

Gametic selection, sex-ratio bias, and transitions between sex-determining systems

Michael F Scott, Matthew M Osmond, Sarah P Otto

Notes:

- 1) The notation in this notebook differs slightly from the text: here we do not use male and female symbols or sub- and super-scripts.
- 2) Throughout we mainly refer to an ancestral XY sex-determination system. By consistently flipping male and female labels, this is equivalent to an ancestral ZW system (noting that the heterogametic sex described in this notebook as XY would refer to heterogametic ZW females).

General

Assumptions used to simplify the results:

$$\text{simpcond} = \text{Reduce}\left[\left\{0 < \text{MAA}, 0 < \text{MAa}, 0 < \text{Maa}, 0 < \text{FAA}, 0 < \text{FAa}, 0 < \text{Faa}, 0 < \text{wAf}, 0 < \text{waf}, 0 < \text{wAm}, 0 < \text{wam}, 0 < \alpha m < 1, 0 < \alpha f < 1, 0 \leq Rm \leq \frac{1}{2}, 0 \leq Rf \leq \frac{1}{2}\right\}\right];$$

Recursion equations

Haploid competition

We start with haploid competition in each sex separately (e.g., pollen competition for eggs). We assume that the allele at an autosomal locus **A** determines competitive ability, with relative fitnesses w_{Af} and w_{af} in females and w_{Am} and w_{am} in males (in the text this is, e.g., w_A^{female}). Let the frequency of each gamete from each sex before selection be denoted by four letters, the first being the allele at the ancestral SDR (X or Y), the second being the allele at the selected locus (A or a), the third being the allele at the novel SDR (M or m), and the fourth indicating which sex the gamete came from (m or f). E.g., XAMf is the frequency of the X-A-M genotype among eggs (in the text this is x_1^{female}). Let the frequencies after haploid selection end in an s, e.g., XAMfs. We then have

```

(*mean fitness of female gametes*)
wbarHapFemale =
  (wAf XAMf + waf XaMf + wAf XAmf + waf Xamf) + (wAf YAMf + waf YaMf + wAf YAmf + waf Yamf);

(*non-mutant eggs*)
XAMfs = wAf XAMf / wbarHapFemale;
XaMfs = waf XaMf / wbarHapFemale;
YAMfs = wAf YAMf / wbarHapFemale;
YaMfs = waf YaMf / wbarHapFemale;

(*mutant eggs*)
XAmfs = wAf XAmf / wbarHapFemale;
Xamfs = waf Xamf / wbarHapFemale;
YAmfs = wAf YAmf / wbarHapFemale;
Yamfs = waf Yamf / wbarHapFemale;

(*mean fitness of male gametes*)
wbarHapMale =
  (wAm XAMm + wam XaMm + wAm XAmm + wam Xamm) + (wAm YAMm + wam YaMm + wAm YAmm + wam Yammm);

(*non-mutant male gametes*)
XAMms = wAm XAMm / wbarHapMale;
XaMms = wam XaMm / wbarHapMale;
YAMms = wAm YAMm / wbarHapMale;
YaMms = wam YaMm / wbarHapMale;

(*mutant male gametes*)
XAmms = wAm XAmm / wbarHapMale;
Xamms = wam Xamm / wbarHapMale;
YAmms = wAm YAmm / wbarHapMale;
Yamms = wam Yammm / wbarHapMale;

```

Check that frequencies sum to one in each sex after haploid selection:

```

1 == XAMms + XaMms + YAMms + YaMms + XAmms + Xammms + YAmms + Yamms // Factor
(*male gametes*)
1 == XAMfs + XaMfs + XAmfs + Xamfs + YAMfs + YaMfs + YAmfs + Yamfs // Factor (*eggs*)
True
True

```

Random mating

Let the probability that a zygote develops into a female be denoted with k followed by four letters: the first two indicating the ancestral SDR genotype, and the second two denoting the novel SDR genotype. For example, kXYMm is the probability that a zygote with XY at the ancestral SDR and Mm at the novel SDR develops into a female. Let the frequency of zygotes be denoted by the two gamete genotypes followed by the sex, e.g., XAMYamfemale is the frequency of zygotes that are female and composed of X-A-M and Y-a-M gametes (it does not matter which came from which parent, i.e., XAM could come from the mother or father). Then, after random mating the frequencies of the diploid genotypes are

```

(*MM Homozygotes*)
(*XM-XM females*)
XAMXAMfemale = kXXMM (XAMfs XAMms);

```

```

XAMXaMfemale = kXXMM (XAMfs XaMms + XaMfs XAMms) ;
XaMXaMfemale = kXXMM (XaMfs XaMms) ;
(*XM-XM males*)
XAMXAMmale = (1 - kXXMM) (XAMfs XAMms) ;
XAMXaMmale = (1 - kXXMM) (XAMfs XaMms + XaMfs XAMms) ;
XaMXaMmale = (1 - kXXMM) (XaMfs XaMms) ;

(*XM-YM females*)
XAMYAMfemale = kXYMM (XAMfs YAMms + YAMfs XAMms) ;
XAMYaMfemale = kXYMM (XAMfs YaMms + YaMfs XAMms) ;
XaMYAMfemale = kXYMM (XaMfs YAMms + YAMfs XaMms) ;
XaMYaMfemale = kXYMM (XaMfs YaMms + YaMfs XaMms) ;
(*XM-YM males*)
XAMYAMmale = (1 - kXYMM) (XAMfs YAMms + YAMfs XAMms) ;
XAMYaMmale = (1 - kXYMM) (XAMfs YaMms + YaMfs XAMms) ;
XaMYAMmale = (1 - kXYMM) (XaMfs YAMms + YAMfs XaMms) ;
XaMYaMmale = (1 - kXYMM) (XaMfs YaMms + YaMfs XaMms) ;

(*YM-YM females*)
YAMYAMfemale = kYYMM (YAMfs YAMms) ;
YAMYaMfemale = kYYMM (YAMfs YaMms + YaMfs YAMms) ;
YaMYaMfemale = kYYMM (YaMfs YaMms) ;
(*YM-YM males*)
YAMYAMmale = (1 - kYYMM) (YAMfs YAMms) ;
YAMYaMmale = (1 - kYYMM) (YAMfs YaMms + YaMfs YAMms) ;
YaMYaMmale = (1 - kYYMM) (YaMfs YaMms) ;

(*Mm heterozygotes*)
(*XM-Xm females*)
XAmXAMfemale = kXXMm (XAmfs XAMms + XAMfs XAmms) ;
XAmXaMfemale = kXXMm (XAmfs XaMms + XaMfs XAmms) ;
XAMXamfemale = kXXMm (XAMfs Xamms + Xamfs XAMms) ;
XamXaMfemale = kXXMm (Xamfs XaMms + XaMfs Xamms) ;
(*XM-Xm males*)
XAmXAMmale = (1 - kXXMm) (XAmfs XAMms + XAMfs XAmms) ;
XAmXaMmale = (1 - kXXMm) (XAmfs XaMms + XaMfs XAmms) ;
XAMXammale = (1 - kXXMm) (XAMfs Xamms + Xamfs XAMms) ;
XamXaMmale = (1 - kXXMm) (Xamfs XaMms + XaMfs Xamms) ;

(*Xm-YM females*)
XAmYAMfemale = kXYMm (XAmfs YAMms + YAMfs XAmms) ;
XAmYaMfemale = kXYMm (XAmfs YaMms + YaMfs XAmms) ;
XamYAMfemale = kXYMm (Xamfs YAMms + YAMfs Xamms) ;
XamYaMfemale = kXYMm (Xamfs YaMms + YaMfs Xamms) ;
(*Xm-YM males*)
XAmYAMmale = (1 - kXYMm) (XAmfs YAMms + YAMfs XAmms) ;
XAmYaMmale = (1 - kXYMm) (XAmfs YaMms + YaMfs XAmms) ;
XamYAMmale = (1 - kXYMm) (Xamfs YAMms + YAMfs Xamms) ;
XamYaMmale = (1 - kXYMm) (Xamfs YaMms + YaMfs Xamms) ;

(*XM-Ym females*)
XAMYAmfemale = kXYMm (XAMfs YAmms + YAmfs XAMms) ;
XAMYamfemale = kXYMm (XAMfs Yamms + Yamfs XAMms) ;
XaMYAmfemale = kXYMm (XaMfs YAmms + YAmfs XaMms) ;
XaMYamfemale = kXYMm (XaMfs Yamms + Yamfs XaMms) ;

```

```

(*XM-Ym females*)
XAMYAmmale = (1 - kXYMm) (XAMfs YAmms + YAmfs XAMms);
XAMYammale = (1 - kXYMm) (XAMfs Yamms + Yamfs XAMms);
XaMYAmmale = (1 - kXYMm) (XaMfs YAmms + YAmfs XaMms);
XaMYammale = (1 - kXYMm) (XaMfs Yamms + Yamfs XaMms);

(*Ym-YM females*)
YAmYAMfemale = kYYMm (YAmfs YAMms + YAMfs YAmms);
(*Ym-YM males*)
YAmYAMmale = (1 - kYYMm) (YAmfs YAMms + YAMfs YAmms);
YAmYaMmale = (1 - kYYMm) (YAmfs YaMms + YaMfs YAMms);
YamYAMmale = (1 - kYYMm) (Yamfs YAMms + YAMfs Yamms);
YamYaMmale = (1 - kYYMm) (Yamfs YaMms + YaMfs Yamms);

(*mm heterozygotes*)
(*Xm-Xm females*)
XAmXAmfemale = kXXmm (XAmfs XAmms);
XAmXamfemale = kXXmm (XAmfs Xamms + Xamfs XAmms);
XamXamfemale = kXXmm (Xamfs Xamms);
(*Xm-Xm males*)
XAmXammale = (1 - kXXmm) (XAmfs XAmms);
XAmXammale = (1 - kXXmm) (XAmfs Xamms + Xamfs XAmms);
XamXammale = (1 - kXXmm) (Xamfs Xamms);

(*Xm-Ym females*)
XAmYAmfemale = kXYmm (XAmfs YAmms + YAmfs XAmms);
(*Xm-Ym males*)
XAmYAmmale = (1 - kXYmm) (XAmfs YAmms + YAmfs XAmms);
XAmYammale = (1 - kXYmm) (XAmfs Yamms + Yamfs XAmms);
XamYAmmale = (1 - kXYmm) (Xamfs YAmms + YAmfs Xamms);
XamYammale = (1 - kXYmm) (Xamfs Yamms + Yamfs Xamms);

(*Ym-Ym females*)
YAmYAmfemale = kYYmm (YAmfs YAmms);
YAmYamfemale = kYYmm (YAmfs Yamms + Yamfs YAmms);
YamYAmfemale = kYYmm (Yamfs Yamms);
(*Ym-Ym males*)
YAmYAmmale = (1 - kYYmm) (YAmfs YAmms);
YAmYammale = (1 - kYYmm) (YAmfs Yamms + Yamfs YAmms);
YamYammale = (1 - kYYmm) (Yamfs Yamms);

```

Check that they sum to one

```

1 == XAMXAMfemale + XAMXaMfemale + XaMXaMfemale +
  XAMXAMmale + XAMXaMmale + XaMXaMmale +
  XAMYAMmale + XAMYaMmale + XaMYAMmale + XaMYaMmale +
  XAMYAMfemale + XAMYaMfemale + XaMYAMfemale + XaMYaMfemale +
  YAMYAMmale + YAMYaMmale + YaMYaMmale +
  YAMYAMfemale + YAMYaMfemale + YaMYaMfemale +
  XAmXAMfemale + XAmXaMfemale + XAMXamfemale + XamXaMfemale +
  XAmXAMmale + XAmXaMmale + XAMXammale + XamXaMmale +
  XAmYAMfemale + XAmYaMfemale + XamYAMfemale + XamYaMfemale +
  XAmYAMmale + XAmYaMmale + XamYAMmale + XamYaMmale +
  XAMYAmfemale + XAMYamfemale + XaMYAmfemale + XaMYamfemale +
  XAMYAmmale + XAMYammale + XaMYAmmale + XaMYammale +
  YAmYAMfemale + YAmYaMfemale + YamYAMfemale + YamYaMfemale +
  YAmYAMmale + YAmYaMmale + YamYAMmale + YamYaMmale +
  XAmXAmfemale + XAmXamfemale + XamXamfemale +
  XAmXAmmale + XAmXammale + XamXammale +
  XAmYAmfemale + XAmYamfemale + XamYAmfemale + XamYamfemale +
  XAmYAmmale + XAmYammale + XamYAmmale + XamYammale +
  YAmYAmfemale + YAmYamfemale + YamYamfemale +
  YAmYAmmale + YAmYammale + YamYammale // Factor

```

True

Diploid selection

We next model selection in diploids, which can differ among the sexes. Let FAA, FAa, and Faa be the relative fitnesses of females homozygous for allele A, heterozygous, or homozygous for allele a at the autosomal locus. Similarly we use MAA, MAa, and Maa in males. Frequencies after selection in diploids are indicated by appending an s to the name

```

(*mean male fitness*)
wbarM =
  MAA XAMXAMmale + MAa XAMXaMmale + Maa XaMXaMmale +
  MAA XAMYAMmale + MAa (XAMYaMmale + XAMYAMmale) + Maa XaMYaMmale +
  MAA YAMYAMmale + MAa YAMYaMmale + Maa YaMYaMmale +
  MAA XAmXAMmale + MAa (XAmXaMmale + XAMXammale) + Maa XamXaMmale +
  MAA XAmYAMmale + MAa (XAmYaMmale + XamYAMmale) + Maa XamYaMmale +
  MAA XAMYAmmale + MAa (XAMYammale + XaMYAmmale) + Maa XaMYammale +
  MAA YAmYAMmale + MAa (YAmYaMmale + YamYAMmale) + Maa YamYaMmale +
  MAA XAmXAmmale + MAa XAmXammale + Maa XamXammale +
  MAA XAmYAmmale + MAa (XAmYammale + XamYAmmale) + Maa XamYammale +
  MAA YAmYAmmale + MAa YAmYammale + Maa YamYammale;

XAMXAMmales = MAA XAMXAMmale / wbarM;
XAMXaMmales = MAa XAMXaMmale / wbarM;
XaMXaMmales = Maa XaMXaMmale / wbarM;

XAMYAMmales = MAA XAMYAMmale / wbarM;
XAMYaMmales = MAa XAMYaMmale / wbarM;
XaMYAMmales = MAa XaMYAMmale / wbarM;
XaMYaMmales = Maa XaMYaMmale / wbarM;

YAMYAMmales = MAA YAMYAMmale / wbarM;
YAMYaMmales = MAa YAMYaMmale / wbarM;

```

```

YaMYaMmales = Maa YaMYaMmale / wbarM;

XAmXAMmales = MAA XAmXAMmale / wbarM;
XAmXaMmales = Maa XAmXaMmale / wbarM;
XAMXammale = MAa XAMXammale / wbarM;
XamXaMmales = Maa XamXaMmale / wbarM;

XAmYAMmales = MAA XAmYAMmale / wbarM;
XAmYaMmales = Maa XAmYaMmale / wbarM;
XamYAMmales = MAa XamYAMmale / wbarM;
XamYaMmales = Maa XamYaMmale / wbarM;

XAMYAmmale = MAA XAMYAmmale / wbarM;
XAMYammale = MAa XAMYammale / wbarM;
XaMYAmmale = MAa XaMYAmmale / wbarM;
XaMYammale = Maa XaMYammale / wbarM;

YAmYAMmales = MAA YAmYAMmale / wbarM;
YAmYaMmales = Maa YAmYaMmale / wbarM;
YamYAMmales = MAa YamYAMmale / wbarM;
YamYaMmales = Maa YamYaMmale / wbarM;

XAmXAmmales = MAA XAmXAmmale / wbarM;
XAmXammale = MAa XAmXammale / wbarM;
XamXammale = Maa XamXammale / wbarM;

XAmYAmmales = MAA XAmYAmmale / wbarM;
XAmYammale = MAa XAmYammale / wbarM;
XamYAmmales = MAa XamYAmmale / wbarM;
XamYammale = Maa XamYammale / wbarM;

YAmYAmmales = MAA YAmYAmmale / wbarM;
YAmYammale = MAa YAmYammale / wbarM;
YamYammale = Maa YamYammale / wbarM;

```

Frequency of males after diploid selection should sum to 1

```

1 == XAMXAMmales + XAMXaMmales + XaMXaMmales +
  XAMYAMmales + XAMYaMmales + XaMYAMmales + XaMYaMmales +
  YAMYAMmales + YAMYaMmales + YaMYaMmales +
  XAmXAMmales + XAmXaMmales + XAMXammale + XamXaMmales +
  XAmYAMmales + XAmYaMmales + XamYAMmales + XamYaMmales +
  XAMYAmmale + XAMYammale + XaMYAmmale + XaMYammale +
  YAmYAMmales + YAmYaMmales + YamYAMmales + YamYaMmales +
  XAmXAmmales + XAmXammale + XamXammale +
  XAmYAmmales + XAmYammale + XamYAmmales + XamYammale +
  YAmYAmmales + YAmYammale + YamYammale // Factor

```

True

```

(*mean female fitness*)
wbarF = FAA XAMXAMfemale + FAa XAMXaMfemale + Faa XaMXaMfemale +
  FAA XAMYAMfemale + FAa (XAMYaMfemale + XaMYAMfemale) + Faa XaMYaMfemale +
  FAA YAMYAMfemale + FAa YAMYaMfemale + Faa YaMYaMfemale +
  FAA XAmXAMfemale + FAa XAmXaMfemale + FAa XAMXAmfemale + Faa XamXaMfemale +
  FAA XAmYAMfemale + FAa XAmYaMfemale + Faa XamYAMfemale + Faa XamYaMfemale +

```

```

FAA XAMYAmfemale + FAa XAMYamfemale + FAa XaMYAmfemale + Faa XaMYamfemale +
FAA YAMYAMfemale + FAa YAmYaMfemale + FAa YamYAMfemale + Faa YamYaMfemale +
FAA XAmXAmfemale + FAa XAmXamfemale + Faa XamXamfemale +
FAA XAmYAmfemale + FAa XAmYamfemale + FAa XamYAMfemale + Faa XamYamfemale +
FAA YAMYAmfemale + FAa YAmYamfemale + Faa YamYAMfemale;

XAMXAMfemales = FAA XAMXAMfemale / wbarF;
XAMXaMfemales = FAa XAMXaMfemale / wbarF;
XaMXaMfemales = Faa XaMXaMfemale / wbarF;

XAMYAMfemales = FAA XAMYAMfemale / wbarF;
XAMYaMfemales = FAa XAMYaMfemale / wbarF;
XaMYAMfemales = FAa XaMYAMfemale / wbarF;
XaMYaMfemales = Faa XaMYaMfemale / wbarF;

YAMYAMfemales = FAA YAMYAMfemale / wbarF;
YAMYaMfemales = FAa YAMYaMfemale / wbarF;
YamYA Mfemales = Faa YamYA Mfemale / wbarF;

XAmXAMfemales = FAA XAmXAMfemale / wbarF;
XAmXaMfemales = FAa XAmXaMfemale / wbarF;
XAMXamfemales = FAa XAMXamfemale / wbarF;
XamXaMfemales = Faa XamXaMfemale / wbarF;

XAmYAMfemales = FAA XAmYAMfemale / wbarF;
XAmYaMfemales = FAa XAmYaMfemale / wbarF;
XamYAMfemales = FAa XamYAMfemale / wbarF;
XamYaMfemales = Faa XamYaMfemale / wbarF;

XAMYAmfemales = FAA XAMYAmfemale / wbarF;
XAMYamfemales = FAa XAMYamfemale / wbarF;
XaMYAmfemales = FAa XaMYAmfemale / wbarF;
XaMYamfemales = Faa XaMYamfemale / wbarF;

YAmYAMfemales = FAA YAmYAMfemale / wbarF;
YAmYaMfemales = FAa YAmYaMfemale / wbarF;
YamYAMfemales = FAa YamYAMfemale / wbarF;
YamYaMfemales = Faa YamYaMfemale / wbarF;

XAmXAmfemales = FAA XAmXAmfemale / wbarF;
XAmXamfemales = FAa XAmXamfemale / wbarF;
XamXamfemales = Faa XamXamfemale / wbarF;

XAmYAmfemales = FAA XAmYAmfemale / wbarF;
XAmYamfemales = FAa XAmYamfemale / wbarF;
XamYAmfemales = FAa XamYAmfemale / wbarF;
XamYamfemales = Faa XamYamfemale / wbarF;

YAmYAmfemales = FAA YAmYAmfemale / wbarF;
YAmYamfemales = FAa YAmYamfemale / wbarF;
YamYAmfemales = Faa YamYAmfemale / wbarF;

```

Frequency of females after diplod selection should sum to 1

```

1 == XAMXAMfemales + XAMXaMfemales + XaMXaMfemales +
  XAMYAMfemales + XAMYaMfemales + XaMYAMfemales + XaMYaMfemales +
  YAMYAMfemales + YAMYaMfemales + YaMYaMfemales +
  XAmXAMfemales + XAmXaMfemales + XAMXamfemales + XamXaMfemales +
  XAmYAMfemales + XAmYaMfemales + XamYAMfemales + XamYaMfemales +
  XAMYAmfemales + XAMYamfemales + XaMYAmfemales + XaMYamfemales +
  YAmyYAMfemales + YAmYaMfemales + YamYAMfemales + YamYaMfemales +
  XAmXAmfemales + XAmXamfemales + XamXamfemales +
  XAmYAmfemales + XAmYamfemales + XamYAmfemales + XamYamfemales +
  YAmYAmfemales + YAmYamfemales + YamYamfemales // Factor

```

True

Meiosis

Translate zygote frequencies into notation in the main text to give recursion equations:

- a) first two letters refer to the ancestral SDR locus genotype (now lower case, e.g., xy)
- b) then two numbers ij give A and M locus genotype where 1=AM, 2=aM, 3=Am, 4=am
- c) then m or f indicate male or female
- d) then s indicates that these are frequencies after selection

e.g., xy41ms is the frequency of zygotes that are male and composed of gametes X-a-m and Y-A-M.
When i and j are interchangeable (in YY and XX individuals) the lowest number is given first.

```

xx11ms = XAMXAMmales;
xx12ms = XAMXaMmales;
xx22ms = XaMXaMmales;

xx13ms = XAmXAMmales;
xx23ms = XAmXaMmales;
xx14ms = XAMXammale;
xx24ms = XamXaMmales;

xx33ms = XAmXAmmale;
xx34ms = XAmXammale;
xx44ms = XamXammale;

xy11ms = XAMYAMmales;
xy12ms = XAMYaMmales;
xy21ms = XaMYAMmales;
xy22ms = XaMYaMmales;

xy31ms = XAmYAMmales;
xy32ms = XAmYaMmales;
xy41ms = XamYAMmales;
xy42ms = XamYaMmales;

xy13ms = XAMYAmmale;
xy14ms = XAMYammale;
xy23ms = XaMYAmmale;
xy24ms = XaMYammale;

xy33ms = XAmYAMmmale;
xy34ms = XAmYammale;
xy43ms = XamYAMmmale;
xy44ms = XamYammale;

yy11ms = YAMYAMmales;
yy12ms = YAMYaMmales;
yy22ms = YaMYaMmales;

yy13ms = YAmYAMmales;
yy23ms = YAmYaMmales;
yy14ms = YamYAMmales;
yy24ms = YamYaMmales;

yy33ms = YAmYAmmale;
yy34ms = YAmYammale;
yy44ms = YamYammale;

```

```

xx11fs = XAMXAMfemales;
xx12fs = XAMXaMfemales;
xx22fs = XaMXaMfemales;

xx13fs = XAmXAMfemales;
xx23fs = XAmXaMfemales;
xx14fs = XAMXamfemales;
xx24fs = XamXaMfemales;

xx33fs = XAmXAmfemales;
xx34fs = XAmXamfemales;
xx44fs = XamXamfemales;

xy11fs = XAMYAMfemales;
xy12fs = XAMYaMfemales;
xy21fs = XaMYAMfemales;
xy22fs = XaMYaMfemales;

xy31fs = XAmYAMfemales;
xy32fs = XAmYaMfemales;
xy41fs = XamYAMfemales;
xy42fs = XamYaMfemales;

xy13fs = XAMYAmfemales;
xy14fs = XAMYamfemales;
xy23fs = XaMYAmfemales;
xy24fs = XaMYamfemales;

xy33fs = XAmYAmfemales;
xy34fs = XAmYamfemales;
xy43fs = XamYAmfemales;
xy44fs = XamYamfemales;

yy11fs = YAMYAMfemales;
yy12fs = YAMYaMfemales;
yy22fs = YaMYaMfemales;

yy13fs = YAmyYAMfemales;
yy23fs = YAmyYaMfemales;
yy14fs = YamYAMfemales;
yy24fs = YamYaMfemales;

yy33fs = YAmyYAmfemales;
yy34fs = YAmyYamfemales;
yy44fs = YamYamfemales;

```

Let the probability of recombination (an odd number of crossover events) between the ancestral SDR and the selected locus be r . Let the probability of recombination between the selected locus and the novel SDR be R . And let the probability of recombination between the ancestral and novel SDRs be ρ .

Let the probability that a heterozygote at the selected locus passes down an A depend on sex, αf or αm . E.g., when $\alpha f=1/2$ there is no meiotic drive in females.

The next generation frequencies in female gametes are then

```

(*X bearing ovules*)
nextXAMf = xx11fs + xx13fs / 2 + (xx12fs + xx14fs * (1 - R) + xx23fs * R) af +
    xy11fs / 2 + (xy12fs (1 - r) + xy21fs (r)) af +
    (xy13fs (1 - ρ) + xy31fs (ρ)) / 2 +
    xy14fs * af +
    (xy14fs (- (R + r + ρ)) +
        xy41fs (r + ρ - R) +
        xy23fs (R + r - ρ) +
        xy32fs (R + ρ - r)) af / 2;

nextXaMf = xx22fs + xx24fs / 2 + (xx12fs + xx23fs * (1 - R) + xx14fs * R) (1 - af) +
    xy22fs / 2 + (xy12fs (r) + xy21fs (1 - r)) (1 - af) +
    (xy24fs (1 - ρ) + xy42fs (ρ)) / 2 +
    xy23fs * (1 - af) +
    (xy14fs (R + r - ρ) +
        xy41fs (R + ρ - r) +
        xy23fs (- (R + r + ρ)) +
        xy32fs (r + ρ - R)) (1 - af) / 2;

nextXAmf = xx33fs + xx13fs / 2 + (xx23fs * (1 - R) + R * xx14fs + xx34fs) af +
    xy33fs / 2 + (xy34fs (1 - r) + xy43fs (r)) af +
    (xy13fs (ρ) + xy31fs (1 - ρ)) / 2 +
    xy32fs * af +
    (xy14fs (R + ρ - r) +
        xy41fs (R + r - ρ) +
        xy23fs (r + ρ - R) +
        xy32fs (- (R + r + ρ))) af / 2;

nextXamf = xx44fs + xx24fs / 2 + (xx14fs * (1 - R) + xx23fs * R + xx34fs) (1 - af) +
    xy44fs / 2 + (xy34fs (r) + xy43fs (1 - r)) (1 - af) +
    (xy24fs (ρ) + xy42fs (1 - ρ)) / 2 +
    xy41fs * (1 - af) +
    (xy14fs (r + ρ - R) +
        xy41fs (- (R + r + ρ)) +
        xy23fs (R + ρ - r) +
        xy32fs (R + r - ρ)) (1 - af) / 2;

(*Y bearing ovules*)
nextYAMf = yy11fs + yy13fs / 2 + (yy12fs + yy14fs * (1 - R) + yy23fs * R) af +
    xy11fs / 2 + (xy12fs (r) + xy21fs (1 - r)) (af) +
    (xy13fs (ρ) + xy31fs (1 - ρ)) / 2 +
    xy41fs * af +
    (xy14fs (r + ρ - R) +
        xy41fs (- (R + r + ρ)) +
        xy23fs (R + ρ - r) +
        xy32fs (R + r - ρ)) af / 2;

nextYaMf = yy22fs + yy24fs / 2 + (yy12fs + yy23fs * (1 - R) + yy14fs * R) (1 - af) +
    xy22fs / 2 + (xy12fs (1 - r) + xy21fs (r)) (1 - af) +
    (xy24fs (ρ) + xy42fs (1 - ρ)) / 2 +
    xy32fs * (1 - af) +
    (xy14fs (R + ρ - r) +
        xy41fs (R + r - ρ) +
        xy23fs (r + ρ - R) +
        xy32fs (- (R + r + ρ))) (1 - af) / 2;

```

```

nextYAmf = yy33fs + yy13fs / 2 + (yy23fs * (1 - R) + R * yy14fs + yy34fs) αf +
xy33fs / 2 + (xy34fs (r) + xy43fs (1 - r)) (αf) +
(xy13fs (1 - ρ) + xy31fs (ρ)) / 2 +
xy23fs * αf +
(xy14fs (R + r - ρ) +
xy41fs (R + ρ - r) +
xy23fs (- (R + r + ρ)) +
xy32fs (r + ρ - R)) αf / 2;

nextYamf = yy44fs + yy24fs / 2 + (yy14fs * (1 - R) + yy23fs * R + yy34fs) (1 - αf) +
xy44fs / 2 + (xy34fs (1 - r) + xy43fs (r)) (1 - αf) +
(xy24fs (1 - ρ) + xy42fs (ρ)) / 2 +
xy14fs * (1 - αf) +
(xy14fs (- (R + r + ρ)) +
xy41fs (r + ρ - R) +
xy23fs (R + r - ρ) +
xy32fs (R + ρ - r)) (1 - αf) / 2;

Factor[
nextXAMf + nextXaMf + nextXAmf + nextXamf + nextYAMf + nextYaMf + nextYAmf + nextYamf]
1

```

And the next generation frequencies in male gametes are

```

(*X bearing pollen*)
nextXAMm = xx11ms + xx13ms / 2 + (xx12ms + xx14ms * (1 - R) + xx23ms * R) αm +
xy11ms / 2 + (xy12ms (1 - r) + xy21ms (r)) αm +
(xy13ms (1 - ρ) + xy31ms (ρ)) / 2 +
xy14ms * αm +
(xy14ms (- (R + r + ρ)) +
xy41ms (r + ρ - R) +
xy23ms (R + r - ρ) +
xy32ms (R + ρ - r)) αm / 2;

nextXaMm = xx22ms + xx24ms / 2 + (xx12ms + xx23ms * (1 - R) + xx14ms * R) (1 - αm) +
xy22ms / 2 + (xy12ms (r) + xy21ms (1 - r)) (1 - αm) +
(xy24ms (1 - ρ) + xy42ms (ρ)) / 2 +
xy23ms * (1 - αm) +
(xy14ms (R + r - ρ) +
xy41ms (R + ρ - r) +
xy23ms (- (R + r + ρ)) +
xy32ms (r + ρ - R)) (1 - αm) / 2;

nextXAmm = xx33ms + xx13ms / 2 + (xx23ms * (1 - R) + R * xx14ms + xx34ms) αm +
xy33ms / 2 + (xy34ms (1 - r) + xy43ms (r)) αm +
(xy13ms (ρ) + xy31ms (1 - ρ)) / 2 +
xy32ms * αm +
(xy14ms (R + ρ - r) +
xy41ms (R + r - ρ) +
xy23ms (r + ρ - R) +
xy32ms (- (R + r + ρ))) αm / 2;

nextXamm = xx44ms + xx24ms / 2 + (xx14ms * (1 - R) + xx23ms * R + xx34ms) (1 - αm) +
xy44ms / 2 + (xy34ms (r) + xy43ms (1 - r)) (1 - αm) +

```

```

(xy24ms ( $\rho$ ) + xy42ms (1 -  $\rho$ )) / 2 +
xy41ms * (1 -  $\alpha m$ ) +
(xy14ms (r +  $\rho$  - R) +
xy41ms (- (R + r +  $\rho$ )) +
xy23ms (R +  $\rho$  - r) +
xy32ms (R + r -  $\rho$ )) (1 -  $\alpha m$ ) / 2;

(*Y bearing pollen*)
nextYAMm = yy11ms + yy13ms / 2 + (yy12ms + yy14ms * (1 - R) + yy23ms * R)  $\alpha m$  +
xy11ms / 2 + (xy12ms (r) + xy21ms (1 - r)) ( $\alpha m$ ) +
(xy13ms ( $\rho$ ) + xy31ms (1 -  $\rho$ )) / 2 +
xy41ms *  $\alpha m$  +
(xy14ms (r +  $\rho$  - R) +
xy41ms (- (R + r +  $\rho$ )) +
xy23ms (R +  $\rho$  - r) +
xy32ms (R + r -  $\rho$ ))  $\alpha m$  / 2;

nextYaMm = yy22ms + yy24ms / 2 + (yy12ms + yy23ms * (1 - R) + yy14ms * R) (1 -  $\alpha m$ ) +
xy22ms / 2 + (xy12ms (1 - r) + xy21ms (r)) (1 -  $\alpha m$ ) +
(xy24ms ( $\rho$ ) + xy42ms (1 -  $\rho$ )) / 2 +
xy32ms * (1 -  $\alpha m$ ) +
(xy14ms (R +  $\rho$  - r) +
xy41ms (R + r -  $\rho$ ) +
xy23ms (r +  $\rho$  - R) +
xy32ms (- (R + r +  $\rho$ ))) (1 -  $\alpha m$ ) / 2;

nextYAmM = yy33ms + yy13ms / 2 + (yy23ms * (1 - R) + R * yy14ms + yy34ms)  $\alpha m$  +
xy33ms / 2 + (xy34ms (r) + xy43ms (1 - r)) ( $\alpha m$ ) +
(xy13ms (1 -  $\rho$ ) + xy31ms ( $\rho$ )) / 2 +
xy23ms *  $\alpha m$  +
(xy14ms (R + r -  $\rho$ ) +
xy41ms (R +  $\rho$  - r) +
xy23ms (- (R + r +  $\rho$ )) +
xy32ms (r +  $\rho$  - R))  $\alpha m$  / 2;

nextYamm = yy44ms + yy24ms / 2 + (yy14ms * (1 - R) + yy23ms * R + yy34ms) (1 -  $\alpha m$ ) +
xy44ms / 2 + (xy34ms (1 - r) + xy43ms (r)) (1 -  $\alpha m$ ) +
(xy24ms (1 -  $\rho$ ) + xy42ms ( $\rho$ )) / 2 +
xy14ms * (1 -  $\alpha m$ ) +
(xy14ms (- (R + r +  $\rho$ )) +
xy41ms (r +  $\rho$  - R) +
xy23ms (R + r -  $\rho$ ) +
xy32ms (R +  $\rho$  - r)) (1 -  $\alpha m$ ) / 2;

Factor[nextXAMm + nextXaMm + nextXAmm +
nextXamm + nextYAMm + nextYaMm + nextYAmM + nextYamm]
1

```

Let the probability a zygote develops as a female be completely determined by the ancestral SDR when a zygote has no m alleles (XX females and XY and YY males) and be k whenever a zygote has at least one m allele:

```

SUBS = {
  kXXMM → 1,
  kXYMM → 0,
  kYYMM → 0,
  kXXMm → k,
  kXYMm → k,
  kYYMm → k,
  kXXmm → k,
  kXYmm → k,
  kYYmm → k
};


```

Resident equilibria and stability

General

We first explore the resident equilibria, where the m allele is absent and sex is only determined by the ancestral sex-determining locus. In this case there no Y alleles in gametes from females and no M alleles in gametes from any sex, leaving us with 6 equations.

```

{nextXAMm, nextXaMm, nextXamm, nextXamm, nextYAMm, nextYaMm, nextYamm, nextYam,
 nextXAmf, nextXaMf, nextXAmf, nextXamf, nextYAmf, nextYaMf, nextYAmf,
 nextYamf} /. XAm → 0 /. Xamm → 0 /. YAm → 0 /. Yam → 0 /. XAmf → 0 /.
 Xamf → 0 /. Yamf → 0 /. Yamf → 0 /. Yamf → 0 /. YaMf → 0 /. SUBS // Simplify

{ (MAA wAf wAm XAMf YAMm + 2 MAA (-(-1+r) wAf wam XAMf YaMm + r waf wAm XaMf YAMm) αm) /
 (2 (Maa waf wam XaMf YaMm + MAA wAf wAm XAMf YaMm) , (Maa waf wam XaMf YaMm -
 MAA wAf wAm XaMf YAMm + MAA wAf wAm XAMf YAMm) ) , (Maa waf wam XaMf YaMm -
 2 MAA (r wAf wam XAMf YaMm + waf wAm XaMf YAMm - r waf wAm XaMf YAMm) (-1 + αm) ) /
 (2 (Maa waf wam XaMf YaMm + MAA wAf wAm XAMf YaMm + MAA waf wAm XaMf YAMm +
 MAA wAf wAm XAMf YAMm) ) , 0, 0, (MAA wAf wAm XAMf YAMm +
 2 MAA (r wAf wam XAMf YaMm + waf wAm XaMf YAMm - r waf wAm XaMf YAMm) αm) /
 (2 (Maa waf wam XaMf YaMm + MAA wAf wAm XAMf YaMm +
 MAA waf wAm XaMf YAMm + MAA wAf wAm XAMf YAMm) ) ,
 (Maa waf wam XaMf YaMm + 2 MAA ((-1+r) wAf wam XAMf YaMm - r waf wAm XaMf YAMm) (-1 + αm) ) /
 (2 (Maa waf wam XaMf YaMm + MAA wAf wAm XAMf YaMm +
 MAA waf wAm XaMf YAMm + MAA wAf wAm XAMf YAMm) ) , 0, 0,
 FAA wAf wAm XAMf XAMm + FAA wAf wam XAMf XaMm αf + FAA waf wAm XaMf XAMm αf
 Faa waf wam XaMf XaMm + FAA wAf wam XAMf XaMm + FAA waf wAm XaMf XAMm + FAA wAf wAm XAMf XAMm
 ,
 Faa waf wam XaMf XaMm - FAA (wAf wam XAMf XaMm + waf wAm XaMf XAMm) (-1 + αf)
 Faa waf wam XaMf XaMm + FAA wAf wam XAMf XaMm + FAA waf wAm XaMf XAMm + FAA wAf wAm XAMf XAMm
 , 0, 0, 0, 0, 0, 0}

```

Letting p_{Xf} be the frequency of A in non-mutant female gametes, p_{Xm} be the frequency of A in non-mutant male gametes with an X allele, p_{Ym} be the frequency of A in non-mutant female gametes with a Y allele, and q be the frequency of Y alleles in male gametes

```

subequil = {
  XAMf → pXf,
  XaMf → (1 - pXf),
  YAMf → 0,
  YaMf → 0,
  XAMm → pXm (1 - q),
  XaMm → (1 - pXm) (1 - q),
  YAMm → pYm * q,
  YaMm → (1 - pYm) * q,
  XAmf → 0,
  Xamf → 0,
  XAmm → 0,
  Xamm → 0,
  YAmm → 0,
  Yamm → 0,
  YAmf → 0,
  Yamf → 0
};

```

this system of six equations reduces to four (because the two female gamete frequencies must sum to one, as must the 4 male gamete frequencies).

We now wish to solve for the equilibrium frequencies of the haploid genotypes in gametes of each sex. Equilibrium is achieved when the following four equations are zero

```

differenceEqs = {
  nextXAMf - XAMf,
  nextXAMm - XAMm,
  nextYAMm - YAMm,
  nextYAMm + nextYaMm - (YAMm + YaMm)
} /. SUBS /. subequil;

```

The first equation is zero when

```

sub1 = Flatten[Solve[differenceEqs[[3]] == 0, pXf]] // Factor
{pXf → (2 pYm waf (-Maa q wam + Maa pYm q wam - MAa pYm q wAm + MAa wAm cm - MAa r wAm cm)) /
(-2 Maa pYm q waf wam + 2 Maa pYm^2 q waf wam + 2 MAa pYm q wAf wam - 2 MAa pYm^2 q wAf wam -
2 MAa pYm^2 q waf wAm - MAA pYm wAf wAm + 2 MAA pYm^2 q wAf wAm - 2 MAa r wAf wam cm +
2 MAa pYm r wAf wam cm + 2 MAa pYm waf wAm cm - 2 MAa pYm r waf wAm cm)}]

```

The second equation is zero when

```

sub2 = Flatten[Solve[Factor[differenceEqs[[2]] /. sub1 == 0, pXm]] // Factor
{pXm →
 - (pYm (-Maa MAA pYm q wam wAm + Maa MAA pYm2 q wam wAm - MAa MAA pYm2 q wAm2 - 2 Maa MAA q
 wam2 cm + 4 Maa MAa pYm q wam2 cm - 2 Maa MAa pYm2 q wam2 cm + 2 Maa MAa q r wam2 cm -
 4 Maa MAa pYm q r wam2 cm + 2 Maa MAa pYm2 q r wam2 cm - 2 MAa2 pYm q wam wAm cm +
 2 MAa2 pYm2 q wam wAm cm + 4 MAa2 pYm q r wam wAm cm - 4 MAa2 pYm2 q r wam wAm cm +
 MAa MAA pYm wAm2 cm - 2 MAa MAA pYm r wAm2 cm + 2 MAa MAA pYm2 q r wAm2 cm + 2 MAa2 wam
 wAm cm2 - 2 MAa2 pYm wam wAm cm2 - 4 MAa2 r wam wAm cm2 + 4 MAa2 pYm r wam wAm cm2) ) /
 ((-1 + q) (-Maa MAA pYm wam wAm + Maa MAA pYm2 wam wAm - MAa MAA pYm2 wAm2 -
 2 Maa MAa r wam2 cm + 4 Maa MAa pYm r wam2 cm - 2 Maa MAa pYm2 r wam2 cm +
 2 MAa2 pYm wam wAm cm - 2 MAa2 pYm2 wam wAm cm - 4 MAa2 pYm r wam wAm cm +
 4 MAa2 pYm2 r wam wAm cm + 2 MAa MAA pYm2 wAm2 cm - 2 MAa MAA pYm2 r wAm2 cm) ) }
}

```

The fourth equation is zero when

```

sub3 = Flatten[Solve[Factor[differenceEqs[[4]] /. sub1 /. sub2 == 0, q]] // Factor
{q → - (Maa MAA pYm wam wAm - Maa MAA pYm2 wam wAm + 2 MAa MAA pYm2 r wAm2 +
 2 MAa MAa r wam2 cm - 4 Maa MAa pYm r wam2 cm + 2 Maa MAa pYm2 r wam2 cm -
 4 MAa2 pYm wam wAm cm + 4 MAa2 pYm2 wam wAm cm + 8 MAa2 pYm r wam wAm cm -
 8 MAa2 pYm2 r wam wAm cm - 2 MAa MAA pYm2 r wAm2 cm + 4 MAa2 pYm wam wAm cm2 -
 4 MAa2 pYm2 wam wAm cm2 - 8 MAa2 pYm r wam wAm cm2 + 8 MAa2 pYm2 r wam wAm cm2) /
 (2 (Maa MAa pYm wam2 - 2 Maa MAa pYm2 wam2 + Maa MAa pYm3 wam2 - 2 Maa MAa pYm r wam2 +
 4 Maa MAa pYm2 r wam2 - 2 Maa MAa pYm3 r wam2 - Maa MAA pYm wam wAm +
 2 MAa2 pYm2 wam wAm + Maa MAA pYm2 wam wAm - 2 MAa2 pYm3 wam wAm -
 4 MAa2 pYm2 r wam wAm + 4 MAa2 pYm3 r wam wAm - MAa MAA pYm2 wAm2 + MAa MAA pYm3 wAm2 -
 2 MAa MAA pYm3 r wAm2 - 2 Maa MAa pYm wam2 cm + 4 Maa MAa pYm2 wam2 cm -
 2 Maa MAa pYm3 wam2 cm - 2 Maa MAa r wam2 cm + 8 Maa MAa pYm r wam2 cm -
 10 Maa MAa pYm2 r wam2 cm + 4 Maa MAa pYm3 r wam2 cm + 2 MAa2 pYm wam wAm cm -
 6 MAa2 pYm2 wam wAm cm + 4 MAa2 pYm3 wam wAm cm - 4 MAa2 pYm r wam wAm cm +
 12 MAa2 pYm2 r wam wAm cm - 8 MAa2 pYm3 r wam wAm cm + 2 MAa MAA pYm2 wAm2 cm -
 2 MAa MAA pYm3 wAm2 cm - 2 MAa MAA pYm2 r wAm2 cm + 4 MAa MAA pYm3 r wAm2 cm) ) }
}

```

Leaving us with an equation for pYm only

```
equilsol = Numerator[Factor[differenceEqs[[1]] /. sub1 /. sub2 /. sub3]];
```

Here we use linear stability analysis to examine when the resident equilibrium (i.e., no mutants) derived above is stable.

The internal stability matrix (i.e., the Jacobian for the residents) is

```

eqs = {nextXAMf, nextXAMm / (nextXAMm + nextXaMm),
       nextYAMm / (nextYAMm + nextYaMm), nextYAMm + nextYaMm} /. SUBS /. subequil;
matIntFull = Transpose[{
  D[eqs, pXf],
  D[eqs, pXm],
  D[eqs, pYm],
  D[eqs, freqYm]
} /. SUBS /. subequil // Factor;

```

The last column of matIntFull is full of zeros, corresponding to an eigenvalue of zero and indicating that the sex ratio in the system equilibrates immediately:

```
Transpose[matIntFull][[4]]
{0, 0, 0, 0}
```

As a consequence, we drop the last row and column in matIntFull:

```
matIntFull = Table[matIntFull[[i, j]], {i, 1, 3}, {j, 1, 3}];
```

The characteristic polynomial is

```
charpolyIntFull = Det[matIntFull - IdentityMatrix[3] * λ];
```

We will explore the equilibria for two special cases, tight linkage and weak selection.

Tight linkage

Equilibria

Here we look at the case where there is tight linkage between the ancestral sex-determining locus and the selected locus.

When there is complete linkage, $r=0$ (ie the selected locus is in the ancestral sex-determining region), the equilibrium frequency of A on Y gametes in males is

```
equilsol /. r → 0 // Factor;
equilsol2 = Solve[% == 0, pYm] // Simplify
{{pYm → 0}, {pYm → 0}, {pYm → 0}, {pYm → 1}, {pYm → 1}, {pYm → 1},
 {pYm → (wam (Faa Maa waf wam (MAA wAm + 2 MAa wam (-1 + αm)) + 2 FAA MAa wAf wAm αm
 (-Maa wam + 2 MAa wAm αm) - FAa (Maa wam + 2 MAa wAm αm) (Maa waf wam (-1 + αf) +
 MAA wAf wAm αf + 2 MAa (wAf wam αf (-1 + αm) - waf wAm (-1 + αf) αm))) /
 (Faa waf wam (MAA wAm + 2 MAa wam (-1 + αm)) (Maa wam + 2 MAa wAm (-1 + αm)) +
 FAA wAf wAm (MAA wAm - 2 MAa wam αm) (Maa wam - 2 MAa wAm αm) -
 FAa (Maa wam^2 - wAm (MAA wAm + MAa wam (2 - 4 αm))) (Maa waf wam (-1 + αf) +
 MAA wAf wAm αf + 2 MAa (wAf wam αf (-1 + αm) - waf wAm (-1 + αf) αm)))})}}
```

First, consider cases where $pYm=0$.

With allele a fixed on Y ($pYm=0$), then $\{pXf, pXm, q\}$ equal:

```
Solve[(differenceEqs /. r → 0 /. pYm → 0 // Factor) == {0, 0, 0, 0}, {pXf, pXm, q}] // Simplify
{ $\left\{\left\{pXf \rightarrow 0, pXm \rightarrow 0, q \rightarrow \frac{1}{2}\right\}, \{pXf \rightarrow 1, pXm \rightarrow 1, q \rightarrow 1 - \alpha m\}, \right.$ 
 $\left. \left\{pXf \rightarrow \frac{waf (Faa Maa waf wam - Faa wAf \alpha f (Maa wam + 2 MAa wAm \alpha m))}{Faa Maa waf^2 wam + wAf (2 FAA MAa wAf wAm \alpha m - Faa waf (Maa wam + 2 MAa wAm \alpha m))}, \right.\right.$ 
 $pXm \rightarrow -(2 MAa \alpha m (-Faa Maa waf wam + Faa wAf \alpha f (Maa wam + 2 MAa wAm \alpha m))) / (2 Maa MAa (Faa waf wam + FAA wAf wAm) \alpha m +$ 
 $Faa (Maa wam + 2 MAa wAm \alpha m) (Maa waf (-1 + \alpha f) - 2 MAa wAf \alpha f \alpha m)),$ 
 $q \rightarrow (Faa (Maa waf (-1 + \alpha f) + 2 MAa wAf \alpha f (-1 + \alpha m)) (Maa wam + 2 MAa wAm \alpha m) +$ 
 $2 Maa MAa (-Faa waf wam (-1 + \alpha m) + FAA wAf wAm \alpha m)) / (2 (Faa (Maa waf (-1 + \alpha f) - MAa wAf \alpha f) (Maa wam + 2 MAa wAm \alpha m) +$ 
 $Maa MAa (Faa waf wam + 2 FAA wAf wAm \alpha m)))\right\}\right\}$ 
```

The last of these has a polymorphism on the X chromosome; we'll call this equilA0 ("0" because this is the leading order term in r...we could take additional order terms to get the linear approximation in r, etc.).

```
equilA0 = {pXf → (pXf /. %[[3]]), pXm → (pXm /. %[[3]]), pYm → 0, q → (q /. %[[3]])}
{pXf →  $waf (Faa Maa waf wam - Faa wAf \alpha f (Maa wam + 2 MAa wAm \alpha m))$ ,
 $Faa Maa waf^2 wam + wAf (2 FAA MAa wAf wAm \alpha m - Faa waf (Maa wam + 2 MAa wAm \alpha m))$ ,  

 $pXm \rightarrow -(2 MAa \alpha m (-Faa Maa waf wam + Faa wAf \alpha f (Maa wam + 2 MAa wAm \alpha m))) / (2 Maa MAa (Faa waf wam + FAA wAf wAm) \alpha m +$ 
 $Faa (Maa wam + 2 MAa wAm \alpha m) (Maa waf (-1 + \alpha f) - 2 MAa wAf \alpha f \alpha m)), pYm \rightarrow 0,$ 
 $q \rightarrow (Faa (Maa waf (-1 + \alpha f) + 2 MAa wAf \alpha f (-1 + \alpha m)) (Maa wam + 2 MAa wAm \alpha m) +$ 
 $2 Maa MAa (-Faa waf wam (-1 + \alpha m) + FAA wAf wAm \alpha m)) / (2 (Faa (Maa waf (-1 + \alpha f) - MAa wAf \alpha f) (Maa wam + 2 MAa wAm \alpha m) +$ 
 $Maa MAa (Faa waf wam + 2 FAA wAf wAm \alpha m)))\right\}$ 
```

The second of these is also polymorphic (Y fixed on a and X fixed on A), which we'll call equilB0.

```
equilB0 = {pXf → (pXf /. %%[[2]]), pXm → (pXm /. %%[[2]]), pYm → 0, q → (q /. %%[[2]])}
{pXf → 1, pXm → 1, pYm → 0, q → 1 - \alpha m}
```

Next, consider cases where pYm=1.

With allele A fixed on Y (pYm=0), then {pXf, pXm, q} equal:

```

Solve[(differenceEqs /. r → 0 /. pYm → 1 // Factor) == {0, 0, 0, 0}, {pXf, pXm, q}] // Simplify
{ {pXf → 0, pXm → 0, q → αm}, {pXf → 1, pXm → 1, q → 1/2}, }

{pXf → (waf (FAa wAf αf (MAA wAm - 2 MAa wam (-1 + αm)) + 2 Faa MAa waf wam (-1 + αm))) / (wAf (-FAA MAA wAf wAm + FAa waf (MAA wAm - 2 MAa wam (-1 + αm))) + 2 Faa MAa waf^2 wam (-1 + αm)), pXm → (MAA (FAa wAf αf (MAA wAm - 2 MAa wam (-1 + αm)) + 2 Faa MAa waf wam (-1 + αm))) / (-FAa (-MAA wAm + 2 MAa wam (-1 + αm)) (MAA wAf αf + 2 MAa waf (-1 + αf) (-1 + αm)) + 2 MAa MAA (Faa waf wam + FAA wAf wAm) (-1 + αm)), q → (2 Faa MAa MAA waf wam (-1 + αm) - 2 FAA MAa MAA wAf wAm αm + Faa (-MAA wAm + 2 MAa wam (-1 + αm)) (-MAA wAf αf + 2 MAa waf (-1 + αf) αm)) / (2 (FAa (MAa waf (-1 + αf) - MAA wAf αf) (-MAA wAm + 2 MAa wam (-1 + αm)) + MAa MAA (-FAA wAf wAm + 2 Faa waf wam (-1 + αm)))) } }

```

The last of these has a polymorphism on the X chromosome; we'll call this equilAp0 ("p" for prime -- the alternate allele is fixed on the Y).

```

equilAp0 = {pXf → (pXf /. %[[3]]), pXm → (pXm /. %[[3]]), pYm → 1, q → (q /. %[[3]])}
{pXf → (waf (FAa wAf αf (MAA wAm - 2 MAa wam (-1 + αm)) + 2 Faa MAa waf wam (-1 + αm))) / (wAf (-FAA MAA wAf wAm + FAa waf (MAA wAm - 2 MAa wam (-1 + αm))) + 2 Faa MAa waf^2 wam (-1 + αm)), pXm → (MAA (FAa wAf αf (MAA wAm - 2 MAa wam (-1 + αm)) + 2 Faa MAa waf wam (-1 + αm))) / (-FAa (-MAA wAm + 2 MAa wam (-1 + αm)) (MAA wAf αf + 2 MAa waf (-1 + αf) (-1 + αm)) + 2 MAa MAA (Faa waf wam + FAA wAf wAm) (-1 + αm)), pYm → 1, q → (2 Faa MAa MAA waf wam (-1 + αm) - 2 FAA MAa MAA wAf wAm αm + Faa (-MAA wAm + 2 MAa wam (-1 + αm)) (-MAA wAf αf + 2 MAa waf (-1 + αf) αm)) / (2 (FAa (MAa waf (-1 + αf) - MAA wAf αf) (-MAA wAm + 2 MAa wam (-1 + αm)) + MAa MAA (-FAA wAf wAm + 2 Faa waf wam (-1 + αm)))) }

```

The first of these is also polymorphic (Y fixed on A and X fixed on a), which we'll call equilBp0.

```

equilBp0 = {pXf → (pXf /. %%[[1]]), pXm → (pXm /. %%[[1]]), pYm → 1, q → (q /. %%[[1]])}
{pXf → 0, pXm → 0, pYm → 1, q → αm}

```

Finally, we have the equilibrium with a polymorphism on the Y (which will turn out not to be stable)

```

Factor[{pXf, pXm, pYm, q} /. sub1 /. sub2 /. sub3 /. r -> 0] /. Last[equilSol2] //
Simplify;
equilC0 = {pXf -> %[[1]], pXm -> %[[2]], pYm -> %[[3]], q -> %[[4]]}

{pXf -> waf (Maa wam - 2 MAA wAm  $\alpha m$ ) /.
Maa waf wam + MAA wAf wAm + 2 MAA (-wAf wam + wAf wam  $\alpha m$  - waf wAm  $\alpha m$ ) ,
pXm -> (wam (Maa wam - 2 MAA wAm  $\alpha m$ )
(-Faa waf (MAA wAm + 2 MAA wam (-1 +  $\alpha m$ )) + Faa (Maa waf wam (-1 +  $\alpha f$ ) +
MAA wAf wAm  $\alpha f$  + 2 MAA (wAf wam  $\alpha f$  (-1 +  $\alpha m$ ) - waf wAm (-1 +  $\alpha f$ )  $\alpha m$ ))) /
(- (Faa waf wam + FAA wAf wAm) (MAA wAm + 2 MAA wam (-1 +  $\alpha m$ )) (Maa wam - 2 MAA wAm  $\alpha m$ ) +
Faa (Maa waf wam (-1 +  $\alpha f$ ) + MAA wAf wAm  $\alpha f$  +
2 MAA (wAf wam  $\alpha f$  (-1 +  $\alpha m$ ) - waf wAm (-1 +  $\alpha f$ )  $\alpha m$ ))
(Maa wam2 - wAm (MAA wAm + 2 MAA wam (-1 + 2  $\alpha m$ )))) ,
pYm -> (wam (Faa Maa waf wam (MAA wAm + 2 MAA wam (-1 +  $\alpha m$ )) + 2 FAA MAA wAf wAm  $\alpha m$ 
(-Maa wam + 2 MAA wAm  $\alpha m$ ) - Faa (Maa wam + 2 MAA wAm  $\alpha m$ ) (Maa waf wam (-1 +  $\alpha f$ ) +
MAA wAf wAm  $\alpha f$  + 2 MAA (wAf wam  $\alpha f$  (-1 +  $\alpha m$ ) - waf wAm (-1 +  $\alpha f$ )  $\alpha m$ ))) /
(Faa waf wam (MAA wAm + 2 MAA wam (-1 +  $\alpha m$ )) (Maa wam + 2 MAA wAm (-1 +  $\alpha m$ )) +
FAA wAf wAm (MAA wAm - 2 MAA wam  $\alpha m$ ) (Maa wam - 2 MAA wAm  $\alpha m$ ) -
Faa (Maa wam2 - wAm (MAA wAm + MAA wam (2 - 4  $\alpha m$ ))) (Maa waf wam (-1 +  $\alpha f$ ) +
MAA wAf wAm  $\alpha f$  + 2 MAA (wAf wam  $\alpha f$  (-1 +  $\alpha m$ ) - waf wAm (-1 +  $\alpha f$ )  $\alpha m$ )) ,
q -> (Faa waf wam (MAA wAm + 2 MAA wam (-1 +  $\alpha m$ )) (Maa wam + 2 MAA wAm (-1 +  $\alpha m$ )) +
FAA wAf wAm (MAA wAm - 2 MAA wam  $\alpha m$ ) (Maa wam - 2 MAA wAm  $\alpha m$ ) -
Faa (Maa wam2 - wAm (MAA wAm + MAA wam (2 - 4  $\alpha m$ ))) (Maa waf wam (-1 +  $\alpha f$ ) +
MAA wAf wAm  $\alpha f$  + 2 MAA (wAf wam  $\alpha f$  (-1 +  $\alpha m$ ) - waf wAm (-1 +  $\alpha f$ )  $\alpha m$ )) /
(2 (Faa waf wam (Maa wam - MAA wAm) (MAA wAm + 2 MAA wam (-1 +  $\alpha m$ )) +
FAA wAf wAm (Maa wam - MAA wAm) (-Maa wam + 2 MAA wAm  $\alpha m$ ) -
Faa (Maa wam2 - MAA wAm2) (Maa waf wam (-1 +  $\alpha f$ ) + MAA wAf wAm  $\alpha f$  +
2 MAA (wAf wam  $\alpha f$  (-1 +  $\alpha m$ ) - waf wAm (-1 +  $\alpha f$ )  $\alpha m$ ))) ) }

```

We'll call this equilC0.

We can define four terms to convert these leading order equilibria into the form found in the main text

```

trySub = {
  φ → (1 + αDf) wAf FAa (wam Maa + (1 + αDm) wAm MAA) / 2 - wam waf Maa Faa,
  ψ → (1 - αDf) waf FAa (wam Maa + (1 + αDm) wAm MAA) / 2 - (1 + αDm) wAm wAf MAA FAA,
  φp → (1 - αDf) waf FAa (wAm MAA + (1 - αDm) wam MAA) / 2 - wAm wAf MAA FAA,
  ψp → (1 + αDf) wAf FAa (wAm MAA + (1 - αDm) wam MAA) / 2 - (1 - αDm) wam waf MAA Faa
};

(q /. equilA0 /. αm → (αDm + 1) / 2 /. αf → (αDf + 1) / 2) -  $\left(1 - \alpha Dm \frac{MAa \phi}{MAa \phi + Maa \psi}\right)$  / 2 /.

  trySub // Factor // Simplify
(pXf /. equilA0 /. αm → (αDm + 1) / 2 /. αf → (αDf + 1) / 2) -
 $\left(\frac{waf \phi}{waf \phi + wAf \psi}\right)$  // Factor

(pXm /. equilA0 /. αm → (αDm + 1) / 2 /. αf → (αDf + 1) / 2) -
 $\left(\frac{(1 + \alpha Dm) MAa \phi}{(1 + \alpha Dm) MAa \phi + Maa \psi}\right)$  // Factor

(q /. equilAp0 /. αm → (αDm + 1) / 2 /. αf → (αDf + 1) / 2) -  $\left(1 + \alpha Dm \frac{MAa \phi p}{MAa \phi p + MAA \psi p}\right)$  / 2 /.

  trySub // Factor // Simplify
(pXf /. equilAp0 /. αm → (αDm + 1) / 2 /. αf → (αDf + 1) / 2) -
 $\left(1 - \frac{wAf \phi p}{wAf \phi p + waf \psi p}\right)$  // Factor

(pXm /. equilAp0 /. αm → (αDm + 1) / 2 /. αf → (αDf + 1) / 2) -
 $\left(1 - \frac{(1 - \alpha Dm) MAa \phi p}{(1 - \alpha Dm) MAa \phi p + MAA \psi p}\right)$  // Factor

0
0
0
0
0

```

When there is no haploid selection these reduce to the equilibria in Otto 2014 JEB 27:1431 (where diploid fitness is relative to heterozygotes, and equilB and equilBp are switched)

```

({{q, pYm, pXm, pXf} /. equilA0 /. cm → 1 / 2 /. af → 1 / 2 /. waf → 1 /. wAf → 1 /. wAm → 1 / .
wam → 1 /. MAa → 1 /. FAa → 1) ==
{1 / 2, 0,  $\frac{1 + Maa - 2 Faa Maa}{(1 + Maa)^2 - 2 (Faa + FAA) Maa}$ ,  $\frac{1 + Maa - 2 Faa Maa}{2 (1 + Maa - FAA - Faa Maa)}$ } // Simplify
({{q, pYm, pXm, pXf} /. equilAp0 /. cm → 1 / 2 /. af → 1 / 2 /. waf → 1 /. wAf → 1 /. wAm → 1 / .
wam → 1 /. MAa → 1 /. FAa → 1) ==
{1 / 2, 1,  $\frac{MAA + MAA^2 - 2 Faa MAA}{(1 + MAA)^2 - 2 (Faa + FAA) MAA}$ ,  $\frac{1 + MAA - 2 Faa}{2 (1 + MAA - Faa - FAA MAA)}$ } // Simplify
({{q, pYm, pXm, pXf} /. equilB0 /. cm → 1 / 2 /. af → 1 / 2 /. waf → 1 /. wAf → 1 /. wAm → 1 / .
wam → 1) == {1 / 2, 0, 1, 1} // Simplify
({{q, pYm, pXm, pXf} /. equilBp0 /. cm → 1 / 2 /. af → 1 / 2 /. waf → 1 /. wAf → 1 /. wAm → 1 / .
wam → 1) == {1 / 2, 1, 0, 0} // Simplify
({{q, pYm, pXm, pXf} /. equilC0 /. cm → 1 / 2 /. af → 1 / 2 /. waf → 1 /. wAf → 1 /. wAm → 1 / .
wam → 1 /. MAa → 1 /. FAa → 1) ==
{1 / 2,  $\frac{2 FAA (1 - Maa) - 2 Faa Maa (1 - MAA) - (1 + Maa) (MAA - Maa)}{(MAA - Maa)^2 + 2 (Faa + FAA) (1 - MAA) (1 - Maa)}$ ,
 $\frac{(1 - Maa) (2 Faa (1 - MAA) + MAA - Maa)}{(MAA - Maa)^2 + 2 (Faa + FAA) (1 - MAA) (1 - Maa)}$ ,  $\frac{1 - Maa}{2 - Maa - MAA}$ } // Simplify
True
True
True
True
True
True

```

Characteristic polynomial

The characteristic polynomial with $r = 0$ is

```
charpolyIntFullr0 = charpolyIntFull /. r → 0;
```

We consider only equilibria A, B (i.e., with a fixed on Y), and C. The conditions required for equilibria A' and B' can be obtained by interchanging the alleles, A and a.

Stability of equilA ($r \sim 0$)

The three allele frequencies at this equilibrium are

```
eq1 = Factor[pXf /. equilA0]
```

$$\frac{waf (Faa Maa waf wam - FAa Maa wAf wam af - 2 FAa MAa wAf wAm af cm)}{Faa Maa waf^2 wam - FAa Maa waf wAf wam - 2 FAa MAa waf wAf wAm cm + 2 FAA MAa wAf^2 wAm cm}$$

```

eq2 = Factor[pXm /. equilA0]
(2 MAA αm (-Faa Maa waf wam + FAa Maa wAf wam αf + 2 FAa MAA wAf wAm αf αm)) /
(FAA Maa2 waf wam - FAa Maa2 waf wam αf - 2 Faa Maa MAA waf wam αm +
 2 FAa Maa MAA waf wAm αm - 2 FAa Maa MAA wAf wAm αm + 2 FAa Maa MAA wAf wam αf αm -
 2 FAa Maa MAA waf wAm αf αm + 4 FAa MAA2 wAf wAm αf αm2)

```

```

eq3 = Factor[pYm /. equilA0]
0

```

Therefore each of the following terms must be positive for the equilibrium to be valid:

```

{{eq1, 1 - eq1}, {eq2, 1 - eq2}} // Simplify
{{waf (Faa Maa waf wam - FAa wAf αf (Maa wam + 2 MAA wAm αm)) ,
  Faa Maa waf2 wam + wAf (2 FAA MAA wAf wAm αm - FAa waf (Maa wam + 2 MAA wAm αm)) ,
  wAf (2 FAA MAA wAf wAm αm + FAa waf (-1 + αf) (Maa wam + 2 MAA wAm αm)) },
  Faa Maa waf2 wam + wAf (2 FAA MAA wAf wAm αm - FAa waf (Maa wam + 2 MAA wAm αm)) ,
  -(2 MAA αm (-Faa Maa waf wam + FAa wAf αf (Maa wam + 2 MAA wAm αm)) /
  (2 Maa MAA (Faa waf wam + FAA wAf wAm) αm +
  FAa (Maa wam + 2 MAA wAm αm) (Maa waf (-1 + αf) - 2 MAA wAf αf αm)) ,
  (Maa (2 FAA MAA wAf wAm αm + FAa waf (-1 + αf) (Maa wam + 2 MAA wAm αm)) ) /
  (2 Maa MAA (Faa waf wam + FAA wAf wAm) αm +
  FAa (Maa wam + 2 MAA wAm αm) (Maa waf (-1 + αf) - 2 MAA wAf αf αm)) } }

```

Notice that the numerators are common in sign for {eq1,1-eq1} and {eq2,1-eq2}. Thus, either the numerators and denominators must be both positive or both negative, the equilibrium will not be valid otherwise. That is, we must have either:

$$\text{Condition 1: } \frac{FAa wAf αf (Maa wam + 2 MAA wAm αm)}{Maa waf wam} > Faa \text{ and } \frac{FAa waf (1-αf) (Maa wam + 2 MAA wAm αm)}{2 MAA wAf wAm αm} > FAA$$

or

$$\text{Condition 2: } \frac{FAa wAf αf (Maa wam + 2 MAA wAm αm)}{Maa waf wam} < Faa \text{ and } \frac{FAa waf (1-αf) (Maa wam + 2 MAA wAm αm)}{2 MAA wAf wAm αm} < FAA$$

```

Reduce[{0 < eq1 < 1, 0 < eq2 < 1, simpcond}, {FAA, Faa}] // Factor


$$\left( 0 \leq R_f \leq \frac{1}{2} \& MAA > 0 \& 0 \leq R_m \leq \frac{1}{2} \& wAf > 0 \& 0 < \alpha_m < 1 \& \right.$$


$$waf > 0 \& Maa > 0 \& MAA > 0 \& 0 < \alpha_f < 1 \& wam > 0 \& wAm > 0 \&$$


$$FAA \cdot waf \cdot (-1 + \alpha_f) \cdot (Maa \cdot wam + 2 \cdot MAA \cdot wAm \cdot \alpha_m) \&$$


$$FAa > 0 \& 0 < FAA < -\frac{FAa \cdot wAf \cdot \alpha_f \cdot (Maa \cdot wam + 2 \cdot MAA \cdot wAm \cdot \alpha_m)}{2 \cdot MAA \cdot wAf \cdot wAm \cdot \alpha_m} \&$$


$$0 < Faa < \frac{FAa \cdot wAf \cdot \alpha_f \cdot (Maa \cdot wam + 2 \cdot MAA \cdot wAm \cdot \alpha_m)}{Maa \cdot waf \cdot wam} \Big) \quad ||$$


$$\left( 0 \leq R_f \leq \frac{1}{2} \& MAA > 0 \& 0 \leq R_m \leq \frac{1}{2} \& wAf > 0 \& 0 < \alpha_m < 1 \& waf > 0 \&$$


$$Maa > 0 \& MAA > 0 \& 0 < \alpha_f < 1 \& wam > 0 \& wAm > 0 \&$$


$$FAA > 0 \& FAA > -\frac{FAa \cdot waf \cdot (-1 + \alpha_f) \cdot (Maa \cdot wam + 2 \cdot MAA \cdot wAm \cdot \alpha_m)}{2 \cdot MAA \cdot wAf \cdot wAm \cdot \alpha_m} \&$$


$$FAa > \frac{FAa \cdot wAf \cdot \alpha_f \cdot (Maa \cdot wam + 2 \cdot MAA \cdot wAm \cdot \alpha_m)}{Maa \cdot waf \cdot wam} \Big)$$


```

That is, equilA0 is valid if:

```

validcondA = \left( \left( FAA < \frac{FAa \cdot waf \cdot (1 - \alpha_f) \cdot (Maa \cdot wam + 2 \cdot MAA \cdot wAm \cdot \alpha_m)}{2 \cdot MAA \cdot wAf \cdot wAm \cdot \alpha_m} \&

$$FAa < \frac{FAa \cdot wAf \cdot \alpha_f \cdot (Maa \cdot wam + 2 \cdot MAA \cdot wAm \cdot \alpha_m)}{Maa \cdot waf \cdot wam} \right) \quad ||$$


$$\left( FAA > \frac{FAa \cdot waf \cdot (1 - \alpha_f) \cdot (Maa \cdot wam + 2 \cdot MAA \cdot wAm \cdot \alpha_m)}{2 \cdot MAA \cdot wAf \cdot wAm \cdot \alpha_m} \&$$


$$FAa > \frac{FAa \cdot wAf \cdot \alpha_f \cdot (Maa \cdot wam + 2 \cdot MAA \cdot wAm \cdot \alpha_m)}{Maa \cdot waf \cdot wam} \right) \Big);$$

```

Making a set of substitutions that might help simplify factors below:

```

subA1 = (Solve[eq1 == pXf, MAA] // Flatten)

$$\left\{ MAA \rightarrow \frac{Maa \cdot waf \cdot wam \cdot (-Faa \cdot waf + Faa \cdot pXf \cdot waf - FAA \cdot pXf \cdot wAf + FAa \cdot wAf \cdot \alpha_f)}{2 \cdot wAf \cdot wAm \cdot (FAa \cdot pXf \cdot waf - FAA \cdot pXf \cdot wAf - FAa \cdot waf \cdot \alpha_f) \cdot \alpha_m} \right\}$$

subA2 = Solve[(eq2 - pXm /. subA1) == 0, FAA] // Flatten

$$\left\{ FAA \rightarrow \frac{(-pXf + pXf^2) \cdot (-Faa \cdot waf \cdot wam + Faa \cdot pXm \cdot waf \cdot wam + FAA \cdot pXm \cdot wAf \cdot wAm)}{(-pXf \cdot wAf \cdot wam + pXf \cdot pXm \cdot wAf \cdot wam - pXm \cdot waf \cdot wAm + pXf \cdot pXm \cdot waf \cdot wAm) \cdot (pXf - \alpha_f)} \right\}$$

subA2b = Solve[(eq2 - pXm /. subA1) == 0, \alpha_f] // Flatten

$$\left\{ \alpha_f \rightarrow (pXf \cdot (-Faa \cdot waf \cdot wam + Faa \cdot pXf \cdot waf \cdot wam + Faa \cdot pXm \cdot waf \cdot wam) - Faa \cdot pXf \cdot pXm \cdot waf \cdot wam - FAA \cdot pXf \cdot wAf \cdot wAm + FAA \cdot pXf \cdot pXm \cdot wAf \cdot wAm + FAA \cdot pXm \cdot wAf \cdot wAm - FAA \cdot pXf \cdot pXm \cdot wAf \cdot wAm) / (FAa \cdot (-pXf \cdot wAf \cdot wam + pXf \cdot pXm \cdot wAf \cdot wam - pXm \cdot waf \cdot wAm + pXf \cdot pXm \cdot waf \cdot wAm)) \right\}$$


```

The equilibrium A is stable if the three eigenvalues of the stability matrix are all less than one in magnitude. We are guaranteed that the eigenvalues are real and that the largest of the eigenvalues is non-negative by the Perron-Frobenius theorem, given that the stability matrix is non-negative.

The characteristic polynomial is

```

charpolyIntFullr0 /. equilAO // Factor

( (- Faa Maa MAA waf wam wAm + FAa Maa MAA wAf wam wAm αf +
  2 FAa Maa MAA waf wam wAm αm - 2 FAa Maa MAA waf wam wAm αf αm +
  2 FAa MAA MAA wAf wAm2 αf αm + 4 FAa MAA2 waf wAm2 αm2 - 4 FAa MAA2 wAf wAm2 αm2 -
  4 FAa MAA2 waf wAm2 αf αm2 - FAa MAA2 waf wam2 λ + 2 Faa Maa MAA waf wam2 λ +
  FAa MAA2 waf wam2 αf λ - 2 FAa Maa MAA wAf wam2 αf λ - 2 Faa Maa MAA waf wam2 αm λ -
  2 FAa Maa MAA waf wam wAm cm λ + 2 FAa Maa MAA wAf wam wAm cm λ +
  2 FAa Maa MAA wAf wam2 αf αm λ + 2 FAa Maa MAA waf wam wAm αf αm λ -
  4 FAa MAA2 wAf wam wAm αf αm λ + 4 FAa MAA2 wAf wam wAm αf αm2 λ)

(- 2 Faa FAa Maa MAA waf2 wam wAm cm + 2 Faa FAa Maa MAA waf wAf wam wAm cm +
  2 Faa FAa Maa MAA waf2 wam wAm αf αm - 2 FAa FAa Maa MAA wAf2 wam wAm αf αm -
  Faa FAa MAA2 waf2 wam2 λ + Faa FAa MAA2 waf2 wam2 αf λ + 2 Faa FAa Maa MAA waf wAf
  wam wAm cm λ - 4 FAa FAa MAA2 wAf2 wAm2 αf αm2 λ + FAa2 MAA2 waf wAf wam2 αf λ2 -
  FAa2 MAA2 waf wAf wam2 αf2 λ2 - 2 Faa FAa Maa MAA waf wAf wam wAm cm λ2 +
  4 FAa2 MAA2 waf wAf wAm2 αf αm2 λ2 - 4 FAa2 MAA2 waf wAf wAm2 αf2 αm2 λ2) ) /
(waf wAf wam (- FAa MAA2 waf wam + 2 Faa Maa MAA waf wam + FAa MAA2 waf wam αf -
  2 FAa Maa MAA wAf wam αf - 2 Faa Maa MAA waf wam αm - 2 FAa Maa MAA waf wAm αm +
  2 FAa Maa MAA wAf wAm αm + 2 Faa Maa MAA wAf wam αf αm +
  2 FAa Maa MAA waf wAm αf αm - 4 FAa MAA2 wAf wAm αf αm + 4 FAa MAA2 wAf wAm αf αm2)

(- FAa2 MAA2 wam2 αf + FAa2 MAA2 wam2 αf2 + 2 Faa FAa Maa MAA waf wAm αm -
  4 FAa2 MAA2 waf wAm αf αm + 4 FAa2 MAA2 waf wAm αf2 αm -
  4 FAa2 MAA2 wAm2 αf αm2 + 4 FAa2 MAA2 wAm2 αf2 αm2) )

```

The first eigenvalue is

```

Numerator[Factor[charpolyIntFull /. r → 0 /. equilAO]][[1]] // Simplify;
eigenA1 = λ /. (Solve[% == 0, λ] // Simplify // Flatten)

(wAm (Faa Maa MAA waf wam + 4 FAa MAA2 wAf wAm αm2 +
  FAa (Maa wam + 2 MAA wAm αm) (-MAA wAf αf + 2 MAA waf (-1 + αf) αm)) ) /
(wam (FAa (Maa waf (-1 + αf) + 2 MAA wAf αf (-1 + αm)) (Maa wam + 2 MAA wAm αm) +
  2 Maa MAA (-Faa waf wam (-1 + αm) + FAa wAf wAm αm)) )

```

This first eigenvalue is consistent with stability if the following is positive:

```

Collect[1 - eigenA1 /. subA1 /. subA2, MAA, Factor]


$$\frac{MAA pXf (-1 + pXm) wAf wAm \alpha m}{Maa (-1 + pXf) waf wam (-pXm - \alpha m + 2 pXm \alpha m)} -$$


$$\frac{pXf pXm wAf wam + pXf wAf wam \alpha m - 2 pXf pXm wAf wam \alpha m - pXm waf wAm \alpha m + pXf pXm waf wAm \alpha m}{pXf wAf wam (-pXm - \alpha m + 2 pXm \alpha m)}$$


```

which we write more clearly as:

```

MAA (1 - pXm) pXf wAf wAm cm
- -----
Maa (1 - pXf) waf wam (pXm (1 - cm) + cm (1 - pXm)) +
pXf wAf wam (pXm (1 - cm) + cm (1 - pXm)) - pXm (1 - pXf) waf wAm cm
pXf wAf wam (pXm (1 - cm) + cm (1 - pXm));
% - %% // FullSimplify
0

```

The first term is never positive, so we require that MAA be sufficiently small,

$MAA < \frac{Maa (1-pXf) waf (pXf wAf wam (pXm (1-cm) + cm (1-pXm)) - pXm (1-pXf) waf wAm cm)}{(1-pXm) pXf^2 wAf^2 wAm cm}$, which itself requires that the

second term is positive (i.e., we must have $pXf wAf wam (pXm(1-cm) + cm(1-pXm)) - pXm(1-pXf) waf wAm cm > 0$ for there to be any parameter set in which MAA can be positive and less than this quantity):

```
Solve[% == 0, MAA] // Simplify
```

```
{ {MAA → (Maa (-1 + pXf) waf
(-pXm waf wAm cm + pXf (wAf wam cm + pXm (wAf wam - 2 wAf wam cm + waf wAm cm))) ) /
(pXf^2 (-1 + pXm) wAf^2 wAm cm) } }
```

The last two eigenvalues are more readily interpreted by analysing the quadratic portion of the characteristic polynomial (with leading $+\lambda^2$ term):

```
Table[matIntFull[[i, j]], {i, 1, 2}, {j, 1, 2}] /. r → 0 /. equilA0 // Factor;
charpolyA = Det[% - λ IdentityMatrix[2]] // Factor;
```

Stability requires that the slope at $\lambda=1$ is positive and that the intercept at $\lambda=1$ is positive, otherwise one or both of these two eigenvalues will be greater than one:

```
slopeA = D[charpolyA, λ] /. λ → 1 /. subA1 /. subA2 // Factor
(-Faa pXf^2 waf wAf wam^2 + 2 Faa pXf^2 pXm waf wAf wam^2 - Faa pXf^2 pXm^2 waf wAf wam^2 +
2 FAA pXf^2 pXm wAf^2 wam wAm - 2 FAA pXf^2 pXm^2 wAf^2 wam wAm + FAA pXf pXm^2 waf wAf wAm^2 -
FAA pXf^2 pXm^2 waf wAf wAm^2 - Faa pXf waf wAf wam^2 αf + 2 Faa pXf^2 waf wAf wam^2 αf +
2 Faa pXf pXm waf wAf wam^2 αf - 4 Faa pXf^2 pXm waf wAf wam^2 αf -
Faa pXf pXm^2 waf wAf wam^2 αf + 2 Faa pXf^2 pXm^2 waf wAf wam^2 αf - 2 Faa pXm waf^2 wam wAm αf +
4 Faa pXf pXm waf^2 wam wAm αf - 2 Faa pXf^2 pXm waf^2 wam wAm αf + 2 Faa pXm^2 waf^2 wam wAm αf -
4 Faa pXf pXm^2 waf^2 wam wAm αf + 2 Faa pXf^2 pXm^2 waf^2 wam wAm αf -
2 FAA pXf^2 pXm wAf^2 wam wAm αf + 2 FAA pXf^2 pXm^2 wAf^2 wam wAm αf + FAA pXm^2 waf wAf wAm^2 αf -
3 FAA pXf pXm^2 waf wAf wAm^2 αf + 2 FAA pXf^2 pXm^2 waf wAf wAm^2 αf) /
((-pXf wAf wam + pXf pXm wAf wam - pXm waf wAm + pXf pXm waf wAm) -
(-FAA pXf pXm wAf wAm + Faa waf wam αf - Faa pXf waf wam αf -
Faa pXm waf wam αf + Faa pXf pXm waf wam αf + FAA pXf pXm wAf wAm αf) )
```

```

slopeA2 = D[charpolyA, λ] /. λ → 1 /. subA1 /. subA2b // Factor
(Faa pXf waf wAf wam2 - 2 Faa pXf2 waf wAf wam2 - 2 Faa pXf pXm waf wAf wam2 +
 4 Faa pXf2 pXm waf wAf wam2 + Faa pXf pXm2 waf wAf wam2 - 2 Faa pXf2 pXm2 waf wAf wam2 +
 2 FAa pXf2 wAf2 wam2 - 4 FAa pXf2 pXm wAf2 wam2 + 2 FAa pXf2 pXm2 wAf2 wam2 +
 2 Faa pXm waf2 wam wAm - 4 Faa pXf pXm waf2 wam wAm + 2 Faa pXf2 pXm waf2 wam wAm -
 2 Faa pXm2 waf2 wam wAm + 4 Faa pXf pXm2 waf2 wam wAm - 2 Faa pXf2 pXm2 waf2 wam wAm +
 4 Faa pXf pXm waf wAm - 4 FAa pXf2 pXm waf wAm wAm -
 4 Faa pXf pXm2 waf wAm wAm + 4 FAa pXf2 pXm2 waf wAm wAm +
 2 FAA pXf2 pXm wAf2 wam wAm - 2 FAA pXf2 pXm2 wAf2 wam wAm + 2 FAa pXm2 waf2 wAm2 -
 4 Faa pXf pXm2 waf2 wAm2 + 2 FAa pXf2 pXm2 waf2 wAm2 - FAA pXm2 waf wAf wAm2 +
 3 FAA pXf pXm2 waf wAf wAm2 - 2 FAA pXf2 pXm2 waf wAf wAm2) /
((-pXf wAf wam + pXf pXm wAf wam - pXm waf wAm + pXf pXm waf wAm) -
 (-Faa waf wam + Faa pXf waf wam + Faa pXm waf wam - Faa pXf pXm waf wam - FAa pXf wAf wam +
  FAa pXf pXm wAf wam - FAa pXm waf wAm + FAa pXf pXm waf wAm - FAA pXf pXm wAf wAm))
interceptA = charpolyA /. λ → 1 /. subA1 /. subA2 // Factor
(Faa pXf waf wam - Faa pXf pXm waf wam - Faa pXf waf wam αf +
 Faa pXf pXm waf wam αf - FAA pXm wAf wAm αf + FAA pXf pXm wAf wAm αf) /
(-FAA pXf pXm wAf wAm + Faa waf wam αf - Faa pXf waf wam αf -
 Faa pXm waf wam αf + Faa pXf pXm waf wam αf + FAA pXf pXm wAf wAm αf)

```

The intercept is easier to interpret using subA2b:

```

interceptA2 = charpolyA /. λ → 1 /. subA1 /. subA2b // Factor
(Faa pXf waf wam - Faa pXf pXm waf wam - FAa pXf wAf wam + Faa pXf pXm wAf wam -
 FAa pXm waf wAm + FAa pXf pXm waf wAm + FAA pXm wAf wAm - FAA pXf pXm wAf wAm) /
(-Faa waf wam + Faa pXf waf wam + Faa pXm waf wam - Faa pXf pXm waf wam - FAa pXf wAf wam +
 FAa pXf pXm wAf wam - FAa pXm waf wAm + FAa pXf pXm waf wAm - FAA pXf pXm wAf wAm)

```

Using Collect, this can be written as:

```

(-Faa pXf (1 - pXm) waf wam - FAA (1 - pXf) pXm wAf wAm +
  Faa (pXf (1 - pXm) wAf wam + pXm (1 - pXf) waf wAm)) / (Faa (1 - pXf) (1 - pXm) waf wam +
  FAA pXf pXm wAf wAm + FAa (pXf (1 - pXm) wAf wam + pXm (1 - pXf) waf wAm));
Factor[interceptA2 /
%]
1

```

The denominator is always positive, so we require the numerator to be positive for stability, that is, FAa must be large enough for the intercept at $\lambda=1$ to be positive,

$$FAa > \frac{Faa pXf (1-pXm) waf wam + FAA (1-pXf) pXm wAf wAm}{pXf (1-pXm) wAf wam + pXm (1-pXf) waf wAm}.$$

Note that the fraction can be interpreted further in light of the validity conditions:

validcondA

$$\left(\begin{array}{l} \text{FAA} < \frac{\text{FAa waf } (1 - \alpha f) \ (Maa wam + 2 MAA wAm \ \alpha m)}{2 MAA wAf wAm \ \alpha m} \ \& \\ \text{FAa} < \frac{\text{FAa wAf } \alpha f \ (Maa wam + 2 MAA wAm \ \alpha m)}{Maa waf wam} \end{array} \right) \ \& \ \left(\begin{array}{l} \text{FAA} > \frac{\text{FAa waf } (1 - \alpha f) \ (Maa wam + 2 MAA wAm \ \alpha m)}{2 MAA wAf wAm \ \alpha m} \ \& \ \text{FAa} > \frac{\text{FAa wAf } \alpha f \ (Maa wam + 2 MAA wAm \ \alpha m)}{Maa waf wam} \end{array} \right)$$

Under the first of these conditions (equivalent to overdominance),

$\frac{\text{FAa pXf } (1-pXm) \ waf \ wam + \text{FAA } (1-pXf) \ pXm \ wAf \ wAm}{pXf \ (1-pXm) \ wAf \ wam + pXm \ (1-pXf) \ waf \ wAm}$ will be less than

$$\left(\begin{array}{l} \frac{\text{FAa wAf } \alpha f \ (Maa wam + 2 MAA wAm \ \alpha m)}{Maa waf wam} \ pXf \ (1 - pXm) \ waf \ wam + \\ \frac{\text{FAa waf } (1 - \alpha f) \ (Maa wam + 2 MAA wAm \ \alpha m)}{2 MAA wAf wAm \ \alpha m} \ (1 - pXf) \ pXm \ wAf \ wAm \end{array} \right) / \\ (\text{pXf } (1 - pXm) \ wAf \ wam + pXm \ (1 - pXf) \ waf \ wAm) /. \text{subA1} /. \text{subA2b} // \text{Factor}$$

FAa

which makes it guaranteed that $\text{FAa} > \frac{\text{FAa pXf } (1-pXm) \ waf \ wam + \text{FAA } (1-pXf) \ pXm \ wAf \ wAm}{pXf \ (1-pXm) \ wAf \ wam + pXm \ (1-pXf) \ waf \ wAm}$ (i.e., the validity conditions in the overdominance-like case ensure intercept condition for stability is satisfied).

