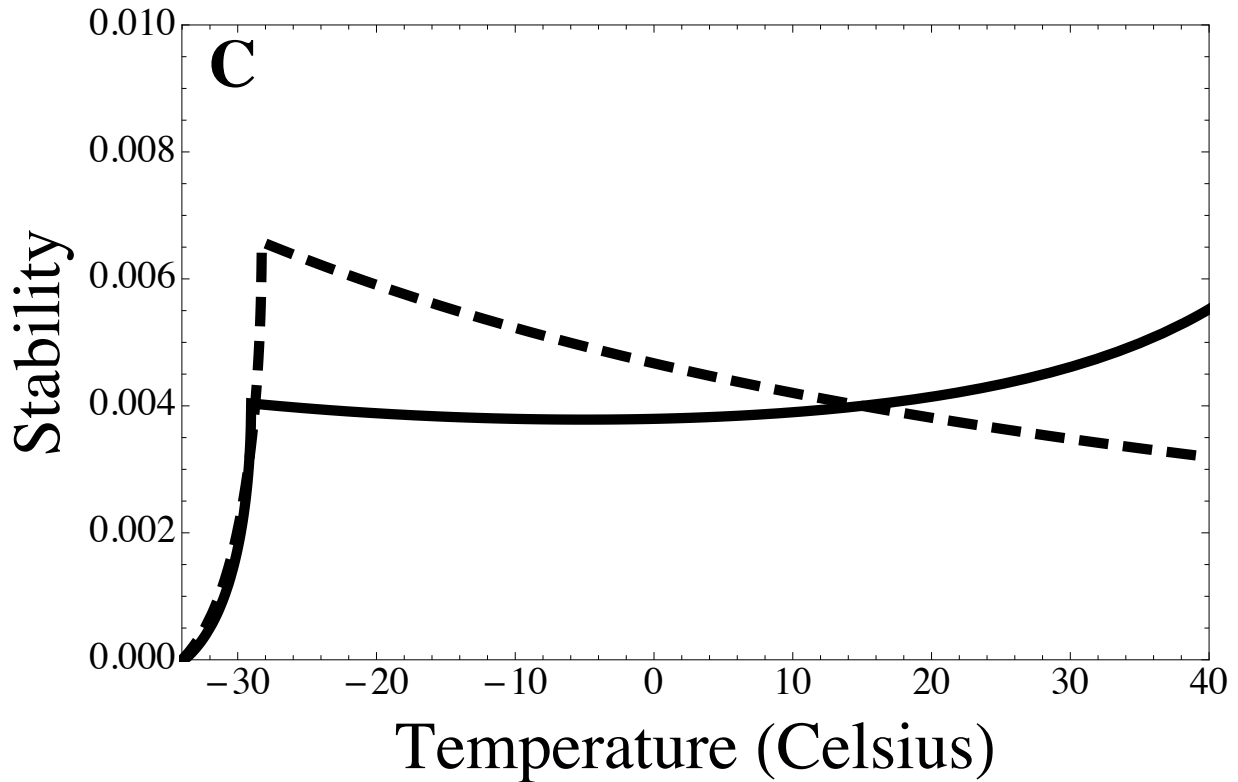
Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>



```
Show[(*Plot[
  -Max[Re[lambda/.K[T]→K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ]/.T[i_]→T+273.15/.K15/.k→8.62*10-5/.
    a[T]→0.1/.e[T]→0.15/.m[T]→0.6/.r[T]→2/.EB→0.32/.ES→0.9]],
  {T,5,30},PlotStyle→{Gray,Thickness[0.01]}],*)Plot[
  -Max[Re[lambda/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15b/.r15b/.
    m15/.EB→0.32/.ES→0.9/.k→8.62*10-5/.Em→0.65/.
    Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.ε→-0.5/.
    μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→T+273.15]],
  {T,-40,40},PlotStyle→{Black,Dashing[Large],Thickness[0.01]},
  Axes→
    False,PlotRange→
      {0,All}],
  Plot[-Max[Re[lambda/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15b/.r15b/.
    m15/.EB→0.32/.ES→0.9/.k→8.62*10-5/.Em→0.65/.
    Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.ε→-0.5/.
    μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15]],
  {T,-40,40},PlotStyle→{Black,Thickness[0.01]},
  PlotRange→
    {0,All}],
  (*Plot[-Max[Re[
    lambda/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.
    ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.
    α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.04/.T[i_]→T+273.15]],
  {T,5,30},PlotStyle→{Blue,Thickness[0.01]}],PlotRange→
    {0,
      All}],*)
  PlotRange→{0,0.01},
  Frame→True,
  FrameLabel→
    {Style["Temperature (Celsius)",LabelSize],Style["Stability",LabelSize],,},
  FrameStyle→Directive[FontSize→TickSize],
  ImagePadding→Pad,
  ImageSize→FigureSize,
  PlotRangePadding→None,
  Epilog→{
    Text[Style["C",LabelSize,Bold],Scaled@letpos]
  }
]
```

(*Export[imagedir<>"StabilityAllTempMassDep.pdf",%];*)



Varying strengths of the temperature-size rule

BCR:

```
Show[
  Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
    0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
    v0[i_] → 1 /. x → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Thickness[0.01], Black}],
  Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
    v0[i_] → 1 /. x → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15, {T, 5, 30},
    PlotStyle → {Thickness[0.01], Black, Dashing[Large]}],
  Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
    v0[i_] → 1 /. x → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Thick, Gray}],
  Plot[Evaluate@Table[
    BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /.
```

```

      Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
      μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → β /. T[i_] → T + 273.15,
      {β, 0.01, 0.1, 0.01}], {T, 5, 30}, PlotStyle → {Thick}],
Plot[Evaluate@Table[
  BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
    0.9 /. ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
    v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /.
    TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15, {β, 0.01, 0.1, 0.01}],
  {T, 5, 30}, PlotStyle → {Dashing[Large]}],
Plot[
  Evaluate@
  Table[
    BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
      {β, 0.01, 0.1, 0.01}], {T, 5, 30}, PlotStyle → {Dotted}],
  Frame → True,
  FrameLabel →
    {Style[("Temperature (Celcius)"), LabelSize], Style["BCR", LabelSize], },
  FrameStyle → Directive[FontSize → TickSize],
  ImagePadding → Pad,
  ImageSize → FigureSize,
  PlotRangePadding → None,
  (*PlotRangeClipping→False,*)
  Epilog → {
    Text[Style["A", LabelSize, Bold], Scaled@letpos],
    (*Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree]*)
  }
]

```

Plot::exclul : $\left\{ \operatorname{Im}\left[e^{-\frac{10672.9}{273.15+T}}\right] - 0, \operatorname{Im}[(1 - 0.01(-15. + T)) M15[C]] - 0, \operatorname{Im}[(1 - 0.01(-15. + T)) M15[R]] - 0 \right\}$

must be a list of equalities or real-valued functions. >>

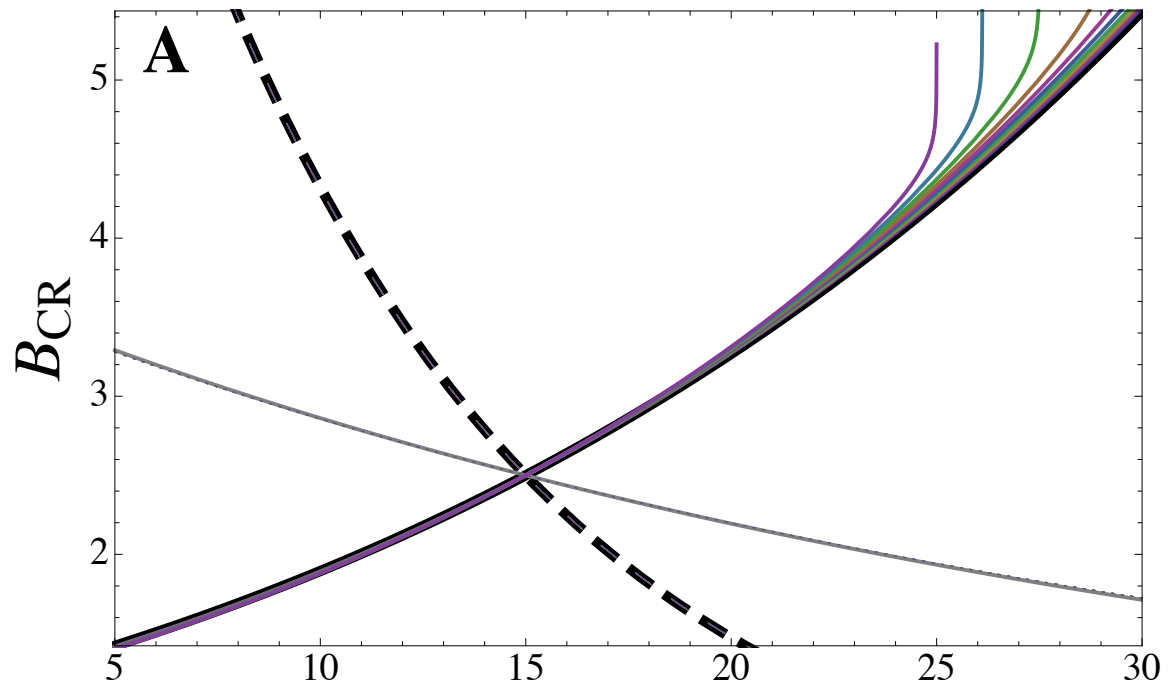
Plot::exclul : $\left\{ \operatorname{Im}\left[e^{-\frac{10672.9}{273.15+T}}\right] - 0, \operatorname{Im}[(1 - 0.02(-15. + T)) M15[C]] - 0, \operatorname{Im}[(1 - 0.02(-15. + T)) M15[R]] - 0 \right\}$

must be a list of equalities or real-valued functions. >>

Plot::exclul : $\left\{ \operatorname{Im}\left[e^{-\frac{10672.9}{273.15+T}}\right] - 0, \operatorname{Im}[(1 - 0.03(-15. + T)) M15[C]] - 0, \operatorname{Im}[(1 - 0.03(-15. + T)) M15[R]] - 0 \right\}$

must be a list of equalities or real-valued functions. >>

General::stop : Further output of Plot::exclul will be suppressed during this calculation. >>



Biomass ratios:

```
Show[
Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
Axes →
False], Plot[Evaluate@
Table[
CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
      0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /.
      TSR /. β[i_] → β /. T[i_] → T + 273.15, {β, 0.01, 0.1, 0.01}],
{T, 5, 30}, PlotStyle → Thick, Axes → False],
PlotRange → {0.2, 0.5},
Frame → True,
FrameLabel →
{Style[(*"Temperature (Celcius)"*)"", LabelSize], Style["C:R", LabelSize], , },
FrameStyle → Directive[FontSize → TickSize],
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
Epilog → {
Text[Style["B", LabelSize, Bold], Scaled@letpos],
Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree]
}
]
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Plot::exclul :

$$\left\{ \operatorname{Im}\left[e^{-\frac{10672.9}{273.15+T}}\right] - 0, \operatorname{Im}[(1 - 0.01(-15. + T)) M15[C]] - 0, \operatorname{Im}[(1 - 0.01(-15. + T)) M15[C]] - 0, \operatorname{Im}\left[e^{-\frac{10672.9}{273.15+T}}\right] - 0, \operatorname{Im}[(1 - 0.01(-15. + T)) M15[C]] - 0, \operatorname{Im}[(1 - 0.01(-15. + T)) M15[R]] - 0 \right\}$$

must be a list of equalities or real-valued functions. >>

Plot::exclul :

$$\left\{ \operatorname{Im}\left[e^{-\frac{10672.9}{273.15+T}}\right] - 0, \operatorname{Im}[(1 - 0.02(-15. + T)) M15[C]] - 0, \operatorname{Im}[(1 - 0.02(-15. + T)) M15[C]] - 0, \operatorname{Im}\left[e^{-\frac{10672.9}{273.15+T}}\right] - 0, \operatorname{Im}[(1 - 0.02(-15. + T)) M15[C]] - 0, \operatorname{Im}[(1 - 0.02(-15. + T)) M15[R]] - 0 \right\}$$

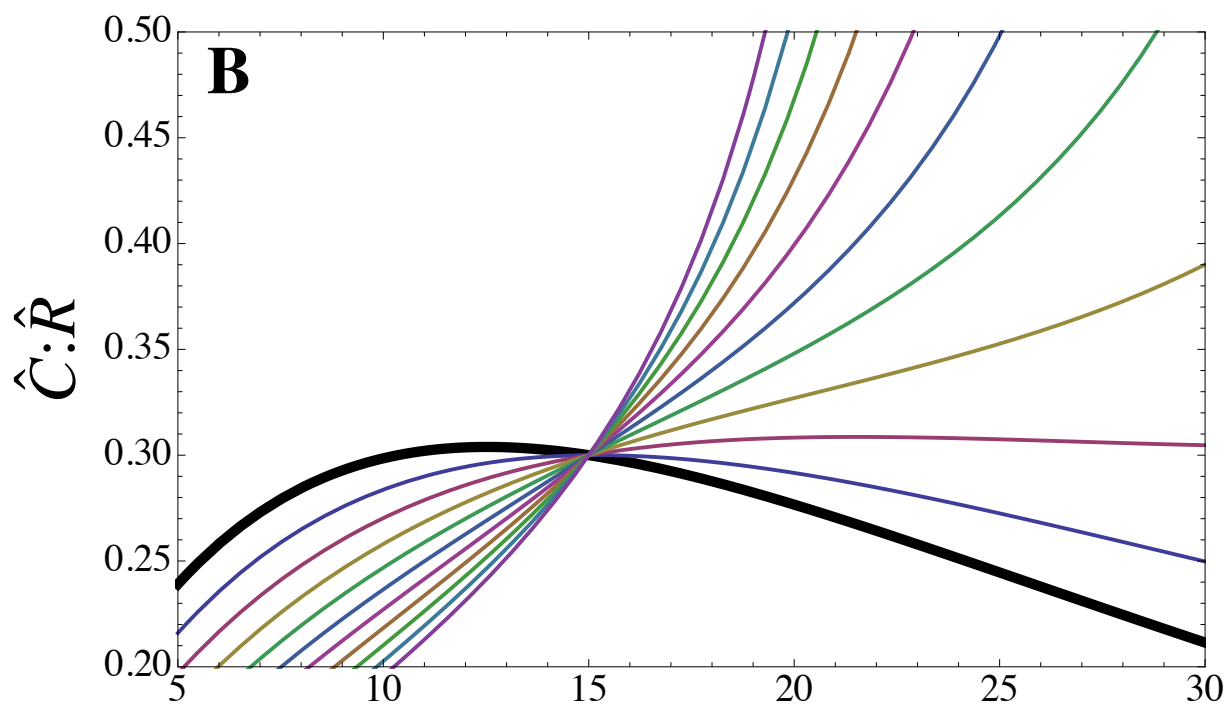
must be a list of equalities or real-valued functions. >>

Plot::exclul :

$$\left\{ \operatorname{Im}\left[e^{-\frac{10672.9}{273.15+T}}\right] - 0, \operatorname{Im}[(1 - 0.03(-15. + T)) M15[C]] - 0, \operatorname{Im}[(1 - 0.03(-15. + T)) M15[C]] - 0, \operatorname{Im}\left[e^{-\frac{10672.9}{273.15+T}}\right] - 0, \operatorname{Im}[(1 - 0.03(-15. + T)) M15[C]] - 0, \operatorname{Im}[(1 - 0.03(-15. + T)) M15[R]] - 0 \right\}$$

must be a list of equalities or real-valued functions. >>

General::stop : Further output of Plot::exclul will be suppressed during this calculation. >>



Stability:

```
Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /.
    Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
    μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
  Axes →
  False,
  PlotRange → {0, All}], Plot[Evaluate@
  Table[-Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /.
    m15 /. EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /.
    Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
    μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → β /. T[i_] → T + 273.15]],
  {β, 0.01, 0.1, 0.01}], {T, 5, 30}, PlotStyle → {Thick}, PlotRange → {0,
  All}],
  PlotRange → {0, 0.75},
  Frame → True,
  FrameLabel →
  {Style["Temperature (Celsius)", LabelSize], Style["Stability", LabelSize], , },
  FrameStyle → Directive[FontSize → TickSize],
  ImagePadding → Pad,
  ImageSize → FigureSize,
  PlotRangePadding → None,
  Epilog → {
    Text[Style["C", LabelSize, Bold], Scaled@letpos]
  }
]
```

Plot::exclul :

$$\left\{ \operatorname{Im}\left[e^{-\frac{10672.9}{273.15+T}}\right] - 0, \operatorname{Im}[(1 - 0.01(-15. + T)) M15[C]] - 0, \ll 25 \gg, \left(3.01934 \times 10^{-9} \operatorname{Re}\left[e^{\operatorname{Plus}[\ll 2 \gg]} M15[\ll 1 \gg]^{0.5} \operatorname{Times}[\ll 2 \gg]^{0.8} \right. \right. \right. \\ \left. \left. \left. - 3.01934 \times 10^{-9} \operatorname{Re}\left[e^{\operatorname{Plus}[\ll 2 \gg]} M15[\ll 1 \gg]^{0.5} \operatorname{Times}[\ll 2 \gg]^{0.81} (\operatorname{Times}[\ll 6 \gg] + \operatorname{Power}[\ll 2 \gg])\right]\right) / \left(\sqrt{\operatorname{Power}[\ll 2 \gg]} \operatorname{Times}[\ll 2 \gg]^{0.5} M15[\ll 1 \gg]^{0.81} \right) \right) - 0 \right\}$$

must be a list of equalities or real-valued functions. >>

Plot::exclul :

$$\left\{ \operatorname{Im}\left[e^{-\frac{10672.9}{273.15+T}}\right] - 0, \operatorname{Im}[(1 - 0.02(-15. + T)) M15[C]] - 0, \ll 25 \gg, \left(3.01934 \times 10^{-9} \operatorname{Re}\left[e^{\operatorname{Plus}[\ll 2 \gg]} M15[\ll 1 \gg]^{0.5} \operatorname{Times}[\ll 2 \gg]^{0.8} \right. \right. \right. \\ \left. \left. \left. - 3.01934 \times 10^{-9} \operatorname{Re}\left[e^{\operatorname{Plus}[\ll 2 \gg]} M15[\ll 1 \gg]^{0.5} \operatorname{Times}[\ll 2 \gg]^{0.81} (\operatorname{Times}[\ll 6 \gg] + \operatorname{Power}[\ll 2 \gg])\right]\right) / \left(\sqrt{\operatorname{Power}[\ll 2 \gg]} \operatorname{Times}[\ll 2 \gg]^{0.5} M15[\ll 1 \gg]^{0.81} \right) \right) - 0 \right\}$$

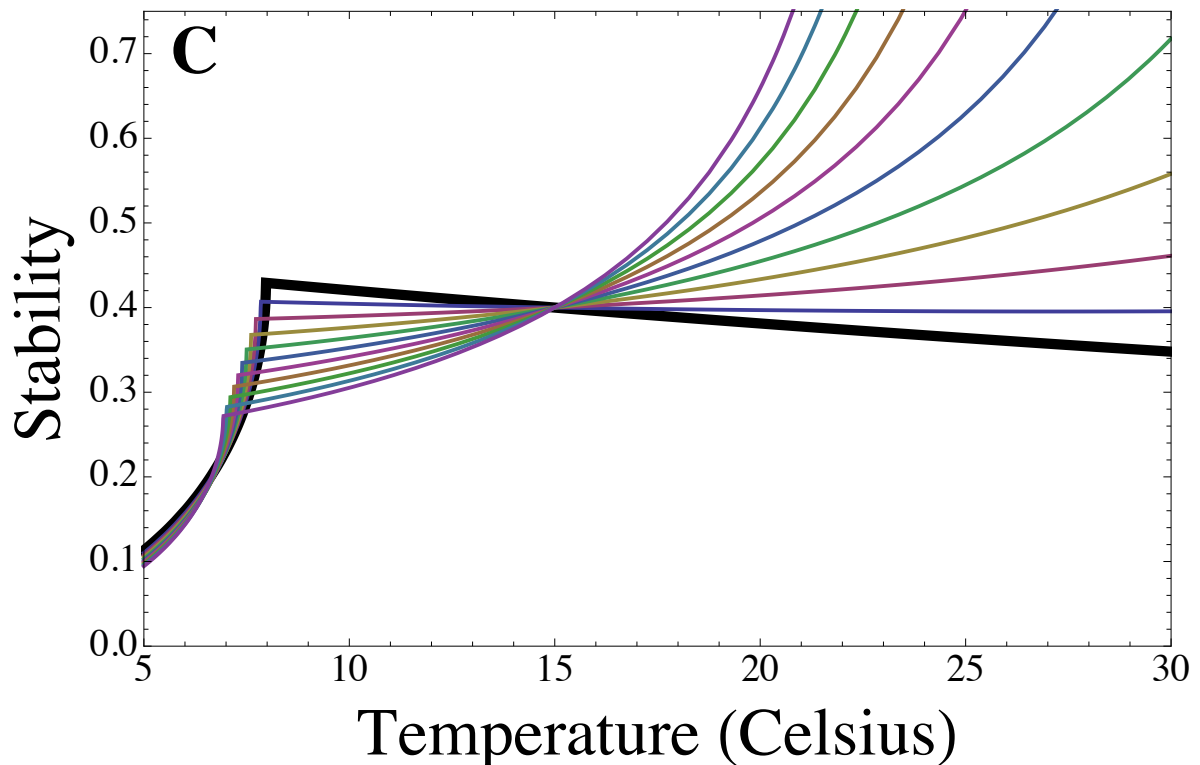
must be a list of equalities or real-valued functions. >>

Plot::exclul :

$$\left\{ \operatorname{Im}\left[e^{-\frac{10672.9}{273.15+T}}\right] - 0, \operatorname{Im}[(1 - 0.03(-15. + T)) M15[C]] - 0, \ll 25 \gg, \left(3.01934 \times 10^{-9} \operatorname{Re}\left[e^{\operatorname{Plus}[\ll 2 \gg]} M15[\ll 1 \gg]^{0.5} \operatorname{Times}[\ll 2 \gg]^{0.8} \right. \right. \right. \\ \left. \left. \left. - 3.01934 \times 10^{-9} \operatorname{Re}\left[e^{\operatorname{Plus}[\ll 2 \gg]} M15[\ll 1 \gg]^{0.5} \operatorname{Times}[\ll 2 \gg]^{0.81} (\operatorname{Times}[\ll 6 \gg] + \operatorname{Power}[\ll 2 \gg])\right]\right) / \left(\sqrt{\operatorname{Power}[\ll 2 \gg]} \operatorname{Times}[\ll 2 \gg]^{0.5} M15[\ll 1 \gg]^{0.81} \right) \right) - 0 \right\}$$

must be a list of equalities or real-valued functions. >>

General::stop : Further output of Plot::exclul will be suppressed during this calculation. >>



So TSRs greater than 1% cause observable (here) differences from predictions not incorporating the TSR.

TSRs greater than roughly 2% cause qualitative differences, with greater TSRs leading to even larger qualitative differences.

Exploring asymmetric temperature-size responses (resource < consumer)

In the previous section we've supposed that the consumer and resource have the same TSR response. Lets relax that. If the consumer is larger than the resource then, according to Forster et al 2012 Figure 2, it might also have a larger TSR response, say maybe a 4% decrease with degree, rather than the 2% for unicells (we are keeping the assumption that the loss is linear).

With a 4% TSR in consumer and a 2% TSR in resource, our predictions become:

```
Show[
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
      0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Thickness[0.01], Gray}],
(*Plot[
BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /. ES → 0.32 /.
```

```

      k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
      ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→T+273.15,
    {T,5,30},PlotStyle→{Thickness[0.01],Black,Dashing[Large]}}],
Plot[
  BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.9/.
      k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
      ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→T+273.15,
    {T,5,30},PlotStyle→{Thickness[0.01],Gray}},*)
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Thickness[0.005], Black}],
(*Plot[
  BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.32/.
      k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
      ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
    {T,5,30},PlotStyle→{Thickness[0.01],Red,Dashing[Large]}}],
Plot[
  BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.9/.
      k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
      ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
    {T,5,30},PlotStyle→{Thickness[0.01],Pink}},*)
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /.
      TSR /. β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → T + 273.15, {T, 5, 30},
    PlotStyle → {Thickness[0.01], Black, Dashing[Large]}}],
(*Plot[
  BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.32/.
      k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
      ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[R]→0.02/.β[C]→0.04/.
      T[i_]→T+273.15,{T,5,30},PlotStyle→{Thickness[0.01],Blue,
      Dashing[Large]}}],Plot[
  BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.9/.
      k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
      ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[R]→0.02/.β[C]→0.04/.
      T[i_]→T+273.15,{T,5,30},PlotStyle→{Thickness[0.01],
      Lighter[Blue]}}],*)
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.04 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Thickness[0.01], Black}],
Frame → True,
FrameLabel →
  {Style[(*"Temperature (Celcius)"*)"", LabelSize], Style["BCR", LabelSize], ,},
FrameStyle → Directive[FontSize → TickSize],
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,

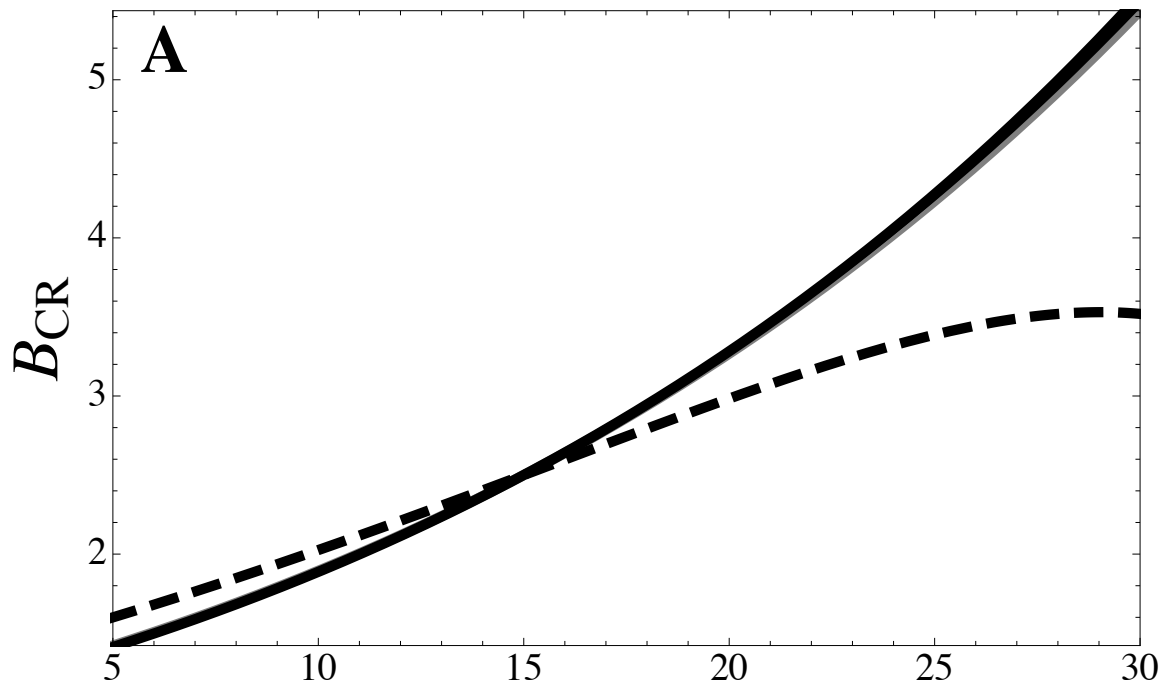
```

```

(*PlotRangeClipping->False,*)
Epilog->{
  Text[Style["A", LabelSize, Bold], Scaled@{1, 5}],
  (*Rotate[Text[Style["BCR", LabelSize], Scaled@{1, 5}], 90 Degree] *)
}
]

