S1 Appendix

Recursion equations

In each generation we census the genotype frequencies in male and female gametes/gametophytes (hereafter, gametes) between meiosis (and any meiotic drive) and gametic competition. At this stage we denote the frequencies of X- and Y-bearing gametes from males and females $x_i^{⚥}$ and $y_i^{⚥}$. The superscript $⚥ \in \{♂, ♀\}$ specifies the sex of the diploid that the gamete came from. The subscript $i \in \{1, 2, 3, 4\}$ specifies the genotype at the selected locus **A** and at the novel sex-determining locus **M**, where $1 = AM$, $2 = aM$, $3 = Am$, and $4 = am$. The gamete frequencies from each sex sum to one, $\sum_i x_i^{⚥} + y_i^{⚥} = 1$.

Competition then occurs among gametes of the same sex (e.g., among eggs and among sperm separately) according to the genotype at the **A** locus ($w_1^{⚥} = w_3^{⚥} = w_A^{⚥}$, $w_2^{⚥} = w_4^{⚥} = w_a^{⚥}$, see Table **??**). The genotype frequencies after gametic competition are $x_i^{⚥,s} = w_i x_i^{⚥} / \bar{w}_H^{⚥}$ and $y_i^{⚥,s} = w_i y_i^{⚥} / \bar{w}_H^{⚥}$, where $\bar{w}_H^{⚥} = \sum_i w_i x_i^{⚥} + w_i y_i^{⚥}$ is the mean fitness of male ($⚥ = ♂$) or female ($⚥ = ♀$) gametes.

Random mating then occurs between gametes to produce diploid zygotes. The frequencies of XX zygotes are then denoted as xx_{ij}, XY zygotes as xy_{ij}, and YY zygotes as yy_{ij}, where **A** and **M** locus genotypes are given by $i, j \in \{1, 2, 3, 4\}$, as above. In XY zygotes, the haplotype inherited from an X-bearing gamete is given by i and the haplotype from a Y-bearing gamete is given by j. In XX and YY zygotes, individuals with diploid genotype ij are equivalent to those with diploid genotype ji; for simplicity, we use xx_{ij} and yy_{ij} with $i \neq j$ to denote the average of these frequencies, $xx_{ij} = (x_i^{♀,s} x_j^{♂,s} + x_j^{♀,s} x_i^{♂,s})/2$ and $yy_{ij} = (y_i^{♀,s} y_j^{♂,s} + y_j^{♀,s} y_i^{♂,s})/2$.

Denoting the **M** locus genotype by $b \in \{MM, Mm, mm\}$ and the **X** locus genotype by $c \in \{XX, XY, YY\}$, zygotes develop as females with probability k_{bc}. Therefore, the frequencies of XX females are given by $xx_{ij}^{♀} = k_{bc} xx_{ij}$, XY females are given by $xy_{ij}^{♀} = k_{bc} xy_{ij}$, and YY females are given by $yy_{ij}^{♀} = k_{bc} yy_{ij}$. Similarly, XX male frequencies are $xx_{ij}^{♂} = (1 - k_{bc}) xx_{ij}$, XY male frequencies are $xy_{ij}^{♂} = (1 - k_{bc}) xy_{ij}$, and YY males frequencies are $yy_{ij}^{♂} = (1 - k_{bc}) yy_{ij}$. This notation allows both the ancestral and novel sex-determining regions to determine zygotic sex according to an XY system, a ZW system, or an environmental sex-determining system. In addition, we can consider any epistatic dominance relationship between the two sex-determining loci. Here, we assume that the ancestral sex-determining system (**X** locus) is XY ($k_{MMXX} = 1$ and $k_{MMXY} = k_{MMYY} = 0$) or ZW ($k_{MMZZ} = 0$ and $k_{MMZW} = k_{MMWW} = 1$) and epistatically recessive to a dominant novel sex-determining locus, **M** ($k_{Mmc} = k_{mmc} = k$).