Under the second of these conditions (equivalent to underdominance), however,

$\frac{\text{FAa pXf } (1-pXm) \ waf \ wam + \text{FAA } (1-pXf) \ pXm \ wAf \ wAm}{pXf \ (1-pXm) \ wAf \ wam + pXm \ (1-pXf) \ waf \ wAm}$ will be greater than FAa by the same calculations, making it impossible to satisfy the intercept condition for stability.

We conclude that only under the first validity conditions (equivalent to overdominance) is stability and validity of the equilibrium possible. Furthermore, under the first validity conditions, we are guaranteed that the intercept is positive at $\lambda=1$.

To complete the proof, we show below that the slope at $\lambda=1$ provides no further conditions on stability:

slopeA2 // Factor

$$\begin{aligned} & (\text{FAa pXf waf wAf wam}^2 - 2 \text{FAa pXf}^2 \ waf \ wAf \ wam^2 - 2 \text{FAa pXf pXm waf wAf wam}^2 + \\ & 4 \text{FAa pXf}^2 \ pXm \ waf \ wAf \ wam^2 + \text{FAa pXf pXm}^2 \ waf \ wAf \ wam^2 - 2 \text{FAa pXf}^2 \ pXm^2 \ waf \ wAf \ wam^2 + \\ & 2 \text{FAa pXf}^2 \ wAf^2 \ wam^2 - 4 \text{FAa pXf}^2 \ pXm \ wAf^2 \ wam^2 + 2 \text{FAa pXf}^2 \ pXm^2 \ wAf^2 \ wam^2 + \\ & 2 \text{FAa pXm waf}^2 \ wam \ wAm - 4 \text{FAa pXf pXm waf}^2 \ wam \ wAm + 2 \text{FAa pXf}^2 \ pXm \ waf}^2 \ wam \ wAm - \\ & 2 \text{FAa pXm}^2 \ waf}^2 \ wam \ wAm + 4 \text{FAa pXf pXm}^2 \ waf}^2 \ wam \ wAm - 2 \text{FAa pXf}^2 \ pXm^2 \ waf}^2 \ wam \ wAm + \\ & 4 \text{FAa pXf pXm waf wAf wam wAm} - 4 \text{FAa pXf}^2 \ pXm waf wAf wam wAm - \\ & 4 \text{FAa pXf pXm}^2 \ waf wAf wam wAm + 4 \text{FAa pXf}^2 \ pXm^2 \ waf wAf wam wAm + \\ & 2 \text{FAA pXf}^2 \ pXm \ wAf^2 \ wam \ wAm - 2 \text{FAA pXf}^2 \ pXm^2 \ wAf^2 \ wam \ wAm + 2 \text{FAA pXm}^2 \ waf}^2 \ wAm^2 - \\ & 4 \text{FAA pXf pXm}^2 \ waf}^2 \ wAm^2 + 2 \text{FAA pXf}^2 \ pXm^2 \ waf}^2 \ wAm^2 - \text{FAA pXm}^2 \ waf wAf wAm^2 + \\ & 3 \text{FAA pXf pXm}^2 \ waf wAf wAm^2 - 2 \text{FAA pXf}^2 \ pXm^2 \ waf wAf wAm^2) / \\ & ((-\text{pXf} \ wAf \ wam + \text{pXf} \ pXm \ wAf \ wam - \text{pXm} \ waf \ wAm + \text{pXf} \ pXm \ waf \ wAm) \\ & (-\text{FAa} \ waf \ wam + \text{FAa} \ pXf \ waf \ wam + \text{FAa} \ pXm \ waf \ wam - \text{FAa} \ pXf \ pXm \ waf \ wam - \text{FAa} \ pXf \ wAf \ wam + \\ & \text{FAa} \ pXf \ pXm \ wAf \ wam - \text{FAa} \ pXm \ waf \ wAm + \text{FAa} \ pXf \ pXm \ waf \ wAm - \text{FAA} \ pXf \ pXm \ wAf \ wAm)) \end{aligned}$$

The proof we use involves rewriting the above in terms of the intercept, shown above to be positive:

```

subtry = Solve[int == interceptA2, FAa] // Flatten

{FAa → (Faa int waf wam + Faa pXf waf wam - Faa int pXf waf wam -
    Faa int pXm waf wam - Faa pXf pXm waf wam + Faa int pXf pXm waf wam +
    FAA pXm wAf wAm - FAA pXf pXm wAf wAm + FAA int pXf pXm wAf wAm) /
   ((-1 + int) (-pXf wAf wam + pXf pXm wAf wam - pXm waf wAm + pXf pXm waf wAm)) }

```

Multiplying slopeA2 by a positive quantity to get rid of the denominator:

```

slopeA2 * ((pXf (1 - pXm) wAf wam + pXm (1 - pXf) waf wAm) (Faa waf wam (1 - pXf) (1 - pXm) +
    FAA wAf wAm pXf pXm + FAa ( wAf wam pXf (1 - pXm) + waf wAm (1 - pXf) pXm))) // Factor

Faa pXf waf wAf wam2 - 2 Faa pXf2 waf wAf wam2 - 2 Faa pXf pXm waf wAf wam2 +
4 Faa pXf2 pXm waf wAf wam2 + Faa pXf pXm2 waf wAf wam2 - 2 Faa pXf2 pXm2 waf wAf wam2 +
2 FAa pXf2 wAf2 wam2 - 4 FAa pXf2 pXm wAf2 wam2 + 2 FAa pXf2 pXm2 wAf2 wam2 +
2 Faa pXm waf2 wam wAm - 4 Faa pXf pXm waf2 wam wAm + 2 Faa pXf2 pXm waf2 wam wAm -
2 Faa pXm2 waf2 wam wAm + 4 Faa pXf pXm2 waf2 wam wAm - 2 Faa pXf2 pXm2 waf2 wam wAm +
4 FAa pXf pXm waf wAf wam wAm - 4 FAa pXf2 pXm waf wAf wam wAm -
4 FAa pXf pXm2 waf wAf wam wAm + 4 FAa pXf2 pXm2 waf wAf wam wAm +
2 FAA pXf2 pXm wAf2 wam wAm - 2 FAA pXf2 pXm2 wAf2 wam wAm +
2 Faa pXm2 waf2 wAm2 - 4 Faa pXf pXm2 waf2 wAm2 + 2 Faa pXf2 pXm2 waf2 wAm2 -
FAA pXm2 waf wAf wAm2 + 3 FAA pXf pXm2 waf wAf wAm2 - 2 FAA pXf2 pXm2 waf wAf wAm2

```

```

Collect[Factor[% (1 - int) /. subtry], {int, Faa, FAa, FAA}, Factor]

- FAA pXm wAf wAm (-2 pXf wAf wam + 2 pXf pXm wAf wam - pXm waf wAm + pXf pXm waf wAm) +
  Faa (-1 + pXm) waf wam (-pXf wAf wam + pXf pXm wAf wam - 2 pXm waf wAm + 2 pXf pXm waf wAm) +
  int (Faa pXf (-1 + pXm)2 waf wAf wam2 - FAA (-1 + pXf) pXm2 waf wAf wAm2)

```

Below, we show that (1-int) is also positive (that is, the intercept at $\lambda=1$ is never greater than +1), the remaining quantity can be written as the following:

```

Faa waf wam (1 - pXm) (pXf (1 - pXm) wAf wam + 2 pXm (1 - pXf) waf wAm) +
    FAA wAf wAm pXm (2 pXf (1 - pXm) wAf wam + pXm (1 - pXf) waf wAm) +
    int (Faa pXf (1 - pXm)2 waf wAf wam2 + FAA (1 - pXf) pXm2 waf wAf wAm2);
%% - % // Factor
0

```

If the intercept is negative, we already know that the system is unstable (above proof). If the intercept is positive, then the slope will also be positive because all of the terms in the above are positive and so is (1-int), shown next:

```

Factor[1 - interceptA2]

- (-Faa waf wam + Faa pXm waf wam - FAA pXm wAf wAm) /
  (Faa waf wam - Faa pXf waf wam - Faa pXm waf wam + Faa pXf pXm waf wam + FAa pXf wAf wam -
  FAa pXf pXm wAf wam + FAa pXm waf wAm - FAa pXf pXm waf wAm + FAA pXf pXm wAf wAm)

```

Using Collect, this can be written as the strictly positive quantity:

```

(Faa waf wam (1 - pXm) + FAA wAf wAm pXm) / (Faa waf wam (1 - pXf) (1 - pXm) +
    FAA wAf wAm pXf pXm + FAa ( wAf wam pXf (1 - pXm) + waf wAm (1 - pXf) pXm));
Factor[(1 - interceptA2) - %]
0

```

Hence the slope at $\lambda=1$ is positive, assuming that the intercept at $\lambda=1$ is positive, and adds no further constraint on the stability conditions.

Putting together the constraints on all of the eigenvalues, as well as the validity conditions, the conditions required for validity and stability are:

$$\begin{aligned} \text{stabcondA} = & \text{Simplify}\left[\text{Reduce}\left[\left\{\text{MAA} < \frac{\text{Maa} (1 - \text{pXf}) \text{waf} (\text{pXf wAf wam} (\text{pXm} (1 - \alpha m) + \alpha m (1 - \text{pXm})) - \text{pXm} (1 - \text{pXf}) \text{waf wAm} \alpha m)}{(1 - \text{pXm}) \text{pXf}^2 \text{wAf}^2 \text{wAm} \alpha m}, \right.\right. \\ & \left.\left. \text{validcondA}[[1]], \text{simpcond}, 0 < \text{pXf} < 1, 0 < \text{pXm} < 1\right]\right] \\ & 0 \leq R_f \leq \frac{1}{2} \& \& 0 \leq R_m \leq \frac{1}{2} \& \& 0 < \alpha m < 1 \& \& 0 < \text{pXm} < 1 \& \& 0 < \text{pXf} < 1 \& \& \\ & \text{wAm} > 0 \& \& \text{wam} > 0 \& \& \text{wAf} > 0 \& \& 0 < \text{waf} < -\frac{\text{pXf wAf wam} (\text{pXm} + \alpha m - 2 \text{pXm} \alpha m)}{(-1 + \text{pXf}) \text{pXm} \text{wAm} \alpha m} \& \& \\ & \text{MAA} > 0 \& \& \text{Maa} > \left(\text{MAA pXf}^2 (-1 + \text{pXm}) \text{wAf}^2 \text{wAm} \alpha m\right) / ((-1 + \text{pXf}) \text{waf} \\ & \quad (-\text{pXm} \text{waf} \text{wAm} \alpha m + \text{pXf} (\text{wAf wam} \alpha m + \text{pXm} (\text{wAf wam} - 2 \text{wAf wam} \alpha m + \text{waf wAm} \alpha m))) \& \& \\ & 0 < \alpha f < 1 \& \& \text{MAa} > 0 \& \& \text{FAA} > 0 \& \& \text{FAa} > -\frac{2 \text{FAA MAa wAf wAm} \alpha m}{\text{waf} (-1 + \alpha f) (\text{Maa wam} + 2 \text{MAa wAm} \alpha m)} \& \& \\ & 0 < \text{Faa} < \frac{\text{FAa wAf} \alpha f (\text{Maa wam} + 2 \text{MAa wAm} \alpha m)}{\text{Maa waf wam}} \end{aligned}$$

We can regain the validity and stability conditions from Otto (2014):

$$\begin{aligned} \text{Condition for validity and stability: } & \frac{1+\text{Maa}}{2 \text{Maa}} > \text{Faa} \text{ and } \frac{1+\text{Maa}}{2} > \text{FAA} \text{ and} \\ & 1 - \frac{(1-\text{Maa}) \left(\frac{1+\text{Maa}}{2} - \text{FAA}\right)}{\text{Maa} \left(\frac{1+\text{Maa}}{2} - \text{Faa}\right)} > \text{MAA}. \end{aligned}$$

by reducing the model to that one ($wAf=waf=wAm=wam=1$, $\alpha m=\alpha f=1/2$, $FAa=MAa=1$):

$$\begin{aligned} \text{validcondA}[[1]] / . & \{wAf \rightarrow 1, waf \rightarrow 1, wAm \rightarrow 1, wam \rightarrow 1, \alpha f \rightarrow 1/2, \alpha m \rightarrow 1/2, FAa \rightarrow 1, MAa \rightarrow 1\} \\ & \text{FAA} < \frac{1 + \text{Maa}}{2} \& \& \text{Faa} < \frac{1 + \text{Maa}}{2 \text{Maa}} \\ & \left(\text{MAA} - \left(\text{Maa} \left(\frac{\text{MAa} - \text{Maa}}{\text{Maa}} \frac{\left(\frac{1}{2} \left(\frac{\text{Maa}}{\text{MAa}} + 1\right) - \frac{\text{FAA}}{\text{FAa}}\right)}{\left(\frac{1}{2} \left(\frac{\text{MAa}}{\text{Maa}} + 1\right) - \frac{\text{Faa}}{\text{FAa}}\right)} + \frac{\text{MAA} - \text{MAa}}{\text{MAa}} \right) \right) / . \text{equilAO} / . \right. \\ & \left. \frac{\text{Maa} (1 - \text{pXf}) \text{waf} (\text{pXf wAf wam} (\text{pXm} (1 - \alpha m) + \alpha m (1 - \text{pXm})) - \text{pXm} (1 - \text{pXf}) \text{waf wAm} \alpha m)}{(1 - \text{pXm}) \text{pXf}^2 \text{wAf}^2 \text{wAm} \alpha m} \right) \\ & / \left(\text{MAa} \left(\frac{\text{MAa} - \text{Maa}}{\text{Maa}} \frac{\left(\frac{1}{2} \left(\frac{\text{Maa}}{\text{MAa}} + 1\right) - \frac{\text{FAA}}{\text{FAa}}\right)}{\left(\frac{1}{2} \left(\frac{\text{MAa}}{\text{Maa}} + 1\right) - \frac{\text{Faa}}{\text{FAa}}\right)} + \frac{\text{MAA} - \text{MAa}}{\text{MAa}} \right) \right) / . \text{equilAO} / . \\ & \{wAf \rightarrow 1, waf \rightarrow 1, wAm \rightarrow 1, wam \rightarrow 1, \alpha f \rightarrow 1/2, \alpha m \rightarrow 1/2\} // \text{Factor} \end{aligned}$$

1

We cannot have underdominance in marginal fitness in females (including ovule selection):

```
Reduce[{stabcondA, FAA wAf > Faa waf, Faa waf > FAA wAf}]
```

False

With haploid selection we need

$$\begin{aligned} & - \frac{\text{Maa} (1 - pXf) waf (pXf wAf wam (pXm (1 - \alpha m) + \alpha m (1 - pXm)) - pXm (1 - pXf) waf wAm \alpha m)}{(1 - pXm) pXf^2 wAf^2 wAm \alpha m} + MAA < \\ & 0 /. \text{equilAO} // \text{Simplify}; \\ & \% /. \text{Faa} \rightarrow (\text{Faa} /. \text{Solve}[\phi == \alpha f wAf \text{FAA} (wam Maa + 2 \alpha m wAm MAA) - wam waf Maa \text{Faa}, \text{Faa}]) // \text{Simplify}; \\ & \% /. \text{FAA} \rightarrow (\text{FAA} /. \text{Solve}[\psi == (1 - \alpha f) waf \text{FAA} (wam Maa + 2 \alpha m wAm MAA) - 2 \alpha m wAm wAf MAA, \text{FAA}]) // \text{Simplify} \\ & \{ \{ -wAm \phi (MAA wAm \phi - Maa wam \psi + 2 MAA (wam (-1 + \alpha m) \phi + wAm \alpha m \psi)) \} \} > 0 \end{aligned}$$

Note that $\phi > 0$ whenever the equilibrium is valid and stable

```
Reduce[{\alpha f wAf \text{FAA} (wam Maa + 2 \alpha m wAm MAA) - wam waf Maa \text{Faa} > 0, simpcond}, \text{Faa}] // Factor
```

$$\begin{aligned} & MAA > 0 \& \& 0 \leq Rf \leq \frac{1}{2} \& \& \text{FAA} > 0 \& \& 0 \leq Rm \leq \frac{1}{2} \& \& 0 < \alpha m < 1 \& \& \\ & 0 < \alpha f < 1 \& \& wAm > 0 \& \& wam > 0 \& \& Maa > 0 \& \& MAA > 0 \& \& wAf > 0 \& \& \\ & waf > 0 \& \& \text{FAa} > 0 \& \& 0 < \text{Faa} < \frac{\text{FAa} wAf \alpha f (Maa wam + 2 MAA wAm \alpha m)}{Maa waf wam} \end{aligned}$$

validcondA[[1, 2]]

$$\text{Faa} < \frac{\text{FAa} wAf \alpha f (Maa wam + 2 MAA wAm \alpha m)}{Maa waf wam}$$

(as is ψ)

```
Reduce[{\{(1 - \alpha f) waf \text{FAA} (wam Maa + 2 \alpha m wAm MAA) - 2 \alpha m wAm wAf MAA \text{FAA} > 0, simpcond}, \text{FAA}] // Factor
```

$$\begin{aligned} & MAA > 0 \& \& 0 \leq Rf \leq \frac{1}{2} \& \& \text{Faa} > 0 \& \& 0 \leq Rm \leq \frac{1}{2} \& \& 0 < \alpha m < 1 \& \& \\ & 0 < \alpha f < 1 \& \& wAm > 0 \& \& wam > 0 \& \& Maa > 0 \& \& MAA > 0 \& \& wAf > 0 \& \& \\ & waf > 0 \& \& \text{FAa} > 0 \& \& 0 < \text{FAA} < - \frac{\text{FAa} waf (-1 + \alpha f) (Maa wam + 2 MAA wAm \alpha m)}{2 MAA wAf wAm \alpha m} \end{aligned}$$

validcondA[[1, 1]]

$$\text{FAA} < \frac{\text{FAa} waf (1 - \alpha f) (Maa wam + 2 MAA wAm \alpha m)}{2 MAA wAf wAm \alpha m}$$

We therefore need only

$$\begin{aligned} & (MAA wAm \phi - Maa wam \psi + 2 MAA (wam (-1 + \alpha m) \phi + wAm \alpha m \psi)) < 0; \\ & \text{Collect}[\%, \{wAm, wam\}] \\ & wam (2 MAA (-1 + \alpha m) \phi - Maa \psi) + wAm (MAA \phi + 2 MAA \alpha m \psi) < 0 \end{aligned}$$

which is the same as $\gamma_{(A)} > 0$ as stated in the main text.

This is the same as ensuring the marginal fitness of Y-a is greater than the marginal fitness of Y-A, as stated in the main text,

```
wam (pXf (1 - αDm) wAf MAA + (1 - pXf) waf Maa)
wAm (pXf wAf MAA + (1 - pXf) waf (1 + αDm) MAA) > 1 /. equilAO /. αDm → 2 αm - 1 /.
αDf → 2 αf - 1 // Simplify;
0 > wAm (pXf wAf MAA + (1 - pXf) waf (1 + αDm) MAA) -
wam (pXf (1 - αDm) wAf MAA + (1 - pXf) waf Maa) /.
equilAO /. αDm → 2 αm - 1 /. αDf → 2 αf - 1 // Simplify;
% /. Faa → (Faa /. Solve[φ == αf wAf FAa (wam Maa + 2 αm wAm MAA) - wam waf Maa Faa, Faa]) // Simplify;
% /. FAA → (FAA /. Solve[ψ == (1 - αf) waf FAa (wam Maa + 2 αm wAm MAA) - 2 αm wAm wAf Maa FAA,
FAA]) // Simplify
{ {waf wAf (waf φ + wAf ψ) (MAA wAm φ - Maa wam ψ + 2 MAA (wam (-1 + αm) φ + wAm αm ψ))} } < 0
```

Stability of equilB ($r \sim 0$)

The eigenvalues are

```
eigenB = Solve[0 == charpolyIntFullr0 /. equilB0, λ] // Simplify
{ {λ → MAA wAm / (2 MAA wam - 2 MAA wam αm)}, {λ → -1 / (2 FAA MAA wAf wAm αm) (FAa MAA waf wAm (-1 + αf) αm +
Sqrt[FAa MAA waf wAm (-1 + αf) αm (-2 FAA Maa wAf wam + FAa MAA waf wAm (-1 + αf) αm)]), λ → 1 / (2 FAA MAA wAf wAm αm) (-FAa MAA waf wAm (-1 + αf) αm +
Sqrt[FAa MAA waf wAm (-1 + αf) αm (-2 FAA Maa wAf wam + FAa MAA waf wAm (-1 + αf) αm)])} }
```

as can be seen from the Jacobian

```
matIntFull /. r → 0 /. equilB0 // Factor;
% // MatrixForm
( -FAa waf (-1+αf) - FAa wam (-1+αf)
FAA wAf
Maa waf
2 MAA wAf αm
0
0
0
0
0
0
0
MAA wAm
2 MAA wam (-1+αm) )
```

I.e., one eigenvalue is $\frac{MAA wAm}{2 MAA wam (1-\alpha m)}$ and the other two are the roots of a 2x2.

Equilibrium B is locally stable when all eigenvalues are less than one in magnitude. We are guaranteed that the eigenvalues are real and that the largest of the eigenvalues is non-negative by the Perron-Frobenius theorem, given that the stability matrix is non-negative.

The last two eigenvalues are more readily interpreted by analysing the quadratic portion of the characteristic polynomial (with leading $+\lambda^2$ term):

```
charpolyB =
Collect[Det[Table[matIntFull[[i, j]] /. r → 0 /. equilB0, {i, 1, 2}, {j, 1, 2}] -
λ * IdentityMatrix[2]], λ, Factor]
FAa Maa waf wam (-1 + αf) - FAa waf (-1 + αf) λ
2 FAA MAA wAf wAm αm + FAA wAf λ^2 + λ^2
```

Stability requires that the slope at $\lambda=1$ is positive and that the intercept at $\lambda=1$ is positive, otherwise one

or both of these two eigenvalues will be greater than one:

$$\begin{aligned} \text{slopeB} &= D[\text{charpolyB}, \lambda] /. \lambda \rightarrow 1 // \text{Factor} \\ &= -\text{FAa waf} + 2 \text{ FAA wAf} + \frac{\text{FAa waf } \alpha f}{\text{FAA wAf}} \\ \text{interceptB} &= \text{charpolyB} /. \lambda \rightarrow 1 // \text{Factor} \\ &= \frac{1}{2 \text{ FAA MAa wAf wAm } \alpha m} (-\text{FAa Maa waf wam} + \text{FAa Maa waf wam } \alpha f - \\ &\quad 2 \text{ FAA MAa waf wAm } \alpha m + 2 \text{ FAA MAa wAf wAm } \alpha m + 2 \text{ FAa MAa waf wAm } \alpha f \alpha m) \end{aligned}$$

slopeB is strictly greater than interceptB, so both will be positive if interceptB is positive:

$$\begin{aligned} \text{slopeB} - \text{interceptB} // \text{Factor} \\ = \frac{\text{FAa Maa waf wam} - \text{FAa Maa waf wam } \alpha f + 2 \text{ FAA MAa wAf wAm } \alpha m}{2 \text{ FAA MAa wAf wAm } \alpha m} \end{aligned}$$

Thus, for equilibrium B to be stable, we must have $\frac{\text{MAA wAm}}{2 \text{ MAa wam } (1 - \alpha m)} < 1$ and

$$\text{FAA} > \frac{\text{FAa waf } (1 - \alpha f) (\text{Maa wam} + 2 \text{ MAa wAm } \alpha m)}{2 \text{ MAa wAf wAm } \alpha m}.$$

stabcondB =

$$\begin{aligned} \text{Reduce} \left[\left\{ \frac{\text{MAA wAm}}{2 \text{ MAa wam } (1 - \alpha m)} < 1, \text{interceptB} > 0, \text{simpcond} \right\}, \{\text{MAa}, \text{Maa}, \text{FAA}\} \right] // \text{Factor} \\ 0 \leq Rf \leq \frac{1}{2} \&& \text{Faa} > 0 \&& 0 \leq Rm \leq \frac{1}{2} \&& 0 < \alpha m < 1 \&& 0 < \alpha f < 1 \&& \text{wAm} > 0 \&& \\ \text{wam} > 0 \&& \text{wAf} > 0 \&& \text{waf} > 0 \&& \text{MAA} > 0 \&& \text{FAa} > 0 \&& \text{MAa} > -\frac{\text{MAA wAm}}{2 \text{ wam } (-1 + \alpha m)} \&& \\ \text{Maa} > 0 \&& \text{FAA} > -\frac{\text{FAa waf } (-1 + \alpha f) (\text{Maa wam} + 2 \text{ MAa wAm } \alpha m)}{2 \text{ MAa wAf wAm } \alpha m} \end{aligned}$$

That the first eigenvalue is less than one is the same as saying $\gamma_{(B)} > 0$, or that the marginal fitness of Y-a is greater than the marginal fitness of Y-A, as stated in the main text

$$\begin{aligned} \frac{\text{wam } (\text{pXf } (1 - \alpha Dm) \text{ wAf MAa} + (1 - \text{pXf}) \text{ waf Maa})}{\text{wAm } (\text{pXf wAf MAA} + (1 - \text{pXf}) \text{ waf } (1 + \alpha Dm) \text{ MAa})} > 1 // \text{. equilB0} /. \alpha Dm \rightarrow 2 \alpha m - 1 // \\ \alpha Df \rightarrow 2 \alpha f - 1 // \text{Simplify}; \\ 0 > \text{wAm } (\text{pXf wAf MAA} + (1 - \text{pXf}) \text{ waf } (1 + \alpha Dm) \text{ MAa}) - \\ \text{wam } (\text{pXf } (1 - \alpha Dm) \text{ wAf MAa} + (1 - \text{pXf}) \text{ waf Maa}) /. \\ \text{equilB0} /. \alpha Dm \rightarrow 2 \alpha m - 1 /. \alpha Df \rightarrow 2 \alpha f - 1 // \text{Simplify}; \\ \% /. \text{Faa} \rightarrow (\text{Faa} /. \text{Solve}[\phi == \alpha f \text{ wAf FAa } (\text{wam Maa} + 2 \alpha m \text{ wAm MAa}) - \text{wam waf Maa Faa}, \text{Faa}]) // \\ \text{Simplify}; \\ \% /. \text{FAA} \rightarrow (\text{FAA} /. \text{Solve}[\psi == (1 - \alpha f) \text{ waf FAa } (\text{wam Maa} + 2 \alpha m \text{ wAm MAa}) - 2 \alpha m \text{ wAm wAf MAa FAa}, \\ \text{FAA}]) // \text{Simplify} \\ \text{wAf } (\text{MAA wAm} + 2 \text{ MAa wam } (-1 + \alpha m)) < 0 \end{aligned}$$

Also notice that ψ must be negative for equilB to be valid and stable

```

Reduce[{(1 - αf) waf FAa (wam Maa + 2 cm wAm MAa) - 2 cm wAm wAf MAa FAA < 0, simpcond},
FAA] // Factor

MAA > 0 && 0 ≤ Rf ≤  $\frac{1}{2}$  && Faa > 0 && 0 ≤ Rm ≤  $\frac{1}{2}$  && 0 < cm < 1 &&
0 < αf < 1 && wAm > 0 && wam > 0 && Maa > 0 && MAa > 0 && wAf > 0 &&
waf > 0 && FAa > 0 && FAA > -  $\frac{FAa waf (-1 + \alpha f) (Maa wam + 2 MAa wAm cm)}{2 MAa wAf wAm cm}$ 

Reduce[{interceptB > 0, simpcond}, FAA] // Factor

MAA > 0 && 0 ≤ Rf ≤  $\frac{1}{2}$  && Faa > 0 && 0 ≤ Rm ≤  $\frac{1}{2}$  && 0 < cm < 1 &&
0 < αf < 1 && wAm > 0 && wam > 0 && Maa > 0 && MAa > 0 && wAf > 0 &&
waf > 0 && FAa > 0 && FAA > -  $\frac{FAa waf (-1 + \alpha f) (Maa wam + 2 MAa wAm cm)}{2 MAa wAf wAm cm}$ 

```

Stability of equilC ($r \sim 0$)

The three allele frequencies at this equilibrium are

```

eq1 = pXf /. equilC0 // Simplify

$$\frac{waf (Maa wam - 2 MAa wAm cm)}{Maa waf wam + MAA wAf wAm + 2 MAa (-wAf wam + wAf wam cm - waf wAm cm)}$$


eq2 = pXm /. equilC0 // Simplify

$$(wam (Maa wam - 2 MAa wAm cm) \\
(-Faa waf (MAA wAm + 2 MAa wam (-1 + cm)) + FAa (Maa waf wam (-1 + αf) + \\
MAA wAf wAm αf + 2 MAa (wAf wam αf (-1 + cm) - waf wAm (-1 + αf) cm))) / \\
(- (Faa waf wam + FAA wAf wAm) (MAA wAm + 2 MAa wam (-1 + cm)) (Maa wam - 2 MAa wAm cm) + \\
FAa (Maa waf wam (-1 + αf) + MAA wAf wAm αf + \\
2 MAa (wAf wam αf (-1 + cm) - waf wAm (-1 + αf) cm)) \\
(Maa wam^2 - wAm (MAA wAm + 2 MAa wam (-1 + 2 cm)))) )$$


eq3 = pYm /. equilC0 // Simplify

$$(wam (Faa Maa waf wam (MAA wAm + 2 MAa wam (-1 + cm)) + \\
2 FAA MAa wAf wAm cm (-Maa wam + 2 MAa wAm cm) - \\
FAa (Maa wam + 2 MAa wAm cm) (Maa waf wam (-1 + αf) + MAA wAf wAm αf + \\
2 MAa (wAf wam αf (-1 + cm) - waf wAm (-1 + αf) cm))) / \\
(Faa waf wam (MAA wAm + 2 MAa wam (-1 + cm)) (Maa wam + 2 MAa wAm (-1 + cm)) + \\
FAA wAf wAm (MAA wAm - 2 MAa wam cm) (Maa wam - 2 MAa wAm cm) - \\
FAa (Maa wam^2 - wAm (MAA wAm + MAa wam (2 - 4 cm))) (Maa waf wam (-1 + αf) + \\
MAA wAf wAm αf + 2 MAa (wAf wam αf (-1 + cm) - waf wAm (-1 + αf) cm))) )$$


```

For pXf to lie between 0 and 1 requires that:

$$pXf = \frac{waf (Maa wam - 2 MAa wAm cm)}{Maa waf wam + MAA wAf wAm + 2 MAa (-wAf wam + wAf wam cm - waf wAm cm)} > 0$$

$$1 - pXf = \frac{wAf (MAA wAm + 2 MAa wam (-1 + cm))}{Maa waf wam + MAA wAf wAm + 2 MAa (-wAf wam + wAf wam cm - waf wAm cm)} > 0$$

so that either the numerators and denominators are both positive (without haploid selection, requires overdominance in males) or both negative (without haploid selection, requires underdominance in males). Thus the equilibrium will not be valid if $\frac{MAA wAm}{2 wam(1-\alpha m)} > MAa > \frac{Maa wam}{2 wAm \alpha m}$ or if $\frac{MAA wAm}{2 wam(1-\alpha m)} < MAa < \frac{Maa wam}{2 wAm \alpha m}$.

Evaluating the characteristic polynomial at $\lambda = 1$ (if this is negative, we know the leading eigenvalue is greater than one because the leading term of the characteristic polynomial is λ^3 and so it must eventually cross the horizontal axis at some $\lambda > 1$):

$$\begin{aligned} \text{stab} = \text{Factor}[\text{Det}[\text{IdentityMatrix}[3] - \text{matIntFull}] /. r \rightarrow 0 /. \text{equilC0}] \\ & ((-2 MAA wam + MAA wAm + 2 MAa wam \alpha m) (Maa wam - 2 MAa wAm \alpha m) \\ & (2 Faa Maa MAa waf wam^2 - 4 Faa MAa^2 waf wam^2 + Faa Maa MAA waf wam wAm + \\ & 2 Faa MAa MAA waf wam wAm - FAA Maa MAA wAf wam wAm - 2 Faa Maa MAa waf wam^2 \alpha f + \\ & 4 Faa MAa^2 wAf wam^2 \alpha f - Faa Maa MAA waf wam wAm \alpha f - Faa MAA^2 wAf wAm^2 \alpha f - \\ & 2 Faa Maa MAa waf wam^2 \alpha m + 8 Faa MAa^2 waf wam^2 \alpha m - 4 Faa MAa^2 waf wam wAm \alpha m - \\ & 2 Faa MAa MAA waf wam wAm \alpha m - 2 Faa MAa MAA waf wAm^2 \alpha m + 2 FAA MAa MAA wAf wAm^2 \alpha m + \\ & 2 Faa Maa MAA waf wam^2 \alpha f \alpha m - 8 Faa MAa^2 wAf wam^2 \alpha f \alpha m + 4 Faa MAa^2 waf wam wAm \alpha f \alpha m + \\ & 2 Faa MAa MAA waf wAm^2 \alpha f \alpha m - 4 Faa MAa^2 waf wam^2 \alpha m^2 + 4 Faa MAa^2 waf wam wAm \alpha m^2 + \\ & 4 Faa MAa^2 wAf wam^2 \alpha f \alpha m^2 - 4 Faa MAa^2 waf wAm \alpha f \alpha m^2)) / \\ & (-Faa Maa^2 waf wam^2 + 2 Faa Maa MAa waf wam^2 - Faa Maa MAA waf wam wAm + \\ & FAa Maa^2 waf wam^2 \alpha f - 2 Faa Maa MAa wAf wam^2 \alpha f + FAa Maa MAA wAf wam wAm \alpha f - \\ & 2 Faa Maa MAa waf wam^2 \alpha m + 2 FAA Maa MAa wAf wam wAm \alpha m + 2 Faa Maa MAa wAf wam^2 \alpha f \alpha m - \\ & 4 Faa MAa^2 wAf wam wAm \alpha f \alpha m + 2 Faa MAa MAA wAf wAm^2 \alpha f \alpha m + 4 Faa MAa^2 waf wAm^2 \alpha m^2 - \\ & 4 FAA MAa^2 wAf wAm^2 \alpha m^2 + 4 Faa MAa^2 wAf wam wAm \alpha f \alpha m^2 - 4 Faa MAa^2 waf wAm^2 \alpha f \alpha m^2)) / \\ & (waf wAf wam^2 wAm^2 (Maa MAA - 4 MAa^2 \alpha m + 4 MAa^2 \alpha m^2)^2 \\ & (-Faa FAA Maa^2 wam^2 + 2 Faa FAA Maa MAa wam^2 - Faa FAA Maa MAA wam wAm + \\ & FAa FAA Maa^2 wam^2 \alpha f - 4 Faa FAa MAa^2 wam^2 \alpha f + 4 Faa FAa Maa MAA wam wAm \alpha f - \\ & Faa FAa MAA^2 wAm^2 \alpha f - 2 Faa FAA Maa MAa wam^2 \alpha m + 4 Faa FAA Maa MAa wam wAm \alpha m - \\ & 4 Faa FAA MAa^2 wam wAm \alpha m + 2 Faa FAA MAa MAA wAm^2 \alpha m + \\ & 8 Faa FAa MAa^2 wam^2 \alpha f \alpha m - 4 Faa FAA Maa MAa wam wAm \alpha f \alpha m - \\ & 4 Faa FAa MAa MAA wam wAm \alpha f \alpha m + 4 Faa FAA MAa^2 wam wAm \alpha m^2 - \\ & 4 Faa FAA MAa^2 wAm^2 \alpha m^2 - 4 Faa FAa MAa^2 wam^2 \alpha f \alpha m^2 + 4 Faa FAA MAa^2 wAm^2 \alpha f \alpha m^2)) \end{aligned}$$

Making a set of substitutions that might help simplify factors below:

$$\begin{aligned} \text{subC1} = (\text{Solve}[\text{eq1} == \text{pXf}, \text{MAa}] // \text{Flatten}) \\ \left\{ \text{MAa} \rightarrow \frac{-Maa waf wam + Maa pXf waf wam + MAA pXf wAf wAm}{2 (pXf wAf wam - pXf wAf wam \alpha m - waf wAm \alpha m + pXf waf wAm \alpha m)} \right\} \end{aligned}$$

```

subC2 = Solve[ $(\text{eq2} - \text{pXm}) / . \text{subC1} == 0, \text{FAA}] // \text{Flatten}$ 
 $\left\{ \text{FAA} \rightarrow \frac{1}{(-\text{pXf} + \text{pXf}^2) \text{pXm wAf wAm}}$ 
 $(-\text{Faa pXf waf wam} + \text{Faa pXf}^2 \text{waf wam} + \text{Faa pXf pXm waf wam} - \text{Faa pXf}^2 \text{pXm waf wam} -$ 
 $\text{FAa pXf}^2 \text{wAf wam} + \text{FAa pXf}^2 \text{pXm wAf wam} - \text{FAa pXf pXm waf wAm} + \text{FAa pXf}^2 \text{pXm waf wAm} + \text{FAa}$ 
 $\text{pXf wAf wam } \alpha f - \text{FAa pXf pXm wAf wam } \alpha f + \text{FAa pXm waf wAm } \alpha f - \text{FAa pXf pXm waf wAm } \alpha f) \right\}$ 

subC3 = Solve[ $(\text{eq3} - \text{pYm}) / . \text{subC1} / . \text{subC2} // \text{Factor} == 0, \text{MAA}] // \text{Flatten}$ 
 $\left\{ \text{MAA} \rightarrow$ 
 $(\text{Maa} (-1 + \text{pXf}) \text{waf wam} (-\text{pXf pXm wAf wam} + \text{pXf pXm pYm waf wAm} + \text{pXm pYm waf wAm} - \text{pXf pXm}$ 
 $\text{pYm waf wAm} - \text{pXf wAf wam } \alpha m + 2 \text{pXf pXm wAf wam } \alpha m + \text{pXf pYm wAf wam } \alpha m -$ 
 $2 \text{pXf pXm pYm wAf wam } \alpha m + \text{pXm waf wAm } \alpha m - \text{pXf pXm waf wAm } \alpha m -$ 
 $2 \text{pXm pYm waf wAm } \alpha m + 2 \text{pXf pXm pYm waf wAm } \alpha m) ) /$ 
 $(\text{pXf wAf wAm} (\text{pXf pYm wAf wam} - \text{pXf pXm pYm wAf wam} - \text{pXm pYm waf wAm} +$ 
 $\text{pXf pXm pYm waf wAm} + \text{pXf wAf wam } \alpha m - \text{pXf pXm wAf wam } \alpha m -$ 
 $2 \text{pXf pYm wAf wam } \alpha m + 2 \text{pXf pXm pYm wAf wam } \alpha m - \text{pYm waf wAm } \alpha m +$ 
 $\text{pXf pYm waf wAm } \alpha m + 2 \text{pXm pYm waf wAm } \alpha m - 2 \text{pXf pXm pYm waf wAm } \alpha m) ) \right\}$ 

```

With these substitutions, the sign of stab reduces to:

```

stab /. subC1 /. subC2 /. subC3 // Factor;
Collect[ $\text{Factor}[\% / ((1 - \text{pYm}) \text{pYm wam} (\text{pXf} (1 - \text{pXm}) \text{wAf wam} - \text{pXm} (1 - \text{pXf}) \text{waf wAm})^2 /$ 
 $(\text{pXm} (\text{pXf} (1 - \text{pYm}) \text{wAf wam} - \text{pYm} (1 - \text{pXf}) \text{waf wAm})^2$ 
 $(\text{Faa} (1 - \text{pXf}) (1 - \text{pXm}) \text{waf wam} + \text{FAa} (\text{pXf} (1 - \text{pXm}) \text{wAf wam} + \text{pXm} (1 - \text{pXf}) \text{waf wAm})$ 
 $(1 - \alpha f)))], \{\text{Faa}, \text{FAa}, \text{FAA}\}, \text{Factor}]$ 

```

$\text{Faa} (-1 + \text{pXf}) \text{waf} - \text{FAa wAf} (\text{pXf} - \alpha f)$

If $(\text{pXf}-\alpha f)$ is positive, then stab is strictly negative and the leading eigenvalue must be greater than one.

The case where $(\text{pXf}-\alpha f)$ is negative is better handled by repeating the above but with a different subC2 (replacing for Faa instead):

```

subC2b = Solve[ $(\text{eq2} - \text{pXm}) / . \text{subC1} == 0, \text{Faa}] // \text{Flatten}$ 
 $\left\{ \text{Faa} \rightarrow \frac{1}{(-\text{pXf} + \text{pXf}^2) (-1 + \text{pXm}) \text{waf wam}}$ 
 $(-\text{FAa pXf}^2 \text{wAf wam} + \text{FAa pXf}^2 \text{pXm wAf wam} - \text{FAa pXf pXm waf wAm} + \text{FAa pXf}^2 \text{pXm waf wAm} +$ 
 $\text{FAA pXf pXm wAf wAm} - \text{FAA pXf}^2 \text{pXm wAf wAm} + \text{FAa pXf wAf wam } \alpha f -$ 
 $\text{FAa pXf pXm wAf wam } \alpha f + \text{FAa pXm waf wAm } \alpha f - \text{FAa pXf pXm waf wAm } \alpha f) \right\}$ 

```

```

subC3b = Solve[(eq3 - pYm /. subC1 /. subC2b // Factor) == 0, MAA] // Flatten
{MAA →
(Maa (-1 + pXf) waf wam (-pXf pXm wAf wam + pXf pXm pYm wAf wam + pXm pYm waf wAm - pXf pXm
pYm waf wAm - pXf wAf wam αm + 2 pXf pXm wAf wam αm + pXf pYm wAf wam αm -
2 pXf pXm pYm wAf wam αm + pXm waf wAm αm - pXf pXm waf wAm αm -
2 pXm pYm waf wAm αm + 2 pXf pXm pYm waf wAm αm) ) /
(pXf wAf wAm (pXf pYm wAf wam - pXf pXm pYm wAf wam - pXm pYm waf wAm +
pXf pXm pYm waf wAm + pXf wAf wam αm - pXf pXm wAf wam αm -
2 pXf pYm wAf wam αm + 2 pXf pXm pYm wAf wam αm - pYm waf wAm αm +
pXf pYm waf wAm αm + 2 pXm pYm waf wAm αm - 2 pXf pXm pYm waf wAm αm) ) }

stab /. subC1 /. subC2b /. subC3b // Factor;
Collect[
Factor[% / ((1 - pYm) pYm wAm (pXf (1 - pXm) wAf wam - pXm (1 - pXf) waf wAm)^2 / ((1 - pXm)
(pXf (1 - pYm) wAf wam - pYm (1 - pXf) waf wAm)^2 (FAA pXf pXm wAf wAm + FAa (pXf
(1 - pXm) wAf wam + pXm (1 - pXf) waf wAm) αf)))], {Faa, FAa, FAA}, Factor]
- FAA pXf wAf + FAa waf (pXf - αf)

```

Hence, if $(pXf - \alpha f)$ is negative, stab is also negative, and therefore the leading eigenvalue must be greater than one.

We conclude that in no case can equilC0 represent a stable equilibrium.

Weak selection

Assumptions

In this section we will assume weak selection in both haploids and diploids

```

weaksel = {
MAA → 1 + sAm ε,
Maa → 1 + hAm sAm ε,
Maa → 1,
FAA → 1 + sAf ε,
FAa → 1 + hAf sAf ε,
Faa → 1,
wAf → 1 + tf ε,
waf → 1,
wAm → 1 + tm ε,
wam → 1,
αm → 1 / 2 + α1m ε,
αf → 1 / 2 + α1f ε
};

```

All selection terms are of order ϵ . We will solve for genotype frequencies using a Taylor Series. First, we write the genotype frequencies as a series of terms, each proportional to increasing orders of ϵ (only necessary to go to ϵ^2 here), which we will then solve for.

```

frequenciessub = {
  pXf → pXf0 + pXf1 ε + pXf2 * ε^2,
  pXm → pXm0 + pXm1 ε + pXm2 * ε^2,
  pYm → pYm0 + pYm1 ε + pYm2 * ε^2,
  q → q + q1 ε + q2 * ε^2
};

```

Stability

Instead of examining when the non-trivial equilibrium (A polymorphic) is stable we will look for when the trivial equilibria (loss or fixation of A) are unstable.

When A is fixed the characteristic polynomial with no selection gives eigenvalues

```

charpolyInt1 = charpolyIntFull /. pXm → 1 /. pXf → 1 /. pYm → 1 /. q → 1 / 2;
Normal[Series[charpolyInt1 /. weaksel, {ε, 0, 0}]] // Factor;
Solve[% == 0, λ] // Factor // Flatten

```

$$\left\{ \lambda \rightarrow 1, \lambda \rightarrow \frac{1}{4} \left(1 - 2r - \sqrt{9 - 20r + 4r^2} \right), \lambda \rightarrow \frac{1}{4} \left(1 - 2r + \sqrt{9 - 20r + 4r^2} \right) \right\}$$

and $\lambda=1$ is the leading eigenvalue.

With weak selection first order term in the eigenvalues is

```

Normal[Series[charpolyInt1 /. weaksel /. λ → 1 + δλ1 * ε, {ε, 0, 1}]] /. ε → 1 // Factor;
δλ1fix = Solve[% == 0, δλ1] // Flatten

```

$$\left\{ \delta\lambda_1 \rightarrow \frac{1}{2} (-sAf + hAf sAf - sAm + hAm sAm - tf - tm - 2\alpha_1f - 2\alpha_1m) \right\}$$

And so the fixation of A is unstable when this expression is positive.

When A is lost the characteristic polynomial with no selection gives eigenvalues

```

charpolyInt0 = charpolyIntFull /. pXm → 0 /. pXf → 0 /. pYm → 0 /. q → 1 / 2;
Normal[Series[charpolyInt0 /. weaksel, {ε, 0, 0}]] // Factor;
Solve[% == 0, λ] // Flatten

```

$$\left\{ \lambda \rightarrow 1, \lambda \rightarrow \frac{1}{4} \left(1 - 2r - \sqrt{9 - 20r + 4r^2} \right), \lambda \rightarrow \frac{1}{4} \left(1 - 2r + \sqrt{9 - 20r + 4r^2} \right) \right\}$$

With weak selection first order term in the eigenvalues is

```

Normal[Series[charpolyInt0 /. weaksel /. λ → 1 + δλ1 * ε, {ε, 0, 1}]] /. ε → 1 // Factor;
δλ1loss = Solve[% == 0, δλ1] // Flatten

```

$$\left\{ \delta\lambda_1 \rightarrow \frac{1}{2} (hAf sAf + hAm sAm + tf + tm + 2\alpha_1f + 2\alpha_1m) \right\}$$

And so the loss of A is unstable when this expression is positive.

Combining the two results, the protected polymorphism (non-trivial equilibrium) requires that

```

stabconds = { (δλ1 /. δλ1fix) > 0, (δλ1 /. δλ1loss) > 0};

% /. αlf → αDf / 2 /. αlm → αDm / 2

{1
2 (-sAf + hAf sAf - sAm + hAm sAm - tf - tm - αDf - αDm) > 0,
1
2 (hAf sAf + hAm sAm + tf + tm + αDf + αDm) > 0}

```

Equilibria

Without selection the recursions for the change in the frequencies are

```

differenceEqs0 = Normal[Series[differenceEqs /. weaksel /. frequenciessub, {ε, 0, 0}]]
```

$$\left\{ \frac{1}{2} (-pXf_0 + pXm_0), \frac{1}{2} (pXf_0 - 2 pXm_0 + 2 pXm_0 q - pXf_0 r + pYm_0 r), \right.$$

$$\left. \frac{1}{2} (pYm_0 - 2 pYm_0 q + pXf_0 r - pYm_0 r), \frac{1}{2} (1 - 2 q) \right\}$$

And at equilibrium we have

```

sol0 = Solve[differenceEqs0 == 0, {pYm0, pXm0, q}]

```

$$\left\{ \left\{ pYm_0 \rightarrow pXf_0, pXm_0 \rightarrow pXf_0, q \rightarrow \frac{1}{2} \right\} \right\}$$

That is, to leading order ($O(1)$), there is no difference in A allele frequency between gamete types ($pYm_0=pXm_0=pXf_0$) and 50% of male gametes are Y-bearing ($q=1/2$). We call the leading order frequency of A alleles pA .

At the next order we have

```

differenceEqs1 =
Collect[Normal[Series[differenceEqs /. weaksel /. frequenciessub /. sol0,
{ε, 0, 1}]], ε, Factor] /. pXf0 → pA // Flatten

```

$$\left\{ \frac{1}{2} (-pXf_1 + pXm_1 + 2 hAf pA sAf + 2 pA^2 sAf - 6 hAf pA^2 sAf - 2 pA^3 sAf + 4 hAf pA^3 sAf + pA tf - pA^2 tf + pA tm - pA^2 tm + 4 pA αlf - 4 pA^2 αlf) ε, \right.$$

$$\left. \frac{1}{2} (pXf_1 - pXm_1 + 2 pA q_1 - pXf_1 r + pYm_1 r + hAm pA sAm + pA^2 sAm - 3 hAm pA^2 sAm - pA^3 sAm + 2 hAm pA^3 sAm + pA tf - pA^2 tf - pA r tf + pA^2 r tf + pA r tm - pA^2 r tm + 2 pA αlm - 2 pA^2 αlm) ε, \right.$$

$$\left. \frac{1}{2} (-2 pA q_1 + pXf_1 r - pYm_1 r + hAm pA sAm + pA^2 sAm - 3 hAm pA^2 sAm - pA^3 sAm + 2 hAm pA^3 sAm + pA r tf - pA^2 r tf + pA tm - pA^2 tm - pA r tm + pA^2 r tm + 2 pA αlm - 2 pA^2 αlm) ε, -q_1 ε \right\}$$

And we can solve for the first order difference in the frequency of A on Xs in males vs females, $dpAXmXf$, and on Ys in males vs Xs in females, $dpAYmXf$

```

realSol = Solve[differenceEqs1 == 0 /. pXm1 → dpAXmXf + pXf1 /. pYm1 → dpAYmXf + pXf1,
  {dpAXmXf, dpAYmXf, pA, q1}] // Factor

{ {dpAXmXf → 0, dpAYmXf → 0, pA → 0, q1 → 0}, {dpAXmXf → 0, dpAYmXf → 0, pA → 1, q1 → 0} ,
  {dpAXmXf → 1 / (-sAf + 2 hAf sAf - sAm + 2 hAm sAm)^3
    (-sAf + hAf sAf - sAm + hAm sAm - tf - tm - 2 α1f - 2 α1m)
    (hAf sAf + hAm sAm + tf + tm + 2 α1f + 2 α1m) (2 hAf sAf sAm - 2 hAm sAf sAm -
    sAf tf + 2 hAf sAf tf + sAm tf - 2 hAm sAm tf - sAf tm + 2 hAf sAf tm + sAm tm -
    2 hAm sAm tm + 4 sAm α1f - 8 hAm sAm α1f - 4 sAf α1m + 8 hAf sAf α1m) ,
  dpAYmXf → 1 / r (-sAf + 2 hAf sAf - sAm + 2 hAm sAm)^3
    (-sAf + hAf sAf - sAm + hAm sAm - tf - tm - 2 α1f - 2 α1m)
    (hAf sAf + hAm sAm + tf + tm + 2 α1f + 2 α1m)
    (hAf sAf sAm - hAm sAf sAm - r sAf tf + 2 hAf r sAf tf + sAm tf - 2 hAm sAm tf -
    r sAm tf + 2 hAm r sAm tf - sAf tm + 2 hAf sAf tm + r sAf tm - 2 hAf r sAf tm +
    r sAm tm - 2 hAm r sAm tm + 2 sAm α1f - 4 hAm sAm α1f - 2 sAf α1m + 4 hAf sAf α1m) ,
  pA → 1 / (-sAf + 2 hAf sAf - sAm + 2 hAm sAm), q1 → 0} }

```

Alternatively we could look at the difference in the frequency of A on Xs in males vs females, dpAXmXf , and on Ys in males vs Xs in males, dpAYmXm

```

realSol2 = Solve[differenceEqs1 == 0 /. pXf1 → dpAXfXm + pXm1 /. pYm1 → dpAYmXm + pXm1,
  {dpAXfXm, dpAYmXm, pA, q1}] // Factor;

```

Given that there is a polymorphs at the selected locus, the zeroth order frequency of A on all backgrounds, pA, can be written

```

pA /. realSol[[3]] /. α1f → αDf / 2 /. α1m → αDm / 2

```

$$\frac{hAf sAf + hAm sAm + tf + tm + αDf + αDm}{-sAf + 2 hAf sAf - sAm + 2 hAm sAm}$$

The first order differences can be re-written in a more readable form using the following terms

```

trySub3 = {Df → (pA sAf + (1 - pA) hAf sAf) - (pA hAf sAf + (1 - pA)),
  Dm → (pA sAm + (1 - pA) hAm sAm) - (pA hAm sAm + (1 - pA)),
  VA → pA (1 - pA), αDm → 2 α1m, αDf → 2 α1f};

```

The difference in frequency of A between Xs in males and Xs in females is then

```

VA (Dm - Df + αDm - αDf) /. trySub3;
(% == dpAXmXf) /. realSol[[3]] // Simplify
True

```

The difference in frequency of A between Ys in males and Xs in females is then

```

VA (Dm - Df + αDm - αDf + (1 - 2 r) (tm - tf)) / (2 r) /. trySub3;
(% == dpAYmXf) /. realSol[[3]] // Simplify
True

```

And the difference in frequency of A between Ys in males and Xs in males is then

```

VA (Dm - Df + αDm - αDf + tm - tf) (1 - 2 r) / (2 r) /. trysub3;
(% == dpAYmXm) /. realsol2[[3]] // Simplify
True

```

To get the frequency of Y bearing male gametes we need to go to the next order of selection (takes a minute)

```

differenceEqs2freqY =
Collect[Normal[Series[differenceEqs[[4]] /. weaksel /. frequencessub /. sol0 /.
pXf0 → pA /. pXm1 → (dpAXmXf + pXf1) /. pYm1 → dpAYmXf + pXf1 /.
realsol[[3]], {ε, 0, 2}]], ε, Factor] // Flatten;

```

And we can solve for the leading order Y bias in males

```
freqYm2sol = Solve[differenceEqs2freqY == 0, {q2}] // Flatten // Simplify;
```

which is simply α_{1m} times the difference in frequency of A between Y-bearing and X-bearing male gametes

```
α1m dpAYmXm == q2 /. freqYm2sol /. realsol2[[3]] // Factor
```

```
True
```

Finally, van Doorn and Kirkpatrick 2007 (SOM, page 6) give the first order frequency of pA as

$$vDK07 = \frac{hAf sAf + hAm sAm}{(-1 + 2 hAf) sAf + (-1 + 2 hAm) sAm};$$

And when there is no haploid selection or drive our result reduces to theirs

```
(pA /. realsol[[3]] /. tf → 0 /. tm → 0 /. α1f → 0 /. α1m → 0) == vDK07 // FullSimplify
```

```
True
```

Invasion

General

Characteristic polynomial

The Jacobian for the mutants is

```

eqsmut = {nextXAmf, nextXamf, nextYAmf,
          nextYamf, nextXamm, nextXamm, nextYamm, nextYamm} /. SUBS;
matExtFull = Transpose[{D[eqsmut, XAmf], D[eqsmut, Xamf], D[eqsmut, YAmf],
                         D[eqsmut, Yamf], D[eqsmut, XAmm], D[eqsmut, Xamm],
                         D[eqsmut, YAmm], D[eqsmut, Yamm]}] /. subequil // Factor;

```

Giving characteristic polynomial

```
charpolyExt = Det[matExtFull - IdentityMatrix[8] * λ];
```

Note that for a neo-W invading an XY system, the Jacobian is not in block triangular form:

```
PossibleZeroQ[matExtFull /. k → 1] /. True → 0 /. False → 1 // MatrixForm
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

But it is for a neo-Y

```
PossibleZeroQ[matExtFull /. k → 0] /. True → 0 /. False → 1 // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

The relevant quadratic

Let us instead track the sum of the mutant-selected allele haplotypes across X and Y backgrounds in each sex, XAm+YAm (call this S), as well as the difference, Xam-Yam (call this D):

```
subsumdif = Flatten[
  Solve[{SAmf == YAmf + XAmf, Samf == Yamf + Xamf, DAmf == XAmf - YAmf, Damf == Xamf - Yamf,
  SAmm == YAmm + XAmm, Samm == Yammm + Xamm, DAmm == XAmm - YAmm, Damm == Xamm - Yammm},
  {XAmf, Xamf, YAmf, Yamf, XAmm, Xamm, YAmm, Yammm}]]
```

$$\left\{ \begin{array}{l} XAmf \rightarrow \frac{DAmf + SAmf}{2}, Xamf \rightarrow \frac{Damf + Samf}{2}, YAmf \rightarrow -\frac{DAmf}{2} + \frac{SAmf}{2}, Yamf \rightarrow -\frac{Damf}{2} + \frac{Samf}{2}, \\ XAmm \rightarrow \frac{DAmm + SAmm}{2}, Xamm \rightarrow \frac{Damm + Samm}{2}, YAmm \rightarrow -\frac{DAmm}{2} + \frac{SAmm}{2}, Yammm \rightarrow -\frac{Damm}{2} + \frac{Samm}{2} \end{array} \right\}$$

This is just a translation of variables with no loss of information.

Calculating the Jacobian matrix for this new set of variables:

```
eqsmutNEW =
{nextXAmf + nextYAmf, nextXamf + nextYamf, nextXAmf - nextYAmf, nextXamf - nextYamf,
nextXAmm + nextYAmm, nextXamm + nextYamm, nextXAmm - nextYAmm,
nextXamm - nextYamm} /. SUBS /. subsumdif;
matExtFullNEW = Transpose[{D[eqsmutNEW, SAmf], D[eqsmutNEW, Samf],
D[eqsmutNEW, DAmf], D[eqsmutNEW, Damf], D[eqsmutNEW, SAmm],
D[eqsmutNEW, Samm], D[eqsmutNEW, DAmm], D[eqsmutNEW, Damm]}] /. subequil /.
{SAmf → 0, Samf → 0, DAmf → 0, Damf → 0, SAmm → 0, Samm → 0,
DAmm → 0, Damm → 0} // Factor;
```

The advantage of this new variable system is that it now has a block (2x2) by (2x2) by (4x4) form for a neo-W into a neo-Y system (a “1” in the following means there is a potential non-zero entry):

```
PossibleZeroQ[matExtFullNEW /. k → 1] /. True → 0 /. False → 1 // MatrixForm
```

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The 2x2 in the top left describes the dynamics of the sum total frequencies of the neo-W-A and neo-W-a in females, summed over the old-X and old-Y backgrounds. The 2x2 below and right of it describes the dynamics of the difference between old-X and old-Y backgrounds in the frequencies of the neo-W-A and neo-W-a types in females.

The same thing goes for a neo-Y invading an XY system.

```
PossibleZeroQ[matExtFullNEW /. k → 0] /. True → 0 /. False → 1 // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Now the bottom right 2x2 describes the dynamics of the difference between old-X and old-Y backgrounds in the frequencies of the neo-Y-A and neo-Y-a types in males. The 2x2 above and to the left it describes the sum total frequencies of the neo-Y-A and neo-Y-a in males, summed over the old-X and old-Y backgrounds.

Now, the difference cannot grow faster than the sum because the new mutant type is assumed rare and cannot become negative in frequency. So at the very worst, the difference might increase until one of the old-X or old-Y types becomes exceedingly rare, but at that point the difference and the sum would have to grow at the same rate. Consequently, the behaviour of the system is determined by the behaviour of the sum. We can therefore look only at the eigenvalues emerging from the 2x2 Jacobian describing the changes in the summed frequencies over X and Y backgrounds of the m-A and m-a haplotypes in the sex that m makes.

The relevant part of the characteristic polynomial for a neo-W is thus

```

{nextXAmf + nextYAmf, nextXamf + nextYamf} /. SUBS /. subsumdif;
Transpose[{D[% , SAmf], D[% , Samf]}] /. subequil /. {SAMf → 0, Samf → 0,
    DAmf → 0, Damf → 0, SAMm → 0, Samm → 0, DAmm → 0, Damm → 0} // Factor;
charpolyα1 = Collect[Det[% - IdentityMatrix[2]*λ /. k → 1], λ, Simplify]
- (waf wAf (Faa (1 + pXm (-1 + q) - pYm q) wam
    (FAA (pXm (-1 + q) - pYm q) wAm + 2 Faa (1 + pXm (-1 + q) - pYm q) (-1 + R) wam αf) +
    2 Faa (pXm (-1 + q) - pYm q) wAm (-1 + αf) (-FAA (pXm (-1 + q) - pYm q) (-1 + R) wAm +
    2 Faa (1 + pXm (-1 + q) - pYm q) (-1 + 2 R) wam αf))) /
    (4 (-1 + q)^2 (Faa (-1 + pXf) (-1 + pXm) waf wam + FAA pXf pXm wAf wAm +
    FAa (pXm waf wAm - pXf ((-1 + pXm) wAf wam + pXm waf wAm)))^2) +
    ((Faa (1 + pXm (-1 + q) - pYm q) waf wam + FAA (pXm - pXm q + pYm q) wAf wAm -
    2 Faa (-1 + R) (wAf wam αf + pXm (-1 + q) (waf wAm (-1 + αf) + wAf wam αf) -
    pYm q (waf wAm (-1 + αf) + wAf wam αf))) λ) /
    (2 (-1 + q) (Faa (-1 + pXf) (-1 + pXm) waf wam + FAA pXf pXm wAf wAm +
    FAa (pXm waf wAm - pXf ((-1 + pXm) wAf wam + pXm waf wAm))) + λ^2

```

And the relevant part of the characteristic polynomial for a neo-W is thus

```

{nextXAMm + nextYAMm, nextXamm + nextYamm} /. SUBS /. subsumdif;
Transpose[{D[% , SAMm], D[% , Samm]}] /. subequil /. {SAMf → 0, Samf → 0,
    DAmf → 0, Damf → 0, SAMm → 0, Samm → 0, DAmm → 0, Damm → 0} // Factor;
charpolyα0 = Collect[Det[% - IdentityMatrix[2]*λ /. k → 0], λ, Simplify]
(wam wAm (-Maa (-1 + pXf) waf (MAA pXf wAf + 2 MAa (-1 + pXf) (-1 + R) waf αm) + 2 MAa pXf wAf
    (-1 + αm) (MAA pXf (-1 + R) wAf + 2 MAa (-1 + pXf + 2 R - 2 pXf R) waf αm))) /
    (4 q^2 (Maa (-1 + pXf) (-1 + pYm) waf wam + MAA pXf pYm wAf wAm +
    MAA (pYm waf wAm - pXf ((-1 + pYm) wAf wam + pYm waf wAm))^2) +
    ((Maa (-1 + pXf) waf wam - MAA pXf wAf wAm - 2 MAa (-1 + R)
    (-waf wAm αm + pXf (wAf wam (-1 + αm) + waf wAm αm))) λ) /
    (2 q (Maa (-1 + pXf) (-1 + pYm) waf wam + MAA pXf pYm wAf wAm +
    MAa (pYm waf wAm - pXf ((-1 + pYm) wAf wam + pYm waf wAm))) + λ^2

```

We confirmed that the alternative quadratic (from the difference instead of the sum) never determined invasion (details available upon request).

Mean fitness

Among the zygotes, the frequency of diploid females is $(1-\zeta)$:

```

freqFemale = XAMXAMfemale + XAMXaMfemale + XaMXaMfemale +
    XAMYAMfemale + (XAMYaMfemale + XaMYAMfemale) + XaMYaMfemale +
    YAMYAMfemale + YAMYaMfemale + YaMYaMfemale +
    XAmXAMfemale + XAmXaMfemale + XAMXamfemale + XamXaMfemale +
    XAmYAMfemale + XAmYaMfemale + XamYAMfemale + XamYaMfemale +
    XAMYAmfemale + XAMYamfemale + XaMYAmfemale + XaMYamfemale +
    YAmYAMfemale + YAmYaMfemale + YamYAMfemale + YamYaMfemale +
    XAmXAmfemale + XAmXamfemale + XamXamfemale +
    XAmYAmfemale + XAmYamfemale + XamYAmfemale + XamYamfemale +
    XAmYAmfemale + YAmYamfemale + YamYamfemale /. SUBS /. subequil // Factor
    (-1 + q) (-wam + pXm wam - pXm wAm)

```

wam - pXm wam + pXm q wam - pYm q wam + pXm wAm - pXm q wAm + pYm q wAm

The frequency of diploid male zygotes is then ζ , as given in Table S.2:

```

freqMale = XAMXAMmale + XAMXaMmale + XaMXaMmale +
           XAMYAMmale + (XAMYaMmale + XaMYAMmale) + XaMYaMmale +
           YAMYAMmale + YAMYaMmale + YaMYaMmale +
           XAmXAMmale + XAmXaMmale + XAMXammale + XamXaMmale +
           XAmYAMmale + XAmYaMmale + XamYAMmale + XamYaMmale +
           XAMYAmmale + XAMYammale + XaMYAmmale + XaMYammale +
           YAmyYAMmale + YAmYaMmale + YamYAMmale + YamYaMmale +
           XAmXammale + XAmXammale + XamXammale +
           XAmYammale + XAmYammale + XamYammale + XamYammale +
           XAmYammale + YAmYammale /. SUBS /. subequil // Factor

$$q (-wam + pYm wam - pYm wAm)$$



---



$$-wam + pXm wam - pXm q wam + pYm q wam - pXm wAm + pXm q wAm - pYm q wAm$$


```

As a check the frequency of females and males sum to one:

```
freqFemale + freqMale // Factor
```

```
1
```

The mean fitness of a diploid female can be written by considering the chance that the female inherited "A" from the mother and either "A" or "a" from the father (first row) or "a" from the mother and either "A" or "a" from the father (second row), conditioning on the fact that an X must have been inherited from the dad (i.e., dividing by the frequency of female zygotes):

$$\begin{aligned}
wbarDipFemale &= pXf \left(F_{AA} \frac{wAf}{wbarHapFemale} \left(\frac{pXm (1-q)}{wbarHapMale} \frac{wAm}{freqFemale} \right) + \right. \\
&\quad F_{Aa} \frac{wAf}{wbarHapFemale} \left(\frac{(1-pXm) (1-q)}{wbarHapMale} \frac{wAm}{freqFemale} \right) + \\
&\quad (1-pXf) \left(F_{AA} \frac{waf}{wbarHapFemale} \left(\frac{pXm (1-q)}{wbarHapMale} \frac{wAm}{freqFemale} \right) + \right. \\
&\quad \left. \left. F_{Aa} \frac{waf}{wbarHapFemale} \left(\frac{(1-pXm) (1-q)}{wbarHapMale} \frac{wAm}{freqFemale} \right) \right) / . \text{SUBS} / . \text{subequil} // \text{Simplify}
\end{aligned}$$

$$\begin{aligned}
&(F_{AA} (-1 + pXf) (-1 + pXm) waf wam + F_{AA} pXf pXm wAf wAm + \\
&\quad F_{AA} (pXm waf wAm - pXf ((-1 + pXm) wAf wam + pXm waf wAm)) / \\
&\quad (((-1 + pXf) waf - pXf wAf) ((-1 + pXm) wam - pXm wAm))
\end{aligned}$$

Similarly, the mean fitness of a diploid male can be written as:

```
wbarDipMale =
pxf 
$$\left( MAA \frac{wAf}{wbarHapFemale} \left( \frac{pYm q \frac{wAm}{wbarHapMale}}{freqMale} \right) + MAa \frac{wAf}{wbarHapFemale} \left( \frac{(1-pYm) q \frac{wam}{wbarHapMale}}{freqMale} \right) \right) +$$


$$(1 - pXf) \left( MAa \frac{waf}{wbarHapFemale} \left( \frac{pYm q \frac{wAm}{wbarHapMale}}{freqMale} \right) +$$


$$MAa \frac{waf}{wbarHapFemale} \left( \frac{(1-pYm) q \frac{wam}{wbarHapMale}}{freqMale} \right) \right) /. SUBS /. subequil // Simplify$$

```

$$(MAa (-1 + pXf) (-1 + pYm) waf wam + MAA pXf pYm wAf wAm +$$

$$MAa (pYm waf wAm - pXf ((-1 + pYm) wAf wam + pYm waf wAm)) /$$

$$((-1 + pXf) waf - pXf wAf) ((-1 + pYm) wam - pYm wAm))$$

Note that if we define mean haploid fitnesses of X and Y bearing male gametes

```
wbarHapMaleX = (1 - q) ((1 - pXm) wam + pXm wAm);  
wbarHapMaleY = q ((1 - pYm) wam + pYm wAm);
```

then

```
freqFemale wbarHapMale == wbarHapMaleX /. SUBS /. subequil // Simplify
```

True

```
freqMale wbarHapMale == wbarHapMaleY /. SUBS /. subequil // Simplify
```

True

The following replaces terms used to stand in for the mean fitnesses with their formulae:

```
reverse =
{meandipM -> wbarDipMale, meandipF -> wbarDipFemale, ξ -> freqMale, meanhapM ->
wbarHapMale, meanhapF -> wbarHapFemale} /. SUBS /. subequil // Simplify;
```

The following removes haploid selection:

```
nohap = {wam → 1, wAm → 1, waf → 1, wAf → 1, αm → 1/2, αf → 1/2};
```

neo-Y characteristic polynomial

As shown above, the relevant part of the characteristic polynomial for a masculinizing mutant is

charpolyα0

$$(wam wAm (-Maa (-1 + pXf) waf (MAA pXf wAf + 2 MAa (-1 + pXf) (-1 + R) waf αm) + 2 MAa pXf wAf (-1 + αm) (MAA pXf (-1 + R) wAf + 2 MAa (-1 + pXf + 2 R - 2 pXf R) waf αm))) /$$

$$(4 q^2 (Maa (-1 + pXf) (-1 + pYm) waf wam + MAA pXf pYm wAf wAm +$$

$$MAa (pYm waf wAm - pXf ((-1 + pYm) wAf wam + pYm waf wAm))^2) +$$

$$((Maa (-1 + pXf) waf wam - MAA pXf wAf wAm - 2 MAa (-1 + R)$$

$$(-waf wAm αm + pXf (wAf wam (-1 + αm) + waf wAm αm))) λ) /$$

$$(2 q (Maa (-1 + pXf) (-1 + pYm) waf wam + MAA pXf pYm wAf wAm +$$

$$MAa (pYm waf wAm - pXf ((-1 + pYm) wAf wam + pYm waf wAm))) + λ^2$$

The coefficients of the relevant quadratic, $λ^2 + bλ + c = 0$, are then:

```

acoef0 = Coefficient[charpoly $\alpha$ 0,  $\lambda^2$ ];
bcoef0 = Coefficient[charpoly $\alpha$ 0,  $\lambda$ ] / acoef0;
ccoeff0 = (charpoly $\alpha$ 0 - acoef0  $\lambda^2$  - acoef0 bcoef0  $\lambda$ ) / acoef0;

```

The b coefficient is

```

bcoef0 * 
$$\left( \frac{wbarDipMale freqMale wbarHapMale wbarHapFemale}{meandipM \zeta meanhapM meanhapF} \right) /. SUBS /. subequil // Simplify$$


```

```

(Maa (-1 + pXf) waf wam - MAA pXf wAf wAm -
 2 Maa (-1 + R) (-waf wAm \alpha m + pXf (wAf wam (-1 + \alpha m) + waf wAm \alpha m))) /
 (2 meandipM meanhapF meanhapM \zeta)

```

The c coefficient is

```

ccoeff0 * 
$$\left( \frac{wbarDipMale freqMale wbarHapMale wbarHapFemale}{meandipM \zeta meanhapM meanhapF} \right)^2 /. SUBS /. subequil // Simplify$$


```

```

- (wam wAm (Maa (-1 + pXf) waf (MAA pXf wAf + 2 Maa (-1 + pXf) (-1 + R) waf \alpha m) +
 2 Maa pXf wAf (-1 + \alpha m) (-MAA pXf (-1 + R) wAf + 2 Maa (-1 + pXf) (-1 + 2 R) waf \alpha m))) /
 (4 meandipM2 meanhapF2 meanhapM2 \zeta2)

```

With no recombination the eigenvalues are

```

noreceigens0 = ( $\lambda$  /. Solve[0 == charpoly $\alpha$ 0 /. R  $\rightarrow$  0,  $\lambda$ ]) *
 
$$\left( \frac{wbarDipMale freqMale wbarHapMale wbarHapFemale}{meandipM \zeta meanhapM meanhapF} \right) /. SUBS /. subequil // Factor // Simplify$$


$$\left\{ -\frac{wam (Maa (-1 + pXf) waf + 2 Maa pXf wAf (-1 + \alpha m))}{2 meandipM meanhapF meanhapM \zeta}, \right.$$


$$\left. \frac{wAm (MAA pXf wAf - 2 Maa (-1 + pXf) waf \alpha m)}{2 meandipM meanhapF meanhapM \zeta} \right\}$$


```

Define the no recombination eigenvalues

```

\lambda_{mA0} = noreceigens0[[2]]
\lambda_{ma0} = noreceigens0[[1]]

```

$$\frac{wAm (MAA pXf wAf - 2 Maa (-1 + pXf) waf \alpha m)}{2 meandipM meanhapF meanhapM \zeta}$$

$$-\frac{wam (Maa (-1 + pXf) waf + 2 Maa pXf wAf (-1 + \alpha m))}{2 meandipM meanhapF meanhapM \zeta}$$

and the rate at which recombination occurs

```


$$\begin{aligned}
& \chi_{mA0} = R \text{MAa Coefficient}[\text{noreceigens0}[[2]], \text{MAa}] \\
& \chi_{ma0} = R \text{MAa Coefficient}[\text{noreceigens0}[[1]], \text{MAa}] \\
& - \frac{\text{MAa} (-1 + pXf) R waf wAm \alpha m}{\text{meandipM meanhapF meanhapM } \zeta} \\
& - \frac{\text{MAa } pXf R wAf wam (-1 + \alpha m)}{\text{meandipM meanhapF meanhapM } \zeta}
\end{aligned}$$


```

The b coefficient can then be written as

```
- ( $\lambda_{mA0} + \lambda_{ma0}$ ) + ( $\chi_{mA0} + \chi_{ma0}$ );
```

```

% - bcoef0 * 
$$\left( \frac{\text{wbarDipMale freqMale wbarHapMale wbarHapFemale}}{\text{meandipM } \zeta \text{ meanhapM meanhapF}} \right) /. \text{SUBS} /. \text{subequil} //$$

Simplify
0

```

and the c coefficient can be written as

```
( $\lambda_{mA0} - \chi_{mA0}$ ) ( $\lambda_{ma0} - \chi_{ma0}$ ) - ( $\chi_{mA0} \chi_{ma0}$ );
```

```

% - ccoef0 * 
$$\left( \frac{\text{wbarDipMale freqMale wbarHapMale wbarHapFemale}}{\text{meandipM } \zeta \text{ meanhapM meanhapF}} \right)^2 /. \text{SUBS} /. \text{subequil} //$$

Simplify
0

```

neo-W characteristic polynomial

As shown above, the relevant part of the characteristic polynomial for a feminizing mutant is

charpoly α 1

```

- (waf wAf (Faa (1 + pXm (-1 + q) - pYm q) wam
  (FAA (pXm (-1 + q) - pYm q) wAm + 2 FAa (1 + pXm (-1 + q) - pYm q) (-1 + R) wam  $\alpha$ f) +
  2 FAa (pXm (-1 + q) - pYm q) wAm (-1 +  $\alpha$ f) (-FAA (pXm (-1 + q) - pYm q) (-1 + R) wAm +
  2 FAa (1 + pXm (-1 + q) - pYm q) (-1 + 2 R) wam  $\alpha$ f))) /
  (4 (-1 + q)2 (Faa (-1 + pXf) (-1 + pXm) waf wam + FAA pXf pXm wAf wAm +
  FAa (pXm waf wAm - pXf ((-1 + pXm) wAf wam + pXm waf wAm))2) +
  ((Faa (1 + pXm (-1 + q) - pYm q) waf wam + FAA (pXm - pXm q + pYm q) wAf wAm -
  2 FAa (-1 + R) (wAf wam  $\alpha$ f + pXm (-1 + q) (waf wAm (-1 +  $\alpha$ f) + wAf wam  $\alpha$ f) -
  pYm q (waf wAm (-1 +  $\alpha$ f) + wAf wam  $\alpha$ f)))  $\lambda$ ) /
  (2 (-1 + q) (Faa (-1 + pXf) (-1 + pXm) waf wam + FAA pXf pXm wAf wAm +
  FAa (pXm waf wAm - pXf ((-1 + pXm) wAf wam + pXm waf wAm)))) +  $\lambda$ 2

```