(*Export[imagedir<>"BCRAllTempMassDepAsymm.pdf", %]; *)

```



```
Show[
Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]},
Axes →
False],
Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Black, Thickness[0.005]},
Axes →
False],
Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /.
      TSR /. β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → T + 273.15, {T, 5, 30},
PlotStyle → {Black, Thickness[0.01], Dashing[Large]},
Axes →
False],
Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.04 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
Axes →
False],
PlotRange → {0.2, 0.5},
Frame → True,
FrameLabel →
{Style[(*"Temperature (Celcius)"*)"", LabelSize], Style["C:R", LabelSize], , },
FrameStyle → Directive[FontSize → TickSize],
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
Epilog → {
Text[Style["B", LabelSize, Bold], Scaled@letpos]
}
]
```

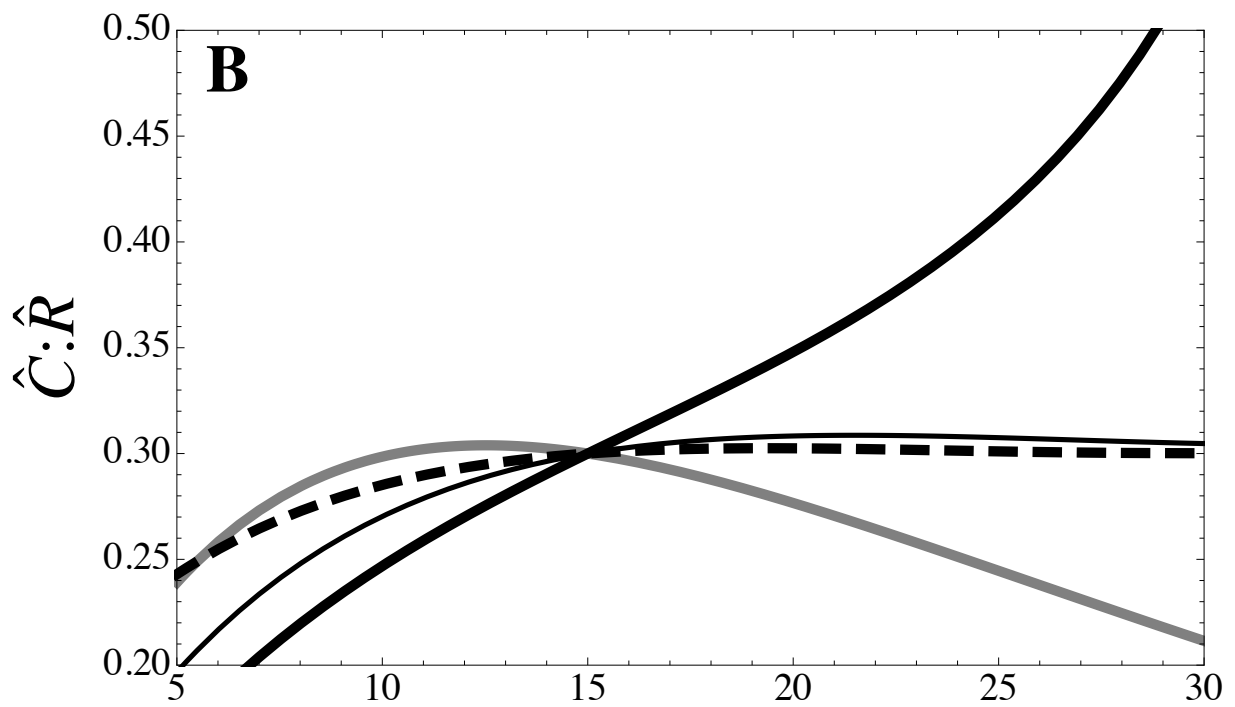
```
(*Export[imagedir<>"CtoRAllTempMassDepAsymm.pdf",%];*)
```

Solve::ratnz: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Solve::ratnz: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

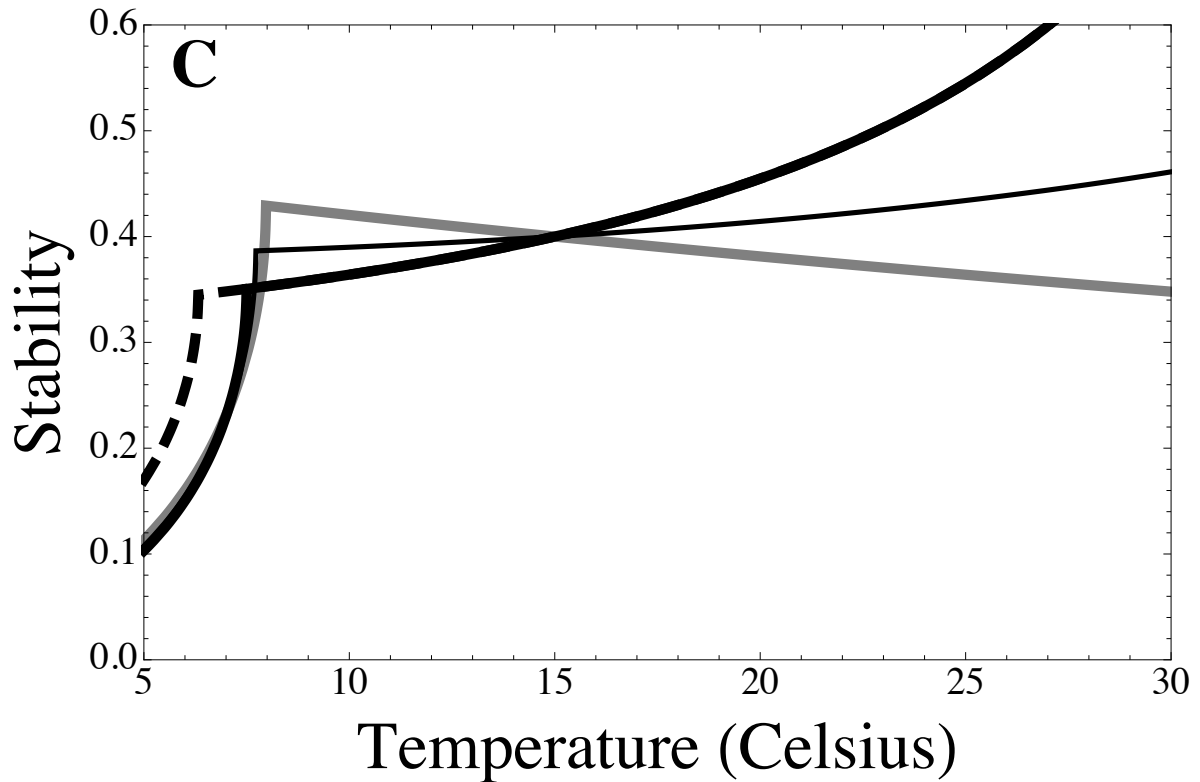
Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

General::stop : Further output of Solve::ratnz will be suppressed during this calculation. >>



```
Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /.
    Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
    μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]},
  Axes →
    False, PlotRange →
      {0, All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] →
    0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thickness[0.005]},
  PlotRange →
    {0, All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] →
    0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thickness[0.01], Dashing[Large]},
  PlotRange →
    {0, All}],
  Plot[
    -Max[
      Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
        0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
        v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
        ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.04 /. T[i_] → T + 273.15]],
      {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
      PlotRange →
        {0,
          All}],
    PlotRange → {0, 0.6},
    Frame → True,
    FrameLabel →
      {Style["Temperature (Celsius)", LabelSize], Style["Stability", LabelSize], },
    FrameStyle → Directive[FontSize → TickSize],
    ImagePadding → Pad,
    ImageSize → FigureSize,
    PlotRangePadding → None,
    Epilog → {
      Text[Style["C", LabelSize, Bold], Scaled@letpos]
    }
  ]
]

(*Export[imagedir<>"StabilityAllTempMassDepAsymm.pdf", %];*)
```



Exploring asymmetric temperature-size responses (resource > consumer)

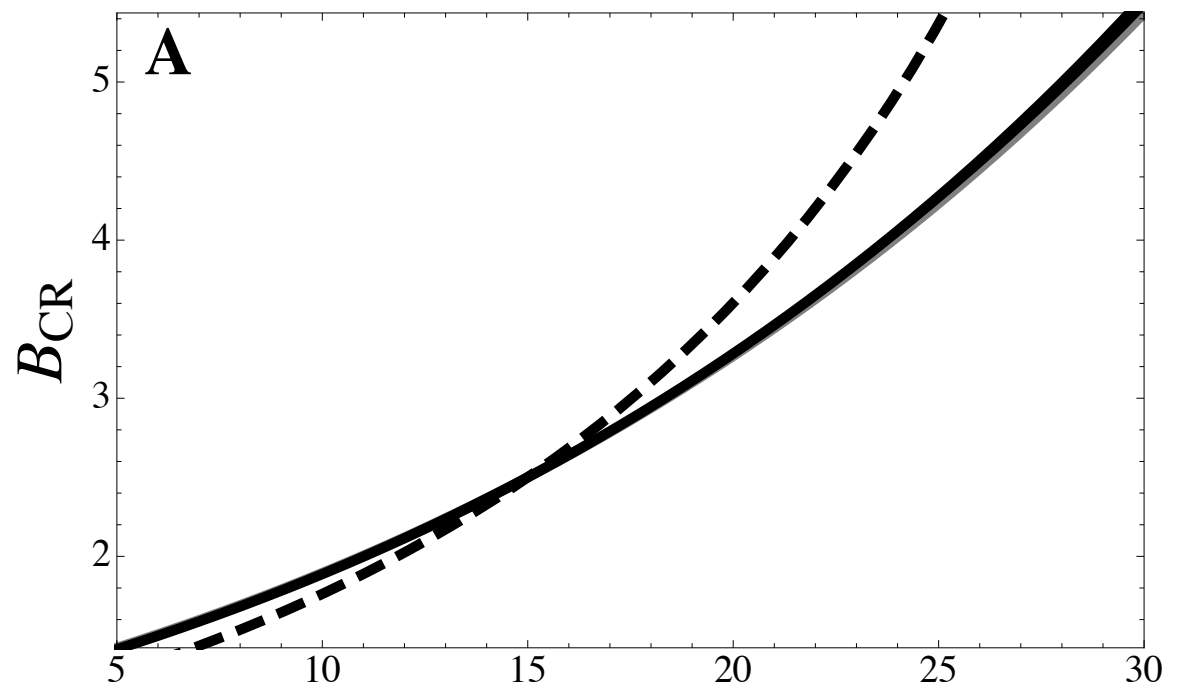
If the asymmetry goes the other way we might see different results. With a 4% TSR in resource and a 2% TSR in consumer, our predictions become:

```
Show[
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
      0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Thickness[0.01], Gray}],
(*Plot[
BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /. ES → 0.32 /.
      k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /.
      ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Thickness[0.01], Black, Dashing[Large]}],
Plot[
BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /. ES → 0.9 /.
      k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /.
      ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Thickness[0.01], Gray}], *)
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
```

```

EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Thickness[0.005], Black}],
(*Plot[
BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /. ES → 0.32 /.
k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /.
ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Thickness[0.01], Red, Dashing[Large]}],
Plot[
BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /. ES → 0.9 /.
k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /.
ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Thickness[0.01], Pink}], *)
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /.
TSR /. β[R] → 0.04 /. β[C] → 0.02 /. T[i_] → T + 273.15, {T, 5, 30},
PlotStyle → {Thickness[0.01], Black, Dashing[Large]}],
(*Plot[
BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /. ES → 0.32 /.
k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /.
ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.04 /.
T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thickness[0.01], Blue,
Dashing[Large]}], Plot[
BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /. ES → 0.9 /.
k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /.
ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.04 /.
T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thickness[0.01],
Lighter[Blue]}], *)
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.04 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Thickness[0.01], Black}],
Frame → True,
FrameLabel →
{Style[(*"Temperature (Celcius)"*)"", LabelSize], Style["BCR", LabelSize], },
FrameStyle → Directive[FontSize → TickSize],
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
(*PlotRangeClipping → False, *)
Epilog → {
Text[Style["A", LabelSize, Bold], Scaled@letpos],
(*Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree] *)
}
]
(*Export[imagedir<"BCRAllTempMassDepAsymm.pdf", %]; *)

```

```
Show[
Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]},
Axes →
False],
Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Black, Thickness[0.005]},
Axes →
False],
Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /.
      TSR /. β[R] → 0.04 /. β[C] → 0.02 /. T[i_] → T + 273.15, {T, 5, 30},
PlotStyle → {Black, Thickness[0.01], Dashing[Large]},
Axes →
False],
Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.04 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
Axes →
False],
PlotRange → {0.2, 0.5},
Frame → True,
FrameLabel →
{Style[(*"Temperature (Celcius)"*)"", LabelSize], Style["C:R", LabelSize], , },
FrameStyle → Directive[FontSize → TickSize],
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
Epilog → {
Text[Style["B", LabelSize, Bold], Scaled@letpos]
}
]
```

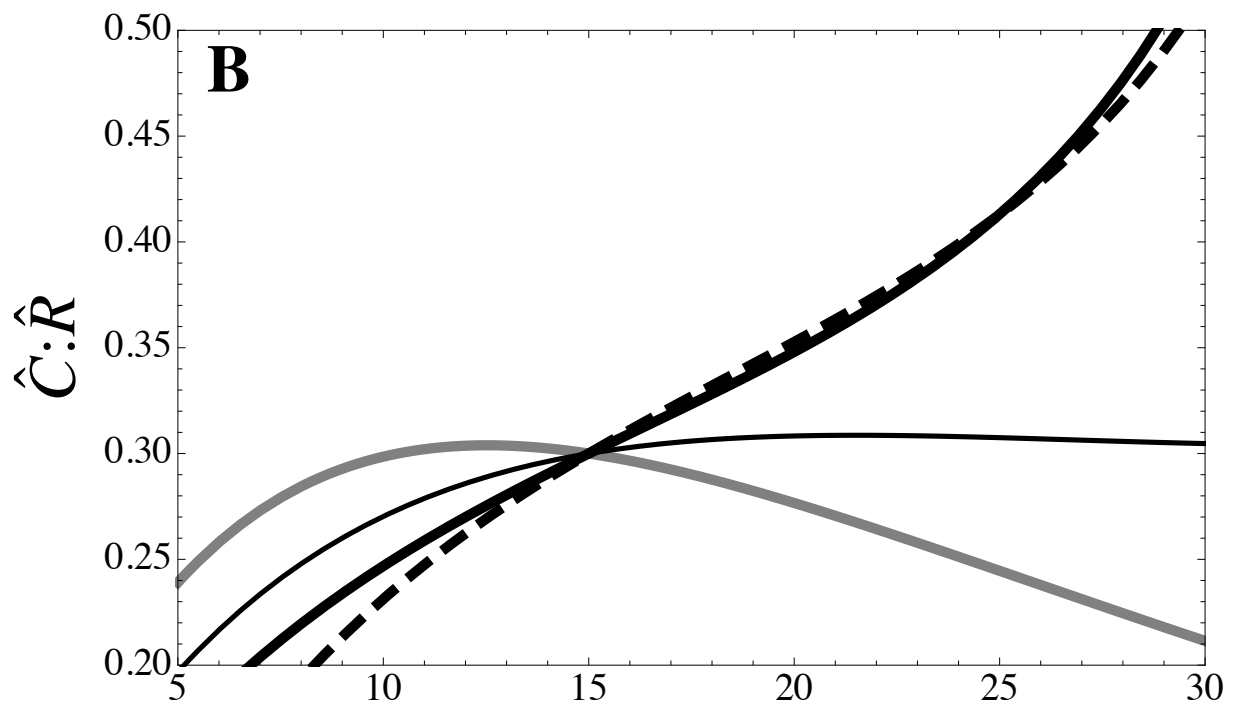
```
(*Export[imagedir<>"CtoRAllTempMassDepAsymm.pdf",%];*)
```

Solve::ratnz: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Solve::ratnz: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

General::stop : Further output of Solve::ratnz will be suppressed during this calculation. >>

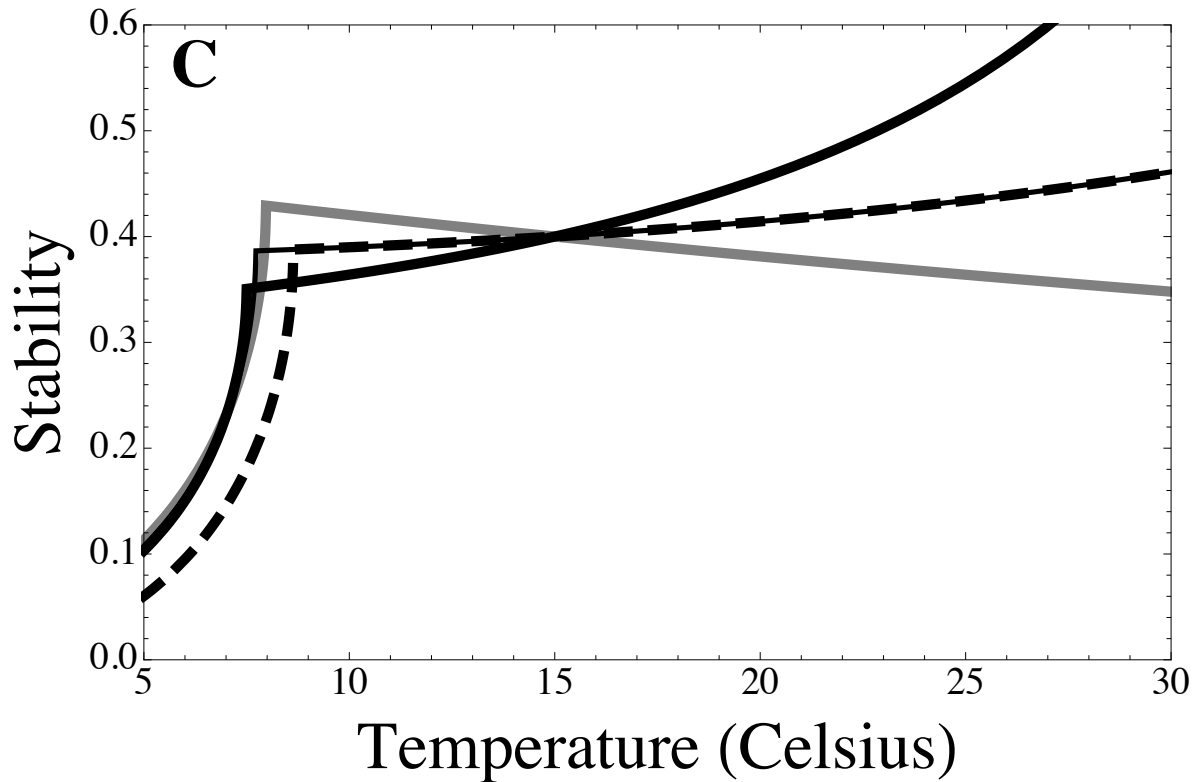


```

Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /.
    Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
    μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]},
  Axes →
    False, PlotRange →
      {0, All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] →
    0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thickness[0.005]},
  PlotRange →
    {0, All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] →
    0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.02 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thickness[0.01], Dashing[Large]},
  PlotRange →
    {0, All}],
  Plot[
    -Max[
      Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
        0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
        v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
        ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.04 /. T[i_] → T + 273.15]],
      {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
      PlotRange →
        {0,
          All}],
    PlotRange → {0, 0.6},
    Frame → True,
    FrameLabel →
      {Style["Temperature (Celsius)", LabelSize], Style["Stability", LabelSize], },
    FrameStyle → Directive[FontSize → TickSize],
    ImagePadding → Pad,
    ImageSize → FigureSize,
    PlotRangePadding → None,
    Epilog → {
      Text[Style["C", LabelSize, Bold], Scaled@letpos]
    }
  ]
]

(*Export[imagedir<>"StabilityAllTempMassDepAsymm.pdf",%];*)

```



Type-II functional response

BCR at a given T , like that defined by Gilbert et al (Eqn 5), but with type-II functional response

$$\text{BCR2}[T_]:= \frac{e[T] f[R, T] K[T]}{m[C, T]} /. f[R, T] \rightarrow \frac{a[T]}{1 + a[T] h[T] R} /. m[C, T] \rightarrow m[T]$$

Equilibrium biomasses at given temperature assuming a type II functional response and density-independent consumer mortality

$$\text{Eq2}[T_]:= \text{Solve}\left[\{0 == \text{dRdt}[R, C, T], 0 == \text{dCdt}[R, C, T]\} /. \right. \\ \left. f[R, T] \rightarrow \frac{a[T]}{1 + a[T] h[T] R} /. m[C, T] \rightarrow m[T], \{R, C\}\right]$$

Equilibrium consumer to resource biomass ratio at a given temperature (at the equilibrium where both populations persist)

$$\text{CR2}[T_]:= \frac{C}{R} /. \text{Eq2}[T][[3]]$$

The Jacobian evaluated at equilibrium (determines stability)

$$\begin{aligned} \text{Jac2} = & \left\{ \left\{ \text{D} \left[\text{dRdt}[\mathbf{R}, \mathbf{C}, \mathbf{T}] /. \mathbf{f}[\mathbf{R}, \mathbf{T}] \rightarrow \frac{\mathbf{a}[\mathbf{T}]}{1 + \mathbf{a}[\mathbf{T}] \mathbf{h}[\mathbf{T}] \mathbf{R}} /. \mathbf{m}[\mathbf{C}, \mathbf{T}] \rightarrow \mathbf{m}[\mathbf{T}], \mathbf{R} \right], \right. \\ & \left. \text{D} \left[\text{dRdt}[\mathbf{R}, \mathbf{C}, \mathbf{T}] /. \mathbf{f}[\mathbf{R}, \mathbf{T}] \rightarrow \frac{\mathbf{a}[\mathbf{T}]}{1 + \mathbf{a}[\mathbf{T}] \mathbf{h}[\mathbf{T}] \mathbf{R}} /. \mathbf{m}[\mathbf{C}, \mathbf{T}] \rightarrow \mathbf{m}[\mathbf{T}], \mathbf{C} \right], \right. \\ & \left\{ \text{D} \left[\text{dCdt}[\mathbf{R}, \mathbf{C}, \mathbf{T}] /. \mathbf{f}[\mathbf{R}, \mathbf{T}] \rightarrow \frac{\mathbf{a}[\mathbf{T}]}{1 + \mathbf{a}[\mathbf{T}] \mathbf{h}[\mathbf{T}] \mathbf{R}} /. \mathbf{m}[\mathbf{C}, \mathbf{T}] \rightarrow \mathbf{m}[\mathbf{T}], \mathbf{R} \right], \right. \\ & \left. \left. \text{D} \left[\text{dCdt}[\mathbf{R}, \mathbf{C}, \mathbf{T}] /. \mathbf{f}[\mathbf{R}, \mathbf{T}] \rightarrow \frac{\mathbf{a}[\mathbf{T}]}{1 + \mathbf{a}[\mathbf{T}] \mathbf{h}[\mathbf{T}] \mathbf{R}} /. \mathbf{m}[\mathbf{C}, \mathbf{T}] \rightarrow \mathbf{m}[\mathbf{T}], \mathbf{C} \right] \right\} \right\} /. \text{Eq2}[\mathbf{T}][[3]]; \end{aligned}$$

The eigenvalues of the Jacobian are

`lambda2 = Eigenvalues[Jac2];`

Now the dependencies. We can use the scalings of the other parameters from Gilbert and DeLong:

`GilbertDeLongT =`

$$\left\{ \mathbf{r}[\mathbf{T}] \rightarrow e^{-\frac{\text{EB}}{k \mathbf{T}[\mathbf{R}]}} \mathbf{r}[\mathbf{M}], \mathbf{K}[\mathbf{T}] \rightarrow e^{\frac{\text{EB}}{k \mathbf{T}[\mathbf{R}]} - \frac{\text{ES}}{k \mathbf{T}[\mathbf{S}]}} \mathbf{K}[\mathbf{M}], \mathbf{m}[\mathbf{T}] \rightarrow e^{-\frac{\text{Em}}{k \mathbf{T}[\mathbf{C}]}} \mathbf{m}[\mathbf{M}], \mathbf{e}[\mathbf{T}] \rightarrow \mathbf{e}[\mathbf{M}] \right\};$$

`GilbertDeLongM = {r[M] → r0 M[R]ρ, K[M] → K0 M[R]κ, e[M] → e0 M[C]ε, m[M] → m0 M[C]μ};`

The attack rate and handling time scalings are (in a 2D environment; Rall et al. 2012)

$$\text{RallT} = \left\{ \mathbf{h}[\mathbf{T}] \rightarrow e^{\frac{-\text{Eh}}{k \mathbf{T}[\mathbf{C}]}} \mathbf{h}[\mathbf{M}], \mathbf{a}[\mathbf{T}] \rightarrow e^{\frac{-\text{Ea}}{k \mathbf{T}[\mathbf{C}]}} \mathbf{a}[\mathbf{M}] \right\};$$

$$\text{RallM} = \left\{ \mathbf{h}[\mathbf{M}] \rightarrow \mathbf{h0} \mathbf{M}[\mathbf{C}]^{\mathbf{hC}} \mathbf{M}[\mathbf{R}]^{\mathbf{hR}}, \mathbf{a}[\mathbf{M}] \rightarrow \mathbf{a0} \mathbf{M}[\mathbf{C}]^{\mathbf{aC}} \mathbf{M}[\mathbf{R}]^{\mathbf{aR}} \right\};$$

To have the same population dynamics parameter values at 15C as in Gilbert et al. Figure 3, we need

```
ah15 = Solve[
  0.1 == Simplify[ $\frac{\mathbf{a}[\mathbf{T}]}{1 + \mathbf{a}[\mathbf{T}] \mathbf{h}[\mathbf{T}] \mathbf{R}}$  /. Eq2[T][[3]]] /. RallT /. RallM /. GilbertDeLongT /.
    GilbertDeLongM /. TSR /. T[i_] → 273.15 + 15, a0] // Flatten
  {a0 → (0.1 e $\frac{0.00347041 \text{ Ea}}{k}$  M15[C]-1. aC M15[R]-1. aR) /
    (1. - 1 / e01. × 1.hC-1. ε+μ e $-\frac{0.00347041 \text{ Eh}}{k} - \frac{0.00347041 \text{ Em}}{k}$  h0 m0 M15[C]hC-1. ε+μ M15[R]hR)}
```

`e15 = Solve[0.15 == e[T] /. GilbertTable1 /. DeLongTable1 /. TSR /.`
`T[i_] → 273.15 + 15, e0] // Flatten`

`{e0 → 0.15 M15[C]-1. ε}`

`k15 =`
`Solve[100 == K[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, K0] //`
`Flatten`

`{K0 → 100. e $-\frac{0.00347041 \text{ EB}}{k} + \frac{0.00347041 \text{ ES}}{k}$ M15[R]-1. κ}`

`r15 =`
`Solve[2 == r[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] → 273.15 + 15, r0] //`
`Flatten`

`{r0 → 2. e $\frac{0.00347041 \text{ EB}}{k}$ M15[R]-1. ρ}`

```

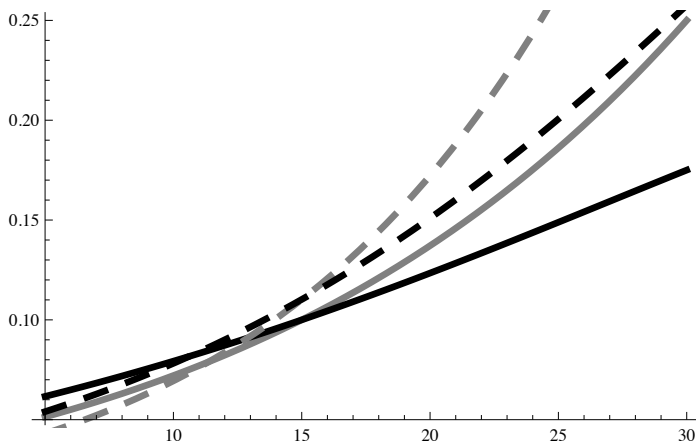
m15 =
  Solve[0.6 == m[T] /. GilbertTable1 /. DeLongTable1 /. TSR /. T[i_] -> 273.15 + 15, m0] //
  Flatten

$$\left\{ m0 \rightarrow 0.6 e^{\frac{0.00347041 \text{ Em}}{K}} M15 [C]^{-1. \mu} \right\}$$


```

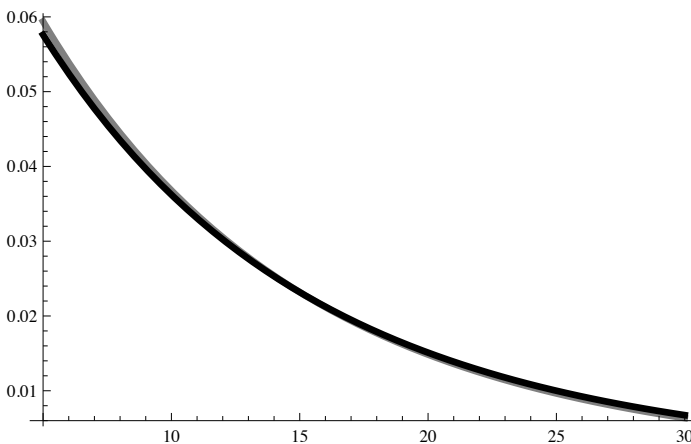
Changes in attack rate (solid) and capture rate (dashed) with temperature, with (black) and without (gray) the TSR