Selection among diploids then occurs according to the diploid genotype at the **A** locus, $l \in \{AA, Aa, aa\}$, for an individual of type ij (see Table **??**). The diploid frequencies after selection in sex ⚥ are given by $xx_{ij}^{⚥,s} = w_l^{⚥} xx_{ij}/\bar{w}^{⚥}$, $xy_{ij}^{⚥,s} = w_l^{⚥} xy_{ij}/\bar{w}^{⚥}$, and $yy_{ij}^{⚥,s} = w_l^{⚥} yy_{ij}/\bar{w}^{⚥}$, where $\bar{w}^{⚥} = \sum_{i=1}^{4}\sum_{j=1}^{4} w_l^{⚥} xx_{ij} + w_l^{⚥} xy_{ij} + w_l^{⚥} yy_{ij}$ is the mean fitness of individuals of sex ⚥.

Finally, these diploids undergo meiosis to produce the next generation of gametes. Recombination and sex-specific meiotic drive occur during meiosis. Here, we allow any relative locations for the SDR, **A**, and **M** loci by using three parameters to describe the recombination rates between them. R is the recombination rate between the **A** locus and the **M** locus, ρ is the recombination rate between the **M** locus and the **X** locus, and r is the recombination rate between the **A** locus and the **X** locus. Table tab:chisubstitutions shows replacements that can be made for each possible ordering of the loci assuming that there is no cross-over interference. During meiosis in sex ⚥, meiotic drive occurs such that, in Aa heterozygotes, a fraction $\alpha^{⚥}$ of gametes produced carry the A allele and $(1-\alpha^{⚥})$ carry the a allele.

Among gametes from sex ⚥, the frequencies of haplotypes (before gametic competition) in the next generation are given by

$$
\begin{aligned}
x_1^{\text{⚥}\prime} =& xx_{11}^{\text{⚥},s} + xx_{13}^{\text{⚥},s}/2 + (xx_{12}^{\text{⚥},s} + xx_{14}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& - R(xx_{14}^{\text{⚥},s} - xx_{23}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& + (xy_{11}^{\text{⚥},s} + xy_{13}^{\text{⚥},s})/2 + (xy_{12}^{\text{⚥},s} + xy_{14}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& - r(xy_{12}^{\text{⚥},s} - xy_{21}^{\text{⚥},s})\alpha^{\text{⚥}} - \rho(xy_{13}^{\text{⚥},s} - xy_{31}^{\text{⚥},s})/2 \\
& + \big[- (R + r + \rho)xy_{14}^{\text{⚥},s} + (R + \rho - r)xy_{41}^{\text{⚥},s} \\
& + (R + r - \rho)xy_{23}^{\text{⚥},s} + (R + \rho - r)xy_{32}^{\text{⚥},s}\big]\alpha^{\text{⚥}}/2
\end{aligned}
\tag{S1.1a}
$$

$$
\begin{aligned}
x_2^{\text{⚥}\prime} =& xx_{22}^{\text{⚥},s} + xx_{24}^{\text{⚥},s}/2 + (xx_{12}^{\text{⚥},s} + xx_{23}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& - R(xx_{23}^{\text{⚥},s} - xx_{14}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& (xy_{22}^{\text{⚥},s} + xy_{24}^{\text{⚥},s})/2 + (xy_{21}^{\text{⚥},s} + xy_{23}^{\text{⚥},s})(1 - \alpha^{\text{⚥}}) \\
& - r(xy_{21}^{\text{⚥},s} - xy_{12}^{\text{⚥},s})(1 - \alpha^{\text{⚥}}) - \rho(xy_{24}^{\text{⚥},s} - xy_{42}^{\text{⚥},s})/2 \\
& + \big[- (R + r + \rho)xy_{23}^{\text{⚥},s} + (R + \rho - r)xy_{32}^{\text{⚥},s} \\
& + (R + r - \rho)xy_{14}^{\text{⚥},s} + (R + \rho - r)xy_{41}^{\text{⚥},s}\big](1 - \alpha^{\text{⚥}})/2
\end{aligned}
\tag{S1.1b}
$$