```

acoef1 = Coefficient[charpoly $\alpha$ 1,  $\lambda$ 2];
bcoef1 = Coefficient[charpoly $\alpha$ 1,  $\lambda$ ] / acoef1;
ccoeff1 = (charpoly $\alpha$ 1 - acoef1  $\lambda$ 2 - acoef1 bcoef1  $\lambda$ ) / acoef1;

```

The b coefficient is

$$\text{bcoef1} * \left(\frac{\text{wbarDipFemale freqFemale wbarHapMale wbarHapFemale}}{\text{meandipF} (1 - \zeta) \text{meanhapM meanhapF}} \right) / . \text{SUBS} / . \text{subequil} // \\ \text{Simplify}$$

1

$$\frac{2 \text{meandipF} \text{meanhapF} \text{meanhapM} (-1 + \zeta)}{(Faa (1 + pXm (-1 + q) - pYm q) waf wam + FAA (pXm - pXm q + pYm q) wAf wAm - 2 FAA (-1 + R) (wAf wam \alpha f + pXm (-1 + q) (waf wAm (-1 + \alpha f) + wAf wam \alpha f) - pYm q (waf wAm (-1 + \alpha f) + wAf wam \alpha f)))}$$

The c coefficient is

$$\text{ccoef1} * \left(\frac{\text{wbarDipFemale freqFemale wbarHapMale wbarHapFemale}}{\text{meandipF} (1 - \zeta) \text{meanhapM meanhapF}} \right)^2 / . \text{SUBS} / . \text{subequil} //$$

Simplify

$$- (waf wAf (Faa (1 + pXm (-1 + q) - pYm q) wam (FAA (pXm (-1 + q) - pYm q) wAm + 2 FAA (1 + pXm (-1 + q) - pYm q) (-1 + R) wam \alpha f) + 2 FAA (pXm (-1 + q) - pYm q) wAm (-1 + \alpha f) (-FAA (pXm (-1 + q) - pYm q) (-1 + R) wAm + 2 FAA (1 + pXm (-1 + q) - pYm q) (-1 + 2 R) wam \alpha f))) / (4 \text{meandipF}^2 \text{meanhapF}^2 \text{meanhapM}^2 (-1 + \zeta)^2))$$

The no recombination eigenvalues are (this takes a couple minutes)

$$\text{noreceigens1} = (\lambda / . \text{Solve}[0 == \text{charpoly}\alpha 1 / . \text{R} \rightarrow 0, \lambda]) * \\ \left(\frac{\text{wbarDipFemale freqFemale wbarHapMale wbarHapFemale}}{\text{meandipF} (1 - \zeta) \text{meanhapM meanhapF}} \right) / . \text{SUBS} / . \\ \text{subequil} / . \text{pXm} \rightarrow \frac{\text{pAveM} - q \text{pYm}}{(1 - q)} // \text{Factor} // \text{Simplify} \\ \left\{ \frac{FAA \text{pAveM} \text{wAf} \text{wAm} + 2 \text{FAa} \text{wAf} \text{wam} \alpha f - 2 \text{FAa} \text{pAveM} \text{wAf} \text{wam} \alpha f}{2 \text{meandipF} \text{meanhapF} \text{meanhapM} - 2 \text{meandipF} \text{meanhapF} \text{meanhapM} \zeta}, \right. \\ \left. \frac{waf (\text{Faa} (-1 + \text{pAveM}) \text{wam} + 2 \text{FAa} \text{pAveM} \text{wAm} (-1 + \alpha f))}{2 \text{meandipF} \text{meanhapF} \text{meanhapM} (-1 + \zeta)} \right\}$$

Define the no recombination eigenvalues

$$\lambda_{\text{m1}} = \text{noreceigens1}[[1]] \\ \lambda_{\text{m1}} = \text{noreceigens1}[[2]] \\ \frac{FAA \text{pAveM} \text{wAf} \text{wAm} + 2 \text{FAa} \text{wAf} \text{wam} \alpha f - 2 \text{FAa} \text{pAveM} \text{wAf} \text{wam} \alpha f}{2 \text{meandipF} \text{meanhapF} \text{meanhapM} - 2 \text{meandipF} \text{meanhapF} \text{meanhapM} \zeta} \\ \frac{waf (\text{Faa} (-1 + \text{pAveM}) \text{wam} + 2 \text{FAa} \text{pAveM} \text{wAm} (-1 + \alpha f))}{2 \text{meandipF} \text{meanhapF} \text{meanhapM} (-1 + \zeta)}$$

and the rate at which recombination occurs

```


$$\begin{aligned} \chi_{mA1} &= R F_{AA} \text{Coefficient}[noreceigens1[[1]], F_{AA}] \\ \chi_{ma1} &= R F_{AA} \text{Coefficient}[noreceigens1[[2]], F_{AA}] \\ &\quad F_{AA} R (2 w_{Af} w_{Am} \alpha_f - 2 p_{AveM} w_{Af} w_{Am} \alpha_f) \\ &\quad \frac{2 \text{meandipF} \text{meanhapF} \text{meanhapM} - 2 \text{meandipF} \text{meanhapF} \text{meanhapM} \zeta}{F_{AA} p_{AveM} R w_{Af} w_{Am} (-1 + \alpha_f)} \\ &\quad \frac{\text{meandipF} \text{meanhapF} \text{meanhapM} (-1 + \zeta)}{\text{meandipF} \text{meanhapF} \text{meanhapM} (-1 + \zeta)} \end{aligned}$$


```

The b coefficient can then be written as

```
- (\lambda_{mA1} + \lambda_{ma1}) + (\chi_{mA1} + \chi_{ma1});
```

```

% - bcoef1 * 
$$\left( \frac{w_{barDipFemale} f_{reqFemale} w_{barHapMale} w_{barHapFemale}}{\text{meandipF} (1 - \zeta) \text{meanhapM} \text{meanhapF}} \right) /. \text{SUBS} /.$$


$$\text{subequil} /. \text{pXm} \rightarrow \frac{p_{AveM} - q p_{Ym}}{(1 - q)} // \text{Simplify}$$


```

0

and the c coefficient can be written as

```
(\lambda_{mA1} - \chi_{mA1}) (\lambda_{ma1} - \chi_{ma1}) // \text{Simplify};
```

```

% - ccoef1 * 
$$\left( \frac{w_{barDipFemale} f_{reqFemale} w_{barHapMale} w_{barHapFemale}}{\text{meandipF} (1 - \zeta) \text{meanhapM} \text{meanhapF}} \right)^2 /. \text{SUBS} /.$$


$$\text{subequil} /. \text{pXm} \rightarrow \frac{p_{AveM} - q p_{Ym}}{(1 - q)} // \text{Simplify}$$


```

0

Invasion conditions

```
Clear[\lambdamA, \lambdama]
```

To recap, with either a neo-Y or a neo-W, invasion is determined by the solution to the quadratic $f(\lambda) = \lambda^2 + b\lambda + c = 0$, where

$$b = -(\lambda_{mA} - \chi_{mA}) - (\lambda_{ma} - \chi_{ma})$$

$$c = (\lambda_{mA} - \chi_{mA}) (\lambda_{ma} - \chi_{ma}) - \chi_{mA} \chi_{ma}.$$

When $R=0$, both $\chi_{mA} = \chi_{ma} = 0$, and the two solutions for λ are λ_{mA} and λ_{ma} . We call these the “haplotype-only growth rates”, as they indicate how rapidly the haplotypes would spread if recombination never altered the background of the neo-SD.

In this section, we show for $R>0$ that:

- * **Lemma 1:** invasion always becomes more difficult as R rises.
- * **Lemma 2:** invasion is always prohibited if both $\lambda_{mA}<1$ and $\lambda_{ma}<1$.
- * **Lemma 3:** invasion is guaranteed for any R if both $\lambda_{mA}>1$ and $\lambda_{ma}>1$. In particular, invasion of a neo-SD on a different chromosome ($R = 1/2$) will occur in this case.
- * **Lemma 4:** invasion becomes more difficult as R rises if only one of λ_{mA} and λ_{ma} is greater than one, requiring that equation (1) in the paper be satisfied.

Preliminaries: We are guaranteed to have both roots of $f(\lambda)$ be greater than one if the slope at $\lambda = 1$ is negative, i.e., if $f'(1) = 2 + b < 0$, or equivalently when $-\frac{b}{2} = \frac{(\lambda m A - \chi m A) + (\lambda m a - \chi m a)}{2} > 1$. This is because the Perron-Frobenius theorem for a stability matrix without negative entries guarantees that the leading eigenvalue is positive and real and we know $f(\lambda)$ is concave up. Thus, if the quadratic has a negative slope at $\lambda=1$ then it will cross zero and cross back at some $\lambda>1$. In words, the neo-SD is guaranteed to spread when the haplotype-only growth rate minus its loss due to recombination averaged for the mA and ma haplotypes is greater than 1. We will see that this condition is a bit stronger than we really need, because it does not consider the fact that recombination also creates mutant haplotypes.

When $-\frac{b}{2} < 1$, then we will still have one $\lambda>1$ when $f(1) = 1 + b + c < 0$. Again, the leading eigenvalue must be real and positive and $f(\lambda)$ is concave up, so if it is negative at $\lambda=1$ it must cross zero at some $\lambda>1$. This condition can be rearranged to give equation (1)

$$\begin{aligned} & (\lambda m A - 1)(\lambda m a - 1) < \chi m a (\lambda m A - 1) + \chi m A (\lambda m a - 1) \\ & (\lambda m A - 1)(\lambda m a - 1) - (\chi m a (\lambda m A - 1) + \chi m A (\lambda m a - 1)) ; \\ & \% - (1 + b + c) / . b \rightarrow -((\lambda m A - \chi m A) + (\lambda m a - \chi m a)) / . \\ & c \rightarrow (\lambda m A - \chi m A)(\lambda m a - \chi m a) - \chi m A \chi m a // \text{Factor} \\ & 0 \end{aligned}$$

Furthermore, the leading eigenvalue, λ_L , is bounded between $\lambda m A$ and $\lambda m a$, given that the quadratic has opposite signs when evaluated at these two values:

$$\begin{aligned} & \lambda^2 + b \lambda + c / . b \rightarrow -((\lambda m A - \chi m A) + (\lambda m a - \chi m a)) / . c \rightarrow (\lambda m A - \chi m A)(\lambda m a - \chi m a) - \chi m A \chi m a ; \\ & \% / . \lambda \rightarrow \lambda m A // \text{Factor} \\ & \% / . \lambda \rightarrow \lambda m a // \text{Factor} \\ & (-\lambda m a + \lambda m A) \chi m A \\ & -(\lambda m a - \lambda m A) \chi m A \end{aligned}$$

when $R = 0$, the χ terms are zero and the larger of $\lambda m a$ and $\lambda m A$ is the leading eigenvalue.

Proof of Lemma 1: We next prove that the leading eigenvalue always declines as R increases. First we note that both $\chi m a$ and $\chi m A$ are proportional to R and we make this explicit in the quadratic $f(\lambda)$:

$$\begin{aligned} & \lambda[R]^2 + b \lambda[R] + c / . b \rightarrow -((\lambda m A - \chi m A) + (\lambda m a - \chi m a)) / . \\ & c \rightarrow (\lambda m A - \chi m A)(\lambda m a - \chi m a) - \chi m A \chi m a / . \chi m A \rightarrow f_a * R / . \chi m a \rightarrow f_a * R \\ & -R^2 f_a f_A + (\lambda m a - R f_a)(\lambda m A - R f_A) + (-\lambda m a - \lambda m A + R f_a + R f_A) \lambda[R] + \lambda[R]^2 \end{aligned}$$

Solving for $\frac{d\lambda}{dR}$ and returning to the χ terms:

$$\begin{aligned} & \text{Solve}[D[\%, R] == 0, \lambda'[R]] / . f_A \rightarrow \chi m A / R / . f_a \rightarrow \chi m a / R / . \lambda[R] \rightarrow \lambda // \text{Flatten} // \\ & \text{Simplify} \\ & \left\{ \lambda'[R] \rightarrow \frac{\lambda m A \chi m a + \lambda m a \chi m A - \lambda (\chi m a + \chi m A)}{R (2 \lambda - \lambda m a - \lambda m A + \chi m a + \chi m A)} \right\} \end{aligned}$$

This can be written as:

$$-\frac{\chi m a + \chi m A}{R \text{ slope}} \left(\lambda_L - \frac{\lambda m A \chi m a + \lambda m a \chi m A}{\chi m a + \chi m A} \right)$$

where “slope” = $(2\lambda_L + b)$ is the slope of the quadratic, $\frac{df(\lambda)}{d\lambda}$, evaluated at the leading eigenvalue λ_L , which must be positive (because $f(\lambda)$ is concave up and we’re asking about the right-most root). Hence the first fraction is positive.

Furthermore, the second fraction must be positive, since the leading eigenvalue is strictly above $\frac{\lambda_{mA} \chi_{mA} + \lambda_{ma} \chi_{mA}}{\chi_{mA} + \chi_{mA}}$. This is because $f'(\frac{\lambda_{mA} \chi_{mA} + \lambda_{ma} \chi_{mA}}{\chi_{mA} + \chi_{mA}})$ is strictly negative and so the leading eigenvalue must occur at a higher value of λ :

$$\begin{aligned} & \lambda^2 + b\lambda + c / . \quad b \rightarrow -((\lambda_{mA} - \chi_{mA}) + (\lambda_{ma} - \chi_{ma})) / . \quad c \rightarrow (\lambda_{mA} - \chi_{mA})(\lambda_{ma} - \chi_{ma}) - \chi_{mA}\chi_{ma}; \\ & \% / . \quad \lambda \rightarrow \frac{\lambda_{mA} \chi_{mA} + \lambda_{ma} \chi_{mA}}{\chi_{mA} + \chi_{mA}} // \text{Factor} \\ & - \frac{(\lambda_{ma} - \lambda_{mA})^2 \chi_{mA} \chi_{mA}}{(\chi_{mA} + \chi_{mA})^2} \end{aligned}$$

We conclude that the leading eigenvalue must fall as R increases.

Corollary of Lemma 1: The non-leading eigenvalue will also decline as R increase, because “slope” is then negative and the term, $(\lambda_N - \frac{\lambda_{mA} \chi_{mA} + \lambda_{ma} \chi_{mA}}{\chi_{mA} + \chi_{mA}})$, will also be negative (because the non-leading eigenvalue must fall below $\frac{\lambda_{mA} \chi_{mA} + \lambda_{ma} \chi_{mA}}{\chi_{mA} + \chi_{mA}}$ is the quadratic is negative at that point).

Proof of Lemma 2: When both the haplotype-only growth rates (λ_{mA} and λ_{ma}) are less than 1, then $-\frac{b}{2} = \frac{(\lambda_{mA} - \chi_{mA}) + (\lambda_{ma} - \chi_{ma})}{2}$ must be less than one (recall that the χ terms are positive) and $1 + b + c$ cannot be negative because $(\lambda_{mA} - 1)(\lambda_{ma} - 1) < \chi_{ma}(\lambda_{mA} - 1) + \chi_{mA}(\lambda_{ma} - 1)$ is false (left hand side is positive while right hand side is negative). Thus, the neo-SD never invades when neither haplotype can grow in the absence of recombination ($R=0$).

Check: Can invasion occur?

```
Simplify[Reduce[
  {(\lambda_{mA} - \chi_{mA} + \lambda_{ma} - \chi_{ma})/2 > 1 (* Slope condition met: f'(1) < 0 *),
   || \chi_{ma}(\lambda_{mA} - 1) + \chi_{mA}(\lambda_{ma} - 1) > (\lambda_{mA} - 1)(\lambda_{ma} - 1),
   (* or intercept condition met: f(1) > 0 *)}]] ,
  {\lambda_{mA} < 1, \lambda_{ma} < 1, 0 < \chi_{mA}, 0 < \chi_{ma}}]
```

False

Proof of Lemma 3: When both the haplotype-only growth rates (λ_{mA} and λ_{ma}) are greater than 1, we consider two sub-cases:

Case 1: Both haplotype-only growth rates are so large that the average of $(\lambda_{mA} - \chi_{mA})$ and $(\lambda_{ma} - \chi_{ma})$ is also greater than one. Then we are guaranteed that both roots of $f(\lambda)$ are greater than one regardless of R since $-\frac{b}{2} = \frac{(\lambda_{mA} - \chi_{mA}) + (\lambda_{ma} - \chi_{ma})}{2} > 1$.

Case 2: If, however, $-\frac{b}{2} < 1$, then invasion will still occur if both λ_{mA} and λ_{ma} are greater than one.

This is because the intercept can be written as:

$$- [(\lambda_{mA} - 1) (2 + b) + (\lambda_{mA} - 1)^2 + (\lambda_{mA} - \lambda_{mA}) \chi_{mA}]$$

and equivalently as:

$$- [(\lambda_{mA} - 1) (2 + b) + (\lambda_{mA} - 1)^2 + (\lambda_{mA} - \lambda_{mA}) \chi_{mA}]$$

Thus, if $\lambda_{mA} \geq \lambda_{mA}$, we choose the first way to write the intercept, which is clearly negative (all terms in square brackets are positive given that both λ_{mA} and λ_{mA} are greater than one and $-\frac{b}{2} < 1$). If if $\lambda_{mA} \geq \lambda_{mA}$, we choose the second way to write the intercept, which is also clearly negative. Either way, invasion is guaranteed regardless of R.

```
- ((\lambda_{mA} - 1) (2 + b) + (\lambda_{mA} - 1)^2 + (\lambda_{mA} - \lambda_{mA}) \chi_{mA});  
% - (1 + b + c) /. b -> - ((\lambda_{mA} - \chi_{mA}) + (\lambda_{mA} - \chi_{mA})) /.  
c -> (\lambda_{mA} - \chi_{mA}) (\lambda_{mA} - \chi_{mA}) - \chi_{mA} \chi_{mA} // Factor
```

0

```
- ((\lambda_{mA} - 1) (2 + b) + (\lambda_{mA} - 1)^2 + (\lambda_{mA} - \lambda_{mA}) \chi_{mA});  
% - (1 + b + c) /. b -> - ((\lambda_{mA} - \chi_{mA}) + (\lambda_{mA} - \chi_{mA})) /.  
c -> (\lambda_{mA} - \chi_{mA}) (\lambda_{mA} - \chi_{mA}) - \chi_{mA} \chi_{mA} // Factor
```

0

Check: Can invasion occur?

```
Simplify[Reduce[  
{\frac{\lambda_{mA} - \chi_{mA} + \lambda_{mA} - \chi_{mA}}{2} > 1 (* Slope condition met: f'(1) < 0 *),  
|| \chi_{mA} (\lambda_{mA} - 1) + \chi_{mA} (\lambda_{mA} - 1) > (\lambda_{mA} - 1) (\lambda_{mA} - 1)  
(* or intercept condition met: f(1) > 0 *)}],  
{\lambda_{mA} > 1, \lambda_{mA} > 1, 0 < \chi_{mA}, 0 < \chi_{mA}}]
```

True

Proof of Lemma 4: When only one of the haplotype-only growth rates (λ_{mA} and λ_{mA}) is greater than 1, we first note that stability is determined solely by the intercept of $f(\lambda)$ at $\lambda = 1$, because the slope at $\lambda = 1$ is immaterial. This is because Lemma 1 and its corollary imply that both roots of $f(\lambda)$ decline as R rises. Thus, if only one root is greater than 1 at $R = 0$ and if both roots decline with R, then we can never have both roots greater than one. With only one root possibly greater than 1, this root will be greater than one if and only if $f(1)$ is negative.

Thus, when only one of λ_{mA} and λ_{mA} is greater than one, invasion will occur according to the intercept condition:

$$(\lambda_{mA}-1)(\lambda_{mA}-1) < \chi_{mA}(\lambda_{mA}-1)+\chi_{mA}(\lambda_{mA}-1)$$

[dividing both sides by $(\lambda_{mA}-1)(\lambda_{mA}-1)$ and switching the inequality because this is negative]:

$$1 > \frac{\chi_{ma}}{\lambda_{ma}-1} + \frac{\chi_{mA}}{\lambda_{mA}-1}$$

which is equation (1). One of the terms on the right is positive and one negative. Because the χ are both positive and proportional to R , the condition is always satisfied when $R = 0$ (right-hand side is zero).

In words, we need the amount of recombination off the growing background, weighted by the growth rate of the growing haplotype, to be small enough relative to the amount of recombination off the shrinking haplotype, weighted by its rate of decline.

Check: Can invasion occur if equation (1) is satisfied?

```
Simplify[Reduce[
{(\lambda_{mA} - \chi_{mA} + \lambda_{ma} - \chi_{ma})/2 > 1 (* Slope condition met: f'(1) < 0 *),
 || \chi_{ma} (\lambda_{mA} - 1) + \chi_{mA} (\lambda_{ma} - 1) > (\lambda_{mA} - 1) (\lambda_{ma} - 1)
 (* or intercept condition met: f(1) > 0 *)},
 {\lambda_{mA} > 1, \lambda_{ma} < 1, 0 < \chi_{mA}, 0 < \chi_{ma}, 1 > \frac{\chi_{ma}}{\lambda_{ma} - 1} + \frac{\chi_{mA}}{\lambda_{mA} - 1} }]
True
```

Tight linkage

neo-Y equilA ($r \sim 0$)

From the above conditions, invasion requires that at least one of the no recombination eigenvalues is greater than one

```
{\lambda_{mA0}, \lambda_{ma0}} /. reverse /. equilAO // Simplify
{ (wAm (Faa Maa MAA waf wam + 4 FAA MAAa2 wAf wAm \alpha m2 +
FAa (Maa wam + 2 MAa wAm \alpha m) (-MAA wAf \alpha f + 2 MAa waf (-1 + \alpha f) \alpha m)) /
(wam (FAa (Maa waf (-1 + \alpha f) + 2 MAa wAf \alpha f (-1 + \alpha m)) (Maa wam + 2 MAa wAm \alpha m) +
2 Maa MAa (-Faa waf wam (-1 + \alpha m) + FAA wAf wAm \alpha m))), 1}
```

The first of these can be written as (1-factor) where factor is positive under the stability condition, demonstrating that a neo-Y cannot invade a system that already has tight linkage:

```

1 -

$$\frac{\left( \frac{Maa (1-pXf) waf (pXf wAf wam (pXm (1-\alpha m) + \alpha m (1-pXm)) - pXm (1-pXf) waf wAm \alpha m)}{(1-pXm) pXf^2 wAf^2 wAm \alpha m} - MAA \right) wAm wAf \alpha m pXf (1 - pXm)}{Maa (1 - pXf) waf wam (pXm (1 - \alpha m) + \alpha m (1 - pXm))} ;$$

(%[[1]] - % /. subA1 /. subA2 // Factor
0
```

Confirming that invasion is not possible:

```
Reduce[
{((λmA0 > 1 || λma0 > 1) /. reverse /. equilA0 // Simplify) && (stabcondA /. equilA0)}]
False
```

neo-Y equilB ($r \sim 0$)

From the above conditions, invasion requires that at least one of the no recombination eigenvalues is greater than one

```
{λmA0, λma0} /. reverse /. equilB0 // Simplify
```

$$\left\{ \frac{\text{MAA wAm}}{2 \text{MAa wam} - 2 \text{MAa wam} \alpha m}, 1 \right\}$$

This equilibrium is internally stable only if $\frac{\text{MAA wAm}}{2 \text{MAa wam} (1-\alpha m)} < 1$, so a neo-Y can be neutral at best and will not invade.

Confirming:

This is not possible given the stability conditions for equilB

```
Reduce[{((λmA0 > 1 || λma0 > 1) /. reverse /. equilB0 // Simplify) && stabcondB}]
False
```

neo-W equilA ($r \sim 0$)

The four terms that define the characteristic polynomial are

```
{λmA1, λma1, xmA1, xma1} /. reverse /. pAveM → q pYm + (1 - q) pXm /. equilA0 // Simplify
{ (Faa FAA Maa MAA waf wam wAm αm +
  2 Faa2 wam αf (Maa waf (-1 + αf) + MAA wAf αf (-1 + αm)) (Maa wam + 2 MAA wAm αm) +
  Faa MAA αf (-2 Faa Maa waf wam2 (-1 + αm) + FAA wAf wAm αm (3 Maa wam - 2 MAA wAm αm)) ) /
  (waf (2 Faa FAA Maa MAA wAm αm + Faa2 (-1 + αf) αf (Maa wam + 2 MAA wAm αm)2) ) ,
  (-Faa2 Maa MAA waf wam2 (-1 + αm) +
  2 Faa2 MAA wAf wAm (-1 + αf) αf αm (Maa wam + 2 MAA wAm αm) +
  Faa wam (2 FAA Maa MAA wAf wAm αm + Faa (Maa2 waf wam (-1 + αf) +
  Maa MAa wAf wam αf (-1 + αm) + 2 MAA2 wAf wAm αf (-1 + αm) αm)) ) /
  (wAf (2 Faa FAA Maa MAA wAm αm + Faa2 (-1 + αf) αf (Maa wam + 2 MAA wAm αm)2) ) ,
  (2 Faa R wam αf (Faa (Maa waf (-1 + αf) + MAA wAf αf (-1 + αm)) (Maa wam + 2 MAA wAm αm) +
  Maa MAa (-Faa waf wam (-1 + αm) + 2 FAA wAf wAm αm)) ) /
  (waf (2 Faa FAA Maa MAA wAm αm + Faa2 (-1 + αf) αf (Maa wam + 2 MAA wAm αm)2) ) ,
  2 Faa MAA R wAm (-1 + αf) αm (-Faa Maa waf wam + Faa wAf αf (Maa wam + 2 MAA wAm αm)) ) /
  wAf (2 Faa FAA Maa MAA wAm αm + Faa2 (-1 + αf) αf (Maa wam + 2 MAA wAm αm)2) }
```

As shown in the equilibria section above, we can write equilA as

$$\text{equilAOsimp} = \left\{ q \rightarrow \left(1 - \alpha Dm \frac{MAa \phi}{MAa \phi + Maa \psi} \right) / 2, \right.$$

$$pXf \rightarrow \frac{waf \phi}{waf \phi + wAf \psi}, pXm \rightarrow \frac{(1 + \alpha Dm) MAa \phi}{(1 + \alpha Dm) MAa \phi + Maa \psi}, pYm \rightarrow 0 \};$$

Define 3 new terms a,b,c

$$\text{trySub2} = \{ (waf \phi + wAf \psi) \rightarrow a, (MAa (-1 + \alpha Dm) \phi - 2 Maa \psi) \rightarrow -c,$$

$$FAA MAa wAm (1 + \alpha Dm) \phi^2 + \psi (FAa (Maa wam + MAa wAm (1 + \alpha Dm)) \phi + Faa Maa wam \psi) \rightarrow b \};$$

$$\text{trySub2back} = \{ a \rightarrow (waf \phi + wAf \psi), c \rightarrow -(MAa (-1 + \alpha Dm) \phi - 2 Maa \psi),$$

$$b \rightarrow FAA MAa wAm (1 + \alpha Dm) \phi^2 + \psi (FAa (Maa wam + MAa wAm (1 + \alpha Dm)) \phi + Faa Maa wam \psi) \};$$

The four terms in the characteristic polynomial can then be written

$$\text{equilAOterms} =$$

$$\text{Simplify}[\{\lambda m A1, \lambda m a1, \chi m A1, \chi m a1\} /. \alpha m \rightarrow (\alpha Dm + 1) / 2 /. \alpha f \rightarrow (\alpha Df + 1) / 2 /. \text{reverse} /.$$

$$pAveM \rightarrow q pYm + (1 - q) pXm /. \text{equilAOsimp} /. \text{trySub2}$$

$$\left\{ \frac{a (c FAa wam (1 + \alpha Df) + FAA MAa wAm (1 + \alpha Dm) \phi)}{2 b waf}, \right.$$

$$- \frac{a (-c Faa wam + FAa MAa wAm (-1 + \alpha Df) (1 + \alpha Dm) \phi)}{2 b wAf},$$

$$\left. \frac{a c FAa R wam (1 + \alpha Df)}{2 b waf}, - \frac{a FAa MAa R wAm (-1 + \alpha Df) (1 + \alpha Dm) \phi}{2 b wAf} \right\}$$

where

$$\text{freqMale} == \frac{wam (c - Maa \psi)}{wam c + wAm MAa (1 + \alpha Dm) \phi} /. \text{equilAOsimp} /. \text{trySub2back} // \text{Simplify}$$

$$\text{freqFemale} == \frac{wam Maa \psi + wAm MAa (1 + \alpha Dm) \phi}{wam c + wAm MAa (1 + \alpha Dm) \phi} /. \text{equilAOsimp} /. \text{trySub2back} // \text{Simplify}$$

$$\left(\frac{1}{2 \text{freqFemale}} == \frac{wam c + wAm MAa (1 + \alpha Dm) \phi}{2 (wam Maa \psi + wAm MAa (1 + \alpha Dm) \phi)} \right) /. \text{equilAOsimp} /. \text{trySub2back} //$$

$$\text{Simplify}$$

True

True

True

In the absence of haploid selection the four terms are

```

equilAOtermsNoHap =
equilAOterms /. trysub2back(*/.trysub*) /. αDm → 2 αm - 1 /. αDf → 2 αf - 1 /. waf → 1 /.
wAf → 1 /. wam → 1 /. wAm → 1 / . αm → 1 / 2 /. αf → 1 / 2 // Simplify
{ (φ + ψ) (FAa MAa φ + FAA MAa φ + 2 FAa Maa ψ)
2 (FAA MAa φ2 + ψ (FAa (Maa + MAa) φ + Faa Maa ψ)) ,
(φ + ψ) (Faa MAa φ + FAa MAa φ + 2 Faa Maa ψ)
2 (FAA MAa φ2 + ψ (FAa (Maa + MAa) φ + Faa Maa ψ)) ,
FAa R (φ + ψ) (MAa φ + 2 Maa ψ)
2 (FAA MAa φ2 + ψ (FAa (Maa + MAa) φ + Faa Maa ψ)) ,
FAa MAa R φ (φ + ψ)
2 (FAA MAa φ2 + ψ (FAa (Maa + MAa) φ + Faa Maa ψ)) }

```

Interpretation:

A neo-W carrying the A allele would have mean representation in the pool of female gametes (accounting for meiotic drive) of:

$$\begin{aligned}
& \left(\frac{\text{wAf}}{\text{wbarHapFemale}} pXm (1 - q) - \frac{\text{wAm}}{\text{wbarHapMale}} + \frac{\alpha f}{1/2} \frac{\text{wAf}}{\text{wbarHapFemale}} \right. \\
& (1 - pXm) (1 - q) \frac{\text{wam}}{\text{wbarHapMale}} + \frac{\text{wAf}}{\text{wbarHapFemale}} pYm q \frac{\text{wAm}}{\text{wbarHapMale}} + \\
& \left. \frac{\alpha f}{1/2} \frac{\text{wAf}}{\text{wbarHapFemale}} (1 - pYm) q \frac{\text{wam}}{\text{wbarHapMale}} \right) / . \text{SUBS} / . \text{subequil} // \text{Factor} \\
& (\text{wAf} (-FAA pXm wAm + FAA pXm q wAm - FAA pYm q wAm - 2 FAa wam αf + 2 FAa pXm wam αf - \\
& 2 FAa pXm q wam αf + 2 FAa pYm q wam αf)) / ((waf - pXf waf + pXf wAf) \\
& (-wam + pXm wam - pXm q wam + pYm q wam - pXm wAm + pXm q wAm - pYm q wAm))
\end{aligned}$$

[Note that this is summed over all gametes donated by males because the W is assumed dominant and always produces females.]

This divided by the mean fitness of the wildtype M bearing females (meanF) divided by 2*freqfemale gives the leading eigenvalue in this case:

```

(% / wbarDipFemale)
2 freqfemale / . SUBS / . subequil // Factor;
% - λmA1 / . freqfemale → 1 - ξ / . SUBS / . reverse / .
pAveM → q pYm + (1 - q) pXm / . equilAO // Factor
0

```

The 2 freqfemale in the denominator describes a sex ratio advantage of being in the rarer sex.

A neo-W carrying the a allele would have mean representation in the pool of female gametes (accounting for meiotic drive) of:

$$\begin{aligned}
& \left(\frac{1 - \alpha f}{1/2} F_{AA} \frac{w_{AF}}{w_{barHapFemale}} \left(p_{Xm} (1 - q) \frac{w_{Am}}{w_{barHapMale}} \right) + F_{AA} \frac{w_{AF}}{w_{barHapFemale}} \right. \\
& \left. \left((1 - p_{Xm}) (1 - q) \frac{w_{Am}}{w_{barHapMale}} \right) + \frac{1 - \alpha f}{1/2} F_{AA} \frac{w_{AF}}{w_{barHapFemale}} \left(p_{Ym} q \frac{w_{Am}}{w_{barHapMale}} \right) + \right. \\
& \left. F_{AA} \frac{w_{AF}}{w_{barHapFemale}} \left((1 - p_{Ym}) q \frac{w_{Am}}{w_{barHapMale}} \right) \right) / . \text{SUBS} / . \text{subequil} // \text{Factor} \\
& - (w_{AF} (F_{AA} w_{Am} - F_{AA} p_{Xm} w_{Am} + F_{AA} p_{Xm} q w_{Am} - F_{AA} p_{Ym} q w_{Am} + 2 F_{AA} p_{Xm} w_{Am} - 2 F_{AA} p_{Xm} q w_{Am} + \\
& 2 F_{AA} p_{Ym} q w_{Am} - 2 F_{AA} p_{Xm} w_{Am} \alpha f + 2 F_{AA} p_{Xm} q w_{Am} \alpha f - 2 F_{AA} p_{Ym} q w_{Am} \alpha f) / \\
& ((-w_{AF} + p_{Xf} w_{AF} - p_{Xf} w_{AF}) (w_{Am} - p_{Xm} w_{Am} + p_{Xm} q w_{Am} - p_{Ym} q w_{Am} + \\
& p_{Xm} w_{Am} - p_{Xm} q w_{Am} + p_{Ym} q w_{Am})))
\end{aligned}$$

This divided by the mean fitness of the wildtype M bearing females (meanF) divided by 2*freqfemale gives the leading eigenvalue in this case:

$$\begin{aligned}
& \frac{(\% / w_{barDipFemale})}{2 \text{ freqfemale}} // \text{Factor}; \\
& \% - \lambda_{M1} / . \text{freqfemale} \rightarrow 1 - \xi / . \text{SUBS} / . \text{reverse} / . \text{pAveM} \rightarrow q p_{Ym} + (1 - q) p_{Xm} / . \text{equilA0} // \text{Factor} \\
& 0
\end{aligned}$$

It is difficult for the neo-W to spread with only sexually antagonistic selection because the neo-W often makes daughters with the Y-a haplotype, increasing the flow of "a" alleles (that are fixed on the Y and in that sense male-beneficial) into the daughters. But if $\frac{\text{fitness}[W-a]/\text{meandipF}}{2 \text{ freqfemale}} > 1$ or if $\frac{\text{fitness}[W-A]/\text{meandipF}}{2 \text{ freqfemale}} > 1$ then invasion is possible.

R near 0:

Consequently, when R = 0 (the χ terms absent), the neo-W-A haplotype spreads when the following is greater than 0

$$\begin{aligned}
& \text{Collect}[\text{Numerator}[\text{equilA0termsNoHap}[[1]]] - \text{Denominator}[\text{equilA0termsNoHap}[[1]]], \\
& \{Maa, MAa\}, \text{Simplify}] \\
& (F_{AA} - F_{AA}) Maa \phi (\phi - \psi) + 2 (-F_{AA} + F_{AA}) Maa \psi^2
\end{aligned}$$

where $\phi > 0$ and $\psi > 0$ for equilA to be valid and stable and $\phi - \psi$ in the absence of haploid selection is

$$\phi - \psi / . \text{trySub} / . \alpha D_m \rightarrow 2 \alpha m - 1 / . \alpha D_f \rightarrow 2 \alpha f - 1 / . w_{AF} \rightarrow 1 / . w_{Am} \rightarrow 1 / . w_{Am} \rightarrow 1 / .$$

$$\alpha m \rightarrow 1/2 / . \alpha f \rightarrow 1/2$$

$$-F_{AA} Maa + F_{AA} MAa$$

Thus the neo-W-A haplotype can spread from equilA when sufficiently tightly linked under purely sexually antagonistic selection, e.g.,

```

Reduce[
{stabcondA, equilAOtermsNoHap[[1]] > 1, Faa < FAa < FAA, MAA < MAa < Maa} /. trysub /.
equilAO /. αDm → 2 αm - 1 /. αDf → 2 αf - 1 /. waf → 1 /.
wAf → 1 /. wam → 1 /. wAm → 1 /. αm → 1 / 2 /. αf → 1 / 2]

0 ≤ Rf ≤  $\frac{1}{2}$  && 0 ≤ Rm ≤  $\frac{1}{2}$  && MAa > 0 && Maa > MAa &&
FAA > 0 &&  $\left( \left( \frac{2 \text{FAA MAa}}{\text{Maa} + \text{MAa}} < \text{FAa} < \frac{3 \text{FAA Maa MAa} - \text{FAA MAa}^2}{\text{Maa}^2 + \text{Maa MAa}} \&\&$ 
 $\frac{\text{FAa}^2 \text{Maa}^2 + \text{FAa}^2 \text{Maa MAa} - 3 \text{FAa FAA Maa MAa} + \text{FAa FAA MAa}^2}{2 \text{FAa Maa MAa} - 2 \text{FAA Maa MAa}} <$ 
 $\text{FAa} \frac{\text{FAa Maa} + \text{FAa MAa}}{2 \text{Maa}} \&\& 0 < \text{MAA} < \text{MAa} \right) \parallel$ 
 $\left( \frac{3 \text{FAA Maa MAa} - \text{FAA MAa}^2}{\text{Maa}^2 + \text{Maa MAa}} \leq \text{FAa} < \text{FAA} \&\& 0 < \text{Faa} < \frac{\text{FAa Maa} + \text{FAa MAa}}{2 \text{Maa}} \&\& 0 < \text{MAA} < \text{MAa} \right)$ 

```

for example

```

Reduce[{stabcondA, validcondA, equilAOtermsNoHap[[1]] > 1, Faa < FAa < FAA,
MAA < MAa < Maa} /. trysub /. equilAO /. αDm → 2 αm - 1 /.
αDf → 2 αf - 1 /. waf → 1 /. wAf → 1 /. wam → 1 /. wAm → 1 /.
αm → 1 / 2 /. αf → 1 / 2 /. Faa → 9 / 10 /. FAa → 1 /.
FAA → 11 / 10 /. MAA → 9 / 10 /. MAa → 1 /. Rf → 0 /. Rm → 0] // N

```

1.21362 < Maa < 1.25

Similarly, the neo-W-a haplotype thus spreads when the following is greater than 0

```

Collect[Numerator[equilAOtermsNoHap[[2]]] -
Denominator[equilAOtermsNoHap[[2]]], {ϕ}, Simplify]
(Faa + FAa - 2 FAA) MAa ϕ² + (Faa - FAa) (2 Maa + MAa) ϕ ψ

```

but this is not possible with purely sexually antagonistic selection

```

Reduce[
{stabcondA, equilAOtermsNoHap[[2]] > 1, Faa < FAa < FAA, MAA < MAa < Maa} /. trysub /.
equilAO /. αDm → 2 αm - 1 /. αDf → 2 αf - 1 /. waf → 1 /.
wAf → 1 /. wam → 1 /. wAm → 1 /. αm → 1 / 2 /. αf → 1 / 2]

```

False

Other forms of selection allow the neo-W to invade an internally stable equilibrium, even without haploid selection:

```

Simplify[
  Reduce[{stabcondA, ((λm1 > 1) || (λma1 > 1))} /. reverse /. pAveM → q pYm + (1 - q) pXm /.
    equilAO
   /. nohap /. Rm → 0 /. Rf → 0]
]

FAa > 0 && MAa > 0 && 0 < MAA &&
MAA < 
$$\frac{2 (Faa \text{Maa} + FAA (\text{Maa} - \text{MAa})) \text{MAa} - Faa \text{Maa} (\text{Maa} + \text{MAa})}{2 \text{Faa} \text{Maa} - Faa (\text{Maa} + \text{MAa})} \&& \left( \begin{array}{l} 0 < \text{Faa} \&& 0 < \text{FAA} \&& \\ \left( \begin{array}{l} \text{Maa} == \text{MAa} \&& \text{FAa} + \frac{\text{FAa} \text{Maa}}{\text{MAa}} > 2 \text{FAA} \&& 2 \text{Faa} < \text{FAa} + \frac{\text{FAa} \text{Maa}}{\text{MAa}} \end{array} \right) \mid\mid \left( \begin{array}{l} \text{Maa} > \text{MAa} \&& \\ \text{FAa} < \frac{\text{FAa} (\text{Maa} + \text{MAa})}{2 \text{Maa}} \&& \text{FAa} + \frac{\text{FAa} \text{Maa} (-2 \text{Faa} \text{MAa} + \text{FAa} (\text{Maa} + \text{MAa}))}{\text{MAa} (2 \text{Faa} \text{Maa} + \text{FAa} (-3 \text{Maa} + \text{MAa}))} < 0 \end{array} \right) \end{array} \right) \mid\mid \\ \left( \begin{array}{l} \text{Maa} < \text{MAa} \&& \left( \begin{array}{l} \text{Faa} > 0 \&& \text{FAA} > 0 \&& \text{FAa} + \frac{\text{FAa} \text{Maa}}{\text{MAa}} > 2 \text{FAA} \&& 0 < \text{Maa} \&& \\ 2 \text{Faa} \leq \text{FAa} + \frac{\text{FAa} \text{Maa}}{\text{MAa}} \end{array} \right) \mid\mid \left( \begin{array}{l} \text{Maa} > 0 \&& 2 \text{Faa} \text{Maa} < \text{FAa} (\text{Maa} + \text{MAa}) \&& \\ 2 \text{FAA} \text{MAa} < \text{FAa} (\text{Maa} + \text{MAa}) \&& 2 (\text{Faa} + \text{FAA}) \text{Maa} \text{MAa} < \\ 2 \text{FAA} \text{MAa}^2 + \text{FAa} \text{Maa} (\text{Maa} + \text{MAa}) \&& \text{FAa} (\text{Maa} + \text{MAa}) < 2 \text{Faa} \text{MAa} \end{array} \right) \end{array} \right) \right)$$


```

such as heterozygote advantage in males with directional selection in females:

```

Simplify[Reduce[{stabcondA, λm1 > 1, (((0 < FAA < FAa < Faa) && (Maa < MAa > MAA > 0))} /.
  reverse /. pAveM → q pYm + (1 - q) pXm /. equilAO /. nohap /. Rm → 0 /. Rf → 0]]
Simplify[Reduce[{stabcondA, λma1 > 1, (((0 < FAA < FAa < Faa) && (Maa < MAa > MAA > 0))} /.
  reverse /. pAveM → q pYm + (1 - q) pXm /. equilAO /. nohap /. Rm → 0 /. Rf → 0]]

MAa > 0 && 0 < Maa < MAa && FAA > 0 && 
$$\frac{2 \text{FAA} \text{MAa}}{\text{Maa} + \text{MAa}} < \text{FAa} < \frac{\text{FAA} \text{MAa}}{\text{Maa}} \&&$$

FAa < Faa < 
$$\frac{\text{FAa} (\text{FAA} \text{MAa} (-3 \text{Maa} + \text{MAa}) + \text{FAa} \text{Maa} (\text{Maa} + \text{MAa}))}{2 (\text{FAa} - \text{FAA}) \text{Maa} \text{MAa}} \&&$$

0 < MAA < 
$$\frac{2 (\text{Faa} \text{Maa} + \text{FAA} (\text{Maa} - \text{MAa})) \text{MAa} - \text{FAa} \text{Maa} (\text{Maa} + \text{MAa})}{2 \text{Faa} \text{Maa} - \text{FAa} (\text{Maa} + \text{MAa})}$$


MAa > 0 && Maa > 0 && FAa > 0 && Faa > 0 && MAA > 0 &&
Maa < MAa && FAa < Faa && 2 Faa Maa < FAa (Maa + MAa) &&
2 (Faa + FAA) Maa MAa < 2 FAA MAa^2 + FAa Maa (Maa + MAa) && 2 FAA MAa < FAa (Maa + MAa) &&
2 (FAA (Maa - MAa) MAa + Faa Maa (MAa - MAA)) < FAa (Maa + MAa) (Maa - MAA)

```

e.g., in the blue region of figure 2d in Otto 2014 JEB (p 1436)

```

Reduce[{\%} /. FAa → 1.05 /. FAa → 1 /. FAA → 0.6 /. MAa → 1 /. MAA → 0.5 /. Maa → 0.5] // N
True

```

Underdominance in females with directional selection in males does not work (it does for equilB, see next section):

```

Simplify[Reduce[{stabcondA, λm1 > 1,
    (((FAA > FAa < Faa > 0) && ((0 < Maa < MAa < MAA) || (Maa > MAa > MAA > 0)))}
    /. reverse /. pAveM → q pYm + (1 - q) pXm /. equilAO /. nohap /. Rm → 0 /. Rf → 0]]
Simplify[Reduce[{stabcondA, λmal > 1,
    (((FAA > FAa < Faa > 0) && ((0 < Maa < MAa < MAA) || (Maa > MAa > MAA > 0)))}
    /. reverse /. pAveM → q pYm + (1 - q) pXm /. equilAO /. nohap /. Rm → 0 /. Rf → 0]]

```

False

False

Heterozygote advantage in females with directional selection in males also works (only $\lambda m A_1$ can be greater than 1):

```

Simplify[Reduce[{stabcondA, λm1 > 1,
    (((FAA < FAa > Faa > 0) && ((0 < Maa < MAa < MAA) || (Maa > MAa > MAA > 0)))}
    /. reverse /. pAveM → q pYm + (1 - q) pXm /. equilA0 /. nohap /. Rm → 0 /. Rf → 0]
Simplify[Reduce[{stabcondA, λm1 > 1,
    (((FAA < FAa > Faa > 0) && ((0 < Maa < MAa < MAA) || (Maa > MAa > MAA > 0)))}
    /. reverse /. pAveM → q pYm + (1 - q) pXm /. equilA0 /. nohap /. Rm → 0 /. Rf → 0]]
MAa > 0 && Maa > MAa && FAA > 0 && FAa > FAA &&
Faa > 0 && 2 Faa < FAa +  $\frac{FAa \cdot MAa}{Maa}$  && MAA > 0 && MAa > MAA

```

False

Haplod selection increases the types of selection in the diploid phase that are consistent with neo-W invasion, including sexual antagonism in diploids:

(*Including meiotic drive in males*)

```
Simplify[
Reduce[{stabcondA, ((λmA1 > 1) || (λma1 > 1)), (((FAA > FAa > Faa > 0) && (Maa > MAa >
MAA > 0)) || ((0 < FAA < FAa < Faa) && (0 < Maa < MAa < MAA))) } /.
reverse /. pAveM → q pYm + (1 - q) pXm /. equilA0 /. wam → 1 /. wAm → 1 /.
waf → 1 /. wAf → 1 /. αf → 1 / 2 /. αm → 4 / 10 /. Rm → 0 /. Rf → 0}]]
```

$\text{FAa} > 0 \ \&\& \ \text{MAa} > 0 \ \&\&$

$$\left(\left(0 < \text{Faa} \& \& 0 < \text{MAA} \& \& \text{Faa} < \frac{1}{10} \text{FAa} \left(5 + \frac{4 \text{ MAA}}{\text{Maa}} \right) \& \& \text{FAa} < \text{FAA} \& \& \text{MAA} < \text{MAa} \& \& \right. \right. \\ \left. \left(\left(\text{FAa} \left(4 + \frac{5 \text{ Maa}}{\text{MAa}} \right) > 8 \text{ FAA} \& \& \text{MAa} < \text{Maa} \& \& 5 \text{ Maa} \leq 8 \text{ MAa} \right) \mid \mid \left(5 \text{ Maa} > 8 \text{ MAa} \& \& \right. \right. \right. \\ \left. \left. \left. \text{FAa} \left(-60 \text{ Faa Maa MAa} + \text{FAa} \left(25 \text{ Maa}^2 + 30 \text{ Maa MAa} + 8 \text{ MAa}^2 \right) \right) \right. \right. \right. \\ \left. \left. \left. \text{FAA} + \frac{4 \text{ MAa} \left(10 \text{ Faa Maa} - 15 \text{ FAa Maa} + 4 \text{ FAa MAa} \right)}{4 \text{ MAa} \left(10 \text{ Faa Maa} - 15 \text{ FAa Maa} + 4 \text{ FAa MAa} \right)} < 0 \right) \right) \right) \mid$$

$$\left(\begin{array}{l} Faa > 0 \& Maa > 0 \& MAA > 0 \& FAa < Faa \& 5 Maa < 4 MAA \& \\ 10 Faa Maa < 5 FAa Maa + 4 FAa MAA \& Maa < MAA \& 8 FAA Maa < 5 FAa Maa + 4 FAa MAA \& \\ 2 Maa (5 Faa Maa + 20 FAA Maa + 6 FAa MAA) < 32 FAA MAA^2 + 5 FAa Maa (5 Maa + MAA) \& \\ 8 FAA (5 Maa - 4 MAA) MAA + 10 Faa Maa (6 MAA - 5 MAA) < \\ Faa (5 Maa + 4 MAA) (5 Maa + 2 MAA - 5 MAA) \end{array} \right)$$

```
(*Including meiotic drive in females*)
Simplify[
  Reduce[{stabcondA, ((λmAl > 1) || (λma1 > 1)), (((FAA > FAa > Faa > 0) && (Maa > MAa > MAA > 0)) || ((0 < FAA < FAa < Faa) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilAO /. wam → 1 /. wAm → 1 /.
    waf → 1 /. wAf → 1 /. af → 1 / 2 /. am → 4 / 10 /. Rm → 0 /. Rf → 0]
FAa > 0 && MAa > 0 &&

$$\left( \left( 0 < Faa \&& 0 < MAA \&& Faa < \frac{1}{10} FAa \left( 5 + \frac{4 MAa}{Maa} \right) \&& FAa < FAA \&& MAA < MAa \&& \left( \left( FAa \left( 4 + \frac{5 Maa}{MAa} \right) > 8 FAA \&& MAa < Maa \&& 5 Maa \leq 8 MAA \right) \mid\mid \left( 5 Maa > 8 MAa \&& FAa + \frac{FAa (-60 Faa Maa MAa + FAa (25 Maa^2 + 30 Maa MAa + 8 MAa^2))}{4 MAa (10 Faa Maa - 15 FAa Maa + 4 FAa MAa)} < 0 \right) \right) \mid\mid \left( Faa > 0 \&& Maa > 0 \&& MAA > 0 \&& FAa < Faa \&& 5 Maa < 4 MAa \&& 10 Faa Maa < 5 FAa Maa + 4 FAa MAa \&& MAa < MAA \&& 8 FAA MAa < 5 FAa Maa + 4 FAa MAa \&& 2 MAa (5 Faa Maa + 20 FAA Maa + 6 FAa MAa) < 32 FAa MAa^2 + 5 FAa Maa (5 Maa + MAa) \&& 8 FAA (5 Maa - 4 MAa) MAa + 10 Faa Maa (6 MAa - 5 MAA) < FAa (5 Maa + 4 MAa) (5 Maa + 2 MAa - 5 MAA) \right) \right)$$

```

and directional selection in diploids (ploidally antagonistic selection):

```
(*Including meiotic drive in males*)
Simplify[
  Reduce[{stabcondA, ((λmAl > 1) || (λma1 > 1)), (((0 < FAA < FAa < Faa) && (Maa > MAa > MAA > 0)) || ((FAA > FAa > Faa > 0) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilAO /.
    wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. af → 1 / 2 /. am → 4 / 10 /. Rm → 0 /.
    Rf → 0]
MAa > 0 &&  $\frac{4 MAa}{5} < Maa < MAa \&& FAa > 0 \&&$ 

$$0 < Faa < \frac{1}{10} FAa \left( 5 + \frac{4 MAa}{Maa} \right) \&& FAa < FAA \&& FAa \left( 4 + \frac{5 Maa}{MAa} \right) > 8 FAA \&& MAa < MAA < \frac{4 MAa (-15 Faa Maa - 10 FAA Maa + 8 FAA MAa) + FAa (25 Maa^2 + 30 Maa MAa + 8 MAa^2)}{-50 Faa Maa + 5 FAa (5 Maa + 4 MAa)}$$

```

```
(*Including meiotic drive in females*)
Simplify[
  Reduce[{\$stabcondA, ((λmA1 > 1) || (λma1 > 1)), (((0 < FAA < FAa < Faa) && (Maa > MAA >
    MAA > 0)) || ((FAA > FAa > Faa > 0) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilAO
    /. wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. af → 6 / 10 /. am → 1 / 2 /. Rm → 0 /.
    Rf → 0]
]

MAa > 0 && MAa <  $\frac{3 \text{MAa}}{2}$  && FAa > 0 &&
FAa < Faa <  $\frac{3 \text{FAa} (\text{Maa} + \text{MAa})}{5 \text{Maa}}$  && 0 < FAA <  $\frac{2 \text{FAa} (\text{Maa} + \text{MAa})}{5 \text{MAa}}$  && 0 < MAA < MAa
```

R near 1/2:

Because sexually antagonistic selection can allow invasion at $R = 0$, with $\lambda mA > 1$ (but $\lambda ma < 1$), we have to check condition (1) to determine if an unlinked neo-W can invade. At the end of this section, we prove:

Lemma 5: An unlinked neo-W ($R=1/2$) cannot invade a system with sexually antagonistic selection if selection is directional in each sex and haploid selection is absent.

Unlinked neo-Ws can invade in several other cases, however. We know that if both λmA and λma are greater than one, then invasion will occur for all R (as the leading eigenvalue is bounded between these two values, see “Invasion Conditions”). Scanning the above requirements, we can thus conclude that it is possible for unlinked neo-W chromosomes to invade with:

* heterozygote advantage in males with directional selection in females (no haploid selection)

Recalling that the eigenvalues must decrease with increasing R (see “Invasion Conditions”), we know that sexually antagonistic selection alone (with opposing directional selection in the two sexes) will not allow the spread of a neo-W at equilibrium B under any linkage conditions because both λmA and λma are less than one.

We also know that if both λmA and λma are greater than one, then invasion will occur for all R (as the leading eigenvalue is bounded between these two values, see “Invasion Conditions”). Scanning the above requirements, we can thus conclude that it is possible for unlinked neo-W chromosomes to invade with:

* heterozygote advantage in males with directional selection in females if haploid selection is absent.

Indeed, overdominance in males is required for both $\lambda mA > 1$ and $\lambda ma > 1$ in the absence of haploid selection:

```
(*Other forms of selection in males*) Simplify[
  Reduce[{stabcondA, ((λmA1 > 1) && (λma1 > 1)), (((Maa < MAa < MAA) || (Maa > MAa < MAA) || (Maa > MAa > MAA))) /. reverse /. pAveM → q pYm + (1 - q) pXm /. equilAO
  /. nohap /. Rm → 0 /. Rf → 0]
(*Overdominance in males*)
Simplify[Reduce[{stabcondA, ((λmA1 > 1) && (λma1 > 1)), (Maa < MAa > MAA)} /. reverse /.
  pAveM → q pYm + (1 - q) pXm /. equilAO
  /. nohap /. Rm → 0 /. Rf → 0]
False

FAa > 0 && MAa > 0 && 0 < Maa < MAa &&
0 < MAA <  $\frac{2 (FAa Maa + FAA (Maa - MAa)) MAa - FAa Maa (Maa + MAa)}{2 Fa a Maa - FAa (Maa + MAa)}$  &&

$$\left( \left( \begin{array}{l} FAA > 0 \& FAa + \frac{FAa Maa}{MAa} > 2 FAA \& Fa a > \frac{FAa Maa}{MAa} \& 2 Fa a \leq FAa + \frac{FAa Maa}{MAa} \end{array} \right) \right) \mid \mid
\left( \left( \begin{array}{l} 2 Fa a > FAa + \frac{FAa Maa}{MAa} \& 2 Fa a < FAa + \frac{FAa Maa}{MAa} \& \\
- \frac{FAa Maa (-2 Fa a MAa + FAa (Maa + MAa))}{MAa (2 Fa a Maa + FAa (-3 Maa + MAa))} < FAa < \frac{FAa (Maa + MAa)}{2 MAa} \end{array} \right) \right)$$

```

Various forms of haploid selection are also consistent with both $\lambda m A$ and $\lambda m a$ being greater than one (ensuring unlinked neo-W can invade). For example, meiotic drive for the Y linked allele in males skews the sex ratio towards males, allowing a neo-W chromosome to invade (which balances the sex ratio). Meiotic drive in females can also work, if balanced by the right type of diploid selection.

Haploid selection coupled for example with sexual antagonism in diploids:

```
(*Including meiotic drive in males*)
Simplify[
  Reduce[{stabcondA, ((λmA1 > 1) && (λma1 > 1)), (((FAA > FAa > Faa > 0) && (Maa > MAa >
    MAA > 0)) || ((0 < FAA < FAa < Faa) && (0 < Maa < MAa < MAA))) /. reverse /.
    pAveM → q pYm + (1 - q) pXm /. equilAO
  /. wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. αm → 4 / 10 /. αf → 1 / 2 /.
  Rm → 1 / 2 /. Rf → 1 / 2]]
FAa > 0 && MAa > 0 &&

$$\left( \left( \begin{array}{l} Maa > MAa \& FAa \left( 4 + \frac{5 Maa}{MAa} \right) > 8 FAA \& 3 MAa + \sqrt{57} \sqrt{MAa^2} > 10 Maa \& 0 < MAA \& & \\
Faa < \frac{1}{10} FAa \left( 5 + \frac{4 MAa}{Maa} \right) \& FAa < FAA \& \frac{5 FAa Maa}{6 MAa} < Faa \& MAA < MAa \end{array} \right) \right) \mid \mid
(Maa > 0 \& MAA > 0 \& FAa < Faa \& 5 Maa < 4 MAa \& 10 Faa Maa < 5 FAa Maa + 4 FAa MAa \& &
MAa < MAA \& 8 FAA MAa < 5 FAa Maa + 4 FAa MAa \& &
2 MAa (5 Faa Maa + 20 FAA Maa + 6 FAa MAa) < 32 FAA MAa^2 + 5 FAa Maa (5 Maa + MAa) \& &
8 FAA (5 Maa - 4 MAa) MAa + 10 Faa Maa (6 MAa - 5 MAA) < \\
FAa (5 Maa + 4 MAa) (5 Maa + 2 MAa - 5 MAA) \end{array} \right) \right)$$

```

but not if there is only meiotic drive in females

[NOT SHOWN GENERALLY, ALSO TRIED $\alpha f \rightarrow 1/10, 4/10, 49/100, 51/100, 6/10, 9/10$]:

```
(*Including meiotic drive in females*)
Simplify[
  Reduce[{stabcondA, ((λmA1 > 1) && (λma1 > 1)), (((FAA > FAa > Faa > 0) && (Maa > MAa >
    MAA > 0)) || ((0 < FAA < FAa < Faa) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilAO
    /. wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. xm → 1 / 2 /. af → 4 / 10 /. Rm → 0 /.
    Rf → 0]
]
```

False

Haploid selection (in either sex) also favors the spread of unlinking neo-W with directional selection in diploids (ploidally antagonistic selection):

```
(*Including meiotic drive in males*)
Simplify[
  Reduce[{stabcondA, ((λmA1 > 1) && (λma1 > 1)), (((0 < FAA < FAa < Faa) && (Maa > MAa >
    MAA > 0)) || ((FAA > FAa > Faa > 0) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilAO
    /. wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. xm → 4 / 10 /. af → 1 / 2 /. Rm → 0 /.
    Rf → 0]
]
```

$MAa > 0 \&& Maa > 0 \&& FAa > 0 \&& FAA > 0 \&& MAA > 0 \&& 4 MAa < 5 Maa \&&$
 $Maa < MAa \&& 5 FAa Maa < 6 Faa MAa \&& 10 Faa Maa < 5 FAa Maa + 4 FAa MAa \&&$
 $FAa < FAA \&& 8 FAA MAa < 5 FAa Maa + 4 FAa MAa \&& Maa < MAA \&&$
 $8 FAA (5 Maa - 4 MAa) MAa + 10 Faa Maa (6 MAa - 5 MAA) <$
 $FAa (5 Maa + 4 MAa) (5 Maa + 2 MAa - 5 MAA)$

(*Including meiotic drive in females*)

```
Simplify[
  Reduce[{stabcondA, ((λmA1 > 1) && (λma1 > 1)), (((0 < FAA < FAa < Faa) && (Maa > MAa >
    MAA > 0)) || ((FAA > FAa > Faa > 0) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilAO
    /. wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. xm → 1 / 2 /. af → 6 / 10 /. Rm → 0 /.
    Rf → 0]
]
```

$FAa > 0 \&& MAa > 0 \&& 0 < FAA < \frac{2 FAa (Maa + MAa)}{5 MAa} \&& 0 < MAA < MAa \&&$
 $Faa < \frac{3 FAa (Maa + MAa)}{5 Maa} \&& \left((FAa < Faa \&& MAa < Maa \&& 4 Maa \leq 5 MAa) \right) \&&$
 $\left(4 Maa > 5 MAa \&& 3 MAa + \sqrt{57} \sqrt{MAa^2} > 8 Maa \&& \frac{4 FAa Maa}{5 MAa} < Faa \right)$

Proof of Lemma 5: An unlinked neo-W ($R=1/2$) cannot invade a system with sexually antagonistic selection if selection is directional in each sex and haploid selection is absent.

We now return explicitly to the case where only one of λmA or λma is greater than one. In this case (see the “Invasion Condition” section), we showed that a neo-W will invade if and only if the intercept of the primary quadratic is negative:

```
(1 + b + c) /. b → -((λmA - χmA) + (λma - χma)) /.
c → (λmA - χmA) (λma - χma) - χmA χma // Factor
1 - λma - λmA + λma λmA + χma - λmA χma + χmA - λma χmA
```

Without haploid selection, this becomes:

```

interceptA = (1 + b + c) /. b -> -((λmA1 - χmA1) + (λma1 - χma1)) /.
  c -> (λmA1 - χmA1) (λma1 - χma1) - χmA1 χma1 /. reverse /.
  pAveM → q pYm + (1 - q) pXm /. equilAO /. nohap /. R → 1/2 // Factor
- ((Faa FAa - 2 Faa FAA + FAa FAA) MAa (-2 Faa Maa + FAa Maa + FAa MAa)
  (-2 FAa Maa2 + 2 Faa Maa MAa - FAa Maa MAa + FAa MAa2) ) /
  (2 (FAa2 Maa2 + 2 FAa2 Maa MAa - 4 Faa FAA Maa MAa + FAa2 MAa2)2)

```

In the following, we will use the equilibrium's validity and stability conditions to define the following two positive terms (see stabcondA without haploid selection):

$$\begin{aligned} pos1 &= \frac{FAa (Maa + MAa)}{2 MAa} - FAA; \\ pos2 &= \frac{FAa (Maa + MAa)}{2 Maa} - Faa; \end{aligned}$$

```

Simplify[
 Reduce[{stabcondA, ((0 < FAA < FAa < Faa) && (0 < Maa < MAa < MAA))} /. equilAO /. nohap],
 {Rf == 1/2, Rm == 1/2}]
Simplify[Reduce[{stabcondA, ((FAA > FAa > Faa > 0) && (Maa > MAa > MAA > 0))} /.
  equilAO /. nohap], {Rf == 1/2, Rm == 1/2}]
False

```

$$\begin{aligned} MAa > 0 \&\& Maa > MAa \&\& FAA > 0 \&\& FAa > \frac{2 FAA MAa}{Maa + MAa} \&\& \\ FAa < FAA \&\& Faa > 0 \&\& 2 Faa < FAa + \frac{FAa MAa}{Maa} \&\& MAA > 0 \&\& MAa > MAA \end{aligned}$$

With pure sexually antagonistic selection with directional selection in each sex, we must also have (FAa Maa-Faa MAa) and (Maa-MAa) positive at this equilibrium (which requires that "a" is favored in males and "A" in females).

Consequently, the intercept can be written as a strictly positive quantity when $R = 1/2$, and invasion of an unlinked neo-W is not possible with pure sexually antagonistic selection:

$$\begin{aligned} & \left(\left(2 Maa MAa pos2 (2 Maa (FAa Maa - Faa MAa) + FAa (Maa - MAa) MAa) \right. \right. \\ & \left. \left. \left(2 \frac{MAa}{(Maa + MAa)} Faa pos1 + 2 \frac{Maa}{(Maa + MAa)} FAA pos2 \right) \right) \right) / \\ & \left(2 (FAa^2 Maa^2 + 2 FAa^2 Maa MAa - 4 Faa FAA Maa MAa + FAa^2 MAa^2)^2 \right); \end{aligned}$$

```
% / interceptA // Factor
```

```
1
```

Corollary of proof: An unlinked neo-W ($R=1/2$) can only invade a system at equilibrium A if selection is overdominant in males when haploid selection is absent [Not true for equilibrium B, see below]

In the absence of haploid selection, overdominance in males is required for invasion of neo-W with $R=1/2$ in the case where only $\lambda mA > 1$ (directional selection or underdominance does not work):

```
(*Other forms of selection in males*)
Simplify[Reduce[{stabcondA, (2 Maa (FAa Maa - Faa MAa) + FAa (Maa - MAa) MAa) < 0,
    λm1 > 1, ((Maa < MAa < MAA) || (Maa > MAa < MAA) || (Maa > MAa > MAA))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilAO /. nohap /. Rm → 0 /. Rf → 0]]

(*Overdominance in males*)
Simplify[Reduce[{stabcondA, (2 Maa (FAa Maa - Faa MAa) + FAa (Maa - MAa) MAa) < 0,
    λm1 > 1, (Maa < MAa > MAA)} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilAO /. nohap /. Rm → 0 /. Rf → 0]]
False


$$\frac{2 (Faa Maa + FAA (Maa - MAa)) MAa - FAa Maa (Maa + MAa)}{2 Faa Maa - FAa (Maa + MAa)} \&&$$


$$\left( \left( 0 < FAA < \frac{FAa (Maa + MAa)}{2 Maa} \& 2 Faa \leq FAa + \frac{FAa Maa}{MAa} \&&$$


$$\left( \left( 2 Maa > MAa \& 2 Faa + \frac{FAa MAa}{Maa} > FAa + \frac{2 FAa Maa}{MAa} \& Maa < MAa \right) \mid\mid$$


$$(Faa > 0 \&& Maa > 0 \&& 2 Maa \leq MAa) \right) \mid\mid$$


$$\left( - \frac{FAa Maa (-2 Faa MAa + FAa (Maa + MAa))}{MAa (2 Faa Maa + FAa (-3 Maa + MAa))} < FAA < \frac{FAa (Maa + MAa)}{2 MAa} \&&$$


$$2 Faa < FAa + \frac{FAa MAa}{Maa} \&& (Maa < MAa \mid\mid 2 Maa \leq MAa) \&&$$


$$2 Faa > FAa + \frac{FAa Maa}{MAa} \&& (Maa > 0 \mid\mid 2 Maa > MAa) \right)$$

```

neo-W equilB ($r \sim 0$)

The four terms that define the characteristic polynomial are

$$\begin{aligned} &\{\lambda m1, \lambda ma1, \chi m1, \chi ma1\} / . \text{reverse} / . \text{pAveM} \rightarrow q pYm + (1 - q) pXm / . \text{equilB0} / / \text{Factor} \\ &\left\{ -\frac{-2 FAa wam \alpha f - FAA wAm \alpha m + 2 FAa wam \alpha f \alpha m}{2 FAA wAm \alpha m}, \right. \\ &\quad -\frac{waf (-Faa wam + Faa wam \alpha m - 2 FAa wAm \alpha m + 2 FAa wAm \alpha f \alpha m)}{2 FAA wAf wAm \alpha m}, \\ &\quad -\frac{FAa R wam \alpha f (-1 + \alpha m)}{FAA wAm \alpha m}, \left. -\frac{FAa R waf (-1 + \alpha f)}{FAA wAf} \right\} \end{aligned}$$

charpolyextB =

$$\begin{aligned} &\lambda^2 + b \lambda + c / . b \rightarrow -((\lambda m A - \chi m A) + (\lambda m a - \chi m a)) / . c \rightarrow (\lambda m A - \chi m A) (\lambda m a - \chi m a) - \\ &\quad \chi m A \chi m a / . \{\lambda m A \rightarrow \lambda m1, \chi m A \rightarrow \chi m1, \lambda m a \rightarrow \lambda m1, \chi m a \rightarrow \chi m1\} / . \\ &\quad \text{reverse} / . \text{pAveM} \rightarrow q pYm + (1 - q) pXm / . \text{equilB0} / / \text{Factor}; \end{aligned}$$

In the following, we will use the following to help interpret the results:

DEFINING A FEW TERMS TO INTERPRET THE ABOVE:

With A fixed in females and a in males (equilB0), the mean fitness in haploids is:

```
wbarHapFemale /. SUBS /. subequil /. equilB0 // Factor
```

wAf

```
wbarHapMale /. SUBS /. subequil /. equilB0 // Factor
```

wam - wam α_m + wAm α_m

The mean fitness in females times the frequency of females (i.e., the normalizing term in the recursions) is:

```
wbarF /. SUBS /. subequil /. equilB0 // Factor
```

$\frac{FAA \cdot wAm \cdot \alpha_m}{wam - wam \cdot \alpha_m + wAm \cdot \alpha_m}$

The frequency of diploid females is:

```
freqFemale /. SUBS /. subequil /. equilB0 // Factor
```

$\frac{wAm \cdot \alpha_m}{wam - wam \cdot \alpha_m + wAm \cdot \alpha_m}$

The mean fitness of diploid females:

```
wbarDipFemale /. equilB0 // Factor
```

FAA

R near 0:

If the neo-W is also closely linked (R near 0 but potentially $R > r$), the quadratic factors to give the two eigenvalues, λ_{mA1} and λ_{ma}

```
charpolyextB /. R → 0 /. equilB0 // Factor
```

$$\frac{1}{4 FAA^2 wAf wAm^2 \alpha_m^2} (-2 FAA wam \alpha_f - FAA wAm \alpha_m + 2 FAA wam \alpha_f \alpha_m + 2 FAA wAm \alpha_m \lambda) \\ (-Faa waf wam + Faa waf wam \alpha_m - 2 FAA waf wAm \alpha_m + 2 FAA waf wAm \alpha_f \alpha_m + 2 FAA wAf wAm \alpha_m \lambda)$$

```
λ /. Solve[% == 0, λ] // Factor
```

$$\left\{ \frac{2 FAA wam \alpha_f + FAA wAm \alpha_m - 2 FAA wam \alpha_f \alpha_m}{2 FAA wAm \alpha_m}, \right. \\ \left. - \frac{waf (-Faa wam + Faa wam \alpha_m - 2 FAA wAm \alpha_m + 2 FAA wAm \alpha_f \alpha_m)}{2 FAA wAf wAm \alpha_m} \right\}$$

Interpretation of the λ_{mA} and λ_{ma} terms:

A neo-W carrying the A allele would have mean representation in the pool of female gametes (accounting for meiotic drive) of:

[Note that this is summed over all gametes donated by males because the W is assumed dominant and always produces females.]

$$\begin{aligned}
 & \left(\frac{\text{FAA} \frac{\text{wAf}}{\text{wbarHapFemale}} pXm (1 - q)}{\text{wbarHapMale}} + \frac{\alpha f}{1/2} \text{FAa} \frac{\text{wAf}}{\text{wbarHapFemale}} (1 - pXm) (1 - q) \right. \\
 & \quad \left. + \frac{\text{wAm}}{\text{wbarHapMale}} + \text{FAA} \frac{\text{wAf}}{\text{wbarHapFemale}} pYm q \frac{\text{wAm}}{\text{wbarHapMale}} + \frac{\alpha f}{1/2} \text{FAa} \frac{\text{wAf}}{\text{wbarHapFemale}} \right. \\
 & \quad \left. (1 - pYm) q \frac{\text{wAm}}{\text{wbarHapMale}} \right) / . \text{SUBS} / . \text{subequil} / . \text{equilB0} // \text{Factor} \\
 & - 2 \text{FAa wam} \alpha f - \text{FAA wAm} \alpha m + 2 \text{FAa wam} \alpha f \alpha m \\
 & \hline
 & - \text{wam} + \text{wam} \alpha m - \text{wAm} \alpha m
 \end{aligned}$$

This divided by the mean fitness of the wildtype M bearing females ($\text{wbarDipFemale}=\text{FAA}$) divided by $2 * \text{freqfemale}$ gives the leading eigenvalue in this case:

$$\begin{aligned}
 & \frac{(\% / \text{wbarDipFemale})}{2 \text{ freqfemale}} / . \text{SUBS} / . \text{subequil} / . \text{equilB0} // \text{Factor}; \\
 & \% - \lambda_{\text{m1}} / . \text{freqfemale} \rightarrow 1 - \xi / . \text{reverse} / . \text{pAveM} \rightarrow q pYm + (1 - q) pXm / . \text{equilB0} // \text{Factor} \\
 & 0
 \end{aligned}$$

The 2 freqfemale in the denominator describes a sex ratio advantage of being in the rarer sex. If the combination of meiotic drive and pollen selection in males causes females to be rare, then the neo-W will gain an advantage.

A neo-W carrying the a allele would have mean representation in the pool of female gametes (accounting for meiotic drive) of:

$$\begin{aligned}
 & \left(\frac{1 - \alpha f}{1/2} \text{FAa} \frac{\text{waf}}{\text{wbarHapFemale}} pXm (1 - q) \frac{\text{wAm}}{\text{wbarHapMale}} + \text{Faa} \frac{\text{waf}}{\text{wbarHapFemale}} (1 - pXm) (1 - q) \right. \\
 & \quad \left. + \frac{\text{wam}}{\text{wbarHapMale}} + \frac{1 - \alpha f}{1/2} \text{FAa} \frac{\text{waf}}{\text{wbarHapFemale}} pYm q \frac{\text{wAm}}{\text{wbarHapMale}} + \text{Faa} \frac{\text{waf}}{\text{wbarHapFemale}} \right. \\
 & \quad \left. (1 - pYm) q \frac{\text{wAm}}{\text{wbarHapMale}} \right) / . \text{SUBS} / . \text{subequil} / . \text{equilB0} // \text{Factor} \\
 & \text{waf} (- \text{Faa wam} + \text{Faa wam} \alpha m - 2 \text{FAa wAm} \alpha m + 2 \text{FAa wAm} \alpha f \alpha m) \\
 & \hline
 & \text{wAf} (- \text{wam} + \text{wam} \alpha m - \text{wAm} \alpha m)
 \end{aligned}$$

This divided by the mean fitness of the wildtype M bearing females ($\text{wbarDipFemale}=\text{FAA}$) divided by $2 * \text{freqfemale}$ gives the leading eigenvalue in this case:

$$\begin{aligned}
 & \frac{(\% / \text{wbarDipFemale})}{2 \text{ freqfemale}} / . \text{SUBS} / . \text{subequil} / . \text{equilB0} // \text{Factor}; \\
 & \% - \lambda_{\text{m1}} / . \text{freqfemale} \rightarrow 1 - \xi / . \text{reverse} / . \text{pAveM} \rightarrow q pYm + (1 - q) pXm / . \text{equilB0} // \text{Factor} \\
 & 0
 \end{aligned}$$

A neo-W arriving into the system can invade under some conditions:

In the absence of haploid selection, invasion occurs if female fitness is higher with the "a" allele, despite that allele being fixed in males:

$$\{\lambda m A1, \lambda m a1\} /. \text{reverse} /. \text{pAveM} \rightarrow q pYm + (1 - q) pXm /. \text{equilB0} /. \text{nohop} // \text{Simplify}$$

$$\left\{ \frac{FAa + FAA}{2 FAA}, \frac{Faa + FAa}{2 FAA} \right\}$$

This is impossible with purely sexually antagonistic selection, where "a" is directionally favored in males (Y-a is fixed) and "A" is directionally favored in females. Essentially, the X is already as specialized as possible for the female beneficial allele (X-A is fixed), and the neo-W often makes daughters with the Y-*a* haplotype, increasing the flow of "a" alleles into the daughters, which reduces fitness of those daughters.

If selection doesn't uniformly favor "A" in females, however, the neo-W can spread at this equilibrium if $FAa > FAA$ (from $\lambda m A$) or if $\frac{Faa}{2} + \frac{FAa}{2} > FAA$ (from $\lambda m a$). If $FAa > FAA$, the implication is that "a" is favored in females despite "A" being fixed on the X, which must require that "X-A" is sufficiently favored in males to drive the frequency of "X-A" to one (specifically, from the stability conditions, we must have

$$\frac{MAa}{(Maa+MAa)/2} > \frac{FAa}{FAA}.$$

Even if $FAa < FAA$, the neo-W can spread alongside the "a" allele if there is sufficiently strong underdominance in females with $Faa > FAa$, such that $\frac{Faa}{2} + \frac{FAa}{2} > FAA$. In this case, "a" is not favored in females near the equilibrium where females are AA (comparing Aa to AA genotypes) and yet the neo-W with "a" can spread because it produces female aa individuals at a high rate by capturing Y-*a* haplotypes and Faa is high enough.

Confirming that sexually antagonistic selection alone does not work:

```
Simplify[Reduce[
  {stabcondB, (( $\lambda m A1 > 1$ ) || ( $\lambda m a1 > 1$ )), ((( $FAA > FAa > Faa > 0$ ) && ( $Maa > MAa > MAA > 0$ )) || (( $0 < FAA < FAa < Faa$ ) && ( $0 < Maa < MAa < MAA$ ))) } /. reverse /.
  pAveM \rightarrow  $q pYm + (1 - q) pXm$  /. equilB0 /. nohop /.  $Rm \rightarrow 0$  /.  $Rf \rightarrow 0$ ]]
```

False

Other forms of selection allow the neo-W to invade an internally stable equilibrium, even without haploid selection:

```
Simplify[
  Reduce[{stabcondB, (( $\lambda m A1 > 1$ ) || ( $\lambda m a1 > 1$ )) } /. reverse /. pAveM \rightarrow  $q pYm + (1 - q) pXm$  /.
  equilB0 /. nohop /.  $Rm \rightarrow 0$  /.  $Rf \rightarrow 0$ ]]
```

$$FAA > 0 \&& MAa > MAA \&& MAA > 0 \&& Maa > 0 \&& \frac{2 FAA MAa}{FAa} > Maa + MAa \&&$$

$$((Faa + FAa > 2 FAA \&& 0 < FAa \leq FAA) \mid\mid (Faa > 0 \&& FAA < FAa < 2 FAA))$$

such as heterozygote advantage in males with directional selection in females:

```

Simplify[Reduce[{stabcondB, λmAl > 1, (((0 < FAA < FAa < Faa) && (Maa < MAa > MAA > 0))} 
  /. reverse /. pAveM → q pYm + (1 - q) pXm /. equilB0 /. nohap /. Rm → 0 /. Rf → 0]]
Simplify[Reduce[{stabcondB, λmal > 1, (((0 < FAA < FAa < Faa) && (Maa < MAa > MAA > 0))} 
  /. reverse /. pAveM → q pYm + (1 - q) pXm /. equilB0 /. nohap /. Rm → 0 /. Rf → 0]]
MAa > 0 && 0 < Maa < MAa && FAA > 0 && FAa <  $\frac{2 \text{FAA MAa}}{\text{Maa} + \text{MAa}}$  && 0 < MAA < MAa && Faa > FAa
FAA > 0 && FAA < FAa < 2 FAA && MAa > 0 &&
Maa > 0 &&  $\frac{2 \text{FAA MAa}}{\text{FAa}} > \text{Maa} + \text{MAa}$  && Faa > FAa && 0 < MAA < MAa

```

or underdominance in females with directional selection in males:

```

Simplify[Reduce[{stabcondB, λmAl > 1,
  (((FAA > FAa < Faa > 0) && ((0 < Maa < MAa < MAA) || (Maa > MAa > MAA > 0)))} 
  /. reverse /. pAveM → q pYm + (1 - q) pXm /. equilB0 /. nohap /. Rm → 0 /. Rf → 0]]
Simplify[Reduce[{stabcondB, λmal > 1,
  (((FAA > FAa < Faa > 0) && ((0 < Maa < MAa < MAA) || (Maa > MAa > MAA > 0)))} 
  /. reverse /. pAveM → q pYm + (1 - q) pXm /. equilB0 /. nohap /. Rm → 0 /. Rf → 0]]
False
FAA > 0 && 0 < FAa < FAA && MAa > 0 && Maa > MAa &&
 $\frac{2 \text{FAA MAa}}{\text{FAa}} > \text{Maa} + \text{MAa}$  && Faa > 2 FAA && 0 < MAA < MAa

```

but not heterozygote advantage in females with directional selection in males (which works for equilibrium A considered above):

```

Simplify[Reduce[{stabcondB, λmAl > 1,
  (((FAA < FAa > Faa > 0) && ((0 < Maa < MAa < MAA) || (Maa > MAa > MAA > 0)))} 
  /. reverse /. pAveM → q pYm + (1 - q) pXm /. equilB0 /. nohap /. Rm → 0 /. Rf → 0]]
Simplify[Reduce[{stabcondB, λmal > 1,
  (((FAA < FAa > Faa > 0) && ((0 < Maa < MAa < MAA) || (Maa > MAa > MAA > 0)))} 
  /. reverse /. pAveM → q pYm + (1 - q) pXm /. equilB0 /. nohap /. Rm → 0 /. Rf → 0]]
False
False

```

Haplod selection also increases the types of selection in the diploid phase that are consistent with neo-W invasion, including sexual antagonism in diploids:

```
(*Including meiotic drive in males*)
Simplify[
  Reduce[{stabcondB, ((λmAl > 1) || (λma1 > 1)), (((FAA > FAa > Faa > 0) && (Maa > MAa >
    MAA > 0)) || ((0 < FAA < FAa < Faa) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilBO
    /. wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. af → 1 / 2 /. Rm → 0 /. Rf → 0]
  FAA > 0 && MAa < 2  $\left(-1 + \frac{2 \text{FAA}}{\text{FAa}}\right)$  MAa & MAA < MAA &&
   $\left(\left(\frac{\text{MAA}}{2 - 2 \text{cm}} < \text{MAa} \& 2 \text{cm} < 1\right) \mid\mid (\text{Faa} > 0 \&& \text{FAa} > 0 \&& \text{Faa} < \text{FAa} \&& \text{FAa} < \text{FAA} \&& \text{MAa} < \text{MAA} \&& \text{MAA} < \text{Maa} \&& (\text{FAa} + \text{FAA}) \text{cm} < \text{FAa} \&& \text{FAa} + 2 \text{FAa} \text{cm} < 4 \text{FAA cm})\right)$ 
(*Including meiotic drive in females*)
Simplify[
  Reduce[{stabcondB, ((λmAl > 1) || (λma1 > 1)), (((FAA > FAa > Faa > 0) && (Maa > MAa >
    MAA > 0)) || ((0 < FAA < FAa < Faa) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilBO
    /. wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. af → 1 / 2 /. Rm → 0 /. Rf → 0]
  FAA > 0 && FAa > 0 && MAa > 0 && Maa > 0 && FAA > 0 && MAA > 0 &&
  FAA < 2 FAa && FAa < FAA && FAA < 2 FAa af && af < 1 && MAa < Maa &&
  FAA MAA + FAa (Maa + MAa) (-1 + af) > 0 && Faa < FAa && MAA < MAa
```

and even directional selection in diploids (ploidally antagonistic selection):

```
(*Including meiotic drive in males*)
Simplify[
  Reduce[{stabcondB, ((λmAl > 1) || (λma1 > 1)), (((0 < FAA < FAa < Faa) && (Maa > MAa >
    MAA > 0)) || ((FAA > FAa > Faa > 0) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilBO
    /. wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. af → 1 / 2 /. Rm → 0 /. Rf → 0]
  FAA > 0 && MAA > 0 &&
   $\left(\left(0 < \text{Faa} \&& 0 < \text{FAa} \&& 0 < \text{Maa} \&& \text{Faa} < \text{FAa} \&& \text{FAa} < \text{FAA} \&& \text{MAa} < \text{MAA} \&& \frac{\text{MAA}}{2 - 2 \text{cm}} < \text{MAa} \&&\right.\right.$ 
   $\left.\left(\left(0 < \text{cm} \&& \text{Maa} < 2 \left(-1 + \frac{2 \text{FAA}}{\text{FAa}}\right) \text{MAa cm} \&& \frac{\text{FAa}}{2 \text{FAa} - 4 \text{FAA}} + \text{cm} \leq 0\right) \mid\mid \right.\right.$ 
   $\left.\left.\left(\frac{\text{FAa}}{2 \text{FAa} - 4 \text{FAA}} + \text{cm} > 0 \&& \text{Maa} < \text{MAa} \&& \text{cm} < \frac{\text{FAa}}{\text{FAa} + \text{FAA}}\right)\right)\right) \mid\mid$ 
   $\left(\text{FAa} > \frac{(\text{FAa} - 2 \text{FAA}) \text{cm}}{-1 + \text{cm}} \&& \text{MAa} > \frac{\text{MAA}}{2 - 2 \text{cm}} \&& 3 \text{FAa} < 4 \text{FAA} \&& \frac{\text{FAa}}{2 \text{FAa} - 4 \text{FAA}} + \text{cm} > 0 \&&\right.$ 
   $\left.\left.\left.\left(\text{FAA} < \text{FAa} \&& \text{Maa} < 2 \left(-1 + \frac{2 \text{FAA}}{\text{FAa}}\right) \text{MAa cm} \&& \text{MAa} < \text{Maa} \&& \text{cm} < 1\right)\right)\right)\right)$ 
```