```
Show[Plot[
  a[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.32 /.
    ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.
    κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
    β[i_] → 0.0 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle →
    {Thickness[0.01], Gray}],
Plot[a[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
  EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
  v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
  ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
  {T, 5, 30}, PlotStyle → {Thickness[0.01], Black}],
Plot[
  a[T] /. Eq2[T][[3]] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /.
    ah15 /. e15 /. k15 /. r15 /. m15 /. T[i_] → T /.
    k → 8.62 * 10-5 /. EB → 0.32 /. ES → 0.9 /. Em → 0.65 /.
    κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /.
    β[i_] → 0.0 /. T → 273.15 + t /. aC → 1/4 + 2/3 /. aR → 1/3 /.
    hC → -2/3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /.
    h0 → 10-13 /. M15[i_] → 1, {t, 5, 30}, PlotStyle →
    {Thickness[
      0.01],
    Gray, Dashing[
      Large] }],
Plot[a[T] /. Eq2[T][[3]] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /.
  TSR /. ah15 /. e15 /. k15 /. r15 /. m15 /. T[i_] → T /.
  k → 8.62 * 10-5 /. EB → 0.32 /. ES → 0.9 /. Em → 0.65 /.
  κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /.
  β[i_] → 0.02 /. T → 273.15 + t /. aC → 1/4 + 2/3 /. aR → 1/3 /.
  hC → -2/3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /.
  h0 → 10-13 /. M15[i_] → 1, {t, 5, 30}, PlotStyle →
  {Thickness[
    0.01],
  Black, Dashing[
    Large] }]]
]
```



Changes in handling time with temperature, with and without the TSR

```
Show[
  Plot[
    h[T] /. Eq2[T][[3]] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /.
      ah15 /. e15 /. k15 /. r15 /. m15 /. T[i_] -> T /. k -> 8.62 *
      10-5 /. EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /.  $\kappa$  -> -0.81 /.
       $\alpha$  -> 1 /.  $\epsilon$  -> -0.5 /.  $\mu$  -> -0.29 /.  $\rho$  -> -0.81 /.  $\beta[i_]$  -> 0.0 /.
      T -> 273.15 + t /. aC -> 1/4 + 2/3 /. aR -> 1/3 /. hC -> -2/3 /.
      hR -> 0.5 /. Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 10-13 /. M15[i_] -> 1,
    {t, 5, 30}, PlotStyle -> {Thickness[0.01], Gray}],
  Plot[
    h[T] /. Eq2[T][[3]] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /.
      ah15 /. e15 /. k15 /. r15 /. m15 /. T[i_] -> T /.
      k -> 8.62 * 10-5 /. EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /.
       $\kappa$  -> -0.81 /.  $\alpha$  -> 1 /.  $\epsilon$  -> -0.5 /.  $\mu$  -> -0.29 /.  $\rho$  -> -0.81 /.
       $\beta[i_]$  -> 0.02 /. T -> 273.15 + t /. aC -> 1/4 + 2/3 /. aR -> 1/3 /.
      hC -> -2/3 /. hR -> 0.5 /. Ea -> 0.65 /. Eh -> -0.65 /.
      h0 -> 10-13 /. M15[i_] -> 1, {t, 5, 30}, PlotStyle ->
    {Thickness[
      0.01], Black}]
]
```



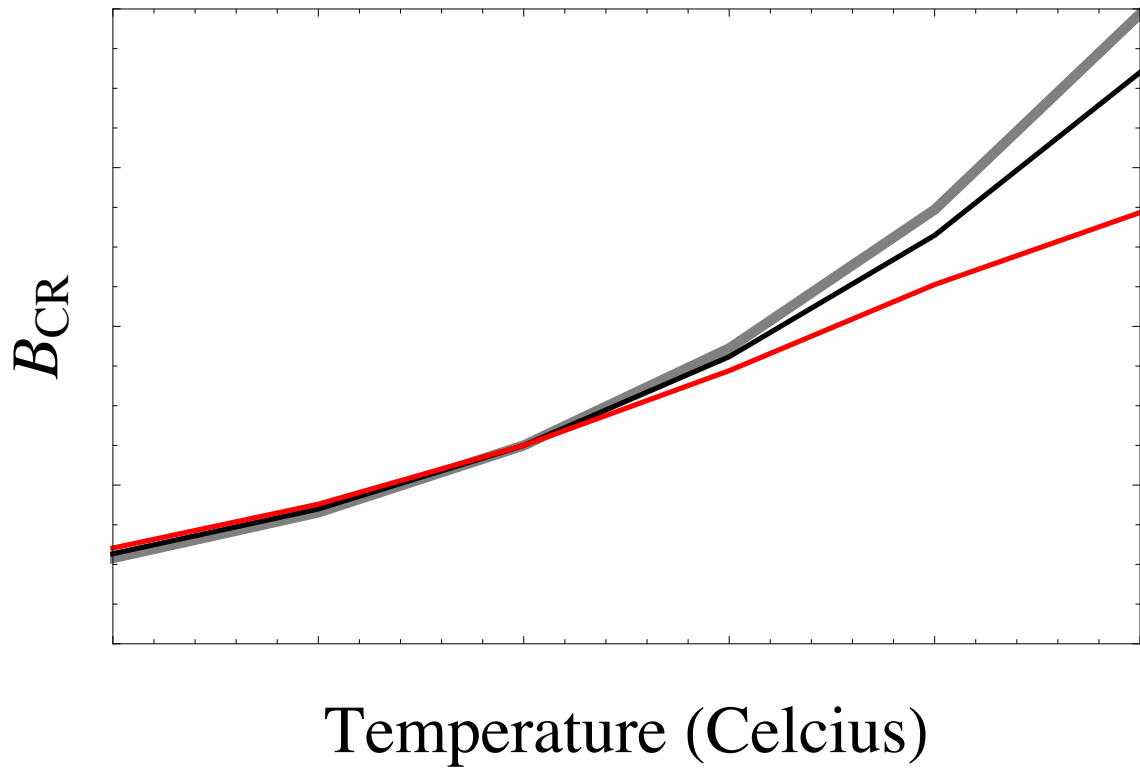
BCR as a function of temperature

```
Show[
  ListPlot[
    Table[{t, Simplify[BCR2[T] /. Eq2[T][[3]]] /. RallT /. RallM /. GilbertDeLongT /.
      GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
      m15 /. T[i_] -> T /. k -> 8.62 * 10-5 /. EB -> 0.32 /.
      ES -> 0.9 /. Em -> 0.65 /.  $\kappa$  -> -0.81 /.  $\alpha$  -> 1 /.  $\epsilon$  -> -0.5 /.
       $\mu$  -> -0.29 /.  $\rho$  -> -0.81 /.  $\beta[i_]$  -> 0.00 /. T -> 273.15 + t /.
      aC -> 1/4 + 2/3 /. aR -> 1/3 /. hC -> -2/3 /. hR -> 0.5 /.
      Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 1 /. M15[i_] -> 1},
    {t, 5, 30, 5}], PlotStyle -> {Gray, Thickness[
    0.01]},
  Axes -> False, Joined ->
```

```

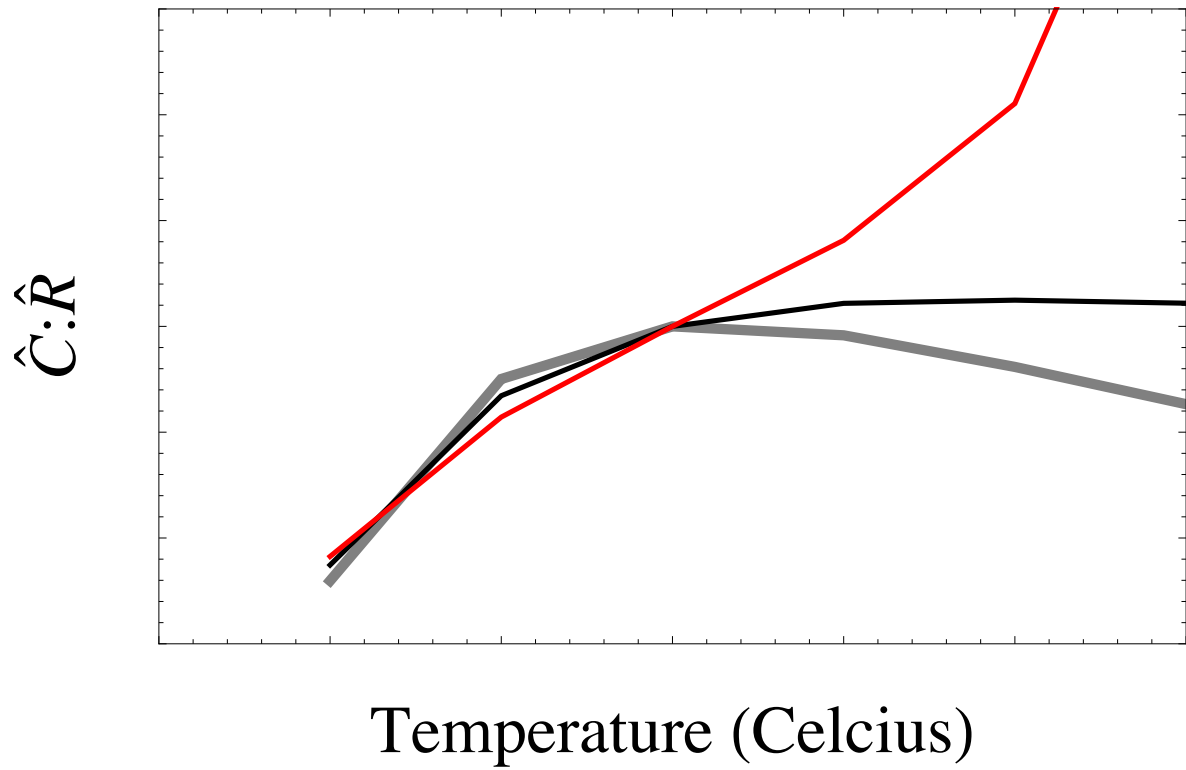
True],
ListPlot[Table[{t,
  Simplify[BCR2[T] /. Eq2[T][[3]]] /. RallT /. RallM /. GilbertDeLongT /.
    GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
      m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.32 /.
        ES → 0.9 /. Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
          μ → -0.29 /. ρ → -0.81 /. β[i_] → 0.02 /. T → 273.15 + t /.
            aC → 1/4 + 2/3 /. aR → 1/3 /. hC → -2/3 /. hR → 0.5 /.
              Ea → 0.65 /. Eh → -0.65 /. h0 → 1 /. M15[i_] → 1}],
  {t, 5, 30, 5}], PlotStyle → {Black,
    Thickness[
      0.005]},
  Axes → False, Joined →
    True],
ListPlot[Table[{t,
  Simplify[BCR2[T] /. Eq2[T][[3]]] /. RallT /. RallM /. GilbertDeLongT /.
    GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
      m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.32 /.
        ES → 0.9 /. Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
          μ → -0.29 /. ρ → -0.81 /. β[i_] → 0.05 /. T → 273.15 + t /.
            aC → 1/4 + 2/3 /. aR → 1/3 /. hC → -2/3 /. hR → 0.5 /.
              Ea → 0.65 /. Eh → -0.65 /. h0 → 1 /. M15[i_] → 1}],
  {t, 5, 30, 5}], PlotStyle → {Red, Thickness[
    0.005]},
  Axes → False, Joined →
    True],
PlotRange → {0, 8},
Frame → True,
FrameLabel →
  {Style["Temperature (Celcius)", LabelSize], Style["BCR", LabelSize], },
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle →
  {{Directive[FontColor → White], Black}, {Directive[FontColor → White], Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None
(*PlotRangeClipping→False,*)
]

```



and the TSR now lowers BCR slightly at high temperatures (it did not with type-I with both $\beta=0.02$).
C:R as a function of temperature

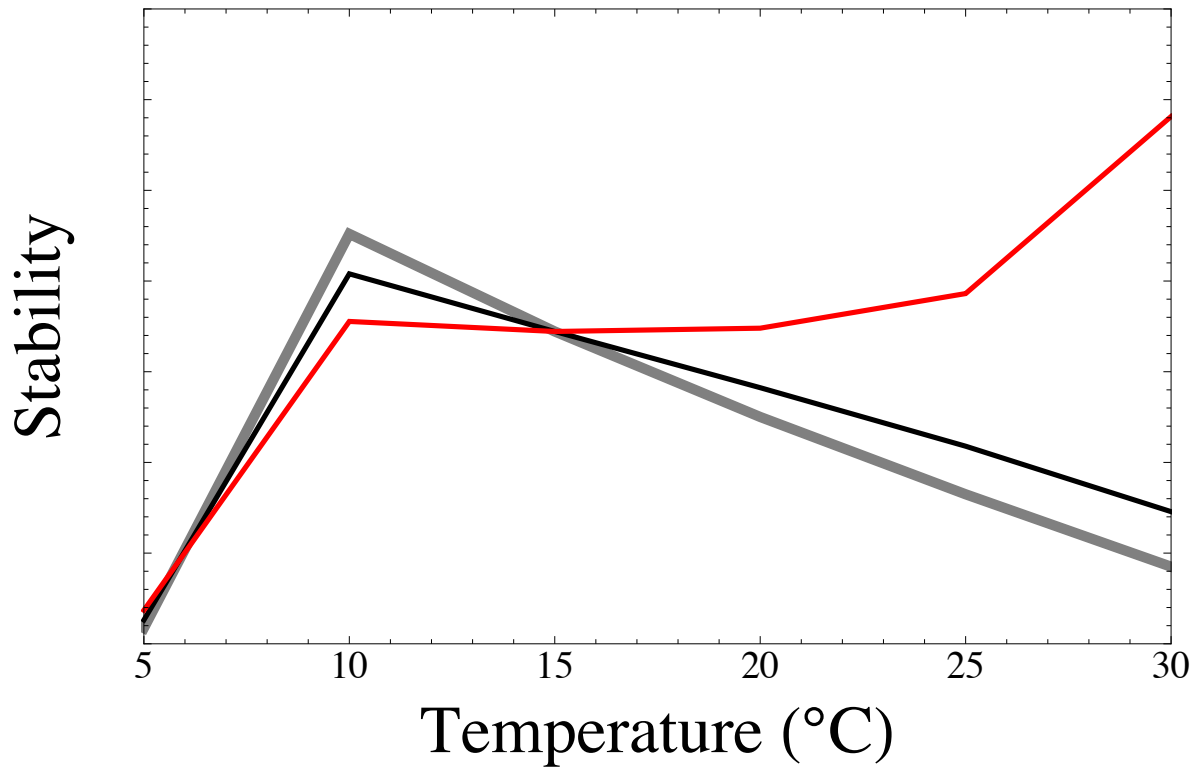
```
Show[
ListPlot[Table[
  {t, CR2[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
    e15 /. k15 /. r15 /. m15 /. T[i_] -> T /. k -> 8.62 * 10-5 /.
    EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /.  $\kappa$  -> -0.81 /.  $\alpha$  -> 1 /.
     $\epsilon$  -> -0.5 /.  $\mu$  -> -0.29 /.  $\rho$  -> -0.81 /.  $\beta$ [i_] -> 0.00 /. T -> 273.15 + t /.
    aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /. hC -> -2 / 3 /. hR -> 0.5 /. Ea -> 0.65 /.
    Eh -> -0.65 /. h0 -> 10-13 /. M15[i_] -> 1}, {t, 5, 30, 5}],
PlotStyle -> {Gray, Thickness[0.01]}, PlotRange ->
{0,
  All}, Joined -> True],
ListPlot[
Table[
  {t,
    CR2[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] -> T /. k -> 8.62 * 10-5 /.
      EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /.  $\kappa$  -> -0.81 /.  $\alpha$  -> 1 /.
       $\epsilon$  -> -0.5 /.  $\mu$  -> -0.29 /.  $\rho$  -> -0.81 /.  $\beta$ [i_] -> 0.02 /. T -> 273.15 + t /.
      aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /. hC -> -2 / 3 /. hR -> 0.5 /. Ea -> 0.65 /.
      Eh -> -0.65 /. h0 -> 10-13 /. M15[i_] -> 1}, {t, 5, 30, 5}],
PlotStyle -> {Black, Thickness[0.005]}, PlotRange ->
{0,
  All}, Joined ->
True],
ListPlot[Table[{t,
  CR2[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
    e15 /. k15 /. r15 /. m15 /. T[i_] -> T /. k -> 8.62 * 10-5 /.
    EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /.  $\kappa$  -> -0.81 /.  $\alpha$  -> 1 /.
     $\epsilon$  -> -0.5 /.  $\mu$  -> -0.29 /.  $\rho$  -> -0.81 /.  $\beta$ [i_] -> 0.05 /. T -> 273.15 + t /.
    aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /. hC -> -2 / 3 /. hR -> 0.5 /. Ea -> 0.65 /.
    Eh -> -0.65 /. h0 -> 10-13 /. M15[i_] -> 1}, {t, 5, 30, 5}],
PlotStyle -> {Red, Thickness[0.005]}, PlotRange ->
{0,
  All}, Joined ->
True],
PlotRange -> {0, 0.6},
Frame -> True,
FrameLabel ->
{Style["Temperature (Celcius)", LabelSize], Style[" $\hat{C}:\hat{R}$ ", LabelSize], },
FrameStyle -> Directive[FontSize -> TickSize],
FrameTicksStyle ->
{{Directive[FontColor -> White], Black}, {Directive[FontColor -> White], Black}},
ImagePadding -> Pad,
ImageSize -> FigureSize,
PlotRangePadding -> None
]
```



this is the same pattern we saw with the type-I.

Stability as a function of temperature

```
Show[ListPlot[
  Table[{t, -Max[Re[lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /.
    TSR /. ah15 /. e15 /. k15 /. r15 /. m15 /. T[i_] →
      T /. k → 8.62 * 10-5 /. EB → 0.32 /. ES → 0.9 /.
        Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
          μ → -0.29 /. ρ → -0.81 /. β[i_] → 0.00 /. T → 273.15 + t /.
            aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /.
              Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /. M15[i_] → 1]}],
    {t, 5, 30, 5}], PlotStyle → {Gray, Thickness[0.01]},
  PlotRange →
    All,
  Joined →
    True],
ListPlot[Table[{t, -Max[Re[
  lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
    e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 *
      10-5 /. EB → 0.32 /. ES → 0.9 /. Em → 0.65 /. κ → -0.81 /.
        α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. β[i_] → 0.02 /.
          T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
            hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /. M15[i_] → 1]}],
    {t, 5, 30, 5}], PlotStyle → {Black, Thickness[
      0.005]}],
  PlotRange → All, Joined →
    True],
ListPlot[Table[{t, -Max[Re[
  lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
    e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 *
      10-5 /. EB → 0.32 /. ES → 0.9 /. Em → 0.65 /. κ → -0.81 /.
        α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. β[i_] → 0.05 /.
          T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
            hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /. M15[i_] → 1]}],
    {t, 5, 30, 5}], PlotStyle → {Red, Thickness[
      0.005]}],
  PlotRange → All, Joined →
    True],
PlotRange → {0, 0.7},
Frame → True,
FrameLabel →
  {Style["Temperature (°C)", LabelSize], Style["Stability", LabelSize], },
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle → {{Directive[FontColor → White], Black}, {Black, Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
PlotRange → All
]
```



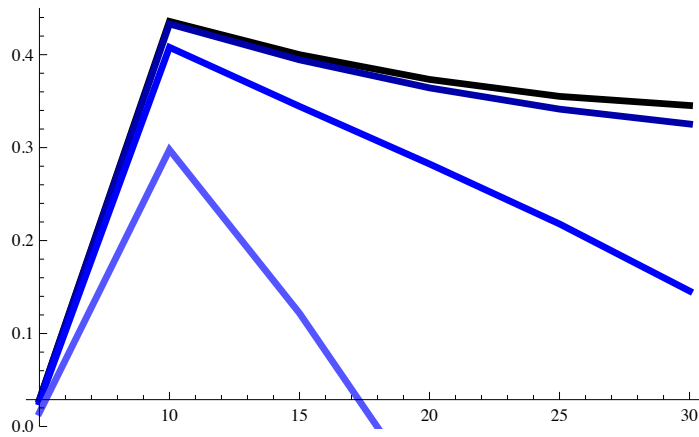
this is the same pattern we saw with the type-I (better stability with TSR at warm temps) but we now need a larger TSR to get increases in stability with temp (red).

Handling time

Note that although we have to give values for handling time h_0 and masses $M_{15[i]}$ at 15 degrees in order to produce the above curves, the results do not depend on them - except in the case of stability, where h_0 has a big effect.

The effect of h_0 on stability with the type-II functional response:

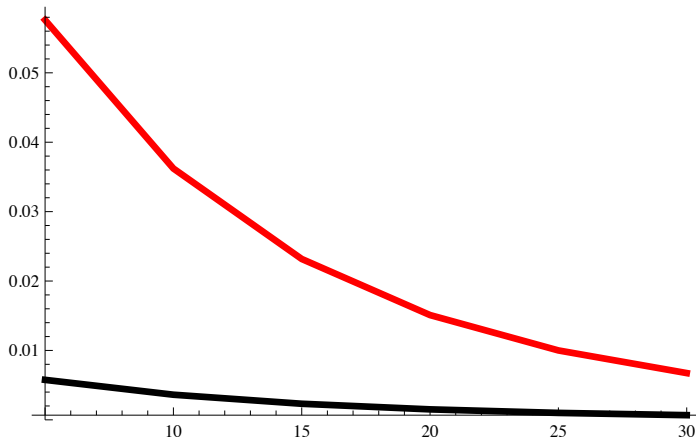
```
Show[
ListPlot[Table[
  {t, -Max[Re[lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /.
    ah15 /. e15 /. k15 /. r15 /. m15 /. T[i_] -> T /. k ->
    8.62 * 10-5 /. EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /.  $\kappa$  ->
    -0.81 /.  $\alpha$  -> 1 /.  $\epsilon$  -> -0.5 /.  $\mu$  -> -0.29 /.  $\rho$  -> -0.81 /.  $\beta$ [i_] ->
    0.02 /. T -> 273.15 + t /. aC -> 1/4 + 2/3 /. aR -> 1/3 /. hC -> -2/3 /.
    hR -> 0.5 /. Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 0 /. M15[i_] -> 1]]],
  {t, 5, 30, 5}], PlotStyle -> {Black, Thickness[0.01]},
PlotRange ->
All,
Joined ->
True],
ListPlot[Table[{t, -Max[Re[
  lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
  e15 /. k15 /. r15 /. m15 /. T[i_] -> T /. k -> 8.62 *
  10-5 /. EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /.  $\kappa$  -> -0.81 /.
   $\alpha$  -> 1 /.  $\epsilon$  -> -0.5 /.  $\mu$  -> -0.29 /.  $\rho$  -> -0.81 /.  $\beta$ [i_] -> 0.02 /.
  T -> 273.15 + t /. aC -> 1/4 + 2/3 /. aR -> 1/3 /. hC -> -2/3 /.
  hR -> 0.5 /. Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 10-14 /. M15[i_] -> 1]]}],
  {t, 5, 30, 5}], PlotStyle -> {Darker[Blue], Thickness[
    0.01]},
PlotRange -> All, Joined ->
True],
ListPlot[Table[{t, -Max[Re[
  lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
  e15 /. k15 /. r15 /. m15 /. T[i_] -> T /. k -> 8.62 *
  10-5 /. EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /.  $\kappa$  -> -0.81 /.
   $\alpha$  -> 1 /.  $\epsilon$  -> -0.5 /.  $\mu$  -> -0.29 /.  $\rho$  -> -0.81 /.  $\beta$ [i_] -> 0.02 /.
  T -> 273.15 + t /. aC -> 1/4 + 2/3 /. aR -> 1/3 /. hC -> -2/3 /.
  hR -> 0.5 /. Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 10-13 /. M15[i_] -> 1]]}],
  {t, 5, 30, 5}], PlotStyle -> {Blue, Thickness[
    0.01]},
PlotRange -> All, Joined ->
True],
ListPlot[Table[{t, -Max[Re[
  lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
  e15 /. k15 /. r15 /. m15 /. T[i_] -> T /. k -> 8.62 *
  10-5 /. EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /.  $\kappa$  -> -0.81 /.
   $\alpha$  -> 1 /.  $\epsilon$  -> -0.5 /.  $\mu$  -> -0.29 /.  $\rho$  -> -0.81 /.  $\beta$ [i_] -> 0.02 /.
  T -> 273.15 + t /. aC -> 1/4 + 2/3 /. aR -> 1/3 /. hC -> -2/3 /.
  hR -> 0.5 /. Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 5 * 10-13 /. M15[i_] -> 1]]}],
  {t, 5, 30, 5}], PlotStyle -> {Lighter[Blue], Thickness[
    0.01]},
PlotRange -> All, Joined ->
True],
PlotRange -> {0, All}
]
```

ie, the greater the handling time the less stable things are, and the faster stability declines with higher temperatures.

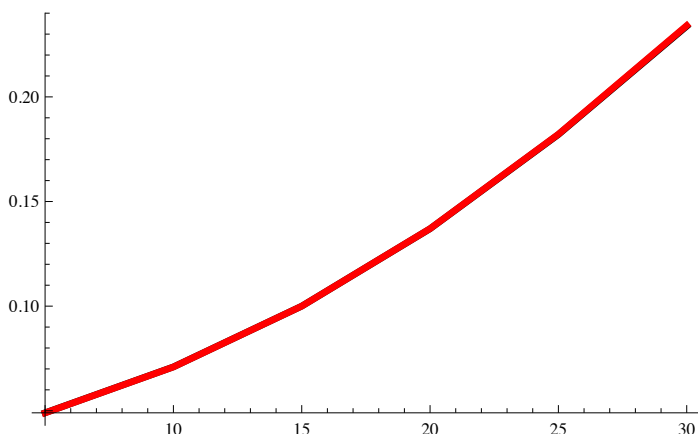
Note that while handling time decreases faster with temperature when h_0 is larger (this should stabilize things)

```
Show[
ListPlot[
Table[{t, h[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] -> T /. k -> 8.62 * 10-5 /.
      EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /.  $\kappa$  -> -0.81 /.  $\alpha$  -> 1 /.
       $\epsilon$  -> -0.5 /.  $\mu$  -> -0.29 /.  $\rho$  -> -0.81 /.  $\beta$ [i_] -> 0.02 /. T -> 273.15 + t /.
      aC -> 1/4 + 2/3 /. aR -> 1/3 /. hC -> -2/3 /. hR -> 0.5 /. Ea -> 0.65 /.
      Eh -> -0.65 /. h0 -> 10-14 /. M15[i_] -> 1}, {t, 5, 30, 5}],
PlotStyle -> {Black, Thickness[0.01]}, PlotRange ->
All,
Joined ->
True],
ListPlot[Table[{t,
      h[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /. e15 /.
      k15 /. r15 /. m15 /. T[i_] -> T /. k -> 8.62 * 10-5 /.
      EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /.  $\kappa$  -> -0.81 /.  $\alpha$  -> 1 /.
       $\epsilon$  -> -0.5 /.  $\mu$  -> -0.29 /.  $\rho$  -> -0.81 /.  $\beta$ [i_] -> 0.02 /. T -> 273.15 + t /.
      aC -> 1/4 + 2/3 /. aR -> 1/3 /. hC -> -2/3 /. hR -> 0.5 /. Ea -> 0.65 /.
      Eh -> -0.65 /. h0 -> 10-13 /. M15[i_] -> 1}, {t, 5, 30, 5}],
PlotStyle -> {Red, Thickness[0.01]}, PlotRange ->
All,
Joined ->
True],
PlotRange -> {0, All}
]
```



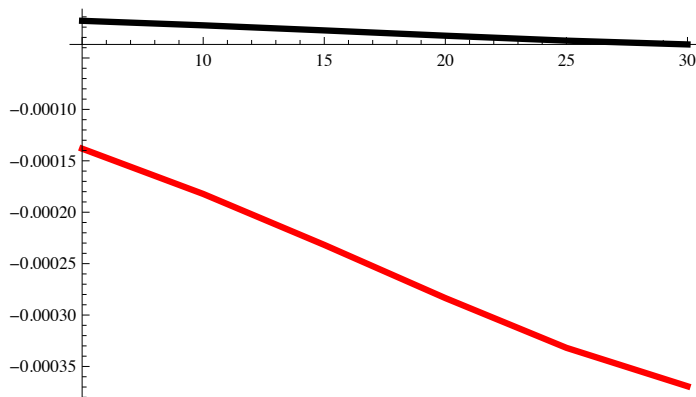
...the functional responses change similarly with temperature (because R must be canceling out the effect of h0)