$$
\begin{aligned}
x_3^{\text{⚥}\prime} =& xx_{33}^{\text{⚥},s} + xx_{13}^{\text{⚥},s}/2 + (xx_{23}^{\text{⚥},s} + xx_{34}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& - R(xx_{23}^{\text{⚥},s} - xx_{14}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& (xy_{33}^{\text{⚥},s} + xy_{31}^{\text{⚥},s})/2 + (xy_{32}^{\text{⚥},s} + xy_{34}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& - r(xy_{34}^{\text{⚥},s} - xy_{43}^{\text{⚥},s})\alpha^{\text{⚥}} - \rho(xy_{31}^{\text{⚥},s} - xy_{13}^{\text{⚥},s})/2 \\
& + \big[- (R + r + \rho)xy_{32}^{\text{⚥},s} + (R + \rho - r)xy_{23}^{\text{⚥},s} \\
& + (R + r - \rho)xy_{41}^{\text{⚥},s} + (R + \rho - r)xy_{14}^{\text{⚥},s}\big]\alpha^{\text{⚥}}/2
\end{aligned}
\tag{S1.1c}
$$

$$
\begin{aligned}
x_4^{\text{⚥}\prime} =& xx_{44}^{\text{⚥},s} + xx_{34}^{\text{⚥},s}/2 + (xx_{14}^{\text{⚥},s} + xx_{24}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& - R(xx_{14}^{\text{⚥},s} - xx_{23}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& (xy_{44}^{\text{⚥},s} + xy_{42}^{\text{⚥},s})/2 + (xy_{41}^{\text{⚥},s} + xy_{43}^{\text{⚥},s})(1-\alpha^{\text{⚥}}) \\
& - r(xy_{43}^{\text{⚥},s} - xy_{34}^{\text{⚥},s})(1-\alpha^{\text{⚥}}) - \rho(xy_{42}^{\text{⚥},s} - xy_{24}^{\text{⚥},s})/2 \\
& + \big[-(R+r+\rho)xy_{41}^{\text{⚥},s} + (R+\rho-r)xy_{14}^{\text{⚥},s} \\
& + (R+r-\rho)xy_{32}^{\text{⚥},s} + (R+\rho-r)xy_{23}^{\text{⚥},s}\big](1-\alpha^{\text{⚥}})/2
\end{aligned}
\tag{S1.1d}
$$

$$
\begin{aligned}
y_1^{\text{⚥}\prime} =& yy_{11}^{\text{⚥},s} + yy_{13}^{\text{⚥},s}/2 + (yy_{12}^{\text{⚥},s} + yy_{14}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& - R(yy_{14}^{\text{⚥},s} - yy_{23}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& (xy_{11}^{\text{⚥},s} + xy_{31}^{\text{⚥},s})/2 + (xy_{21}^{\text{⚥},s} + xy_{41}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& - r(xy_{21}^{\text{⚥},s} - xy_{12}^{\text{⚥},s})\alpha^{\text{⚥}} - \rho(xy_{31}^{\text{⚥},s} - xy_{13}^{\text{⚥},s})/2 \\
& + \big[-(R+r+\rho)xy_{41}^{\text{⚥},s} + (R+\rho-r)xy_{14}^{\text{⚥},s} \\
& + (R+r-\rho)xy_{32}^{\text{⚥},s} + (R+\rho-r)xy_{23}^{\text{⚥},s}\big]\alpha^{\text{⚥}}/2
\end{aligned}
\tag{S1.1e}
$$