```
(*Including meiotic drive in females*)
Simplify[
  Reduce[{stabcondB, ((λmA1 > 1) || (λma1 > 1)), (((0 < FAA < FAa < Faa) && (Maa > MAa >
    MAa > 0)) || ((FAA > FAa > Faa > 0) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilB0
  /. wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. am → 1 / 2 /. Rm → 0 /. Rf → 0]]


$$\frac{FAA}{FAa} + 2 \alpha f > 2 \text{ && } \alpha f < 1 \text{ && } MAa > 0 \text{ && }$$


$$MAa < Maa < -\frac{MAa (FAA + FAa (-1 + \alpha f))}{FAa (-1 + \alpha f)} \text{ && } Faa > FAa \text{ && } 0 < MAA < MAa$$

```

R near 1/2:

Recalling that the eigenvalues must decrease with increasing R (see “Invasion Conditions”), we know that sexually antagonistic selection alone (with opposing directional selection in the two sexes) will not allow the spread of a neo-W at equilibrium B under any linkage conditions because both λ_{mA} and λ_{ma} are less than one.

We also know that if both λ_{mA} and λ_{ma} are greater than one, then invasion will occur for all R (as the leading eigenvalue is bounded between these two values, see “Invasion Conditions”). Scanning the above requirements, we can thus conclude that it is possible for unlinked neo-W chromosomes to invade with:

* heterozygote advantage in males with directional selection in females

Indeed, in the absence of haploid selection, overdominance in males is required for both $\lambda_{mA}>1$ and $\lambda_{ma}>1$:

```
(*Other forms of selection in males*)Simplify[
  Reduce[{stabcondB, ((λmA1 > 1) && (λma1 > 1)), ((Maa < MAa < MAA) || (Maa > MAa < MAA) ||
    (Maa > MAa > MAA))} /. reverse /. pAveM → q pYm + (1 - q) pXm /. equilB0
  /. nohap /. Rm → 0 /. Rf → 0]

(*Overdominance in males*)
Simplify[Reduce[{stabcondB, ((λmA1 > 1) && (λma1 > 1)), (Maa < MAa > MAA)} /. reverse /.
  pAveM → q pYm + (1 - q) pXm /. equilB0
  /. nohap /. Rm → 0 /. Rf → 0]

False


$$FAA > 0 \text{ && } FAA < FAa < 2 FAA \text{ && } MAa > 0 \text{ && }$$


$$\frac{2 FAA MAa}{FAa} > Maa + MAa \text{ && } 0 < MAA < MAa \text{ && } Faa + FAa > 2 FAA$$

```

Various forms of haploid selection are also consistent with both λ_{mA} and λ_{ma} being greater than one (ensuring unlinked neo-W can invade). For example, meiotic drive for the Y linked allele in males skews the sex ratio towards males, allowing a neo-W chromosome to invade (which balances the sex ratio). Meiotic drive in females can also work, if balanced by the right type of diploid selection.

Haplod selection coupled for example with sexual antagonism in diploids:

```
(*Including meiotic drive in males*)
Simplify[
  Reduce[{stabcondB, ((λm1 > 1) && (λma1 > 1)), (((FAA > FAa > Faa > 0) && (Maa > MAa >
    MAA > 0)) || ((0 < FAA < FAa < Faa) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilBO
  /. wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. af → 1 / 2 /. Rm → 1 / 2 /. Rf → 1 / 2]
  FAA > 0 && MAA > 0 && Maa < 2  $\left( -1 + \frac{2 \text{FAA}}{\text{FAa}} \right)$  MAa < MAA &&
   $\left( \left( \text{MAa} > \text{MAA} \& \& 0 < \text{FAa} \& \& \text{Faa} < \text{FAa} \& \& \text{FAa} < \text{FAA} \& \& \frac{\text{FAa}}{2 \text{FAa} - 4 \text{FAA}} > 0 \&& \text{MAa} < \text{Maa} \& \& 0 < \text{cm} \&& \text{FAa} < 2 \text{FAA} \&& \text{FAA} < \text{FAa} \&& \text{MAa} < \text{MAA} \& \& \frac{\text{MAA}}{2 - 2 \text{cm}} < \text{MAa} \&& 2 \text{cm} < 1 \right) \right)$ 
```

but not if there is only meiotic drive in females:

```
(*Including meiotic drive in females*)
Simplify[
  Reduce[{stabcondB, ((λm1 > 1) && (λma1 > 1)), (((FAA > FAa > Faa > 0) && (Maa > MAa >
    MAA > 0)) || ((0 < FAA < FAa < Faa) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilBO
  /. wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. af → 1 / 2 /. Rm → 1 / 2 /. Rf → 1 / 2]
False
```

Haploid selection (in either sex) also favors the spread of unlinking neo-W with directional selection in diploids (ploidally antagonistic selection):

```
(*Including meiotic drive in males*)
Simplify[
  Reduce[{stabcondB, ((λm1 > 1) && (λma1 > 1)), (((0 < FAA < FAa < Faa) && (Maa > MAa >
    MAA > 0)) || ((FAA > FAa > Faa > 0) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilBO
  /. wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. af → 1 / 2 /. Rm → 1 / 2 /. Rf → 1 / 2]
  FAA > 0 && MAA > 0 && 0 < FAa < FAA &&  $\frac{\text{MAA}}{2 - 2 \text{cm}} < \text{MAa} < \text{MAA} \&& \frac{(\text{FAa} - 2 \text{FAA}) \text{cm}}{-1 + \text{cm}} < \text{Faa} < \text{FAa} \&&$ 
   $\left( \left( \text{Maa} > 0 \&& \text{Maa} < \text{MAa} \& \& \frac{\text{FAa}}{2 \text{FAa} - 4 \text{FAA}} + \text{cm} > 0 \&& \frac{\text{FAa}}{\text{FAA}} > 2 \text{cm} \right) \right) \&&$ 
   $\left( 0 < \text{Maa} < 2 \left( -1 + \frac{2 \text{FAA}}{\text{FAa}} \right) \text{MAa} \text{cm} \&& 0 < \text{cm} \leq -\frac{\text{FAa}}{2 \text{FAa} - 4 \text{FAA}} \right)$ 
```

```
(*Including meiotic drive in females*)
Simplify[
  Reduce[{stabcondB, ((λmA1 > 1) && (λma1 > 1)), (((0 < FAA < FAa < Faa) && (Maa > MAa >
    MAA > 0)) || ((FAA > FAa > Faa > 0) && (0 < Maa < MAa < MAA)))} /.
    reverse /. pAveM → q pYm + (1 - q) pXm /. equilB0
    /. wam → 1 /. wAm → 1 /. waf → 1 /. wAf → 1 /. am → 1 / 2 /. Rm → 1 / 2 /. Rf → 1 / 2]
FAA > 0 && MAa > 0 && 0 < MAA &&
FAa (FAA MAa + FAa (Maa + MAa) (-1 + αf)) (-1 + αf) < 0 && MAa < Maa && MAA < MAa &&
((2 FAa < 3 FAA && FAA < FAa && ((FAa > 2 (FAA + FAa (-1 + αf)) &&  $\frac{FAA}{FAa} + \alpha f > \frac{3}{2}$  && αf < 1) ||
  (FAa > FAa &&  $\frac{FAA}{FAa} + 2 \alpha f > 2$  &&  $\frac{FAA}{FAa} + \alpha f \leq \frac{3}{2}$ )) ||
  (FAa < 2 FAA && 3 FAA ≤ 2 FAa && ((FAa > 2 (FAA + FAa (-1 + αf)) &&  $\frac{FAA}{FAa} + \alpha f \geq \frac{3}{2}$  && αf < 1) ||
  (FAa > FAa &&  $\frac{FAA}{FAa} + 2 \alpha f > 2$  &&  $\frac{FAA}{FAa} + \alpha f < \frac{3}{2}$ )) ||
  (FAa ≥ 2 FAA && αf < 1 && FAa > FAa &&  $\frac{FAA}{FAa} + 2 \alpha f > 2$ )))
]
```

Even if only one of λ_{mA} or λ_{ma} is greater than one, invasion is still possible with $R = 1/2$, as explored in the Figures. We show below that this can allow the invasion of unlinked neo-W alleles without haploid selection with underdominance in females or overdominance in males:

Lemma 6: Without haploid selection, an unlinked neo-W ($R=1/2$) can invade only if there is underdominance in females or overdominance in males.

An example where a tightly linked XY can be invaded by an unlinked neo-W with ploidally antagonistic selection:

```
tryFAA = 1.1;
tryFAa = 1;
tryFaa = 0.9;
tryMAA = 1.1;
tryMAa = 1;
tryMaa = 0.9;
trywAf = 1;
trywaf = 1;
trywam = 1;
tryaf = 0.5;
tryam = 0.5;
tryR = 1 / 2;
subpar = {FAA → tryFAA, FAa → tryFAa, Faa → tryFaa,
  MAA → tryMAA, MAa → tryMAa, Maa → tryMaa, wAf → trywAf, waf → trywaf,
  wam → trywam, af → tryaf, am → tryam, R → tryR, Rm → tryR, Rf → tryR};
Reduce[{λ == 1.1, charpolyextB == 0, simpcond} /. subpar]
(wAm == 0.258014 || wAm == 0.688166) && λ == 1.1
```

An example where a tightly linked XY can be invaded by an unlinked neo-W with overdominant selection in males (no haploid selection):

```

tryFAa = 1;
tryFaa = 1.1;
tryMAA = 0.8;
tryMaa = 1;
tryMaa = 0.8;
trywAf = 1;
trywaf = 1;
trywAm = 1;
trywam = 1;
tryaf = 0.5;
tryam = 0.5;
tryR = 1 / 2;
subpar = {FAa → tryFAa, Faa → tryFaa, MAA → tryMAA,
    MAa → tryMaa, Maa → tryMaa, wAf → trywAf, waf → trywaf, wAm → trywAm,
    wam → trywam, af → tryaf, am → tryam, R → tryR, Rm → tryR, Rf → tryR};
Reduce[{λ == 1.1, charpolyextB == 0, simpcond, FAA < 1, Faa > 1} /. subpar]
(FAA == 0.227273 || FAA == 0.916667) && λ == 1.1

```

An example where a tightly linked XY can be invaded by an unlinked neo-W with sexually antagonistic selection and haploid selection:

```

tryFAA = 1.1;
tryFAa = 1;
tryFaa = 0.98;
tryMAA = 0.8;
tryMaa = 1;
tryMaa = 1.01;
trywAf = 1;
trywaf = 1;
trywAm = 1;
trywam = 1;
tryaf = 0.5;
tryam = 0.5;
tryR = 1 / 2;
subpar = {FAA → tryFAA, FAa → tryFAa, Faa → tryFaa,
    MAA → tryMAA, MAa → tryMaa, Maa → tryMaa, wAf → trywAf, waf → trywaf,
    wam → trywam, af → tryaf, am → tryam, R → tryR, Rm → tryR, Rf → tryR};
Reduce[{λ == 1.1, charpolyextB == 0, interceptB < 0, simpcond} /. subpar]
(wAm == 0.268933 || wAm == 0.718914) && λ == 1.1

```

Proof of Lemma 6: Without haploid selection, an unlinked neo-W ($R=1/2$) can invade only if there is underdominance in females or overdominance in males.

We showed above that for both λ_{mA} and λ_{ma} to be greater than one without haploid selection requires overdominance in males. This allows the invasion of unlinked neo-W chromosomes.

Here we consider when an unlinked neo-W can spread with only one of λ_{mA} or λ_{ma} greater than one. In this case (see the “Invasion Condition” section), we showed that a neo-W will invade if and only if the intercept of the primary quadratic is negative:

```
(1 + b + c) /. b -> -((λmA - χmA) + (λma - χma)) /.
c -> (λmA - χmA) (λma - χma) - χmA χma // Factor
1 - λma - λmA + λma λmA + χma - λmA χma + χmA - λma χmA
```

Without haploid selection, this becomes:

```
interceptB = (1 + b + c) /. b -> -((λmA1 - χmA1) + (λma1 - χma1)) /.
c -> (λmA1 - χmA1) (λma1 - χma1) - χmA1 χma1 /. reverse /.
pAveM → q pYm + (1 - q) pXm /. equilB0 /. nohap /. R → 1 / 2 // Factor
Faa FAa - 2 Faa FAA - 3 Faa FAA + 4 FAA2
────────────────────────────────────────────────────────────────────────────────
8 FAA2
```

For invasion, the intercept must be negative, requiring that FAa be larger than:

```
Solve[interceptB == 0, FAa] // Flatten
```

$$\left\{ \text{FAa} \rightarrow \frac{2 (\text{FAa} - 2 \text{FAA}) \text{FAA}}{\text{FAa} - 3 \text{FAA}} \right\}$$

The invasion of an unlinked neo-W into a stable system at equilibrium B is thus possible without over-dominance in males, but only if there is underdominance in females:

```
(*Underdominance in females without overdominance in males*) Simplify[
Reduce[{stabcondB,  $\frac{Faa \cdot FAa - 2 \cdot Faa \cdot FAA - 3 \cdot FAa \cdot FAA + 4 \cdot FAA^2}{8 \cdot FAA^2} < 0$ ,  $((\lambda m A1 > 1) \text{ || } (\lambda m a1 > 1))$ ,
 $((FAa > FAa < FAA) \& \& ((Maa < MAa < MAA) \text{ || } (Maa > MAa < MAA) \text{ || }$ 
 $(Maa > MAa > MAA))) \text{ /. reverse /. pAveM} \rightarrow q \cdot pYm + (1 - q) \cdot pXm \text{ /. equilBO}$ 
/. nohap /. Rm \rightarrow 0 /. Rf \rightarrow 0}]]
```

(*Overdominance in females without overdominance in males*)

```
Simplify[Reduce[{stabcondB,  $\frac{Faa \cdot FAa - 2 \cdot Faa \cdot FAA - 3 \cdot FAa \cdot FAA + 4 \cdot FAA^2}{8 \cdot FAA^2} < 0$ ,
 $((\lambda m A1 > 1) \text{ || } (\lambda m a1 > 1))$ ,  $((FAa < FAa > FAA) \& \&$ 
 $((Maa < MAa < MAA) \text{ || } (Maa > MAa < MAA) \text{ || } (Maa > MAa > MAA))) \text{ /. reverse /.}$ 
 $pAveM \rightarrow q \cdot pYm + (1 - q) \cdot pXm \text{ /. equilBO /. nohap /. Rm \rightarrow 0 /. Rf \rightarrow 0} \text{ } \}]$ 
```

(*Directional selection in females without overdominance in males*)

```
Simplify[Reduce[{stabcondB,  $\frac{Faa \cdot FAa - 2 \cdot Faa \cdot FAA - 3 \cdot FAa \cdot FAA + 4 \cdot FAA^2}{8 \cdot FAA^2} < 0$ ,
 $((\lambda m A1 > 1) \text{ || } (\lambda m a1 > 1))$ ,  $((((FAa < FAa < FAA) \text{ || } (FAa > FAa > FAA)) \& \&$ 
 $((Maa < MAa < MAA) \text{ || } (Maa > MAa < MAA) \text{ || } (Maa > MAa > MAA))) \text{ /. reverse /.}$ 
 $pAveM \rightarrow q \cdot pYm + (1 - q) \cdot pXm \text{ /. equilBO /. nohap /. Rm \rightarrow 0 /. Rf \rightarrow 0} \text{ } \}]$ 
```

$$\begin{aligned} & \text{FAA} > 0 \& \& 0 < FAa < FAA \& \& Faa > \frac{(3 \cdot FAa - 4 \cdot FAA) \cdot FAA}{FAa - 2 \cdot FAA} \& \& \\ & \text{MAa} > 0 \& \& \text{Maa} > \text{MAa} \& \& \frac{2 \cdot FAA \cdot MAa}{FAa} > \text{Maa} + \text{MAa} \& \& 0 < \text{MAA} < \text{MAa} \end{aligned}$$

False

False

With overdominance in males, any form of selection in females can generate conditions allowing the spread of a neo-W:

```
(*Underdominance in females with overdominance in males*)Simplify[
Reduce[{stabcondB,  $\frac{Faa \text{FAa} - 2 Faa \text{FAA} - 3 Faa \text{FAA} + 4 \text{FAA}^2}{8 \text{FAA}^2} < 0, ((\lambda m A1 > 1) || (\lambda m a1 > 1)),$ 
 $((\text{Faa} > \text{FAa} < \text{FAA}) \&\& (\text{Maa} < \text{MAa} > \text{MAA})) \} /. \text{reverse} /.$ 
 $pAveM \rightarrow q pYm + (1 - q) pXm /. \text{equilB0} /. \text{nohap} /. \text{Rm} \rightarrow 0 /. \text{Rf} \rightarrow 0]$ }]]

(*Overdominance in females with overdominance in males*)Simplify[
Reduce[{stabcondB,  $\frac{Faa \text{FAa} - 2 Faa \text{FAA} - 3 Faa \text{FAA} + 4 \text{FAA}^2}{8 \text{FAA}^2} < 0, ((\lambda m A1 > 1) || (\lambda m a1 > 1)),$ 
 $((\text{Faa} < \text{FAa} > \text{FAA}) \&\& (\text{Maa} < \text{MAa} > \text{MAA})) \} /. \text{reverse} /.$ 
 $pAveM \rightarrow q pYm + (1 - q) pXm /. \text{equilB0} /. \text{nohap} /. \text{Rm} \rightarrow 0 /. \text{Rf} \rightarrow 0]$ }]]

(*Directional selection in females with overdominance in males*)Simplify[
Reduce[{stabcondB,  $\frac{Faa \text{FAa} - 2 Faa \text{FAA} - 3 Faa \text{FAA} + 4 \text{FAA}^2}{8 \text{FAA}^2} < 0, ((\lambda m A1 > 1) || (\lambda m a1 > 1)),$ 
 $(((\text{Faa} < \text{FAa} < \text{FAA}) || (\text{Faa} > \text{FAa} > \text{FAA})) \&\& (\text{Maa} < \text{MAa} > \text{MAA})) \} /. \text{reverse} /.$ 
 $pAveM \rightarrow q pYm + (1 - q) pXm /. \text{equilB0} /. \text{nohap} /. \text{Rm} \rightarrow 0 /. \text{Rf} \rightarrow 0]$ }]]

FAA > 0 && FAa > 0 && MAa > 0 && Maa > 0 && MAA > 0 && FAa < FAA &&
FAa FAa + 4 FAa^2 < 2 Faa FAA + 3 Faa FAA && Maa < MAa && MAA < MAa

FAA > 0 && MAa > 0 && Maa > 0 &&  $\frac{2 \text{FAA MAa}}{\text{FAa}} > \text{Maa} + \text{MAa}$  &&
0 < MAA < MAa && FAa < FAa &&  $\left( (0 < \text{Faa} \&\& \text{FAa} < 2 \text{FAA} \&\& 4 \text{FAA} \leq 3 \text{FAa}) ||$ 
 $\left( 3 \text{FAa} < 4 \text{FAA} \&\& \text{FAA} < \text{FAa} \&\& \frac{(3 \text{FAa} - 4 \text{FAA}) \text{FAA}}{\text{FAa} - 2 \text{FAA}} < \text{Faa} \right) \right)$ 

FAA > 0 && FAa < 2 FAa && Faa > FAa &&
 $\frac{2 \text{FAA MAa}}{\text{FAa}} > \text{Maa} + \text{MAa}$  && 0 < MAA < MAa
```

Weak selection

(This section runs slow, so the inputs have been commented out and the main product pasted as “lambda2diffsExplicit”)

Solving for the eigenvalues with no selection gives

```
(*Normal[Series[charpolyExt/.weaksel/.q->1/2+q1*\epsilon,{\epsilon,0,0}]]//Factor;
Solve[%==0,\lambda]//Flatten*)
```

So on the zeroth order the leading eigenvalue is $\lambda=1$.

On the first order the eigenvalue is

```
(*Normal[Series[charpolyExt/.weaksel/.λ→1+δλ*ε/.frequenciesub/.sol0/.pxf0→pA,
{ε,0,1}]]//Factor;
Solve[%==0,δλ];
Collect[(δλ/.%) /((1-pA) pA /VA),α1,Simplify]*)
```

which at equilibrium is zero (because at first order there is no Y bias in male gametes, q1=0)

```
(*%/.realsol[[3]]*)
```

On second order we have

```
(*Normal[
Series[charpolyExt/.weaksel/.λ→1+δλ2*ε^2/.frequenciesub/.q1→0/.sol0/.pxf0→pA/.
pXm1→dpAXmXf+pXf1/.pYm1→dpAYmXf+pXf1,{ε,0,2}]]//Factor;
Solve[%==0,δλ2];
lambda2diffs=Collect[δλ2/.%,{dpAXmXf,dpAYmXf},Simplify]*)

lambda2diffs =
{-1/2 dpAXmXf k (2 hAf (-1+k) (-1+2 pA) sAf - 2 hAm sAm + 2 hAm k sAm - 2 (-1+k)
pA (sAf - sAm + 2 hAm sAm) - k tf + k tm) + dpAYmXf (hAf k^2 (1-2 pA) sAf +
1/2 (2 hAm (-1+2 pA) sAm + k^2 (-2 hAm sAm + 2 pA (sAf - sAm + 2 hAm sAm) + tf - tm) -
2 (pA sAm + tm) + 4 k (pA sAm + hAm (sAm - 2 pA sAm) + tm)) ) +
1/R (2 (-1+2 k) q2 R - (-1+pA) pA (hAf^2 k^2 (1-2 pA)^2 sAf^2 + hAm^2 sAm^2 +
2 hAm pA sAm^2 - 4 hAm^2 pA sAm^2 + pA^2 sAm^2 - 4 hAm pA^2 sAm^2 + 4 hAm^2 pA^2 sAm^2 +
hAm R sAm tf + pA R sAm tf - 2 hAm pA R sAm tf + 2 hAm sAm tm + 2 pA sAm tm - 4 hAm pA sAm tm - hAm R sAm tm - pA R sAm tm + 2 hAm pA R sAm tm + R tf tm +
tm^2 - R tm^2 + 4 hAm sAm α1m + 4 pA sAm α1m - 8 hAm pA sAm α1m - 4 hAm R sAm α1m - 4 pA R sAm α1m + 8 hAm pA R sAm α1m + 4 tm α1m - 4 R tm α1m + 4 α1m^2 - 8 R α1m^2 +
k^2 (hAm^2 sAm^2 + pA^2 (sAf + (-1+2 hAm) sAm)^2 + tf^2 - 2 R tf^2 - 2 tf tm + 4 R tf tm +
tm^2 - 2 R tm^2 + 4 tf α1f - 6 R tf α1f - 4 tm α1f + 6 R tm α1f + 4 α1f^2 - 8 R α1f^2 + hAm sAm ((-2+3 R) tf + (2-3 R) tm + 4 (-1+R) (α1f - α1m)) - 4 tf α1m + 6 R tf α1m + 4 tm α1m - 6 R tm α1m - 8 α1f α1m +
16 R α1f α1m + 4 α1m^2 - 8 R α1m^2 - pA (sAf + (-1+2 hAm) sAm) - (2 hAm sAm + (-2+3 R) tf + 2 tm - 3 R tm - 4 α1f + 4 R α1f + 4 α1m - 4 R α1m) ) - hAf k (-1+2 pA) sAf (2 (pA sAm + hAm (sAm - 2 pA sAm) + R tf + tm - R tm +
2 α1m - 2 R α1m) + k (-2 hAm sAm + 2 pA (sAf - sAm + 2 hAm sAm) + 2 tf - 3 R tf - 2 tm + 3 R tm + 4 α1f - 4 R α1f - 4 α1m + 4 R α1m) ) +
k (2 pA^2 (sAf - sAm) sAm - 2 hAm^2 (1-2 pA)^2 sAm^2 + R tf^2 + 2 tf tm - 4 R tf tm - 2 tm^2 + 3 R tm^2 + 2 R tf α1f + 4 tm α1f - 6 R tm α1f + 4 tf α1m - 6 R tf α1m - 8 tm α1m + 10 R tm α1m + 8 α1f α1m - 16 R α1f α1m - 8 α1m^2 +
16 R α1m^2 + 2 pA (R (sAf (tf - tm - 2 α1m) - 2 sAm (tf - tm + α1f - 2 α1m)) + sAm (tf - 2 tm + 2 α1f - 4 α1m) + sAf (tm + 2 α1m)) - 2 hAm (-1+2 pA) sAm (pA (sAf - 2 sAm) + tf - 2 R tf + 2 (-1+R) (tm - α1f + 2 α1m))))});
```

Then we can put in all the differences

```

lambda2diffsExplicit = lambda2diffs /. realsol[[3]] /. freqYm2sol // Simplify


$$\left\{ -\frac{1}{2 r R (sAf - 2 hAf sAf + sAm - 2 hAm sAm)^4} \right.$$


$$(hAf^3 sAf^3 (sAm + 2 tm + 4 \alpha1m) - hAf^2 sAf^2 (-(-1 + 2 hAm) sAm$$


$$(sAm - tf + 2 tm - 2 \alpha1f + 4 \alpha1m) + sAf (sAm + hAm sAm + 3 tm + 6 \alpha1m)) -$$


$$hAf sAf (2 sAm^2 tf + sAm tf^2 + sAm^2 tm + 4 sAm tf tm + 2 tf^2 tm + 3 sAm tm^2 +$$


$$4 tf tm^2 + 2 tm^3 + 4 sAm^2 \alpha1f + 4 sAm tf \alpha1f + 8 sAm tm \alpha1f + 8 tf tm \alpha1f + 8 tm^2 \alpha1f +$$


$$4 sAm \alpha1f^2 + 8 tm \alpha1f^2 + 2 sAm^2 \alpha1m + 8 sAm tf \alpha1m + 4 tf^2 \alpha1m + 12 sAm tm \alpha1m +$$


$$16 tf tm \alpha1m + 12 tm^2 \alpha1m + 16 sAm \alpha1f \alpha1m + 16 tf \alpha1f \alpha1m + 32 tm \alpha1f \alpha1m +$$


$$16 \alpha1f^2 \alpha1m + 12 sAm \alpha1m^2 + 16 tf \alpha1m^2 + 24 tm \alpha1m^2 + 32 \alpha1f \alpha1m^2 + 16 \alpha1m^3 -$$


$$sAf^2 (tm + 2 \alpha1m) - hAm^2 sAm^2 (-2 sAf + sAm - 4 tf + 2 tm - 8 \alpha1f + 4 \alpha1m) + hAm sAm$$


$$(-sAf^2 - 2 sAf (tf - 2 tm + 2 \alpha1f - 4 \alpha1m) + sAm (sAm - 4 tf + 2 tm - 8 \alpha1f + 4 \alpha1m)) +$$


$$2 sAf (sAm (tf + 2 \alpha1f) + (tm + 2 \alpha1m) (tf + tm + 2 (\alpha1f + \alpha1m))) -$$


$$(hAm sAm + tf + tm + 2 \alpha1f + 2 \alpha1m) (hAm^2 sAm^2 (sAf + 2 tf + 4 \alpha1f) -$$


$$sAf^2 (tm + 2 \alpha1m) + sAm (tf + 2 \alpha1f) (sAm + tf + tm + 2 \alpha1f + 2 \alpha1m) +$$


$$sAf (sAm (tf - tm + 2 \alpha1f - 2 \alpha1m) - (tm + 2 \alpha1m) (tf + tm + 2 (\alpha1f + \alpha1m))) -$$


$$hAm sAm (sAf^2 + sAf (sAm + 3 tf + 6 \alpha1f) +$$


$$(tf + 2 \alpha1f) (3 sAm + 2 (tf + tm + 2 (\alpha1f + \alpha1m))))))$$


$$(-k^2 R sAf tf + 2 k^2 r R sAf tf - 2 r sAm tf + 8 k r sAm tf - 8 k^2 r sAm tf +$$


$$2 R sAm tf - 4 k R sAm tf + 3 k^2 R sAm tf - 8 k r R sAm tf +$$


$$10 k^2 r R sAm tf + 2 r sAf tm - 8 k r sAf tm + 8 k^2 r sAf tm -$$


$$2 R sAf tm + 4 k R sAf tm - 3 k^2 R sAf tm + 8 k r R sAf tm -$$


$$10 k^2 r R sAf tm + k^2 R sAm tm - 2 k^2 r R sAm tm - 4 k^2 R sAf \alpha1f +$$


$$8 k^2 r R sAf \alpha1f - 4 r sAm \alpha1f + 16 k r sAm \alpha1f - 16 k^2 r sAm \alpha1f +$$


$$4 R sAm \alpha1f - 8 k R sAm \alpha1f + 4 k^2 R sAm \alpha1f - 16 k r R sAm \alpha1f +$$


$$24 k^2 r R sAm \alpha1f + 4 r sAf \alpha1m - 16 k r sAf \alpha1m + 16 k^2 r sAf \alpha1m -$$


$$4 R sAf \alpha1m + 8 k R sAf \alpha1m - 4 k^2 R sAf \alpha1m + 16 k r R sAf \alpha1m -$$


$$24 k^2 r R sAf \alpha1m + 4 k^2 R sAm \alpha1m - 8 k^2 r R sAm \alpha1m +$$


$$2 hAf sAf (R ((1 - 2 k + 2 k^2) sAm + 2 (tm + 2 \alpha1m) -$$


$$4 k (tm + 2 \alpha1m) + k^2 (tf + 3 tm + 4 (\alpha1f + \alpha1m))) +$$


$$r ((-1 - 4 k (-1 + R) + 4 k^2 (-1 + R)) sAm - 2 (tm + 2 \alpha1m + 4 k (-1 + R) (tm + 2 \alpha1m) +$$


$$k^2 (R (tf - 5 tm + 4 \alpha1f - 12 \alpha1m) + 4 (tm + 2 \alpha1m))) - 2 hAm sAm$$


$$(R ((1 - 2 k + 2 k^2) sAf + (2 - 4 k + 3 k^2) tf + k^2 tm + 4 \alpha1f - 8 k \alpha1f + 4 k^2 \alpha1f + 4 k^2 \alpha1m) +$$


$$r ((-1 - 4 k (-1 + R) + 4 k^2 (-1 + R)) sAf + 2 ((-1 - 4 k (-1 + R) + k^2 (-4 + 5 R))$$


$$tf - 2 \alpha1f - 8 k (-1 + R) \alpha1f - k^2 (8 \alpha1f + R (tm - 12 \alpha1f + 4 \alpha1m)))))) \}$$


```

For any k we can write this as

```


$$\frac{(1 - 2 k)^2}{4} VA SA^2 \frac{r - R}{r R} + \frac{k dpAYmXm}{2} (k (2 \alpha Dm - 2 \alpha Df + tm - tf) - 2 (1 - k) SA) /.$$


$$SA \rightarrow (Dm + \alpha Dm + tm) - (Df + \alpha Df + tf) /. \text{trySub3} /. \text{realsol2}[[3]];$$


$$\{0\} == \% - lambda2diffsExplicit // Factor$$


$$\text{True}$$


```

For a neo-Y this reduces to

```


$$\frac{1}{4} \frac{VA SA^2}{r R} \frac{r - R}{2} / . SA \rightarrow (Dm + \alpha Dm + tm) - (Df + \alpha Df + tf) / . \text{trysub3} / . \text{realsol2}[[3]];$$


$$\{0\} == \% - \text{lambda2diffsExplicit} / . k \rightarrow 0 // \text{Factor}$$

True

```

And for a neo-W this reduces to

```


$$\frac{1}{4} \frac{VA SA^2}{r R} \frac{r - R}{2} + \frac{\text{dpAYmXm}}{2} (2 \alpha Dm - 2 \alpha Df + tm - tf) / . SA \rightarrow (Dm + \alpha Dm + tm) - (Df + \alpha Df + tf) / .$$


$$\text{trysub3} / . \text{realsol2}[[3]];$$


$$\{0\} == \% - \text{lambda2diffsExplicit} / . k \rightarrow 1 // \text{Factor}$$

True

```

When $r=1/2$ the neo-W case reduces to the neo-Y case as

```


$$\text{dpAYmXm} / . \text{realsol2}[[3]] / . r \rightarrow 1/2$$

0

```

Now consider only $R=1/2$ and $r<1/2$ (a new sex chromosome arises).

When the only haploid selection is male drive a neo-W can invade only when the following is positive

```


$$\frac{1}{4} \frac{VA SA^2}{r R} \frac{r - R}{2} (2 \alpha Dm - 2 \alpha Df + tm - tf) / . SA \rightarrow (Dm + \alpha Dm + tm) - (Df + \alpha Df + tf) / .$$


$$\text{trysub3} / . \text{realsol2}[[3]] / . hAf \rightarrow hAm / . hAm \rightarrow hA / .$$


$$R \rightarrow 1/2 / . tf \rightarrow 0 / . tm \rightarrow 0 / . \alpha 1f \rightarrow 0 // \text{Simplify}$$


$$- \frac{4 (-1 + 2 r) sAf sAm ((-1 + hA) sAf + (-1 + hA) sAm - 2 \alpha 1m) \alpha 1m^2 (hA (sAf + sAm) + 2 \alpha 1m)}{(1 - 2 hA)^2 r (sAf + sAm)^4}$$


```

The denominator is always positive. The two selection terms in parentheses in the numerator are positive when there is a stable polymorphism

```


$$\text{stabconds} / . hAf \rightarrow hAm / . hAm \rightarrow hA / . R \rightarrow 1/2 / . tf \rightarrow 0 / . tm \rightarrow 0 / . \alpha 1f \rightarrow 0 // \text{Simplify}$$


$$\{ (-1 + hA) (sAf + sAm) > 2 \alpha 1m, hA (sAf + sAm) + 2 \alpha 1m > 0 \}$$


```

Thus we need $sAf sAm > 0$ for a neo-W to invade when the only haploid selection is male drive.

When the only haploid selection is female drive a neo-W can invade only when the following is positive

```


$$\frac{1}{4} \frac{VA SA^2}{r R} \frac{r - R}{2} (2 \alpha Dm - 2 \alpha Df + tm - tf) / . SA \rightarrow (Dm + \alpha Dm + tm) - (Df + \alpha Df + tf) / .$$


$$\text{trysub3} / . \text{realsol2}[[3]] / . hAf \rightarrow hAm / . hAm \rightarrow hA / .$$


$$R \rightarrow 1/2 / . tf \rightarrow 0 / . tm \rightarrow 0 / . \alpha 1m \rightarrow 0 // \text{Simplify}$$


$$- \frac{4 (-1 + 2 r) sAf sAm ((-1 + hA) sAf + (-1 + hA) sAm - 2 \alpha 1f) \alpha 1f^2 (hA (sAf + sAm) + 2 \alpha 1f)}{(1 - 2 hA)^2 r (sAf + sAm)^4}$$


```

The denominator is always positive. The two selection terms in parentheses in the numerator are positive when there is a stable polymorphism

```


$$\text{stabconds} / . hAf \rightarrow hAm / . hAm \rightarrow hA / . R \rightarrow 1/2 / . tf \rightarrow 0 / . tm \rightarrow 0 / . \alpha 1m \rightarrow 0 // \text{Simplify}$$


$$\{ (-1 + hA) (sAf + sAm) > 2 \alpha 1f, hA (sAf + sAm) + 2 \alpha 1f > 0 \}$$


```

Thus we need $sAf sAm > 0$ for a neo-W to invade when the only haploid selection is female drive.

When the only haploid selection is male gamete competition a neo-W can invade only when the following is positive

$$\begin{aligned} \frac{1}{4} \frac{VA SA^2}{r R} \frac{r - R}{2} & \left(2 \alpha Dm - 2 \alpha Df + tm - tf \right) / . SA \rightarrow (Dm + \alpha Dm + tm) - (Df + \alpha Df + tf) / . \\ & \text{trySub3 /. realSol2[[3]] /. hAf \rightarrow hAm /.} \\ & hAm \rightarrow hA /. tf \rightarrow 0 /. \alpha 1m \rightarrow 0 /. \alpha 1f \rightarrow 0 // \text{Simplify} \\ - \left(sAf (R (sAf - sAm) + 2 r ((-1 + R) sAf + R sAm)) \right. & \left. ((-1 + hA) sAf + (-1 + hA) sAm - tm) \right. \\ & \left. tm^2 (hA (sAf + sAm) + tm) \right) / (2 (1 - 2 hA)^2 r R (sAf + sAm)^4) \end{aligned}$$

The denominator is always positive. The two selection terms in parentheses in the numerator are positive when there is a stable polymorphism

$$\begin{aligned} \text{stabconds /. hAf \rightarrow hAm /. hAm \rightarrow hA /. R \rightarrow 1/2 /. tf \rightarrow 0 /. \alpha 1m \rightarrow 0 /. \alpha 1f \rightarrow 0 // Simplify} \\ \{(-1 + hA) (sAf + sAm) > tm, hA (sAf + sAm) + tm > 0\} \end{aligned}$$

Thus we need $sAf (sAf - sAm) < 0$ for a neo-W to invade when the only haploid selection is male gamete competition.

When the only haploid selection is female gamete competition a neo-W can invade only when the following is positive

$$\begin{aligned} \frac{1}{4} \frac{VA SA^2}{r R} \frac{r - R}{2} & \left(2 \alpha Dm - 2 \alpha Df + tm - tf \right) / . SA \rightarrow (Dm + \alpha Dm + tm) - (Df + \alpha Df + tf) / . \\ & \text{trySub3 /. realSol2[[3]] /. hAf \rightarrow hAm /. hAm \rightarrow hA /.} \\ & R \rightarrow 1/2 /. tm \rightarrow 0 /. \alpha 1m \rightarrow 0 /. \alpha 1f \rightarrow 0 // \text{Simplify} \\ - \frac{(-1 + 2 r) (sAf - sAm) sAm ((-1 + hA) sAf + (-1 + hA) sAm - tf) tf^2 (hA (sAf + sAm) + tf)}{2 (1 - 2 hA)^2 r (sAf + sAm)^4} & \end{aligned}$$

The denominator is always positive. The two selection terms in parentheses in the numerator are positive when there is a stable polymorphism

$$\begin{aligned} \text{stabconds /. hAf \rightarrow hAm /. hAm \rightarrow hA /. R \rightarrow 1/2 /. tm \rightarrow 0 /. \alpha 1m \rightarrow 0 /. \alpha 1f \rightarrow 0 // Simplify} \\ \{(-1 + hA) (sAf + sAm) > tf, hA (sAf + sAm) + tf > 0\} \end{aligned}$$

Thus we need $(sAf - sAm) sAm > 0$ for a neo-W to invade when the only haploid selection is male gamete competition.

Notice that the invasion fitness of a neo-W into a XY system is the same as a neo-Y into a ZW system (at least up to order ϵ^2)

```


$$\frac{1}{4} \text{VA } \text{SA}^2 \frac{r - R}{r R} + \frac{\text{dpAYmXm}}{2} (2 \alpha Dm - 2 \alpha Df + tm - tf) /. \text{SA} \rightarrow (Dm + \alpha Dm + tm) - (Df + \alpha Df + tf) /.$$

trySub3 /. realSol2[[3]];

$$\frac{1}{4} \text{VA } \text{SA}^2 \frac{r - R}{r R} + \frac{\text{dpAYmXm}}{2} (2 \alpha Dm - 2 \alpha Df + tm - tf) /. \text{SA} \rightarrow (Dm + \alpha Dm + tm) - (Df + \alpha Df + tf) /.$$

trySub3 /. realSol2[[3]] /. sAf \[Rule] temp /. sAm \[Rule] sAf /. temp \[Rule] sAm /.
hAf \[Rule] temp /. hAm \[Rule] hAf /. temp \[Rule] hAm /. tf \[Rule] temp /. tm \[Rule] tf /.
temp \[Rule] tm /. alm \[Rule] temp /. alf \[Rule] alm /. temp \[Rule] alf;
% ==
%% //
Simplify
True

```

Finally, note that the k=1/2 invasion fitness can be written as a function of the neo-Y and neo-W invasion fitnesses evaluated at R=1/2

```


$$\frac{1}{2} \left( \frac{1}{2} \left( \left( \frac{1}{4} \text{VA } \text{SA}^2 \frac{r - R}{r R} /. R \rightarrow 1/2 \right) + \right. \right. \\ \left. \left. \left( \frac{1}{4} \text{VA } \text{SA}^2 \frac{r - R}{r R} + \frac{\text{dpAYmXm}}{2} (2 \alpha Dm - 2 \alpha Df + tm - tf) /. R \rightarrow 1/2 \right) \right) \right) /.$$


$$\text{SA} \rightarrow (Dm + \alpha Dm + tm) - (Df + \alpha Df + tf) /. \text{trySub3} /. \text{realSol2}[[3]];
\{0\} == \% - lambda2diffsExplicit /. k \[Rule] 1/2 // Simplify$$


```

True

and this is the same regardless of whether the ancestral SD system is XY or ZW

```


$$\frac{1}{2} \left( \frac{1}{2} \left( \left( \frac{1}{4} \text{VA } \text{SA}^2 \frac{r - R}{r R} /. R \rightarrow 1/2 \right) + \right. \right. \\ \left. \left. \left( \frac{1}{4} \text{VA } \text{SA}^2 \frac{r - R}{r R} + \frac{\text{dpAYmXm}}{2} (2 \alpha Dm - 2 \alpha Df + tm - tf) /. R \rightarrow 1/2 \right) \right) \right) /.$$


$$\text{SA} \rightarrow (Dm + \alpha Dm + tm) - (Df + \alpha Df + tf) /. \text{trySub3} /. \text{realSol2}[[3]];

$$\frac{1}{2} \left( \frac{1}{2} \left( \left( \frac{1}{4} \text{VA } \text{SA}^2 \frac{r - R}{r R} /. R \rightarrow 1/2 \right) + \left( \frac{1}{4} \text{VA } \text{SA}^2 \frac{r - R}{r R} + \frac{\text{dpAYmXm}}{2} (2 \alpha Dm - 2 \alpha Df + tm - tf) /. R \rightarrow 1/2 \right) \right) \right) /. \text{SA} \rightarrow (Dm + \alpha Dm + tm) - (Df + \alpha Df + tf) /. \text{trySub3} /.$$

realSol2[[3]] /. sAf \[Rule] temp /. sAm \[Rule] sAf /. temp \[Rule] sAm /.
hAf \[Rule] temp /. hAm \[Rule] hAf /. temp \[Rule] hAm /. tf \[Rule] temp /. tm \[Rule] tf /.
temp \[Rule] tm /. alm \[Rule] temp /. alf \[Rule] alm /. temp \[Rule] alf;
% ==
%% //
Simplify
True$$

```

Figures

Plotting parameters

Where to save plots

```
SetDirectory[NotebookDirectory[]];
(*sets current directory to be location of this file*)
plotdir = "Plots/"; (*directory to save figures in*)
```

Remove lines from region plots when saving

```
rasterTrick[plot_] := Show[plot,
  Prolog -> {Opacity[0], Texture[{{0, 0, 0, 0}}], VertexTextureCoordinates ->
    {{0, 0}, {1, 0}, {1, 1}}, Polygon[{{0, 0}, {.1, 0}, {.1, .1}}]}]
```

Universal aesthetics

```
lwd = 0.006; (*line thickness as fraction of plot width*)
letpos = {0.05, 0.95}; (*relative location of panel letter*)
ylabpos = {-0.16, 0.5}; (*relative location of y axis label*)
ylabposregion = {-0.1, 0.5}; (*for region plots*)
inches = 72;
xsize = 5 inches; (*width of panel*)
aspectratio = 1 / Sqrt[2]; (*aspect ratio of panel*)
ticksizes = {0, 0.01}; (*size of tick marks*)
pad = {{50, 20}, {40, 20}}; (*white space around plot {{left,right},{bottom,top}}*)
padregion = {{40, 10}, {40, 30}};
(*white space around region plot {{left,right},{bottom,top}}*)
posplotscale = 1;
```

Basic plot style (λ vs cM plots)

```
Clear[plotstyle]
plotstyle[xmin_, xmax_, xtickmin_, xtickmax_,
  xint_, ymin_, ymax_, ytickmin_, ytickmax_, yint_] := {
  PlotRange -> {{xmin, xmax}, {ymin, ymax}},
  ImageSize -> {xsize * posplotscale, xsize aspectratio * posplotscale},
  AspectRatio -> aspectratio,
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksizes}, {x, xtickmin, xtickmax, xint}],
    Table[{y, y, ticksizes}, {y, ytickmin, ytickmax, yint}]},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]},
    {Directive[Black, Thickness[lwd]], Directive[Black, Thickness[lwd]]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]}, None, None},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14 * posplotscale},
  ImagePadding -> pad,
  PlotRangeClipping -> False
}
```

Log-linear plot style (λ vs R plots)

```

Clear[loglinearplotstyle]
loglinearplotstyle[xmin_, xmax_, xmarks_,
  ymin_, ymax_, ytickmin_, ytickmax_, yint_, recreate_] := {
  PlotRange → {Log@{xmin, xmax}, {ymin, ymax}},
  ImageSize → {xsize * posplotscale, xsize aspectratio * posplotscale},
  AspectRatio → aspectratio,
  PlotRangePadding → 0,
  Frame → {{True, False}, {True, False}},
  FrameTicks → {{Table[{y, y, ticksize}, {y, ytickmin, ytickmax, yint}], None},
    {Append[Table[{Log@x, x, ticksize}, {x, xmarks}], {Log@recreate, "r", ticksize}], None}},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd], FontColor → Black],
    Directive[Black, Thickness[lwd]]},
    {Directive[Black, Thickness[lwd]], Directive[Black, Thickness[lwd]]}}},
  FrameStyle → Directive[Black, Thickness[lwd]],
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14 * posplotscale},
  ImagePadding → pad
}

```

Log-linear inset plot style (frequency dynamics)

```

Clear[insetloglinearplotstyle]
insetloglinearplotstyle[xmin_, xmax_,
  xmarks_, ymin_, ymax_, ytickmin_, ytickmax_, yint_] := {
  PlotRange → {Log@{xmin, xmax}, {ymin, ymax}},
  ImageSize → {xsize, xsize aspectratio },
  AspectRatio → aspectratio,
  PlotRangePadding → 0,
  Frame → {{True, False}, {True, False}},
  FrameTicks → {{Table[{y, y, ticksize}, {y, ytickmin, ytickmax, yint}], None},
    {Table[{Log@x, x, ticksize}, {x, xmarks}], None}},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd], FontColor → Black],
    Directive[Black, Thickness[lwd]]},
    {Directive[Black, Thickness[lwd]], Directive[Black, Thickness[lwd]]}}},
  FrameStyle → Directive[Black, Thickness[lwd]],
  BaseStyle → {FontFamily → "Helvetica", FontSize → 8 * 4 / 5},
  ImagePadding → pad
}

```

Region plot style (where neo-W invades)

```

Clear[regionplotstyle]
regionplotstyle[xmin_, xmax_, xtickmin_,
  xtickmax_, xint_, ymin_, ymax_, ytickmin_, ytickmax_, yint_] := {
  PlotRange → {{xmin, xmax}, {ymin, ymax}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, xtickmin, xtickmax, xint}],
    Table[{y, y, ticksize}, {y, ytickmin, ytickmax, yint}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → padregion,
  PlotRangeClipping → False
}

```

Useful functions

Haldane's mapping function

Haldane's (1919) mapping function: probability of an odd number of crossovers between positions y and z (for y and z sufficiently far apart; Eq 3), where y and z are measured in centiMorgans

$$\text{pcross}[y_, z_] := \frac{1 - \text{Exp}[-2 * \sqrt{(y - z)^2} / 100]}{2} // N$$

Recombination probabilities (no interference)

```

setr[x_, a_, m_] :=
If[
  x < m < a || a < m < x,
  pcross[x, m] (1 - pcross[m, a]) + (1 - pcross[x, m]) pcross[m, a],
  pcross[x, a]
]

setR[x_, a_, m_] :=
If[
  a < x < m || m < x < a,
  pcross[a, x] (1 - pcross[x, m]) + (1 - pcross[a, x]) pcross[x, m],
  pcross[a, m]
]

setρ[x_, a_, m_] :=
If[
  x < a < m || m < a < x,
  pcross[x, a] (1 - pcross[a, m]) + (1 - pcross[x, a]) pcross[a, m],
  pcross[x, m]
]

```

XY resident equilibrium and stability

Resident equilibria

```
(*differenceEqs//Simplify*)

Clear[equilXY]
equilXY[FAA_, FAa_, Faa_, MAA_, MAa_, Maa_, wAf_, waf_, wAm_, wam_, af_, am_, r_] :=
  equilXY[FAA, FAa, Faa, MAA, MAa, Maa, wAf, waf, wAm, wam, af, am, r] =
  {pxf, pxm, pYm, q} /.
  NSolve[{(Faa pXf (-1 + pXf + pxm - pXf pxm) waf wam - FAA (-1 + pXf) pXf pxm wAf wAm +
    FAa (-pXm waf wAm + pXf ((-1 + pxm) wAf wam + pxm waf wAm)) (pXf - af)) /
    (Faa (-1 + pXf) (-1 + pxm) waf wam + FAA pXf pxm wAf wAm +
    FAa (pxm waf wAm - pXf ((-1 + pxm) wAf wam + pxm waf wAm))), (2 Maa (-1 + pXf) pxm (-1 + pYm) (-1 + q) waf wam + MAA pXf pYm (1 + 2 pxm (-1 + q)) wAf wAm -
    2 MAa (pYm waf wAm (pxm - pxm q - r am) + pXf (pxm (-1 + q) ((-1 + pYm) wAf wam +
      pYm waf wAm) + ((-1 + pYm + r - pYm r) wAf wam + pYm r waf wAm) am))) /
    (2 (Maa (-1 + pXf) (-1 + pYm) waf wam + MAA pXf pYm wAf wAm +
      MAa (pYm waf wAm - pXf ((-1 + pYm) wAf wam + pYm waf wAm))), (-2 Maa (-1 + pXf) (-1 + pYm) pYm q waf wam + MAA pXf pYm (1 - 2 pYm q) wAf wAm +
      2 MAa (pYm waf wAm (-pYm q + am - r am) + pXf (pYm^2 q (wAf wam + waf wAm) + r wAf wam am - pYm (q wAf wam + r wAf wam am + waf wAm am - r waf wAm am))) /
      (2 (Maa (-1 + pXf) (-1 + pYm) waf wam + MAA pXf pYm wAf wAm +
        MAa (pYm waf wAm - pXf ((-1 + pYm) wAf wam + pYm waf wAm))), (-Maa (-1 + pXf) (-1 + pYm) (-1 + 2 q) waf wam + MAA pXf pYm (1 - 2 q) wAf wAm +
        2 MAa (pYm waf wAm (-q + r + am - 2 r am) + pXf ((-1 + pYm) wAf wam (-1 + q + r + am - 2 r am) + pYm waf wAm (q - am + r (-1 + 2 am)))))) /
      (2 (Maa (-1 + pXf) (-1 + pYm) waf wam + MAA pXf pYm wAf wAm + MAa (pYm waf wAm - pXf ((-1 + pYm) wAf wam + pYm waf wAm))))} == {0, 0, 0, 0}, {pxf, pxm, pYm, q}]
```

Eigenvalues of internal stability matrix

```
(*matIntFull//Simplify*)
```

```

Clear[internstabXY];
internstabXY[FAA_, FAa_, Faa_, MAA_, MAa_, Maa_,
wAf_, waf_, wAm_, wam_, af_, am_, r_, pXf_, pXm_, pYm_, q_] :=
internstabXY[FAA, FAa, Faa, MAA, MAa, Maa, wAf, waf, wAm, wam, af, am, r,
pXf, pXm, pYm, q] = Eigenvalues[{\{(waf wAf (-FAa FAA pXm^2 wAm^2 (-1 + af) -
Faa (-1 + pXm) wam (FAA pXm wAm - FAa (-1 + pXm) wam af))) /
(Faa (-1 + pXf) (-1 + pXm) waf wam + FAA pXf pXm wAf wAm +
FAa (pXm waf wAm - pXf ((-1 + pXm) wAf wam + pXm waf wAm)))^2, (wam wAm (-FAa FAA
pXf^2 wAf^2 (-1 + af) - Faa (-1 + pXf) waf (FAA pXf wAf - FAa (-1 + pXf) waf af))) /
(Faa (-1 + pXf) (-1 + pXm) waf wam + FAA pXf pXm wAf wAm +
FAa (pXm waf wAm - pXf ((-1 + pXm) wAf wam + pXm waf wAm)))^2, 0\}, \{-(waf wAf
(Maa (-1 + pYm) wam (MAA pYm wAm + 2 MAa (-1 + pYm) (-1 + r) wam am) + 2 MAa pYm
wAm (-1 + am) (-MAA pYm (-1 + r) wAm + 2 MAa (-1 + pYm) (-1 + 2 r) wam am))) /
(Maa (-1 + pXf) (-1 + pYm) waf wam + MAA pXf pYm wAf wAm + 2 MAa
(pYm waf wAm (1 - am + r (-1 + 2 am)) + pXf (wAf wam am + r ((-1 + pYm) wAf wam -
pYm waf wAm) (-1 + 2 am) + pYm (waf wAm (-1 + am) - wAf wam am)))^2,
0, (wam wAm (-Maa (-1 + pXf) waf (MAA pXf wAf - 2 MAa (-1 + pXf) r waf am) +
2 MAa pXf wAf (-1 + am) (-MAA pXf r wAf + 2 MAa (-1 + pXf) (-1 + 2 r) waf am))) /
(Maa (-1 + pXf) (-1 + pYm) waf wam + MAA pXf pYm wAf wAm + 2 MAa
(pYm waf wAm (1 - am + r (-1 + 2 am)) + pXf (wAf wam am + r ((-1 + pYm) wAf wam -
pYm waf wAm) (-1 + 2 am) + pYm (waf wAm (-1 + am) - wAf wam am)))^2\},
\{(waf wAf (-Maa (-1 + pYm) wam (MAA pYm wAm - 2 MAa (-1 + pYm) r wam am) +
2 MAa pYm wAm (-1 + am) (-MAA pYm r wAm + 2 MAa (-1 + pYm) (-1 + 2 r) wam am))) /
(Maa (-1 + pXf) (-1 + pYm) waf wam + MAA pXf pYm wAf wAm -
2 MAa (pYm waf wAm (-am + r (-1 + 2 am)) + pXf
(pYm waf wAm (r + am - 2 r am) + (-1 + pYm) wAf wam (1 - r - am + 2 r am)))^2,
0, -(wam wAm (Maa (-1 + pXf) waf (MAA pXf wAf + 2 MAa (-1 + pXf) (-1 + r) waf am) +
2 MAa pXf wAf (-1 + am)
(-MAA pXf (-1 + r) wAf + 2 MAa (-1 + pXf) (-1 + 2 r) waf am))) /
(Maa (-1 + pXf) (-1 + pYm) waf wam + MAA pXf pYm wAf wAm -
2 MAa (pYm waf wAm (-am + r (-1 + 2 am)) + pXf
(pYm waf wAm (r + am - 2 r am) + (-1 + pYm) wAf wam (1 - r - am + 2 r am)))^2\}\}]\}

```

The following uses this cutoff to avoid numerical problems

```
cutoff = 10^(-12);
```

Restricting answers to cases of polymorphic equilibria that are internally stable

```

Clear[sieveXY]
sieveXY[FAA_, FAa_, Faa_, MAA_, MAa_, Maa_, wAf_, waf_, wAm_, wam_, af_, am_, r_] :=
  sieveXY[FAA, FAa, Faa, MAA, MAa, Maa, wAf, waf, wAm, wam, af, am, r] =
  Block[{}, For[i = 1; write = {}, i <= (max = Length[eq =
    equilXY[FAA, FAa, Faa, MAA, MAa, Maa, wAf, waf, wAm, wam, af, am, r]]), i++,
    If[Length[test = Cases[eq[[i]], x_ /; ((-cutoff <= Re[x] <= 1 + cutoff) &&
      Abs[Im[x]] < cutoff)]]] == 4 &&
    (Chop[eq[[i]]] != {0, 0, 0, 0}) && (Chop[eq[[i]] - {1, 1, 1, 1}] != {0, 0, 0, 0}) &&
    (Max[Abs[internstabXY[FAA, FAa, Faa, MAA, MAa, Maa, wAf, waf, wAm,
      wam, af, am, r, eq[[i, 1]], eq[[i, 2]], eq[[i, 3]], eq[[i, 4]]]]] <
     1), write = Append[write, eq[[i]]]];
  Sort[write]
]

```

Invasion of neo-W into XY

Invasion fitness (external stability)

(*charpolyk1//Simplify*)

```

Clear[invasionplotXY]
invasionplotXY[tryFAA_, tryFAa_, tryFaa_, tryMAA_, tryMAa_, tryMaa_,
  trywAf_, trywaf_, trywAm_, trywam_, tryaf_, tryam_, tryr_, tryR_, tryρ_] :=
  Block[{valideq, temp, pXf, pXm, pYf, pYm, q, subs, r, R, ρ},
    valideq = sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
      tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr];
    If[Length[valideq] == 1,
      temp = Flatten[valideq];
      pXf = temp[[1]];
      pXm = temp[[2]];
      pYf = 0;
      pYm = temp[[3]];
      q = temp[[4]];
      subs =
        {FAA → tryFAA, FAa → tryFAa, Faa → tryFaa, MAA → tryMAA, MAa → tryMAa, Maa → tryMaa,
         wAf → trywAf, waf → trywaf, wAm → trywAm, wam → trywam, af → tryaf, am → tryam};
      r = tryr;
      R = tryR;
      ρ = tryρ;
      Max[
        λ /. NSolve[0 == (λ^4 (2 Faa^2 (-1 + pXf) (-1 + pXm) (-1 + q) waf^2 wam^2 λ (1 - pYm q - 2 λ +
          2 pXf λ + 2 q λ - 2 pXf q λ + pXm (-1 + q) (1 + 2 (-1 + pXf) λ)) -
          Faa waf wam (FAA wAf wAm (pYm q (-1 + pYm q + 2 (-1 + pXf) (-1 + q) λ) - pXm^2
            (-1 + q)^2 (-1 + 2 λ + 8 (-1 + pXf) pXf λ^2) + pXm (-1 + q) (1 + 2 pYm q
            (-1 + λ) - 2 (1 + (-1 + pXf) q) λ + 8 (-1 + pXf) pXf (-1 + q) λ^2)) +
            2 Faa (wAf wam ((1 + pXm (-1 + q) - pYm q) (-1 + R) af (1 - pYm q - 2 λ +
              2 pXf λ + 2 q λ - 2 pXf q λ + pXm (-1 + q) (1 + 2 (-1 + pXf) λ)) +
              pXf (-1 + pXm) (-1 + q) λ (1 - pYm q - 4 λ + 4 pXf λ +
              4 q λ - 4 pXf q λ + pXm (-1 + q) (1 + 4 (-1 + pXf) λ))) +
              (-1 + pXf) (-1 + q) waf wAm λ (2 pYm q (-1 + R) (-1 + af) +
              pXm^2 (-1 + q) (3 + 2 R (-1 + af) - 2 af - 4 λ + 4 pXf λ) +
              pXm (3 + 2 R (-1 + af) - 2 af - 4 λ + 4 pXf λ + q (pYm (-3 + 2 R + 2 af -

```

$$\begin{aligned}
& 2 R \alpha f) + 2 (-1 + R + \alpha f - R \alpha f + 2 \lambda - 2 pXf \lambda))) \}) \} + \\
& 2 (Faa^2 pXf pXm (-1 + q) wAf^2 wAm^2 \lambda (pYm q + pXm (-1 + q) (-1 + 2 pXf \lambda)) - \\
& Faa FAA wAf wAm (pYm q (pYm q waf wAm (-1 + R + \alpha f - R \alpha f) - pXf (-1 + q) \\
& wAf wam \lambda) + pXm^2 (-1 + q)^2 (pXf wAf wam \lambda (-1 + 2 (-1 + R) \alpha f + \\
& 4 pXf \lambda) + waf wAm (-1 + \alpha f + \lambda + pXf \lambda - 2 pXf \alpha f \lambda - 4 pXf \lambda^2 + \\
& 4 pXf^2 \lambda^2 + R (-1 + \alpha f) (-1 + 2 pXf \lambda)) - pXm (-1 + q) \\
& (pXf wAf wam \lambda (1 + 2 \alpha f - 2 R \alpha f - 4 pXf \lambda + q (-1 + 4 pXf \lambda)) + \\
& pYm q (pXf wAf wam (-1 + 2 (-1 + R) \alpha f) \lambda + waf wAm (-2 + 2 \alpha f + \\
& \lambda + pXf \lambda - 2 pXf \alpha f \lambda + 2 R (-1 + \alpha f) (-1 + pXf \lambda)))) \} + \\
& 2 Faa^2 (wAf wam (pYm^2 q^2 waf wAm \alpha f (-1 - 2 R (-1 + \alpha f) + \alpha f) + \\
& pXf (-1 + q) wAf wam \lambda (\alpha f - R \alpha f + pXf (-1 + q) \lambda) + \\
& pYm q (pXf (-1 + q) (-1 + R) wAf wam \alpha f \lambda + waf wAm \\
& (-1 + \alpha f) ((-1 + 2 R) \alpha f + pXf (-1 + q) (-1 + R) \lambda)) \} + \\
& pXm^2 (-1 + q)^2 (pXf wAf^2 wam^2 \lambda ((-1 + R) \alpha f + pXf \lambda) + \\
& (-1 + pXf) waf^2 wAm^2 \lambda (1 + R (-1 + \alpha f) - \alpha f - \lambda + pXf \lambda) + \\
& waf wAf wam wAm ((1 - 2 R) \alpha f^2 + pXf \lambda (1 - R + 2 (-1 + pXf) \lambda) + \\
& \alpha f (-1 + \lambda - 2 pXf \lambda + R (2 + (-1 + 2 pXf) \lambda)) \} - \\
& pXm (-1 + q) ((-1 + pXf) pYm q (-1 + R) waf^2 wAm^2 (-1 + \alpha f) \lambda + \\
& pXf wAf^2 wam^2 \lambda ((-2 + q + pYm q) (-1 + R) \alpha f + 2 pXf (-1 + q) \lambda) + \\
& waf wAf wam wAm (-(-1 + 2 pYm q) (-1 + 2 R) \alpha f^2 + pXf \lambda \\
& (-1 + R + 2 \lambda - 2 pXf \lambda + q (1 + pYm - R - pYm R - 2 \lambda + 2 pXf \lambda)) + \\
& \alpha f (1 - \lambda + 2 pXf \lambda - pXf q \lambda + R (-2 + \lambda - 2 pXf \lambda + pXf q \lambda) + \\
& pYm q (-2 + \lambda - 2 pXf \lambda + R (4 - \lambda + 2 pXf \lambda)))) \} \} \} + \\
& (2 Faa^2 (-1 + pXf) (-1 + pXm) (-1 + q) waf^2 wam^2 \lambda (1 - 2 \lambda + 2 pXf \lambda + \\
& 2 q \lambda - 2 pXf q \lambda + pXm (-1 + q) (1 + 2 (-1 + pXf) \lambda - \rho) + \\
& pYm q (-1 + \rho) - \rho) + 2 Faa^2 pXf pXm (-1 + q) wAf^2 \\
& wAm^2 \lambda (-pYm q (-1 + \rho) + pXm (-1 + q) (-1 + 2 pXf \lambda + \rho)) - \\
& Faa FAA wAf wAm (pYm q (-1 + \rho) (2 pXf (-1 + q) wAf wam \lambda + \\
& pYm q waf wAm (-1 + \alpha f) (-2 + r + R + \rho)) + pXm^2 (-1 + q)^2 \\
& (2 pXf wAf wam \lambda (-1 + 4 pXf \lambda + \rho + \alpha f (-2 + r + R + \rho)) + waf wAm \\
& (-2 + 2 \alpha f + 2 \lambda + 2 pXf \lambda - 4 pXf \alpha f \lambda - 8 pXf \lambda^2 + 8 pXf^2 \lambda^2 + 3 \rho - 3 \alpha f \\
& \rho - 2 \lambda \rho + 2 pXf \alpha f \lambda \rho - \rho^2 + \alpha f \rho^2 + r (-1 + \alpha f) (-1 + 2 pXf \lambda + \rho) + \\
& R (-1 + \alpha f) (-1 + 2 pXf \lambda + \rho)) \} - 2 pXm (-1 + q) (pXf wAf wam \\
& \lambda (1 - 4 pXf \lambda - \rho - \alpha f (-2 + r + R + \rho) + q (-1 + 4 pXf \lambda + \rho)) + \\
& pYm q (pXf wAf wam \lambda (-1 + \rho + \alpha f (-2 + r + R + \rho)) + waf wAm (-2 + 2 \alpha f + \\
& \lambda + pXf \lambda - 2 pXf \alpha f \lambda + 3 \rho - 3 \alpha f \rho - \lambda \rho + pXf \alpha f \lambda \rho - \rho^2 + \alpha f \rho^2 + \\
& r (-1 + \alpha f) (-1 + pXf \lambda + \rho) + R (-1 + \alpha f) (-1 + pXf \lambda + \rho)) \} \} \} + \\
& 2 Faa^2 (pXm^2 (-1 + q)^2 ((-1 + pXf) waf^2 wAm^2 \lambda (2 + r (-1 + \alpha f) + \\
& R (-1 + \alpha f) - 2 \alpha f - 2 \lambda + 2 pXf \lambda - \rho + \alpha f \rho) + \\
& pXf wAf^2 wam^2 \lambda (2 pXf \lambda + \alpha f (-2 + r + R + \rho)) + waf wAf wam \\
& wAm (pXf \lambda (2 - r - R - 4 \lambda + 4 pXf \lambda - \rho) + 2 (-1 + r + R) \alpha f^2 (-1 + \rho) + \\
& \alpha f (-2 + 2 \lambda - 4 pXf \lambda + R (2 - \lambda + 2 pXf \lambda - 2 \rho) + \\
& r (2 + (-1 + 2 pXf) \lambda - 2 \rho) + 2 \rho - \lambda \rho + 2 pXf \lambda \rho) \} \} + \\
& wAf wam (2 pYm^2 q^2 (-1 + r + R) waf wAm (-1 + \alpha f) \alpha f (-1 + \rho) - \\
& pXf (-1 + q) wAf wam \lambda (-2 pXf (-1 + q) \lambda + \alpha f (-2 + r + R + \rho)) +
\end{aligned}$$

$$\begin{aligned}
& pYm q (pXf (-1 + q) wAf wam \alpha f \lambda (-2 + r + R + \rho) - waf wAm (-1 + \alpha f) \\
& (2 (-1 + r + R) \alpha f (-1 + \rho) - pXf (-1 + q) \lambda (-2 + r + R + \rho))) - \\
& pXm (-1 + q) ((-1 + pXf) pYm q waf^2 wAm^2 (-1 + \alpha f) \lambda (-2 + r + R + \rho)) + \\
& pXf wAf^2 wam^2 \lambda (4 pXf (-1 + q) \lambda + (-2 + q + pYm q) \alpha f (-2 + r + R + \rho)) + \\
& waf wAf wam wAm (2 (-1 + 2 pYm q) (-1 + r + R) \alpha f^2 (-1 + \rho)) + \\
& \alpha f (2 - 2 R - 2 \lambda + 4 pXf \lambda - 2 pXf q \lambda + R \lambda - 2 pXf R \lambda + pXf q R \lambda - 2 \rho + \\
& 2 R \rho + \lambda \rho - 2 pXf \lambda \rho + pXf q \lambda \rho + r (-2 + \lambda - 2 pXf \lambda + pXf q \lambda + \\
& 2 \rho) + pYm q (-4 + 2 \lambda - 4 pXf \lambda + R (4 - \lambda + 2 pXf \lambda - 4 \rho) + \\
& r (4 + (-1 + 2 pXf) \lambda - 4 \rho) + 4 \rho - \lambda \rho + 2 pXf \lambda \rho)) + pXf \lambda \\
& (-2 + r + R + 4 \lambda - 4 pXf \lambda + \rho - q (-2 + r + R + 4 \lambda - 4 pXf \lambda + \\
& \rho + pYm (-2 + r + R + \rho))) + Faa waf wam (FAA wAf wAm \\
& (pXm^2 (-1 + q)^2 (8 (-1 + pXf) pXf \lambda^2 - 2 \lambda (-1 + \rho) - (-1 + \rho)^2) - \\
& pYm q (1 + 2 (-1 + pXf + q - pXf q) \lambda + pYm q (-1 + \rho) - \rho) (-1 + \rho) + \\
& pXm (-1 + q) (-8 (-1 + pXf) pXf (-1 + q) \lambda^2 - 2 (1 + (-1 + pXf) q) \\
& \lambda (-1 + \rho) - (-1 + \rho)^2 + 2 pYm q (-1 + \rho) (-1 + \lambda + \rho))) + \\
& FAa (wAf wam (-2 pXf (-1 + pXm) (-1 + q) \lambda (1 - 4 \lambda + 4 pXf \lambda + 4 q \lambda - \\
& 4 pXf q \lambda + pXm (-1 + q) (1 + 4 (-1 + pXf) \lambda - \rho) + pYm q (-1 + \rho) - \\
& \rho) + (1 + pXm (-1 + q) - pYm q) \alpha f (-2 + r + R + \rho) (-1 + 2 \lambda - \\
& 2 pXf \lambda - 2 q \lambda + 2 pXf q \lambda - pXm (-1 + q) (1 + 2 (-1 + pXf) \lambda - \rho) + \\
& \rho + pYm (q - q \rho))) - 2 (-1 + pXf) (-1 + q) waf \\
& wAm \lambda (pYm q (-1 + \alpha f) (-2 + r + R + \rho) + pXm^2 (-1 + q) \\
& (3 + r (-1 + \alpha f) + R (-1 + \alpha f) - 2 \alpha f - 4 \lambda + 4 pXf \lambda - 2 \rho + \alpha f \rho) - \\
& pXm (-3 + r + R + 2 \alpha f - r \alpha f - R \alpha f + 4 \lambda - 4 pXf \lambda + 2 \rho - \alpha f \rho + \\
& q (2 - R + r (-1 + \alpha f) - 2 \alpha f + R \alpha f - 4 \lambda + 4 pXf \lambda - \rho + \alpha f \rho + pYm \\
& (3 + r (-1 + \alpha f) + R (-1 + \alpha f) - 2 \alpha f - 2 \rho + \alpha f \rho))))))) / \\
& (16 (-1 + q)^4 (Faa (-1 + pXf) (-1 + pXm) waf wam + FAA pXf pXm wAf wAm + \\
& FAa (pXm waf wAm - pXf ((-1 + pXm) wAf wam + \\
& pXm waf wAm)))^4) /. subs, \lambda]] - 1 \\
&] \\
&]
\end{aligned}$$

Invasion of neo-W into XY (allows two equilibria)

```

Clear[invasionplotXY2]
invasionplotXY2[
{tryFAA_, tryFAa_, tryFaa_, tryMAA_, tryMaa_, tryFA_, tryFa_, tryMA_,
 tryMa_, tryFdA_, tryMdA_, tryr_, tryR_, tryo_}, {pXf_, pXm_, pYm_, q_}] :=
Block[{subs, r, R, \rho},
subs =
{FAA \rightarrow tryFAA, FAa \rightarrow tryFAa, Faa \rightarrow tryFaa, MAA \rightarrow tryMAA, MAa \rightarrow tryMaa, Maa \rightarrow tryMaa,
wAf \rightarrow tryFA, waf \rightarrow tryFa, wAm \rightarrow tryMA, wam \rightarrow tryMa, \alpha f \rightarrow tryFdA, \alpha m \rightarrow tryMdA};
r = tryr;
R = tryR;
\rho = tryo;
Max[
\lambda /. NSolve[0 == 0 == (\lambda^4 (2 Faa^2 (-1 + pXf) (-1 + pXm) (-1 + q) waf^2 wam^2 \lambda (1 - pYm q - 2 \lambda +
2 pXf \lambda + 2 q \lambda - 2 pXf q \lambda + pXm (-1 + q) (1 + 2 (-1 + pXf) \lambda)) -
Faa waf wam (FAA wAf wAm (pYm q (-1 + pYm q + 2 (-1 + pXf) (-1 + q) \lambda) -
```

$$\begin{aligned}
& pXm^2 (-1 + q)^2 (-1 + 2 \lambda + 8 (-1 + pXf) pXf \lambda^2) + pXm (-1 + q) (1 + 2 pYm q \\
& (-1 + \lambda) - 2 (1 + (-1 + pXf) q) \lambda + 8 (-1 + pXf) pXf (-1 + q) \lambda^2)) + \\
& 2 Faa (wAf wam ((1 + pXm (-1 + q) - pYm q) (-1 + R) \alpha f (1 - pYm q - 2 \lambda + \\
& 2 pXf \lambda + 2 q \lambda - 2 pXf q \lambda + pXm (-1 + q) (1 + 2 (-1 + pXf) \lambda)) + \\
& pXf (-1 + pXm) (-1 + q) \lambda (1 - pYm q - 4 \lambda + 4 pXf \lambda + 4 q \lambda - \\
& 4 pXf q \lambda + pXm (-1 + q) (1 + 4 (-1 + pXf) \lambda))) + \\
& (-1 + pXf) (-1 + q) waf wAm \lambda (2 pYm q (-1 + R) (-1 + \alpha f) + \\
& pXm^2 (-1 + q) (3 + 2 R (-1 + \alpha f) - 2 \alpha f - 4 \lambda + 4 pXf \lambda) + \\
& pXm (3 + 2 R (-1 + \alpha f) - 2 \alpha f - 4 \lambda + 4 pXf \lambda + q (pYm (-3 + 2 R + 2 \alpha f - \\
& 2 R \alpha f) + 2 (-1 + R + \alpha f - R \alpha f + 2 \lambda - 2 pXf \lambda))))) + \\
& 2 (Faa^2 pXf pXm (-1 + q) wAf^2 wAm^2 \lambda (pYm q + pXm (-1 + q) (-1 + 2 pXf \lambda)) - \\
& Faa FAA wAf wAm (pYm q (pYm q waf wAm (-1 + R + \alpha f - R \alpha f) - pXf (-1 + q) \\
& wAf wam \lambda) + pXm^2 (-1 + q)^2 (pXf wAf wam \lambda (-1 + 2 (-1 + R) \alpha f + \\
& 4 pXf \lambda) + waf wAm (-1 + \alpha f + \lambda + pXf \lambda - 2 pXf \alpha f \lambda - 4 pXf \lambda^2 + \\
& 4 pXf^2 \lambda^2 + R (-1 + \alpha f) (-1 + 2 pXf \lambda))) - pXm (-1 + q) \\
& (pXf wAf wam \lambda (1 + 2 \alpha f - 2 R \alpha f - 4 pXf \lambda + q (-1 + 4 pXf \lambda)) + \\
& pYm q (pXf wAf wam (-1 + 2 (-1 + R) \alpha f) \lambda + waf wAm (-2 + 2 \alpha f + \\
& \lambda + pXf \lambda - 2 pXf \alpha f \lambda + 2 R (-1 + \alpha f) (-1 + pXf \lambda)))) + \\
& 2 Faa^2 (wAf wam (pYm^2 q^2 waf wAm \alpha f (-1 - 2 R (-1 + \alpha f) + \alpha f) + \\
& pXf (-1 + q) wAf wam \lambda (\alpha f - R \alpha f + pXf (-1 + q) \lambda) + \\
& pYm q (pXf (-1 + q) (-1 + R) wAf wam \alpha f \lambda + waf wAm \\
& (-1 + \alpha f) ((-1 + 2 R) \alpha f + pXf (-1 + q) (-1 + R) \lambda))) + \\
& pXm^2 (-1 + q)^2 (pXf wAf^2 wam^2 \lambda ((-1 + R) \alpha f + pXf \lambda) + \\
& (-1 + pXf) waf^2 wAm^2 \lambda (1 + R (-1 + \alpha f) - \alpha f - \lambda + pXf \lambda) + \\
& waf wAf wam wAm ((1 - 2 R) \alpha f^2 + pXf \lambda (1 - R + 2 (-1 + pXf) \lambda) + \\
& \alpha f (-1 + \lambda - 2 pXf \lambda + R (2 + (-1 + 2 pXf) \lambda)))) - \\
& pXm (-1 + q) ((-1 + pXf) pYm q (-1 + R) waf^2 wAm^2 (-1 + \alpha f) \lambda + \\
& pXf wAf^2 wam^2 \lambda ((-2 + q + pYm q) (-1 + R) \alpha f + 2 pXf (-1 + q) \lambda) + \\
& waf wAf wam wAm (-(-1 + 2 pYm q) (-1 + 2 R) \alpha f^2 + pXf \lambda \\
& (-1 + R + 2 \lambda - 2 pXf \lambda + q (1 + pYm - R - pYm R - 2 \lambda + 2 pXf \lambda)) + \\
& \alpha f (1 - \lambda + 2 pXf \lambda - pXf q \lambda + R (-2 + \lambda - 2 pXf \lambda + pXf q \lambda) + \\
& pYm q (-2 + \lambda - 2 pXf \lambda + R (4 - \lambda + 2 pXf \lambda)))))) \\
& (2 Faa^2 (-1 + pXf) (-1 + pXm) (-1 + q) waf^2 wam^2 \lambda (1 - 2 \lambda + 2 pXf \lambda + \\
& 2 q \lambda - 2 pXf q \lambda + pXm (-1 + q) (1 + 2 (-1 + pXf) \lambda - \rho) + \\
& pYm q (-1 + \rho) - \rho) + 2 Faa^2 pXf pXm (-1 + q) wAf^2 \\
& wAm^2 \lambda (-pYm q (-1 + \rho) + pXm (-1 + q) (-1 + 2 pXf \lambda + \rho)) - \\
& Faa FAA wAf wAm (pYm q (-1 + \rho) (2 pXf (-1 + q) wAf wam \lambda + \\
& pYm q waf wAm (-1 + \alpha f) (-2 + r + R + \rho))) + \\
& pXm^2 (-1 + q)^2 (2 pXf wAf wam \lambda (-1 + 4 pXf \lambda + \rho + \alpha f (-2 + r + R + \rho)) + \\
& waf wAm (-2 + 2 \alpha f + 2 \lambda + 2 pXf \lambda - 4 pXf \alpha f \lambda - 8 pXf \lambda^2 + 8 pXf^2 \lambda^2 + \\
& 3 \rho - 3 \alpha f \rho - 2 \lambda \rho + 2 pXf \alpha f \lambda \rho - \rho^2 + \alpha f \rho^2 + r (-1 + \alpha f) \\
& (-1 + 2 pXf \lambda + \rho) + R (-1 + \alpha f) (-1 + 2 pXf \lambda + \rho))) - 2 pXm (-1 + q) \\
& (pXf wAf wam \lambda (1 - 4 pXf \lambda - \rho - \alpha f (-2 + r + R + \rho) + q (-1 + 4 pXf \lambda + \rho)) + \\
& pYm q (pXf wAf wam \lambda (-1 + \rho + \alpha f (-2 + r + R + \rho)) + waf wAm (-2 + 2 \alpha f + \\
& \lambda + pXf \lambda - 2 pXf \alpha f \lambda + 3 \rho - 3 \alpha f \rho - \lambda \rho + pXf \alpha f \lambda \rho - \rho^2 + \alpha f \rho^2 +
\end{aligned}$$

$$\begin{aligned}
& r (-1 + \alpha f) (-1 + pXf \lambda + \rho) + R (-1 + \alpha f) (-1 + pXf \lambda + \rho))))) + \\
& 2 Faa^2 (pXm^2 (-1 + q)^2 ((-1 + pXf) waf^2 wAm^2 \lambda (2 + r (-1 + \alpha f) + \\
& R (-1 + \alpha f) - 2 \alpha f - 2 \lambda + 2 pXf \lambda - \rho + \alpha f \rho) + \\
& pXf wAf^2 wam^2 \lambda (2 pXf \lambda + \alpha f (-2 + r + R + \rho)) + waf wAf wam \\
& wAm (pXf \lambda (2 - r - R - 4 \lambda + 4 pXf \lambda - \rho) + 2 (-1 + r + R) \alpha f^2 (-1 + \rho) + \\
& \alpha f (-2 + 2 \lambda - 4 pXf \lambda + R (2 - \lambda + 2 pXf \lambda - 2 \rho) + \\
& r (2 + (-1 + 2 pXf) \lambda - 2 \rho) + 2 \rho - \lambda \rho + 2 pXf \lambda \rho))) + \\
& wAf wam (2 pYm^2 q^2 (-1 + r + R) waf wAm (-1 + \alpha f) \alpha f (-1 + \rho) - \\
& pXf (-1 + q) wAf wam \lambda (-2 pXf (-1 + q) \lambda + \alpha f (-2 + r + R + \rho)) + \\
& pYm q (pXf (-1 + q) wAf wam \alpha f \lambda (-2 + r + R + \rho) - waf wAm (-1 + \alpha f) \\
& (2 (-1 + r + R) \alpha f (-1 + \rho) - pXf (-1 + q) \lambda (-2 + r + R + \rho)))) - \\
& pXm (-1 + q) ((-1 + pXf) pYm q waf^2 wAm^2 (-1 + \alpha f) \lambda (-2 + r + R + \rho) + \\
& pXf wAf^2 wam^2 \lambda (4 pXf (-1 + q) \lambda + (-2 + q + pYm q) \alpha f (-2 + r + R + \rho)) + \\
& waf wAf wam wAm (2 (-1 + 2 pYm q) (-1 + r + R) \alpha f^2 (-1 + \rho) + \\
& \alpha f (2 - 2 R - 2 \lambda + 4 pXf \lambda - 2 pXf q \lambda + R \lambda - 2 pXf R \lambda + pXf q R \lambda - 2 \rho + \\
& 2 R \rho + \lambda \rho - 2 pXf \lambda \rho + pXf q \lambda \rho + r (-2 + \lambda - 2 pXf \lambda + pXf q \lambda + \\
& 2 \rho) + pYm q (-4 + 2 \lambda - 4 pXf \lambda + R (4 - \lambda + 2 pXf \lambda - 4 \rho) + \\
& r (4 + (-1 + 2 pXf) \lambda - 4 \rho) + 4 \rho - \lambda \rho + 2 pXf \lambda \rho)) + pXf \lambda \\
& (-2 + r + R + 4 \lambda - 4 pXf \lambda + \rho - q (-2 + r + R + 4 \lambda - 4 pXf \lambda + \\
& \rho + pYm (-2 + r + R + \rho)))) + Faa waf wam (Faa wAf wAm \\
& (pXm^2 (-1 + q)^2 (8 (-1 + pXf) pXf \lambda^2 - 2 \lambda (-1 + \rho) - (-1 + \rho)^2) - \\
& pYm q (1 + 2 (-1 + pXf + q - pXf q) \lambda + pYm q (-1 + \rho) - \rho) (-1 + \rho) + \\
& pXm (-1 + q) (-8 (-1 + pXf) pXf (-1 + q) \lambda^2 - 2 (1 + (-1 + pXf) q) \\
& \lambda (-1 + \rho) - (-1 + \rho)^2 + 2 pYm q (-1 + \rho) (-1 + \lambda + \rho)))) + \\
& Faa (wAf wam (-2 pXf (-1 + pXm) (-1 + q) \lambda (1 - 4 \lambda + 4 pXf \lambda + \\
& 4 q \lambda - 4 pXf q \lambda + pXm (-1 + q) (1 + 4 (-1 + pXf) \lambda - \rho) + \\
& pYm q (-1 + \rho) - \rho) + (1 + pXm (-1 + q) - pYm q) \alpha f \\
& (-2 + r + R + \rho) (-1 + 2 \lambda - 2 pXf \lambda - 2 q \lambda + 2 pXf q \lambda - \\
& pXm (-1 + q) (1 + 2 (-1 + pXf) \lambda - \rho) + \rho + pYm (q - q \rho))) - \\
& 2 (-1 + pXf) (-1 + q) waf wAm \lambda (pYm q (-1 + \alpha f) (-2 + r + R + \rho) + \\
& pXm^2 (-1 + q) (3 + r (-1 + \alpha f) + R (-1 + \alpha f) - 2 \alpha f - 4 \lambda + 4 pXf \lambda - 2 \rho + \\
& \alpha f \rho) - pXm (-3 + r + R + 2 \alpha f - r \alpha f - R \alpha f + 4 \lambda - 4 pXf \lambda + 2 \rho - \\
& \alpha f \rho + q (2 - R + r (-1 + \alpha f) - 2 \alpha f + R \alpha f - 4 \lambda + 4 pXf \lambda - \rho + \alpha f \rho + \\
& pYm (3 + r (-1 + \alpha f) + R (-1 + \alpha f) - 2 \alpha f - 2 \rho + \alpha f \rho))))))) / \\
& (16 (-1 + q)^4 (Faa (-1 + pXf) (-1 + pXm) waf wam + Faa pXf pXm wAf wAm + Faa \\
& (pXm waf wAm - pXf ((-1 + pXm) wAf wam + pXm waf wAm))^4) /. subs, \lambda]] - 1 \\
&]
\end{aligned}$$

Recursions

Code to convert allele frequencies (from the equilibrium calculations) into haplotype frequencies, as needed for the simulations:

[Assumes that all haplotypes convert from M to m at a small rate, pm. This will cause imbalanced sex ratios at first, so ignore the first few generations, as it reequilibrates.]