```
Show[
ListPlot[
Table[{t,  $\frac{a[T]}{1 + a[T] h[T] R}$  /. Eq2[T][[3]] /. RallT /. RallM /. GilbertDeLongT /.
      GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
      m15 /. T[i_] -> T /. k -> 8.62 * 10-5 /. EB -> 0.32 /.
      ES -> 0.9 /. Em -> 0.65 /. x -> -0.81 /. α -> 1 /. ε -> -0.5 /.
      μ -> -0.29 /. ρ -> -0.81 /. β[i_] -> 0.02 /. T -> 273.15 + t /.
      aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /. hC -> -2 / 3 /. hR -> 0.5 /.
      Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 10-14 /. M15[i_] -> 1},
{t, 5, 30, 5}], PlotStyle -> {Black, Thickness[
  0.01]},
PlotRange -> All, Joined ->
True],
ListPlot[Table[{t,
   $\frac{a[T]}{1 + a[T] h[T] R}$  /. Eq2[T][[3]] /. RallT /. RallM /. GilbertDeLongT /.
      GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
      m15 /. T[i_] -> T /. k -> 8.62 * 10-5 /. EB -> 0.32 /.
      ES -> 0.9 /. Em -> 0.65 /. x -> -0.81 /. α -> 1 /. ε -> -0.5 /.
      μ -> -0.29 /. ρ -> -0.81 /. β[i_] -> 0.02 /. T -> 273.15 + t /.
      aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /. hC -> -2 / 3 /. hR -> 0.5 /.
      Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 10-13 /. M15[i_] -> 1},
{t, 5, 30, 5}], PlotStyle -> {Red, Thickness[
  0.01]},
PlotRange -> All, Joined ->
True],
PlotRange -> All
]
```



The difference is in the derivative (which is important for stability - we take derivatives to get the Jacobian)

```
Show[
  ListPlot[
    Table[{t, D[ $\frac{a[T]}{1 + a[T] h[T] R}$ , R] /. Eq2[T][[3]] /. RallT /. RallM /. GilbertDeLongT /.
      GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
      m15 /. T[i_] -> T /. k -> 8.62 * 10-5 /. EB -> 0.32 /.
      ES -> 0.9 /. Em -> 0.65 /. x -> -0.81 /. α -> 1 /. ε -> -0.5 /.
      μ -> -0.29 /. ρ -> -0.81 /. β[i_] -> 0.02 /. T -> 273.15 + t /.
      aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /. hC -> -2 / 3 /. hR -> 0.5 /.
      Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 10-14 /. M15[i_] -> 1}],
    {t, 5, 30, 5}], PlotStyle -> {Black, Thickness[
      0.01]}],
    PlotRange -> All, Joined ->
    True],
  ListPlot[Table[{t,
    D[ $\frac{a[T]}{1 + a[T] h[T] R}$ , R] /. Eq2[T][[3]] /. RallT /. RallM /. GilbertDeLongT /.
      GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
      m15 /. T[i_] -> T /. k -> 8.62 * 10-5 /. EB -> 0.32 /.
      ES -> 0.9 /. Em -> 0.65 /. x -> -0.81 /. α -> 1 /. ε -> -0.5 /.
      μ -> -0.29 /. ρ -> -0.81 /. β[i_] -> 0.02 /. T -> 273.15 + t /.
      aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /. hC -> -2 / 3 /. hR -> 0.5 /.
      Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 10-13 /. M15[i_] -> 1}],
    {t, 5, 30, 5}], PlotStyle -> {Red, Thickness[
      0.01]}],
    PlotRange -> All, Joined ->
    True],
  PlotRange -> All
]
```

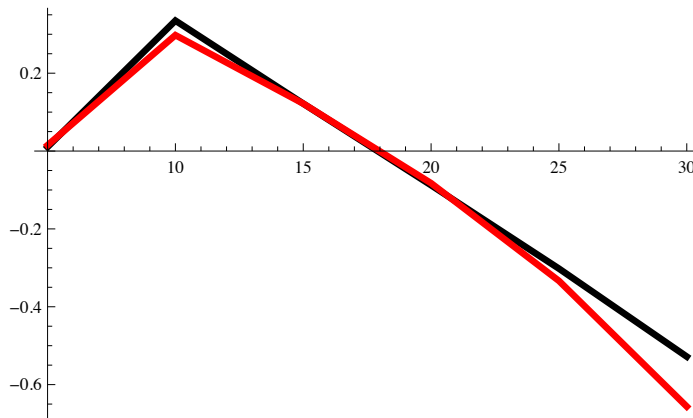


Here we see that the larger h_0 means faster declines in the derivative of the functional response with respect to R as temperatures warm, meaning the system is more sensitive to perturbation.

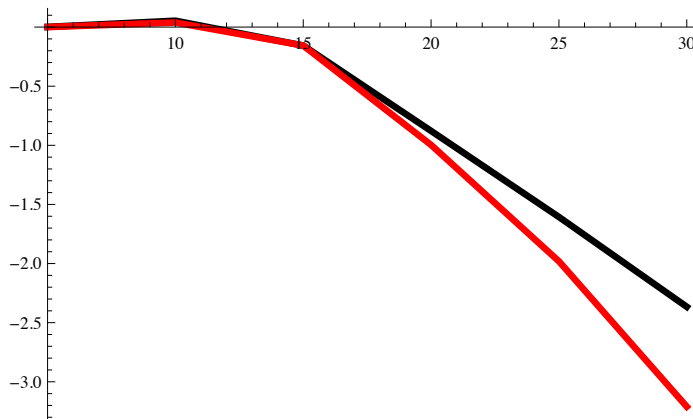
When h_0 is large enough to cause instability, the TSR causes instability to increase faster with

temperature

```
Show[
ListPlot[Table[
  {t, -Max[Re[lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /.
    ah15 /. e15 /. k15 /. r15 /. m15 /. T[i_] -> T /.
    k -> 8.62 * 10^-5 /. EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /.
    κ -> -0.81 /. α -> 1 /. ε -> -0.5 /. μ -> -0.29 /. ρ -> -0.81 /.
    β[i_] -> 0.0 /. T -> 273.15 + t /. aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /.
    hC -> -2 / 3 /. hR -> 0.5 /. Ea -> 0.65 /. Eh -> -0.65 /.
    h0 -> 5 * 10^-13 /. M15[i_] -> 1]]], {t, 5, 30, 5}],
PlotStyle -> {Black, Thickness[0.01]}, PlotRange ->
All,
Joined ->
True],
ListPlot[Table[{t, -Max[Re[
  lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
  e15 /. k15 /. r15 /. m15 /. T[i_] -> T /. k -> 8.62 *
  10^-5 /. EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /. κ -> -0.81 /.
  α -> 1 /. ε -> -0.5 /. μ -> -0.29 /. ρ -> -0.81 /. β[i_] -> 0.02 /.
  T -> 273.15 + t /. aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /. hC -> -2 / 3 /.
  hR -> 0.5 /. Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 5 * 10^-13 /. M15[i_] -> 1]]}],
{t, 5, 30, 5}], PlotStyle -> {Red, Thickness[
0.01]},
PlotRange -> All, Joined ->
True],
PlotRange -> All
]
```



```
Show[
ListPlot[
Table[{t, -Max[Re[lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /.
TSR /. ah15 /. e15 /. k15 /. r15 /. m15 /. T[i_] ->
T /. k -> 8.62 * 10-5 /. EB -> 0.32 /. ES -> 0.9 /.
Em -> 0.65 /. x -> -0.81 /. alpha -> 1 /. epsilon -> -0.5 /.
mu -> -0.29 /. rho -> -0.81 /. beta[i_] -> 0.0 /. T -> 273.15 + t /.
aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /. hC -> -2 / 3 /. hR -> 0.5 /.
Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 10-12 /. M15[i_] -> 1]}],
{t, 5, 30, 5}], PlotStyle -> {Black, Thickness[0.01]},
PlotRange ->
All,
Joined ->
True],
ListPlot[Table[{t, -Max[Re[
lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
e15 /. k15 /. r15 /. m15 /. T[i_] -> T /. k -> 8.62 *
10-5 /. EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /. x -> -0.81 /.
alpha -> 1 /. epsilon -> -0.5 /. mu -> -0.29 /. rho -> -0.81 /. beta[i_] -> 0.02 /.
T -> 273.15 + t /. aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /. hC -> -2 / 3 /.
hR -> 0.5 /. Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 10-12 /. M15[i_] -> 1]}],
{t, 5, 30, 5}], PlotStyle -> {Red, Thickness[
0.01]},
PlotRange -> All, Joined ->
True],
PlotRange -> All
]
```

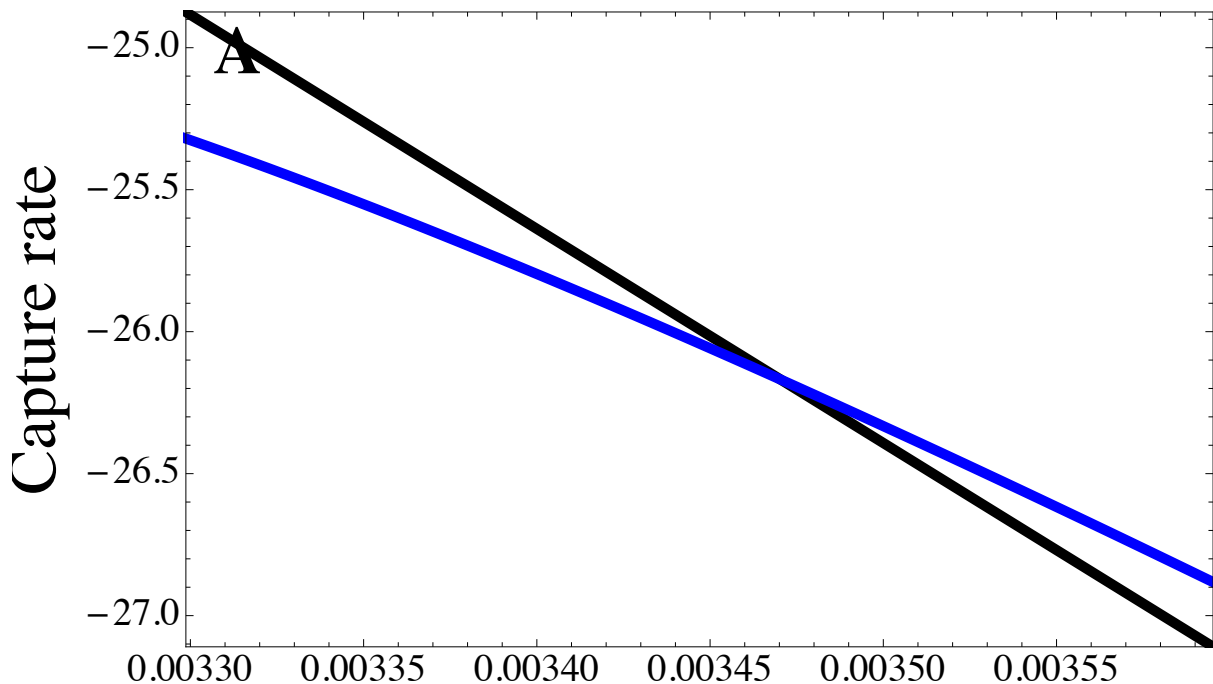


Type-II functional response continued: Improving Rall's fit

Response of capture rate to temperature, with (blue) and without (black) the TSR:

```
Show[
Plot[
  Log[a[T] / a0] /. RallT /. RallM /. k → 8.62 * 10-5 /. M[i_] → 1 /. aC → 1 / 4 + 2 / 3 /. aR →
    1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. T[i_] → 1 / t,
  {t, 1 / (273.15 + 5), 1 / (273.15 + 30)}, PlotStyle → {Black, Thickness[0.01]}],
Plot[Log[a[T] / a0] /. RallT /. RallM /. TSR /. k → 8.62 * 10-5 /. aC → 1 / 4 + 2 / 3 /.
  aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. M15[i_] → 1 /.
  β[i_] → 0.02 /. T[i_] → 1 / t, {t, 1 / (273.15 + 5), 1 / (273.15 + 30)},
  PlotStyle → {Blue, Thickness[0.01]}],
Frame → True,
FrameLabel → {Style[(*"Temperature-1 (Kelvins-1)"*)"", LabelSize],
  Style["Capture rate", LabelSize], , },
FrameStyle → Directive[FontSize → TickSize],
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
(*PlotRangeClipping→False,*)
Epilog → {
  Text[Style["A", LabelSize, Bold], Scaled@letpos],
  (*Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree] *)
}
]

(*Export[imagedir<>"CaptureTSRSymm.pdf", %];*)
```



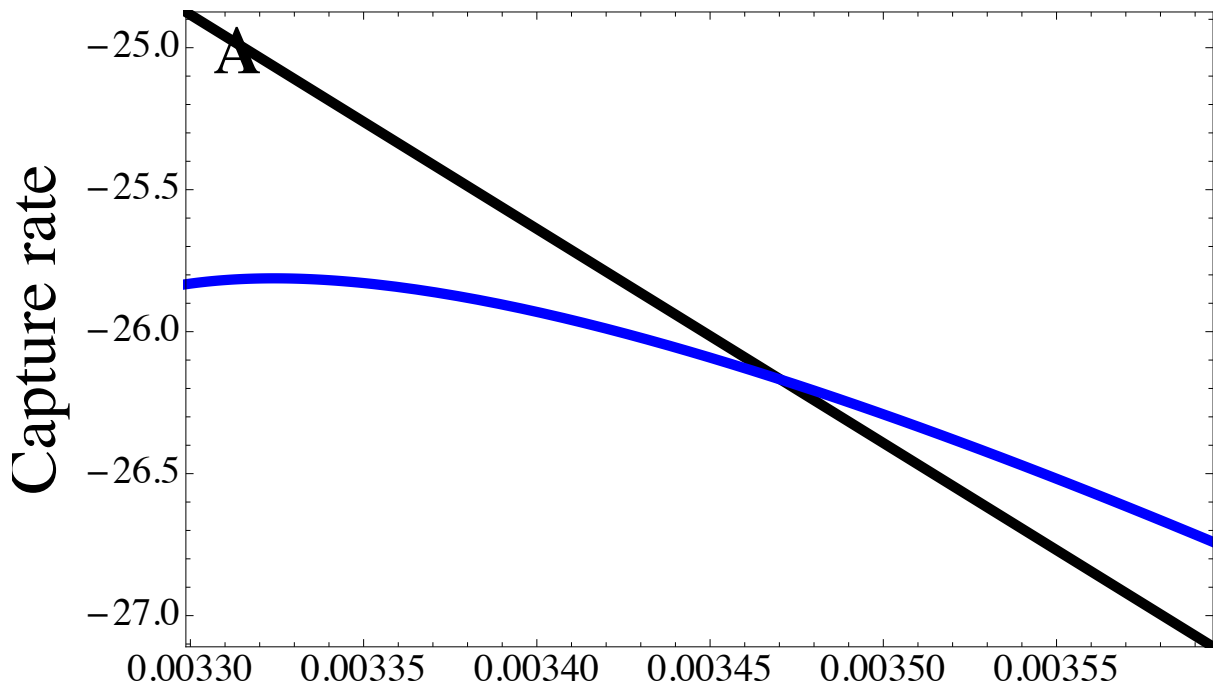
The TSR lowers the activation energy of capture rate, which goes in the same direction as the discrep-

ancy seen in Rall (see Figure 3a in Rall et al 2012).

With an asymmetric TSR this response is even stronger :


```
Show[
Plot[
  Log[a[T] / a0] /. RallT /. RallM /. k → 8.62 * 10-5 /. M[i_] → 1 /. aC → 1 / 4 + 2 / 3 /. aR →
    1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. T[i_] → 1 / t,
  {t, 1 / (273.15 + 5), 1 / (273.15 + 30)}, PlotStyle → {Black, Thickness[0.01]}],
Plot[Log[a[T] / a0] /. RallT /. RallM /. TSR /. k → 8.62 * 10-5 /. aC → 1 / 4 + 2 / 3 /.
  aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. M15[i_] → 1 /.
  β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → 1 / t, {t, 1 / (273.15 + 5),
  1 / (273.15 + 30)}, PlotStyle → {Blue, Thickness[0.01]}],
Frame → True,
FrameLabel → {Style[(*"Temperature-1 (Kelvins-1)"*)"", LabelSize],
  Style["Capture rate", LabelSize], , },
FrameStyle → Directive[FontSize → TickSize],
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
(*PlotRangeClipping → False, *)
Epilog → {
  Text[Style["A", LabelSize, Bold], Scaled@letpos],
  (*Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree] *)
}
]

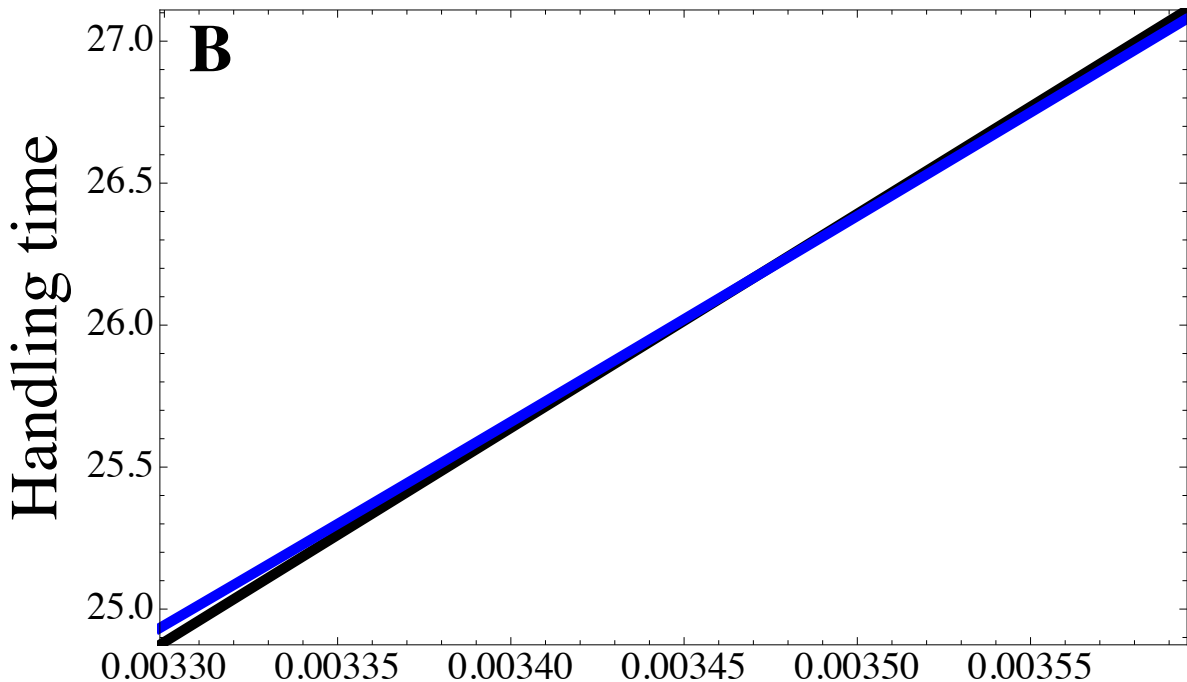
(*Export[imagedir<>"CaptureTSRAsymm.pdf", %]; *)
```



And for handling time:

```
Show[
Plot[
  Log[h[T] / h0] /. RallT /. RallM /. k → 8.62 * 10-5 /. M[i_] → 1 /. aC → 1 / 4 + 2 / 3 /. aR →
    1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. T[i_] → 1 / t,
  {t, 1 / (273.15 + 5), 1 / (273.15 + 30)}, PlotStyle → {Black, Thickness[0.01]}],
Plot[Log[h[T] / h0] /. RallT /. RallM /. TSR /. k → 8.62 * 10-5 /. aC → 1 / 4 + 2 / 3 /.
  aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. M15[i_] → 1 /.
  β[i_] → 0.02 /. T[i_] → 1 / t, {t, 1 / (273.15 + 5), 1 / (273.15 + 30)},
  PlotStyle → {Blue, Thickness[0.01]}],
Frame → True,
FrameLabel → {Style[(*"Temperature-1 (Kelvins-1)"*)"", LabelSize],
  Style["Handling time", LabelSize], },
FrameStyle → Directive[FontSize → TickSize],
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
(*PlotRangeClipping→False,*)
Epilog → {
  Text[Style["B", LabelSize, Bold], Scaled@letpos],
  (*Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree] *)
}
]

(*Export[imagedir<>"HandlingTSRSymm.pdf", %];*)
```

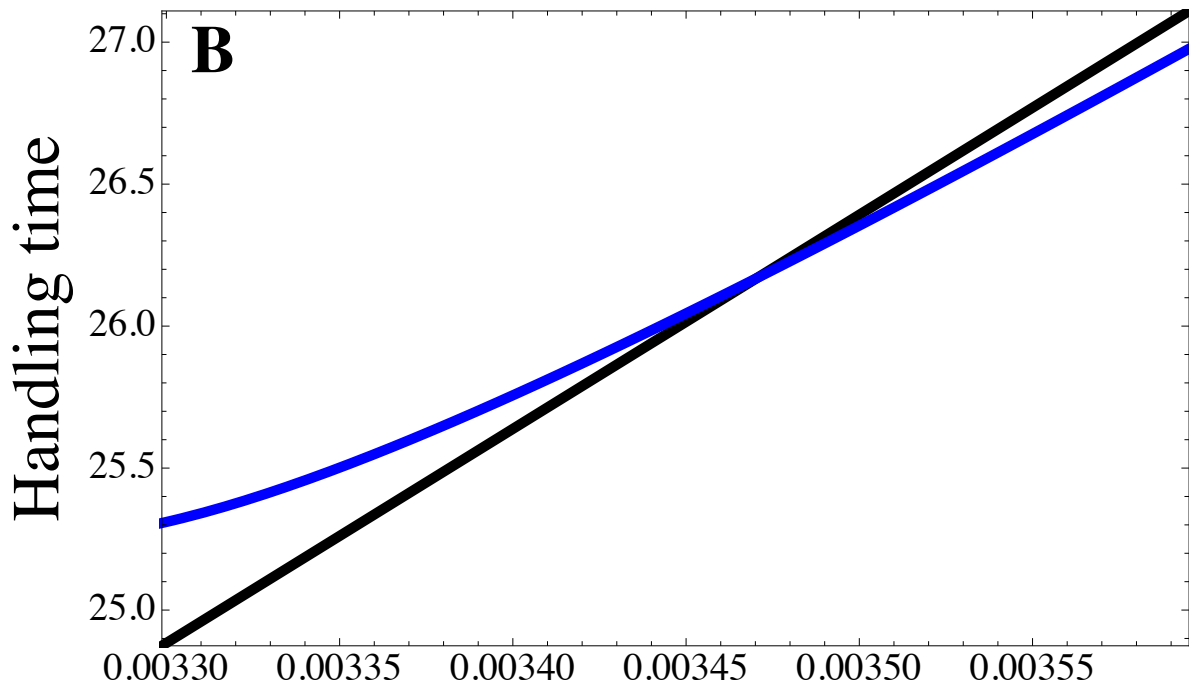


Here the TSR (slightly) decreases the activation energy of handling time. But this is with same TSR in

resource and consumer. If we allow the consumer to have a larger TSR, then we have

```
Show[
  Plot[
    Log[h[T] / h0] /. RallT /. RallM /. k → 8.62 * 10-5 /. M[i_] → 1 /. aC → 1 / 4 + 2 / 3 /. aR →
      1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. T[i_] → 1 / t,
    {t, 1 / (273.15 + 5), 1 / (273.15 + 30)}, PlotStyle → {Black, Thickness[0.01]}],
  Plot[Log[h[T] / h0] /. RallT /. RallM /. TSR /. k → 8.62 * 10-5 /. aC → 1 / 4 + 2 / 3 /.
    aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /.
    M15[R] → 1 /. M15[C] → 1 /. β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → 1 / t,
    {t, 1 / (273.15 + 5), 1 / (273.15 + 30)}, PlotStyle →
      {Blue, Thickness[0.01]}],
  Frame → True,
  FrameLabel → {Style[(*"Temperature-1 (Kelvins-1)"*)"", LabelSize],
    Style["Handling time", LabelSize], , },
  FrameStyle → Directive[FontSize → TickSize],
  ImagePadding → Pad,
  ImageSize → FigureSize,
  PlotRangePadding → None,
  (*PlotRangeClipping→False, *)
  Epilog → {
    Text[Style["B", LabelSize, Bold], Scaled@letpos],
    (*Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree] *)
  }
]
```

(*Export[imagedir<>"HandlingTSRAsymm.pdf", %];*)



which makes the activation energy of handling time smaller, as in Rall (see Figure 3d).

A functional response that depends on the body size ratio

Summary

Functional responses tend to be roughly type-II when consumers and resources have similar body sizes, but become more sigmoidal when consumers are much bigger than their resource, as resources are then harder to find when rare Kalinkat et al (2013). To capture this shift Kalinkat et al (2013) use the generalized functional response $f(R) = \frac{b}{1 + b h R^{1+q}}$, where $q = q_{\max} D^2 / (q_0^2 + D^2)$ depends on the body size ratio of consumer to resource, $D = M_C / M_R$. The scaling parameters q_{\max} and q_0^2 are the asymptotic value and half-saturation constant of q , respectively. The important thing to note is that when the consumer is much smaller than the resource ($D \approx 0$) then $q \approx 0$ and the functional response is roughly type-II. As D increases so does q , and when $q > 0$ the functional response is type-III. Without a temperature-size response the body mass ratio remains constant with temperature and therefore the form of the functional response is not expected to change. However, with a temperature-size response that differs between resource and consumer, the ratio of body sizes will change and influence the form of the functional response. Because consumers are often larger than their resource, and because larger aquatic organisms are expected to have greater reductions in body size with temperature (Forster et al., 2012; Horne et al., 2015), the ratio of consumer to resource body size is expected to decrease with temperature. As stated above, lower consumer to resource body size ratios produce type-II functional responses ($\lim_{D \rightarrow 0} q = 0$), which are less stable than type-III functional responses (McCann, 2011;

Murdoch et al., 2003). Hence, the temperature-size rule can be said to destabilize the consumer-resource dynamics at high temperatures by promoting type-II functional responses. However, the amount by which the shape of the functional response is adjusted by the temperature-size rule does not appear to be large and therefore the stabilizing effects discussed in previous sections will likely prevail (see Analyses section directly below for details).

Analyses

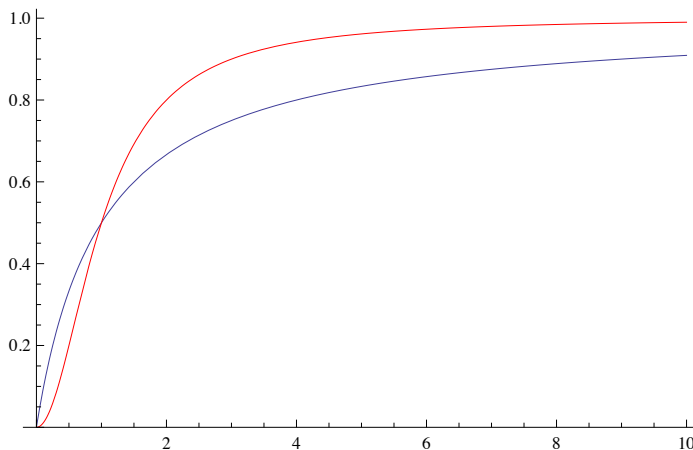
This section is based on Kalinkat et al. 2013. who discuss how the functional response changes with the ratio of body size between consumer and resources.

In particular, we let the functional response be

$$g[R_, T_] := \frac{b[T] R^{q[M]}}{1 + h[T] b[T] R^{1+q[M]}}$$

such that when $q=0$ we have a type-2 functional response and when $q>0$ we have a type-3 (sigmoidal):