$$
\begin{aligned}
y_2^{\text{⚥}\prime} =& yy_{22}^{\text{⚥},s} + yy_{24}^{\text{⚥},s}/2 + (yy_{12}^{\text{⚥},s} + yy_{23}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& - R(yy_{23}^{\text{⚥},s} - yy_{14}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& (xy_{22}^{\text{⚥},s} + xy_{42}^{\text{⚥},s})/2 + (xy_{12}^{\text{⚥},s} + xy_{32}^{\text{⚥},s})(1-\alpha^{\text{⚥}}) \\
& - r(xy_{12}^{\text{⚥},s} - xy_{21}^{\text{⚥},s})(1-\alpha^{\text{⚥}}) - \rho(xy_{42}^{\text{⚥},s} - xy_{24}^{\text{⚥},s})/2 \\
& + \big[-(R+r+\rho)xy_{32}^{\text{⚥},s} + (R+\rho-r)xy_{23}^{\text{⚥},s} \\
& + (R+r-\rho)xy_{41}^{\text{⚥},s} + (R+\rho-r)xy_{14}^{\text{⚥},s}\big](1-\alpha^{\text{⚥}})/2
\end{aligned}
\tag{S1.1f}
$$

$$
\begin{aligned}
y_3^{\text{⚥}\prime} =& yy_{33}^{\text{⚥},s} + yy_{13}^{\text{⚥},s}/2 + (yy_{23}^{\text{⚥},s} + yy_{34}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& - R(yy_{23}^{\text{⚥},s} - yy_{14}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& (xy_{33}^{\text{⚥},s} + xy_{13}^{\text{⚥},s})/2 + (xy_{23}^{\text{⚥},s} + xy_{43}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& - r(xy_{43}^{\text{⚥},s} - xy_{34}^{\text{⚥},s})\alpha^{\text{⚥}} - \rho(xy_{13}^{\text{⚥},s} - xy_{31}^{\text{⚥},s})/2 \\
& + \big[-(R+r+\rho)xy_{23}^{\text{⚥},s} + (R+\rho-r)xy_{32}^{\text{⚥},s} \\
& + (R+r-\rho)xy_{14}^{\text{⚥},s} + (R+\rho-r)xy_{41}^{\text{⚥},s}\big]\alpha^{\text{⚥}}/2
\end{aligned}
\tag{S1.1g}
$$

$$
\begin{aligned}
y_4^{\text{⚥}\prime} = & yy_{44}^{\text{⚥},s} + yy_{34}^{\text{⚥},s}/2 + (yy_{14}^{\text{⚥},s} + yy_{24}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& - R(yy_{14}^{\text{⚥},s} - yy_{23}^{\text{⚥},s})\alpha^{\text{⚥}} \\
& (xy_{44}^{\text{⚥},s} + xy_{24}^{\text{⚥},s})/2 + (xy_{14}^{\text{⚥},s} + xy_{34}^{\text{⚥},s})(1 - \alpha^{\text{⚥}}) \\
& - r(xy_{34}^{\text{⚥},s} - xy_{43}^{\text{⚥},s})(1 - \alpha^{\text{⚥}}) - \rho(xy_{24}^{\text{⚥},s} - xy_{42}^{\text{⚥},s})/2 \\
& + \big[- (R + r + \rho)xy_{14}^{\text{⚥},s} + (R + \rho - r)xy_{41}^{\text{⚥},s} \\
& + (R + r - \rho)xy_{23}^{\text{⚥},s} + (R + \rho - r)xy_{32}^{\text{⚥},s}\big](1 - \alpha^{\text{⚥}})/2
\end{aligned}
\tag{S1.1h}
$$

The full system is therefore described by 16 recurrence equations (three diallelic loci in two sexes, $2^3 \times 2 = 16$). However, not all diploid types are produced under certain sex-determination systems. For example, with the M allele fixed and an ancestral XY sex-determining system, there are XX males, XY females, or YY females ($x_3^{\text{⚥}} = x_4^{\text{⚥}} = y_4^{\text{⚥}} = y_3^{\text{⚥}} = y_i^{\female} = 0$). In this case, the system only involves six recursion equations, which we assume below to calculate the equilibria.