```

Clear[startgen]
startgen[{pXf_, pXm_, pYm_, q_}, pm_] :=
  startgen[{pXf, pXm, pYm, q}, pm] =
    {XAMf, XaMf, XAmf, Xamf, YAMf, YaMf, YAmf,
     Yamf, XAMm, XaMm, XAmm, Xamm, YAMm, YaMm, YAmm} /. {
      XAMf → pXf (1 - pm),
      XaMf → (1 - pXf) (1 - pm),
      YAMf → 0 (1 - pm),
      YaMf → 0 (1 - pm),
      XAMm → (1 - q) pXm (1 - pm),
      XaMm → (1 - q) (1 - pXm) (1 - pm),
      YAMm → q pYm (1 - pm),
      YaMm → q (1 - pYm) (1 - pm),
      XAmf → pXf (pm),
      Xamf → (1 - pXf) (pm),
      YAmf → 0 (pm),
      Yamf → 0 (pm),
      XAmm → (1 - q) pXm (pm),
      Xamm → (1 - q) (1 - pXm) (pm),
      YAmm → q pYm (pm),
      Yamm → q (1 - pYm) (pm)
    }
}

```

Recursion for haplotype frequencies

```

Clear[generation];
generation[{FAA_, FAa_, Faa_, MAA_, MAa_, Maa_, wAf_, waf_, wAm_, wam_,
  af_, am_, r_, R_, ρ_, k_}, {XAMf_, XaMf_, XAmf_, Xamf_, YAMf_, YaMf_,
  YAmf_, XAMm_, XaMm_, XAmm_, Xamm_, YAMm_, YaMm_, YAmm_, Yamm_}] :=
  generation[{FAA, FAa, Faa, MAA, MAa, Maa, wAf, waf, wAm, wam, af, am, r, R, ρ, k},
  {XAMf, XaMf, XAmf, Xamf, YAMf, YaMf, YAmf, XAMm, XaMm, XAmm, Xamm, YAMm, YaMm, YAmm, Yamm}] =
  Block[
  {
    kXXMM = 1,
    kXYMM = 0,
    kYYMM = 0,
    kXXMm = k,
    kXYMm = k,
    kYYMm = k,
    kXXmm = k,
    kXYmm = k,
    kYYmm = k
  },
  (*HAPLOID COMPETITION*)

  (*mean fitness of female gametes*)
  wbarHapFemale = (wAf XAMf + waf XaMf + wAf XAmf + waf Xamf) +
    (wAf YAMf + waf YaMf + wAf YAmf + waf Yamf);

  (*non-mutant eggs*)
  XAMfs = wAf XAMf / wbarHapFemale;
  XaMfs = waf XaMf / wbarHapFemale;
  YAMfs = wAf YAMf / wbarHapFemale;

```

```

YaMfs = waf YaMf / wbarHapFemale;

(*mutant eggs*)
XAmfs = wAf XAmf / wbarHapFemale;
Xamfs = waf Xamf / wbarHapFemale;
YAmfs = wAf YAmf / wbarHapFemale;
Yamfs = waf Yamf / wbarHapFemale;

(*mean fitness of male gametes*)
wbarHapMale = (wAm XAMm + wam XaMm + wAm XAmm + wam Xamm) +
(wAm YAMm + wam YaMm + wAm YAmm + wam Yamm);

(*non-mutant male gametes*)
XAMms = wAm XAMm / wbarHapMale;
XaMms = wam XaMm / wbarHapMale;
YAMms = wAm YAMm / wbarHapMale;
YaMms = wam YaMm / wbarHapMale;

(*mutant male gametes*)
XAmms = wAm XAmm / wbarHapMale;
Xamms = wam Xamm / wbarHapMale;
YAmms = wAm YAmm / wbarHapMale;
Yamms = wam Yamm / wbarHapMale;

(*RANDOM MATING*)

(*MM Homozygotes*)
(*XM-XM females*)
XAMXAMfemale = kXXMM (XAMfs XAMms);
XAMXaMfemale = kXXMM (XAMfs XaMms + XaMfs XAMms);
XaMXaMfemale = kXXMM (XaMfs XaMms);
(*XM-XM males*)
XAMXAMmale = (1 - kXXMM) (XAMfs XAMms);
XAMXaMmale = (1 - kXXMM) (XAMfs XaMms + XaMfs XAMms);
XaMXaMmale = (1 - kXXMM) (XaMfs XaMms);

(*XM-YM females*)
XAMYAMfemale = kXYMM (XAMfs YAMms + YAMfs XAMms);
(*XM-YM males*)
XAMYAMmale = (1 - kXYMM) (XAMfs YAMms + YAMfs XAMms);
XAMYaMmale = (1 - kXYMM) (XAMfs YaMms + YaMfs XAMms);
XaMYAMmale = (1 - kXYMM) (XaMfs YAMms + YAMfs XaMms);
XaMYaMmale = (1 - kXYMM) (XaMfs YaMms + YaMfs XaMms);

(*YM-YM females*)
YAMYAMfemale = kYYMM (YAMfs YAMms);
YAMYaMfemale = kYYMM (YAMfs YaMms + YaMfs YAMms);
YaMYaMfemale = kYYMM (YaMfs YaMms);
(*YM-YM males*)
YAMYAMmale = (1 - kYYMM) (YAMfs YAMms);
YAMYaMmale = (1 - kYYMM) (YAMfs YaMms + YaMfs YAMms);
YaMYaMmale = (1 - kYYMM) (YaMfs YaMms);

```

```

(*Mm heterozygotes*)
(*XM-Xm females*)
XAmXAMfemale = kXXMm (XAmfs XAMms + XAMfs XAmms);
XAmXaMfemale = kXXMm (XAmfs XaMms + XaMfs XAmms );
XAMXamfemale = kXXMm (XAMfs Xamms + Xamfs XAMms);
XamXaMfemale = kXXMm (Xamfs XaMms + XaMfs Xamms );
(*XM-Xm males*)
XAmXAMmale = (1 - kXXMm) (XAmfs XAMms + XAMfs XAmms);
XAmXaMmale = (1 - kXXMm) (XAmfs XaMms + XaMfs XAmms );
XAMXammale = (1 - kXXMm) (XAMfs Xamms + Xamfs XAMms);
XamXaMmale = (1 - kXXMm) (Xamfs XaMms + XaMfs Xamms);

(*Xm-YM females*)
XAmYAMfemale = kXYMm (XAmfs YAMms + YAMfs XAmms);
XAmYaMfemale = kXYMm (XAmfs YaMms + YaMfs XAmms );
XamYAMfemale = kXYMm (Xamfs YAMms + YAMfs Xamms );
XamYaMfemale = kXYMm (Xamfs YaMms + YaMfs Xamms );
(*Xm-YM males*)
XAmYAMmale = (1 - kXYMm) (XAmfs YAMms + YAMfs XAmms);
XAmYaMmale = (1 - kXYMm) (XAmfs YaMms + YaMfs XAmms );
XamYAMmale = (1 - kXYMm) (Xamfs YAMms + YAMfs Xamms );
XamYaMmale = (1 - kXYMm) (Xamfs YaMms + YaMfs Xamms);

(*XM-Ym females*)
XAMYAmfemale = kXYMm (XAMfs YAmms + YAmfs XAMms);
XAMYamfemale = kXYMm (XAMfs Yamms + Yamfs XAMms );
XaMYAmfemale = kXYMm (XaMfs YAmms + YAmfs XaMms );
XaMYamfemale = kXYMm (XaMfs Yamms + Yamfs XaMms );
(*XM-Ym females*)
XAMYAmmale = (1 - kXYMm) (XAMfs YAmms + YAmfs XAMms);
XAMYammale = (1 - kXYMm) (XAMfs Yamms + Yamfs XAMms );
XaMYAmmale = (1 - kXYMm) (XaMfs YAmms + YAmfs XaMms );
XaMYammale = (1 - kXYMm) (XaMfs Yamms + Yamfs XaMms);

(*Ym-YM females*)
YAmYAMfemale = kYYMm (YAmfs YAMms + YAMfs YAmms);
YAmYaMfemale = kYYMm (YAmfs YaMms + YaMfs YAmms );
YamYAMfemale = kYYMm (Yamfs YAMms + YAMfs Yamms );
YamYaMfemale = kYYMm (Yamfs YaMms + YaMfs Yamms );
(*Ym-YM males*)
YAmYAMmale = (1 - kYYMm) (YAmfs YAMms + YAMfs YAmms);
YAmYaMmale = (1 - kYYMm) (YAmfs YaMms + YaMfs YAmms );
YamYAMmale = (1 - kYYMm) (Yamfs YAMms + YAMfs Yamms );
YamYaMmale = (1 - kYYMm) (Yamfs YaMms + YaMfs Yamms );

(*mm heterozygotes*)
(*Xm-Xm females*)
XAmXAmfemale = kXXmm (XAmfs XAmms);
XAmXamfemale = kXXmm (XAmfs Xamms + Xamfs XAmms );
XamXamfemale = kXXmm (Xamfs Xamms );
(*Xm-Xm males*)
XAmXAmmale = (1 - kXXmm) (XAmfs XAmms);
XAmXammale = (1 - kXXmm) (XAmfs Xamms + Xamfs XAmms );
XamXammale = (1 - kXXmm) (Xamfs Xamms );

```

```

(*Xm-Ym females*)
XAmYAmfemale = kXYmm (XAmfs YAmms + YAmfs XAmms );
XAmYamfemale = kXYmm (XAmfs Yamms + Yamfs XAmms );
XamYAmfemale = kXYmm (Xamfs YAmms + YAmfs XAmms );
XamYamfemale = kXYmm (Xamfs Yamms + Yamfs XAmms );
(*Xm-Ym males*)
XAmYAmmale = (1 - kXYmm) (XAmfs YAmms + YAmfs XAmms );
XAmYammale = (1 - kXYmm) (XAmfs Yamms + Yamfs XAmms );
XamYAmmale = (1 - kXYmm) (Xamfs YAmms + YAmfs XAmms );
XamYammale = (1 - kXYmm) (Xamfs Yamms + Yamfs XAmms );

(*Ym-Ym females*)
YAmYAmfemale = kYYmm (YAmfs YAmms);
YAmYamfemale = kYYmm (YAmfs Yamms + Yamfs YAmms);
YamYAmfemale = kYYmm (Yamfs Yamms);
(*Ym-Ym males*)
YAmYAmmale = (1 - kYYmm) (YAmfs YAmms);
YAmYammale = (1 - kYYmm) (YAmfs Yamms + Yamfs YAmms);
YamYammale = (1 - kYYmm) (Yamfs Yamms);

(*DIPLOID SELECTION*)

(*mean male fitness*)
wbarM =
MAA XAMXAMmale + MAA XAMXaMmale + Maa XaMXaMmale +
MAA XAMYAMmale + MAA (XAMYAMmale + XaMYAMmale) + Maa XaMYaMmale +
MAA YAMYAMmale + MAA YAMYaMmale + Maa YaMYaMmale +
MAA XAmXAMmale + MAA (XAmXaMmale + XAMXammale) + Maa XamXaMmale +
MAA XAmYAMmale + MAA (XAmYaMmale + XamYAMmale) + Maa XamYaMmale +
MAA XAMYAMmale + MAA (XAMYammale + XaMYammale) + Maa XaMYammale +
MAA YAmYAMmale + MAA (YAmYaMmale + YamYAMmale) + Maa YamYaMmale +
MAA XAmXammale + MAA XAmXammale + Maa XamXammale +
MAA XAmYammale + MAA (XAmYammale + XamYAmmale) + Maa XamYammale +
MAA YAmYammale + MAA YAmYammale + Maa YamYammale;

XAMXAMmales = MAA XAMXAMmale / wbarM;
XAMXaMmales = MAA XAMXaMmale / wbarM;
XaMXaMmales = Maa XaMXaMmale / wbarM;

XAMYAMmales = MAA XAMYAMmale / wbarM;
XAMYaMmales = MAA XAMYaMmale / wbarM;
XaMYAMmales = MAA XaMYAMmale / wbarM;
XaMYaMmales = Maa XaMYaMmale / wbarM;

YAMYAMmales = MAA YAMYAMmale / wbarM;
YAMYaMmales = MAA YAMYaMmale / wbarM;
YaMYaMmales = Maa YaMYaMmale / wbarM;

XAmXAMmales = MAA XAmXAMmale / wbarM;
XAmXaMmales = MAA XAmXaMmale / wbarM;
XAMXammale = MAA XAMXammale / wbarM;
XamXaMmales = Maa XamXaMmale / wbarM;

XAmYAMmales = MAA XAmYAMmale / wbarM;

```

```

XAmYaMmales = MAa XAmYaMmale / wbarM;
XamYAMmales = MAa XamYAMmale / wbarM;
XamYaMmales = Maa XamYaMmale / wbarM;

XAMYAmmales = MAA XAMYAmmale / wbarM;
XAMYammale = MAa XAMYammale / wbarM;
XaMYAmmale = MAa XaMYAmmale / wbarM;
XaMYammale = Maa XaMYammale / wbarM;

YAmYAMmales = MAA YAmYAMmale / wbarM;
YAmYaMmales = MAa YAmYaMmale / wbarM;
YamYAMmales = MAa YamYAMmale / wbarM;
YamYaMmales = Maa YamYaMmale / wbarM;

XAmXammale = MAA XAmXammale / wbarM;
XAmXammale = MAa XAmXammale / wbarM;
XamXammale = Maa XamXammale / wbarM;

XAmYammale = MAA XAmYammale / wbarM;
XAmYammale = MAa XAmYammale / wbarM;
XamYammale = Maa XamYammale / wbarM;
XamYammale = Maa XamYammale / wbarM;

YAmYammale = MAA YAmYammale / wbarM;
YAmYammale = MAa YAmYammale / wbarM;
YamYammale = Maa YamYammale / wbarM;

(*mean female fitness*)
wbarF = FAA XAMXAMfemale + FAa XAMXaMfemale + Faa XaMXaMfemale +
       FAA XAMYAMfemale + FAa (XAMYaMfemale + XaMYAMfemale) + Faa XaMYaMfemale +
       FAA YAMYAMfemale + FAa YAMYaMfemale + Faa YaMYaMfemale +
       FAA XAmXAMfemale + FAa XAmXaMfemale + FAa XAMXamfemale + Faa XamXaMfemale +
       FAA XAmYAMfemale + FAa XAmYaMfemale + FAa XamYAMfemale + Faa XamYaMfemale +
       FAA XAMYAmfemale + FAa XAMYamfemale + FAa XaMYAmfemale + Faa XaMYamfemale +
       FAA YAmYAMfemale + FAa YAmYaMfemale + FAa YamYAMfemale + Faa YamYaMfemale +
       FAA XAmXAmfemale + FAa XAmXamfemale + Faa XamXamfemale +
       FAA XAmYAmfemale + FAa XAmYamfemale + FAa XamYAmfemale + Faa XamYamfemale +
       FAA YAmYAmfemale + FAa YAmYamfemale + Faa YamYamfemale;

XAMXAMfemales = FAA XAMXAMfemale / wbarF;
XAMXaMfemales = FAa XAMXaMfemale / wbarF;
XaMXaMfemales = Faa XaMXaMfemale / wbarF;

XAMYAMfemales = FAA XAMYAMfemale / wbarF;
XAMYaMfemales = FAa XAMYaMfemale / wbarF;
XaMYAMfemales = FAa XaMYAMfemale / wbarF;
XaMYaMfemales = Faa XaMYaMfemale / wbarF;

YAMYAMfemales = FAA YAMYAMfemale / wbarF;
YAMYaMfemales = FAa YAMYaMfemale / wbarF;
YaMYaMfemales = Faa YaMYaMfemale / wbarF;

XAmXAMfemales = FAA XAmXAMfemale / wbarF;
XAmXaMfemales = FAa XAmXaMfemale / wbarF;
XAMXamfemales = FAa XAMXamfemale / wbarF;

```

```

XamXaMfemales = Faa XamXaMfemale / wbarF;

XAmYAMfemales = FAA XAmYAMfemale / wbarF;
XAmYaMfemales = FAa XAmYaMfemale / wbarF;
XamYAMfemales = FAa XamYAMfemale / wbarF;
XamYaMfemales = Faa XamYaMfemale / wbarF;

XAMYAmfemales = FAA XAMYAmfemale / wbarF;
XAMYamfemales = FAa XAMYamfemale / wbarF;
XaMYAmfemales = FAa XaMYAmfemale / wbarF;
XaMYamfemales = Faa XaMYamfemale / wbarF;

YAmYAMfemales = FAA YAmYAMfemale / wbarF;
YAmYaMfemales = FAa YAmYaMfemale / wbarF;
YamYAMfemales = FAa YamYAMfemale / wbarF;
YamYaMfemales = Faa YamYaMfemale / wbarF;

XAmXAmfemales = FAA XAmXAmfemale / wbarF;
XAmXamfemales = FAa XAmXamfemale / wbarF;
XamXamfemales = Faa XamXamfemale / wbarF;

XAmYAmfemales = FAA XAmYAmfemale / wbarF;
XAmYamfemales = FAa XAmYamfemale / wbarF;
XamYAmfemales = FAa XamYAmfemale / wbarF;
XamYamfemales = Faa XamYamfemale / wbarF;

YAmYAmfemales = FAA YAmYAmfemale / wbarF;
YAmYamfemales = FAa YAmYamfemale / wbarF;
YamYamfemales = Faa YamYamfemale / wbarF;

(*MEIOSIS*)

xx11ms = XAMXAMmales;
xx12ms = XAMXaMmales;
xx22ms = XaMXaMmales;

xx13ms = XAmXAMmales;
xx23ms = XAmXaMmales;
xx14ms = XAMXammales;
xx24ms = XamXaMmales;

xx33ms = XAmXAMmales;
xx34ms = XAmXammales;
xx44ms = XamXammales;

xy11ms = XAMYAMmales;
xy12ms = XAMYaMmales;
xy21ms = XaMYAMmales;
xy22ms = XaMYaMmales;

xy31ms = XAmYAMmales;
xy32ms = XAmYaMmales;
xy41ms = XamYAMmales;
xy42ms = XamYaMmales;

```

```

xy13ms = XAMYAmmales;
xy14ms = XAMYammales;
xy23ms = XaMYAmmales;
xy24ms = XaMYammales;

xy33ms = XAmYAmmales;
xy34ms = XAmYammales;
xy43ms = XamYAmmales;
xy44ms = XamYammales;

yy11ms = YAMYAMmales;
yy12ms = YAMYaMmales;
yy22ms = YaMYaMmales;

yy13ms = YAmYAMmales;
yy23ms = YAmYaMmales;
yy14ms = YamYAMmales;
yy24ms = YamYaMmales;

yy33ms = YAmYAmmales;
yy34ms = YAmYammales;
yy44ms = YamYammales;

xx11fs = XAMXAMfemales;
xx12fs = XAMXaMfemales;
xx22fs = XaMXaMfemales;

xx13fs = XAmXAMfemales;
xx23fs = XAmXaMfemales;
xx14fs = XAMXamfemales;
xx24fs = XamXaMfemales;

xx33fs = XAmXAmfemales;
xx34fs = XAmXamfemales;
xx44fs = XamXamfemales;

xy11fs = XAMYAMfemales;
xy12fs = XAMYaMfemales;
xy21fs = XaMYAMfemales;
xy22fs = XaMYaMfemales;

xy31fs = XAmYAMfemales;
xy32fs = XAmYaMfemales;
xy41fs = XamYAMfemales;
xy42fs = XamYaMfemales;

xy13fs = XAMYAmfemales;
xy14fs = XAMYamfemales;
xy23fs = XaMYAmfemales;
xy24fs = XaMYamfemales;

xy33fs = XAmYAmfemales;
xy34fs = XAmYamfemales;
xy43fs = XamYAmfemales;
xy44fs = XamYamfemales;

```

```

yy11fs = YAMYAMfemales;
yy12fs = YAMYaMfemales;
yy22fs = YaMYaMfemales;

yy13fs = YAmYAMfemales;
yy23fs = YAmYaMfemales;
yy14fs = YamYAMfemales;
yy24fs = YamYaMfemales;

yy33fs = YAmYAmfemales;
yy34fs = YAmYamfemales;
yy44fs = YamYamfemales;

(*X bearing ovules*)
nextXAMf = xx11fs + xx13fs / 2 + (xx12fs + xx14fs * (1 - R) + xx23fs * R) αf +
    xy11fs / 2 + (xy12fs (1 - r) + xy21fs (r)) αf +
    (xy13fs (1 - ρ) + xy31fs (ρ)) / 2 +
    xy14fs * αf +
    (xy14fs (- (R + r + ρ)) +
        xy41fs (r + ρ - R) +
        xy23fs (R + r - ρ) +
        xy32fs (R + ρ - r)) αf / 2;

nextXaMf = xx22fs + xx24fs / 2 + (xx12fs + xx23fs * (1 - R) + xx14fs * R) (1 - αf) +
    xy22fs / 2 + (xy12fs (r) + xy21fs (1 - r)) (1 - αf) +
    (xy24fs (1 - ρ) + xy42fs (ρ)) / 2 +
    xy23fs * (1 - αf) +
    (xy14fs (R + r - ρ) +
        xy41fs (R + ρ - r) +
        xy23fs (- (R + r + ρ)) +
        xy32fs (r + ρ - R)) (1 - αf) / 2;

nextXAmf = xx33fs + xx13fs / 2 + (xx23fs * (1 - R) + R * xx14fs + xx34fs) αf +
    xy33fs / 2 + (xy34fs (1 - r) + xy43fs (r)) αf +
    (xy13fs (ρ) + xy31fs (1 - ρ)) / 2 +
    xy32fs * αf +
    (xy14fs (R + ρ - r) +
        xy41fs (R + r - ρ) +
        xy23fs (r + ρ - R) +
        xy32fs (- (R + r + ρ))) αf / 2;

nextXamf = xx44fs + xx24fs / 2 + (xx14fs * (1 - R) + xx23fs * R + xx34fs) (1 - αf) +
    xy44fs / 2 + (xy34fs (r) + xy43fs (1 - r)) (1 - αf) +
    (xy24fs (ρ) + xy42fs (1 - ρ)) / 2 +
    xy41fs * (1 - αf) +
    (xy14fs (r + ρ - R) +
        xy41fs (- (R + r + ρ)) +
        xy23fs (R + ρ - r) +
        xy32fs (R + r - ρ)) (1 - αf) / 2;

(*Y bearing ovules*)
nextYAMf = yy11fs + yy13fs / 2 + (yy12fs + yy14fs * (1 - R) + yy23fs * R) αf +
    xy11fs / 2 + (xy12fs (r) + xy21fs (1 - r)) (αf) +
    (xy13fs (ρ) + xy31fs (1 - ρ)) / 2 +

```

```

xy41fs * af +
(xy14fs (r + ρ - R) +
 xy41fs (- (R + r + ρ)) +
 xy23fs (R + ρ - r) +
 xy32fs (R + r - ρ)) af / 2;

nextYaMf = yy22fs + yy24fs / 2 + (yy12fs + yy23fs * (1 - R) + yy14fs * R) (1 - af) +
 xy22fs / 2 + (xy12fs (1 - r) + xy21fs (r)) (1 - af) +
 (xy24fs (ρ) + xy42fs (1 - ρ)) / 2 +
 xy32fs * (1 - af) +
 (xy14fs (R + ρ - r) +
 xy41fs (R + r - ρ) +
 xy23fs (r + ρ - R) +
 xy32fs (- (R + r + ρ))) (1 - af) / 2;

nextYAmf = yy33fs + yy13fs / 2 + (yy23fs * (1 - R) + R * yy14fs + yy34fs) af +
 xy33fs / 2 + (xy34fs (r) + xy43fs (1 - r)) (af) +
 (xy13fs (1 - ρ) + xy31fs (ρ)) / 2 +
 xy23fs * af +
 (xy14fs (R + r - ρ) +
 xy41fs (R + ρ - r) +
 xy23fs (- (R + r + ρ)) +
 xy32fs (r + ρ - R)) af / 2;

nextYamf = yy44fs + yy24fs / 2 + (yy14fs * (1 - R) + yy23fs * R + yy34fs) (1 - af) +
 xy44fs / 2 + (xy34fs (1 - r) + xy43fs (r)) (1 - af) +
 (xy24fs (1 - ρ) + xy42fs (ρ)) / 2 +
 xy14fs * (1 - af) +
 (xy14fs (- (R + r + ρ)) +
 xy41fs (r + ρ - R) +
 xy23fs (R + r - ρ) +
 xy32fs (R + ρ - r)) (1 - af) / 2;

(*X bearing pollen*)
nextXAMm = xx11ms + xx13ms / 2 + (xx12ms + xx14ms * (1 - R) + xx23ms * R) am +
 xy11ms / 2 + (xy12ms (1 - r) + xy21ms (r)) am +
 (xy13ms (1 - ρ) + xy31ms (ρ)) / 2 +
 xy14ms * am +
 (xy14ms (- (R + r + ρ)) +
 xy41ms (r + ρ - R) +
 xy23ms (R + r - ρ) +
 xy32ms (R + ρ - r)) am / 2;

nextXaMm = xx22ms + xx24ms / 2 + (xx12ms + xx23ms * (1 - R) + xx14ms * R) (1 - am) +
 xy22ms / 2 + (xy12ms (r) + xy21ms (1 - r)) (1 - am) +
 (xy24ms (1 - ρ) + xy42ms (ρ)) / 2 +
 xy23ms * (1 - am) +
 (xy14ms (R + r - ρ) +
 xy41ms (R + ρ - r) +
 xy23ms (- (R + r + ρ)) +
 xy32ms (r + ρ - R)) (1 - am) / 2;

nextXAmm = xx33ms + xx13ms / 2 + (xx23ms * (1 - R) + R * xx14ms + xx34ms) am +
 xy33ms / 2 + (xy34ms (1 - r) + xy43ms (r)) am +
 (xy13ms (ρ) + xy31ms (1 - ρ)) / 2 +

```

```

xy32ms *  $\alpha_m$  +
(xy14ms (R +  $\rho$  - r) +
 xy41ms (R + r -  $\rho$ ) +
 xy23ms (r +  $\rho$  - R) +
 xy32ms (- (R + r +  $\rho$ )))  $\alpha_m$  / 2;

nextXamm = xx44ms + xx24ms / 2 + (xx14ms * (1 - R) + xx23ms * R + xx34ms) (1 -  $\alpha_m$ ) +
 xy44ms / 2 + (xy34ms (r) + xy43ms (1 - r)) (1 -  $\alpha_m$ ) +
 (xy24ms ( $\rho$ ) + xy42ms (1 -  $\rho$ )) / 2 +
 xy41ms * (1 -  $\alpha_m$ ) +
 (xy14ms (r +  $\rho$  - R) +
 xy41ms (- (R + r +  $\rho$ )) +
 xy23ms (R +  $\rho$  - r) +
 xy32ms (R + r -  $\rho$ )) (1 -  $\alpha_m$ ) / 2;

(*Y bearing pollen*)
nextYAm = yy11ms + yy13ms / 2 + (yy12ms + yy14ms * (1 - R) + yy23ms * R)  $\alpha_m$  +
 xy11ms / 2 + (xy12ms (r) + xy21ms (1 - r)) ( $\alpha_m$ ) +
 (xy13ms ( $\rho$ ) + xy31ms (1 -  $\rho$ )) / 2 +
 xy41ms *  $\alpha_m$  +
 (xy14ms (r +  $\rho$  - R) +
 xy41ms (- (R + r +  $\rho$ )) +
 xy23ms (R +  $\rho$  - r) +
 xy32ms (R + r -  $\rho$ ))  $\alpha_m$  / 2;

nextYaMm = yy22ms + yy24ms / 2 + (yy12ms + yy23ms * (1 - R) + yy14ms * R) (1 -  $\alpha_m$ ) +
 xy22ms / 2 + (xy12ms (1 - r) + xy21ms (r)) (1 -  $\alpha_m$ ) +
 (xy24ms ( $\rho$ ) + xy42ms (1 -  $\rho$ )) / 2 +
 xy32ms * (1 -  $\alpha_m$ ) +
 (xy14ms (R +  $\rho$  - r) +
 xy41ms (R + r -  $\rho$ ) +
 xy23ms (r +  $\rho$  - R) +
 xy32ms (- (R + r +  $\rho$ ))) (1 -  $\alpha_m$ ) / 2;

nextYAmM = yy33ms + yy13ms / 2 + (yy23ms * (1 - R) + R * yy14ms + yy34ms)  $\alpha_m$  +
 xy33ms / 2 + (xy34ms (r) + xy43ms (1 - r)) ( $\alpha_m$ ) +
 (xy13ms (1 -  $\rho$ ) + xy31ms ( $\rho$ )) / 2 +
 xy23ms *  $\alpha_m$  +
 (xy14ms (R + r -  $\rho$ ) +
 xy41ms (R +  $\rho$  - r) +
 xy23ms (- (R + r +  $\rho$ )) +
 xy32ms (r +  $\rho$  - R))  $\alpha_m$  / 2;

nextYamm = yy44ms + yy24ms / 2 + (yy14ms * (1 - R) + yy23ms * R + yy34ms) (1 -  $\alpha_m$ ) +
 xy44ms / 2 + (xy34ms (1 - r) + xy43ms (r)) (1 -  $\alpha_m$ ) +
 (xy24ms (1 -  $\rho$ ) + xy42ms ( $\rho$ )) / 2 +
 xy14ms * (1 -  $\alpha_m$ ) +
 (xy14ms (- (R + r +  $\rho$ )) +
 xy41ms (r +  $\rho$  - R) +
 xy23ms (R + r -  $\rho$ ) +
 xy32ms (R +  $\rho$  - r)) (1 -  $\alpha_m$ ) / 2;

{nextXAMf, nextXaMf, nextXAmf, nextXamf, nextYAMf, nextYaMf, nextYAmf,
 nextXAMm, nextXaMm, nextXAmm, nextXamm, nextYAMm, nextYaMm, nextYAmM, nextYamm}
]
```

Sex ratio

Fraction males among zygotes

```
(*fraction female*)
(*XAMXAMfemale+XAMXaMfemale+XaMXaMfemale+
 XAMYAMfemale+XAMYaMfemale+XaMYAMfemale+XaMYaMfemale+
 YAMYAMfemale+YAMYaMfemale+YaMYaMfemale+
 XAmXAMfemale+XAmXaMfemale+XAMXamfemale+XamXaMfemale+
 XAmYAMfemale+XAmYaMfemale+XamYAMfemale+XamYaMfemale+
 XAMYAmfemale+XAMYamfemale+XaMYAmfemale+XaMYamfemale+
 YAmYAMfemale+YAmYaMfemale+YamYAMfemale+YamYaMfemale+
 XAmXAmfemale+XAmXamfemale+XamXamfemale+
 XAmYAmfemale+XAmYamfemale+XamYAmfemale+XamYamfemale+
 YAmYAmfemale+YAmYamfemale+YamYamfemale/.SUBS//Factor*)

Clear[sexratio]
sexratio[{FAA_, FAa_, Faa_, MAA_, MAa_, Maa_, wAf_, waf_, wAm_, wam_, af_,
 am_, r_, R_, ρ_, k_}, {XAMf_, XaMf_, XAmf_, Yamf_, YaMf_, YAmf_,
 Yamf_, XAMm_, XaMm_, XAmm_, Yamm_, YaMm_, YAmm_, Yamm_}] :=
sexratio[{FAA, FAa, Faa, MAA, MAa, Maa, wAf, waf, wAm, wam, af, am, r, R, ρ, k},
 {XAMf, XaMf, XAmf, Yamf, YaMf, YAmf, Yamf,
 XAMm, XaMm, XAmm, Yamm, YaMm, YAmm, Yamm}] =
1 - (k waf wam Xamf Xamm + k waf wam XaMf Xamm + k wAf wam XAmf Xamm + k wAf wam XAMf Xamm +
 k waf wam Xamf XaMm + waf wam XaMf XaMm + k wAf wam XAmf XaMm + wAf wam XAMf XaMm +
 k waf wAm Xamf XAmm + k waf wAm XaMf XAmm + k wAf wAm XAmf XAmm +
 k wAf wAm XAMf XAmm + k waf wAm Xamf XAMm + waf wAm XaMf XAMm + k wAf wAm XAmf XAMm +
 wAf wAm XAMf XAMm + k waf wam Xamm Yamf + k waf wam XaMm Yamf + k waf wAm XAmm Yamf +
 k waf wAm XAMm Yamf + k waf wam Xamm YaMf + k waf wAm XAmm YaMf + k wAf wam Xamm YAmf +
 k wAf wAm XaMm YAmf + k wAf wAm XAmm YAmf + k wAf wAm XAMm YAmf + k wAf wam Xamm YAMf +
 k wAf wAm XAMm YAMf + k waf wam Xamf Yamm + k waf wam XaMf Yamm + k wAf wam XAmf Yamm +
 k wAf wam XAMf Yamm + k waf wam Yamf Yamm + k waf wam YaMf Yamm + k wAf wam YAmf Yamm +
 k wAf wam YAMf Yamm + k waf wam Xamf YaMm + k wAf wam XAmf YaMm + k waf wam Yamf YaMm +
 k wAf wam YAMf YaMm + k waf wAm Xamf YAMm + k waf wAm XaMf YAMm + k wAf wAm XAmf YAMm +
 k wAf wAm XAMf YAMm + k waf wAm Yamf YAMm + k waf wAm YaMf YAMm + k wAf wAm YAmf YAMm +
 k wAf wAm XAmf YAMm + k waf wAm Yamf YAMm + k wAf wAm YAMf YAMm) /
((waf Xamf + waf XaMf + wAf XAmf + wAf XAMf + waf Yamf + waf YaMf + wAf YAmf + wAf YAMf)
 (wam Xamm + wam XaMm + wAm XAmm + wAm XAMm +
 wam Yamm + wam YaMm + wAm YAMm + wAm YAMm)) ;
```

Figure 2 - region plot of neo-W haplotype invasion with tight linkage and no haploid selection

Plotting parameters

```
xmin = 0;
xmax = 1.5;
xtickmin = 0;
xtickmax = 1;
xint = 1;
ymin = xmin;
ymax = xmax;
ytickmin = xtickmin;
ytickmax = xtickmax;
yint = xint;
```

Panel A - A favoured in females

Parameters

```
params = {
  wAm → 1, wam → 1, wAf → 1, waf → 1,
  αm → 1 / 2, αf → 1 / 2,
  MAa → 1, FAa → 1,
  Faa → 0.85,
  FAA → 1.05
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
  Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)}
```

```

1 < WAinvA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{(
validcondA /. params) &&
(stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) &&
1 < WainvA
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{(
stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
1 < WAinvB(*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{(
stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
1 < WainvB
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{(
stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
(stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{(

```

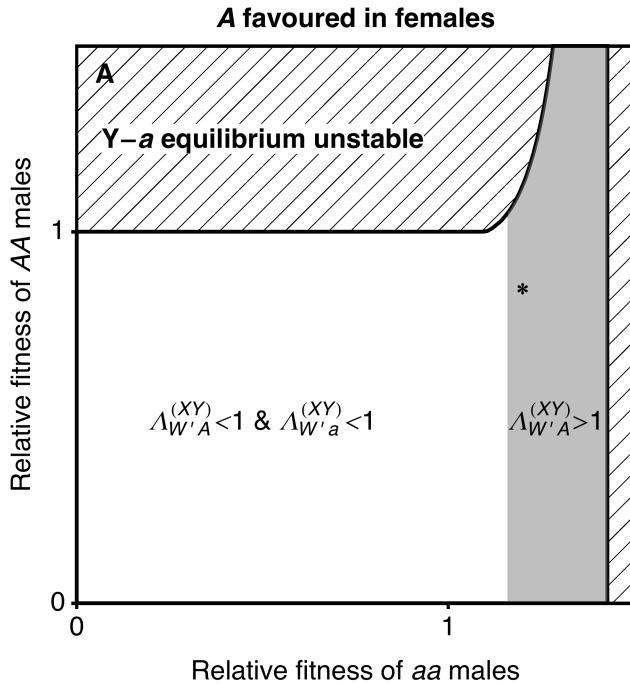
```

1 < 2
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

plotA =
Show[
plotYaUnstable,
plotYaStable,
plotWAinvA,
plotWainvA,
plotWAinvB,
plotWainvB,

FrameLabel -> {"Relative fitness of aa males", ""},
Epilog -> {
Text[Style["A", Bold], Scaled@{0.05, 0.95}],
Text[Style[" $\Lambda_{w'A}^{(xx)} > 1$ ", {1.29, 0.5}],
Text[Style[" $\Lambda_{w'A}^{(xx)} < 1 \text{ & } \Lambda_{w'a}^{(xx)} < 1$ ", {0.5, 0.5}],
Text[Style["Y-a equilibrium unstable", Bold],
{0.5, 1.25}, Background -> White],
Text[Style["*", Bold], {1.2, 0.85}],
Rotate[Text[
Style["Relative fitness of AA males"], Scaled@ylabposregion], 90 Degree],
Text[Style["A favoured in females", Bold], Scaled@{0.5, 1.05}]
},
regionplotstyle[xmin, xmax, xtickmin,
xtickmax, xint, ymin, ymax, ytickmin, ytickmax, yint]
]
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

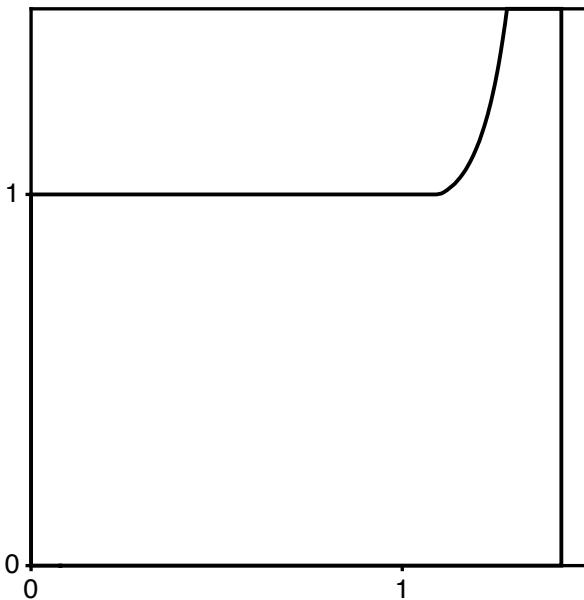
(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  regionplotstyle[xmin, xmax, xtickmin,
    xtickmax, xint, ymin, ymax, ytickmin, ytickmax, yint]
]

```



Panel B - a favoured in females

Parameters

```

params = {
  wAm → 1, wam → 1, wAf → 1, waf → 1,
  αm → 1 / 2, αf → 1 / 2,
  MAA → 1, FAA → 1,
  Faa → 1.05,
  FAA → 0.85
};

```

No recombination eigenvalues

```

WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;

```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```

λWsolA =
Max[Abs[λ /. Solve[0 == charpolyExt /. k → 1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /.
params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyExt /. k → 1 /. r → 0 /. R → 0 /. ρ → 0 /.
equilB0 /. params, λ] // Simplify]];

```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Plot

Region plots of invasion

```

(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{(
validcondA /. params) && (*valid*)
(stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
1 < WAinvA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{(
validcondA /. params) &&
(stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
1 < WainvA
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{(
stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{(
stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) &&
1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},

```

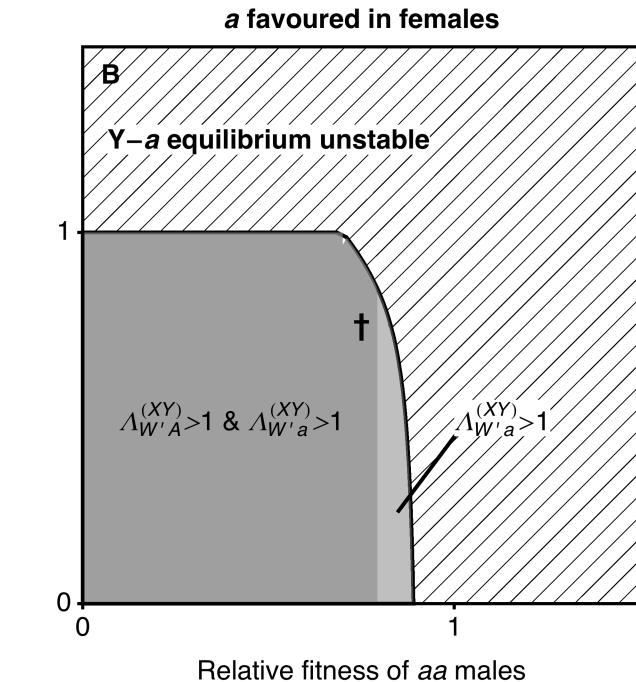
```
PlotStyle -> {Gray, Opacity[0.5]},  
BoundaryStyle -> None  
];  
  
(*Ya equilibrium internally stable*)  
plotYaStable =  
RegionPlot[{  
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilAO /. params) ||  
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilBO /. params)  
,  
 {Maa, 0, 3/2}, {MAA, 0, 3/2},  
 PlotStyle -> White,  
 BoundaryStyle -> {Black, Thick}  
];  
  
(*Ya equilibrium internally unstable*)  
plotYaUnstable =  
RegionPlot[{  
  1 < 2  
,  
 {Maa, 0, 3/2}, {MAA, 0, 3/2},  
 PlotStyle -> None,  
 BoundaryStyle -> None,  
 Mesh -> 50,  
 MeshFunctions -> {#1 - #2 &},  
 MeshStyle -> Black  
};
```

```

plotB =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{0.85, 0.25}, {1, 0.45}}]}],
  FrameLabel -> {"Relative fitness of aa males", ""},
  Epilog -> {
    Text[Style["B", Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(XY)} > 1$ ", {1, 0.5}, {-1, 0}, Background -> White],
    Text[Style[" $\Lambda_{W'A}^{(XY)} > 1 \text{ & } \Lambda_{W'a}^{(XY)} > 1$ ", {0.4, 0.5}],
    Text[Style["Y-a equilibrium unstable", Bold], {0.5, 1.25}, Background -> White],
    Text[Style["†", 16, Bold], {0.75, 0.75}],
    Text[Style["a favoured in females", Bold], Scaled@{0.5, 1.05}]
  },
  regionplotstyle[xmin, xmax, xtickmin,
  xtickmax, xint, ymin, ymax, ytickmin, ytickmax, yint]
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

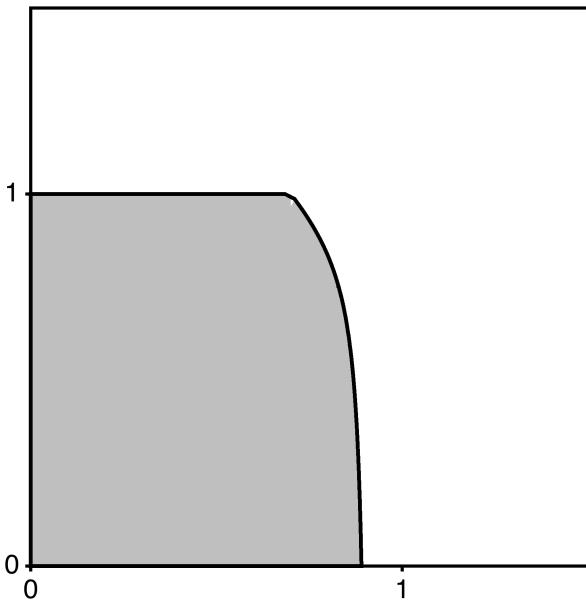
(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  regionplotstyle[xmin, xmax, xtickmin,
    xtickmax, xint, ymin, ymax, ytickmin, ytickmax, yint]
]

```



Panel C - overdominance in females

Parameters

```

params = {
  wAm → 1, wam → 1, wAf → 1, waf → 1,
  αm → 1 / 2, αf → 1 / 2,
  MAa → 1, FAa → 1,
  Faa → 0.6,
  FAA → 0.6
};

```

No recombination eigenvalues

```

WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;

```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```

λWsolA =
Max[Abs[λ /. Solve[0 == charpolyExt /. k → 1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /.
params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyExt /. k → 1 /. r → 0 /. R → 0 /. ρ → 0 /.
equilB0 /. params, λ] // Simplify]];

```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

Plots

Region plots of invasion

```

(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{(
validcondA /. params) && (*valid*)
(stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
1 < WAinvA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{(
validcondA /. params) &&
(stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
1 < WainvA
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{(
stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{(
stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) &&
1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},

```

```

PlotStyle -> {Gray, Opacity[0.5]},  

BoundaryStyle -> None  

];  
  

(*Ya equilibrium internally stable*)  

plotYaStable =  

RegionPlot[{  

  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilAO /. params) ||  

  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilBO /. params)  

},  

{Maa, 0, 3/2}, {MAA, 0, 3/2},  

PlotStyle -> White,  

BoundaryStyle -> {Black, Thick}  

];  
  

(*Ya equilibrium internally unstable*)  

plotYaUnstable =  

RegionPlot[{  

  1 < 2  

},  

{Maa, 0, 3/2}, {MAA, 0, 3/2},  

PlotStyle -> None,  

BoundaryStyle -> None,  

Mesh -> 50,  

MeshFunctions -> {#1 - #2 &},  

MeshStyle -> Black  

];

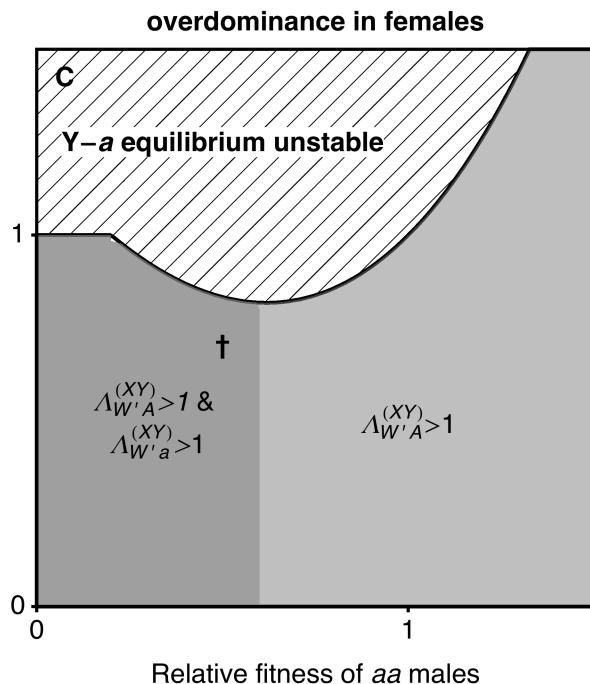
```

```

plotC =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  FrameLabel -> {"Relative fitness of aa males", ""},
  Epilog -> {
    Text[Style["C", Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(XY)} > 1$ ", {1, 0.5}],
    Text[Style[" $\Lambda_{W'a}^{(XY)} > 1 \text{ &} \Lambda_{W'a}^{(XY)} > 1$ ", {0.325, 0.5}],
    Text[Style["Y-a equilibrium unstable", Bold],
      {0.5, 1.25}, Background -> White],
    Text[Style["†", Bold], {0.5, 0.7}],
    Text[Style["overdominance in females", Bold], Scaled@{0.5, 1.05}]
  },
  regionplotstyle[xmin, xmax, xtickmin,
  xtickmax, xint, ymin, ymax, ytickmin, yint]
]

```



? Text

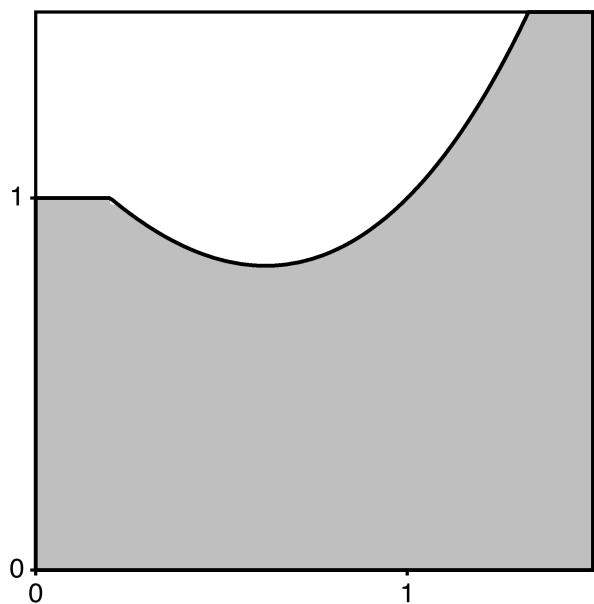
Text[*expr*] displays with *expr* in plain text format.
Text[*expr*, *coords*] is a graphics primitive that
 displays the textual form of *expr* centered at the point specified by *coords*. \gg

Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];
```

```
Show[  
  plotWinvA,  
  plotWinvB,  
  plotYaStable,  
  
  regionplotstyle[xmin, xmax, xtickmin,  
  xtickmax, xint, ymin, ymax, ytickmin, ytickmax, yint]  
]
```



All panels

```
GraphicsRow[
{plotA, plotB, plotC},
Spacings -> -45
]

Export[plotdir <> "Region_plot_combined.eps", % // rasterTrick];
```

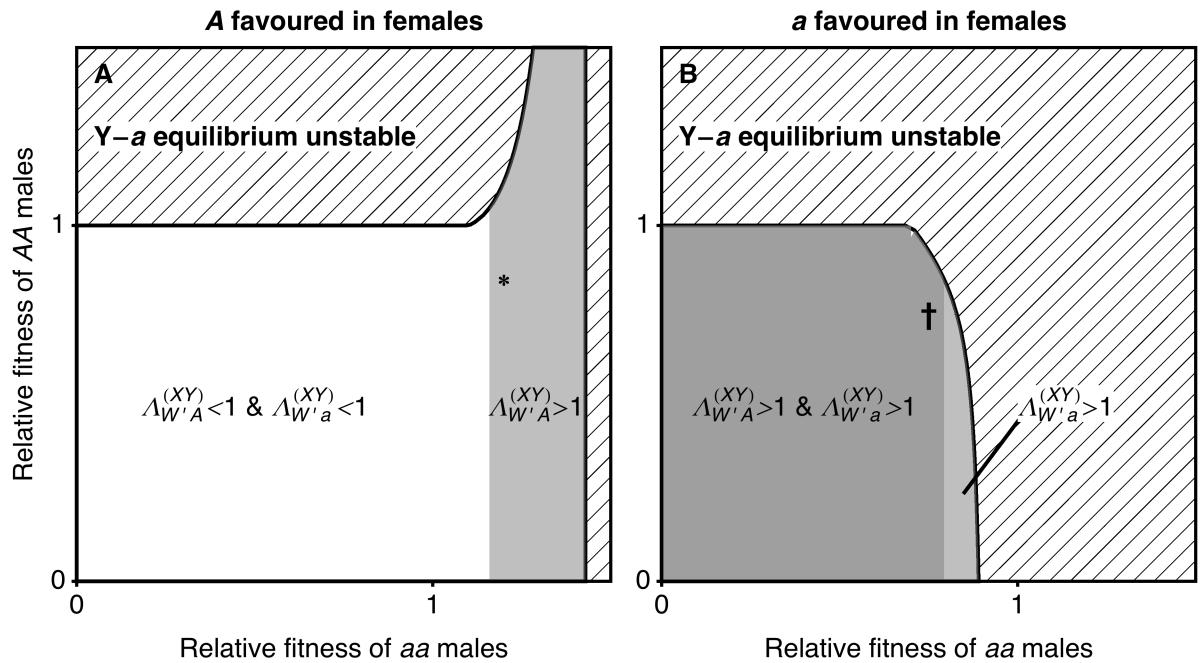


Figure 3 - neo-W invasion with sexual antagonism for loose and tight linkage

Panel A - neo-W invasion fitness as function of selected locus location, with tight linkage

Parameters

```
(*selection parameters*)
tryFAA = 1.05;
tryFAa = 1;
tryFaa = 0.85;
tryMAA = 0.85;
tryMAa = 1;
tryMaa = 1.2;
trywAf = 1;
trywaf = 1;
trywAm = 1;
trywam = 1;
tryaf = 0.5;
tryam = 0.5;

(*position of X and M loci*)
x = -5;
m = 5;

(*x limits*)
xmin = -10;
xmax = 10;
xtickmin = -10;
xtickmax = 10;
xint = 5;

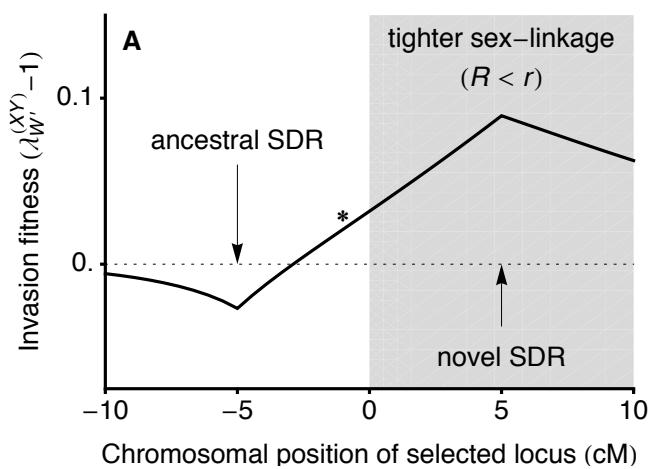
npoints = 36;
```

Plot

```

Clear[a]
positionplotA =
Show[
  (*region of tighter linkage*)
RegionPlot[
  x > 0,
  {x, xmin, xmax},
  {y, ymin, ymax},
  PlotStyle -> LightGray,
  BoundaryStyle -> None,
  AxesOrigin -> {xmin, ymin},
  Frame -> {True, True, False, False}
] ,
  (*neo-W into XY*)
ListPlot[
  Table[
    {
      a,
      invasionplotXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf,
        trywAm, trywam, tryaf, tryam, setr[x, a, m], setR[x, a, m], seto[x, a, m]]
    },
    {a, xmin, xmax, (xmax - xmin) / (npoints)}
  ],
  Joined -> True,
  PlotRange -> All,
  PlotStyle -> Directive[Black, Thickness[lwd]],
  AxesOrigin -> {xmin, ymin},
  Frame -> {True, True, False, False}
] ,
  Plot[0, {x, xmin, xmax}, PlotStyle -> {Black, Dotted}],
  FrameLabel -> {"Chromosomal position of selected locus (cM)", ""},
  Epilog -> {
    Text[Style["A", Bold], Scaled@letpos],
    Rotate[Text["Invasion fitness ( $\lambda_{\text{W}}^{(\text{XY})}$ -1)", Scaled@{-0.15, 0.5}], 90 Degree],
    Text["tighter sex-linkage", {m, ymax * 0.9}],
    Text["( $R < r$ )", {m, ymax * 0.75}],
    Text["ancestral SDR", {x, ymax * 0.5}],
    Text["novel SDR", {m, ymin * 0.75}],
    Arrow[{x, ymax * 0.4}, {x, 0}],
    Arrow[{m, ymin * 0.5}, {m, 0}],
    Text[Style["*", Bold], {-1, 0.03}]
  },
  plotstyle[xmin, xmax, xtickmin,
    xtickmax, xint, ymin, ymax, ytickmin, ytickmax, yint]
]

```



Panel B - neo-W invasion fitness as function of selected locus location, with loose linkage

Parameters

```
(*selection parameters*)
tryFAA = 1.05;
tryFAa = 1;
tryFaa = 0.85;
tryMAA = 0.85;
tryMAa = 1;
tryMaa = 1.2;
trywAf = 1;
trywaf = 1;
trywAm = 1;
trywam = 1;
tryaf = 0.5;
tryam = 0.5;

(*position of X and M loci*)
x = -50;
m = 50;

(*x limits*)
xmin = -100;
xmax = 100;
xtickmin = -100;
xtickmax = 100;
xint = 50;

(*y limits*)
ymin = -0.075;
ymax = 0.15;
ytickmin = -0.1;
ytickmax = 0.1;
yint = 0.1;

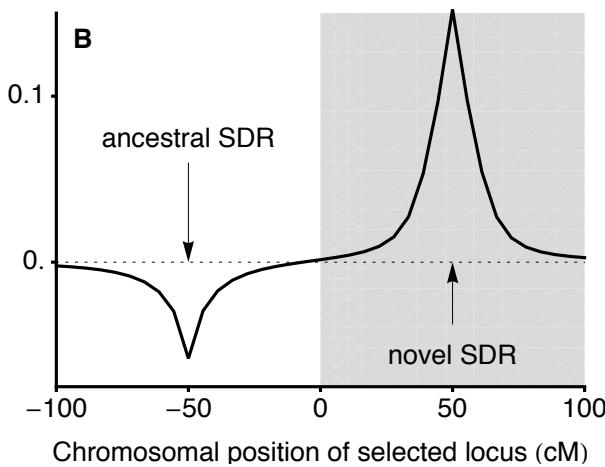
(*number of data points*)
npoints = 36;
```

Plot

```

Clear[a]
positionplotB =
Show[
  (*region of tighter linkage*)
RegionPlot[
  x > 0,
  {x, xmin, xmax},
  {y, ymin, ymax},
  PlotStyle -> LightGray,
  BoundaryStyle -> None,
  AxesOrigin -> {xmin, ymin},
  Frame -> {True, True, False, False}
],
  (*neo-W into XY*)
ListPlot[
  Table[
    {
      a,
      invasionplotXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf,
        trywAm, trywam, tryaf, tryam, setr[x, a, m], setR[x, a, m], seto[x, a, m]]
    },
    {a, xmin, xmax, (xmax - xmin) / npoints}
  ],
  Joined -> True,
  PlotRange -> All,
  PlotStyle -> Directive[Black, Thickness[lwd]],
  AxesOrigin -> {xmin, ymin},
  Frame -> {True, True, False, False}
],
  Plot[0, {x, xmin, xmax}, PlotStyle -> {Black, Dotted}],
  FrameLabel -> {"Chromosomal position of selected locus (cM)", ""},
  Epilog -> {
    Text[Style["B", Bold], Scaled@letpos],
    (*Rotate[Text["Invasion fitness (\lambda_w^{XY}-1)", Scaled@ylabpos], 90 Degree], *)
    Text["ancestral SDR", {x, ymax * 0.5}],
    Text["novel SDR", {m, ymin * 0.75}],
    Arrow[{{x, ymax * 0.4}, {x, 0}}],
    Arrow[{{m, ymin * 0.5}, {m, 0}}]
  },
  plotstyle[xmin, xmax, xtickmin,
  xtickmax, xint, ymin, ymax, ytickmin, ytickmax, yint]
]

```



Panel C - neo-W invasion fitness as function of recombination with selected locus, with inset of neo-W temporal dynamics

Parameters

Collect selection parameters used above

```
subpar = {FAA → tryFAA, FAa → tryFAa, Faa → tryFaa,
MAA → tryMAA, MAa → tryMAa, Maa → tryMaa, wAf → trywAf,
waf → trywaf, wAm → trywAm, wam → trywam, af → tryaf, am → tryam};
```

Set recombination rates

```
tryr = 0.005;
tryRs = {0.001, 0.02, 0.1, 0.5};

(*x limits*)
xmin = 0.0008;
xmax = 0.5;
xmarks = tryRs;
```

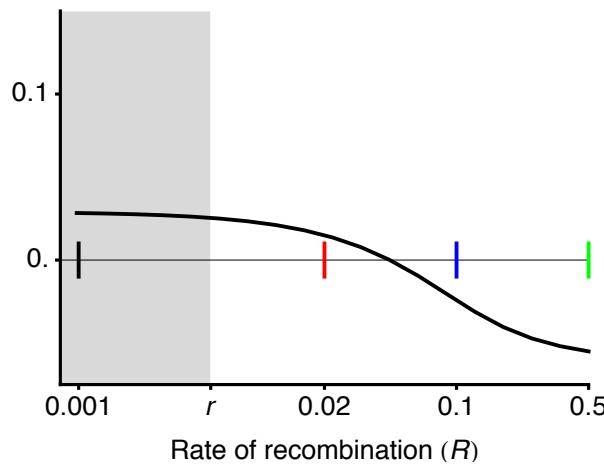
Main plot

```
tab = Table[
equil = sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]];
λ /. Solve[(λ^2 + (- (λmA1 + λma1) + (χmA1 + χma1)) λ + (λmA1 - χmA1) (λma1 - χma1) -
χmA1 χma1 /. pAveM → (1 - q) pXm + q pYm /. reverse /. subpar /. R → 0.5^b /.
{pxf → equil[[1]], pxm → equil[[2]], pym → equil[[3]],
q → equil[[4]]}) == 0, λ], {b, 10, 1, -1/2}];
eigen1 = Transpose[Join[{Table[0.5^b, {b, 10, 1, -1/2}]}, {
Transpose[tab][[1]] - 1}]];
eigen2 = Transpose[Join[{Table[0.5^b, {b, 10, 1, -1/2}]}, {
Transpose[tab][[2]] - 1}]];
```

```

temp = Show[
  (*region of tighter linkage*)
  ListLogLinearPlot[{{xmin, ymax}, {tryr, ymax}},
    Joined → True, PlotStyle → {White, Thin}, Filling → ymin,
    FillingStyle → LightGray, PlotRange → {{xmin, xmax}, {ymin, ymax}}],
  ListLogLinearPlot[{{tryr, ymax}, {tryr, ymin}}],
    Joined → True, PlotStyle → {White, Thin}],
  (*neo-W invasion fitness*)
  Show[
    ListLogLinearPlot[eigen2,
      Joined → True, PlotStyle → {Thick, Black}, PlotRange → All]
  ],
  (*0 line*)
  ListLogLinearPlot[{{xmin, 0}, {xmax, 0}}, Joined → True, PlotStyle → {Black}],
  (*mark recombination rates used in inset*)
  ListLogLinearPlot[{{0.001, -0.01}, {0.001, 0.01}},
    Joined → True, PlotStyle → {Black, Thick}],
  ListLogLinearPlot[{{0.02, -0.01}, {0.02, 0.01}},
    Joined → True, PlotStyle → {Red, Thick}],
  ListLogLinearPlot[{{0.1, -0.01}, {0.1, 0.01}},
    Joined → True, PlotStyle → {Blue, Thick}],
  ListLogLinearPlot[{{0.5, -0.01}, {0.5, 0.01}},
    Joined → True, PlotStyle → {Green, Thick}],
  FrameLabel → {"Rate of recombination ( $R$ )", ""},
  loglinearplotstyle[xmin, xmax, xmarks, ymin, ymax, ytickmin, ytickmax, yint, tryr]
]

```

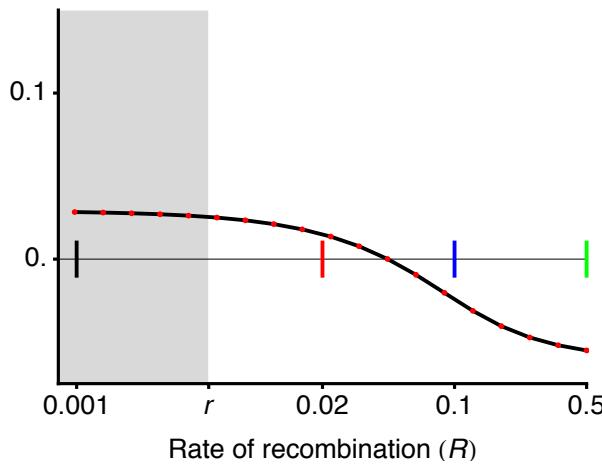


Make sure we get the same answer with the full characteristic polynomial

```

plot2 = ListLogLinearPlot[
  Table[{0.5^b, invasionplotXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
    tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr,
    0.5^b, tryr (1 - 0.5^b) + 0.5^b (1 - tryr)]}, {b, 10, 1, -1/2}],
  PlotStyle -> {Red}, PlotRange -> {{xmin, xmax}, {ymin, ymax}}];
Show[
  temp,
  plot2]

```



Inset parameters

```

trypm = 0.01; (*initial mutant frequency*)
tryk = 1; (*neo-W*)
startplot = 5; (*start plotting at this generation*)
endtime = 5000; (*number of generations to simulate*)
modvec = {0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0};
(*use to get freq of neo-W amongst females from recursion simulation output*)

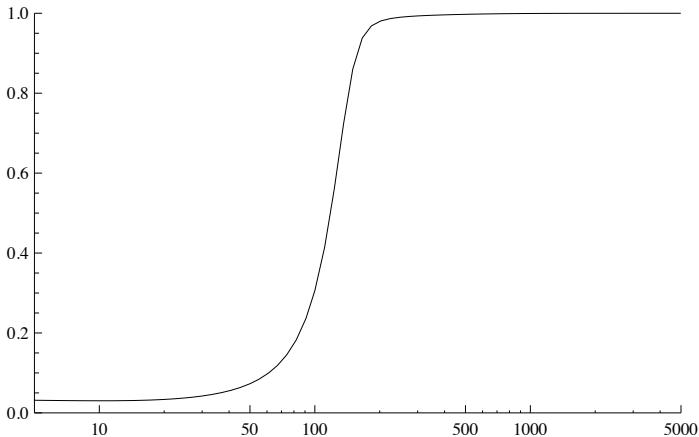
```

Inset

```

tryR = tryRs[[1]];
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtablBlack =
Table[{Round[Exp[exptime]],  $\frac{\text{generation}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]\text{.modvec}}{\text{sexratio}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]}]$ },
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Black = ListLogLinearPlot[MODtablBlack, Joined → True,
  PlotRange → {{startplot, endtime}, {0, 1}}, PlotStyle → {Black}]

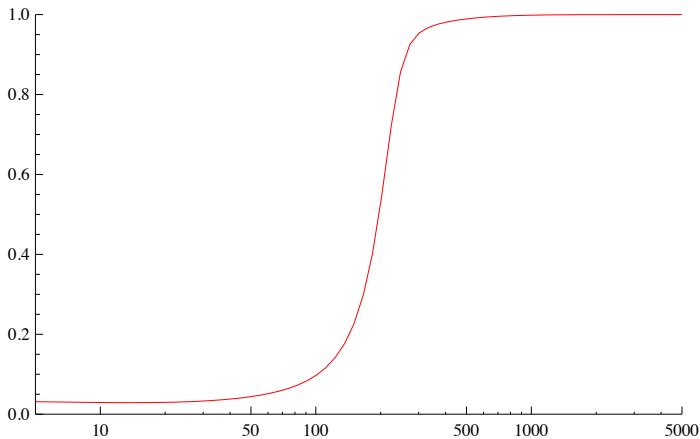
```



```

tryR = tryRs[[2]];
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtablRed =
Table[{Round[Exp[exptime]],  $\frac{\text{generation}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]].\text{modvec}}{\text{sexratio}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]}$ },
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Red = ListLogLinearPlot[MODtablRed, Joined → True,
PlotRange → {{startplot, endtime}, {0, 1}}, PlotStyle → {Red}]

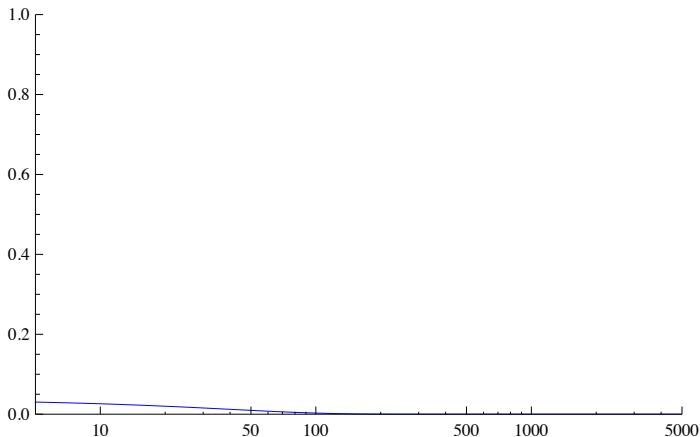
```



```

tryR = tryRs[[3]];
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Blue =
Table[{Round[Exp[exptime]],  $\frac{\text{generation}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]].\text{modvec}}{\text{sexratio}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]}}$ },
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Blue = ListLogLinearPlot[MODtab1Blue, Joined → True,
    PlotRange → {{startplot, endtime}, {0, 1}}, PlotStyle → {Blue}]

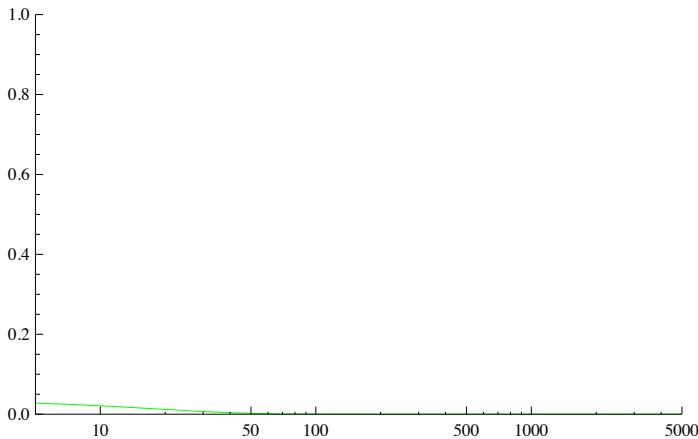
```



```

tryR = tryRs[[4]];
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtablGreen =
Table[{Round[Exp[exptime]],  $\frac{\text{generation}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]].\text{modvec}}{\text{sexratio}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]}$ },
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Green = ListLogLinearPlot[MODtablGreen, Joined → True,
PlotRange → {{startplot, endtime}, {0, 1}}, PlotStyle → {Green}]

```

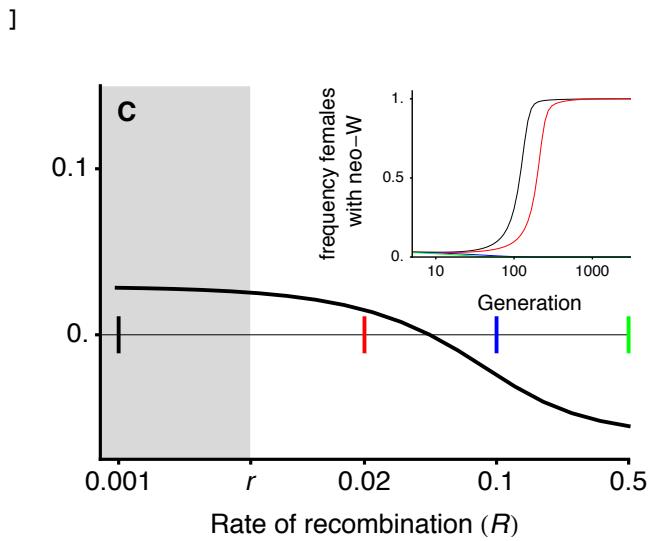


Main plot + inset

```

tempcomb =
Show[
  temp,
  Epilog -> {
    Inset[
      Show[
        MOD1Black, MOD1Red, MOD1Blue, MOD1Green,
        FrameLabel -> {Style["Generation", 11], Style["frequency females
with neo-W", 11]},
        FrameTicksStyle -> Directive[8],
        insetloglinearplotstyle[
          startplot, endtime, {10, 100, 1000}, 0, 1.05, 0, 1, 0.5]
      ],
      {-0, 0.17}, {Right, Top}, {3.6 * 5 / 4, 3.6 * 5 / 4 * Sqrt[2]}
    ],
    Text[Style["C", Bold], Scaled@letpos]
  }
]

```



? FrameTicksStyle

FrameTicksStyle is an option for 2D graphics functions that specifies how frame ticks should be rendered. >>

All panels

```

GraphicsRow[
 {positionplotA, positionplotB, tempcomb},
 Spacings -> -30
]

Export[plotdir <> "PositionPlot_SexAntagTighter.eps",
 Magnify[%, rasterTrick], 9.5 / 10], ImageSize -> {10 inches, 2 inches}];

Export[plotdir <> "PositionPlot_SexAntagTighter.pdf",
 Magnify[%, rasterTrick], 9.5 / 10], ImageSize -> {10 inches, 2 inches}];

```

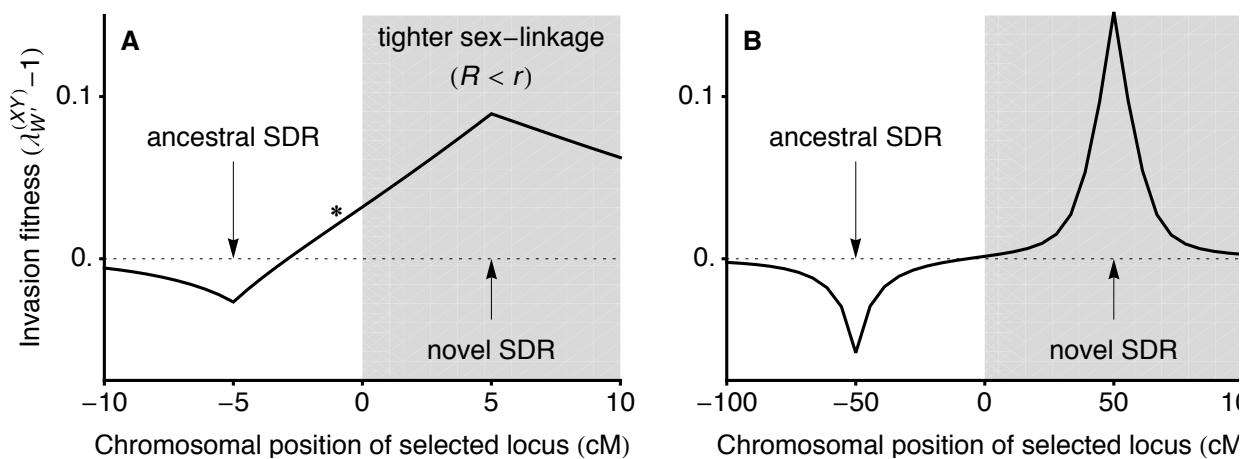


Figure 4 - plot of neo-W invasion fitness as function of selected locus location, with haploid selection

Plotting parameters

```

(*Pad={{40,20},{50,30}}; (*space around plot*)
xplotmin = -50;
xplotmax = 50;
xplotinterval = 25;
yplotmin = -0.04;
yplotmax = 0.07;
yplotinterval = 0.02;
(*ylabpos={-0.1,0.5}; (*relative location of y axis label position*)
ytickmin = -0.1;
ytickmax = 0.1;
yint = 0.1;

npoints = 120;

```

Panel A - meiotic drive in males

Parameters

```
x = -25;
m = 25;

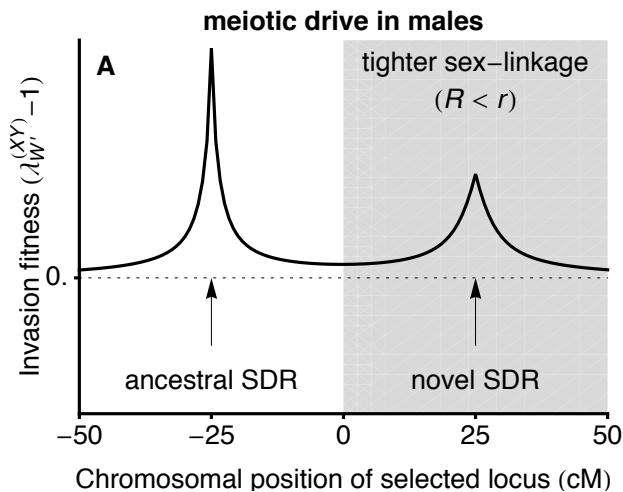
trysAf = 0.1;
tryhAf = 0.7;
tryhAm = 0.7;
trysAm = 0.1;
tryt = 0;
tryFAA = 1 + trysAf;
tryFAa = 1 + tryhAf trysAf;
tryFaa = 1;
tryMAA = 1 + trysAm;
tryMAa = 1 + tryhAm trysAm;
tryMaa = 1;
trywAm = 1 + tryt;
trywam = 1;
trywAf = 1;
trywaf = 1;
tryam = 0.45;
tryaf = 0.5;
```

Plot

```

plotA =
Show[
(*region of tighter linkage*)
RegionPlot[x > 0, {x, xplotmin, xplotmax}, {y, yplotmin, yplotmax},
PlotStyle -> {LightGray},
BoundaryStyle -> None,
Frame -> {True, True, False, False}
] ,
(*neo-W into XY*)
ListPlot[DeleteCases[Table[{a,
invasionplotXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf,
trywAm, trywam, tryaf, tryam, setr[x, a, m], setR[x, a, m], seto[x, a, m]}],
{a, xplotmin, xplotmax, (xplotmax - xplotmin) / npoints}], {n_, Null}, 2],
Joined -> True,
PlotRange -> {yplotmin, All},
PlotStyle -> Directive[Black, Thickness[lwd]]
] ,
Plot[0, {x, xplotmin, xplotmax}, PlotStyle -> {Black, Dotted}],
Epilog -> {
Text[Style["A", Bold], Scaled@letpos],
Rotate[
Text[Style["Invasion fitness ( $\lambda_w^{(xy)} - 1$ )"], Scaled@{-0.1, 0.5}], 90 Degree],
Text[Style["tighter sex-linkage"], {25, yplotmax * 0.9}],
Text["( $R < r$ )", {25, yplotmax * 0.75}],
Text[Style["meiotic drive in males", Bold],
(*{xplotmax,yplotmin*0.75}*)Scaled@{0.5, 1.05}, {0, 0}],
Text["ancestral SDR", {x, yplotmin * 0.75}],
Text["novel SDR", {m, yplotmin * 0.75}],
Arrow[{{-25, yplotmin * 0.5}, {-25, 0}}],
Arrow[{{25, yplotmin * 0.5}, {25, 0}}]
],
FrameLabel -> {"Chromosomal position of selected locus (cM)", ""},
plotstyle[xplotmin, xplotmax, xplotmin, xplotmax,
xplotinterval, yplotmin, yplotmax, ytickmin, ytickmax, yint]
]
]

```



Panel B - haploid competition in males

Parameters

```

x = -25;
m = 25;

trysAf = 0.05;
tryhAf = 0.7;
tryhAm = 0.7;
trysAm = 0.15;
tryt = -0.1;
tryFAA = 1 + trysAf;
tryFAa = 1 + tryhAf trysAf;
tryFaa = 1;
tryMAA = 1 + trysAm;
tryMAa = 1 + tryhAm trysAm;
tryMaa = 1;
trywAm = 1 + tryt;
trywam = 1;
trywAf = 1;
trywaf = 1;
tryam = 0.5;
tryaf = 0.5;

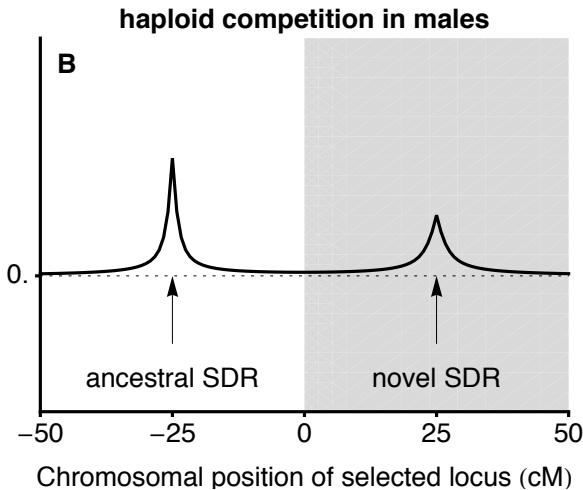
```