```
Show[
  Plot[g[R, T] R /. b[T] → 1 /. h[T] → 1 /. q[M] → 0, {R, 0, 10}, PlotRange → {0, All}],
  Plot[g[R, T] R /. b[T] → 1 /. h[T] → 1 /. q[M] → 1,
    {R, 0, 10}, PlotRange → {0, All}, PlotStyle → Red],
  PlotRange → {0, All}
]
```

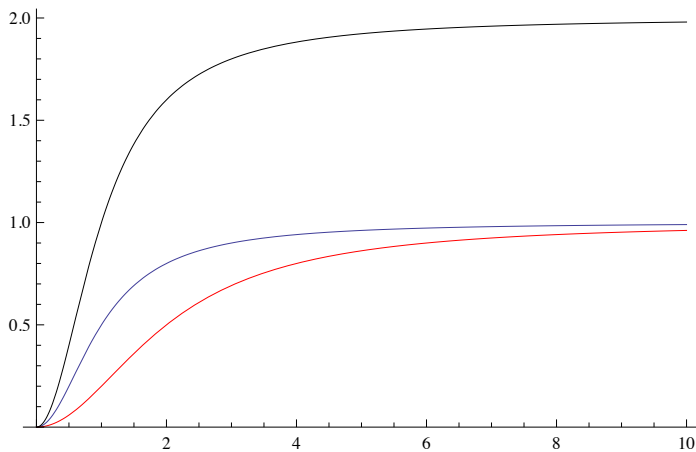


We say q is a function of body mass M , because it depends on the ratio of consumer to resource body size

$$q[M_] := \frac{q_{\max} \left(\frac{M[C]}{M[R]} \right)^2}{q_0^2 + \left(\frac{M[C]}{M[R]} \right)^2}$$

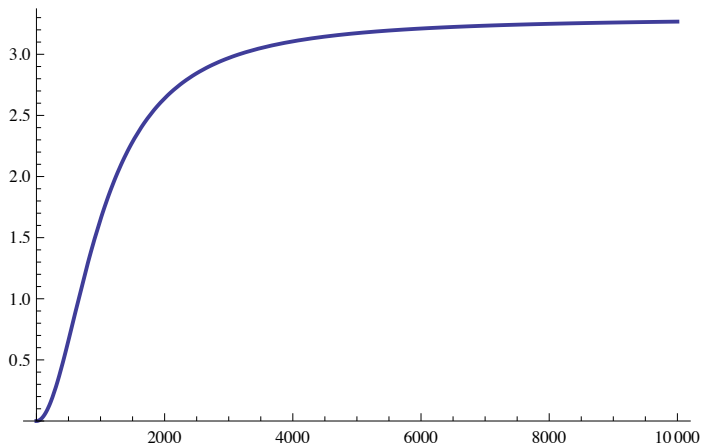
where q_{\max} and q_0 are scaling parameters that determine the shape of the sigmoidal response (q_{\max} is the asymptote, q_0^2 is the half-saturation constant):

```
Show[
  Plot[q[M] /. M[C] → a M[R] /. qmax → 1 /. q0 → 1, {a, 0, 10}, PlotRange → {0, All}],
  Plot[q[M] /. M[C] → a M[R] /. qmax → 1 /. q0 → 2,
    {a, 0, 10}, PlotRange → {0, All}, PlotStyle → Red],
  Plot[q[M] /. M[C] → a M[R] /. qmax → 2 /. q0 → 1, {a, 0, 10},
    PlotRange → {0, All}, PlotStyle → Black]
]
```



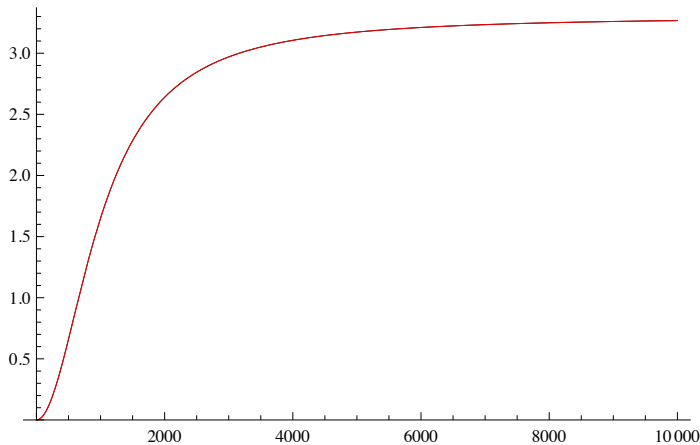
The fitted values in Kalinkat give the following curve

```
Plot[q[M] /. M[C] → a M[R] /. qmax → 3.3 /. q0 → 103,
  {a, 0, 104}, PlotRange → {0, All}, PlotStyle → Thick]
```



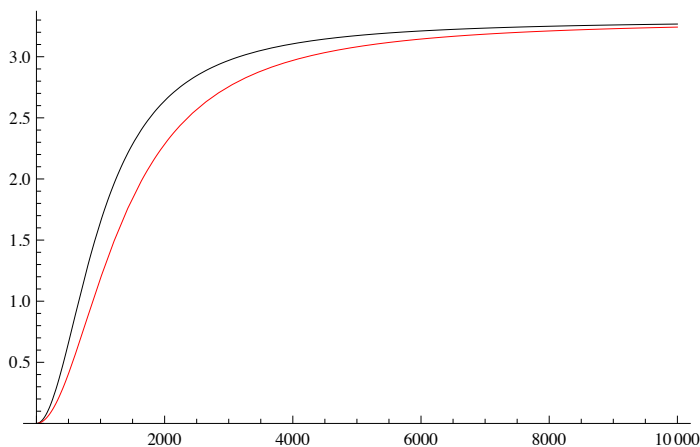
What effect does the TSR have on q?

```
Show[
  Plot[q[M] /. qmax → 3.3 /. q0 → 103 /. TSR /. T[i_] → T /. β[R] → 0 /. β[C] → 0 /.
    M15[C] → a M15[R] /. T → 273.15 + 25,
    {a, 0, 104}, PlotRange → {0, All}, PlotStyle → Black],
  Plot[q[M] /. qmax → 3.3 /. q0 → 103 /. TSR /. T[i_] → T /. β[R] → 0.02 /. β[C] → 0.02 /.
    M15[C] → a M15[R] /. T → 273.15 + 25,
    {a, 0, 104}, PlotRange → {0, All}, PlotStyle → Red]
]
```



As we might expect, because q depends on the ratio of body sizes, if both resource and consumer have the same TSR, then the TSR has no effect on q .

```
Show[
  Plot[q[M] /. qmax → 3.3 /. q0 → 103 /. TSR /. T[i_] → T /. β[R] → 0 /. β[C] → 0 /.
    M15[C] → a M15[R] /. T → 273.15 + 25,
    {a, 0, 104}, PlotRange → {0, All}, PlotStyle → Black],
  Plot[q[M] /. qmax → 3.3 /. q0 → 103 /. TSR /. T[i_] → T /. β[R] → 0.02 /. β[C] → 0.04 /.
    M15[C] → a M15[R] /. T → 273.15 + 25,
    {a, 0, 104}, PlotRange → {0, All}, PlotStyle → Red]
]
```



However, when the consumer has a larger TSR than the resource (4% vs 2%), the body mass ratio decreases with temperature, decreasing q and therefore slowing the switch from type-2 to type-3.

Because type-3 is more stable, the TSR may be said to reduce stability by this mechanism (however, as we can see in the above plot, with the TSR (red) q is not reduced by much).

Figure I (A,B,C) [ES>EB, type-I]

```

Legended[
  Show[
    (*Plot[
      BCR[T] /. K[T] → K0 Exp[ $\frac{EB}{k_{T[R]}} - \frac{ES}{k_{T[S]}}$ ] /. T[i_] → T + 273.15 /. K15 /. k → 8.62 * 10-5 /. a[T] → 0.1 /.
      e[T] → 0.15 /. m[T] → 0.6 /. r[T] → 2 /. EB → 0.32 /. ES → 0.9,
      {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]}], *)
    Plot[
      BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /.  $\kappa$  → -0.81 /.  $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.
       $\rho$  → -0.81 /. TSR /.  $\beta$ [i_] → 0 /. T[i_] → T + 273.15,
      {T, 5, 30}, PlotStyle → {Thickness[0.01], Gray},
      Axes →
      False, PlotRangePadding →
      None],
    (*Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /.
      ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.  $\kappa$  → -0.81 /.
       $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /. TSR /.  $\beta$ [i_] → 0 /. T[i_] → T + 273.15,
      {T, 5, 30}, PlotStyle → {Thickness[0.01], Black, Dashing[Large]}],
    Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /.
      ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.
       $\kappa$  → -0.81 /.  $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /. TSR /.  $\beta$ [i_] → 0 /.
      T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick, Gray}], *)
    Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /.  $\kappa$  → -0.81 /.  $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.
       $\rho$  → -0.81 /. TSR /.  $\beta$ [i_] → 0.02 /. T[i_] → T + 273.15,
      {T, 5, 30}, PlotStyle → {Thickness[0.005], Black},
      PlotRangePadding →
      None],
    (*Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /.
      ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.  $\kappa$  → -0.81 /.
       $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /. TSR /.  $\beta$ [i_] → 0.02 /. T[i_] → T + 273.15,
      {T, 5, 30}, PlotStyle → {Thickness[0.01], Red, Dashing[Large]}], Plot[
      BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /. ES → 0.9 /.
      k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.  $\kappa$  → -0.81 /.  $\alpha$  → 1 /.
       $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /. TSR /.  $\beta$ [i_] → 0.02 /. T[i_] → T + 273.15,
      {T, 5, 30}, PlotStyle → {Thickness[0.01], Pink}], *)
    Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
      0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /.  $\kappa$  → -0.81 /.  $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.
  ]
  ]
  ]

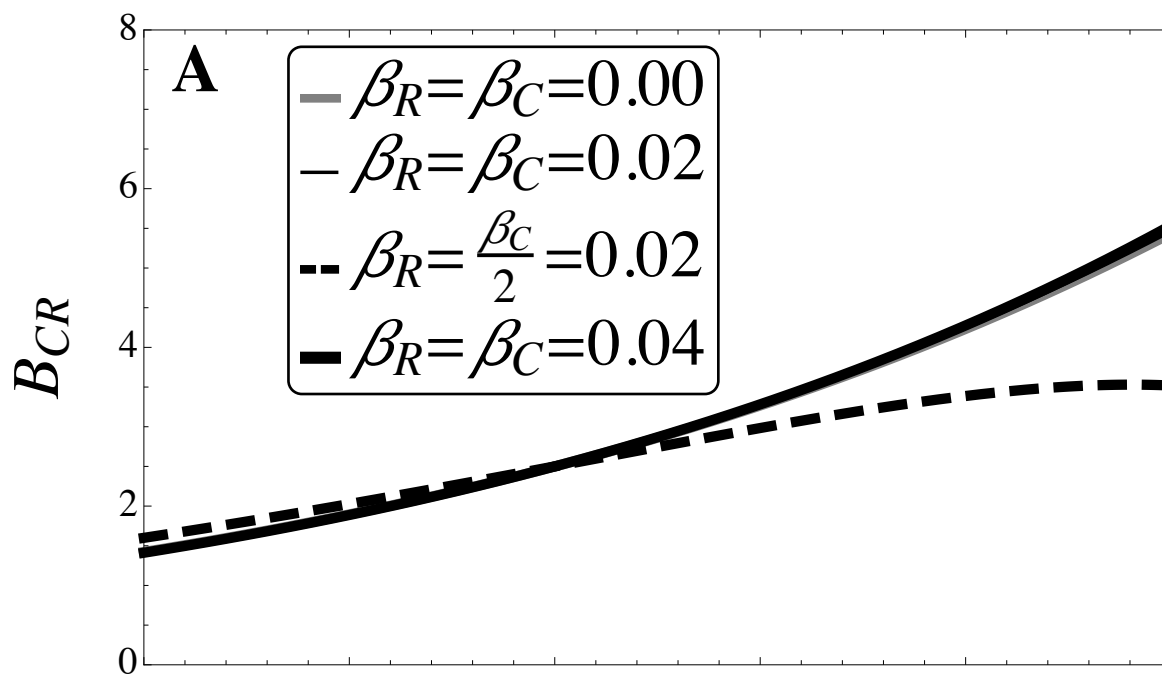
```



```

       $\rho \rightarrow -0.81$  /. TSR /.  $\beta[R] \rightarrow 0.02$  /.  $\beta[C] \rightarrow 0.04$  /.  $T[i_] \rightarrow T + 273.15$ ,
    {T, 5, 30}, PlotStyle → {Thickness[0.01], Black, Dashing[Large]},
    PlotRangePadding →
      None],
    Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
      0.32 /. ES → 0.9 /.  $k \rightarrow 8.62 \times 10^{-5}$  /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /.  $\kappa \rightarrow -0.81$  /.  $\alpha \rightarrow 1$  /.  $\epsilon \rightarrow -0.5$  /.  $\mu \rightarrow -0.29$  /.
       $\rho \rightarrow -0.81$  /. TSR /.  $\beta[R] \rightarrow 0.04$  /.  $\beta[C] \rightarrow 0.04$  /.  $T[i_] \rightarrow T + 273.15$ ,
    {T, 5, 30}, PlotStyle → {Thickness[0.01], Black},
    PlotRangePadding →
      None],
    PlotRange → {0, 8},
    Frame → True,
    FrameLabel → {Style[(*"Temperature (Celcius)"*)"", LabelSize],
      Style[(*"BCR"*)"", LabelSize], },
    FrameStyle → Directive[FontSize → TickSize],
    FrameTicksStyle → {{Black, Black}, {Directive[FontColor → White], Black}},
    ImagePadding → Pad,
    ImageSize → FigureSize,
    PlotRangePadding → None,
    PlotRangeClipping → False,
    Epilog → {
      Text[Style["A", LabelSize, Bold], Scaled@letpos],
      Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree]
    }
  ],
  Placed[
    LineLegend[{
      Directive[Gray, Thickness[0.25]],
      Directive[Black, Thickness[0.1]],
      Directive[Black, Dashing[Medium], Thickness[0.25]],
      Directive[Black, Thickness[0.35]]
    },
    {
      Style[" $\beta_R = \beta_C = 0.00$ ", LabelSize],
      Style[" $\beta_R = \beta_C = 0.02$ ", LabelSize],
      Style[" $\beta_R = \frac{\beta_C}{2} = 0.02$ ", LabelSize],
      Style[" $\beta_R = \beta_C = 0.04$ ", LabelSize]
    },
    LegendFunction → "Frame",
    LegendLayout → "Column"
  ],
  {0.35, 0.7}
]
]
Export[imagedir <> "BCRType1.pdf", %];

```



```
Show[
Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]},
Axes →
False],
Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Black, Thickness[0.005]},
Axes →
False],
Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /.
      TSR /. β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → T + 273.15, {T, 5, 30},
PlotStyle → {Black, Dashing[Large], Thickness[0.01]},
Axes →
False],
Plot[CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.04 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
Axes →
False],
PlotRange → {0, 0.6},
Frame → True,
FrameLabel → {Style[(*"Temperature (Celcius)"*)"", LabelSize],
  Style[(*"C:R"*)"", LabelSize], },
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle → {{Black, Black}, {Directive[FontColor → White], Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
PlotRangeClipping → False,
Epilog → {
  Text[Style["B", LabelSize, Bold], Scaled@letpos],
  Rotate[Text[Style["C:R", LabelSize], Scaled@ylabpos], 90 Degree]
}
]
```

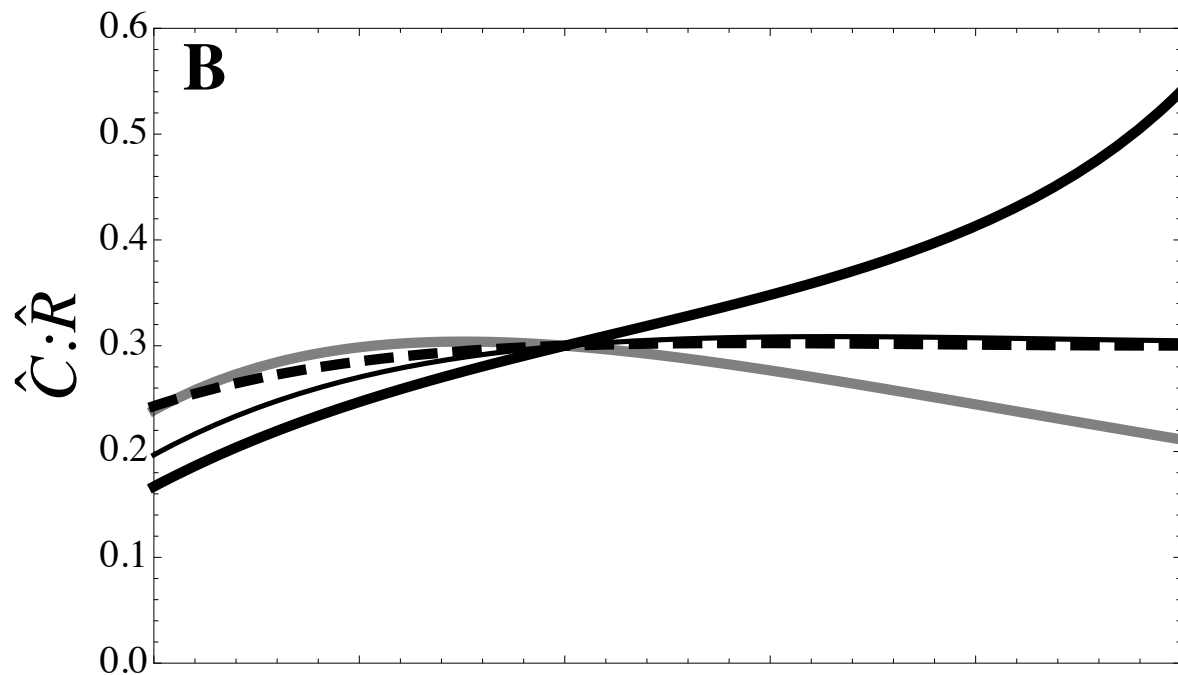
```
Export[imagedir <> "CtoRType1.pdf", %];
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

General::stop : Further output of Solve::ratnz will be suppressed during this calculation. >>



```
Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /.
    Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
    μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]},
  Axes →
    False, PlotRange →
      {0, All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] →
    0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thickness[0.005]},
  PlotRange →
    {0, All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] →
    0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Dashing[Large], Thickness[0.01]},
  PlotRange →
    {0, 0.7}],
  Plot[
    -Max[
      Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
        0.32 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
        v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
        ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.04 /. T[i_] → T + 273.15]],
      {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
      PlotRange →
        {0,
          0.7}],
    PlotRange → {0, 0.7},
    Frame → True,
    FrameLabel → {Style["Temperature (°C)", LabelSize],
      Style[(*"Stability"*)"", LabelSize], ,},
    FrameStyle → Directive[FontSize → TickSize],
    ImagePadding → Pad,
    ImageSize → FigureSize,
    PlotRangePadding → None,
    PlotRangeClipping → False,
    Epilog → {
      Text[Style["C", LabelSize, Bold], Scaled@letpos],
      Rotate[Text[Style["Stability (-λ)", LabelSize], Scaled@ylabpos], 90 Degree]
    }
  ]
]

Export[imagedir <> "StabilityType1.pdf", %];
```

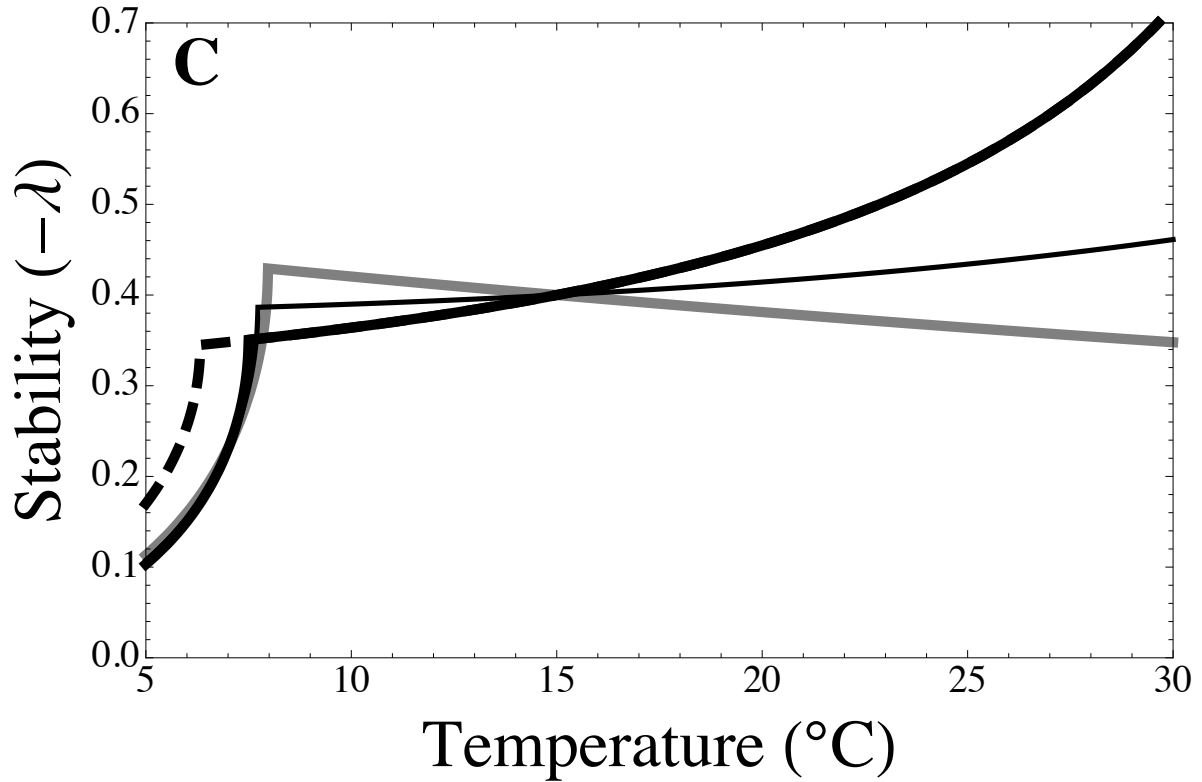


Figure I (D,E,F) [ES>EB, type-II]

```
(*Legended[*)
Show[
(*Plot[
BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.ES→0.9/.
k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→273.15+t,
{t,5,30},PlotStyle→{Black,Thickness[0.01]},
Axes→False],
(*Plot[
BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.32/.
k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→273.15+t,
{t,5,30},PlotStyle→{Black,Thickness[0.01],Dashing[Large]}],
Plot[
BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.9/.
k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→273.15+t,
{t,5,30},PlotStyle→{Gray,Thickness[0.01]}],*),
Plot[
BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.ES→0.9/.
k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
{T,5,30},PlotStyle→{Thickness[0.01],Red},
```

```

Axes→
False],*)
(*Plot[BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→
0.32/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.
α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
{T,5,30},PlotStyle→{Thickness[0.01],Red,Dashing[Large]}],
Plot[
BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.9/.
k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
{T,5,30},PlotStyle→{Thickness[0.01],Pink}],*)
Plot[Simplify[BCR2[T] /. Eq2[T] [[3]]] /. RallT /. RallM /. GilbertDeLongT /.
GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.32 /. ES → 0.9 /.
Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
ρ → -0.81 /. β[i_] → 0.00 /. T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /.
aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /.
h0 → 1 /. M15[i_] → 1, {t, 5, 30}, PlotStyle →
{Gray,
Thickness[
0.01]}], Axes → False],
Plot[Simplify[BCR2[T] /. Eq2[T] [[3]]] /. RallT /. RallM /. GilbertDeLongT /.
GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.32 /. ES → 0.9 /.
Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
ρ → -0.81 /. β[i_] → 0.02 /. T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /.
aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /.
h0 → 1 /. M15[i_] → 1, {t, 5, 30}, PlotStyle →
{Black,
Thickness[
0.005]}], Axes → False],
Plot[Simplify[BCR2[T] /. Eq2[T] [[3]]] /. RallT /. RallM /. GilbertDeLongT /.
GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.32 /. ES → 0.9 /.
Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
ρ → -0.81 /. β[R] → 0.02 /. β[C] → 0.04 /. T → 273.15 + t /.
aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /.
Ea → 0.65 /. Eh → -0.65 /. h0 → 1 /. M15[i_] →
1, {t, 5, 30}, PlotStyle →
{Black,
Dashing[
Large],
Thickness[0.01]}], Axes → False],
Plot[Simplify[BCR2[T] /. Eq2[T] [[3]]] /. RallT /. RallM /. GilbertDeLongT /.
GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.32 /. ES → 0.9 /.
Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
ρ → -0.81 /. β[i_] → 0.04 /. T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /.
aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /.
h0 → 1 /. M15[i_] → 1, {t, 5, 30}, PlotStyle →
{Black,

```

```

Thickness[
  0.01]], Axes → False],
(*Plot[Simplify[BCR2[T]/.Eq2[T][[3]]/.RallT/.RallM/.GilbertDeLongT/.
      GilbertDeLongM/.TSR/.ah15/.el5/.k15/.r15/.m15/.
      T[i_]→T/.k→8.62*10-5/.EB→0.9/.ES→0.32/.Em→0.65/.
      κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.β[i_]→0.02/.
      T→273.15+t/.aC→1/4+2/3/.aR→1/3/.hC→-2/3/.hR→0.5/.Ea→0.65/.
      Eh→-0.65/.h0→1/.M15[i_]→1,{t,5,30},PlotStyle→{Darker[
      Blue],
      Thickness[
      0.01],Dashing[
      Large]}}],
Plot[Simplify[BCR2[T]/.Eq2[T][[3]]/.RallT/.RallM/.GilbertDeLongT/.
      GilbertDeLongM/.TSR/.ah15/.el5/.k15/.r15/.m15/.
      T[i_]→T/.k→8.62*10-5/.EB→0.9/.ES→0.9/.Em→0.65/.
      κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.β[i_]→0.02/.
      T→273.15+t/.aC→1/4+2/3/.aR→1/3/.hC→-2/3/.hR→0.5/.Ea→0.65/.
      Eh→-0.65/.h0→1/.M15[i_]→1,{t,5,30},PlotStyle→{Lighter[
      Blue],
      Thickness[
      0.01]}}],*)
PlotRange → {0, 8},
Frame → True,
FrameLabel → {Style[(*"Temperature (Celcius)"*)"", LabelSize],
  Style[(*"BCR"*)"", LabelSize], ,},
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle →
  {{Directive[FontColor → White], Black}, {Directive[FontColor → White], Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
(*PlotRangeClipping→False,*)
Epilog → {
  Text[Style["D", LabelSize, Bold], Scaled@letpos],
  (*Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree]*)
}
] (*,
Placed[
  LineLegend[{
    Directive[Black,Dashing[Medium],Thickness[0.25]],
    Directive[Black,Thickness[0.25]],
    Directive[Gray,Thickness[0.25]],
    Directive[Gray,Dashing[Medium],Thickness[0.25]]
  },
  {
    Style["βR=βC=0.00, type-II",LabelSize],
    Style["βR=βC=0.02, type-II",LabelSize],
    Style["βR= $\frac{\beta_C}{2}$ =0.02, type-II",LabelSize],
    Style["βR=βC=0.04, type-II",LabelSize]
  },

```

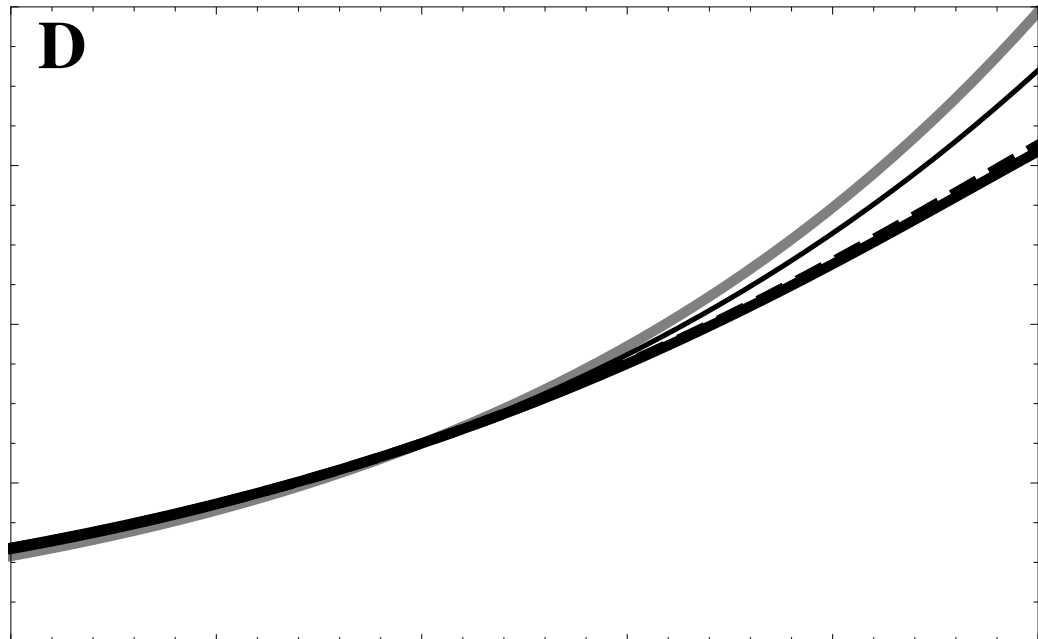


```

LegendFunction->"Frame",
LegendLayout->"Column"
],
{0.35,0.7}
]
]*)

Export[imagedir <> "BCRType2.pdf", %];

```



```

Show[
(*Plot[
CR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB->0.32/.ES->0.9/.
k->8.62*10-5/.Em->0.65/.Ev[i_-]>0.46/.v0[i_-]>1/.κ->-0.81/.α->1/.
ε->-0.5/.μ->-0.29/.ρ->-0.81/.TSR/.β[i_-]>0/.T[i_-]>T+273.15,
{T,5,30},PlotStyle->{Black,Thickness[0.01]},
Axes->False,
PlotRange->{0,All}],
Plot[
CR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB->0.32/.ES->0.9/.
k->8.62*10-5/.Em->0.65/.Ev[i_-]>0.46/.v0[i_-]>1/.κ->-0.81/.α->1/.
ε->-0.5/.μ->-0.29/.ρ->-0.81/.TSR/.β[i_-]>0.02/.T[i_-]>T+273.15,
{T,5,30},PlotStyle->{Red,Thickness[0.01]},
Axes->
False,PlotRange->
{0,All}],*)

```

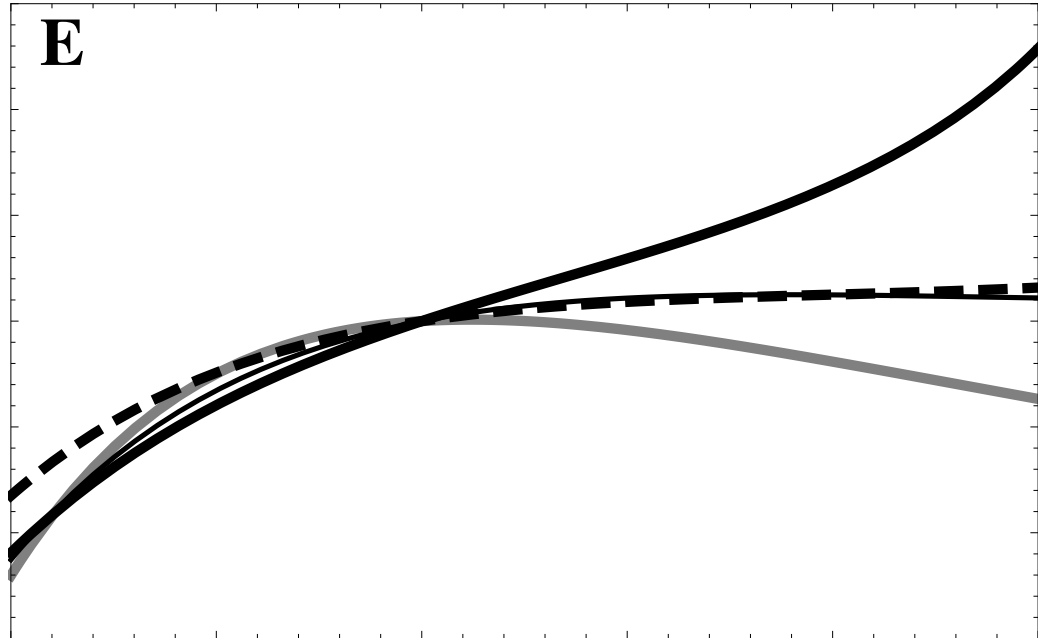
```

Plot[CR2[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.32 /. ES → 0.9 /. Em → 0.65 /.  $\kappa$  → -0.81 /.
       $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[i_]$  → 0.00 /.
      T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /.
M15[i_] → 1, {t, 5, 30}, PlotStyle → {Gray,
  Thickness[
    0.01]}, PlotRange → {0, All}],
Plot[CR2[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.32 /. ES → 0.9 /. Em → 0.65 /.  $\kappa$  → -0.81 /.
       $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[i_]$  → 0.02 /.
      T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /.
M15[i_] → 1, {t, 5, 30}, PlotStyle → {Black,
  Thickness[
    0.005]}, PlotRange → {0, All}],
Plot[CR2[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.32 /. ES → 0.9 /. Em → 0.65 /.  $\kappa$  → -0.81 /.  $\alpha$  → 1 /.
       $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[R]$  → 0.02 /.  $\beta[C]$  → 0.04 /.
      T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /.
M15[i_] → 1, {t, 5, 30}, PlotStyle →
{Black,
  Dashing[
    Large],
  Thickness[0.01]}, PlotRange → {0, All}],
Plot[CR2[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.32 /. ES → 0.9 /. Em → 0.65 /.  $\kappa$  → -0.81 /.
       $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[i_]$  → 0.04 /.
      T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /.
M15[i_] → 1, {t, 5, 30}, PlotStyle → {Black,
  Thickness[
    0.01]}, PlotRange → {0, All}],
PlotRange → {0, 0.6},
Frame → True,
FrameLabel → {Style[(*"Temperature (Celcius)"*)"", LabelSize],
  Style[(*"C:R"*)"", LabelSize], },
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle →
  {{Directive[FontColor → White], Black}, {Directive[FontColor → White], Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
Epilog → {
  Text[Style["E", LabelSize, Bold], Scaled@letpos]
}

```

```
}
]
```

```
Export[imagedir <> "CtoRType2.pdf", %];
```



```
Show[(*Plot[
  -Max[Re[lambda/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.
    ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.
    α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→T+273.15]],
  {T,5,30},PlotStyle→{Black,Thickness[0.01]},
  Axes→
  False,PlotRange→
  {0,All}],Plot[
  -Max[Re[lambda/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.
    ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.
    α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15]],
  {T,5,30},PlotStyle→{Red,Thickness[0.01]},PlotRange→
  {0,All}],*)
Plot[
  -Max[
    Re[lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.32 /. ES → 0.9 /. Em → 0.65 /. κ → -0.81 /. α → 1 /.
      ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. β[i_] → 0.00 /.
      T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /. M15[i_] → 1]],
  {t, 5, 30}, PlotStyle → {Gray, Thickness[0.01]}, PlotRange →
```

```

All],
Plot[
  -Max[
    Re[lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] -> T /. k -> 8.62 * 10-5 /.
      EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /. κ -> -0.81 /. α -> 1 /.
      ε -> -0.5 /. μ -> -0.29 /. ρ -> -0.81 /. β[i_] -> 0.02 /.
      T -> 273.15 + t /. aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /. hC -> -2 / 3 /.
      hR -> 0.5 /. Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 10-13 /. M15[i_] -> 1]],
    {t, 5, 30}, PlotStyle -> {Black, Thickness[0.005]}, PlotRange ->
    All],
Plot[
  -Max[
    Re[lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] -> T /.
      k -> 8.62 * 10-5 /. EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /.
      κ -> -0.81 /. α -> 1 /. ε -> -0.5 /. μ -> -0.29 /.
      ρ -> -0.81 /. β[R] -> 0.02 /. β[C] -> 0.04 /. T -> 273.15 + t /.
      aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /. hC -> -2 / 3 /. hR -> 0.5 /.
      Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 10-13 /. M15[i_] -> 1]],
    {t, 5, 30}, PlotStyle -> {Black, Dashing[Large],
      Thickness[
        0.01]}, PlotRange -> All],
Plot[
  -Max[
    Re[
      lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] -> T /. k -> 8.62 * 10-5 /.
      EB -> 0.32 /. ES -> 0.9 /. Em -> 0.65 /. κ -> -0.81 /. α -> 1 /.
      ε -> -0.5 /. μ -> -0.29 /. ρ -> -0.81 /. β[i_] -> 0.04 /.
      T -> 273.15 + t /. aC -> 1 / 4 + 2 / 3 /. aR -> 1 / 3 /. hC -> -2 / 3 /.
      hR -> 0.5 /. Ea -> 0.65 /. Eh -> -0.65 /. h0 -> 10-13 /. M15[i_] -> 1]],
    {t, 5, 30}, PlotStyle -> {Black, Thickness[
      0.01]},
    PlotRange -> All],
PlotRange -> {0, 0.7},
Frame -> True,
FrameLabel -> {Style["Temperature (°C)", LabelSize],
  Style[(*"Stability"*)"", LabelSize], },
FrameStyle -> Directive[FontSize -> TickSize],
FrameTicksStyle -> {{Directive[FontColor -> White], Black}, {Black, Black}},
ImagePadding -> Pad,
ImageSize -> FigureSize,
PlotRangePadding -> None,
Epilog -> {
  Text[Style["F", LabelSize, Bold], Scaled@letpos]
},
PlotRange -> All
]

```

```
Export[imagedir <> "StabilityType2.pdf", %];
```

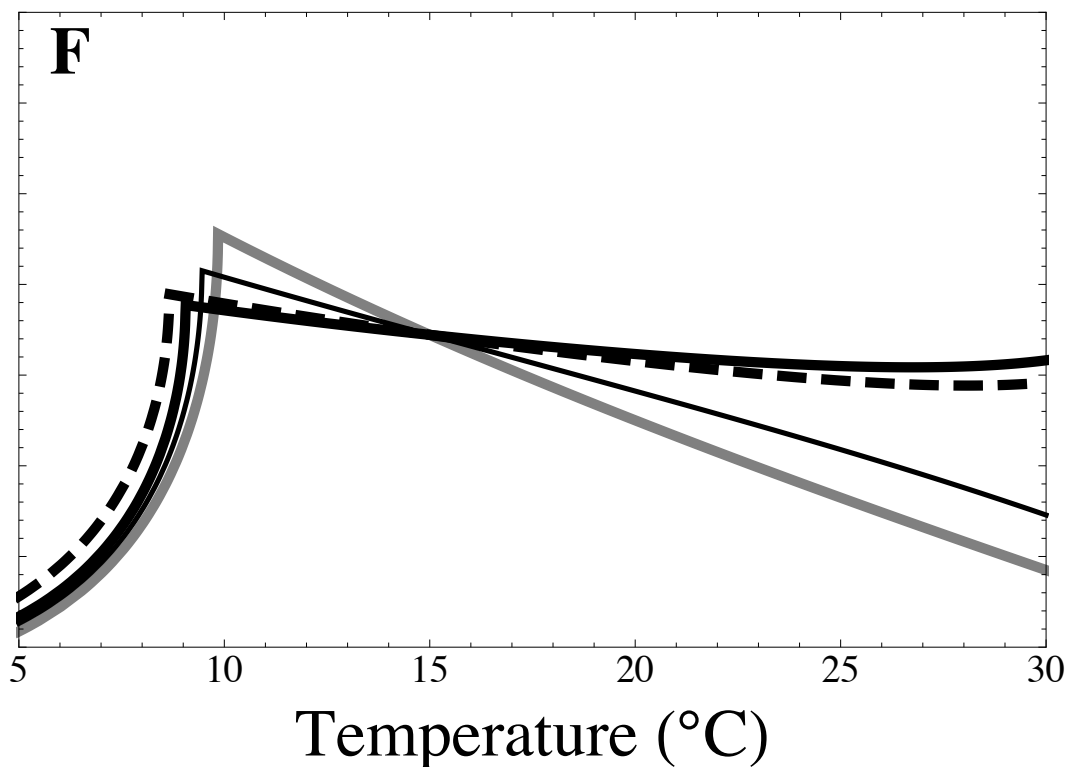


Figure SI (A,B,C) [ES<EB, type-I]

```

Legended[
  Show[
    (*Plot[
      BCR[T]/.K[T]→K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ]/.T[i_]→T+273.15/.K15/.k→8.62*10-5/.a[T]→0.1/.
      e[T]→0.15/.m[T]→0.6/.r[T]→2/.EB→0.32/.ES→0.9,
      {T,5,30},PlotStyle→{Gray,Thickness[0.01]}],*)
    Plot[
      BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.
      EB→0.9/.ES→0.32/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.
      v0[i_]→1/.κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.
      ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→T+273.15,
      {T, 5, 30}, PlotStyle→{Thickness[0.01], Gray},
      Axes→
        False, PlotRangePadding→
        None],
    (*Plot[BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.
      ES→0.32/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.

```

```

 $\alpha \rightarrow 1 / . \epsilon \rightarrow -0.5 / . \mu \rightarrow -0.29 / . \rho \rightarrow -0.81 / . \text{TSR} / . \beta[i\_]\rightarrow 0 / . T[i\_]\rightarrow T+273.15,$ 
{ T, 5, 30}, PlotStyle  $\rightarrow$  {Thickness[0.01], Black, Dashing[Large]}},
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB  $\rightarrow$  0.9 /.
ES  $\rightarrow$  0.9 /. k  $\rightarrow$   $8.62 \cdot 10^{-5}$  /. Em  $\rightarrow$  0.65 /. Ev[i_]  $\rightarrow$  0.46 /. v0[i_]  $\rightarrow$  1 /.
 $\kappa \rightarrow -0.81 / . \alpha \rightarrow 1 / . \epsilon \rightarrow -0.5 / . \mu \rightarrow -0.29 / . \rho \rightarrow -0.81 / . \text{TSR} / . \beta[i\_]\rightarrow 0 / .$ 
T[i_]  $\rightarrow$  T+273.15, {T, 5, 30}, PlotStyle  $\rightarrow$  {Thick, Gray}], *)
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
EB  $\rightarrow$  0.9 /. ES  $\rightarrow$  0.32 /. k  $\rightarrow$   $8.62 \cdot 10^{-5}$  /. Em  $\rightarrow$  0.65 /. Ev[i_]  $\rightarrow$  0.46 /.
v0[i_]  $\rightarrow$  1 /.  $\kappa \rightarrow -0.81 / . \alpha \rightarrow 1 / . \epsilon \rightarrow -0.5 / . \mu \rightarrow -0.29 / .$ 
 $\rho \rightarrow -0.81 / . \text{TSR} / . \beta[i_] \rightarrow 0.02 / . T[i_] \rightarrow T+273.15,$ 
{ T, 5, 30}, PlotStyle  $\rightarrow$  {Thickness[0.005], Black},
PlotRangePadding  $\rightarrow$ 
None],
(*Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB  $\rightarrow$  0.9 /.
ES  $\rightarrow$  0.32 /. k  $\rightarrow$   $8.62 \cdot 10^{-5}$  /. Em  $\rightarrow$  0.65 /. Ev[i_]  $\rightarrow$  0.46 /. v0[i_]  $\rightarrow$  1 /.  $\kappa \rightarrow -0.81 / .$ 
 $\alpha \rightarrow 1 / . \epsilon \rightarrow -0.5 / . \mu \rightarrow -0.29 / . \rho \rightarrow -0.81 / . \text{TSR} / . \beta[i_] \rightarrow 0.02 / . T[i_] \rightarrow T+273.15,$ 
{ T, 5, 30}, PlotStyle  $\rightarrow$  {Thickness[0.01], Red, Dashing[Large]}], Plot[
BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB  $\rightarrow$  0.9 /. ES  $\rightarrow$  0.9 /.
k  $\rightarrow$   $8.62 \cdot 10^{-5}$  /. Em  $\rightarrow$  0.65 /. Ev[i_]  $\rightarrow$  0.46 /. v0[i_]  $\rightarrow$  1 /.  $\kappa \rightarrow -0.81 / . \alpha \rightarrow 1 / .$ 
 $\epsilon \rightarrow -0.5 / . \mu \rightarrow -0.29 / . \rho \rightarrow -0.81 / . \text{TSR} / . \beta[i_] \rightarrow 0.02 / . T[i_] \rightarrow T+273.15,$ 
{ T, 5, 30}, PlotStyle  $\rightarrow$  {Thickness[0.01], Pink}], *)
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB  $\rightarrow$ 
0.9 /. ES  $\rightarrow$  0.32 /. k  $\rightarrow$   $8.62 \cdot 10^{-5}$  /. Em  $\rightarrow$  0.65 /. Ev[i_]  $\rightarrow$  0.46 /.
v0[i_]  $\rightarrow$  1 /.  $\kappa \rightarrow -0.81 / . \alpha \rightarrow 1 / . \epsilon \rightarrow -0.5 / . \mu \rightarrow -0.29 / .$ 
 $\rho \rightarrow -0.81 / . \text{TSR} / . \beta[R] \rightarrow 0.02 / . \beta[C] \rightarrow 0.04 / . T[i_] \rightarrow T+273.15,$ 
{ T, 5, 30}, PlotStyle  $\rightarrow$  {Thickness[0.01], Black, Dashing[Large]}],
PlotRangePadding  $\rightarrow$ 
None],
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB  $\rightarrow$ 
0.9 /. ES  $\rightarrow$  0.32 /. k  $\rightarrow$   $8.62 \cdot 10^{-5}$  /. Em  $\rightarrow$  0.65 /. Ev[i_]  $\rightarrow$  0.46 /.
v0[i_]  $\rightarrow$  1 /.  $\kappa \rightarrow -0.81 / . \alpha \rightarrow 1 / . \epsilon \rightarrow -0.5 / . \mu \rightarrow -0.29 / .$ 
 $\rho \rightarrow -0.81 / . \text{TSR} / . \beta[R] \rightarrow 0.04 / . \beta[C] \rightarrow 0.04 / . T[i_] \rightarrow T+273.15,$ 
{ T, 5, 30}, PlotStyle  $\rightarrow$  {Thickness[0.01], Black},
PlotRangePadding  $\rightarrow$ 
None],
PlotRange  $\rightarrow$  {0, 10},
Frame  $\rightarrow$  True,
FrameLabel  $\rightarrow$  {Style[(*"Temperature (Celcius)"*)"", LabelSize],
Style[(*"BCR"*)"", LabelSize], },
FrameStyle  $\rightarrow$  Directive[FontSize  $\rightarrow$  TickSize],
FrameTicksStyle  $\rightarrow$  {{Black, Black}, {Directive[FontColor  $\rightarrow$  White], Black}},
ImagePadding  $\rightarrow$  Pad,
ImageSize  $\rightarrow$  FigureSize,
PlotRangePadding  $\rightarrow$  None,
PlotRangeClipping  $\rightarrow$  False,
Epilog  $\rightarrow$  {
Text[Style["A", LabelSize, Bold], Scaled@letpos],
Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree]
}
],

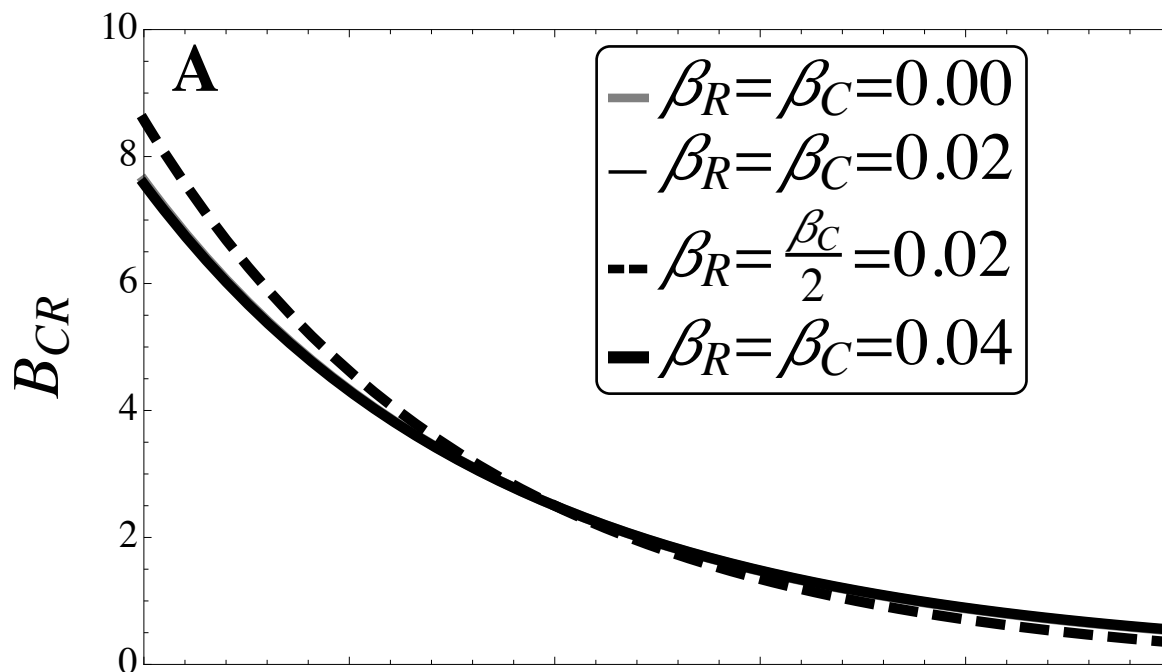
```

```

Placed[
  LineLegend[{
    Directive[Gray, Thickness[0.25]],
    Directive[Black, Thickness[0.1]],
    Directive[Black, Dashing[Medium], Thickness[0.25]],
    Directive[Black, Thickness[0.35]]
  },
  {
    Style[" $\beta_R=\beta_C=0.00$ ", LabelSize],
    Style[" $\beta_R=\beta_C=0.02$ ", LabelSize],
    Style[" $\beta_R=\frac{\beta_C}{2}=0.02$ ", LabelSize],
    Style[" $\beta_R=\beta_C=0.04$ ", LabelSize]
  },
  LegendFunction -> "Frame",
  LegendLayout -> "Column"
],
{0.65, 0.7}
]
]

```

```
(*Export[imagedir<>"BCRType1.pdf",%];*)
```



```

Show[
Plot[
CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /.
      ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.
      κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
      β[i_] → 0 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle →
{Gray, Thickness[0.01]}, Axes → False, PlotRange →
{0, All}],
Plot[ CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Black, Thickness[0.005]},
Axes →
False, PlotRange →
{0, All}],
Plot[ CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /.
      TSR /. β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → T + 273.15, {T, 5, 30},
PlotStyle → {Black, Dashing[Large], Thickness[0.01]},
Axes →
False, PlotRange →
{0, All}],
Plot[ CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.04 /. T[i_] → T + 273.15,
{T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
Axes →
False, PlotRange →
{0, All}],
PlotRange → {0, 0.4},
Frame → True,
FrameLabel → {Style[(*"Temperature (Celcius)"*)"", LabelSize],
Style[(*"C:R"*)"", LabelSize], },
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle → {{Black, Black}, {Directive[FontColor → White], Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
PlotRangeClipping → False,
Epilog → {
Text[Style["B", LabelSize, Bold], Scaled@letpos],
Rotate[Text[Style["C:R", LabelSize], Scaled@ylabpos], 90 Degree]
}
]

(*Export[imagedir<>"CtoRType1.pdf",%];*)

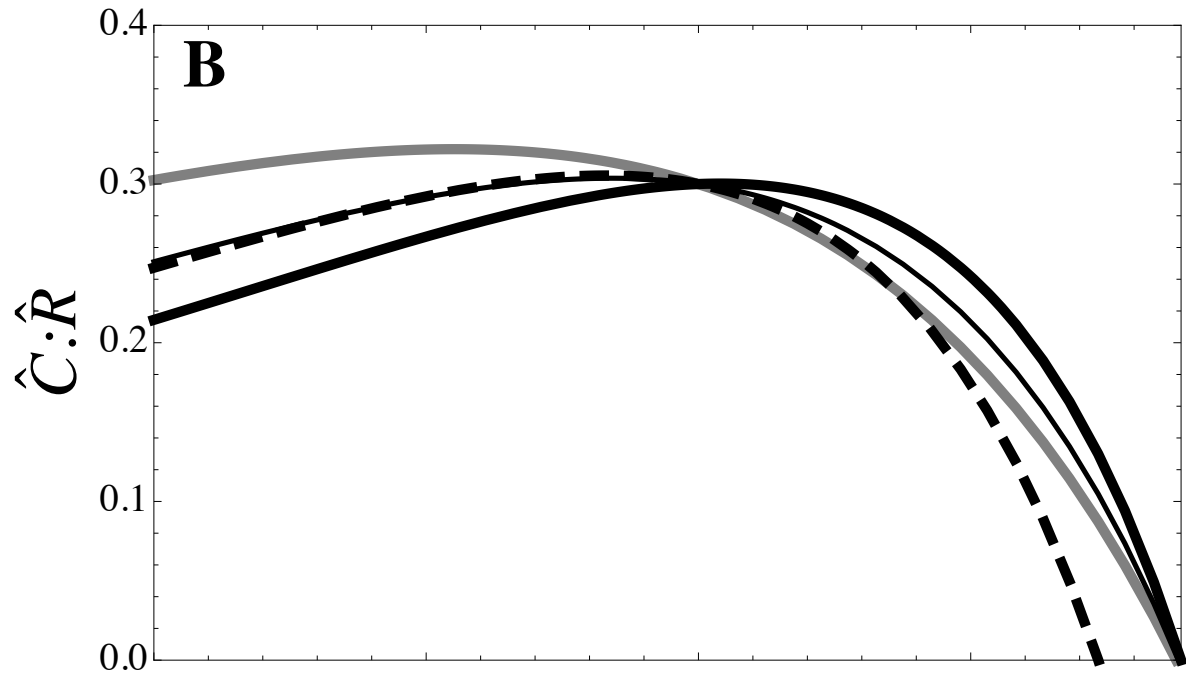
```


Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

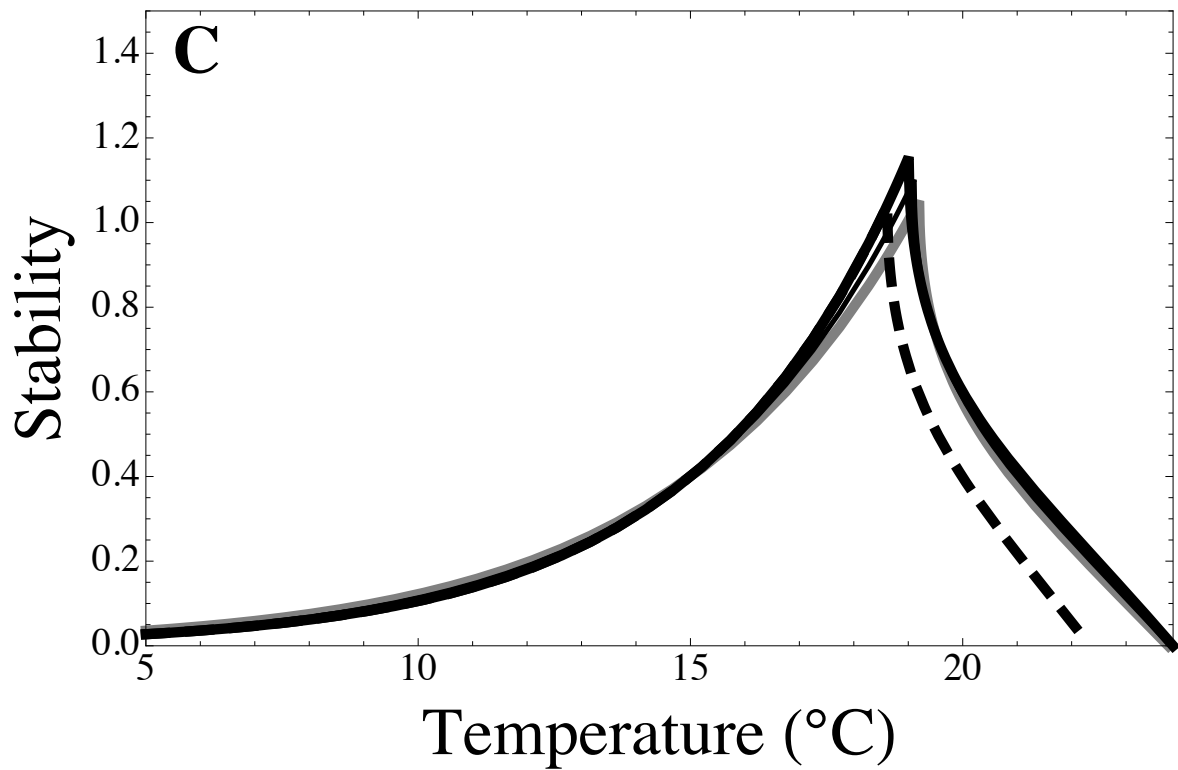
Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