Plot

```

plotB =
Show[
(*region of tighter linkage*)
RegionPlot[x > 0, {x, xplotmin, xplotmax}, {y, yplotmin, yplotmax},
PlotStyle -> {LightGray},
BoundaryStyle -> None,
Frame -> {True, True, False, False}
],
(*neo-W into XY*)
ListPlot[DeleteCases[Table[{a,
invasionplotXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMaa, trywAf, trywaf,
trywAm, trywam, tryaf, tryam, setr[x, a, m], setR[x, a, m], seto[x, a, m]]},
{a, xplotmin, xplotmax, (xplotmax - xplotmin) / npoints}], {n_, Null}, 2],
Joined -> True,
PlotRange -> {yplotmin, All},
PlotStyle -> Directive[Black, Thickness[lwd]]
],
Plot[0, {x, xplotmin, xplotmax}, PlotStyle -> {Black, Dotted}],
Epilog -> {
Text[Style["B", Bold], Scaled@letpos],
(*Rotate[
Text[Style["Invasion fitness (\lambda-1)", 14], Scaled@ylabpos], 90 Degree], *),
Text[Style["haploid competition in males", Bold],
(*{xplotmax, yplotmin*0.75}*) Scaled@{0.5, 1.05}, {0, 0}],
Text["ancestral SDR", {x, yplotmin * 0.75}],
Text["novel SDR", {m, yplotmin * 0.75}],
Arrow[{{x, yplotmin * 0.5}, {x, 0}}],
Arrow[{{m, yplotmin * 0.5}, {m, 0}}]
},
FrameLabel -> {"Chromosomal position of selected locus (cM)", ""},
plotstyle[xplotmin, xplotmax, xplotmin, xplotmax,
xplotinterval, yplotmin, yplotmax, ytickmin, ytickmax, yint]
]
]

```



All panels

```
GraphicsRow[
 {plotA, plotB},
 Spacings → -30
 ]
Export[plotdir <> "PositionPlot.eps", %(*//rasterTrick*)];
```

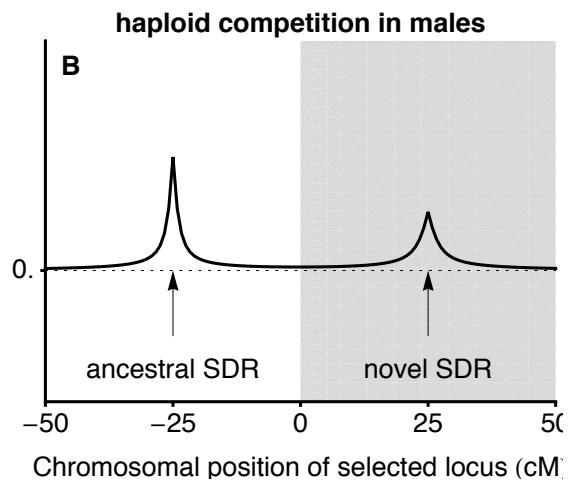
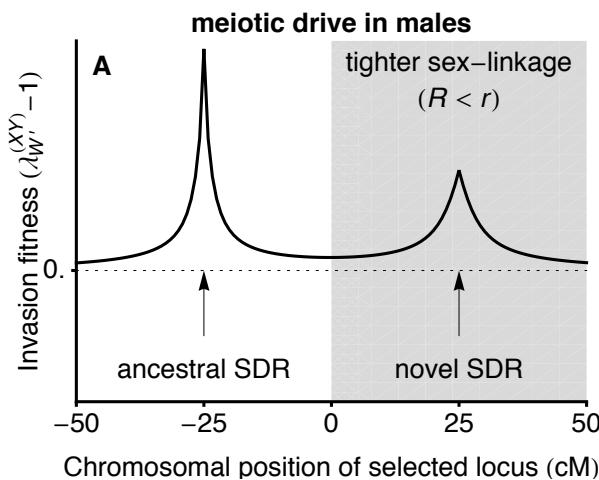


Figure 5 - sex ratio is not a good predictor of neo-SD invasion

Parameters

Plotting parameters

```
yplotmin = -0.075;
yplotmax = 0.15;
yplotinterval = 0.02;

Pad = {{50, 20}, {40, 30}}; (*space around plot*)
letpos = {0.05, .93}; (*relative location of letter, eg A, in figure*)
ylabpos = {-0.175, 0.5}; (*relative location of y axis label position*)
```

Extra parameters

```
modvec = {0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0}; (*fraction female with W*)
modvec2 = {1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0};(*fraction female*)

loglinearplotINVoptions = {Frame → {{True, False}, {True, False}}},
  PlotRangeClipping → False,
  ImagePadding → Pad,
  FrameStyle → Directive[Black, Thickness[lwd]],
  FrameTicksStyle → {{Directive[Black, Thickness[lwd], FontColor → Black],
    Directive[Black, Thickness[lwd]]},
   {Directive[Black, Thickness[lwd]], Directive[Black, Thickness[lwd]]}}},
  FrameTicks → {{{{5, 5, {0, 0.01}}, {10, 10, {0, 0.01}}, {100, 100, {0, 0.01}},
    {1000, 1000, {0, 0.01}}}}, Table[{y, y, ticksize}, {y, 0, 1, 0.5}]},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 12},
  ImageSize → {xsize, xsize aspectratio},
  AspectRatio → aspectratio,
  Axes → True};

loglinearplotINVSoptions = {Frame → {{True, False}, {True, False}}},
  PlotRangeClipping → False,
  ImagePadding → Pad,
  FrameStyle → Directive[Black, Thickness[lwd]],
  FrameTicksStyle → {{Directive[Black, Thickness[lwd], FontColor → Black],
    Directive[Black, Thickness[lwd]]},
   {Directive[Black, Thickness[lwd]], Directive[Black, Thickness[lwd]]}}},
  FrameTicks → {{{{5, 5, {0, 0.01}}, {10, 10, {0, 0.01}}, {50, 50, {0, 0.01}},
    {100, 100, {0, 0.01}}, {500, 500, {0, 0.01}}, {1000, 1000, {0, 0.01}}}},
    Table[{y, y, ticksize}, {y, 0.4, 0.6, 0.1}]},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 12},
  ImageSize → {xsize, xsize aspectratio },
  AspectRatio → aspectratio,
  Axes → True};
```

```

startplot = 5;
endtime = 1000;
tryR = 0;
trypm = 0.01;
tryk = 1;

```

Panel A - neo-W invading XY

Parameters

```

trys = 0.2;
tryh = 0.7;
tryaDm = -0.1;
tryFAA = 1 + trys ;
tryFAa = 1 + trys tryh;
tryFaa = 1;
tryMAA = 1 + trys;
tryMaa = 1 + trys tryh;
tryMaa = 1;
trywAf = 1;
trywaf = 1;
trywAm = 1;
trywam = 1;
tryaf = 0.5;
tryam = 0.5 + tryaDm;
tryr = 0.02;
subpar1 =
{FAA → tryFAA, FAa → tryFAa, Faa → tryFaa, MAA → tryMAA, MAa → tryMaa, Maa → tryMaa,
 wAf → trywAf, waf → trywaf, wAm → trywAm, wam → trywam, af → tryaf, am → tryam};

```

Plots

```

tryR = 0.001;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMaa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMaa,
                                         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time ≤ endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab2Black =
Table[{Round[Exp[exptime]],  $\frac{\text{generation}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]].\text{modvec}}{\text{sexratio}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]}}$ ,
{exptime, Log[startplot], Log[endtime], 0.1}];
MODBlack = ListLogLinearPlot[MODtab2Black, Joined → True,
                           PlotRange → {{startplot, endtime}, {0, 1.05}},
                           PlotStyle → {Black}, loglinearplotINVOptions];
SRtabBlack = Table[{Round[Exp[exptime]], sexratio[param, run[Round[Exp[exptime]]]]},
                   {exptime, Log[startplot], Log[endtime], 0.1}];
SRBlack = ListLogLinearPlot[SRtabBlack, Joined → True,
                           PlotRange → {{startplot, endtime}, {0.4, 0.601}},
                           PlotStyle → {Black, Thick}, loglinearplotINVOptions];

```

```

tryR = 0.02;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtabRed =
Table[{Round[Exp[exptime]],  $\frac{\text{generation}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]] \cdot \text{modvec}}{\text{sexratio}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]}]$ },
{exptime, Log[startplot], Log[endtime], 0.1}];
MODRed = ListLogLinearPlot[MODtabRed, Joined → True, PlotRange →
{{startplot, endtime}, {0, 1.05}}, PlotStyle → {Red}, loglinearplotINVOptions];
SRtabRed = Table[{Round[Exp[exptime]], sexratio[param, run[Round[Exp[exptime]]]]},
{exptime, Log[startplot], Log[endtime], 0.1}];
SRRed = ListLogLinearPlot[SRtabRed, Joined → True,
PlotRange → {{startplot, endtime}, {0.4, 0.601}},
PlotStyle → {Red, Thick}, loglinearplotINVSRoptions];

tryR = 0.1;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtabBlue =
Table[{Round[Exp[exptime]],  $\frac{\text{generation}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]] \cdot \text{modvec}}{\text{sexratio}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]}]$ },
{exptime, Log[startplot], Log[endtime], 0.1}];
MODBlue = ListLogLinearPlot[MODtabBlue, Joined → True, PlotRange →
{{startplot, endtime}, {0, 1.05}}, PlotStyle → {Blue}, loglinearplotINVOptions];
SRtabBlue = Table[{Round[Exp[exptime]], sexratio[param, run[Round[Exp[exptime]]]]},
{exptime, Log[startplot], Log[endtime], 0.1}];
SRBlue = ListLogLinearPlot[SRtabBlue, Joined → True,
PlotRange → {{startplot, endtime}, {0.4, 0.601}},
PlotStyle → {Blue, Thick}, loglinearplotINVSRoptions];

```

```

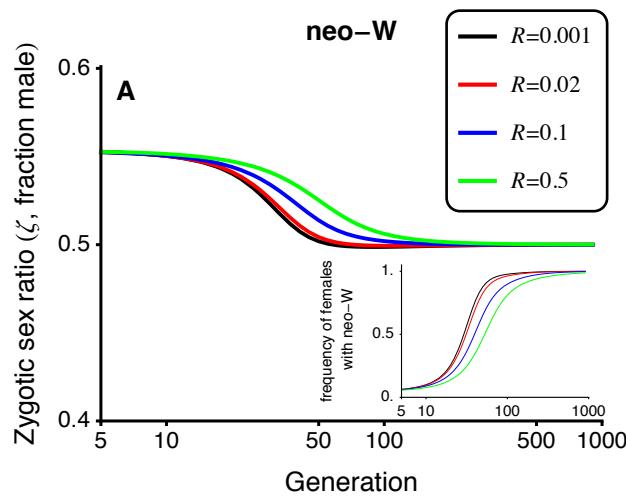
tryR = 0.5;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
          trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
          tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtabGreen =
Table[{Round[Exp[exptime]],  $\frac{\text{generation}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]].\text{modvec}}{\text{sexratio}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]}}$ },
{exptime, Log[startplot], Log[endtime], 0.1}];
MODGreen = ListLogLinearPlot[MODtabGreen, Joined → True,
  PlotRange → {{startplot, endtime}, {0, 1.05}},
  PlotStyle → {Green}, loglinearplotINVOptions];
SRtabGreen = Table[{Round[Exp[exptime]], sexratio[param, run[Round[Exp[exptime]]]]},
{exptime, Log[startplot], Log[endtime], 0.1}];
SRGreen = ListLogLinearPlot[SRtabGreen, Joined → True,
  PlotRange → {{startplot, endtime}, {0.4, 0.601}},
  PlotStyle → {Green, Thick}, loglinearplotINVOptions];

```

```

plotA = Legended[Show[SRBlack, SRRRed, SRBlue, SRGreen,
  ImagePadding -> Pad,
  FrameLabel -> {{Style["Zygotic sex ratio ( $\zeta$ , fraction male)", FontSize -> 14],
    ""}, {Style["Generation", FontSize -> 14], ""}},
  Epilog -> {Inset[Show[MODBlack, MODRed, MODBlue, MODGreen, (*ImageSize -> 150, *)
    BaseStyle -> {TextStyle -> Bold, FontFamily -> "Helvetica", FontSize -> 8},
    FrameLabel -> {Style["frequency of females
with neo-W", FontSize -> 8], ""}], {3.4, 3.4 * Sqrt[2]}],
    Text[Style["A", 14, Bold], Scaled@letpos],
    Text[Style["neo-W", Bold, 14], Scaled@{0.5, 1.1}]]},
  Placed[
  LineLegend[
    {Directive[Thickness[lwd * 20], Black],
      Directive[Thickness[lwd * 20], Red],
      Directive[Thickness[lwd * 20], Blue],
      Directive[Thickness[lwd * 20], Green]
    },
    {Style["R=0.001", 12],
      Style["R=0.02", 12],
      Style["R=0.1", 12],
      Style["R=0.5", 12]
    },
    LegendFunction -> "Frame",
    LegendLayout -> "Column"
  ],
  Scaled@{0.85, 0.875}
]
]
]

```



Panel B - neo-Y invading ZW

Parameters

Here we are going to adjust the fitness parameters only to change definition of sex.
Also have to adjust sex ratio to 1-sexratio because it gives the frequency of the ancestrally heterozygous sex.

```

trys = 0.2;
tryh = 0.7;
tryαDm = -0.1;
tryFAA = 1 + trys ;
tryFAa = 1 + trys tryh;
tryFaa = 1;
tryMAA = 1 + trys;
tryMAa = 1 + trys tryh;
tryMaa = 1;
trywAf = 1;
trywaf = 1;
trywAm = 1;
trywam = 1;
tryaf = 0.5 + tryαDm;
tryam = 0.5;
tryr = 0.02;
subpar1 =
{FAA → tryFAA, FAa → tryFAa, Faa → tryFaa, MAA → tryMAA, MAa → tryMAa, Maa → tryMaa,
 wAf → trywAf, waf → trywaf, wAm → trywAm, wam → trywam, af → tryaf, am → tryam};

```

Plots

```

tryR = 0.001;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time ≤ endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab2Black =
Table[{Round[Exp[exptime]],  $\frac{\text{generation}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]].\text{modvec}}{1 - \text{sexratio}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]}}$ },
{exptime, Log[startplot], Log[endtime], 0.1}];
MODBlack = ListLogLinearPlot[MODtab2Black, Joined → True,
  PlotRange → {{startplot, endtime}, {0, 1.05}},
  PlotStyle → {Black}, loglinearplotINVOptions];
SRtabBlack = Table[{Round[Exp[exptime]],
  1 - sexratio[param, run[Round[Exp[exptime]]]]},
{exptime, Log[startplot], Log[endtime], 0.1}];
SRBlack = ListLogLinearPlot[SRtabBlack, Joined → True,
  PlotRange → {{startplot, endtime}, {0.4, 0.601}},
  PlotStyle → {Black, Thick}, loglinearplotINVOptions];

```

```

tryR = 0.02;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtabRed =
Table[{Round[Exp[exptime]],  $\frac{\text{generation}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]\text{.modvec}}{1 - \text{sexratio}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]}}$ },
{exptime, Log[startplot], Log[endtime], 0.1}];
MODRed = ListLogLinearPlot[MODtabRed, Joined → True, PlotRange →
{{startplot, endtime}, {0, 1.05}}, PlotStyle → {Red}, loglinearplotINVOptions];
SRtabRed = Table[{Round[Exp[exptime]],
1 - sexratio[param, run[Round[Exp[exptime]]]]},
{exptime, Log[startplot], Log[endtime], 0.1}];
SRRed = ListLogLinearPlot[SRtabRed, Joined → True,
PlotRange → {{startplot, endtime}, {0.4, 0.601}},
PlotStyle → {Red, Thick}, loglinearplotINVSRoptions];

tryR = 0.1;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtabBlue =
Table[{Round[Exp[exptime]],  $\frac{\text{generation}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]\text{.modvec}}{1 - \text{sexratio}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]}}$ },
{exptime, Log[startplot], Log[endtime], 0.1}];
MODBlue = ListLogLinearPlot[MODtabBlue, Joined → True, PlotRange →
{{startplot, endtime}, {0, 1.05}}, PlotStyle → {Blue}, loglinearplotINVOptions];
SRtabBlue = Table[{Round[Exp[exptime]],
1 - sexratio[param, run[Round[Exp[exptime]]]]},
{exptime, Log[startplot], Log[endtime], 0.1}];
SRBlue = ListLogLinearPlot[SRtabBlue, Joined → True,
PlotRange → {{startplot, endtime}, {0.4, 0.601}},
PlotStyle → {Blue, Thick}, loglinearplotINVSRoptions];

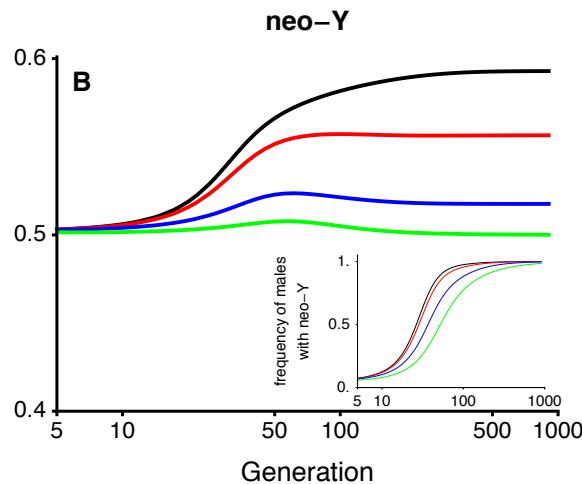
```

```

tryR = 0.5;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
          trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
          tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtabGreen =
Table[{Round[Exp[exptime]],  $\frac{\text{generation}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]\text{.modvec}}{1 - \text{sexratio}[\text{param}, \text{run}[\text{Round}[\text{Exp}[\text{exptime}]]]]}}$ },
{exptime, Log[startplot], Log[endtime], 0.1}];
MODGreen = ListLogLinearPlot[MODtabGreen, Joined → True,
PlotRange → {{startplot, endtime}, {0, 1.05}},
PlotStyle → {Green}, loglinearplotINVOptions];
SRtabGreen = Table[{Round[Exp[exptime]],
1 - sexratio[param, run[Round[Exp[exptime]]]]},
{exptime, Log[startplot], Log[endtime], 0.1}];
SRGreen = ListLogLinearPlot[SRtabGreen, Joined → True,
PlotRange → {{startplot, endtime}, {0.4, 0.601}},
PlotStyle → {Green, Thick}, loglinearplotINVOptions];

plotB = Show[SRBlack, SRRed, SRBlue, SRGreen,
ImagePadding → Pad,
FrameLabel →
{{Style["", FontSize → 14], ""}, {Style["Generation", FontSize → 14], ""}},
Epilog → {Inset[Show[MODBlack, MODRed, MODBlue, MODGreen, (*ImageSize→150,*)
BaseStyle → {TextStyle → Bold, FontFamily → "Helvetica", FontSize → 8},
FrameLabel → {{Style["frequency of males
with neo-Y", FontSize → 8], ""}, {Style["", FontSize → 14], ""}}],
Scaled@{1.05, -0.15}, {Right, Bottom}, {3.4, 3.4 * Sqrt[2]}],
Text[Style["B", 14, Bold], Scaled@letpos],
Text[Style["neo-Y", Bold, 14], Scaled@{0.5, 1.1}]}
]

```



All panels

```
GraphicsRow[{plotA, plotB}, Spacings -> -35]
Export[plotdir <> "Temporal_SR.eps", %(*//rasterTrick*)];
```

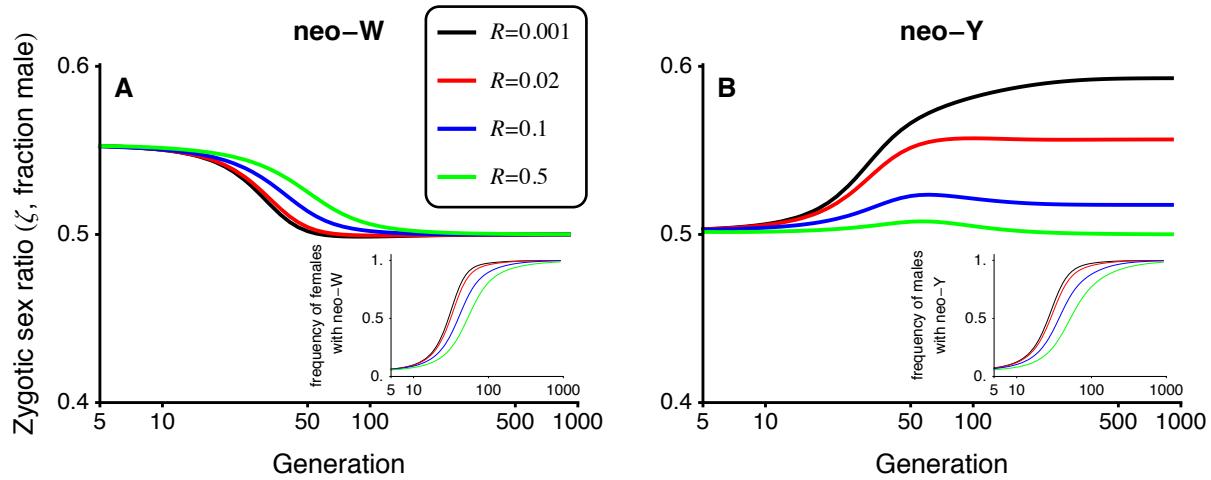


Figure S.1 - neo-W invasion fitness as a function of the position of the selected locus with overdominance in males, no haploid selection

Plotting parameters

```
(*Pad={{50,20},{50,20}}; (*space around plot*)
yplotmin = -0.075;
yplotmax = 0.15;
yplotinterval = 0.02;
(*ylabpos={-0.15,0.5}; (*relative location of y axis label position*))
```

Panel A - loose linkage; overdominance in males and directional selection for a in females

Parameters

```
x = -25;
m = 25;

tryt = 0;
tryFAA = 0.85;
tryFAa = 1;
tryFaa = 1.05;
tryMAA = 0.75;
tryMaa = 1;
tryMaa = 0.75;
tryMA = 1 + tryt;
tryMa = 1;
tryFA = 1;
tryFa = 1;
tryMdA = 0.5;
tryFdA = 0.5;

xplotmin = -50;
xplotmax = 50;
xplotinterval = 25;
npoints = 32;
```

Plot

```
positionplotA =
Legended[
Show[
(*region of tighter linkage*)
RegionPlot[x > 0, {x, xplotmin, xplotmax}, {y, yplotmin, yplotmax},
PlotStyle -> {LightGray},
BoundaryStyle -> None,
PlotRange -> {yplotmin, yplotmax},
AxesOrigin -> {xplotmin, yplotmin},
Frame -> {True, True, False, False}],
],
(*neo-W into XY*)
ListPlot[
Table[{a, invasionplotXY2[{tryFAA, tryFAa, tryFaa, tryMAA, tryMaa, tryMaa,
tryFA, tryFa, tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m], setR[x, a, m],
seto[x, a, m]}, sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMaa, tryMaa,
tryFA, tryFa, tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m]][[1]]]},
{a, xplotmin, xplotmax, (xplotmax - xplotmin) / npoints}],
Joined -> True,
PlotRange -> {yplotmin, All},
PlotStyle -> Directive[Blue, Thickness[lwd]]],
```

```

AxesOrigin -> {xplotmin, yplotmin},
Frame -> {True, True, False, False}
],
ListPlot[Table[{a,
    invasionplotXY2[{tryFAA, tryFAa, tryFaa, tryMAA, tryMaa, tryMaa, tryFA, tryFa,
        tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m], setR[x, a, m], setp[x, a, m]}, 
        sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMaa, tryMaa, tryFA, tryFa,
            tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m]][[2]]]}, {a, -26, -24, 0.25}],
Joined -> True,
PlotRange -> {yplotmin, All},
PlotStyle -> Directive[Black, Thickness[lwd]],
AxesOrigin -> {xplotmin, yplotmin},
Frame -> {True, True, False, False}
],
Plot[0, {x, xplotmin, xplotmax}, PlotStyle -> {Black, Dotted}],
plotstyle[xplotmin, xplotmax, xplotmin, xplotmax,
    xplotinterval, yplotmin, yplotmax, ytickmin, ytickmax, yint],
Epilog -> {
    Text[Style["A", Bold], Scaled@letpos],
    Rotate[
        Text[Style["Invasion fitness ( $\lambda_w^{(xx)} - 1$ )"], Scaled@{-0.15, 0.5}], 90 Degree]
    (*,Text[Style["tighter sex-linkage",14],{m,yplotmax*0.9}],*)
    (*,Text["ancestral SDR",{x,yplotmin*0.75}],
    Text["novel SDR",{m,yplotmin*0.75}],
    Arrow[{{x,yplotmin*0.5},{x,0}}],
    Arrow[{{m,yplotmin*0.5},{m,0}}]*)
}
],
Placed[
LineLegend[
{Directive[Thickness[0.1], Black],
 Directive[Thickness[0.1], Blue]
},
{Style["Y-a equilibrium", 14 * posplotscale],
 Style["Y-A equilibrium", 14 * posplotscale]
},
LegendFunction -> "Frame",
LegendLayout -> "Column"
],
Scaled@{0.7, 0.75}
]
]
]

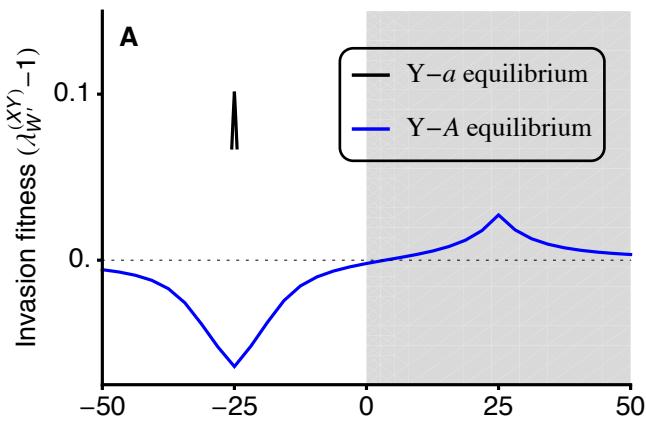
```

Part::partw : Part 2 of {{0.0283897, 0.0313209, 0.959256, 0.5}} does not exist. >>

Part::partw : Part 2 of {{0.0213283, 0.0235138, 0.969581, 0.5}} does not exist. >>

Part::partw : Part 2 of {{0.0213283, 0.0235138, 0.969581, 0.5}} does not exist. >>

General::stop : Further output of Part::partw will be suppressed during this calculation. >>



Panel B - tight linkage; overdominance in males and directional selection for a in females

Parameters

```
x = -5;
m = 5;

tryt = 0;
tryFAA = 0.85;
tryFaa = 1;
tryFaa = 1.05;
tryMAA = 0.75;
tryMaa = 1;
tryMaa = 0.75;
tryMA = 1 + tryt;
tryMa = 1;
tryFA = 1;
tryFa = 1;
tryMdA = 0.5;
tryFdA = 0.5;

xplotmin = -10;
xplotmax = 10;
xplotinterval = 5;
npoints = 32;
```

Plot

```
positionplotB =
(*Legended[*)
Show[
(*region of tighter linkage*)
RegionPlot[x > 0, {x, xplotmin, xplotmax}, {y, yplotmin, yplotmax},
PlotStyle -> {LightGray},
```

```

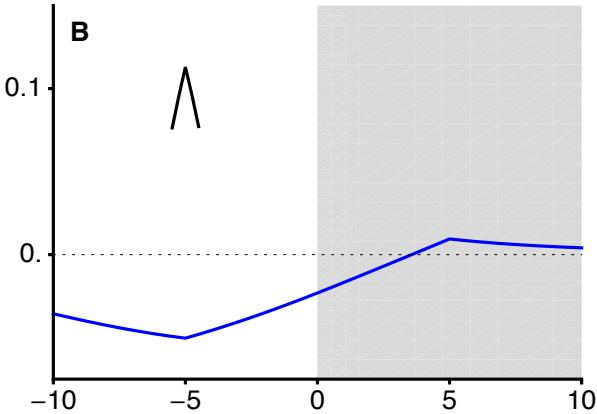
BoundaryStyle -> None,
PlotRange -> {yplotmin, yplotmax},
AxesOrigin -> {xplotmin, yplotmin},
Frame -> {True, True, False, False}
], 

(*neo-W into XY*)
ListPlot[
Table[{a, invasionplotXY2[{tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa,
    tryFA, tryFa, tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m], setR[x, a, m],
    seto[x, a, m]}, sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa,
    tryFA, tryFa, tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m]][[1]]]},
{a, xplotmin, xplotmax, (xplotmax - xplotmin) / npoints}],
Joined -> True,
PlotRange -> {yplotmin, All},
PlotStyle -> Directive[Blue, Thickness[lwd]],
AxesOrigin -> {xplotmin, yplotmin},
Frame -> {True, True, False, False}
],
ListPlot[Table[{a,
    invasionplotXY2[{tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, tryFA, tryFa,
        tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m], setR[x, a, m], seto[x, a, m]},
        sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, tryFA, tryFa,
        tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m]][[2]]}], {a, -6, -4, 0.25}],
Joined -> True,
PlotRange -> {yplotmin, All},
PlotStyle -> Directive[Black, Thickness[lwd]],
AxesOrigin -> {xplotmin, yplotmin},
Frame -> {True, True, False, False}
],
Plot[0, {x, xplotmin, xplotmax}, PlotStyle -> {Black, Dotted}],
plotstyle[xplotmin, xplotmax, xplotmin, xplotmax,
    xplotinterval, yplotmin, yplotmax, ytickmin, ytickmax, yint],
Epilog -> {
    Text[Style["B", Bold], Scaled@letpos]
},
PlotRangeClipping -> False
](*,
Placed[
LineLegend[
{Directive[Thickness[0.005], Black],
 Directive[Thickness[0.005], Blue]
},
{Style["Ya equilibrium", 16],
 Style["YA equilibrium", 16]
},
(*LegendFunction -> "Frame", *)
LegendLayout -> "Column"
],
Scaled@{0.72, 0.65}
]
]*)
]
```

```

Part::partw : Part 2 of {{0.0283897, 0.0313209, 0.959256, 0.5}} does not exist. >>
Part::partw : Part 2 of {{0.0213283, 0.0235138, 0.969581, 0.5}} does not exist. >>
Part::partw : Part 2 of {{0.0213283, 0.0235138, 0.969581, 0.5}} does not exist. >>
General::stop : Further output of Part::partw will be suppressed during this calculation. >>

```



Panel C - loose linkage; overdominance in both sexes

Parameters

```

x = -25;
m = 25;

tryt = 0;
tryFAA = 0.6;
tryFAa = 1;
tryFaa = 0.6;
tryMAA = 0.7;
tryMAa = 1;
tryMaa = 0.5;
tryMA = 1 + tryt;
tryMa = 1;
tryFA = 1;
tryFa = 1;
tryMdA = 0.5;
tryFdA = 0.5;

xplotmin = -50;
xplotmax = 50;
xplotinterval = 25;
npoints = 36;

```

Plot

```

positionplotC =
Show[

```

```

(*region of tighter linkage*)
RegionPlot[x > 0, {x, xplotmin, xplotmax}, {y, yplotmin, yplotmax},
  PlotStyle -> {LightGray},
  BoundaryStyle -> None,
  PlotRange -> {yplotmin, yplotmax},
  AxesOrigin -> {xplotmin, yplotmin},
  Frame -> {True, True, False, False}
], 

(*neo-W into XY*)
ListPlot[
Table[{a, invasionplotXY2[{tryFAA, tryFAa, tryFaa, tryMAA, tryMaa, tryMaa,
  tryFA, tryFa, tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m], setR[x, a, m],
  seto[x, a, m]}, sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMaa, tryMaa,
  tryFA, tryFa, tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m]][[1]]]},
{a, xplotmin, xplotmax, (xplotmax - xplotmin) / npoints}],
Joined -> True,
PlotRange -> {yplotmin, All},
PlotStyle -> Directive[Blue, Thickness[lwd]],
AxesOrigin -> {xplotmin, yplotmin},
Frame -> {True, True, False, False}
], 
ListPlot[Table[{a,
  invasionplotXY2[{tryFAA, tryFAa, tryFaa, tryMAA, tryMaa, tryMaa, tryFA, tryFa,
  tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m], setR[x, a, m], seto[x, a, m]},
  sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMaa, tryMaa, tryFA, tryFa,
  tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m]][[2]]}], {a, -27, -23, 0.25}],
Joined -> True,
PlotRange -> {yplotmin, All},
PlotStyle -> Directive[Black, Thickness[lwd]],
AxesOrigin -> {xplotmin, yplotmin},
Frame -> {True, True, False, False}
], 

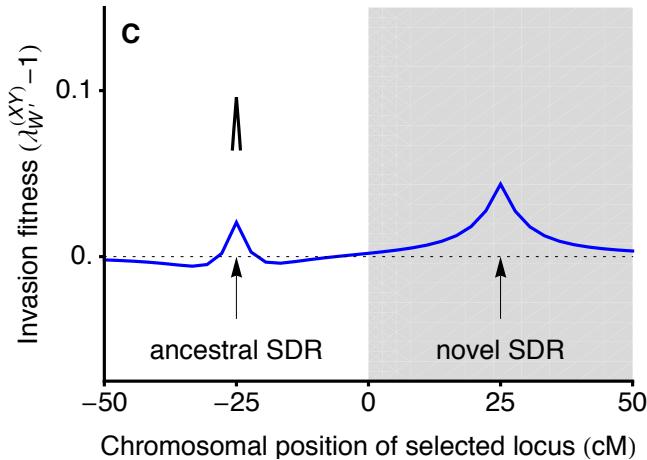
Plot[0, {x, xplotmin, xplotmax}, PlotStyle -> {Black, Dotted}],
plotstyle[xplotmin, xplotmax, xplotmin, xplotmax,
 xplotinterval, yplotmin, yplotmax, ytickmin, ytickmax, yint],
FrameLabel -> {"Chromosomal position of selected locus (cM)", ""},
Epilog -> {
  Text[Style["C", Bold], Scaled@letpos],
  Rotate[
    Text[Style["Invasion fitness ( $\lambda_w^{(XY)} - 1$ )"], Scaled@{-0.15, 0.5}], 90 Degree]
  (*,Text[Style["tighter sex-linkage",14],{m,yplotmax*0.9}],*)
  , Text["ancestral SDR", {x, yplotmin * 0.75}],
  Text["novel SDR", {m, yplotmin * 0.75}],
  Arrow[{{x, yplotmin * 0.5}, {x, 0}}],
  Arrow[{{m, yplotmin * 0.5}, {m, 0}}]
  ],
  PlotRangeClipping -> False
]
]

```

```

Part::partw : Part 2 of {{0.40879, 0.351282, 0.936149, 0.5}} does not exist. >>
Part::partw : Part 2 of {{0.404106, 0.344002, 0.944328, 0.5}} does not exist. >>
Part::partw : Part 2 of {{0.399301, 0.336588, 0.952469, 0.5}} does not exist. >>
General::stop : Further output of Part::partw will be suppressed during this calculation. >>

```



Panel D - tight linkage and overdominance in both sexes

Parameters

```

x = -5;
m = 5;

tryt = 0;
tryFAA = 0.6;
tryFAa = 1;
tryFaa = 0.6;
tryMAA = 0.7;
tryMAa = 1;
tryMaa = 0.5;
tryMA = 1 + tryt;
tryMa = 1;
tryFA = 1;
tryFa = 1;
tryMdA = 0.5;
tryFdA = 0.5;

xplotmin = -10;
xplotmax = 10;
xplotinterval = 5;
npoints = 36;

```

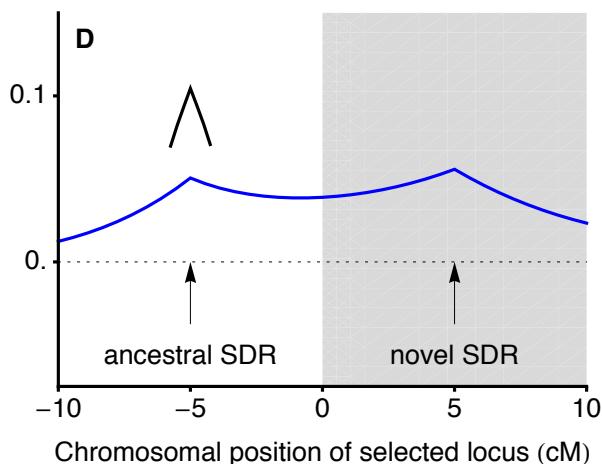
Plot

```

positionplotD =
Show[
  (*region of tighter linkage*)
  RegionPlot[x > 0, {x, xplotmin, xplotmax}, {y, yplotmin, yplotmax},
    PlotStyle -> {LightGray},
    BoundaryStyle -> None,
    PlotRange -> {yplotmin, yplotmax},
    AxesOrigin -> {xplotmin, yplotmin},
    Frame -> {True, True, False, False}
  ],
  (*neo-W into XY*)
  ListPlot[
    Table[{a, invasionplotXY2[{tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa,
      tryFA, tryFa, tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m], setR[x, a, m],
      seto[x, a, m]}, sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa,
      tryFA, tryFa, tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m]][[1]]]},
      {a, xplotmin, xplotmax, (xplotmax - xplotmin) / npoints}],
    Joined -> True,
    PlotRange -> {yplotmin, All},
    PlotStyle -> Directive[Blue, Thickness[lwd]],
    AxesOrigin -> {xplotmin, yplotmin},
    Frame -> {True, True, False, False}
  ],
  ListPlot[Table[{a,
    invasionplotXY2[{tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, tryFA, tryFa,
      tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m], setR[x, a, m], seto[x, a, m]},
      sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, tryFA, tryFa,
      tryMA, tryMa, tryFdA, tryMdA, setr[x, a, m]][[2]]}], {a, -6, -4, 0.25}],
    Joined -> True,
    PlotRange -> {yplotmin, All},
    PlotStyle -> Directive[Black, Thickness[lwd]],
    AxesOrigin -> {xplotmin, yplotmin},
    Frame -> {True, True, False, False}
  ],
  Plot[0, {x, xplotmin, xplotmax}, PlotStyle -> {Black, Dotted}],
  plotstyle[xplotmin, xplotmax, xplotmin, xplotmax,
    xplotinterval, yplotmin, yplotmax, ytickmin, ytickmax, yint],
  FrameLabel -> {"Chromosomal position of selected locus (cM)", ""},
  Epilog -> {
    Text[Style["D", Bold], Scaled@letpos],
    (*Rotate[Text[Style["Invasion fitness (\lambda-1)", 14], Scaled@ylabpos], 90 Degree],
    Text[Style["tighter sex-linkage", 14], {m, yplotmax * 0.9}], *)
    Text["ancestral SDR", {x, yplotmin * 0.75}],
    Text["novel SDR", {m, yplotmin * 0.75}],
    Arrow[{{x, yplotmin * 0.5}, {x, 0}}],
    Arrow[{{m, yplotmin * 0.5}, {m, 0}}]
  },
  PlotRangeClipping -> False
]

```

```
Part::partw : Part 2 of {{0.389312, 0.321357, 0.968596, 0.5}} does not exist. >>
Part::partw : Part 2 of {{0.389312, 0.321357, 0.968596, 0.5}} does not exist. >>
```



Combined

```

GraphicsGrid[{{positionplotA, positionplotB}, {positionplotC, positionplotD}} ,
  Spacings -> {-36, -30},
  Epilog -> {
    Rotate[Text[Style["overdominance in males", 14 * posplotscale,
      Bold, FontFamily -> "Helvetica"], Scaled@{1.5, 1.05}], -π/2],
    Rotate[Text[Style["overdominance in both sexes", 14 * posplotscale,
      Bold, FontFamily -> "Helvetica"], Scaled@{1.5, 0.1}], -π/2],
    Rotate[Text[Style["loose linkage", 14 * posplotscale, Bold,
      FontFamily -> "Helvetica"], Scaled@{0.05, 1.475}], 0],
    Rotate[Text[Style["tight linkage", 14 * posplotscale, Bold,
      FontFamily -> "Helvetica"], Scaled@{1.05, 1.475}], 0]
  }
]

Export[plotdir <> "PositionPlot_Overdominance.eps", %(*//rasterTrick*)];

```

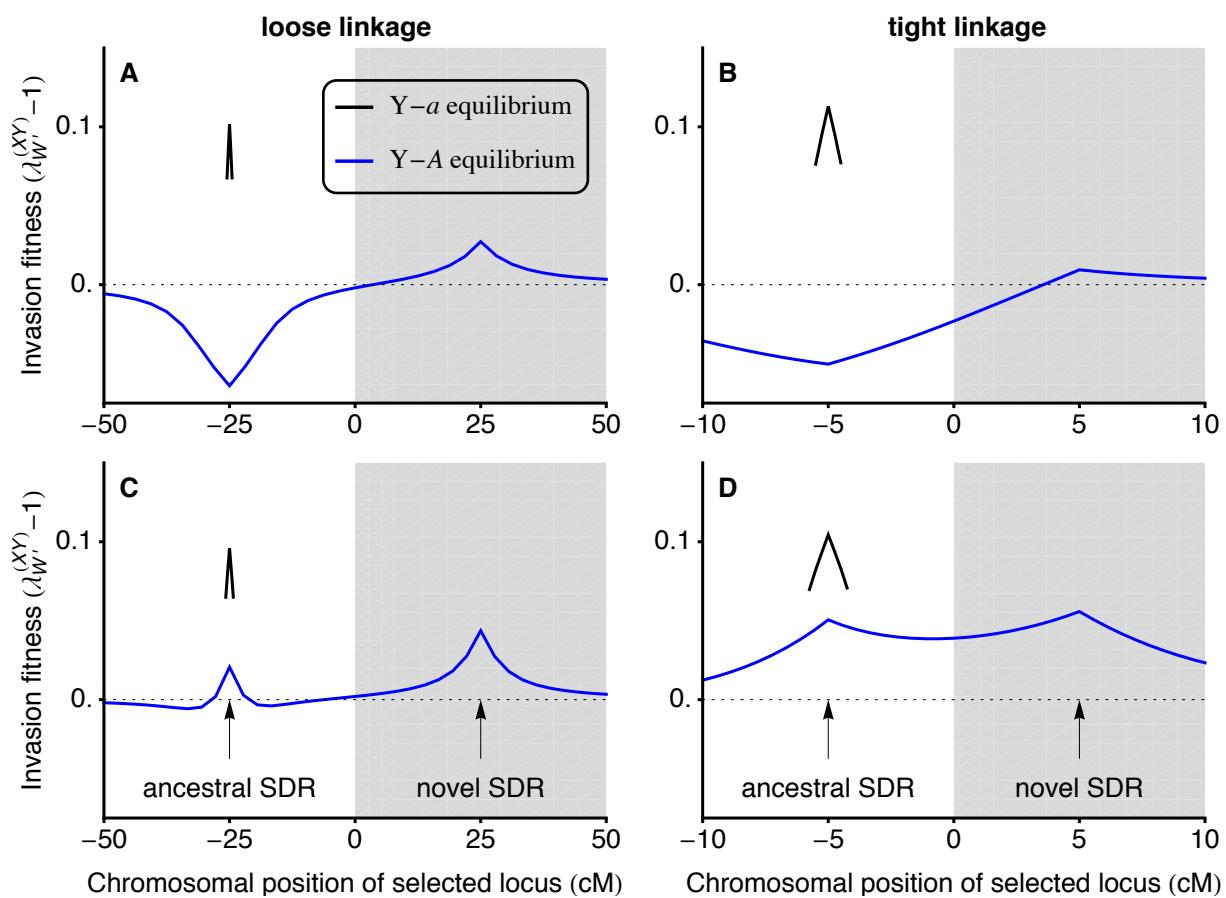


Figure S.2 - neo-W invasion dynamics for tight linkage ($r \sim 0$) with overdominance in males, no haploid selection

Plotting parameters

```
Pad = {{50, 20}, {50, 30}}; (*space around plot*)
yplotmin = -0.075;
yplotmax = 0.15;
yplotinterval = 0.02;
ylabpos = {-0.13, 0.5}; (*relative location of y axis label position*)
```

Parameters

```
tryFAA2 = 0.85;
tryFAa2 = 1;
tryFaa2 = 1.05;
tryMAA2 = 0.75;
tryMaa2 = 1;
tryMaa2 = 0.75;
trywAf2 = 1;
trywaf2 = 1;
trywAm2 = 1;
trywam2 = 1;
tryaf2 = 0.5;
tryam2 = 0.5;
tryr2 = 0.005;
tryR2 = 0;
subpar2 = {FAA -> tryFAA2, FAa -> tryFAa2, Faa -> tryFaa2, MAA -> tryMAA2,
           MAa -> tryMaa2, Maa -> tryMaa2, wAf -> trywAf2, waf -> trywaf2,
           wAm -> trywAm2, wam -> trywam2, af -> tryaf2, am -> tryam2};

tryFAA3 = 0.6;
tryFAa3 = 1;
tryFaa3 = 0.6;
tryMAA3 = 0.7;
tryMaa3 = 1;
tryMaa3 = 0.5;
trywAf3 = 1;
trywaf3 = 1;
trywAm3 = 1;
trywam3 = 1;
tryaf3 = 0.5;
tryam3 = 0.5;
tryr3 = 0.005;
tryR3 = 0;
subpar3 = {FAA -> tryFAA3, FAa -> tryFAa3, Faa -> tryFaa3, MAA -> tryMAA3,
           MAa -> tryMaa3, Maa -> tryMaa3, wAf -> trywAf3, waf -> trywaf3,
           wAm -> trywAm3, wam -> trywam3, af -> tryaf3, am -> tryam3};
```

Extra parameters

```

modvec = {0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0};

loglinearplotINVoptions = {Frame -> {{True, False}, {True, False}},
  PlotRangeClipping -> False,
  ImagePadding -> Pad,
  FrameStyle -> Directive[Black, Thickness[lwd]],
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]], FontColor -> Black],
    Directive[Black, Thickness[lwd]]},
  {Directive[Black, Thickness[lwd]], Directive[Black, Thickness[lwd]]}},
  FrameTicks -> {{{{5, 5, {0, 0.01}}, {10, 10, {0, 0.01}}, {50, 50, {0, 0.01}}},
    {100, 100, {0, 0.01}}, {500, 500, {0, 0.01}}, {1000, 1000, {0, 0.01}},
    {5000, 5000, {0, 0.01}}}}, Table[{y, y, ticksize}, {y, 0, 0.5, 0.25}]},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 12},
  ImageSize -> {xsize, xsize aspectratio},
  AspectRatio -> aspectratio,
  Axes -> True};

startplot = 5;
endtime = 5000;
tryR = 0;
trypm = 0.01;
tryk = 1;

```

Panel A - a favoured in females, r = 0

```

tryr2 = 0;

tryR = 0.001;
param =
  {tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2, trywaf2, trywAm2,
   trywam2, tryaf2, tryam2, tryr2, tryR, tryr2 (1 - tryR) + tryR (1 - tryr2), tryk};
run[0] = generation[param, startgen[
  sieveXY[tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2,
   trywaf2, trywAm2, trywam2, tryaf2, tryam2, tryr2][[2]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab2Blackr0 =
  Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
    {exptime, Log[startplot], Log[endtime], 0.1}];
MOD2Blackr0 = ListLogLinearPlot[MODtab2Blackr0, Joined -> True,
  PlotRange -> {{startplot, endtime}, {0, 0.5}},
  PlotStyle -> {Black, Thick}, loglinearplotINVoptions];

```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The
answer was obtained by solving a corresponding exact system and numericizing the result. >>

```

tryR = 0.02;
param =
{tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2, trywaf2, trywAm2,
 trywam2, tryaf2, tryam2, tryr2, tryR, tryr2 (1 - tryR) + tryR (1 - tryr2), tryk};
run[0] = generation[param, startgen[
  sieveXY[tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2,
  trywaf2, trywAm2, trywam2, tryaf2, tryam2, tryr2][[2]], trypm]];
For[time = 1, time <= endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab2Redr0 =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
 {exptime, Log[startplot], Log[endtime], 0.1}];
MOD2Redr0 = ListLogLinearPlot[MODtab2Redr0, Joined → True,
 PlotRange → {{startplot, endtime}, {0, 0.5}},
 PlotStyle → {Red, Thick}, loglinearplotINVoptions];

tryR = 0.1;
param =
{tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2, trywaf2, trywAm2,
 trywam2, tryaf2, tryam2, tryr2, tryR, tryr2 (1 - tryR) + tryR (1 - tryr2), tryk};
run[0] = generation[param, startgen[
  sieveXY[tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2,
  trywaf2, trywAm2, trywam2, tryaf2, tryam2, tryr2][[2]], trypm]];
For[time = 1, time <= endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab2Bluer0 =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
 {exptime, Log[startplot], Log[endtime], 0.1}];
MOD2Bluer0 = ListLogLinearPlot[MODtab2Bluer0, Joined → True,
 PlotRange → {{startplot, endtime}, {0, 0.5}},
 PlotStyle → {Blue, Thick}, loglinearplotINVoptions];

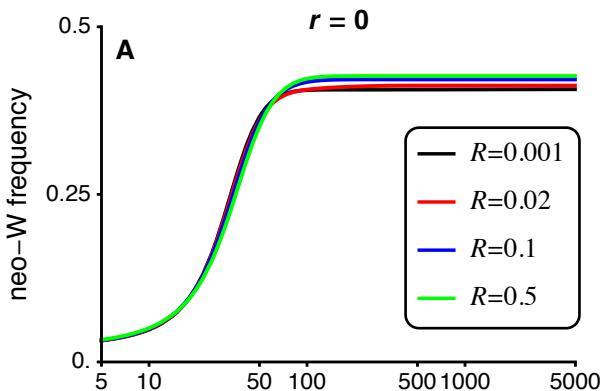
tryR = 0.5;
param =
{tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2, trywaf2, trywAm2,
 trywam2, tryaf2, tryam2, tryr2, tryR, tryr2 (1 - tryR) + tryR (1 - tryr2), tryk};
run[0] = generation[param, startgen[
  sieveXY[tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2,
  trywaf2, trywAm2, trywam2, tryaf2, tryam2, tryr2][[2]], trypm]];
For[time = 1, time <= endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab2Greenr0 =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
 {exptime, Log[startplot], Log[endtime], 0.1}];
MOD2Greenr0 = ListLogLinearPlot[MODtab2Greenr0, Joined → True,
 PlotRange → {{startplot, endtime}, {0, 0.5}},
 PlotStyle → {Green, Thick}, loglinearplotINVoptions];

```

```

partA = Legended[Show[MOD2Blackr0, MOD2Redr0, MOD2Bluer0, MOD2Greenr0,
FrameLabel →
{{Style["neo-W frequency", FontSize → 14], ""}, {Style["", FontSize → 14], ""}},
Epilog → {
Text[Style["A", 14, Bold], Scaled@letpos],
Text[Style["r = 0", 14, Bold], Scaled@{0.5, 1.02}]
}],
Placed[
LineLegend[
{Directive[Thickness[0.1], Black],
Directive[Thickness[0.1], Red],
Directive[Thickness[0.1], Blue],
Directive[Thickness[0.1], Green]
},
{Style["R=0.001", 14],
Style["R=0.02", 14],
Style["R=0.1", 14],
Style["R=0.5", 14]
},
LegendFunction → "Frame",
LegendLayout → "Column"
],
Scaled@{0.825, 0.4}
]
]
]

```



Panel B - a favoured in females, r = 0.005

```
tryr2 = 0.005;
```

```

tryR = 0.001;
param =
{tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2, trywaf2, trywAm2,
 trywam2, tryaf2, tryam2, tryr2, tryR, tryr2 (1 - tryR) + tryR (1 - tryr2), tryk};
run[0] = generation[param, startgen[
  sieveXY[tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2,
  trywaf2, trywAm2, trywam2, tryaf2, tryam2, tryr2][[2]], trypm]];
For[time = 1, time <= endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab2Black =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD2Black = ListLogLinearPlot[MODtab2Black, Joined → True,
 PlotRange → {{startplot, endtime}, {0, 0.5}},
 PlotStyle → {Black, Thick}, loglinearplotINVoptions];

tryR = 0.02;
param =
{tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2, trywaf2, trywAm2,
 trywam2, tryaf2, tryam2, tryr2, tryR, tryr2 (1 - tryR) + tryR (1 - tryr2), tryk};
run[0] = generation[param, startgen[
  sieveXY[tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2,
  trywaf2, trywAm2, trywam2, tryaf2, tryam2, tryr2][[2]], trypm]];
For[time = 1, time <= endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab2Red =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD2Red = ListLogLinearPlot[MODtab2Red, Joined → True,
 PlotRange → {{startplot, endtime}, {0, 0.5}},
 PlotStyle → {Red, Thick}, loglinearplotINVoptions];

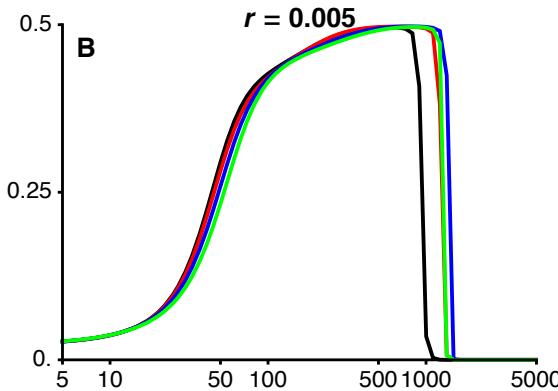
tryR = 0.1;
param =
{tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2, trywaf2, trywAm2,
 trywam2, tryaf2, tryam2, tryr2, tryR, tryr2 (1 - tryR) + tryR (1 - tryr2), tryk};
run[0] = generation[param, startgen[
  sieveXY[tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2,
  trywaf2, trywAm2, trywam2, tryaf2, tryam2, tryr2][[2]], trypm]];
For[time = 1, time <= endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab2Blue =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD2Blue = ListLogLinearPlot[MODtab2Blue, Joined → True,
 PlotRange → {{startplot, endtime}, {0, 0.5}},
 PlotStyle → {Blue, Thick}, loglinearplotINVoptions];

```

```

tryR = 0.5;
param =
{tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2, trywaf2, trywAm2,
 trywam2, tryaf2, tryam2, tryr2, tryR, tryr2 (1 - tryR) + tryR (1 - tryr2), tryk};
run[0] = generation[param, startgen]
sieveXY[tryFAA2, tryFAa2, tryFaa2, tryMAA2, tryMaa2, trywAf2,
        trywaf2, trywAm2, trywam2, tryaf2, tryam2, tryr2][[2]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab2Green =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD2Green = ListLogLinearPlot[MODtab2Green, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.5}},
PlotStyle → {Green, Thick}, loglinearplotINVoptions];
partB = Show[MOD2Black, MOD2Red, MOD2Blue, MOD2Green,
(*FrameLabel → {{Style["neo-W frequency", FontSize → 14], ""},
{Style["Generation", FontSize → 14], ""}}}, *]
Epilog → {
Text[Style["B", 14, Bold], Scaled@letpos],
Text[Style["r = 0.005", 14, Bold], Scaled@{0.5, 1.02}]
}]

```



Panel C - overdominance in females, r = 0

```
tryr3 = 0;
```

```

tryR = 0.001;
param =
{tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMAa3, tryMaa3, trywAf3, trywaf3, trywAm3,
 trywam3, tryaf3, tryam3, tryr3, tryR, tryr3 (1 - tryR) + tryR (1 - tryr3), tryk};
run[0] = generation[param, startgen[
  sieveXY[tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMAa3, tryMaa3, trywAf3,
  trywaf3, trywAm3, trywam3, tryaf3, tryam3, tryr3][[2]], trypm]];
For[time = 1, time <= endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab3Blackr0 =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
 {exptime, Log[startplot], Log[endtime], 0.1}];
MOD3Blackr0 = ListLogLinearPlot[MODtab3Blackr0, Joined → True,
 PlotRange → {{startplot, endtime}, {0, 0.5}},
 PlotStyle → {Black, Thick}, loglinearplotINVoptions];

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The
answer was obtained by solving a corresponding exact system and numericizing the result. >>

tryR = 0.02;
param =
{tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMAa3, tryMaa3, trywAf3, trywaf3, trywAm3,
 trywam3, tryaf3, tryam3, tryr3, tryR, tryr3 (1 - tryR) + tryR (1 - tryr3), tryk};
run[0] = generation[param, startgen[
  sieveXY[tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMAa3, tryMaa3, trywAf3,
  trywaf3, trywAm3, trywam3, tryaf3, tryam3, tryr3][[2]], trypm]];
For[time = 1, time <= endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab3Redr0 =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
 {exptime, Log[startplot], Log[endtime], 0.1}];
MOD3Redr0 = ListLogLinearPlot[MODtab3Redr0, Joined → True,
 PlotRange → {{startplot, endtime}, {0, 0.5}},
 PlotStyle → {Red, Thick}, loglinearplotINVoptions];

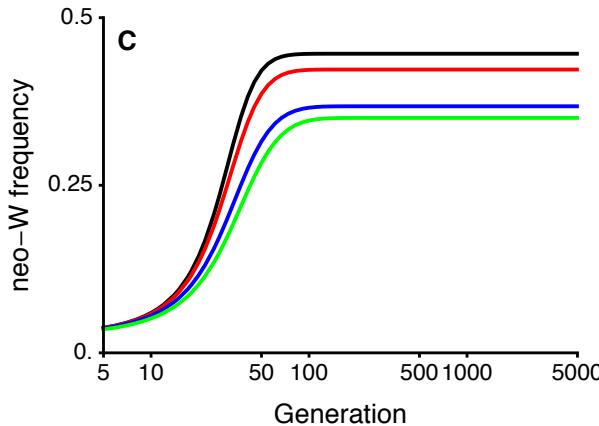
tryR = 0.1;
param =
{tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMAa3, tryMaa3, trywAf3, trywaf3, trywAm3,
 trywam3, tryaf3, tryam3, tryr3, tryR, tryr3 (1 - tryR) + tryR (1 - tryr3), tryk};
run[0] = generation[param, startgen[
  sieveXY[tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMAa3, tryMaa3, trywAf3,
  trywaf3, trywAm3, trywam3, tryaf3, tryam3, tryr3][[2]], trypm]];
For[time = 1, time <= endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab3Bluer0 =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
 {exptime, Log[startplot], Log[endtime], 0.1}];
MOD3Bluer0 = ListLogLinearPlot[MODtab3Bluer0, Joined → True,
 PlotRange → {{startplot, endtime}, {0, 0.5}},
 PlotStyle → {Blue, Thick}, loglinearplotINVoptions];

```

```

tryR = 0.5;
param =
{tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMaa3, trywAf3, trywaf3, trywAm3,
 trywam3, tryaf3, tryam3, tryr3, tryR, tryr3 (1 - tryR) + tryR (1 - tryr3), tryk};
run[0] = generation[param, startgen[
    sieveXY[tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMaa3, trywAf3,
    trywaf3, trywAm3, trywam3, tryaf3, tryam3, tryr3][[2]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab3Greenr0 =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD3Greenr0 = ListLogLinearPlot[MODtab3Greenr0, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.5}},
PlotStyle → {Green, Thick}, loglinearplotINVoptions];
partC = Show[MOD3Blackr0, MOD3Redr0, MOD3Bluer0, MOD3Greenr0,
FrameLabel → {{Style["neo-W frequency", FontSize → 14], ""},
{Style["Generation", FontSize → 14], ""}},
Epilog → {
Text[Style["C", 14, Bold], Scaled@letpos]*,
Text[Style["r=0", 14, Bold], Scaled@{0.5, 1.02}]*}
}]

```



Panel D - overdominance in females, r = 0.005

```
tryr3 = 0.005;
```

```

tryR = 0.001;
param =
{tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMaa3, trywAf3, trywaf3, trywAm3,
 trywam3, tryaf3, tryam3, tryr3, tryR, tryr3 (1 - tryR) + tryR (1 - tryr3), tryk};
run[0] = generation[param, startgen[
  sieveXY[tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMaa3, trywAf3,
  trywaf3, trywAm3, trywam3, tryaf3, tryam3, tryr3][[2]], trypm]];
For[time = 1, time <= endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab3Black =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD3Black = ListLogLinearPlot[MODtab3Black, Joined → True,
 PlotRange → {{startplot, endtime}, {0, 0.5}},
 PlotStyle → {Black, Thick}, loglinearplotINVoptions];

tryR = 0.02;
param =
{tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMaa3, trywAf3, trywaf3, trywAm3,
 trywam3, tryaf3, tryam3, tryr3, tryR, tryr3 (1 - tryR) + tryR (1 - tryr3), tryk};
run[0] = generation[param, startgen[
  sieveXY[tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMaa3, trywAf3,
  trywaf3, trywAm3, trywam3, tryaf3, tryam3, tryr3][[2]], trypm]];
For[time = 1, time <= endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab3Red =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD3Red = ListLogLinearPlot[MODtab3Red, Joined → True,
 PlotRange → {{startplot, endtime}, {0, 0.5}},
 PlotStyle → {Red, Thick}, loglinearplotINVoptions];

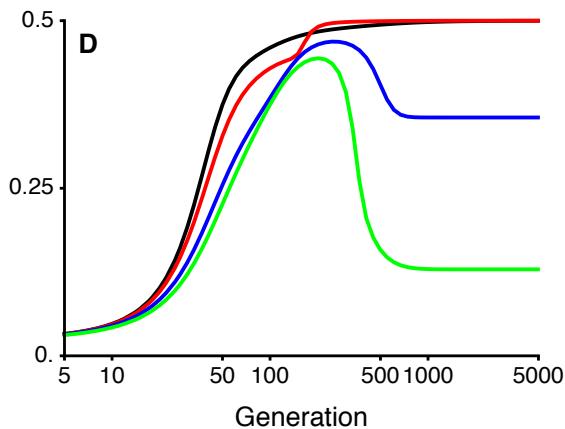
tryR = 0.1;
param =
{tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMaa3, trywAf3, trywaf3, trywAm3,
 trywam3, tryaf3, tryam3, tryr3, tryR, tryr3 (1 - tryR) + tryR (1 - tryr3), tryk};
run[0] = generation[param, startgen[
  sieveXY[tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMaa3, trywAf3,
  trywaf3, trywAm3, trywam3, tryaf3, tryam3, tryr3][[2]], trypm]];
For[time = 1, time <= endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab3Blue =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD3Blue = ListLogLinearPlot[MODtab3Blue, Joined → True,
 PlotRange → {{startplot, endtime}, {0, 0.5}},
 PlotStyle → {Blue, Thick}, loglinearplotINVoptions];

```

```

tryR = 0.5;
param =
{tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMaa3, trywAf3, trywaf3, trywAm3,
 trywam3, tryaf3, tryam3, tryr3, tryR, tryr3 (1 - tryR) + tryR (1 - tryr3), tryk};
run[0] = generation[param, startgen[
    sieveXY[tryFAA3, tryFAa3, tryFaa3, tryMAA3, tryMaa3, trywAf3,
    trywaf3, trywAm3, trywam3, tryaf3, tryam3, tryr3][[2]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab3Green =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD3Green = ListLogLinearPlot[MODtab3Green, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.5}},
PlotStyle → {Green, Thick}, loglinearplotINVoptions];
partD = Show[MOD3Black, MOD3Red, MOD3Blue, MOD3Green,
FrameLabel →
{{Style["", FontSize → 14], ""}, {Style["Generation", FontSize → 14], ""}},
Epilog →
{Text[Style["D", 14, Bold], Scaled@letpos]*,
Text[Style["r=0.005", 14, Bold], Scaled@{0.5, 1.02}]*}
}]

```



All panels

```
GraphicsGrid[{{partA, partB}, {partC, partD}}, Spacings -> -30]
Export[plotdir <> "Temporal_Overdominance.eps", %(*//rasterTrick*)];
```

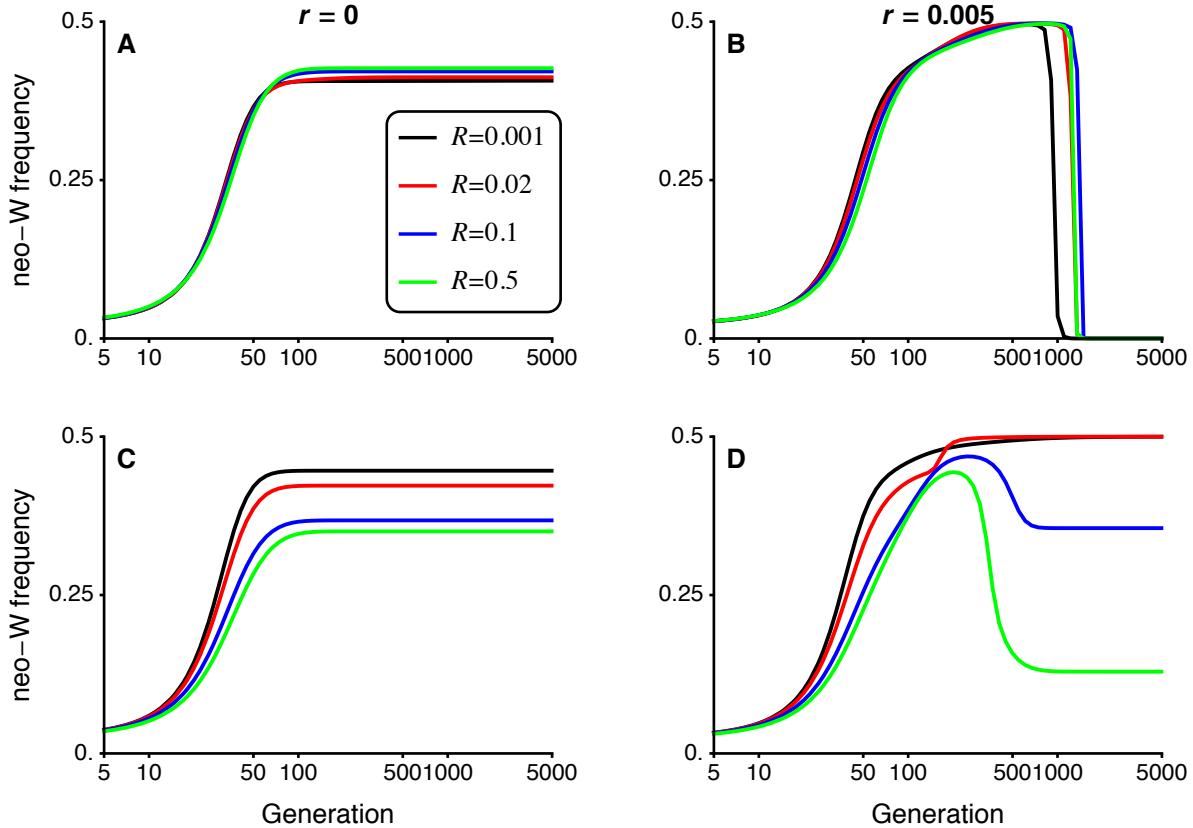


Figure S.3 - sexual antagonism and haploid selection allows less closely linked neo-W to invade

Plotting parameters

```
Pad = {{50, 30}, {50, 30}}; (*space around plot*)
yplotmin = -0.075;
yplotmax = 0.15;
yplotinterval = 0.02;
ylabpos = {-0.15, 0.5}; (*relative location of y axis label position*)
```

```

startplot = 5;
endtime = 5000;
tryR = 0;
trypm = 0.01;
tryk = 1;

loglinearplotINOptions = {Frame -> {{True, False}, {True, False}}},
  FrameStyle -> Directive[Black, Thickness[lwd]],
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]], FontColor -> Black],
    Directive[Black, Thickness[lwd]]},
    Directive[Black, Thickness[lwd]], Directive[Black, Thickness[lwd]]}},
  FrameTicks -> {{{5, 5, {0, 0.01}}, {10, 10, {0, 0.01}}, {50, 50, {0, 0.01}}},
    {100, 100, {0, 0.01}}, ({500, 500, {0, 0.01}}, *){1000, 1000, {0, 0.01}}}
    (*,{5000,5000,{0,0.01}})}, Table[{y, y, ticksize}, {y, 0, 0.5, 0.5}]},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 12},
  ImageSize -> {xsize, xsize aspectratio},
  AspectRatio -> aspectratio,
  Axes -> True,
  FrameLabel -> {{Style["neo-W frequency", FontSize -> 14], ""},
    {Style["Generation", FontSize -> 14], ""}}};

loglinearplotROptions = {Frame -> {{True, False}, {True, False}}},
  ImageSize -> {xsize, xsize aspectratio},
  AspectRatio -> aspectratio,
  PlotRangePadding -> 0,
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  PlotRangeClipping -> False,
  FrameStyle -> Directive[Black, Thickness[lwd]],
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]], FontColor -> Black],
    Directive[Black, Thickness[lwd]]},
    Directive[Black, Thickness[lwd]], Directive[Black, Thickness[lwd]]}},
  FrameTicks -> {{{{0.001, 0.001, {0, 0.01}}, {0.005, "r", {0, 0.01}}},
    {0.01, 0.01, {0, 0.01}}, {0.05, 0.05, {0, 0.01}}, {0.5, 0.5, {0, 0.01}}},
    Table[{y, y, ticksize}, {y, -0.1, 0.2, 0.1}]},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 12},
  ImageSize -> {xsize, xsize aspectratio},
  AspectRatio -> aspectratio,
  Axes -> True,
  FrameLabel ->
    {"", "", {Style["Rate of recombination (R)", FontSize -> 14], ""}}};

letposB = {0.05, 0.95};

```

Panel A

Parameters

```

x = -5;
m = 5;

tryFAA = 1 + 1 / 20;
tryFAa = 1;
tryFaa = 1 - 3 / 20;
tryMAA = 1 - 3 / 20;
tryMAa = 1;
tryMaa = 1 + 1 / 20;
trywAf = 1;
trywaf = 1;
trywAm = 1;
trywam = 1;
tryaf = 1 / 2;
tryam = 1 / 2 - 8 / 100;

xplotmin = -10;
xplotmax = 10;
xplotinterval = 5;
npoints = 28;

tryr = 0.005;

```

Plot

```

positionplot =
Show[
  (*region of tighter linkage*)
  RegionPlot[x > 0, {x, xplotmin, xplotmax}, {y, yplotmin, yplotmax},
    PlotStyle -> {LightGray},
    BoundaryStyle -> None,
    PlotRange -> {yplotmin, yplotmax},
    AxesOrigin -> {xplotmin, yplotmin},
    Frame -> {True, True, False, False}
  ],
  (*neo-W into XY*)
  ListPlot[Table[{a,
    invasionplotXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf,
      trywAm, trywam, tryaf, tryam, setr[x, a, m], setR[x, a, m], setP[x, a, m]]},
    {a, xplotmin, xplotmax, (xplotmax - xplotmin) / npoints}],
    Joined -> True,
    PlotRange -> {yplotmin, All},
    PlotStyle -> Directive[Black, Thickness[lwd]],
    AxesOrigin -> {xplotmin, yplotmin},
    Frame -> {True, True, False, False}
  ],

```

```

Plot[0, {x, xplotmin, xplotmax}, PlotStyle -> {Black, Dotted}],  

PlotRange -> {{xplotmin, xplotmax}, {yplotmin, yplotmax}},  

ImageSize -> {xsize, xsize aspectratio},  

AspectRatio -> aspectratio,  

PlotRangePadding -> 0,  

FrameTicks -> {Table[{x, x, ticksize}, {x, xplotmin, xplotmax, xplotinterval}],  

  Table[{y, y, ticksize}, {y, -0.1, 0.2, 0.1}]},  

FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],  

  Directive[Black, Thickness[lwd]]},  

  {Directive[Black, Thickness[lwd]], Directive[Black, Thickness[lwd]]}},  

FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]}, None, None},  

FrameLabel -> {"Chromosomal position of selected locus (cM)", ""},  

BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},  

ImagePadding -> Pad,  

Epilog -> {
  Text[Style["A", 14 * 4 / 5, Bold], Scaled@letposB],  

  Rotate[Text[Style[(* "Invasion fitness (\lambda-1)" *) "", 14 * 4 / 5],  

    Scaled@ylabpos], 90 Degree],  

  Rotate[Text[Style["Invasion fitness (\lambda_w^{xy}-1)", 14], Scaled@ylabpos],  

    90 Degree],  

  (*Text[Style["tighter sex-linkage", 14], {m, yplotmax*0.9}],*)  

  Text["ancestral SDR", {x, yplotmin * 0.75}],  

  Text["novel SDR", {m, yplotmin * 0.75}],  

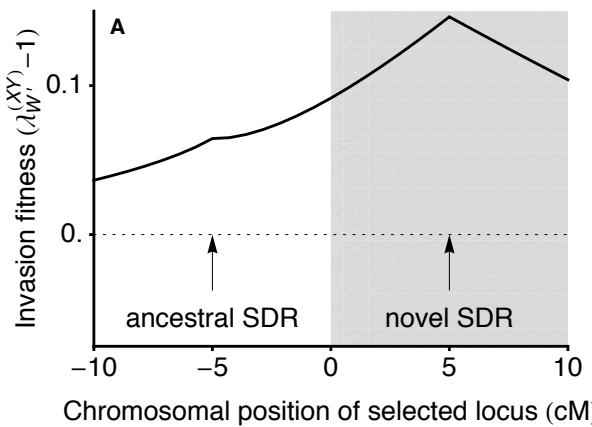
  Arrow[{x, yplotmin * 0.5}, {x, 0}],  

  Arrow[{m, yplotmin * 0.5}, {m, 0}]}
},
PlotRangeClipping -> False
]

(*Export[plotdir<>"PositionPlotB.pdf",%];*)

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The
answer was obtained by solving a corresponding exact system and numericizing the result. >

```



Panel B

Plot

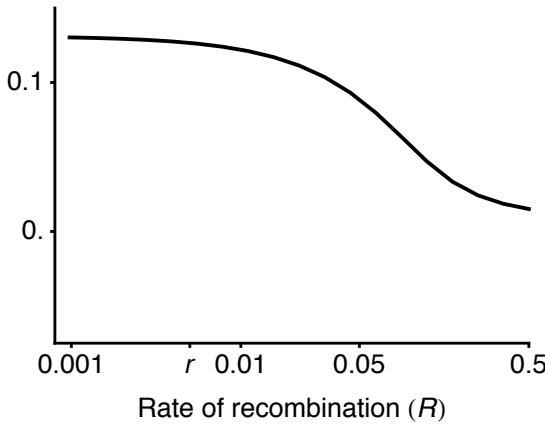
```

subpar1 =
{FAA → tryFAA, FAa → tryFAa, Faa → tryFaa, MAA → tryMAA, MAa → tryMAa, Maa → tryMaa,
 wAf → trywAf, waf → trywaf, wAm → trywAm, wam → trywam, af → tryaf, am → tryam};

tab = Table[
  equil = sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
    tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]];
  λ /. Solve[(λ^2 + (- (λmA1 + λma1) + (χmA1 + χma1)) λ + (λmA1 - χmA1) (λma1 - χma1) -
    χmA1 χma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. subpar1 /. R → 0.5^b . .
    {pxf → equil[[1]], pxm → equil[[2]], pym → equil[[3]],
    q → equil[[4]]}) == 0, λ], {b, 10, 1, -1/2}];

eigen1 = Transpose[Join[{Table[0.5^b, {b, 10, 1, -1/2}]},
  {Transpose[tab][[1]] - 1}]];
eigen2 = Transpose[Join[{Table[0.5^b, {b, 10, 1, -1/2}]},
  {Transpose[tab][[2]] - 1}]];
plot1 = Show[
  ListLogLinearPlot[eigen2, Joined → True, PlotStyle → {Thick, Black},
    PlotRange → {{0.0008, 0.5}, {yplotmin, yplotmax}},
    loglinearplotRλoptions]
]

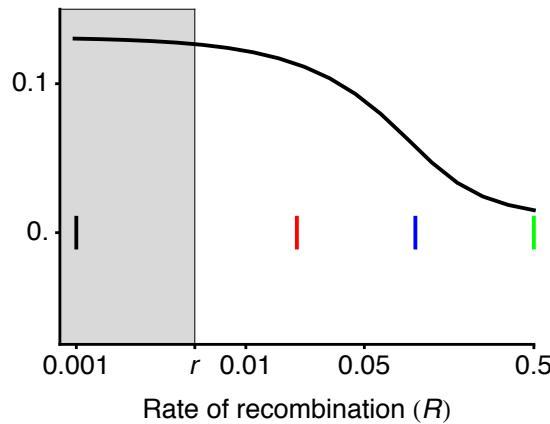
```



```

temp = Show[
  ListLogLinearPlot[{{0.0008, yplotmax}, {tryr, yplotmax}}, Joined → True,
    PlotStyle → {Black, Thin}, Filling → yplotmin, FillingStyle → LightGray,
    loglinearplotR\!options, PlotRange -> {{0.0008, 0.5}, {yplotmin, yplotmax}}],
  plot1,
  ListLogLinearPlot[{{tryr, yplotmax}, {tryr, yplotmin}}, Joined → True, PlotStyle → {Black, Thin}],
  ListLogLinearPlot[{{0.001, -0.01}, {0.001, 0.01}}, Joined → True, PlotStyle → {Black, Thick}],
  ListLogLinearPlot[{{0.02, -0.01}, {0.02, 0.01}}, Joined → True, PlotStyle → {Red, Thick}],
  ListLogLinearPlot[{{0.1, -0.01}, {0.1, 0.01}}, Joined → True, PlotStyle → {Blue, Thick}],
  ListLogLinearPlot[{{0.5, -0.01}, {0.5, 0.01}}, Joined → True, PlotStyle → {Green, Thick}],
  GridLines → {{0.005}, {1}}(*,
  GridLinesStyle→Black*)]

```

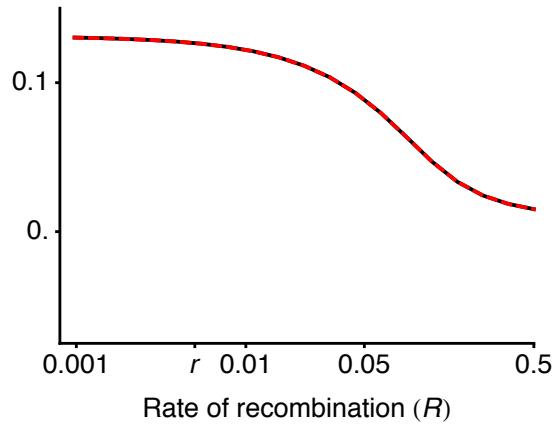


Ensure this is consistent with the full characteristic polynomial result

```

plot2 = ListLogLinearPlot[Table[{0.5^b,
    invasionplotXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf,
    trywAm, trywam, tryaf, tryam, tryr, 0.5^b, tryr (1 - 0.5^b) + 0.5^b (1 - tryr)]},
    {b, 10, 1, -1/2}], Joined → True, PlotStyle → {Thick, Red, Dashed},
PlotRange → {{0.0008, 0.5}, {yplotmin, yplotmax}},
loglinearplotRλoptions];
Show[plot1, plot2]

```



Panel C

```

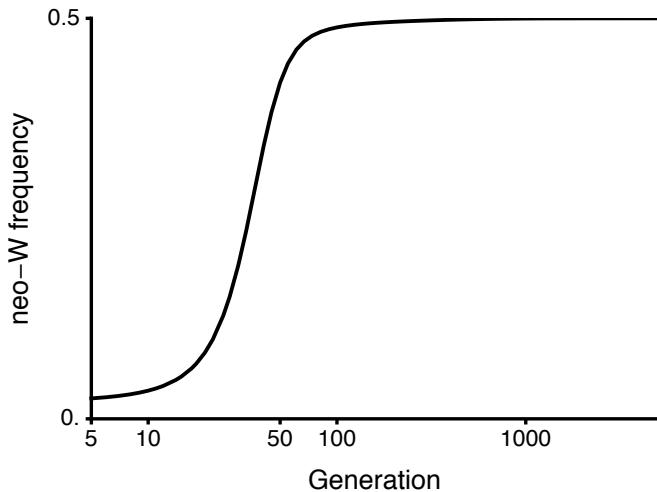
modvec = {0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0};

```

```

tryR = 0.001;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Black =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Black = ListLogLinearPlot[MODtab1Black, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.5}},
PlotStyle → {Black, Thick}, loglinearplotINOptions]

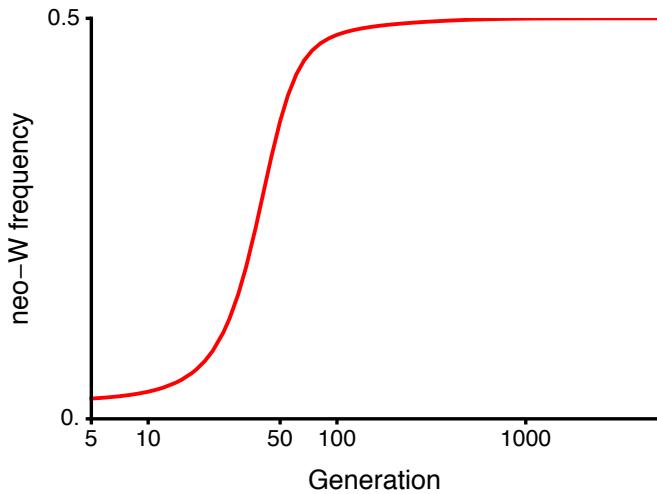
```



```

tryR = 0.02;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Red =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Red = ListLogLinearPlot[MODtab1Red, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.5}},
PlotStyle → {Red, Thick}, loglinearplotINVoptions]

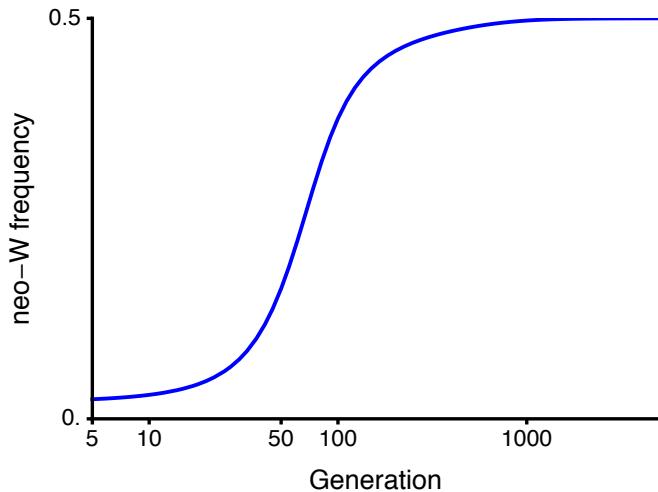
```



```

tryR = 0.1;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Blue =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Blue = ListLogLinearPlot[MODtab1Blue, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.5}},
PlotStyle → {Blue, Thick}, loglinearplotINVoptions]

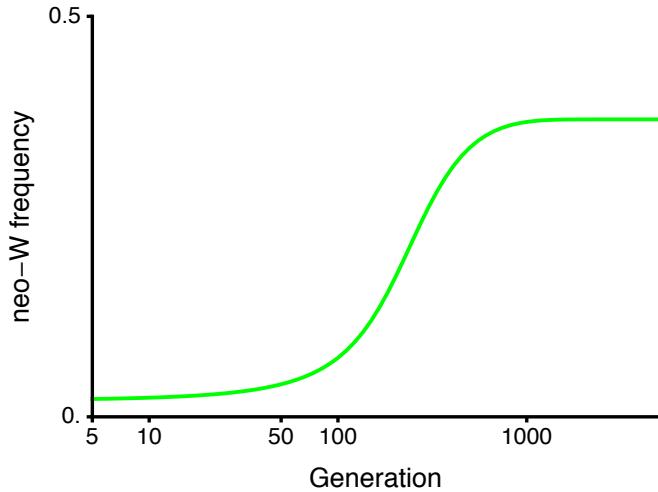
```



```

tryR = 0.5;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Green =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Green = ListLogLinearPlot[MODtab1Green, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.5}},
PlotStyle → {Green, Thick}, loglinearplotINOptions]