General::stop : Further output of Solve::ratnz will be suppressed during this calculation. >>



```
Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /.
    Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
    μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]},
  Axes →
    False, PlotRange →
      {0, All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] →
    0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thickness[0.005]},
  PlotRange →
    {0, All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] →
    0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Dashing[Large], Thickness[0.01]},
  PlotRange →
    {0, All}],
Plot[
  -Max[
    Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
      0.9 /. ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.04 /. T[i_] → T + 273.15]],
    {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
    PlotRange →
      {0,
        All}],
PlotRange → {0, 1.5},
Frame → True,
FrameLabel → {Style["Temperature (°C)", LabelSize],
  Style[(*"Stability"*)"", LabelSize], ,},
FrameStyle → Directive[FontSize → TickSize],
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
PlotRangeClipping → False,
Epilog → {
  Text[Style["C", LabelSize, Bold], Scaled@letpos],
  Rotate[Text[Style["Stability", LabelSize], Scaled@ylabpos], 90 Degree]
}
]

(*Export[imagedir<"StabilityType1.pdf",%];*)
```



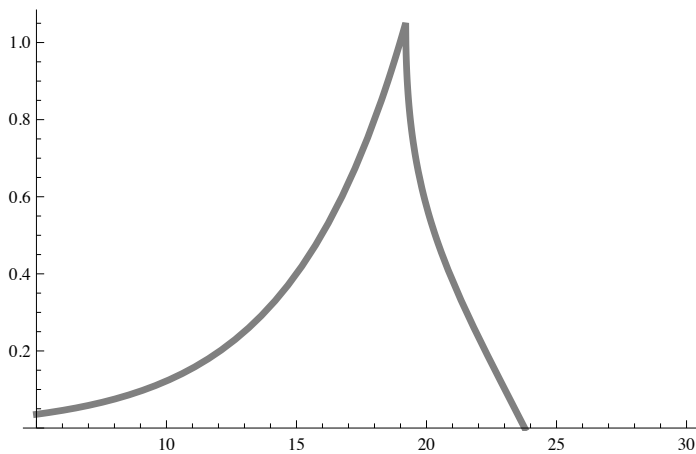
Paradox of enrichment in reverse (why stability increases and then declines)

We see the sudden drop in stability around 19 degrees C because this is where population cycles end (ie, where the eigenvalues become real). Below 19C the carrying capacity K is relatively large and there are population cycles. Decreasing K in this region increases stability (paradox of enrichment in reverse). Above 19C K is relatively small and there are no population cycles. Decreasing K in this region decreases stability.

```
Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /.
    Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
    μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]},
  PlotRange →
  {0,
    All}]
```

```
Plot[
  -Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
    0.9 /. ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
    v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15],
  {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]},
  PlotRange →
  {0,
    All}]
```

```
Plot[Im[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
  EB → 0.9 /. ES → 0.32 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
  v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
  ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15],
  {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]},
  PlotRange →
  {0,
    All}]
```



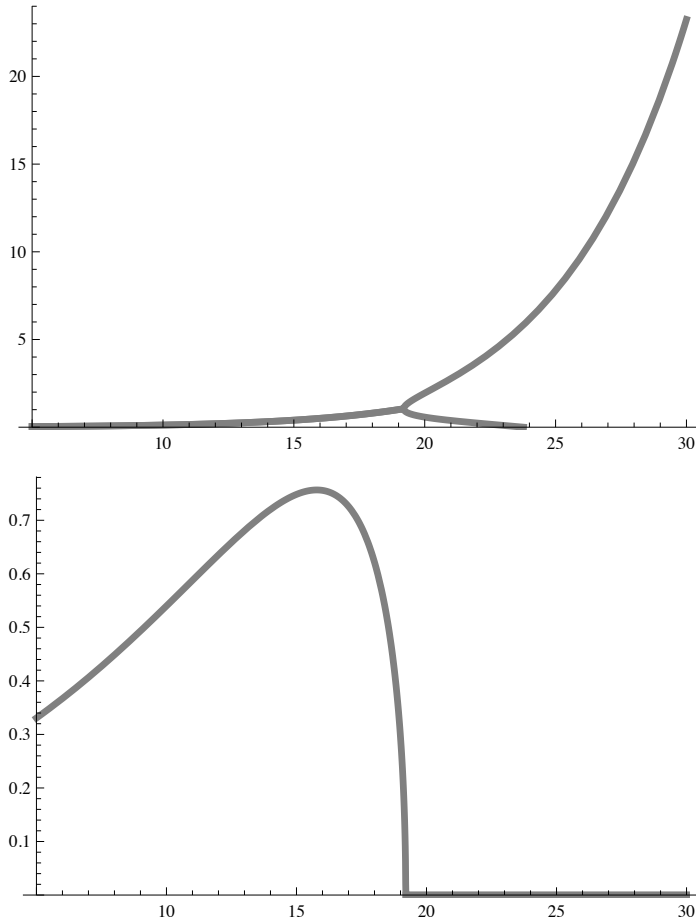


Figure SI (D,E,F) [ES<EB, type-II]

```
(*Legended[*)
Show[
(*Plot[
BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.ES→0.9/.
k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→273.15+t,
{t,5,30},PlotStyle→{Black,Thickness[0.01]},
Axes→False],
(*Plot[
BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.32/.
k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→273.15+t,
{t,5,30},PlotStyle→{Black,Thickness[0.01],Dashing[Large]}],
Plot[
BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.9/.
k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→273.15+t,
{t,5,30},PlotStyle→{Gray,Thickness[0.01]}],*)
Plot[
```

```

BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.ES→0.9/.
      k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
      ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
{ T, 5, 30}, PlotStyle→{Thickness[0.01], Red},
Axes→
  False], *)
(*Plot[BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→
      0.32/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.
      α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
{ T, 5, 30}, PlotStyle→{Thickness[0.01], Red, Dashing[Large]}],
Plot[
  BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.9/.
      k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
      ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
{ T, 5, 30}, PlotStyle→{Thickness[0.01], Pink}], *)
Plot[Simplify[BCR2[T] /. Eq2[T] [[3]]] /. RallT /. RallM /. GilbertDeLongT /.
      GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
      m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.9 /. ES → 0.32 /.
      Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. β[i_] → 0.00 /. T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /.
      aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /.
      h0 → 1 /. M15[i_] → 1, {t, 5, 30}, PlotStyle →
{Gray,
  Thickness[
    0.01]], Axes → False],
Plot[Simplify[BCR2[T] /. Eq2[T] [[3]]] /. RallT /. RallM /. GilbertDeLongT /.
      GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
      m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.9 /. ES → 0.32 /.
      Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. β[i_] → 0.02 /. T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /.
      aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /.
      h0 → 1 /. M15[i_] → 1, {t, 5, 30}, PlotStyle →
{Black,
  Thickness[
    0.005]], Axes → False],
Plot[Simplify[BCR2[T] /. Eq2[T] [[3]]] /. RallT /. RallM /. GilbertDeLongT /.
      GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
      m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.9 /. ES → 0.32 /.
      Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. β[R] → 0.02 /. β[C] → 0.04 /. T → 273.15 + t /.
      aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /.
      Ea → 0.65 /. Eh → -0.65 /. h0 → 1 /. M15[i_] →
      1, {t, 5, 30}, PlotStyle →
{Black,
  Dashing[
    Large],
  Thickness[0.01]], Axes → False],
Plot[Simplify[BCR2[T] /. Eq2[T] [[3]]] /. RallT /. RallM /. GilbertDeLongT /.
      GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
      m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.9 /. ES → 0.32 /.
      Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.

```

```

       $\rho \rightarrow -0.81 / . \beta[i\_] \rightarrow 0.04 / . T \rightarrow 273.15 + t / . aC \rightarrow 1 / 4 + 2 / 3 / .$ 
       $aR \rightarrow 1 / 3 / . hC \rightarrow -2 / 3 / . hR \rightarrow 0.5 / . Ea \rightarrow 0.65 / . Eh \rightarrow -0.65 / .$ 
       $h0 \rightarrow 1 / . M15[i\_] \rightarrow 1, \{t, 5, 30\}, PlotStyle \rightarrow$ 
      {Black,
      Thickness[
      0.01]}, Axes  $\rightarrow$  False],
(*Plot[Simplify[BCR2[T]/.Eq2[T][[3]]]/.RallT/.RallM/.GilbertDeLongT/.
      GilbertDeLongM/.TSR/.ah15/.e15/.k15/.r15/.m15/.
      T[i_]  $\rightarrow$  T/.k $\rightarrow$ 8.62*10-5/.EB $\rightarrow$ 0.9/.ES $\rightarrow$ 0.32/.Em $\rightarrow$ 0.65/.
       $\kappa \rightarrow -0.81 / . \alpha \rightarrow 1 / . \epsilon \rightarrow -0.5 / . \mu \rightarrow -0.29 / . \rho \rightarrow -0.81 / . \beta[i_] \rightarrow 0.02 / .$ 
       $T \rightarrow 273.15 + t / . aC \rightarrow 1 / 4 + 2 / 3 / . aR \rightarrow 1 / 3 / . hC \rightarrow -2 / 3 / . hR \rightarrow 0.5 / . Ea \rightarrow 0.65 / .$ 
       $Eh \rightarrow -0.65 / . h0 \rightarrow 1 / . M15[i_] \rightarrow 1, \{t, 5, 30\}, PlotStyle \rightarrow$  {Darker[
      Blue],
      Thickness[
      0.01], Dashing[
      Large]}],
Plot[Simplify[BCR2[T]/.Eq2[T][[3]]]/.RallT/.RallM/.GilbertDeLongT/.
      GilbertDeLongM/.TSR/.ah15/.e15/.k15/.r15/.m15/.
      T[i_]  $\rightarrow$  T/.k $\rightarrow$ 8.62*10-5/.EB $\rightarrow$ 0.9/.ES $\rightarrow$ 0.9/.Em $\rightarrow$ 0.65/.
       $\kappa \rightarrow -0.81 / . \alpha \rightarrow 1 / . \epsilon \rightarrow -0.5 / . \mu \rightarrow -0.29 / . \rho \rightarrow -0.81 / . \beta[i_] \rightarrow 0.02 / .$ 
       $T \rightarrow 273.15 + t / . aC \rightarrow 1 / 4 + 2 / 3 / . aR \rightarrow 1 / 3 / . hC \rightarrow -2 / 3 / . hR \rightarrow 0.5 / . Ea \rightarrow 0.65 / .$ 
       $Eh \rightarrow -0.65 / . h0 \rightarrow 1 / . M15[i_] \rightarrow 1, \{t, 5, 30\}, PlotStyle \rightarrow$  {Lighter[
      Blue],
      Thickness[
      0.01]}], *)
PlotRange  $\rightarrow$  {0, 10},
Frame  $\rightarrow$  True,
FrameLabel  $\rightarrow$  {Style[(*"Temperature (Celcius)"*)"", LabelSize],
      Style[(*"BCR"*)"", LabelSize], ,},
FrameStyle  $\rightarrow$  Directive[FontSize  $\rightarrow$  TickSize],
FrameTicksStyle  $\rightarrow$ 
  {{Directive[FontColor  $\rightarrow$  White], Black}, {Directive[FontColor  $\rightarrow$  White], Black}},
ImagePadding  $\rightarrow$  Pad,
ImageSize  $\rightarrow$  FigureSize,
PlotRangePadding  $\rightarrow$  None,
(*PlotRangeClipping $\rightarrow$ False,*)
Epilog  $\rightarrow$  {
  Text[Style["D", LabelSize, Bold], Scaled@letpos],
  (*Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree]*)
}
] (*,
Placed[
  LineLegend[{
    Directive[Black, Dashing[Medium], Thickness[0.25]],
    Directive[Black, Thickness[0.25]],
    Directive[Gray, Thickness[0.25]],
    Directive[Gray, Dashing[Medium], Thickness[0.25]]
  },
  {
    Style[" $\beta_R = \beta_C = 0.00$ , type-II", LabelSize],
    Style[" $\beta_R = \beta_C = 0.02$ , type-II", LabelSize],
  }

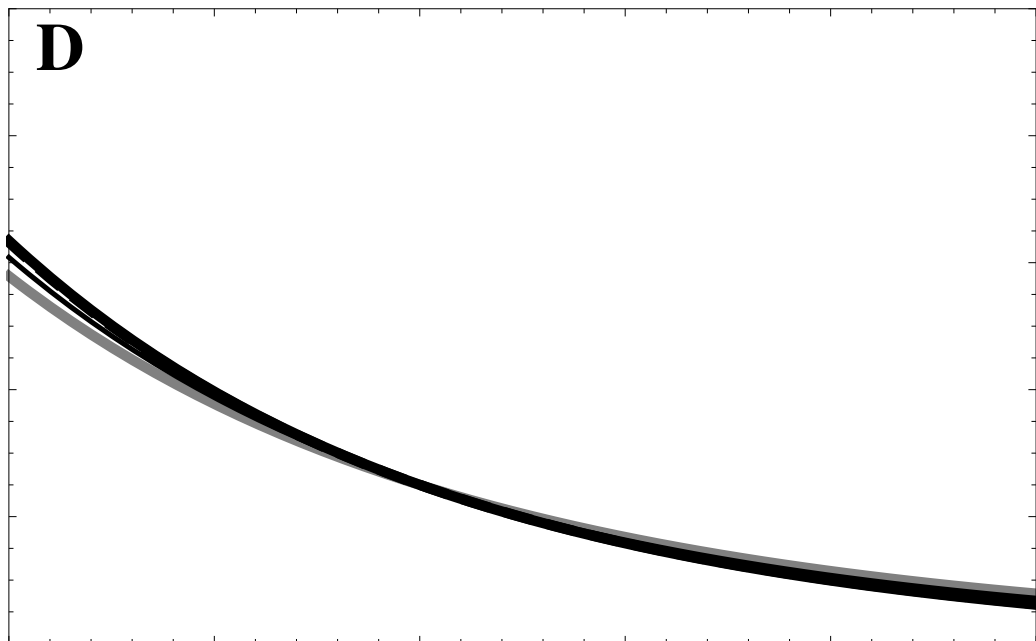
```

```

Style[" $\beta_R = \frac{\beta_C}{2} = 0.02$ , type-II", LabelSize],
Style[" $\beta_R = \beta_C = 0.04$ , type-II", LabelSize]
},
LegendFunction->"Frame",
LegendLayout->"Column"
],
{0.35, 0.7}
]
] *)

(*Export[imageDir<>"BCRType2.pdf", %]; *)

```



```

Show[
(*Plot[
CR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB->0.32/.ES->0.9/.
k->8.62*10^-5/.Em->0.65/.Ev[i_]->0.46/.v0[i_]->1/.x->-0.81/.alpha->1/.
epsilon->-0.5/.mu->-0.29/.rho->-0.81/.TSR/.beta[i_]->0/.T[i_]->T+273.15,
{T, 5, 30}, PlotStyle->{Black, Thickness[0.01]},
Axes->False,
PlotRange->{0, All}],
Plot[
CR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB->0.32/.ES->0.9/.
k->8.62*10^-5/.Em->0.65/.Ev[i_]->0.46/.v0[i_]->1/.x->-0.81/.alpha->1/.
epsilon->-0.5/.mu->-0.29/.rho->-0.81/.TSR/.beta[i_]->0.02/.T[i_]->T+273.15,

```



```

{t, 5, 30}, PlotStyle → {Red, Thickness[0.01]},
Axes →
  False, PlotRange →
    {0, All}], *)
Plot[CR2[t] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.9 /. ES → 0.32 /. Em → 0.65 /.  $\kappa$  → -0.81 /.
       $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[i_]$  → 0.00 /.
      T → 273.15 + t /. aC → 1/4 + 2/3 /. aR → 1/3 /. hC → -2/3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /.
M15[i_] → 1, {t, 5, 30}, PlotStyle →
  {Gray,
  Thickness[
    0.01]}, PlotRange → {0, All}],
Plot[CR2[t] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.9 /. ES → 0.32 /. Em → 0.65 /.  $\kappa$  → -0.81 /.
       $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[i_]$  → 0.02 /.
      T → 273.15 + t /. aC → 1/4 + 2/3 /. aR → 1/3 /. hC → -2/3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /.
M15[i_] → 1, {t, 5, 30}, PlotStyle → {Black,
  Thickness[
    0.005]}, PlotRange → {0, All}],
Plot[CR2[t] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.9 /. ES → 0.32 /. Em → 0.65 /.  $\kappa$  → -0.81 /.  $\alpha$  → 1 /.
       $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[R]$  → 0.02 /.  $\beta[C]$  → 0.04 /.
      T → 273.15 + t /. aC → 1/4 + 2/3 /. aR → 1/3 /. hC → -2/3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /.
M15[i_] → 1, {t, 5, 30}, PlotStyle →
  {Black,
  Dashing[
    Large],
  Thickness[0.01]}, PlotRange → {0, All}],
Plot[CR2[t] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.9 /. ES → 0.32 /. Em → 0.65 /.  $\kappa$  → -0.81 /.
       $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[i_]$  → 0.04 /.
      T → 273.15 + t /. aC → 1/4 + 2/3 /. aR → 1/3 /. hC → -2/3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /.
M15[i_] → 1, {t, 5, 30}, PlotStyle → {Black,
  Thickness[
    0.01]}, PlotRange → {0, All}],
PlotRange → {0, 0.4},
Frame → True,
FrameLabel → {Style[(*"Temperature (Celcius)"*)"", LabelSize],
  Style[(*"C:R"*)"", LabelSize], },
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle →
  {{Directive[FontColor → White], Black}, {Directive[FontColor → White], Black}},

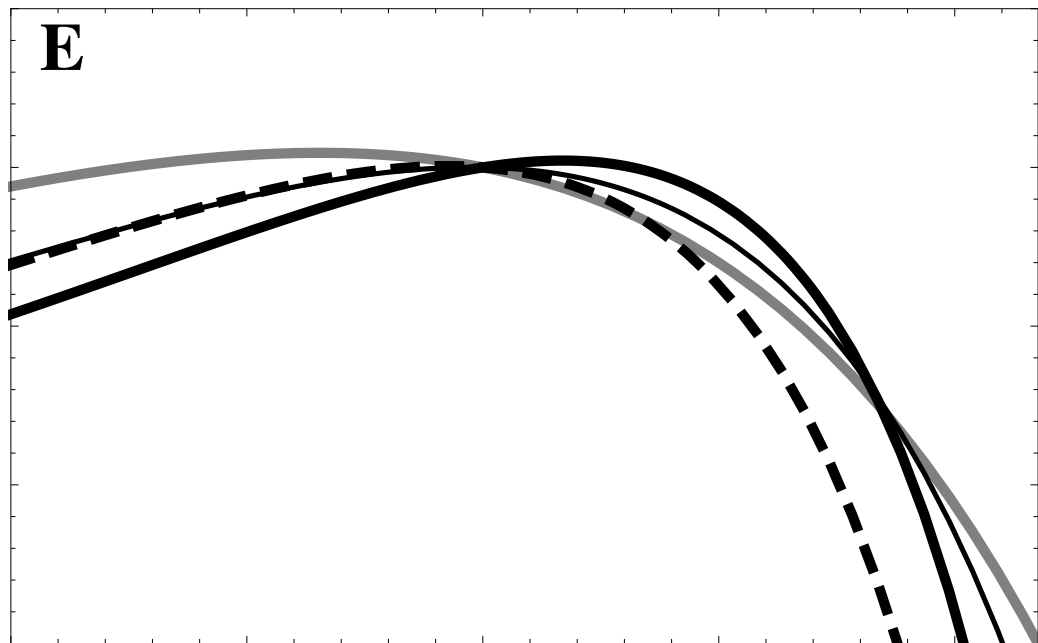
```

```

ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
Epilog → {
  Text[Style["E", LabelSize, Bold], Scaled@1etpos]
}
]

(*Export[imagedir<>"CtoRType2.pdf",%];*)

```



```

Show[(*Plot[
  -Max[Re[lambda/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.
    ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.x→-0.81/.
    α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→T+273.15]],
  {T,5,30},PlotStyle→{Black,Thickness[0.01]},
  Axes→
  False,PlotRange→
  {0,All}],Plot[
  -Max[Re[lambda/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.
    ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.x→-0.81/.
    α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15]],
  {T,5,30},PlotStyle→{Red,Thickness[0.01]},PlotRange→
  {0,All}],*)
Plot[
  -Max[
    Re[lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
    e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.

```

```

EB → 0.9 /. ES → 0.32 /. Em → 0.65 /.  $\kappa$  → -0.81 /.  $\alpha$  → 1 /.
 $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[i\_]$  → 0.00 /.
T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /. M15[i_] → 1]],
{t, 5, 30}, PlotStyle → {Gray, Thickness[0.01]}, PlotRange →
All],
Plot[
-Max[
Re[lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
EB → 0.9 /. ES → 0.32 /. Em → 0.65 /.  $\kappa$  → -0.81 /.  $\alpha$  → 1 /.
 $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[i\_]$  → 0.02 /.
T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /. M15[i_] → 1]],
{t, 5, 30}, PlotStyle → {Black, Thickness[0.005]}, PlotRange →
All],
Plot[
-Max[
Re[lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
e15 /. k15 /. r15 /. m15 /. T[i_] → T /.
k → 8.62 * 10-5 /. EB → 0.9 /. ES → 0.32 /. Em → 0.65 /.
 $\kappa$  → -0.81 /.  $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.
 $\rho$  → -0.81 /.  $\beta[R]$  → 0.02 /.  $\beta[C]$  → 0.04 /. T → 273.15 + t /.
aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /.
Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /. M15[i_] → 1]],
{t, 5, 30}, PlotStyle → {Black, Dashing[Large],
Thickness[
0.01]}, PlotRange → All],
Plot[
-Max[
Re[
lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
EB → 0.9 /. ES → 0.32 /. Em → 0.65 /.  $\kappa$  → -0.81 /.  $\alpha$  → 1 /.
 $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[i\_]$  → 0.04 /.
T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /. M15[i_] → 1]],
{t, 5, 30}, PlotStyle → {Black, Thickness[
0.01]},
PlotRange → All],
PlotRange → {0, 1.5},
Frame → True,
FrameLabel → {Style["Temperature (°C)", LabelSize],
Style[(*"Stability"*)"", LabelSize], ,},
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle → {{Directive[FontColor → White], Black}, {Black, Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,

```

```

Epilog → {
  Text[Style["F", LabelSize, Bold], Scaled@letpos]
},
PlotRange → All
]

(*Export[imagedir<>"StabilityType2.pdf",%];*)

```

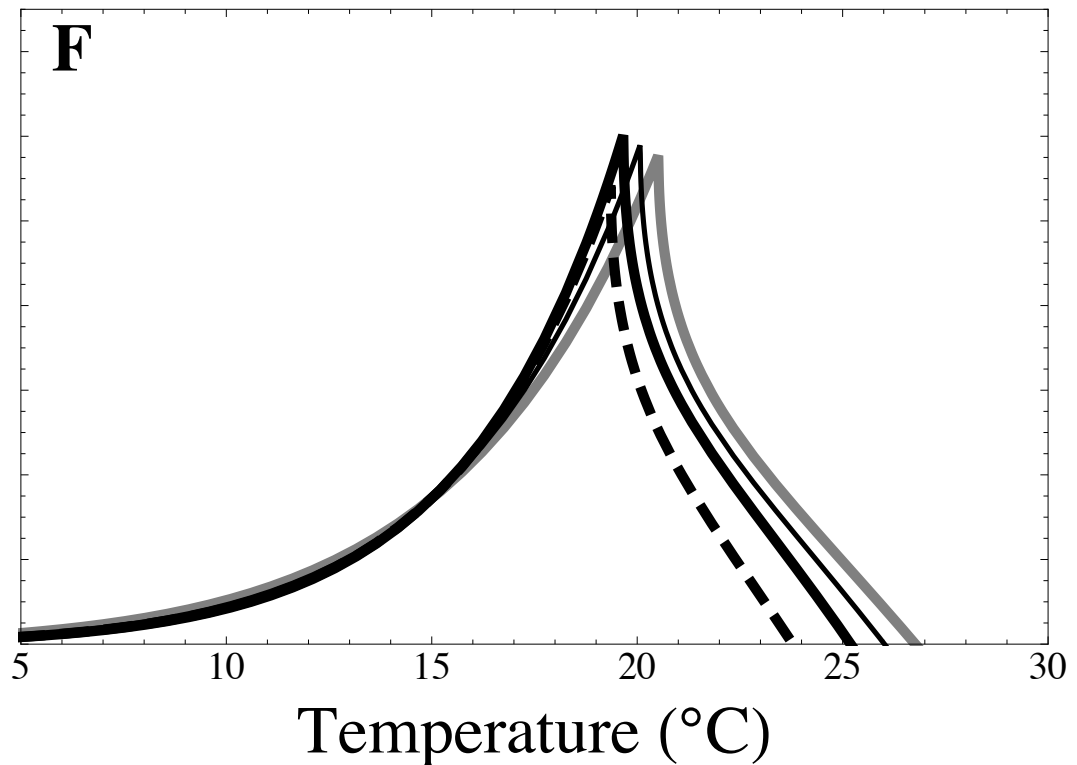


Figure S2 (A,B,C) [ES=EB, type-I]

```

Legended[
  Show[
    (*Plot[
      BCR[T]/.K[T]→K0 Exp[ $\frac{EB}{k T[R]} - \frac{ES}{k T[S]}$ ]/.T[i_]→T+273.15/.K15/.k→8.62*10-5/.a[T]→0.1/.
      e[T]→0.15/.m[T]→0.6/.r[T]→2/.EB→0.32/.ES→0.9,
      {T,5,30},PlotStyle→{Gray,Thickness[0.01]}],*)
    Plot[
      BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
      {T, 5, 30}, PlotStyle → {Thickness[0.01], Gray},

```

```

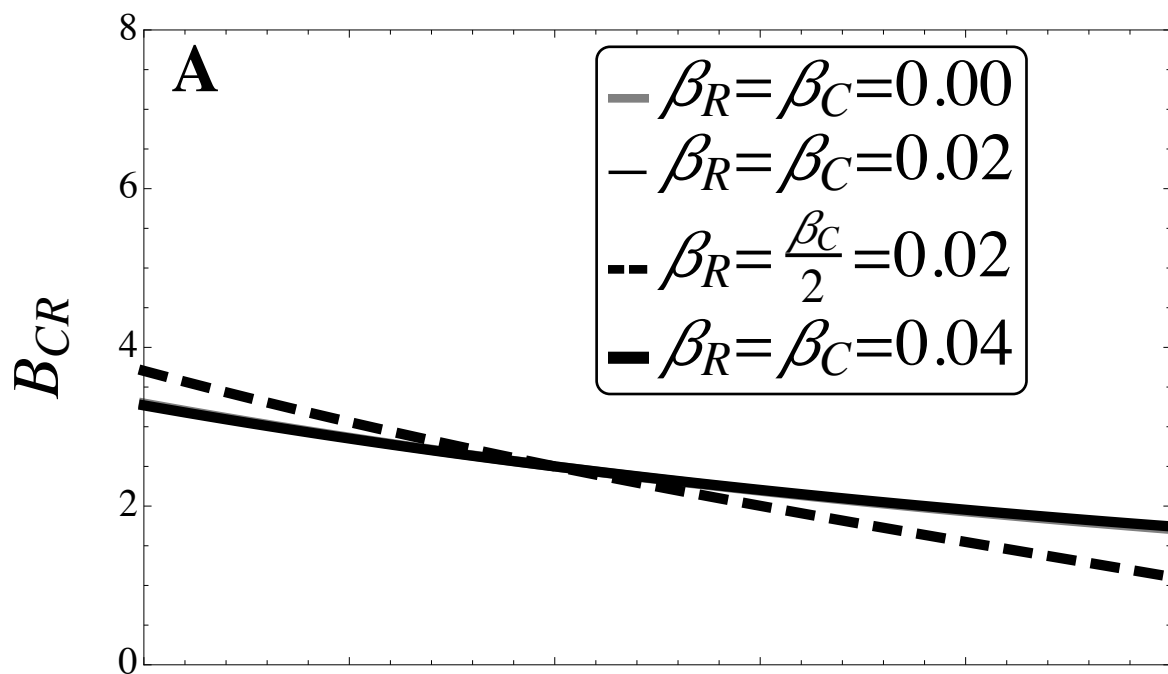
Axes →
  False, PlotRangePadding →
    None],
(*Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /.
    ES → 0.32 /. k → 8.62*10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /.
    α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Thickness[0.01], Black, Dashing[Large]}],
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /.
    ES → 0.9 /. k → 8.62*10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.
    κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /.
    T[i_] → T + 273.15, {T, 5, 30}, PlotStyle → {Thick, Gray}], *)
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.9 /. ES → 0.9 /. k → 8.62*10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
    v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Thickness[0.005], Black},
    PlotRangePadding →
      None],
(*Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /.
    ES → 0.32 /. k → 8.62*10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /.
    α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Thickness[0.01], Red, Dashing[Large]}], Plot[
    BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /. ES → 0.9 /.
    k → 8.62*10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /.
    ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Thickness[0.01], Pink}], *)
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.9 /. ES → 0.9 /. k → 8.62*10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
    v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Thickness[0.01], Black, Dashing[Large]}],
    PlotRangePadding →
      None],
Plot[BCR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.9 /. ES → 0.9 /. k → 8.62*10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
    v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.04 /. T[i_] → T + 273.15,
    {T, 5, 30}, PlotStyle → {Thickness[0.01], Black},
    PlotRangePadding →
      None],
PlotRange → {0, 8},
Frame → True,
FrameLabel → {Style[(*"Temperature (Celcius)"*)"", LabelSize],
    Style[(*"BCR"*)"", LabelSize], },
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle → {{Black, Black}, {Directive[FontColor → White], Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
PlotRangeClipping → False,
Epilog → {