```



All panels

```

GraphicsRow[{
  positionplot,
  Show[temp, Graphics[Text[Style["B", 14, Bold], Scaled@letposB]],
    LogLinearPlot[0, {x, 0.0008, 0.5}, PlotStyle -> {Black, Dotted}]], 
  Show[MOD1Black, MOD1Red, MOD1Blue, MOD1Green,
    Graphics[Text[Style["C", 14, Bold], Scaled@letposB]],
    Epilog -> Rotate[Text[Style["neo-W frequency", 14], Scaled@ylabpos], 90 Degree],
    ImagePadding -> Pad]
},
Spacings -> -60
]

Export[plotdir <> "PositionPlot_SexAntagTighter_MaleDrive.eps",
Magnify[%(*//rasterTrick*), 8 / 10]];

```

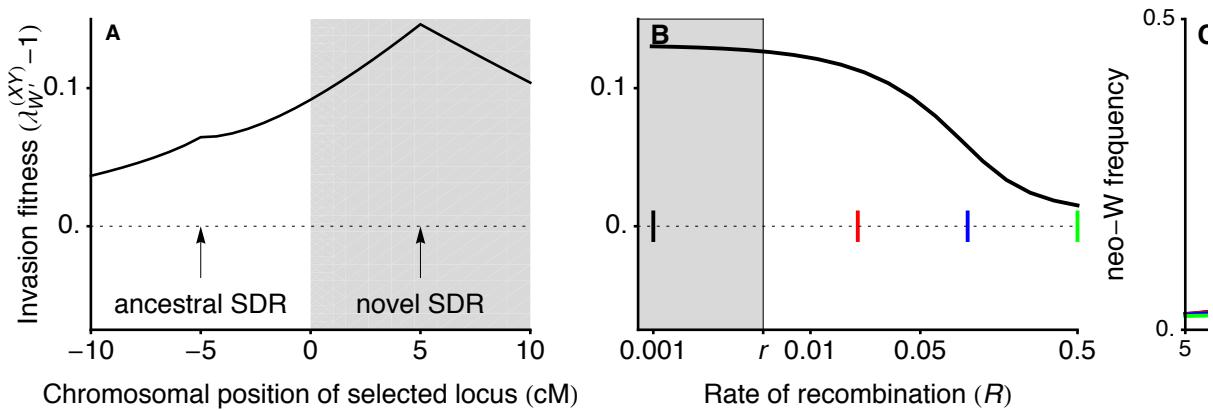


Figure S.4 - neo-W haplotype invasion with meiotic drive in males

Panel A - A favoured in females, a drives in males

Parameters

```
params = {
  wAm → 1, wam → 1, wAf → 1, waf → 1,
  αm → 1 / 2 - 8 / 100,
  αf → 1 / 2,
  MAA → 1, FAA → 1,
  Faa → 1 - 3 / 20,
  FAA → 1 + 1 / 20
};
```

Making the sex-ratio

```
freqMale /. equilB0 /. params // N
0.58
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
  Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
}];

(*neo-Wa invades XY from equilA*)
```

```

plotWainvA =
RegionPlot[{
  (validcondA /. params) &&
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilAO /. params) &&
  1 < WainvA
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilBO /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilBO /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilAO /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilBO /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 + 1 == 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

```

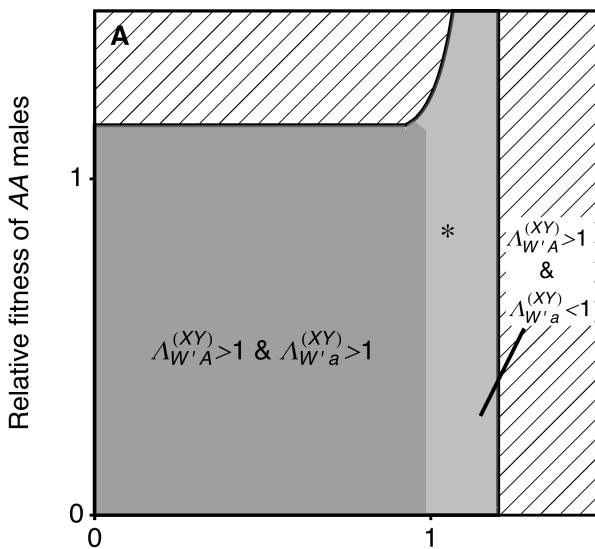
```

plotA =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{1.15, 0.3}, {1.275, 0.55}}]}],
  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  (*FrameLabel→{"Relative fitness of aa males","",""},*)
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["A", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $A_{w'A}^{xy} > 1 \& A_{w'a}^{xy} > 1$ ", 14], {0.5, 0.5}],
    Text[Style[" $A_{w'A}^{xy} > 1$  &
 $A_{w'a}^{xy} < 1$ ", 12], {1.355, 0.725}, Background → White],
    Text[Style["*", 18], {1.05, 0.85}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
  },
  PlotLabel → Style["A favoured in females", 16, Black, Bold],
  PlotRangeClipping → False
]

```

A favoured in females



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

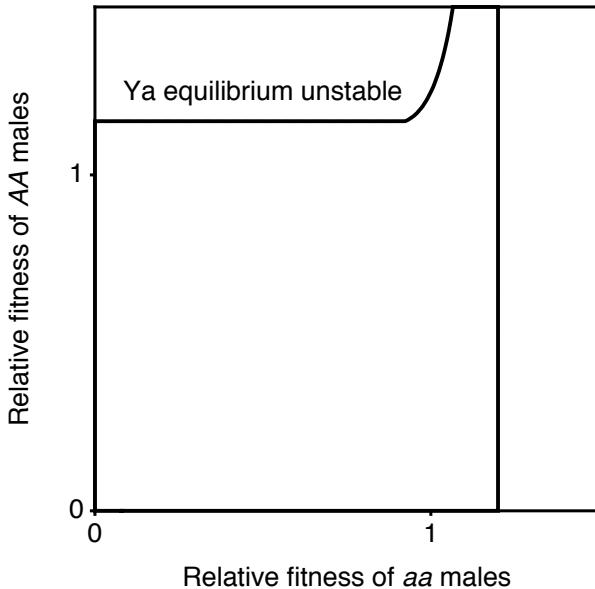
(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];
```

```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
    PlotLabel -> Style["A favoured in females", 16, Black, Bold],
    PlotRangeClipping -> False
  }
]

```

A favoured in females

Panel B - a favoured in females, a drives in males

Parameters

```
params = {
    wAm → 1, wam → 1, wAf → 1, waf → 1,
    cm → 1 / 2 - 8 / 100,
    af → 1 / 2,
    MAa → 1, FAa → 1,
    FAA → 1 - 3 / 20,
    Faa → 1 + 1 / 20
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsola =
Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{
    (validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]}, 
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]}, 
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 5
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

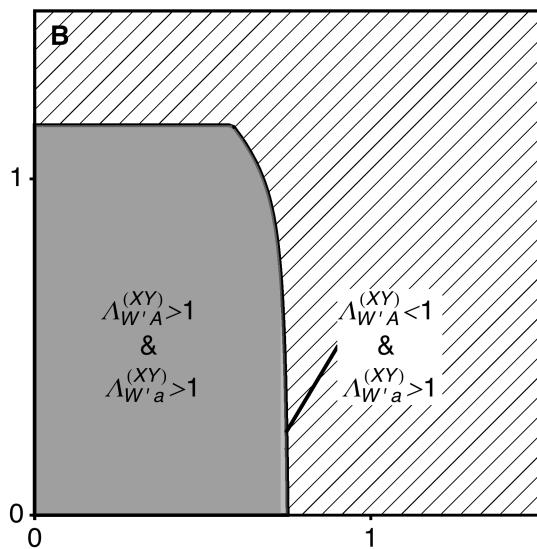
```

```

plotB =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{0.9, 0.5}, {0.75, 0.25}}]}],
  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
    (*FrameLabel→{"Relative fitness of aa males","",""},*)
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["B", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1$ ", 14],
       $\Lambda_{W'A}^{(xy)} > 1$ , {0.35, 0.5}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} < 1$ ", 14],
       $\Lambda_{W'A}^{(xy)} < 1$ , {0.9, 0.5}, {-1.0, 0}, Background → White](*,
      Rotate[
        Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]),
    PlotLabel → Style["a favoured in females", 16, Black, Bold],
    PlotRangeClipping → False
  }
]

```

a favoured in females

Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

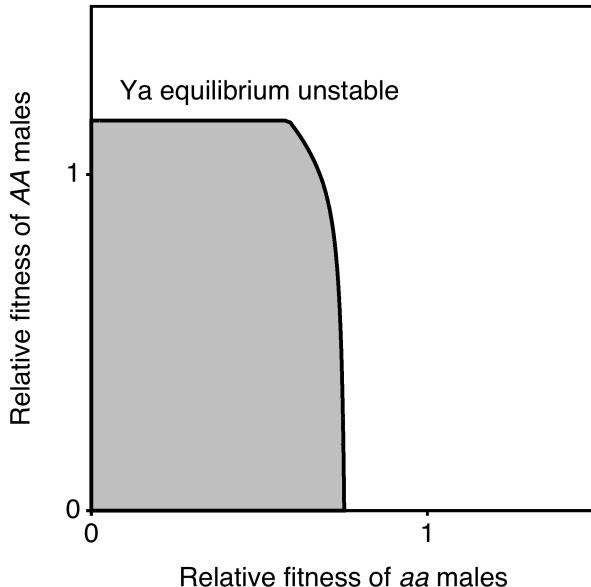
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
  PlotLabel → Style["a favoured in females", 16, Black, Bold],
  PlotRangeClipping → False
]

```

***a* favoured in females**



Panel C - overdominance in females, a drives in males

Parameters

```
params = {
    wAm → 1, wam → 1, wAf → 1, waf → 1,
    cm → 1 / 2 - 8 / 100,
    af → 1 / 2,
    MAA → 1, Faa → 1,
    FAA → 1 - 4 / 10,
    Faa → 1 - 4 / 10
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsola =
    Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{
    (validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]}, 
BoundaryStyle → None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]}, 
BoundaryStyle → None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) ||
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → White,
BoundaryStyle → {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  2 < 3
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → None,
BoundaryStyle → None,
Mesh → 50,
MeshFunctions → {#1 - #2 &},
MeshStyle → Black
];

```

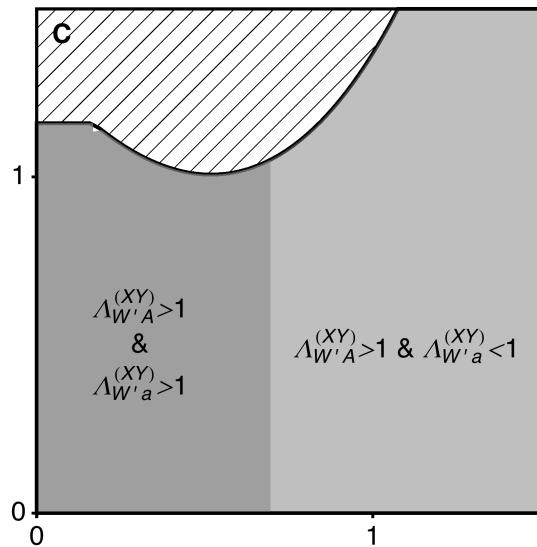
```

plotC =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  (*FrameLabel → {"Relative fitness of aa males", "", *})
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["C", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1$ ", 14],
      {0.45, 0.5}, {1, 0}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1 \& \Lambda_{W'a}^{(xy)} < 1$ ", 14],
      {1.1, 0.5}] (*,
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree] *)
  },
  PlotLabel → Style["overdominance in females", 16, Black, Bold],
  PlotRangeClipping → False
]

```

overdominance in females



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

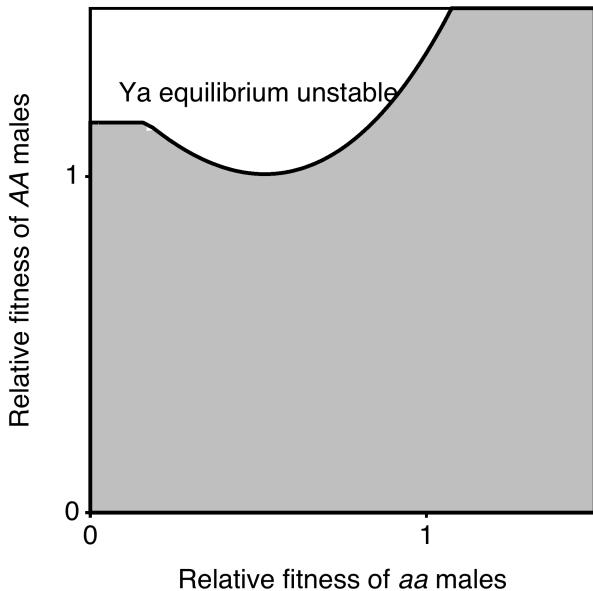
(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];
```

```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
  PlotLabel -> Style["A favoured in females", 16, Black, Bold],
  PlotRangeClipping -> False
]

```

A favoured in females

Panel D - A favoured in females, A drives in males

Parameters

```
params = {
    wAm → 1, wam → 1, wAf → 1, waf → 1,
    αm → 1 / 2 + 8 / 100,
    αf → 1 / 2,
    MAa → 1, FAa → 1,
    Faa → 1 - 3 / 20,
    FAA → 1 + 1 / 20
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsola =
    Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{
    (validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]}, 
BoundaryStyle → None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]}, 
BoundaryStyle → None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) ||
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → White,
BoundaryStyle → {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → None,
BoundaryStyle → None,
Mesh → 50,
MeshFunctions → {#1 - #2 &},
MeshStyle → Black
];

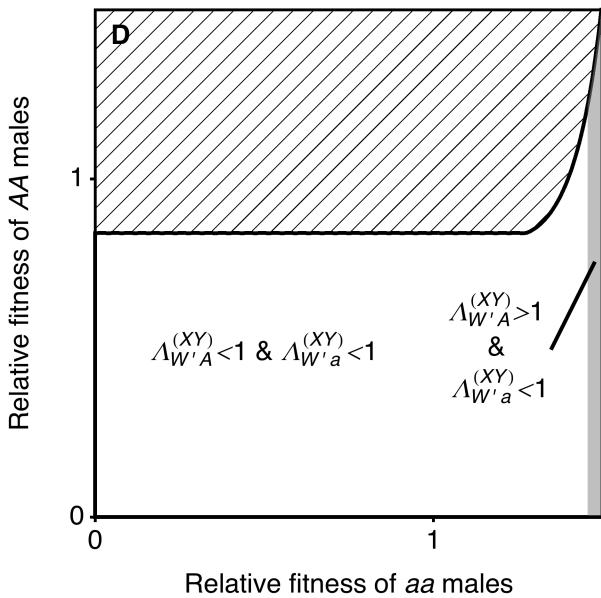
```

```

plotD =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{1.35, 0.5}, {1.475, 0.75}}]}],
  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["D", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} < 1 \& \Lambda_{W'a}^{(xy)} < 1$ ", 14], {0.5, 0.5}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1$  &
 $\Lambda_{W'a}^{(xy)} < 1$ ", 14], {1.35, 0.5}, {1, 0}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
  },
  PlotLabel -> Style[(*"A favoured in females"*) "", 16, Black, Bold],
  PlotRangeClipping -> False
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

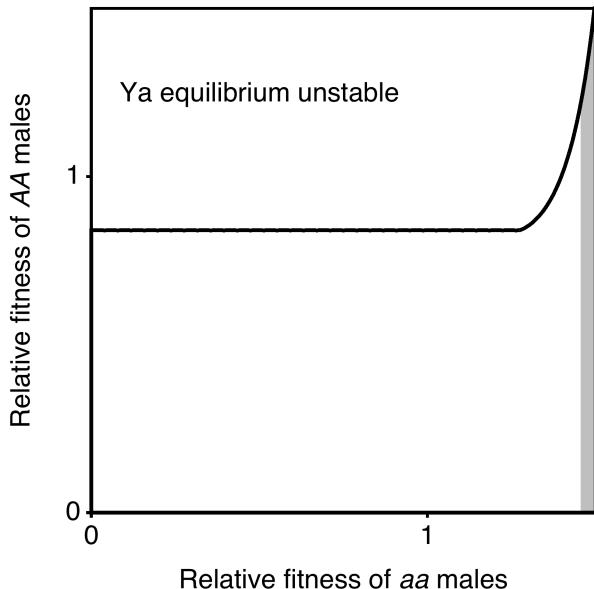
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
  PlotLabel -> Style["A favoured in females", 16, Black, Bold],
  PlotRangeClipping -> False
]

```

A favoured in females



Panel E - a favoured in females, A drives in males

Parameters

```
params = {
    wAm → 1, wam → 1, wAf → 1, waf → 1,
    cm → 1 / 2 + 8 / 100,
    cf → 1 / 2,
    MAa → 1, FAc → 1,
    FAA → 1 - 3 / 20,
    Faa → 1 + 1 / 20
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsola =
Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{
    (validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]}, 
BoundaryStyle → None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]}, 
BoundaryStyle → None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) ||
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → White,
BoundaryStyle → {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 > 0
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → None,
BoundaryStyle → None,
Mesh → 50,
MeshFunctions → {#1 - #2 &},
MeshStyle → Black
];

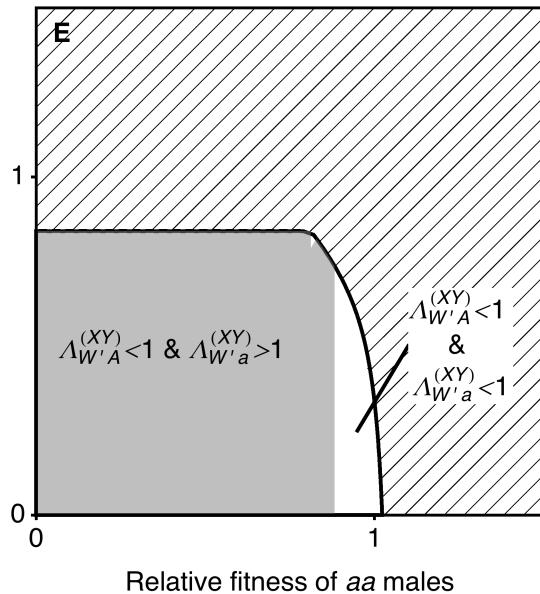
```

```

plotE =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{1.1, 0.5}, {0.95, 0.25}}]}],
  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["E", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{w'A}^{(xy)} < 1 \& \Lambda_{w'a}^{(xy)} > 1$ ", 14], {0.4, 0.5}],
    Text[Style[" $\Lambda_{w'A}^{(xy)} < 1$ " &
 $\Lambda_{w'a}^{(xy)} < 1$ ", 14], {1.1, 0.5}, {-1, 0}, Background -> White] (*,
      Rotate[
        Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]*),
    PlotLabel -> Style[(*"a favoured in females"*) "", 16, Black, Bold],
    PlotRangeClipping -> False
  }
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

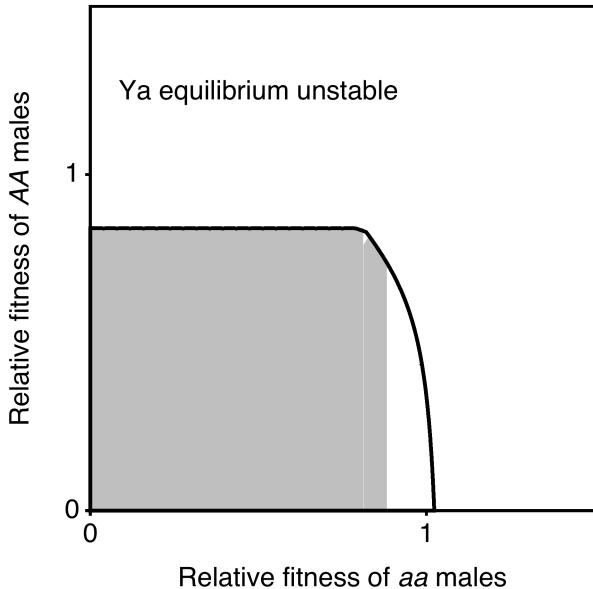
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
    PlotLabel -> Style["a favoured in females", 16, Black, Bold],
    PlotRangeClipping -> False
  }
]

```

***a* favoured in females**



Panel F - overdominance in females, A drives in males

Parameters

```
params = {
    wAm → 1, wam → 1, wAf → 1, waf → 1,
    cm → 1 / 2 + 8 / 100,
    cf → 1 / 2,
    MAa → 1, FAa → 1,
    FAA → 1 - 4 / 10,
    Faa → 1 - 4 / 10
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsola =
    Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{
    (validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]}, 
BoundaryStyle → None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]}, 
BoundaryStyle → None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) ||
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → White,
BoundaryStyle → {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 + 1 < 3
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → None,
BoundaryStyle → None,
Mesh → 50,
MeshFunctions → {#1 - #2 &},
MeshStyle → Black
];

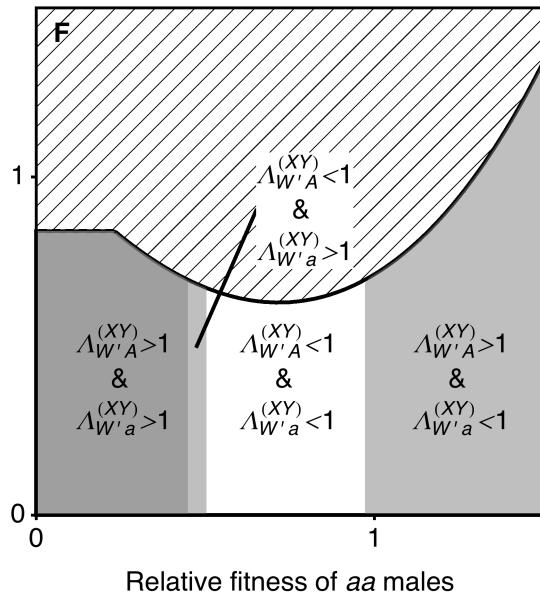
```

```

plotF =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{0.65, 0.9}, {0.475, 0.5}}]}],
  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["F", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $A_{W'A}^{(xy)}$  >1
&
 $A_{W'a}^{(xy)} >1$ ", 14], {0.4, 0.4}, {1, 0}],
    Text[Style[" $A_{W'A}^{(xy)}$  >1
&
 $A_{W'a}^{(xy)} <1$ ", 14], {1.4, 0.4}, {1, 0}],
    Text[Style[" $A_{W'A}^{(xy)}$  <1
&
 $A_{W'a}^{(xy)} <1$ ", 14], {0.6, 0.4}, {-1, 0}],
    Text[Style[" $A_{W'A}^{(xy)}$  >1", 14], {0.65, 0.9}, {-1, 0}, Background → White](*,
      Rotate[
        Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]*),
    },
  PlotLabel → Style[(*"overdominance in females"*) "", 16, Black, Bold],
  PlotRangeClipping → False
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

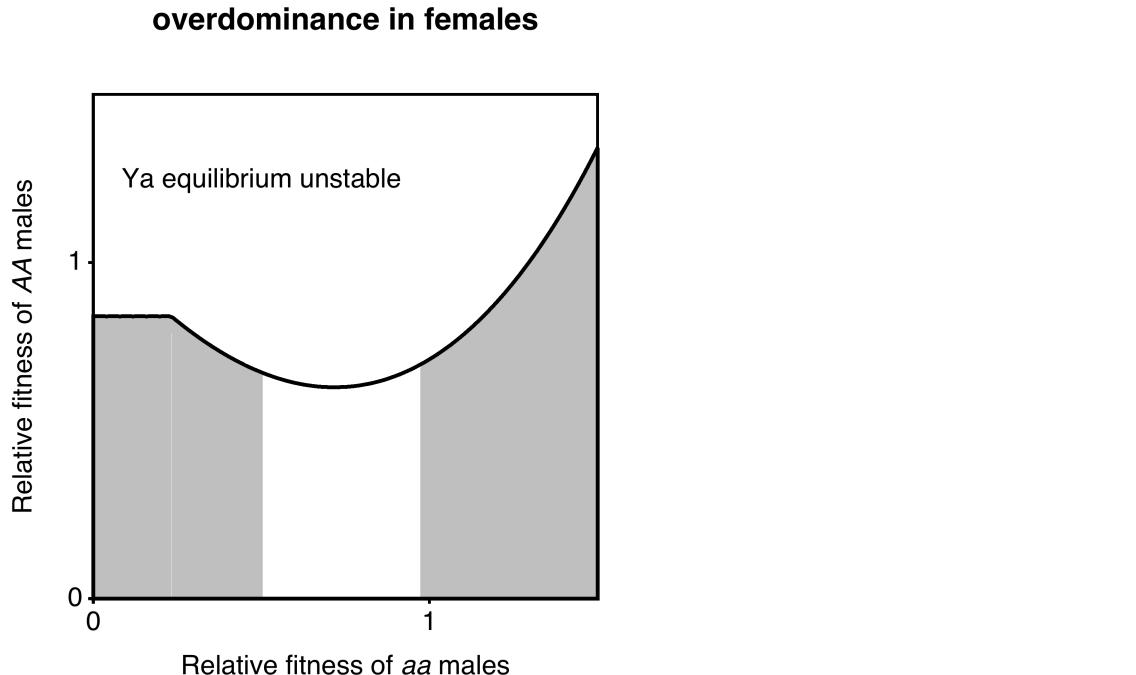
(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
  PlotLabel → Style["overdominance in females", 16, Black, Bold],
  PlotRangeClipping → False
]

```



All Panels

```

GraphicsGrid[{
  {plotA, plotB, plotC},
  {plotD, plotE, plotF}
],
Spacings -> -50
]

Export[plotdir <> "Region_plot_combined_MaleDrive.eps", % // rasterTrick];

```

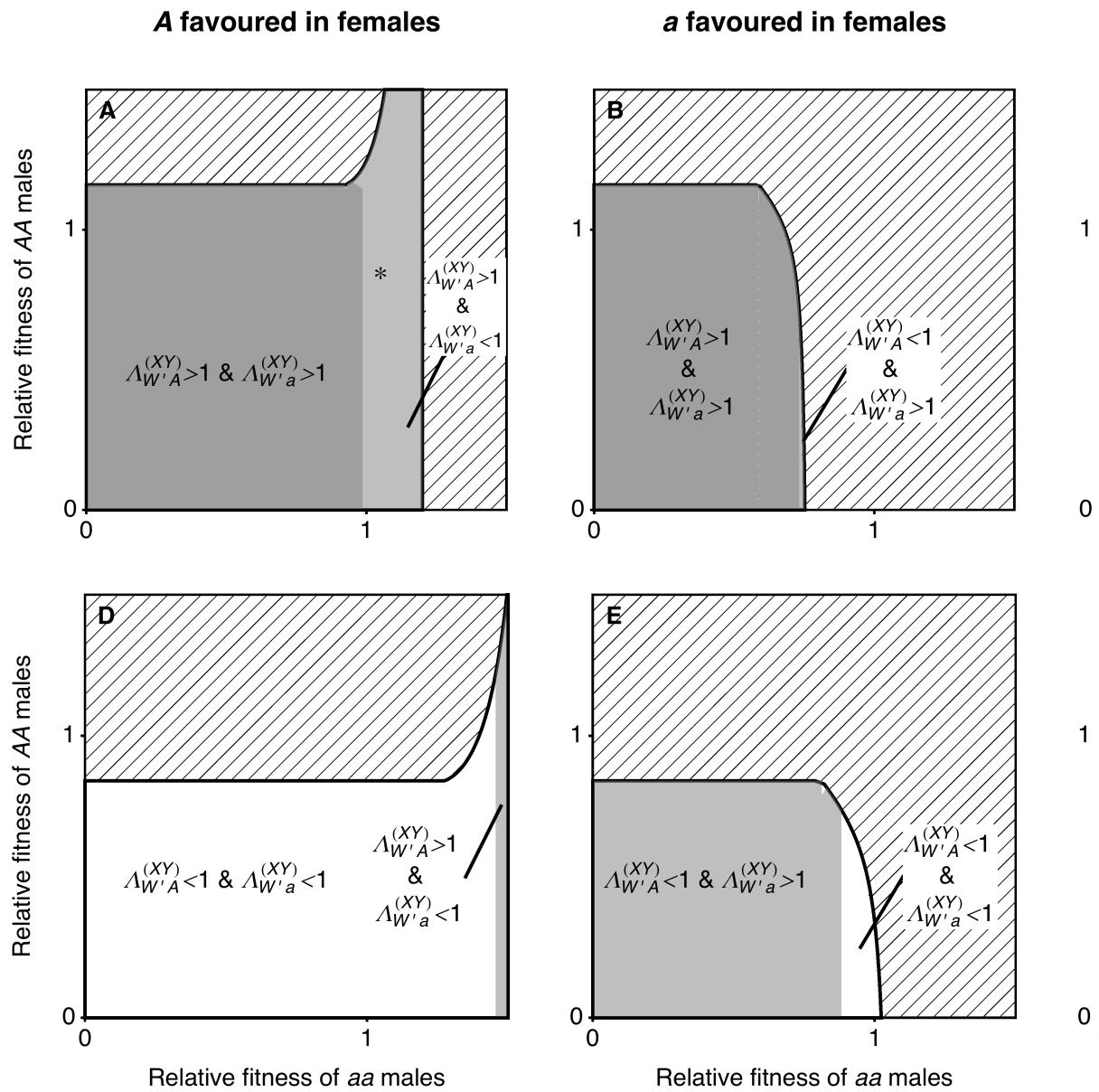


Figure S.5 - neo-W haplotype invasion with haploid competition in males

Panel A - A favoured in females, haploid competition favours a in males

Parameters

```
params = {
  wAm → 1, wam → 1 + 8 / 50, wAf → 1, waf → 1,
  αm → 1 / 2, αf → 1 / 2,
  MAa → 1, FAa → 1,
  Faa → 1 - 3 / 20,
  FAA → 1 + 1 / 20
};
```

Making the sex-ratio

```
freqMale /. equilB0 /. params // N
0.537037
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
  Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];
```

```

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{
  (stabcondA /. params) &&
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilAO /. params) &&
  1 < WainvA
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]}, 
BoundaryStyle -> None
];

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilBO /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]}, 
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilBO /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]}, 
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilAO /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilBO /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},

```

```

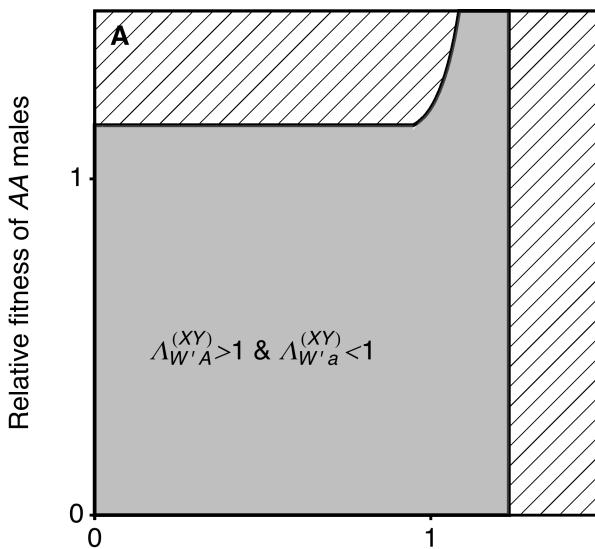
MeshStyle -> Black
];

plotA =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

PlotRange -> {{0, 1.5}, {0, 1.5}},
ImageSize -> {xsize, xsize},
PlotRangePadding -> 0,
FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
  Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
  Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
  Directive[Black, Thickness[lwd], FontColor -> White]}},
FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
  {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
(*FrameLabel -> {"Relative fitness of aa males", "", *})
BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
ImagePadding -> Pad,
Epilog -> {
  Text[Style["A", 14, Bold], Scaled@{0.05, 0.95}],
  Text[Style[" $\Lambda_w^{(xy)} > 1 \text{ & } \Lambda_w^{(xy)} < 1$ ", 14], {0.5, 0.5}],
  Rotate[
    Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
  },
  PlotLabel -> Style["A favoured in females", 16, Black, Bold],
  PlotRangeClipping -> False
]
]

```

A favoured in females



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];
```

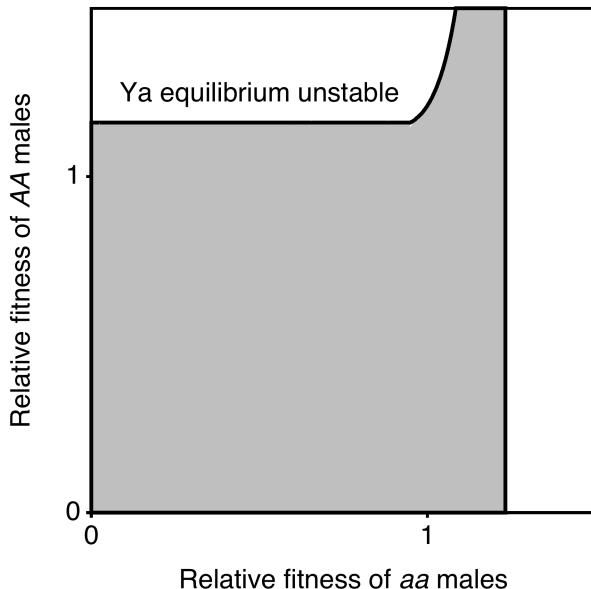
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
  PlotLabel → Style["A favoured in females", 16, Black, Bold],
  PlotRangeClipping → False
]

```

A favoured in females



Panel B - a favoured in females, haploid competition favours a in males

Parameters

```
params = {
  wAm → 1, wam → 1 + 8 / 50, wAf → 1, waf → 1,
  cm → 1 / 2, αf → 1 / 2,
  MAa → 1, FAa → 1,
  FAA → 1 - 3 / 20,
  Faa → 1 + 1 / 20
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
  Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{(
    validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
  RegionPlot[{(
    validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

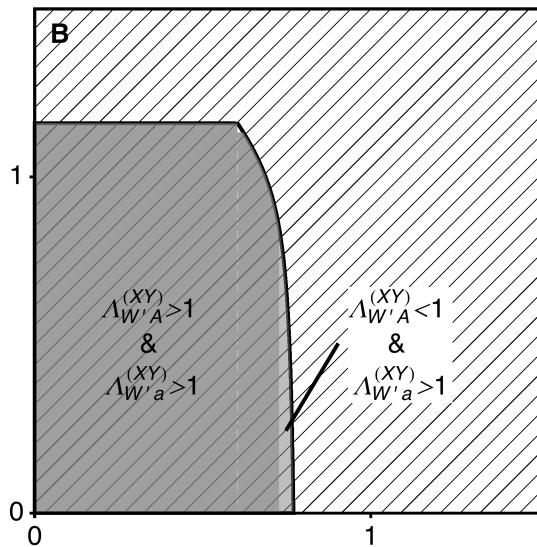
```

```

plotB =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{0.9, 0.5}, {0.75, 0.25}}]}],
  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
    (*FrameLabel→{"Relative fitness of aa males","",""},*)
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["B", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $A_{W'A}^{(xy)} > 1$ ", 14],
      {0.35, 0.5}],
    Text[Style[" $A_{W'A}^{(xy)} < 1$ ", 14],
      {0.9, 0.5}, {-1.2, 0}, Background → White] (*,
      Rotate[
        Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]*),
    },
    PlotLabel → Style["a favoured in females", 16, Black, Bold],
    PlotRangeClipping → False
  }
]

```

a favoured in females

Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];
```

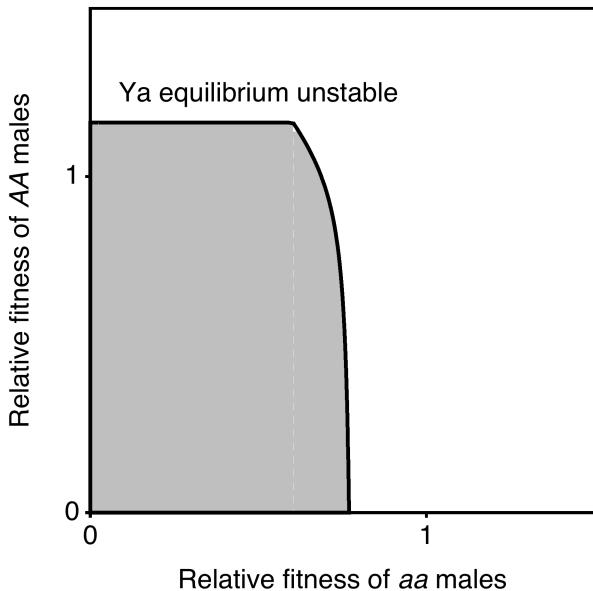
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
    PlotLabel → Style["a favoured in females", 16, Black, Bold],
    PlotRangeClipping → False
  }
]

```

***a* favoured in females**



Panel C - overdominance in females, haploid competition favours a in males

Parameters

```
params = {
  wAm → 1, wam → 1 + 8 / 50, wAf → 1, waf → 1,
  cm → 1 / 2, αf → 1 / 2,
  MAa → 1, FAa → 1,
  FAA → 1 - 4 / 10,
  Faa → 1 - 4 / 10
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilAO /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilAO /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilBO /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilBO /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
  Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilAO /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilBO /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{(
    validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
  RegionPlot[{(
    validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) &&
    1 < WainvA
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

```

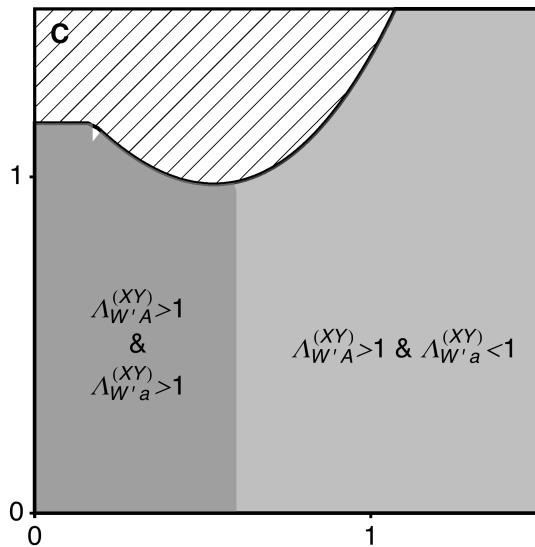
```

plotC =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  (*FrameLabel→{"Relative fitness of aa males","",""},*)
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["C", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1$ "]],
    &
    Text[Style[" $\Lambda_{W'a}^{(xy)} > 1$ ", 14], {0.45, 0.5}, {1, 0}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1 \& \Lambda_{W'a}^{(xy)} < 1$ ", 14], {1.1, 0.5}](*,
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]*),
  },
  PlotLabel → Style["overdominance in females", 16, Black, Bold],
  PlotRangeClipping → False
]

```

overdominance in females



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];
```

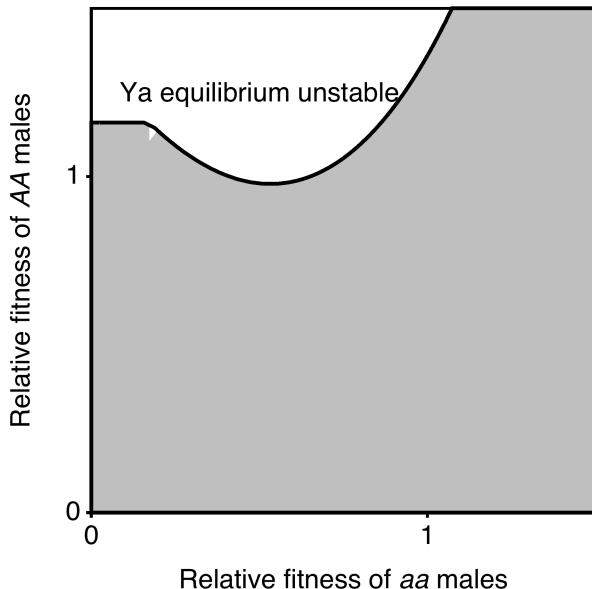
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
  PlotLabel → Style["A favoured in females", 16, Black, Bold],
  PlotRangeClipping → False
]

```

A favoured in females



Panel D - A favoured in females, haploid competition favours A in males

Parameters

```
params = {
    wAm → 1 + 8 / 50, wam → 1, wAf → 1, waf → 1,
    cm → 1 / 2, αf → 1 / 2,
    MAa → 1, FAa → 1,
    Faa → 1 - 3 / 20,
    FAA → 1 + 1 / 20
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilAO /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilAO /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilBO /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilBO /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
  Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilAO /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilBO /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{(
    validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
  RegionPlot[{(
    validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) &&
    1 < WainvA
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

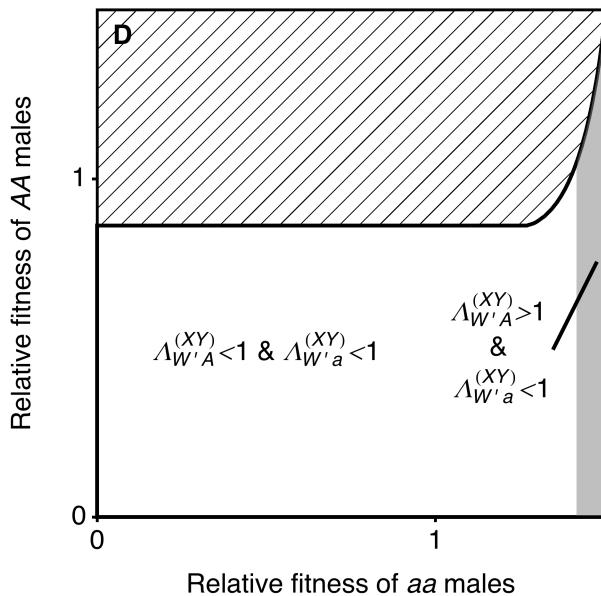
```

```

plotD =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{1.35, 0.5}, {1.475, 0.75}}]}],
  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["D", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{w'A}^{(xy)} < 1 \& \Lambda_{w'a}^{(xy)} < 1$ ", 14], {0.5, 0.5}],
    Text[Style[" $\Lambda_{w'A}^{(xy)} > 1$  &
 $\Lambda_{w'a}^{(xy)} < 1$ ", 14], {1.35, 0.5}, {1, 0}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
  },
  PlotLabel -> Style[(*"A favoured in females"*) "", 16, Black, Bold],
  PlotRangeClipping -> False
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

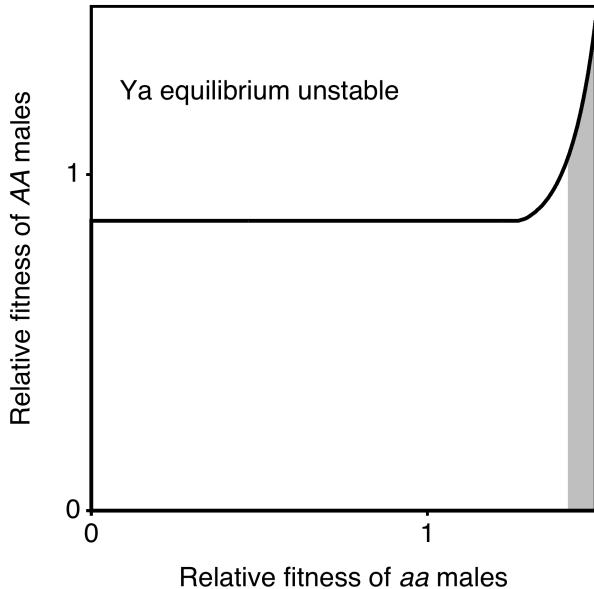
(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
    PlotLabel → Style["A favoured in females", 16, Black, Bold],
    PlotRangeClipping → False
  }
]

```

A favoured in females

Panel E - a favoured in females, haploid competition favours A in males

Parameters

```
params = {
    wAm → 1 + 8 / 50, wam → 1, wAf → 1, waf → 1,
    cm → 1 / 2, αf → 1 / 2,
    MAa → 1, FAa → 1,
    FAA → 1 - 3 / 20,
    Faa → 1 + 1 / 20
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilAO /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilAO /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilBO /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilBO /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilAO /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilBO /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{(
validcondA /. params) && (*valid*)
(stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) && (*internally stable*)
1 < WAinvA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{(
validcondA /. params) &&
(stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) &&
1 < WainvA
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

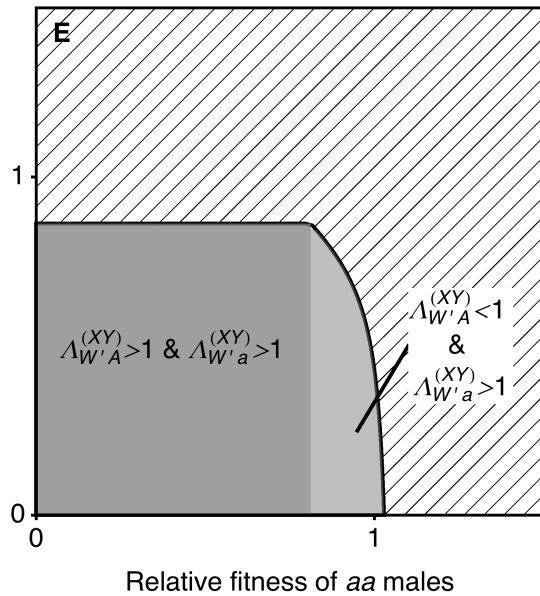
```

```

plotE =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{1.1, 0.5}, {0.95, 0.25}}]}],
  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["E", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1 \text{ & } \Lambda_{W'a}^{(xy)} > 1$ ", 14], {0.4, 0.5}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} < 1$ &
 $\Lambda_{W'a}^{(xy)} > 1$ ", 14], {1.1, 0.5}, {-1, 0}, Background -> White] (*,
      Rotate[
        Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]*),
    PlotLabel -> Style[(*"a favoured in females"*) "", 16, Black, Bold],
    PlotRangeClipping -> False
  }
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

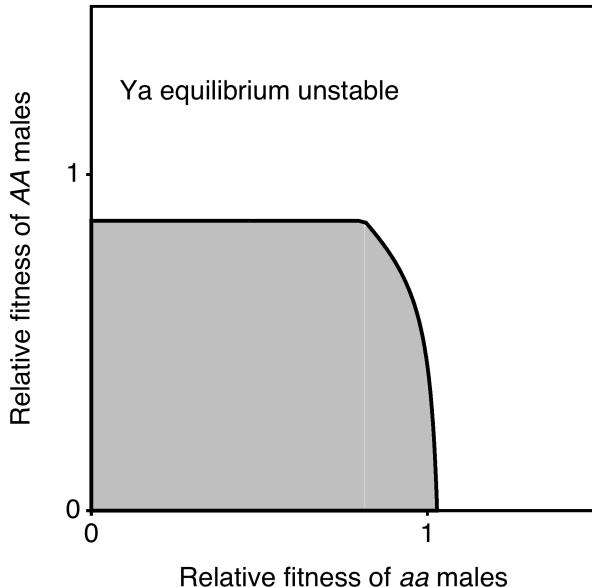
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
    PlotLabel -> Style["a favoured in females", 16, Black, Bold],
    PlotRangeClipping -> False
  }
]

```

***a* favoured in females**



Panel F - overdominance in females, haploid competition favours A in males

Parameters

```
params = {
    wAm → 1 + 8 / 50, wam → 1, wAf → 1, waf → 1,
    cm → 1 / 2, αf → 1 / 2,
    MAa → 1, FAa → 1,
    FAA → 1 - 4 / 10,
    Faa → 1 - 4 / 10
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{(
validcondA /. params) && (*valid*)
(stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
1 < WAinvA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{(
validcondA /. params) &&
(stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
1 < WainvA
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) ||
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → White,
BoundaryStyle → {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → None,
BoundaryStyle → None,
Mesh → 50,
MeshFunctions → {#1 - #2 &},
MeshStyle → Black
];

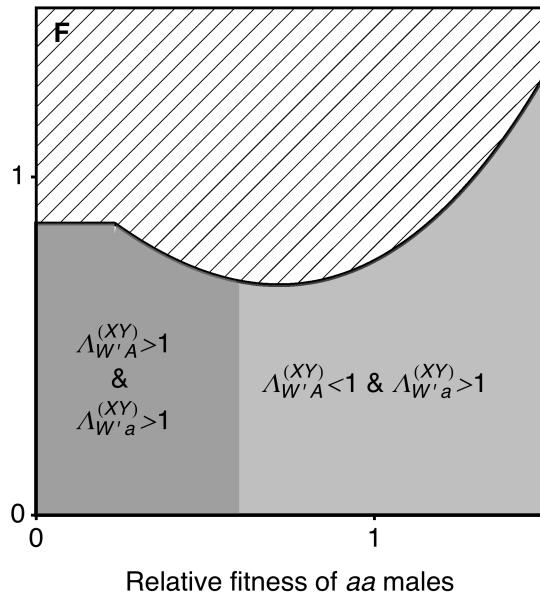
```

```

plotF =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["F", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1$ ", 14],
      {0.4, 0.4}, {1, 0}],
    Text[Style[" $\Lambda_{W'a}^{(xy)} > 1$ ", 14], {1, 0.4}] (*,
      Rotate[
        Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree] *),
    PlotLabel → Style[(*"overdominance in females"*) "", 16, Black, Bold],
    PlotRangeClipping → False
  }
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];
```

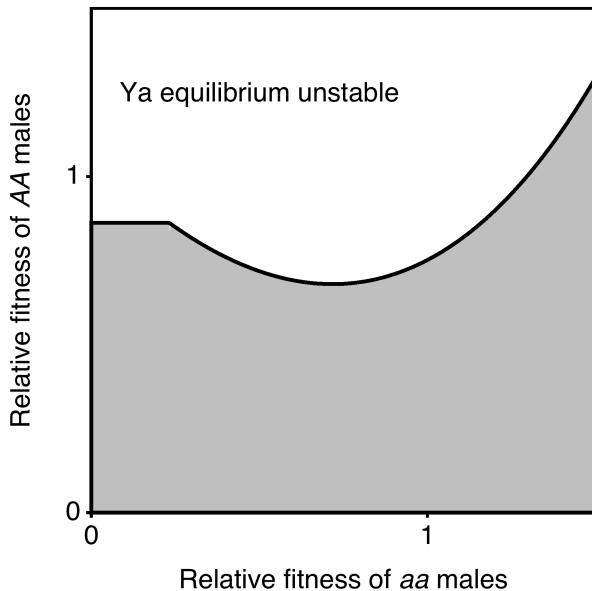
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
  PlotLabel -> Style["overdominance in females", 16, Black, Bold],
  PlotRangeClipping -> False
]

```

overdominance in females



All Panels

```

GraphicsGrid[{
  {plotA, plotB, plotC},
  {plotD, plotE, plotF}
],
Spacings -> -50
]

Export[plotdir <> "Region_plot_combined_MaleGS.eps", % // rasterTrick];

```

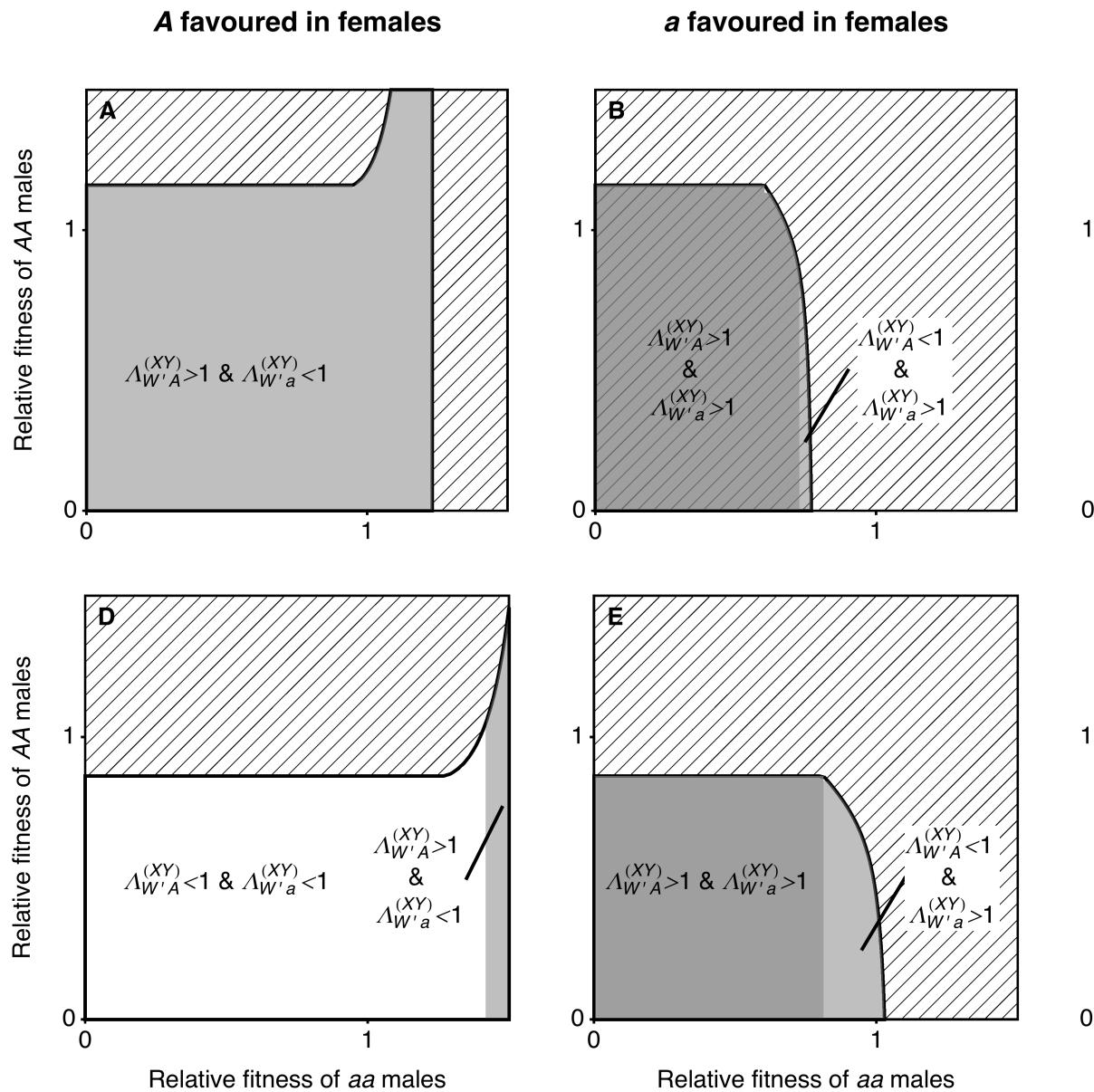


Figure S.6 - neo-W haplotype invasion with meiotic drive in females

Panel A - A favoured in females, a drives in females

Parameters

```
params = {
  wAm → 1, wam → 1, wAf → 1, waf → 1,
  af → 1 / 2 - 8 / 100,
  am → 1 / 2,
  MAA → 1, Faa → 1,
  Faa → 1 - 3 / 20,
  FAA → 1 + 1 / 20
};
```

Making the sex-ratio

```
freqMale /. equilB0 /. params // N
0.5
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
  Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{(
    validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];
```

```

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{
  (validcondA /. params) &&
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
  1 < WainvA
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) ||
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → White,
BoundaryStyle → {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → None,
BoundaryStyle → None,
Mesh → 50,

```

```

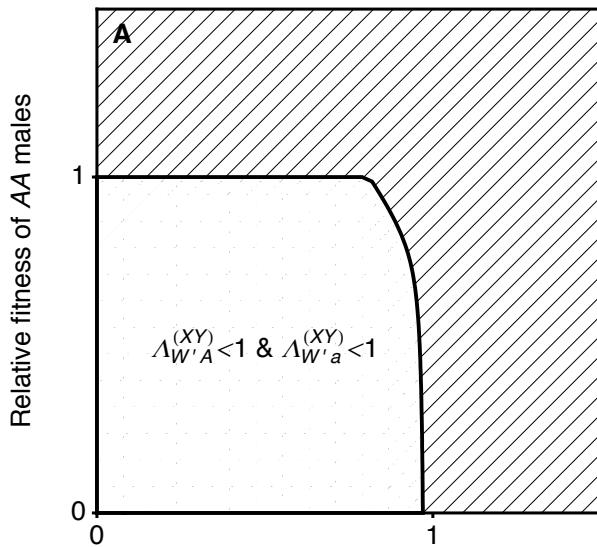
MeshFunctions → {#1 - #2 &},
MeshStyle → Black
];

plotA =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
    (*FrameLabel→{"Relative fitness of aa males","",""},*)
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["A", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_w^{(xy)} < 1 \wedge \Lambda_w^{(xy)} < 1$ ", 14], {0.5, 0.5}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    },
  PlotLabel → Style["A favoured in females", 16, Black, Bold],
  PlotRangeClipping → False
]
]

```

A favoured in females



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

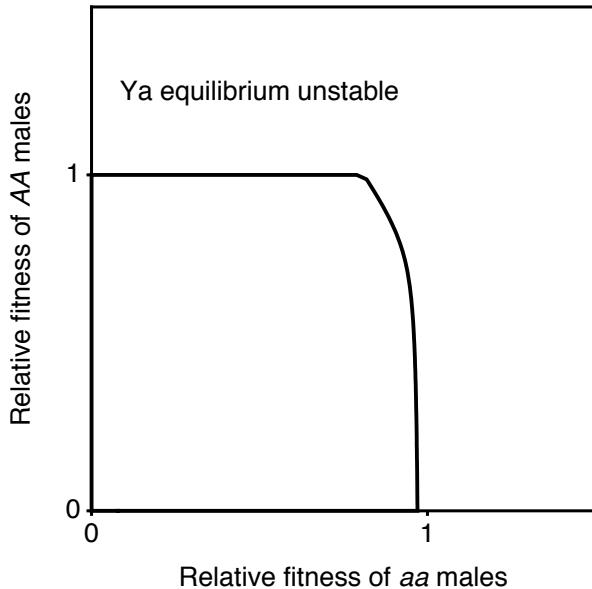
(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
    PlotLabel → Style["A favoured in females", 16, Black, Bold],
    PlotRangeClipping → False
  }
]

```

A favoured in females

Panel B - a favoured in females, a drives in females

Parameters

```
params = {
    wAm → 1, wam → 1, wAf → 1, waf → 1,
    af → 1 / 2 - 8 / 100,
    am → 1 / 2,
    MAA → 1, Faa → 1,
    FAA → 1 - 3 / 20,
    Faa → 1 + 1 / 20
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsola =
  Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < WAinvA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{
  (validcondA /. params) &&
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
  1 < WainvA
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]}, 
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]}, 
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

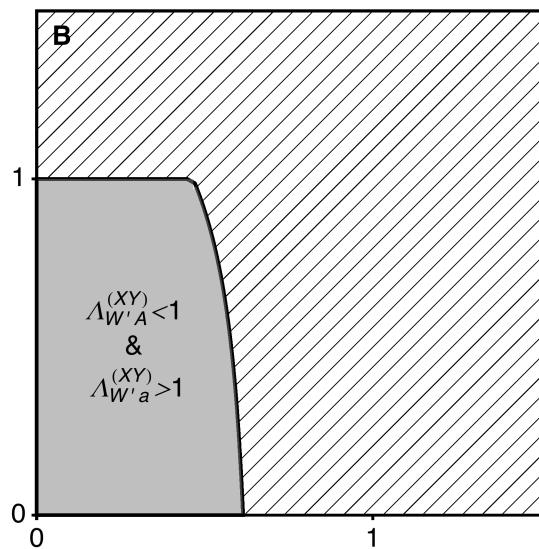
```

```

plotB =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  (*FrameLabel→{"Relative fitness of aa males","",""},*)
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["B", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} < 1$ ", 14], {0.3, 0.5}] (*,
      Rotate[
        Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree] *)
    },
  PlotLabel → Style["a favoured in females", 16, Black, Bold],
  PlotRangeClipping → False
]

```

a favoured in females

Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];
```

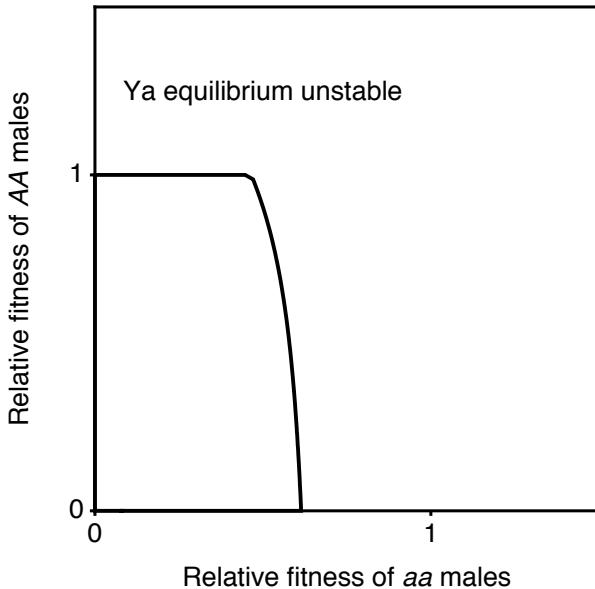
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
  PlotLabel → Style["a favoured in females", 16, Black, Bold],
  PlotRangeClipping → False
]

```

a favoured in females



Panel C - overdominance in females, a drives in females

Parameters

```
params = {
    wAm → 1, wam → 1, wAf → 1, waf → 1,
    af → 1 / 2 - 8 / 100,
    am → 1 / 2,
    MAA → 1, Faa → 1,
    FAA → 1 - 4 / 10,
    Faa → 1 - 4 / 10
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsola =
    Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{(
    validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
),
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
}];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{(
    validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
),
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
}];
```

```
(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]}, 
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]}, 
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];
```

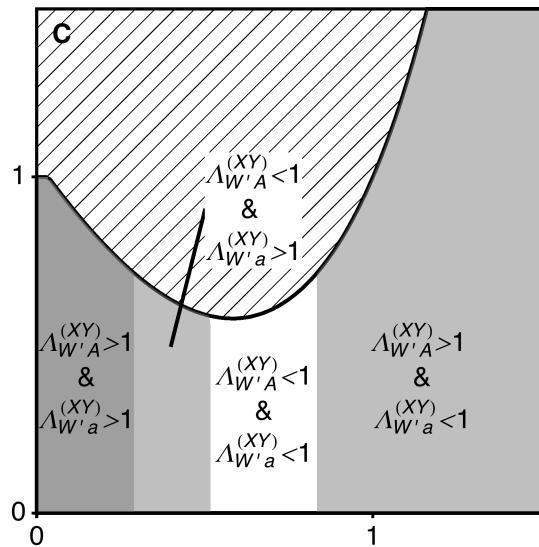
```

plotC =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{0.5, 0.9}, {0.4, 0.5}}]}],
  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
    (*FrameLabel→{"Relative fitness of aa males","",""},*)
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["C", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1$ ", 14],
      {0.29, 0.4}, {1, 0}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} < 1$ ", 14],
      {0.5, 0.9}, {-1, 0}, Background → White],
    Text[Style[" $\Lambda_{W'A}^{(xy)} < 1$ ", 14],
      {0.82, 0.3}, {1, 0}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1$ ", 14],
      {1.3, 0.4}, {1, 0}](*,
      Rotate[
        Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]*),
    PlotLabel → Style["overdominance in females", 16, Black, Bold],
    PlotRangeClipping → False
  }
]

```

overdominance in females



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

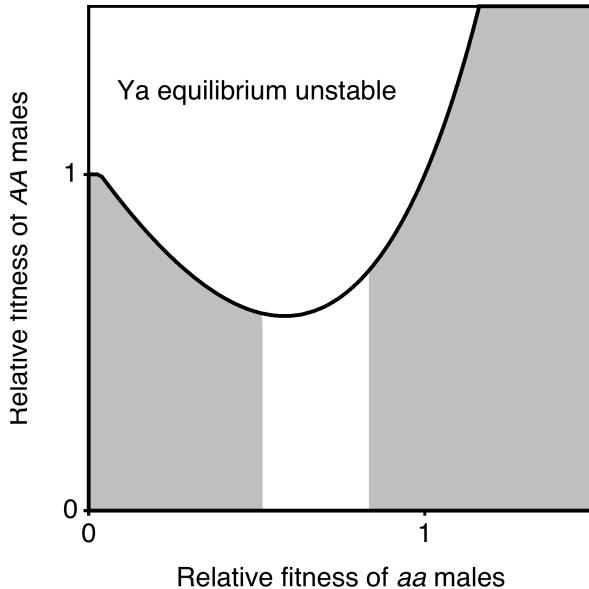
(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
    PlotLabel -> Style["A favoured in females", 16, Black, Bold],
    PlotRangeClipping -> False
  }
]

```

A favoured in females

Panel D - A favoured in females, A drives in females

Parameters

```
params = {
    wAm → 1, wam → 1, wAf → 1, waf → 1,
    af → 1 / 2 + 8 / 100,
    am → 1 / 2,
    MAa → 1, FAa → 1,
    Faa → 1 - 3 / 20,
    FAA → 1 + 1 / 20
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsola =
Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{
    (validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]}, 
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]}, 
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

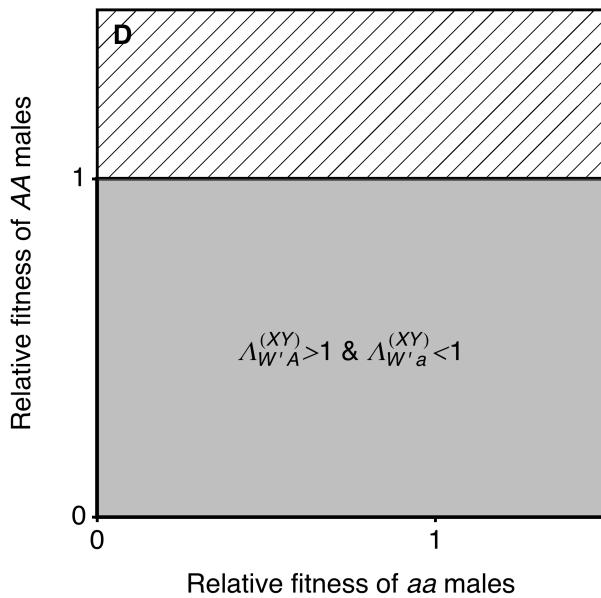
```

```

plotD =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["D", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{w'A}^{(xy)} > 1 \text{ & } \Lambda_{w'a}^{(xy)} < 1$ ", 14], {0.75, 0.5}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
  },
  PlotLabel → Style[(*"A favoured in females"*) "", 16, Black, Bold],
  PlotRangeClipping → False
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];
```

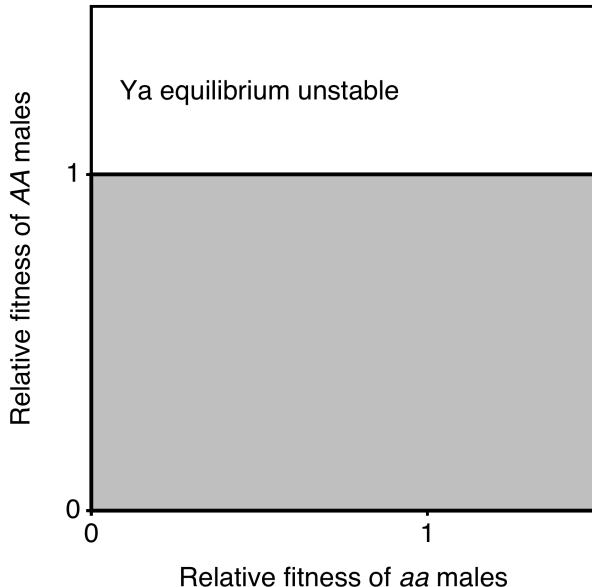
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
  PlotLabel → Style["A favoured in females", 16, Black, Bold],
  PlotRangeClipping → False
]

```

A favoured in females



Panel E - a favoured in females, A drives in females

Parameters

```
params = {
    wAm → 1, wam → 1, wAf → 1, waf → 1,
    af → 1 / 2 + 8 / 100,
    am → 1 / 2,
    MAa → 1, FAA → 1,
    FAA → 1 - 3 / 20,
    Faa → 1 + 1 / 20
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsola =
    Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{
    (validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]}, 
BoundaryStyle → None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]}, 
BoundaryStyle → None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) ||
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → White,
BoundaryStyle → {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → None,
BoundaryStyle → None,
Mesh → 50,
MeshFunctions → {#1 - #2 &},
MeshStyle → Black
];

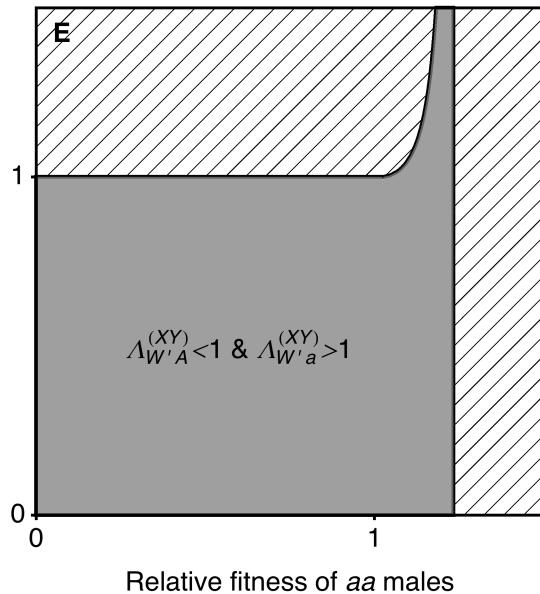
```

```

plotE =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["E", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} < 1 \text{ & } \Lambda_{W'a}^{(xy)} > 1$ ", 14], {0.6, 0.5}] (*,
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree] *)
  },
  PlotLabel → Style[(*"a favoured in females"*) "", 16, Black, Bold],
  PlotRangeClipping → False
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

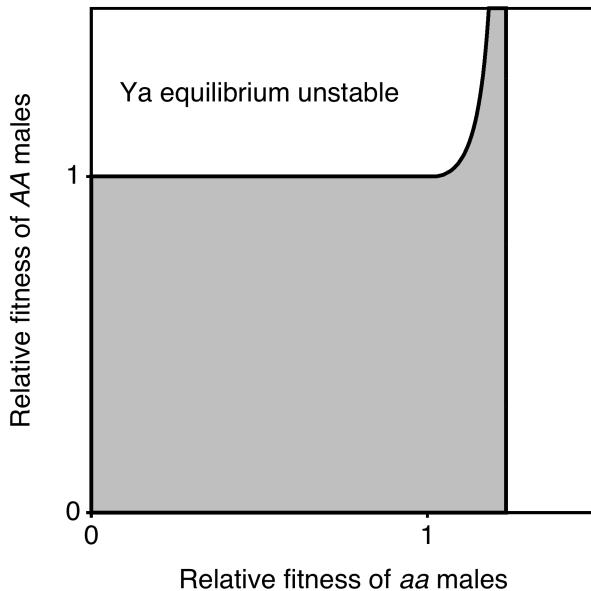
(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
    PlotLabel -> Style["a favoured in females", 16, Black, Bold],
    PlotRangeClipping -> False
  }
]

```

***a* favoured in females**

Panel F - overdominance in females, A drives in females

Parameters

```
params = {
    wAm → 1, wam → 1, wAf → 1, waf → 1,
    af → 1 / 2 + 8 / 100,
    am → 1 / 2,
    MAA → 1, FAA → 1,
    FAA → 1 - 4 / 10,
    Faa → 1 - 4 / 10
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsola =
    Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{
    (validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]}, 
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]}, 
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

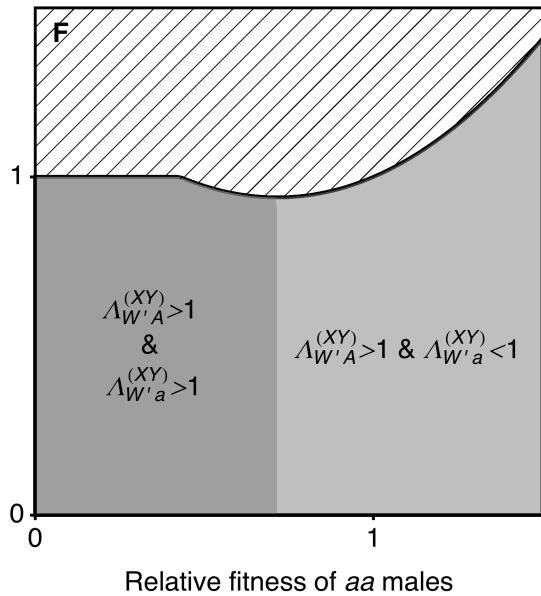
```

```

plotF =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["F", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1$ ", 14],
       $\Lambda_{W'a}^{(xy)} > 1$ , 14], {0.35, 0.5}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1 \& \Lambda_{W'a}^{(xy)} < 1$ ", 14], {1.1, 0.5}] (*,
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree] *)
  },
  PlotLabel → Style[(*"overdominance in females"*) "", 16, Black, Bold],
  PlotRangeClipping → False
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

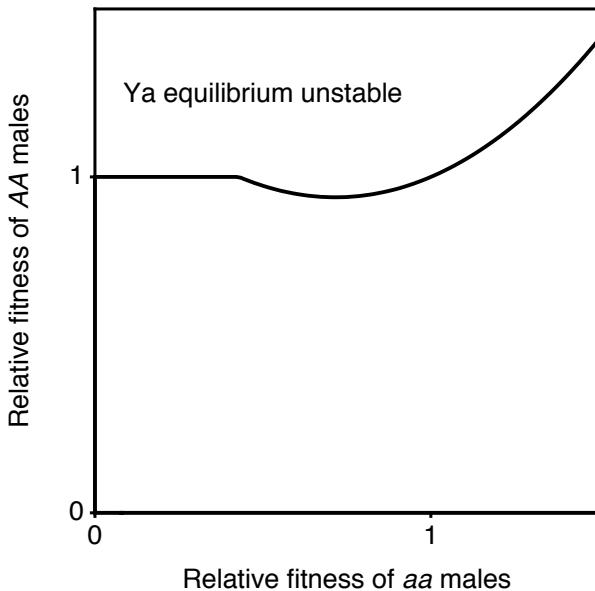
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
  PlotLabel -> Style["overdominance in females", 16, Black, Bold],
  PlotRangeClipping -> False
]

```

overdominance in females



All Panels

```

GraphicsGrid[{
  {plotA, plotB, plotC},
  {plotD, plotE, plotF}
],
Spacings -> -50
]

Export[plotdir <> "Region_plot_combined_FemaleDrive.eps", % // rasterTrick];

```

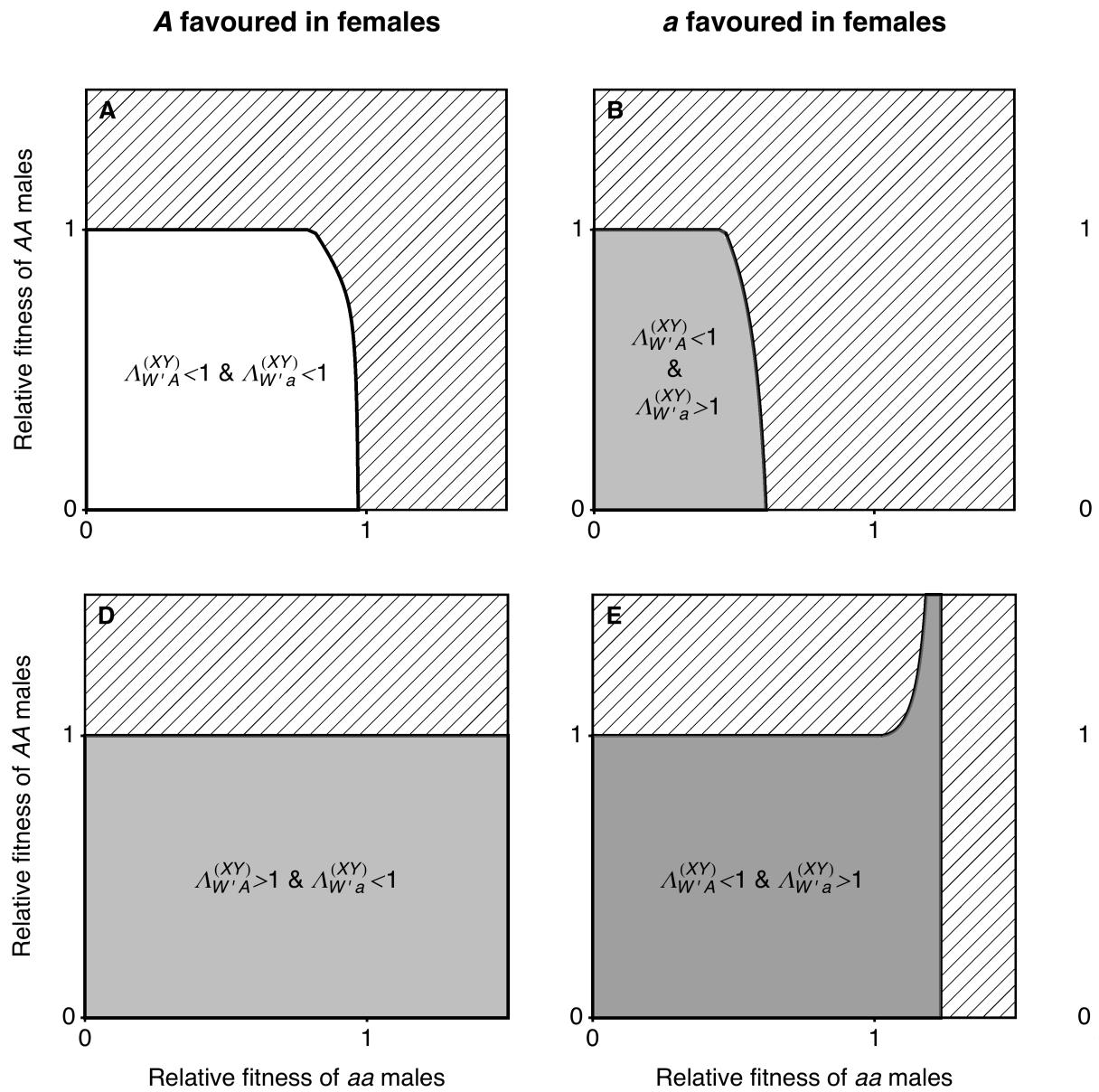


Figure S.7 - neo-W haplotype invasion with haploid competition in females

Panel A - A favoured in females, haploid competition favours a in females

Parameters

```
params = {
  wAm → 1, wam → 1, wAf → 1, waf → 1 + 8 / 50,
  αm → 1 / 2, αf → 1 / 2,
  MAa → 1, FAa → 1,
  Faa → 1 - 3 / 20,
  FAA → 1 + 1 / 20
};
```

Making the sex-ratio

```
freqMale /. equilB0 /. params // N
0.5
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
  Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];
```

```

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{
  (validcondA /. params) &&
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) &&
  1 < WainvA
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]}, 
BoundaryStyle → None
];

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilBO /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]}, 
BoundaryStyle → None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilBO /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]}, 
BoundaryStyle → None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) ||
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilBO /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → White,
BoundaryStyle → {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → None,
BoundaryStyle → None,
Mesh → 50,
MeshFunctions → {#1 - #2 &},

```

```

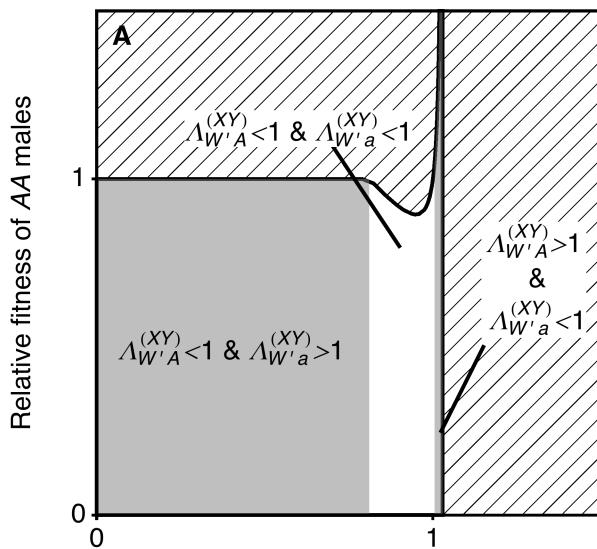
MeshStyle -> Black
];

plotA =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{1.15, 0.5}, {1.025, 0.25}}]}],
  Graphics[{Thick, Black, Line[{{0.9, 0.8}, {0.7, 1.1}}]}],
  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  (*FrameLabel -> {"Relative fitness of aa males", "", *})
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["A", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} < 1 \text{ & } \Lambda_{W'a}^{(xy)} > 1$ ", 14], {0.4, 0.5}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1$ ", 14], {1.3, 0.7}, Background -> White],
    Text[Style[" $\Lambda_{W'A}^{(xy)} < 1 \text{ & } \Lambda_{W'a}^{(xy)} < 1$ ", 14], {0.6, 1.15}, Background -> White],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
  },
  PlotLabel -> Style["A favoured in females", 16, Black, Bold],
  PlotRangeClipping -> False
]

```

A favoured in females



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];
```

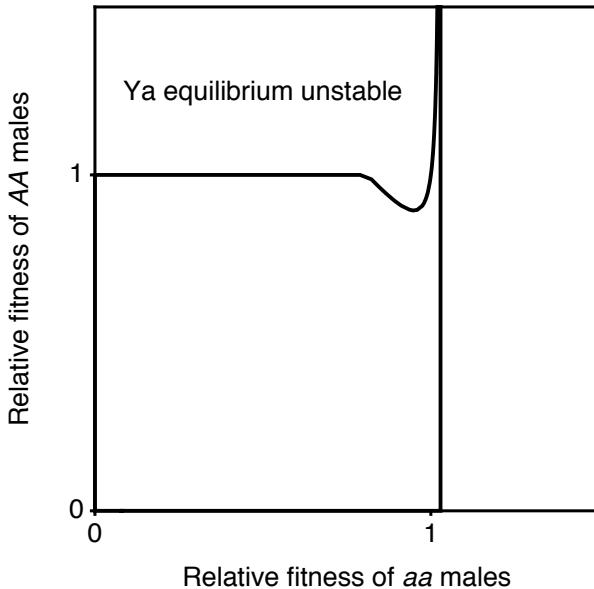
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
  PlotLabel -> Style["A favoured in females", 16, Black, Bold],
  PlotRangeClipping -> False
]

```

A favoured in females



Panel B - a favoured in females, haploid competition favours a in females

Parameters