```

```

Text[Style["A", LabelSize, Bold], Scaled@letpos],
Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree]
}
],
Placed[
LineLegend[{
Directive[Gray, Thickness[0.25]],
Directive[Black, Thickness[0.1]],
Directive[Black, Dashing[Medium], Thickness[0.25]],
Directive[Black, Thickness[0.35]]
},
{
Style[" $\beta_R = \beta_C = 0.00$ ", LabelSize],
Style[" $\beta_R = \beta_C = 0.02$ ", LabelSize],
Style[" $\beta_R = \frac{\beta_C}{2} = 0.02$ ", LabelSize],
Style[" $\beta_R = \beta_C = 0.04$ ", LabelSize]
},
LegendFunction → "Frame",
LegendLayout → "Column"
],
{0.65, 0.7}
]
]
(*Export[imagedir<>"BCRType1.pdf",%];*)

```



```

Show[
Plot[
CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB → 0.9 /.
      ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /. v0[i_] → 1 /.
      κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. TSR /.
      β[i_] → 0 /. T[i_] → T + 273.15, {T, 5, 30}, PlotStyle →
      {Gray, Thickness[0.01]}, Axes → False],
Plot[ CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15,
      {T, 5, 30}, PlotStyle → {Black, Thickness[0.005]},
      Axes →
      False],
Plot[ CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /.
      TSR /. β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → T + 273.15, {T, 5, 30},
      PlotStyle → {Black, Dashing[Large], Thickness[0.01]},
      Axes →
      False],
Plot[ CR[T] /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
      EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
      v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.04 /. T[i_] → T + 273.15,
      {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
      Axes →
      False],
PlotRange → {0, 1},
Frame → True,
FrameLabel → {Style[(*"Temperature (Celcius)"*)"", LabelSize],
      Style[(*"C:R"*)"", LabelSize], },
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle → {{Black, Black}, {Directive[FontColor → White], Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
PlotRangeClipping → False,
Epilog → {
      Text[Style["B", LabelSize, Bold], Scaled@letpos],
      Rotate[Text[Style["C:R", LabelSize], Scaled@ylabpos], 90 Degree]
}
]

(*Export[imagedir<>"CtoRType1.pdf",%];*)

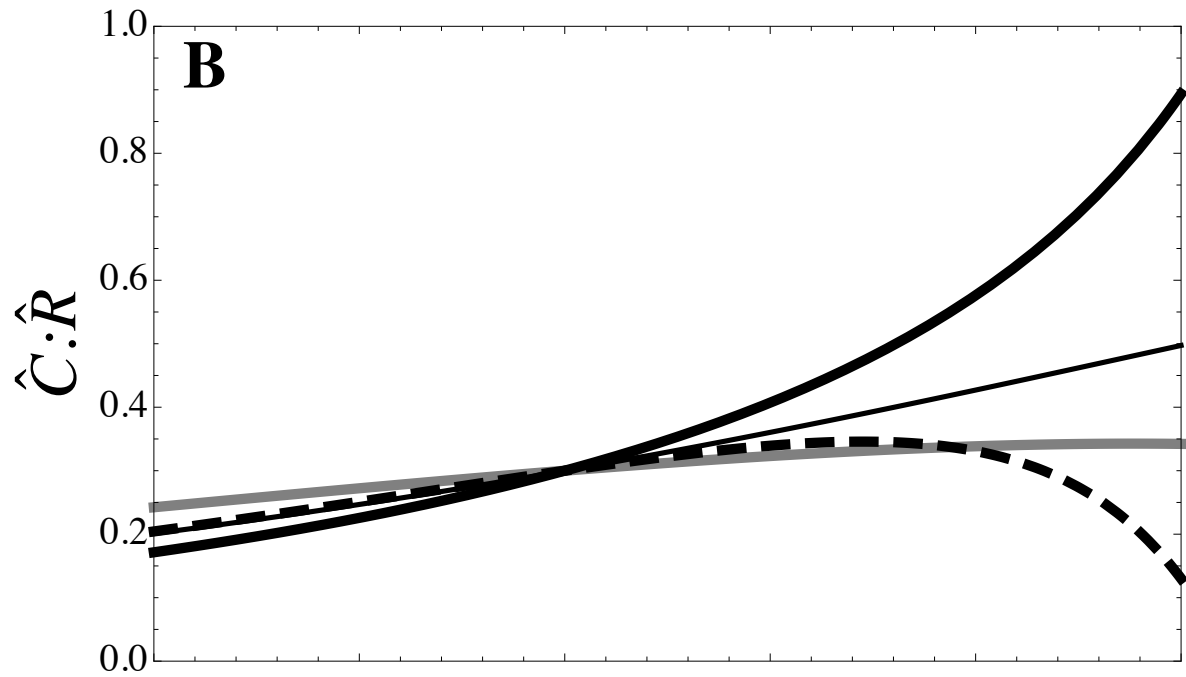
```

Solve::ratnz: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

General::stop : Further output of Solve::ratnz will be suppressed during this calculation. >>



```
Show[Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /.
    Ev[i_] → 0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /.
    μ → -0.29 /. ρ → -0.81 /. TSR /. β[i_] → 0 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Gray, Thickness[0.01]},
  Axes →
    False, PlotRange →
      {0, All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] →
    0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[i_] → 0.02 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Thickness[0.005]},
  PlotRange →
    {0, All}], Plot[
  -Max[Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /.
    EB → 0.9 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] →
    0.46 /. v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
    ρ → -0.81 /. TSR /. β[R] → 0.02 /. β[C] → 0.04 /. T[i_] → T + 273.15]],
  {T, 5, 30}, PlotStyle → {Black, Dashing[Large], Thickness[0.01]},
  PlotRange →
    {0, All}],
  Plot[
    -Max[
      Re[lambda /. GilbertTable1 /. DeLongTable1 /. a15 /. e15 /. k15 /. r15 /. m15 /. EB →
        0.9 /. ES → 0.9 /. k → 8.62 * 10-5 /. Em → 0.65 /. Ev[i_] → 0.46 /.
        v0[i_] → 1 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
        ρ → -0.81 /. TSR /. β[R] → 0.04 /. β[C] → 0.04 /. T[i_] → T + 273.15]],
      {T, 5, 30}, PlotStyle → {Black, Thickness[0.01]},
      PlotRange →
        {0,
          All}],
    PlotRange → {0, 3.5},
    Frame → True,
    FrameLabel → {Style["Temperature (°C)", LabelSize],
      Style[(*"Stability"*)"", LabelSize], , },
    FrameStyle → Directive[FontSize → TickSize],
    ImagePadding → Pad,
    ImageSize → FigureSize,
    PlotRangePadding → None,
    PlotRangeClipping → False,
    Epilog → {
      Text[Style["C", LabelSize, Bold], Scaled@letpos],
      Rotate[Text[Style["Stability", LabelSize], Scaled@ylabpos], 90 Degree]
    }
  ]
]

(*Export[imagedir<"StabilityType1.pdf", %];*)
```

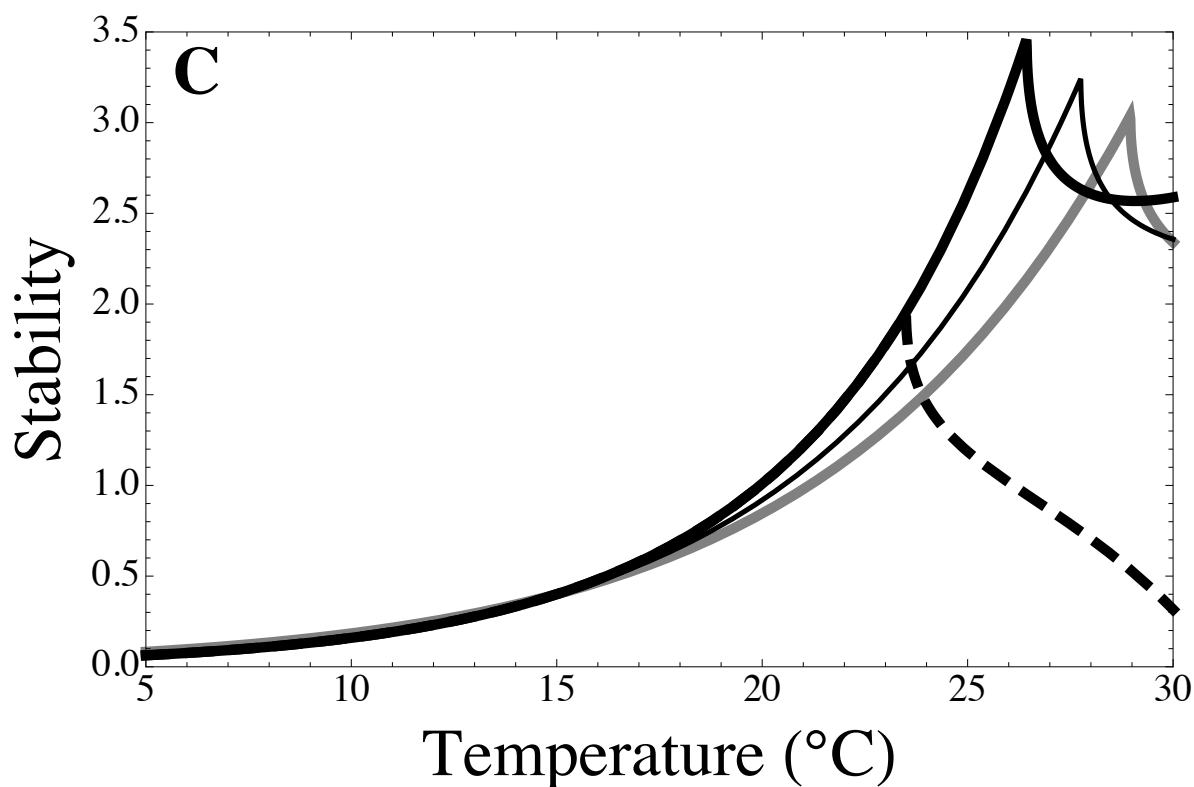


Figure S2 (D,E,F) [ES=EB, type-II]

```
(*Legended[*)
Show[
(*Plot[
BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.ES→0.9/.
k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.x→-0.81/.α→1/.
ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→273.15+t,
{t,5,30},PlotStyle→{Black,Thickness[0.01]},
Axes→False],
(*Plot[
BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.32/.
k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.x→-0.81/.α→1/.
ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→273.15+t,
{t,5,30},PlotStyle→{Black,Thickness[0.01],Dashing[Large]}],
Plot[
BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.9/.
k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.x→-0.81/.α→1/.
ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→273.15+t,
{t,5,30},PlotStyle→{Gray,Thickness[0.01]}],*)
Plot[
BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.ES→0.9/.
k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.x→-0.81/.α→1/.
ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
{T,5,30},PlotStyle→{Thickness[0.01],Red},
```

```

Axes→
False],*)
(*Plot[BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→
0.32/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.
α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
{T,5,30},PlotStyle→{Thickness[0.01],Red,Dashing[Large]}],
Plot[
BCR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.9/.ES→0.9/.
k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.κ→-0.81/.α→1/.
ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15,
{T,5,30},PlotStyle→{Thickness[0.01],Pink}],*)
Plot[Simplify[BCR2[T] /. Eq2[T] [[3]]] /. RallT /. RallM /. GilbertDeLongT /.
GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.9 /. ES → 0.9 /.
Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
ρ → -0.81 /. β[i_] → 0.00 /. T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /.
aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /.
h0 → 1 /. M15[i_] → 1, {t, 5, 30}, PlotStyle →
{Gray,
Thickness[
0.01]}], Axes → False],
Plot[Simplify[BCR2[T] /. Eq2[T] [[3]]] /. RallT /. RallM /. GilbertDeLongT /.
GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.9 /. ES → 0.9 /.
Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
ρ → -0.81 /. β[i_] → 0.02 /. T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /.
aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /.
h0 → 1 /. M15[i_] → 1, {t, 5, 30}, PlotStyle →
{Black,
Thickness[
0.005]}], Axes → False],
Plot[Simplify[BCR2[T] /. Eq2[T] [[3]]] /. RallT /. RallM /. GilbertDeLongT /.
GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.9 /. ES → 0.9 /.
Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
ρ → -0.81 /. β[R] → 0.02 /. β[C] → 0.04 /. T → 273.15 + t /.
aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /.
Ea → 0.65 /. Eh → -0.65 /. h0 → 1 /. M15[i_] →
1, {t, 5, 30}, PlotStyle →
{Black,
Dashing[
Large],
Thickness[0.01]}], Axes → False],
Plot[Simplify[BCR2[T] /. Eq2[T] [[3]]] /. RallT /. RallM /. GilbertDeLongT /.
GilbertDeLongM /. TSR /. ah15 /. e15 /. k15 /. r15 /.
m15 /. T[i_] → T /. k → 8.62 * 10-5 /. EB → 0.9 /. ES → 0.9 /.
Em → 0.65 /. κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
ρ → -0.81 /. β[i_] → 0.04 /. T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /.
aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /.
h0 → 1 /. M15[i_] → 1, {t, 5, 30}, PlotStyle →
{Black,

```

```

Thickness[
  0.01]], Axes → False],
(*Plot[Simplify[BCR2[T]/.Eq2[T][[3]]/.RallT/.RallM/.GilbertDeLongT/.
      GilbertDeLongM/.TSR/.ah15/.el5/.k15/.r15/.m15/.
      T[i_]→T/.k→8.62*10-5/.EB→0.9/.ES→0.32/.Em→0.65/.
      κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.β[i_]→0.02/.
      T→273.15+t/.aC→1/4+2/3/.aR→1/3/.hC→-2/3/.hR→0.5/.Ea→0.65/.
      Eh→-0.65/.h0→1/.M15[i_]→1,{t,5,30},PlotStyle→{Darker[
      Blue],
      Thickness[
      0.01],Dashing[
      Large]}}],
Plot[Simplify[BCR2[T]/.Eq2[T][[3]]/.RallT/.RallM/.GilbertDeLongT/.
      GilbertDeLongM/.TSR/.ah15/.el5/.k15/.r15/.m15/.
      T[i_]→T/.k→8.62*10-5/.EB→0.9/.ES→0.9/.Em→0.65/.
      κ→-0.81/.α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.β[i_]→0.02/.
      T→273.15+t/.aC→1/4+2/3/.aR→1/3/.hC→-2/3/.hR→0.5/.Ea→0.65/.
      Eh→-0.65/.h0→1/.M15[i_]→1,{t,5,30},PlotStyle→{Lighter[
      Blue],
      Thickness[
      0.01]}}],*)
PlotRange → {0, 8},
Frame → True,
FrameLabel → {Style[(*"Temperature (Celcius)"*)"", LabelSize],
  Style[(*"BCR"*)"", LabelSize], ,},
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle →
  {{Directive[FontColor → White], Black}, {Directive[FontColor → White], Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
(*PlotRangeClipping→False,*)
Epilog → {
  Text[Style["D", LabelSize, Bold], Scaled@letpos],
  (*Rotate[Text[Style["BCR", LabelSize], Scaled@ylabpos], 90 Degree]*)
}
] (*,
Placed[
  LineLegend[{
    Directive[Black,Dashing[Medium],Thickness[0.25]],
    Directive[Black,Thickness[0.25]],
    Directive[Gray,Thickness[0.25]],
    Directive[Gray,Dashing[Medium],Thickness[0.25]]
  },
  {
    Style["βR=βC=0.00, type-II",LabelSize],
    Style["βR=βC=0.02, type-II",LabelSize],
    Style["βR= $\frac{\beta_C}{2}$ =0.02, type-II",LabelSize],
    Style["βR=βC=0.04, type-II",LabelSize]
  },

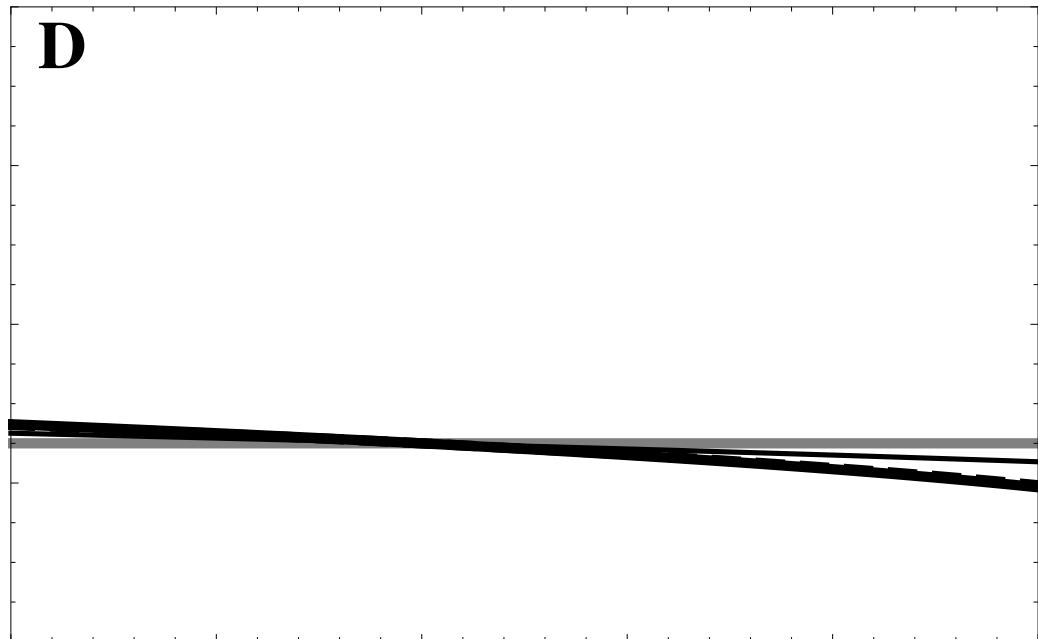
```

```

      LegendFunction->"Frame",
      LegendLayout->"Column"
    ],
    {0.35,0.7}
  ]
] *)

(*Export[imagedir<>"BCRType2.pdf",%];*)

```



```

Show[
  (*Plot[
    CR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB->0.32/.ES->0.9/.
      k->8.62*10-5/.Em->0.65/.Ev[i_]->0.46/.v0[i_]->1/.κ->-0.81/.α->1/.
      ε->-0.5/.μ->-0.29/.ρ->-0.81/.TSR/.β[i_]->0/.T[i_]->T+273.15,
    {T,5,30},PlotStyle->{Black,Thickness[0.01]},
    Axes->False,
    PlotRange->{0,All}],
  Plot[
    CR[T]/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB->0.32/.ES->0.9/.
      k->8.62*10-5/.Em->0.65/.Ev[i_]->0.46/.v0[i_]->1/.κ->-0.81/.α->1/.
      ε->-0.5/.μ->-0.29/.ρ->-0.81/.TSR/.β[i_]->0.02/.T[i_]->T+273.15,
    {T,5,30},PlotStyle->{Red,Thickness[0.01]},
    Axes->
      False,PlotRange->
        {0,All}],*)

```

```

Plot[CR2[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.9 /. ES → 0.9 /. Em → 0.65 /.  $\kappa$  → -0.81 /.
       $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[i_]$  → 0.00 /.
      T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /.
M15[i_] → 1, {t, 5, 30}, PlotStyle →
{Gray,
  Thickness[
    0.01]}, PlotRange → {0, All}],
Plot[CR2[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.9 /. ES → 0.9 /. Em → 0.65 /.  $\kappa$  → -0.81 /.
       $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[i_]$  → 0.02 /.
      T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /.
M15[i_] → 1, {t, 5, 30}, PlotStyle → {Black,
  Thickness[
    0.005]}, PlotRange → {0, All}],
Plot[CR2[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.9 /. ES → 0.9 /. Em → 0.65 /.  $\kappa$  → -0.81 /.  $\alpha$  → 1 /.
       $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[R]$  → 0.02 /.  $\beta[C]$  → 0.04 /.
      T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /.
M15[i_] → 1, {t, 5, 30}, PlotStyle →
{Black,
  Dashing[
    Large],
  Thickness[0.01]}, PlotRange → {0, All}],
Plot[CR2[T] /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.9 /. ES → 0.9 /. Em → 0.65 /.  $\kappa$  → -0.81 /.
       $\alpha$  → 1 /.  $\epsilon$  → -0.5 /.  $\mu$  → -0.29 /.  $\rho$  → -0.81 /.  $\beta[i_]$  → 0.04 /.
      T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /.
M15[i_] → 1, {t, 5, 30}, PlotStyle → {Black,
  Thickness[
    0.01]}, PlotRange → {0, All}],
PlotRange → {0, 1},
Frame → True,
FrameLabel → {Style[(*"Temperature (Celcius)"*)"", LabelSize],
  Style[(*"C:R"*)"", LabelSize], },
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle →
  {{Directive[FontColor → White], Black}, {Directive[FontColor → White], Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
Epilog → {

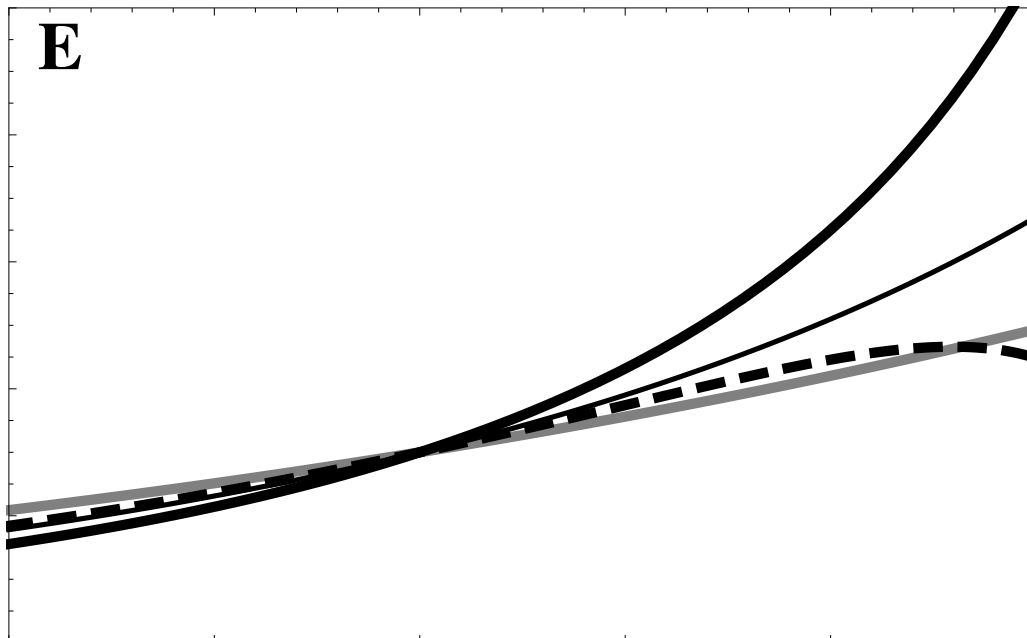
```

```

Text[Style["E", LabelSize, Bold], Scaled@letpos]
}
]

(*Export[imagedir<>"CtoRType2.pdf",%];*)

```



```

Show[(*Plot[
  -Max[Re[lambda/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.
    ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.x→-0.81/.
    α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0/.T[i_]→T+273.15]],
  {T,5,30},PlotStyle→{Black,Thickness[0.01]},
  Axes→
  False,PlotRange→
  {0,All}],Plot[
  -Max[Re[lambda/.GilbertTable1/.DeLongTable1/.a15/.e15/.k15/.r15/.m15/.EB→0.32/.
    ES→0.9/.k→8.62*10-5/.Em→0.65/.Ev[i_]→0.46/.v0[i_]→1/.x→-0.81/.
    α→1/.ε→-0.5/.μ→-0.29/.ρ→-0.81/.TSR/.β[i_]→0.02/.T[i_]→T+273.15]],
  {T,5,30},PlotStyle→{Red,Thickness[0.01]},PlotRange→
  {0,All}],*)
Plot[
  -Max[
    Re[lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.9 /. ES → 0.9 /. Em → 0.65 /. x → -0.81 /. α → 1 /.
      ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. β[i_] → 0.00 /.
      T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /. M15[i_] → 1]],

```



```

{t, 5, 30}, PlotStyle → {Gray, Thickness[0.01]}, PlotRange →
  All],
Plot[
  -Max[
    Re[lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.9 /. ES → 0.9 /. Em → 0.65 /. κ → -0.81 /. α → 1 /.
      ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. β[i_] → 0.02 /.
      T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /. M15[i_] → 1]],
{t, 5, 30}, PlotStyle → {Black, Thickness[0.005]}, PlotRange →
  All],
Plot[
  -Max[
    Re[lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /.
      k → 8.62 * 10-5 /. EB → 0.9 /. ES → 0.9 /. Em → 0.65 /.
      κ → -0.81 /. α → 1 /. ε → -0.5 /. μ → -0.29 /.
      ρ → -0.81 /. β[R] → 0.02 /. β[C] → 0.04 /. T → 273.15 + t /.
      aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /. hR → 0.5 /.
      Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /. M15[i_] → 1]],
{t, 5, 30}, PlotStyle → {Black, Dashing[Large],
  Thickness[
    0.01]}, PlotRange → All],
Plot[
  -Max[
    Re[
      lambda2 /. RallT /. RallM /. GilbertDeLongT /. GilbertDeLongM /. TSR /. ah15 /.
      e15 /. k15 /. r15 /. m15 /. T[i_] → T /. k → 8.62 * 10-5 /.
      EB → 0.9 /. ES → 0.9 /. Em → 0.65 /. κ → -0.81 /. α → 1 /.
      ε → -0.5 /. μ → -0.29 /. ρ → -0.81 /. β[i_] → 0.04 /.
      T → 273.15 + t /. aC → 1 / 4 + 2 / 3 /. aR → 1 / 3 /. hC → -2 / 3 /.
      hR → 0.5 /. Ea → 0.65 /. Eh → -0.65 /. h0 → 10-13 /. M15[i_] → 1]],
{t, 5, 30}, PlotStyle → {Black, Thickness[
  0.01]},
PlotRange → All],
PlotRange → {0, 3.5},
Frame → True,
FrameLabel → {Style["Temperature (°C)", LabelSize],
  Style[(*"Stability"*)"", LabelSize], ,},
FrameStyle → Directive[FontSize → TickSize],
FrameTicksStyle → {{Directive[FontColor → White], Black}, {Black, Black}},
ImagePadding → Pad,
ImageSize → FigureSize,
PlotRangePadding → None,
Epilog → {
  Text[Style["F", LabelSize, Bold], Scaled@letpos]
},
PlotRange → All

```

]

```
(*Export[imagedir<>"StabilityType2.pdf",%];*)
```