```
params = {
    wAm → 1, wam → 1, wAf → 1, waf → 1 + 8 / 50,
    cm → 1 / 2, αf → 1 / 2,
    MAa → 1, FAa → 1,
    FAA → 1 - 3 / 20,
    Faa → 1 + 1 / 20
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
  Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{(
    validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
  RegionPlot[{(
    validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

```

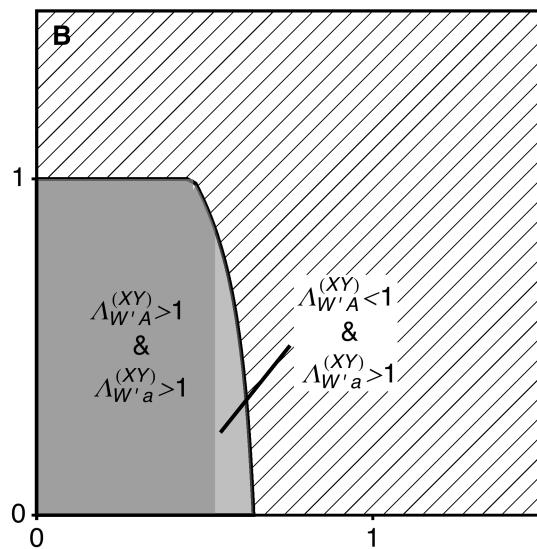
```

plotB =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{0.75, 0.5}, {0.55, 0.25}}]}],
  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  (*FrameLabel→{"Relative fitness of aa males","",""},*)
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["B", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1$ ", 14],
      &
       $\Lambda_{W'a}^{(xy)} > 1$ , 14], {0.3, 0.5}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} < 1$ ", 14],
      &
       $\Lambda_{W'a}^{(xy)} > 1$ , 14], {0.75, 0.55}, {-1.1, 0.05}, Background → White] (*,
      Rotate[
        Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]*),
    },
    PlotLabel → Style["a favoured in females", 16, Black, Bold],
    PlotRangeClipping → False
  }
]

```

a favoured in females



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];
```

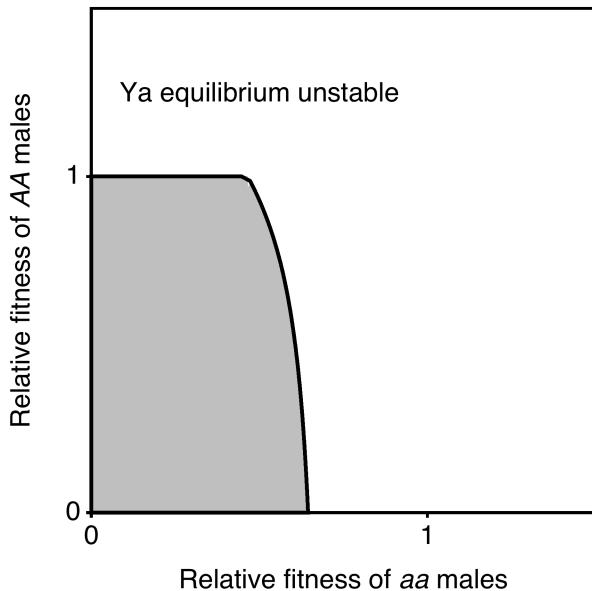
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
    PlotLabel → Style["a favoured in females", 16, Black, Bold],
    PlotRangeClipping → False
  }
]

```

***a* favoured in females**



Panel C - overdominance in females, haploid competition favours a in females

Parameters

```
params = {
    wAm → 1, wam → 1, wAf → 1, waf → 1 + 8 / 50,
    cm → 1 / 2, αf → 1 / 2,
    MAa → 1, FAa → 1,
    FAA → 1 - 4 / 10,
    Faa → 1 - 4 / 10
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilAO /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilAO /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilBO /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilBO /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilAO /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilBO /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{(
validcondA /. params) && (*valid*)
(stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) && (*internally stable*)
1 < WAinvA (*invasion*)
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{(
validcondA /. params) &&
(stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) &&
1 < WainvA
},
{Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

```

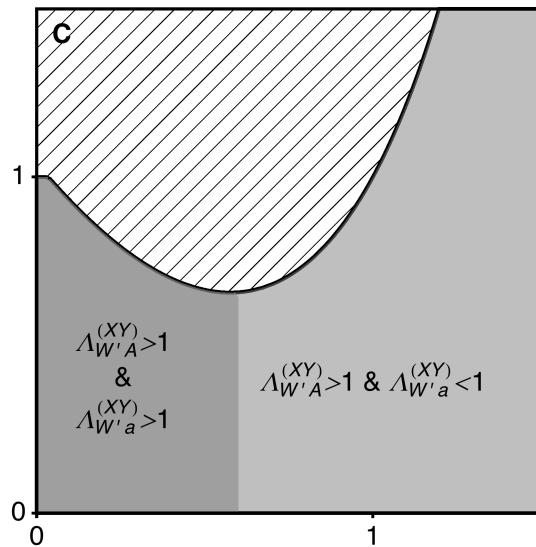
```

plotC =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  (*FrameLabel→{"Relative fitness of aa males","",""},*)
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["C", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1$ ", 14],
      {0.4, 0.4}, {1, 0}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1 \& \Lambda_{W'a}^{(xy)} < 1$ ", 14],
      {1, 0.4}] (*,
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]*),
    },
  PlotLabel → Style["overdominance in females", 16, Black, Bold],
  PlotRangeClipping → False
]

```

overdominance in females



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];
```

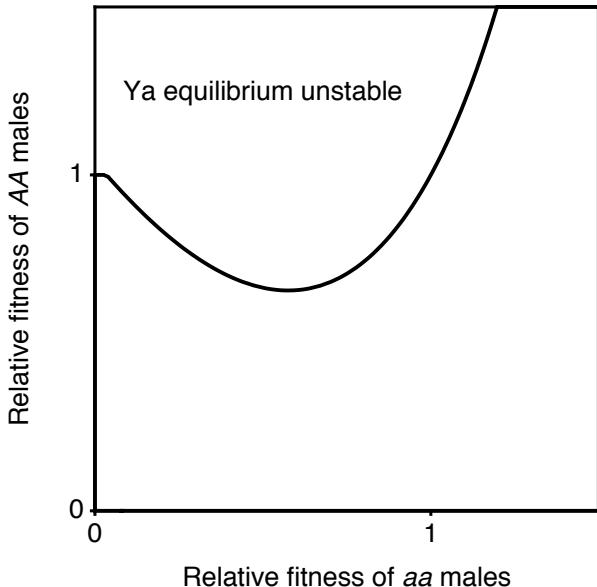
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
  PlotLabel -> Style["A favoured in females", 16, Black, Bold],
  PlotRangeClipping -> False
]

```

A favoured in females



Panel D - A favoured in females, haploid competition favours A in females

Parameters

```
params = {
    wAm → 1, wam → 1, wAf → 1 + 8 / 50, waf → 1,
    cm → 1 / 2, αf → 1 / 2,
    MAa → 1, FAa → 1,
    Faa → 1 - 3 / 20,
    FAA → 1 + 1 / 20
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilAO /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilAO /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilBO /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilBO /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
  Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilAO /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilBO /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{(
    validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
  RegionPlot[{(
    validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) &&
    1 < WainvA
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

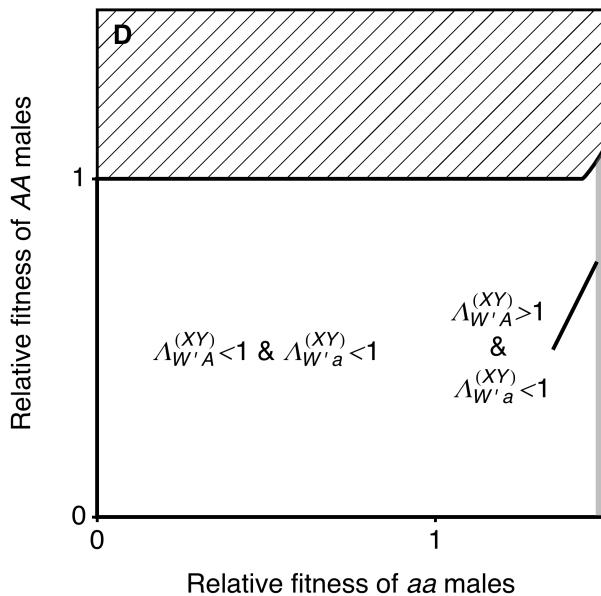
```

```

plotD =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{1.35, 0.5}, {1.475, 0.75}}]}],
  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["D", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{w'A}^{(xy)} < 1 \text{ & } \Lambda_{w'a}^{(xy)} < 1$ ", 14], {0.5, 0.5}],
    Text[Style[" $\Lambda_{w'A}^{(xy)} > 1$  &
 $\Lambda_{w'a}^{(xy)} < 1$ ", 14], {1.35, 0.5}, {1, 0}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    },
    PlotLabel -> Style[(*"A favoured in females"*) "", 16, Black, Bold],
    PlotRangeClipping -> False
  }
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

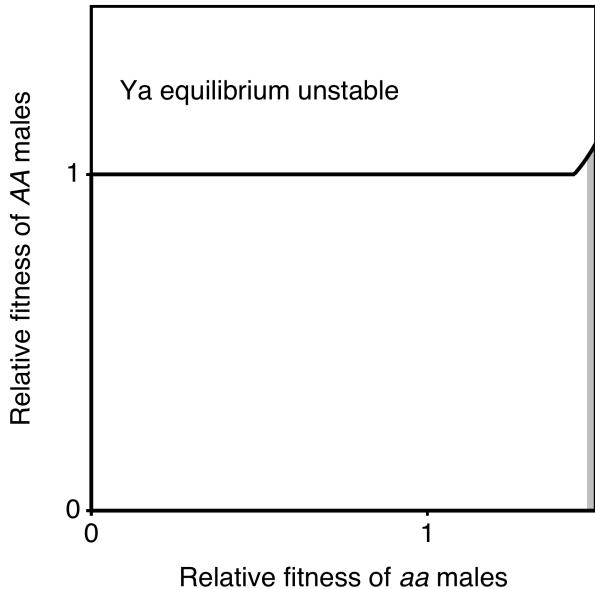
(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
    PlotLabel → Style["A favoured in females", 16, Black, Bold],
    PlotRangeClipping → False
  }
]

```

A favoured in females

Panel E - a favoured in females, haploid competition favours A in females

Parameters

```
params = {
  wAm → 1, wam → 1, wAf → 1 + 8 / 50, waf → 1,
  cm → 1 / 2, αf → 1 / 2,
  MAa → 1, FAa → 1,
  FAA → 1 - 3 / 20,
  Faa → 1 + 1 / 20
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilAO /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilAO /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilBO /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilBO /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
  Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilAO /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilBO /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{(
    validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
  RegionPlot[{(
    validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilAO /. params) &&
    1 < WainvA
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

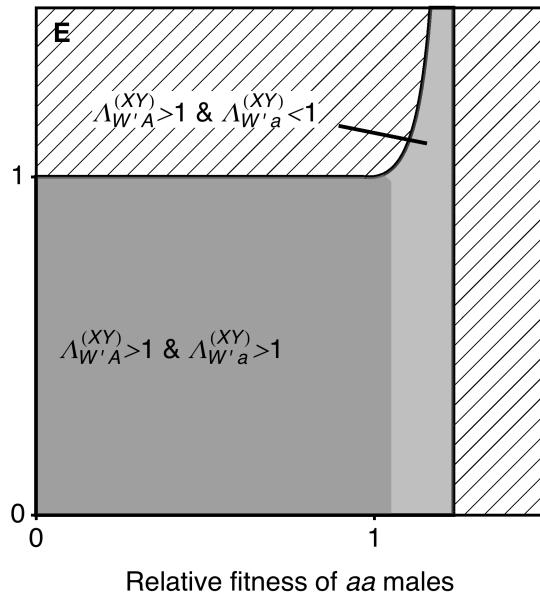
```

```

plotE =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{1.15, 1.1}, {0.9, 1.15}}]}],
  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["E", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{w'A}^{(xy)} > 1 \text{ & } \Lambda_{w'a}^{(xy)} > 1$ ", 14], {0.4, 0.5}],
    Text[Style[" $\Lambda_{w'A}^{(xy)} > 1 \text{ & } \Lambda_{w'a}^{(xy)} < 1$ ", 14], {0.5, 1.2}, Background -> White] (*,
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree] *)
  },
  PlotLabel -> Style[(*"a favoured in females"*) "", 16, Black, Bold],
  PlotRangeClipping -> False
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

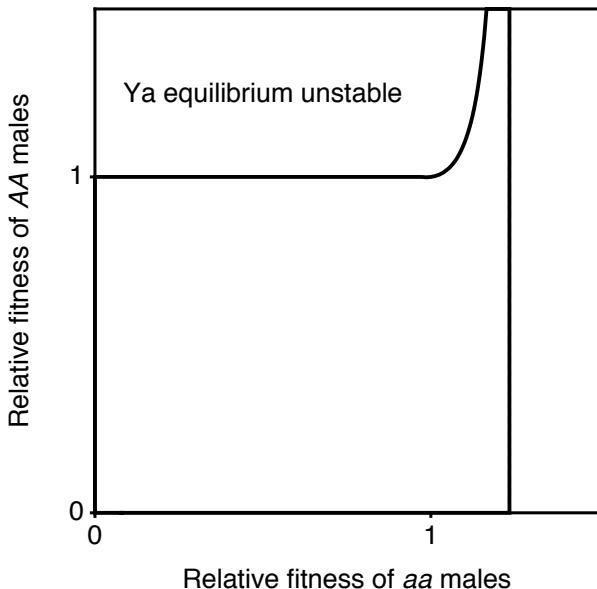
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
    PlotLabel -> Style["a favoured in females", 16, Black, Bold],
    PlotRangeClipping -> False
  }
]

```

a favoured in females



Panel F - overdominance in females, haploid competition favours A in females

Parameters

```
params = {
  wAm → 1, wam → 1, wAf → 1 + 8 / 50, waf → 1,
  cm → 1 / 2, αf → 1 / 2,
  MAa → 1, FAa → 1,
  FAA → 1 - 4 / 10,
  Faa → 1 - 4 / 10
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA =
  Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params, λ] // Simplify]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /. params, λ] // Simplify]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{(
    validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
  RegionPlot[{(
    validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
  },
  {Maa, 0, 3 / 2}, {MAA, 0, 3 / 2},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];
```

```

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

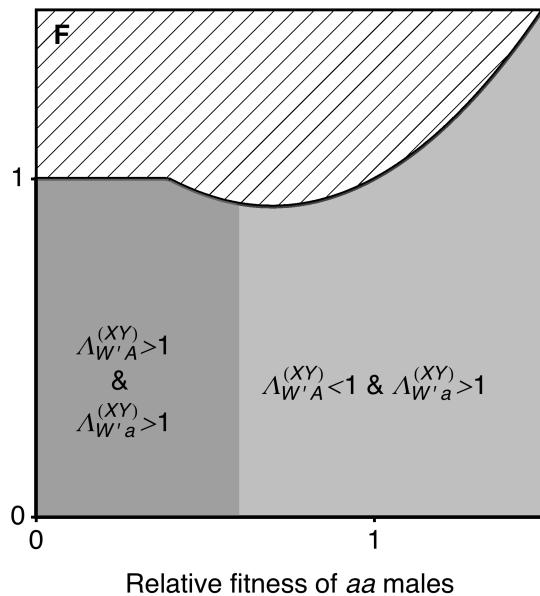
```

```

plotF =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  PlotRange → {{0, 1.5}, {0, 1.5}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Relative fitness of aa males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["F", 14, Bold], Scaled@{0.05, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1$ ", 14],
      {0.4, 0.4}, {1, 0}],
    Text[Style[" $\Lambda_{W'a}^{(xy)} > 1$ ", 14], {1, 0.4}] (*,
      Rotate[
        Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree] *),
    PlotLabel → Style[(*"overdominance in females"*) "", 16, Black, Bold],
    PlotRangeClipping → False
  }
]

```



Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB (*invasion*)
},
{Maa, 0, 3/2}, {MAA, 0, 3/2},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

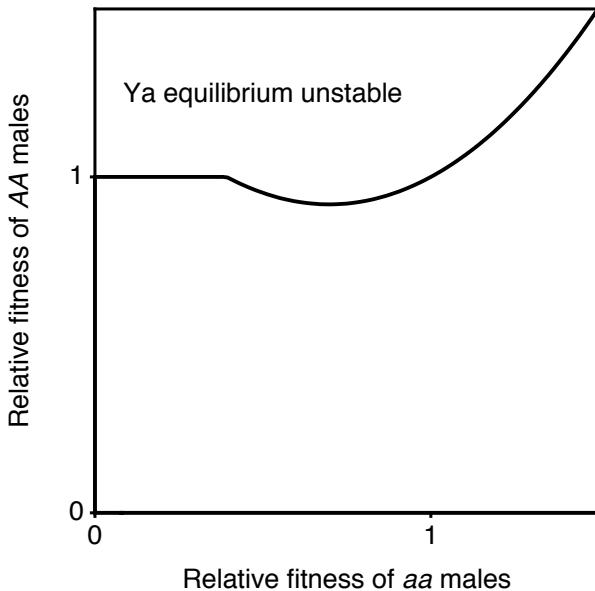
```

Show[
  plotWinvA,
  plotWinvB,
  plotYaStable,

  PlotRange -> {{0, 1.5}, {0, 1.5}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, 0, 1, 1}],
    Table[{y, y, ticksize}, {y, 0, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Relative fitness of aa males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["Ya equilibrium unstable", 14], {0.5, 1.25}],
    Rotate[
      Text[Style["Relative fitness of AA males", 14], Scaled@ylabpos], 90 Degree]
    ],
  PlotLabel -> Style["overdominance in females", 16, Black, Bold],
  PlotRangeClipping -> False
]

```

overdominance in females



All Panels

```

GraphicsGrid[{
  {plotA, plotB, plotC},
  {plotD, plotE, plotF}
],
Spacings -> -50
]

Export[plotdir <> "Region_plot_combined_FemaleGS.eps", % // rasterTrick];

```

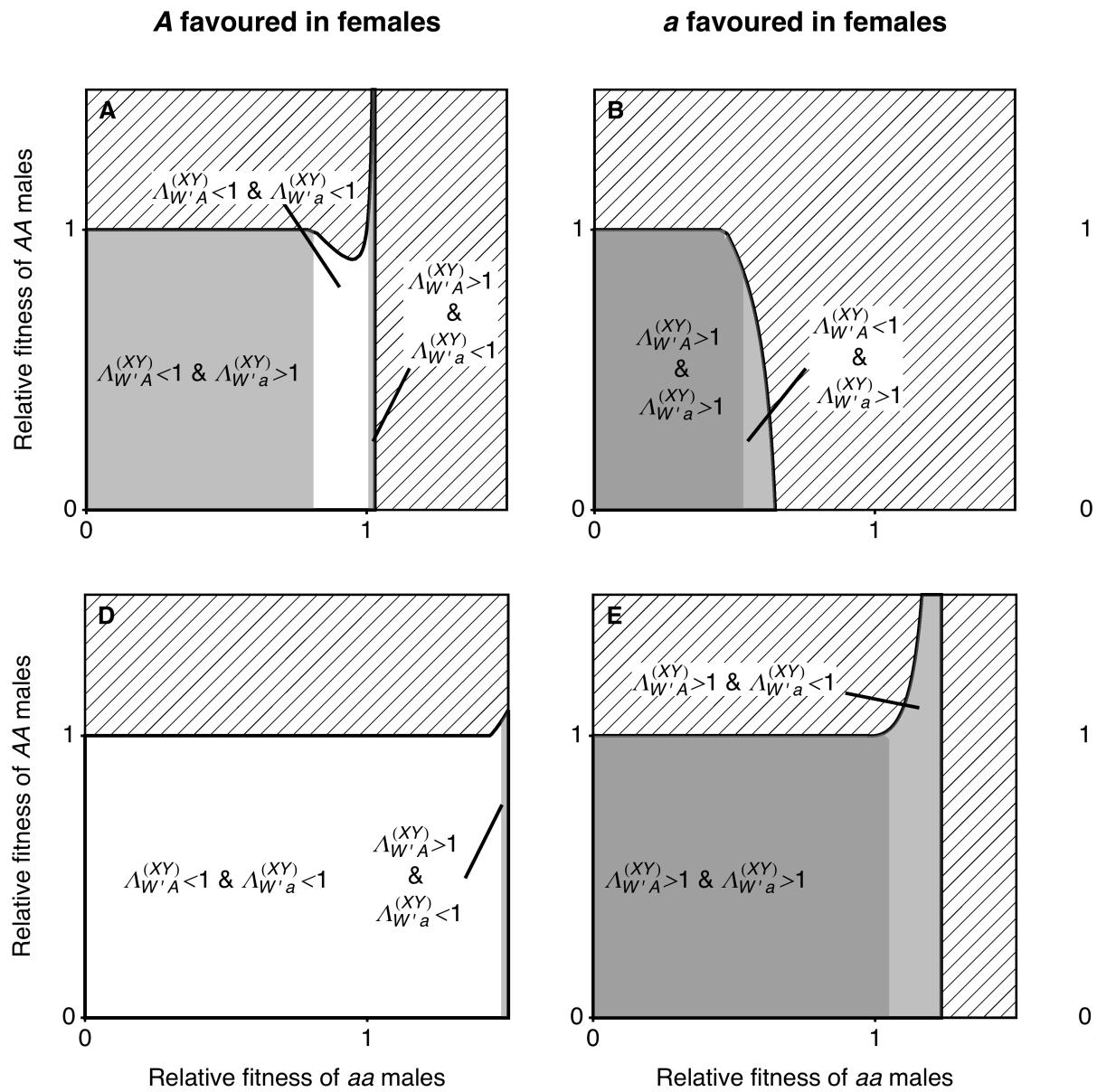


Figure S.8 - neo-W haplotype invasion with ploidally-antagonistic selection

extra parameters (move to top)

```

tryr = 1 / 200;
tryk = 1;
trypm = 0.01;

startplot = 5;
endtime = 15 000;

loglinearplotINVoptions = {
  Frame → {{True, False}, {True, False}},
  FrameStyle → Directive[Black, Thickness[lwd]],
  FrameTicksStyle → {{Directive[Black, Thickness[lwd], FontColor → Black],
    Directive[Black, Thickness[lwd]]], Directive[Black, Thickness[lwd]]}},
  FrameTicks → {{{(*{5, 5, {0, 0.01}}), *}{{10, 10, {0, 0.01}}, {*}{{50, 50, {0, 0.01}}}, {*}
    {100, 100, {0, 0.01}}, {*}{{500, 500, {0, 0.01}}, {*}{{1000, 1000, {0, 0.01}}
    {*,{5000, 5000, {0, 0.01}}}, {10000, 10000, {0, 0.01}}}}},
  Table[{y, y, ticksz}, {y, 0, 0.5, 0.5}]},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImageSize → {xsize, xsize aspectratio},
  AspectRatio → aspectratio,
  Axes → True,
  FrameLabel → {Style["Generation", FontSize → 14], },
  ImagePadding → Pad
};

modvec = {0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0};

```

Panel A - invasion region with drive in males

Parameters

```

params = {
  wAm → 1, wAm → 1, wAf → 1, waf → 1,
  αm → (1 + αmd) / 2, αf → 1 / 2,
  Maa → 1, Faa → 1,
  MAa → 1 + (3 / 4) s, FAa → 1 + (3 / 4) s,
  MAA → 1 + s, FAA → 1 + s
};

```

No recombination eigenvalues

```

WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;

```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
 $\lambda_{WsolA} = \text{Max}[\text{Abs}[\lambda /. \text{Solve}[0 == \text{charpolyk1} /. r \rightarrow 0 /. R \rightarrow 0 /. \rho \rightarrow 0 /. \text{equilA0} /. \text{params} // \text{Simplify}, \lambda]]];$ 
 $\lambda_{WsolB} = \text{Max}[\text{Abs}[\lambda /. \text{Solve}[0 == \text{charpolyk1} /. r \rightarrow 0 /. R \rightarrow 0 /. \rho \rightarrow 0 /. \text{equilB0} /. \text{params} // \text{Simplify}, \lambda]]];$ 
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf \[Rule] 0 /. Rm \[Rule] 0 /. equilA0 /. params) && (*internally stable*)
  1 < WAinvA (*invasion*)
},
{amd, -1, 1}, {s, -1, 1},
PlotStyle \[Rule] {Gray, Opacity[0.5]},
BoundaryStyle \[Rule] None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
RegionPlot[{
  (validcondA /. params) &&
  (stabcondA /. Rf \[Rule] 0 /. Rm \[Rule] 0 /. equilA0 /. params) &&
  1 < WainvA
},
{amd, -1, 1}, {s, -1, 1},
PlotStyle \[Rule] {Gray, Opacity[0.5]},
BoundaryStyle \[Rule] None
];

(*neo-WA invades XY from equilB*)
plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf \[Rule] 0 /. Rm \[Rule] 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB (*invasion*)
},
{amd, -1, 1}, {s, -1, 1},
PlotStyle \[Rule] {Gray, Opacity[0.5]},
BoundaryStyle \[Rule] None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf \[Rule] 0 /. Rm \[Rule] 0 /. equilB0 /. params) &&
  1 < WainvB
},
{amd, -1, 1}, {s, -1, 1},
PlotStyle \[Rule] {Gray, Opacity[0.5]},
BoundaryStyle \[Rule] None
];
```

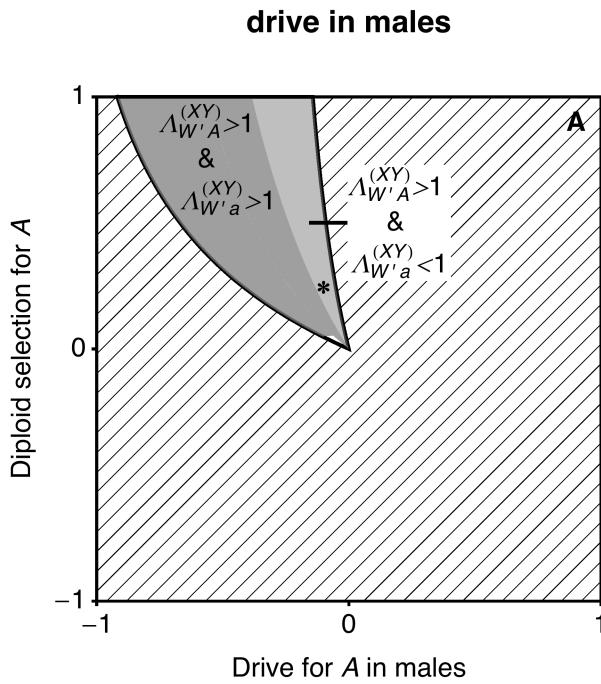
```
];
(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{αmd, -1, 1}, {s, -1, 1},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];
(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{αmd, -1, 1}, {s, -1, 1},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];
```

```

plotA =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{-0.15, 0.5}, {0.1, 0.5}}]}],
  PlotRange -> {{-1, 1}, {-1, 1}},
  ImageSize -> {xsize, xsize},
  PlotRangePadding -> 0,
  FrameTicks -> {Table[{x, x, ticksize}, {x, -1, 1, 1}],
    Table[{y, y, ticksize}, {y, -1, 1, 1}], None, None},
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor -> White]}},
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel -> {"Drive for A in males", ""},
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},
  ImagePadding -> Pad,
  Epilog -> {
    Text[Style["A", 14, Bold], Scaled@{0.95, 0.95}],
    Text[Style[" $A_{W'A}^{(xy)}$  > 1
  &
   $A_{W'a}^{(xy)} > 1$ ", 14], {-0.55, 0.75}],
    Text[Style[" $A_{W'A}^{(xy)}$  > 1
  &
   $A_{W'a}^{(xy)} < 1$ ", 14], {0.2, 0.5}, Background -> White],
    Text[Style["*", 16, Bold], {-0.1, 0.25}],
    Rotate[Text[Style["Diploid selection for A", 14], Scaled@ylabpos], 90 Degree]
  },
  PlotLabel -> Style["drive in males", 16, Black, Bold],
  PlotRangeClipping -> False
]

```

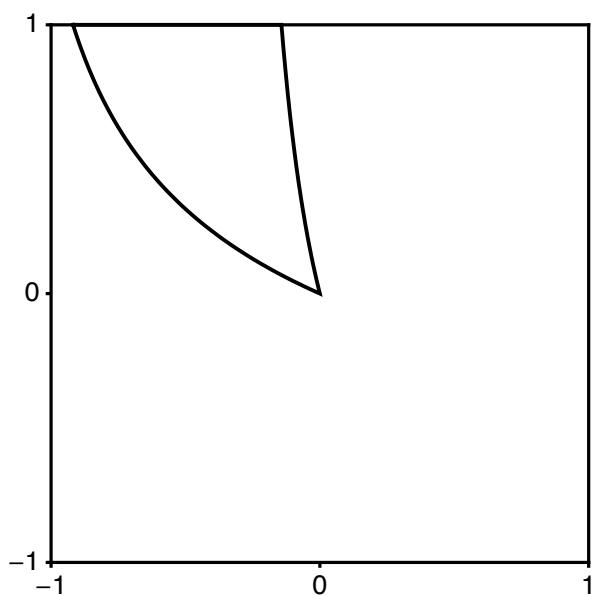


Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA(*invasion*)
},
{αmd, -1, 1}, {s, -1, 1},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB(*invasion*)
},
{αmd, -1, 1}, {s, -1, 1},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```
Show[  
  plotWinvA,  
  plotWinvB,  
  plotYaStable,  
  
  PlotRange -> {{-1, 1}, {-1, 1}},  
  ImageSize -> {xsize, xsize},  
  PlotRangePadding -> 0,  
  FrameTicks -> {Table[{x, x, ticksize}, {x, -1, 1, 1}],  
    Table[{y, y, ticksize}, {y, -1, 1, 1}], None, None},  
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],  
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],  
    Directive[Black, Thickness[lwd], FontColor -> White]}},  
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},  
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},  
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},  
  ImagePadding -> Pad,  
  PlotRangeClipping -> False  
]
```



Panel B - invasion region with haploid competition in males

Parameters

```
params = {
  wAm → 1 + t, wam → 1, wAf → 1, waf → 1,
  cm → 1 / 2, αf → 1 / 2,
  Maa → 1, Faa → 1,
  MAa → 1 + (3 / 4) s, FAa → 1 + (3 / 4) s,
  MAA → 1 + s, FAA → 1 + s
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA = Max[Abs[λ /. Solve[
  0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params // Simplify, λ]]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /.
  params // Simplify, λ]]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  }},
  {t, -1, 1}, {s, -1, 1},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
  RegionPlot[{{
    (validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
  }},
  {t, -1, 1}, {s, -1, 1},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];

(*neo-WA invades XY from equilB*)
```

```

plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{t, -1, 1}, {s, -1, 1},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{t, -1, 1}, {s, -1, 1},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{t, -1, 1}, {s, -1, 1},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{αmd, -1, 1}, {s, -1, 1},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

```

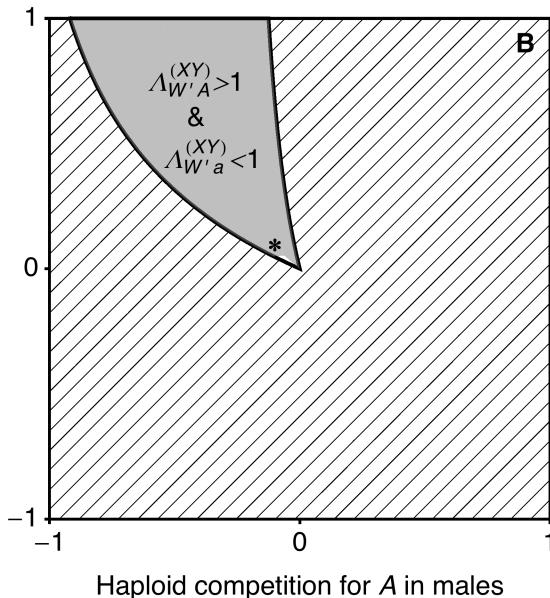
```

plotB =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  PlotRange → {{-1, 1}, {-1, 1}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, -1, 1, 1}],
    Table[{y, y, ticksize}, {y, -1, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Haploid competition for A in males", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["B", 14, Bold], Scaled@{0.95, 0.95}],
    Text[Style[" $\Lambda_{w'A}^{(xy)} > 1$ ", 14],
      {-0.4, 0.6}],
    Text[Style["*", 16, Bold], {-0.1, 0.1}]
  },
  PlotLabel → Style["haploid competition in males", 16, Black, Bold],
  PlotRangeClipping → False
]

```

haploid competition in males

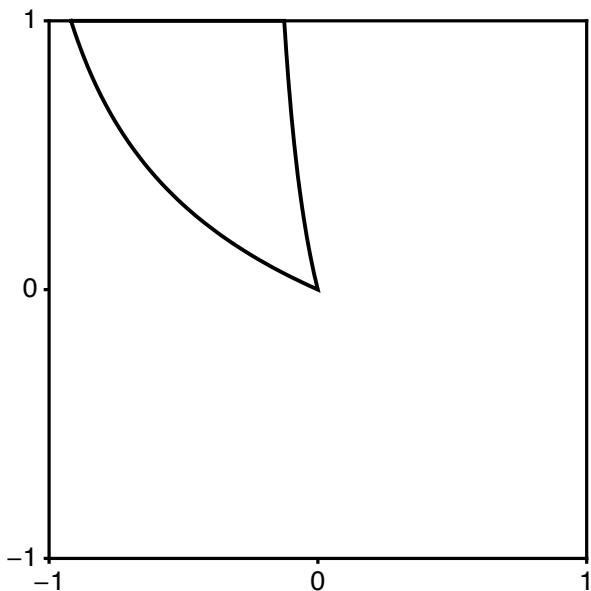


Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA(*invasion*)
},
{t, -1, 1}, {s, -1, 1},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB(*invasion*)
},
{t, -1, 1}, {s, -1, 1},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```
Show[  
  plotWinvA,  
  plotWinvB,  
  plotYaStable,  
  
  PlotRange -> {{-1, 1}, {-1, 1}},  
  ImageSize -> {xsize, xsize},  
  PlotRangePadding -> 0,  
  FrameTicks -> {Table[{x, x, ticksize}, {x, -1, 1, 1}],  
    Table[{y, y, ticksize}, {y, -1, 1, 1}], None, None},  
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],  
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],  
    Directive[Black, Thickness[lwd], FontColor -> White]}},  
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},  
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},  
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},  
  ImagePadding -> Pad,  
  PlotRangeClipping -> False  
]
```



Panel C - invasion region with drive in females

Parameters

```
params = {
  wAm → 1, wam → 1, wAf → 1, waf → 1,
  αm → 1 / 2, αf → (1 + αmd) / 2,
  Maa → 1, Faa → 1,
  MAa → 1 + (3 / 4) s, FAa → 1 + (3 / 4) s,
  MAA → 1 + s, FAA → 1 + s
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA = Max[Abs[λ /. Solve[
  0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params // Simplify, λ]]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /.
  params // Simplify, λ]]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  },
  {αmd, -1, 1}, {s, -1, 1},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
}];

(*neo-Wa invades XY from equilA*)
plotWainvA =
  RegionPlot[{{
    (validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
  },
  {αmd, -1, 1}, {s, -1, 1},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
}];

(*neo-WA invades XY from equilB*)
```

```

plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{αmd, -1, 1}, {s, -1, 1},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{αmd, -1, 1}, {s, -1, 1},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{αmd, -1, 1}, {s, -1, 1},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{αmd, -1, 1}, {s, -1, 1},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

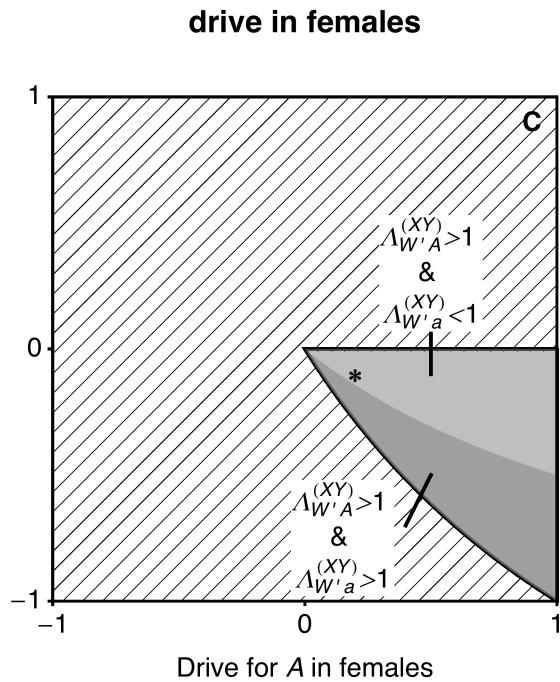
```

```

plotC =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  Graphics[{Thick, Black, Line[{{0.5, -0.1}, {0.5, 0.1}}]}],
  Graphics[{Thick, Black, Line[{{0.5, -0.5}, {0.4, -0.7}}]}]],
PlotRange → {{-1, 1}, {-1, 1}},
ImageSize → {xsize, xsize},
PlotRangePadding → 0,
FrameTicks → {Table[{x, x, ticksize}, {x, -1, 1, 1}],
  Table[{y, y, ticksize}, {y, -1, 1, 1}], None, None},
FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
  Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
  Directive[Black, Thickness[lwd], FontColor → White]}},
FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
  {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
FrameLabel → {"Drive for A in females", ""},
BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
ImagePadding → Pad,
Epilog → {
  Text[Style["C", 14, Bold], Scaled@{0.95, 0.95}],
  Text[Style[" $A_{W'A}^{(xy)}$  > 1
&
 $A_{W'a}^{(xy)} < 1$ ", 14], {0.5, 0.3}, Background → White],
  Text[Style[" $A_{W'A}^{(xy)}$  > 1
&
 $A_{W'a}^{(xy)} > 1$ ", 14], {0.15, -0.75}, Background → White],
  Text[Style["*", 16, Bold], {0.2, -0.1}]
},
PlotLabel → Style["drive in females", 16, Black, Bold],
PlotRangeClipping → False
]
]

```

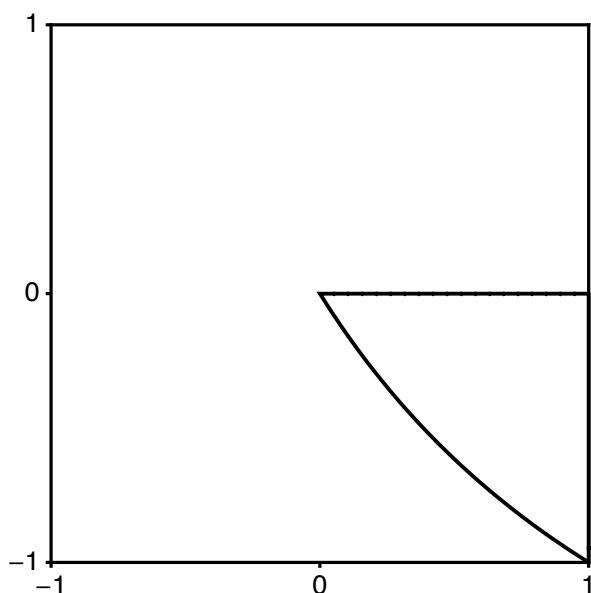


Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA(*invasion*)
},
{αmd, -1, 1}, {s, -1, 1},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB(*invasion*)
},
{αmd, -1, 1}, {s, -1, 1},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```
Show[  
  plotWinvA,  
  plotWinvB,  
  plotYaStable,  
  
  PlotRange -> {{-1, 1}, {-1, 1}},  
  ImageSize -> {xsize, xsize},  
  PlotRangePadding -> 0,  
  FrameTicks -> {Table[{x, x, ticksize}, {x, -1, 1, 1}],  
    Table[{y, y, ticksize}, {y, -1, 1, 1}], None, None},  
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],  
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],  
    Directive[Black, Thickness[lwd], FontColor -> White]}},  
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},  
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},  
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},  
  ImagePadding -> Pad,  
  PlotRangeClipping -> False  
]
```



Panel D - invasion region with haploid competition in females

Parameters

```
params = {
  wAm → 1, wam → 1, wAf → 1 + t, waf → 1,
  cm → 1 / 2, αf → 1 / 2,
  Maa → 1, Faa → 1,
  MAa → 1 + (3 / 4) s, FAa → 1 + (3 / 4) s,
  MAA → 1 + s, FAA → 1 + s
};
```

No recombination eigenvalues

```
WAinvA = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WainvA = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilA0 /. params // Simplify;
WAinvB = λmA1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
WainvB = λma1 /. reverse /. pAveM → (1 - q) pXm + q pYm /. equilB0 /. params // Simplify;
```

Maximum absolute no recombination eigenvalue from the full characteristic polynomial

```
λWsolA = Max[Abs[λ /. Solve[
  0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilA0 /. params // Simplify, λ]]];
λWsolB = Max[Abs[λ /. Solve[0 == charpolyk1 /. r → 0 /. R → 0 /. ρ → 0 /. equilB0 /.
  params // Simplify, λ]]];
```

Plot

Region plots of invasion

```
(*neo-WA invades XY from equilA*)
plotWAinvA =
  RegionPlot[{{
    (validcondA /. params) && (*valid*)
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
    1 < WAinvA (*invasion*)
  }},
  {t, -1, 1}, {s, -1, 1},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];

(*neo-Wa invades XY from equilA*)
plotWainvA =
  RegionPlot[{{
    (validcondA /. params) &&
    (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) &&
    1 < WainvA
  }},
  {t, -1, 1}, {s, -1, 1},
  PlotStyle → {Gray, Opacity[0.5]},
  BoundaryStyle → None
];

(*neo-WA invades XY from equilB*)
```

```

plotWAinvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) && (*internally stable*)
  1 < WAinvB(*invasion*)
},
{t, -1, 1}, {s, -1, 1},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*neo-Wa invades XY from equilB*)
plotWainvB =
RegionPlot[{
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params) &&
  1 < WainvB
},
{t, -1, 1}, {s, -1, 1},
PlotStyle -> {Gray, Opacity[0.5]},
BoundaryStyle -> None
];

(*Ya equilibrium internally stable*)
plotYaStable =
RegionPlot[{
  (stabcondA /. Rf -> 0 /. Rm -> 0 /. equilA0 /. params) ||
  (stabcondB /. Rf -> 0 /. Rm -> 0 /. equilB0 /. params)
},
{t, -1, 1}, {s, -1, 1},
PlotStyle -> White,
BoundaryStyle -> {Black, Thick}
];

(*Ya equilibrium internally unstable*)
plotYaUnstable =
RegionPlot[{
  1 < 2
},
{αmd, -1, 1}, {s, -1, 1},
PlotStyle -> None,
BoundaryStyle -> None,
Mesh -> 50,
MeshFunctions -> {#1 - #2 &},
MeshStyle -> Black
];

```

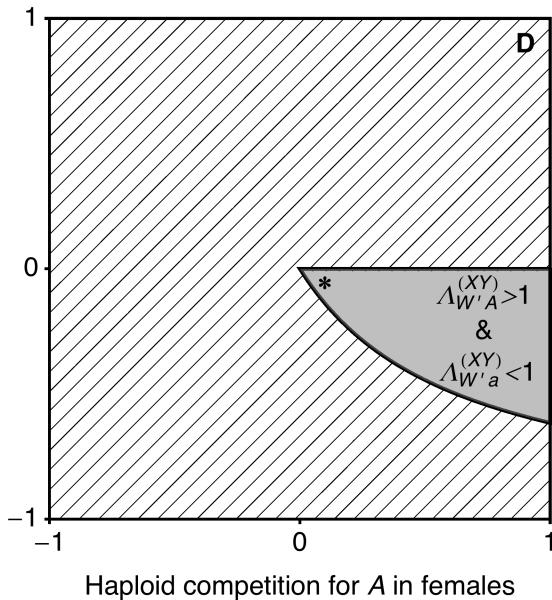
```

plotD =
Show[
  plotYaUnstable,
  plotYaStable,
  plotWAinvA,
  plotWainvA,
  plotWAinvB,
  plotWainvB,

  PlotRange → {{-1, 1}, {-1, 1}},
  ImageSize → {xsize, xsize},
  PlotRangePadding → 0,
  FrameTicks → {Table[{x, x, ticksize}, {x, -1, 1, 1}],
    Table[{y, y, ticksize}, {y, -1, 1, 1}], None, None},
  FrameTicksStyle → {{Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],
    Directive[Black, Thickness[lwd], FontColor → White]}},
  FrameStyle → {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},
  FrameLabel → {"Haploid competition for A in females", ""},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImagePadding → Pad,
  Epilog → {
    Text[Style["D", 14, Bold], Scaled@{0.95, 0.95}],
    Text[Style[" $\Lambda_{W'A}^{(xy)} > 1$ ", 14],
       $\Lambda_{W'A}^{(xy)} < 1$ , 14], {0.75, -0.25}],
    Text[Style["*", 16, Bold], {0.1, -0.05}]
  },
  PlotLabel → Style["haploid competition in females", 16, Black, Bold],
  PlotRangeClipping → False
]

```

haploid competition in females

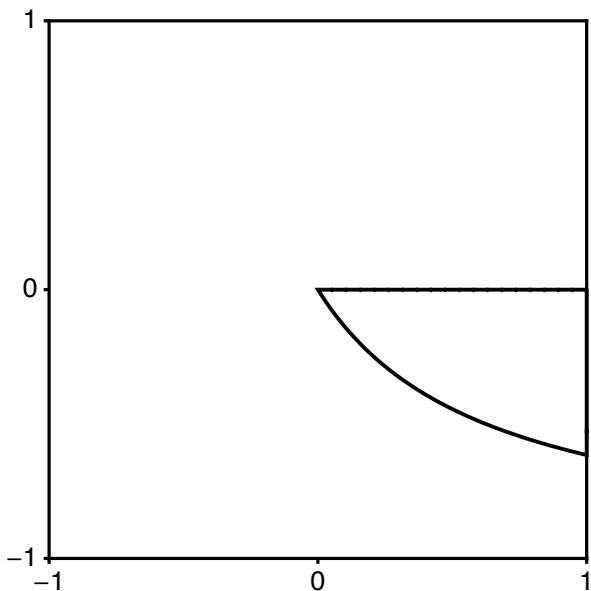


Notice that this is consistent with the solution from the full characteristic polynomial (but here we don't know which eigenvalue belongs to which haplotype)

```
(*neo-W invades XY from equilA*)
plotWinvA =
RegionPlot[{
  (validcondA /. params) && (*valid*)
  (stabcondA /. Rf → 0 /. Rm → 0 /. equilA0 /. params) && (*internally stable*)
  1 < λWsolA(*invasion*)
},
{t, -1, 1}, {s, -1, 1},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];

(*neo-W invades XY from equilB*)
plotWinvB =
RegionPlot[{
  (stabcondB /. Rf → 0 /. Rm → 0 /. equilB0 /. params) && (*internally stable*)
  1 < λWsolB(*invasion*)
},
{t, -1, 1}, {s, -1, 1},
PlotStyle → {Gray, Opacity[0.5]},
BoundaryStyle → None
];
```

```
Show[  
  plotWinvA,  
  plotWinvB,  
  plotYaStable,  
  
  PlotRange -> {{-1, 1}, {-1, 1}},  
  ImageSize -> {xsize, xsize},  
  PlotRangePadding -> 0,  
  FrameTicks -> {Table[{x, x, ticksize}, {x, -1, 1, 1}],  
    Table[{y, y, ticksize}, {y, -1, 1, 1}], None, None},  
  FrameTicksStyle -> {{Directive[Black, Thickness[lwd]],  
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]],  
    Directive[Black, Thickness[lwd], FontColor -> White]}},  
  FrameStyle -> {{Black, Thickness[lwd]}, {Black, Thickness[lwd]},  
    {Black, Thickness[lwd]}, {Black, Thickness[lwd]}},  
  BaseStyle -> {FontFamily -> "Helvetica", FontSize -> 14},  
  ImagePadding -> Pad,  
  PlotRangeClipping -> False  
]
```



Panel E - temporal invasion dynamic with meiotic drive in males

Parameters

```

params = {
  wAm → 1, wam → 1, wAf → 1, waf → 1,
  αm → (1 + αmd) / 2, αf → 1 / 2,
  Maa → 1, Faa → 1,
  MAa → 1 + (3 / 4) s, FAa → 1 + (3 / 4) s,
  MAA → 1 + s, FAA → 1 + s
};

moreparams = {
  s → 1 / 4,
  αmd → -1 / 10
};

tryFAA = FAA /. params /. moreparams;
tryFAa = FAa /. params /. moreparams;
tryFaa = Faa /. params /. moreparams;
tryMAA = MAA /. params /. moreparams;
tryMAa = MAa /. params /. moreparams;
tryMaa = Maa /. params /. moreparams;
trywAf = wAf /. params /. moreparams;
trywaf = waf /. params /. moreparams;
trywAm = wAm /. params /. moreparams;
trywam = wam /. params /. moreparams;
tryαf = αf /. params /. moreparams;
tryαm = αm /. params /. moreparams;

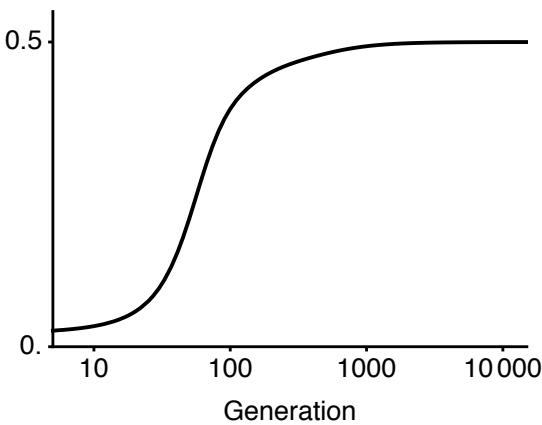
```

Plots

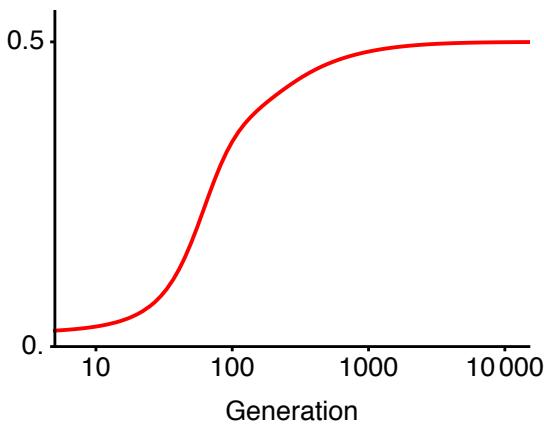
```

tryR = 0.001;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Black =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Black = ListLogLinearPlot[MODtab1Black, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.55}},
PlotStyle → {Black, Thick}, loglinearplotINVoptions]

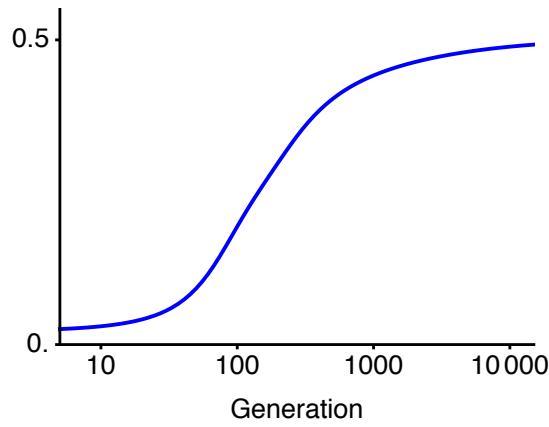
```



```
tryR = 0.02;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtablRed =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Red = ListLogLinearPlot[MODtablRed, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.55}},
PlotStyle → {Red, Thick}, loglinearplotINVoptions]
```



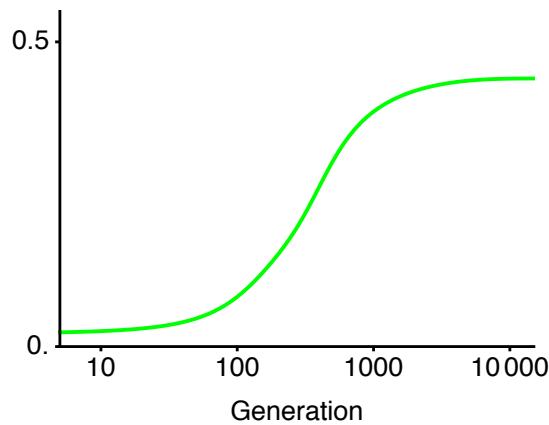
```
tryR = 0.1;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Blue =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Blue = ListLogLinearPlot[MODtab1Blue, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.55}},
PlotStyle → {Blue, Thick}, loglinearplotINVoptions]
```



```

tryR = 0.5;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Green =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Green = ListLogLinearPlot[MODtab1Green, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.55}},
PlotStyle → {Green, Thick}, loglinearplotINVoptions]

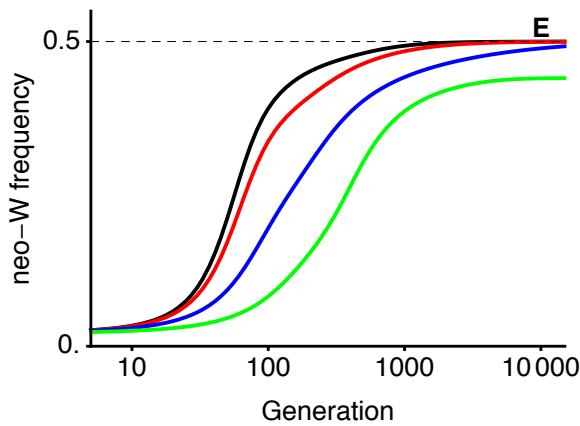
```



```

plotE = Show[MOD1Black, MOD1Red, MOD1Blue, MOD1Green,
  LogLinearPlot[0.5, {x, 5, endtime}, PlotStyle → {Black, Dashed}],
  Epilog → {
    Rotate[Text[Style["neo-W frequency", 14], Scaled@ylabpos], 90 Degree],
    Text[Style["E", 14, Bold], Scaled@{0.95, 0.95}]
  },
  PlotRangeClipping → False]

```



Panel F - temporal invasion dynamic with haploid competition in males

Parameters

```

params = {
  wAm → 1 + t, wam → 1, wAf → 1, waf → 1,
  αm → 1 / 2, αf → 1 / 2,
  Maa → 1, Faa → 1,
  MAa → 1 + (3 / 4) s, FAa → 1 + (3 / 4) s,
  MAA → 1 + s, FAA → 1 + s
};

moreparams = {
  s → 1 / 10,
  t → -1 / 10
};

tryFAA = FAA /. params /. moreparams;
tryFAa = FAa /. params /. moreparams;
tryFaa = Faa /. params /. moreparams;
tryMAA = MAA /. params /. moreparams;
tryMAa = MAa /. params /. moreparams;
tryMaa = Maa /. params /. moreparams;
trywAf = wAf /. params /. moreparams;
trywaf = waf /. params /. moreparams;
trywAm = wAm /. params /. moreparams;
trywam = wam /. params /. moreparams;
tryαf = αf /. params /. moreparams;
tryαm = αm /. params /. moreparams;

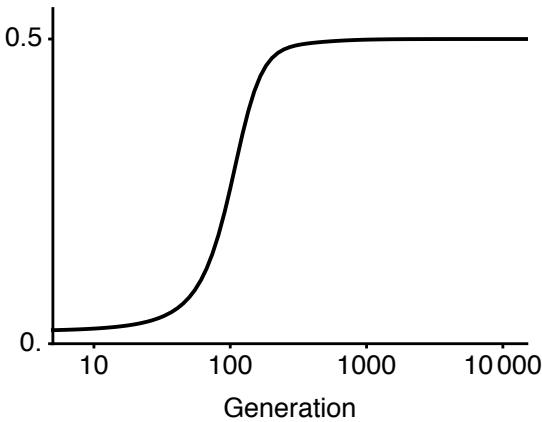
```

Plots

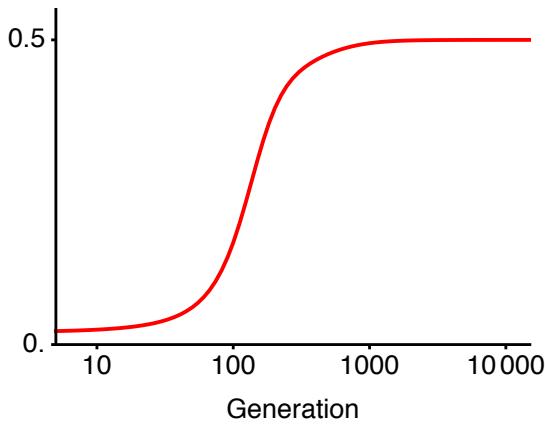
```

tryR = 0.001;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
          trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
          tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Black =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Black = ListLogLinearPlot[MODtab1Black, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.55}},
PlotStyle → {Black, Thick}, loglinearplotINVoptions]

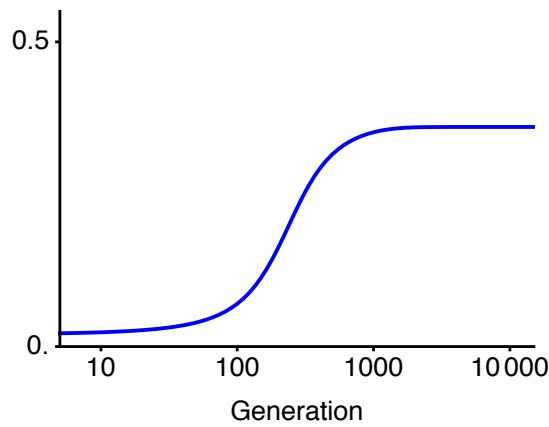
```



```
tryR = 0.02;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Red =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Red = ListLogLinearPlot[MODtab1Red, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.55}},
PlotStyle → {Red, Thick}, loglinearplotINVoptions]
```



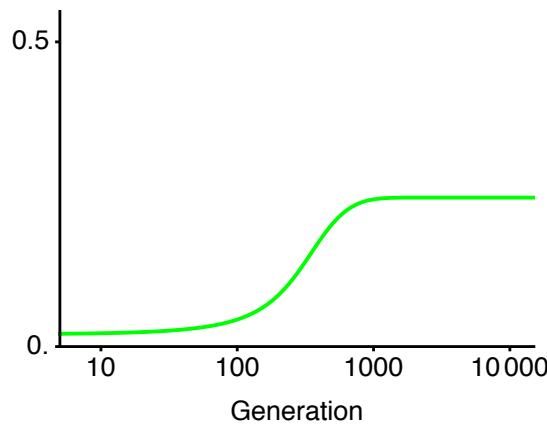
```
tryR = 0.1;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Blue =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Blue = ListLogLinearPlot[MODtab1Blue, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.55}},
PlotStyle → {Blue, Thick}, loglinearplotINVoptions]
```



```

tryR = 0.5;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Green =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Green = ListLogLinearPlot[MODtab1Green, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.55}},
PlotStyle → {Green, Thick}, loglinearplotINVoptions]

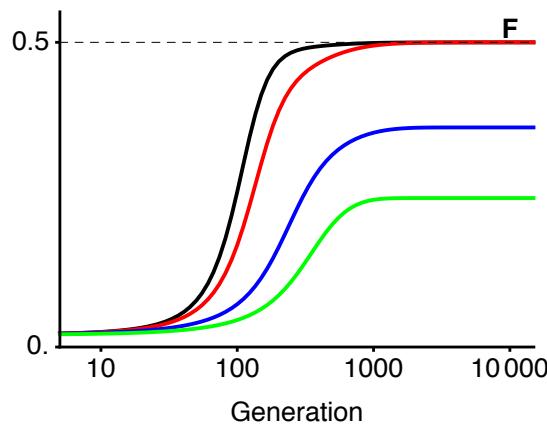
```



```

plotF = Show[MOD1Black, MOD1Red, MOD1Blue, MOD1Green,
  LogLinearPlot[0.5, {x, 5, endtime}, PlotStyle → {Black, Dashed}],
  Epilog → {
    Text[Style["F", 14, Bold], Scaled@{0.95, 0.95}]
  },
  PlotRangeClipping → False]

```



Panel G - temporal invasion dynamic with meiotic drive in females

Parameters

```

params = {
  wAm → 1, wam → 1, wAf → 1, waf → 1,
  αm → 1 / 2, αf → (1 + αmd) / 2,
  Maa → 1, Faa → 1,
  MAa → 1 + (3 / 4) s, FAa → 1 + (3 / 4) s,
  MAA → 1 + s, FAA → 1 + s
};

moreparams = {
  s → -1 / 10,
  αmd → 1 / 5
};

tryFAA = FAA /. params /. moreparams;
tryFAa = FAa /. params /. moreparams;
tryFaa = Faa /. params /. moreparams;
tryMAA = MAA /. params /. moreparams;
tryMAa = MAa /. params /. moreparams;
tryMaa = Maa /. params /. moreparams;
trywAf = wAf /. params /. moreparams;
trywaf = waf /. params /. moreparams;
trywAm = wAm /. params /. moreparams;
trywam = wam /. params /. moreparams;
tryαf = αf /. params /. moreparams;
tryαm = αm /. params /. moreparams;

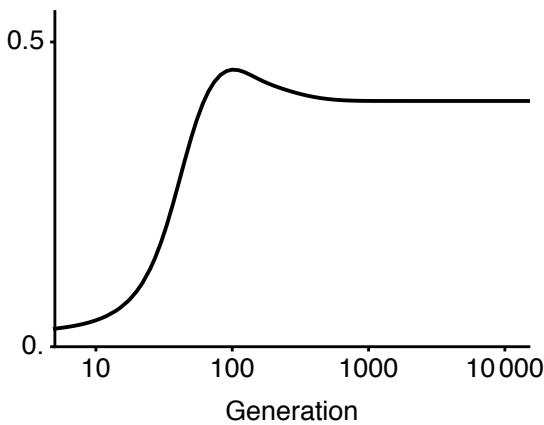
```

Plots

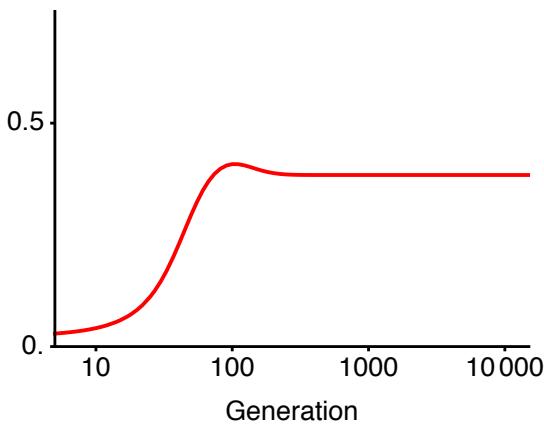
```

tryR = 0.001;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Black =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Black = ListLogLinearPlot[MODtab1Black, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.55}},
PlotStyle → {Black, Thick}, loglinearplotINVoptions]

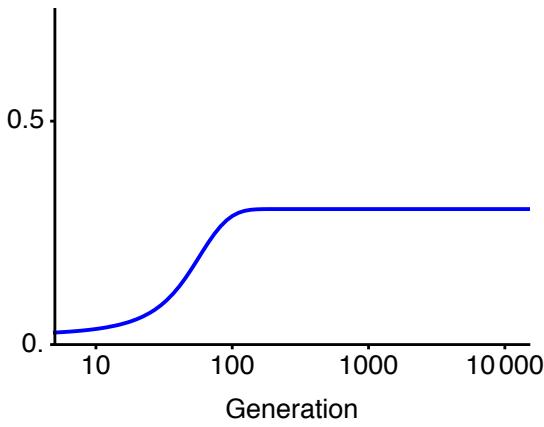
```



```
tryR = 0.02;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Red =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Red = ListLogLinearPlot[MODtab1Red, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.75}},
PlotStyle → {Red, Thick}, loglinearplotINVOptions]
```



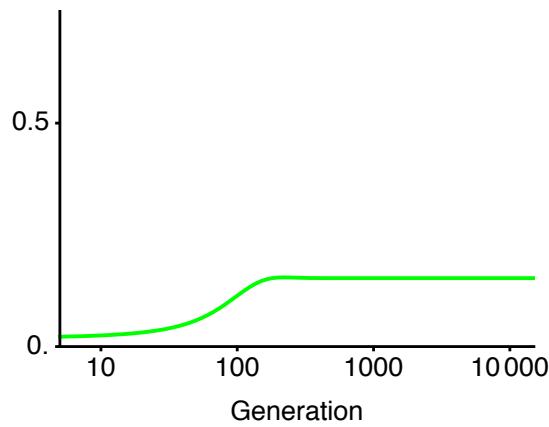
```
tryR = 0.1;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Blue =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Blue = ListLogLinearPlot[MODtab1Blue, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.75}},
PlotStyle → {Blue, Thick}, loglinearplotINVoptions]
```



```

tryR = 0.5;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Green =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Green = ListLogLinearPlot[MODtab1Green, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.75}},
PlotStyle → {Green, Thick}, loglinearplotINVoptions]

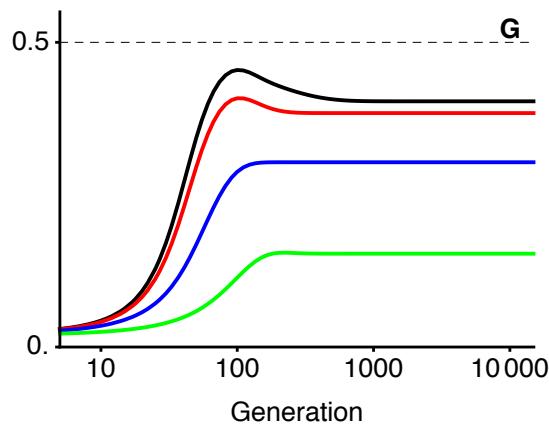
```



```

plotG = Show[MOD1Black, MOD1Red, MOD1Blue, MOD1Green,
  LogLinearPlot[0.5, {x, 5, endtime}, PlotStyle → {Black, Dashed}],
  Epilog → {
    Text[Style["G", 14, Bold], Scaled@{0.95, 0.95}]
  },
  PlotRangeClipping → False]

```



Panel H - temporal invasion dynamic with haploid competition in females

Parameters

```

params = {
  wAm → 1, wam → 1, wAf → 1 + t, waf → 1,
  cm → 1 / 2, αf → 1 / 2,
  Maa → 1, Faa → 1,
  MAa → 1 + (3 / 4) s, FAa → 1 + (3 / 4) s,
  MAA → 1 + s, FAA → 1 + s
};

moreparams = {
  s → -1 / 20,
  t → 1 / 10
};

tryFAA = FAA /. params /. moreparams;
tryFAa = FAa /. params /. moreparams;
tryFaa = Faa /. params /. moreparams;
tryMAA = MAA /. params /. moreparams;
tryMAa = MAa /. params /. moreparams;
tryMaa = Maa /. params /. moreparams;
trywAf = wAf /. params /. moreparams;
trywaf = waf /. params /. moreparams;
trywAm = wAm /. params /. moreparams;
trywam = wam /. params /. moreparams;
tryαf = αf /. params /. moreparams;
trycm = cm /. params /. moreparams;

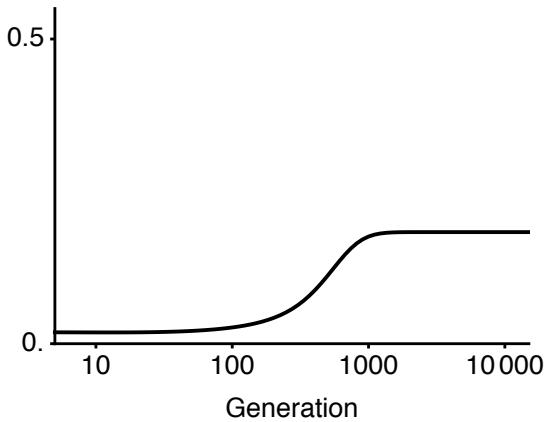
```

Plots

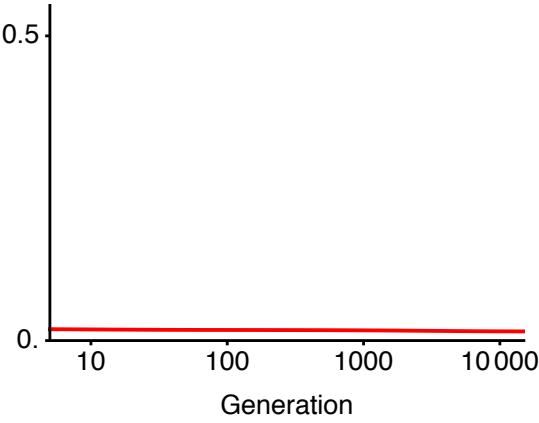
```

tryR = 0.001;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
          trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
          tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Black =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Black = ListLogLinearPlot[MODtab1Black, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.55}},
PlotStyle → {Black, Thick}, loglinearplotINVoptions]

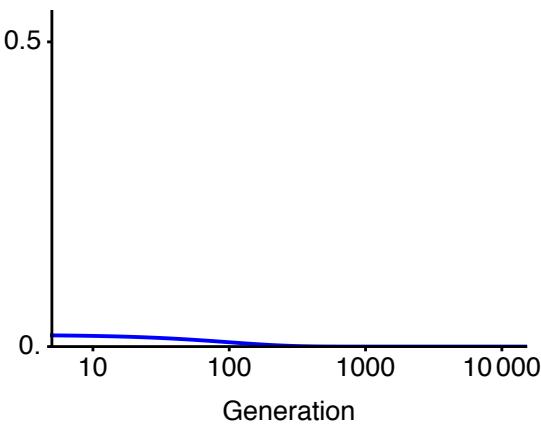
```



```
tryR = 0.02;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Red =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Red = ListLogLinearPlot[MODtab1Red, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.55}},
PlotStyle → {Red, Thick}, loglinearplotINVoptions]
```



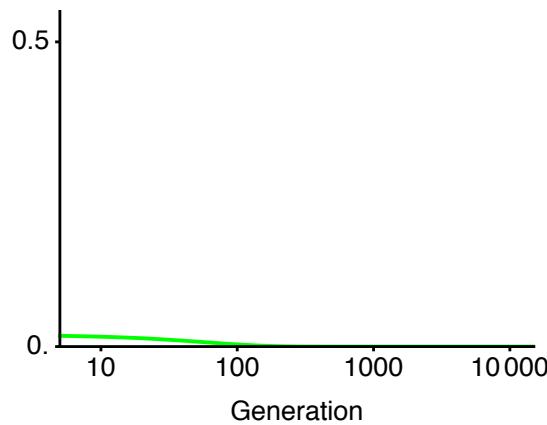
```
tryR = 0.1;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Blue =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Blue = ListLogLinearPlot[MODtab1Blue, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.55}},
PlotStyle → {Blue, Thick}, loglinearplotINVoptions]
```



```

tryR = 0.5;
param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
run[0] = generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
         tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]
]
MODtab1Green =
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].modvec},
{exptime, Log[startplot], Log[endtime], 0.1}];
MOD1Green = ListLogLinearPlot[MODtab1Green, Joined → True,
PlotRange → {{startplot, endtime}, {0, 0.55}},
PlotStyle → {Green, Thick}, loglinearplotINVoptions]

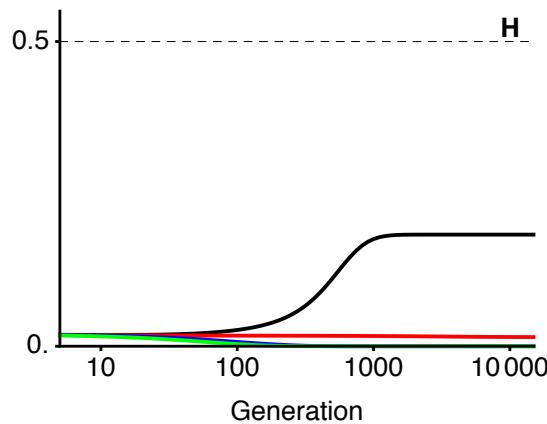
```



```

plotH = Show[MOD1Black, MOD1Red, MOD1Blue, MOD1Green,
  LogLinearPlot[0.5, {x, 5, endtime}, PlotStyle → {Black, Dashed}],
  Epilog → {
    Rotate[Text[Style["", 14], Scaled@ylabpos], 90 Degree],
    Text[Style["H", 14, Bold], Scaled@{0.95, 0.95}]
  },
  PlotRangeClipping → False]

```



All panels

```

GraphicsGrid[{
  {plotA, plotB, plotC, plotD},
  {plotE, plotF, plotG, plotH}
],
Spacings -> {-25, -50}

Export[plotdir <> "All_plot_combined_PloidAntag.eps", % // rasterTrick];

```

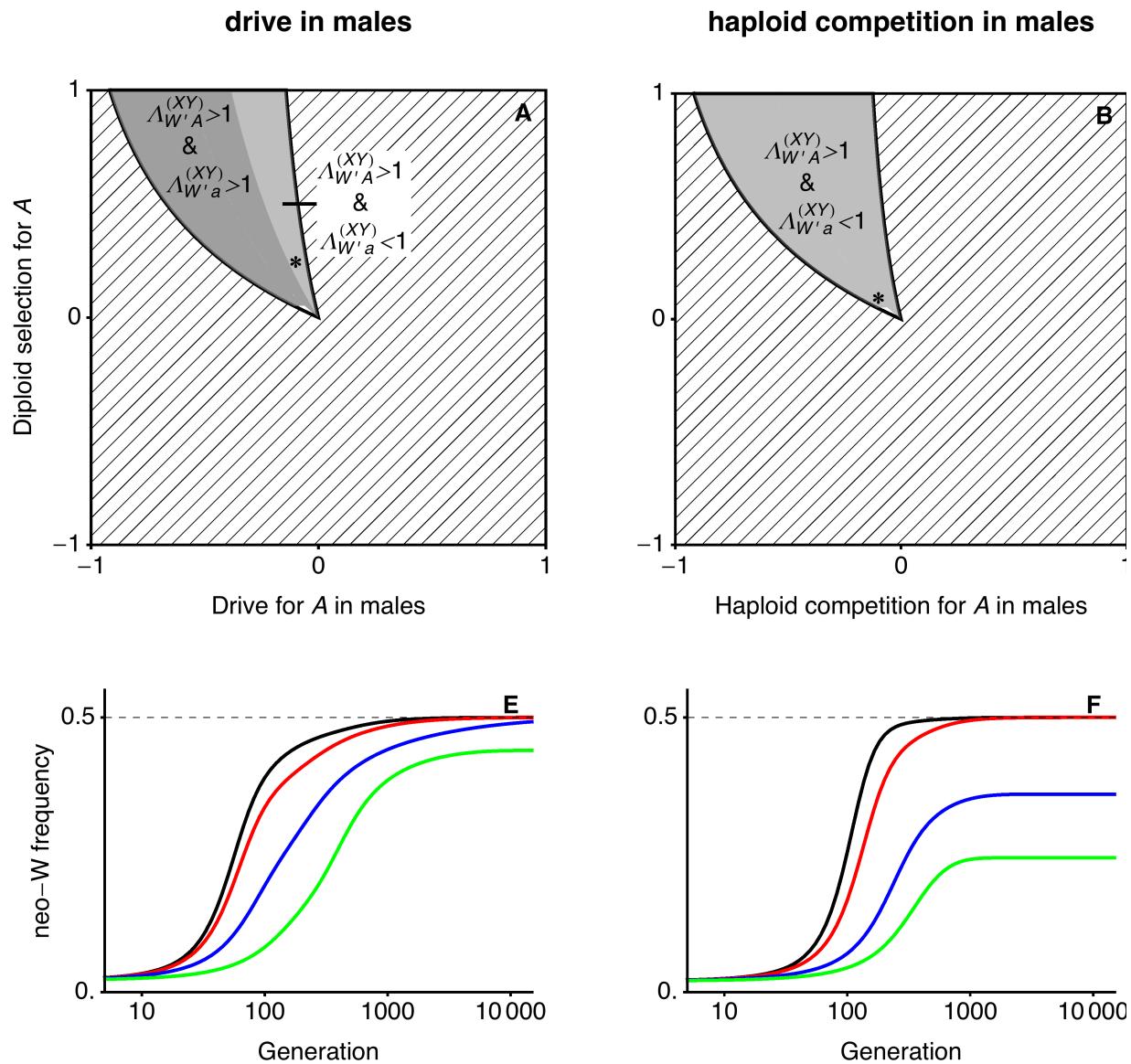


Figure S.9 - polymorphic sex determination can be stable

extra params

```

startplot = 5;
endtime = 5000;
trypm = 0.01;
tryk = 1;

Pad = {{50, 10}, {40, 30}};

loglinearplotINOptions = {
  Frame → {{True, False}, {True, False}},
  FrameStyle → Directive[Black, Thickness[lwd]],
  FrameTicksStyle → {{Directive[Black, Thickness[lwd], FontColor → Black],
    Directive[Black, Thickness[lwd]]}, {Directive[Black, Thickness[lwd]], Directive[Black, Thickness[lwd]]}}},
  FrameTicks → {{{5, 5, {0, 0.01}}, {10, 10, {0, 0.01}}, {50, 50, {0, 0.01}}, {100, 100, {0, 0.01}}, (*{500,500,{0,0.01}},*){1000, 1000, {0, 0.01}}(*,{5000,5000,{0,0.01}}*)}, Table[{y, y, ticksize}, {y, 0, 1, 0.5}]},
  BaseStyle → {FontFamily → "Helvetica", FontSize → 14},
  ImageSize → {xsize, xsize aspectratio},
  AspectRatio → aspectratio,
  Axes → True,
  FrameLabel → {Style["Generation", FontSize → 14], },
  ImagePadding → Pad
};

```

Panel A - ploidy-antagonism with male drive (parameters from fig 4B green)

Parameters

```

tryFAA = 1 + (1 / 5);
tryFAa = 1 + (7 / 10) (1 / 5);
tryFaa = 1;
tryMAA = 1 + (1 / 5);
tryMAa = 1 + (7 / 10) (1 / 5);
tryMaa = 1;
trywAf = 1;
trywaf = 1;
trywAm = 1;
trywam = 1;
tryaf = 1 / 2;
tryam = 1 / 2 - (1 / 10);
tryr = 1 / 50;
tryR = 1 / 2;

param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
  trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};

```

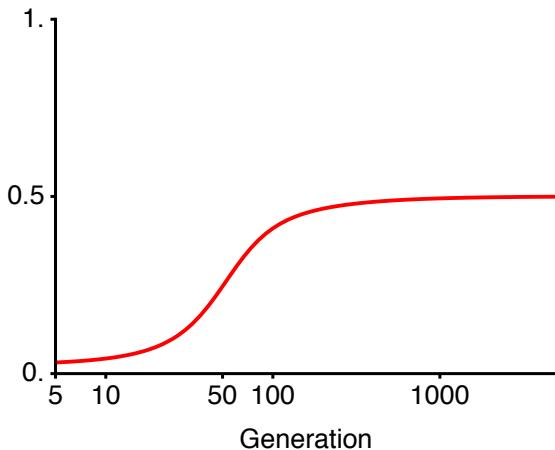
Simulation

```
sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
  tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]]
{0.65153, 0.615375, 0.0840379, 0.556532}

run[0] =
  generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa,
    trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]]
```

Plot neo-W frequency in females

```
Wfemvec = {0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0};
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Wfemvec},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Wfem = ListLogLinearPlot[%, Joined → True, PlotRange → {{startplot, endtime}, {0, 1}},
  PlotStyle → {Red, Thick}, loglinearplotINVoptions]
```

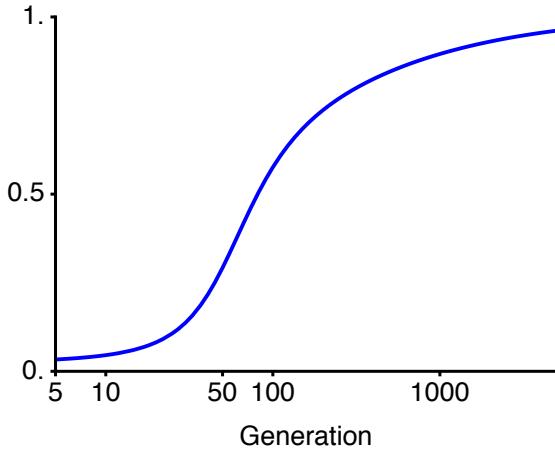


Plot Y frequency in females

```

Yfemvec = {0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0};
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Yfemvec},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Yfem = ListLogLinearPlot[% , Joined → True, PlotRange → {{startplot, endtime}, {0, 1}},
  PlotStyle → {Blue, Thick}, loglinearplotINVoptions]

```

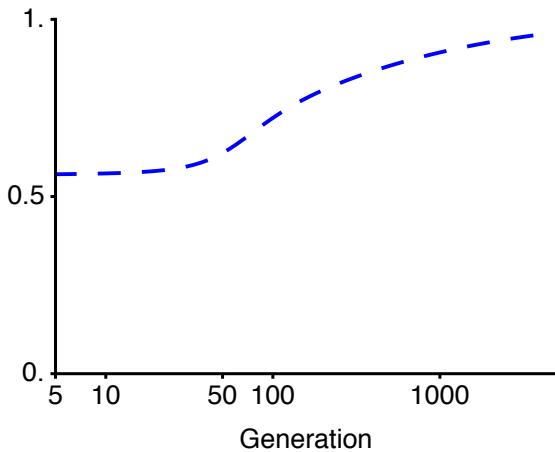


Plot Y frequency in males

```

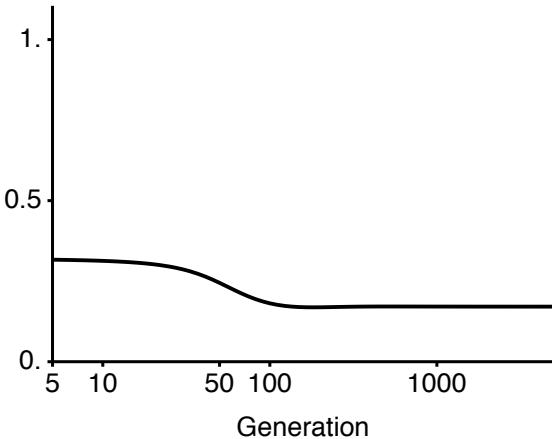
Ymalvec = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1};
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Ymalvec},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Ymal = ListLogLinearPlot[% , Joined → True, PlotRange → {{startplot, endtime}, {0, 1}},
  PlotStyle → {Blue, Thick, Dashing[{Large}]}, loglinearplotINVoptions]

```



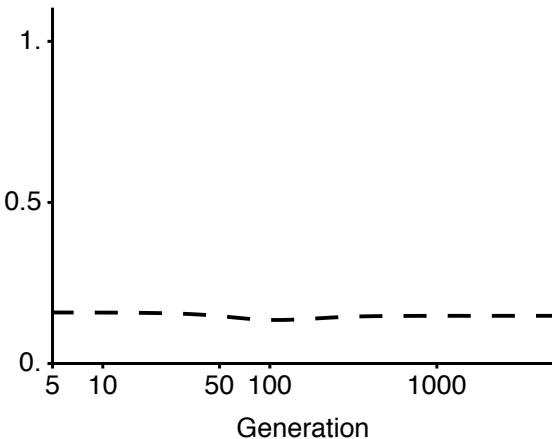
Plot A frequency in females

```
Afemvec = {1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0};
Table[
  {Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Afemvec / 2},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Afem = ListLogLinearPlot[%, Joined → True,
  PlotRange → {{startplot, endtime}, {0, 1.1}},
  PlotStyle → {Black, Thick}, loglinearplotINVoptions]
```



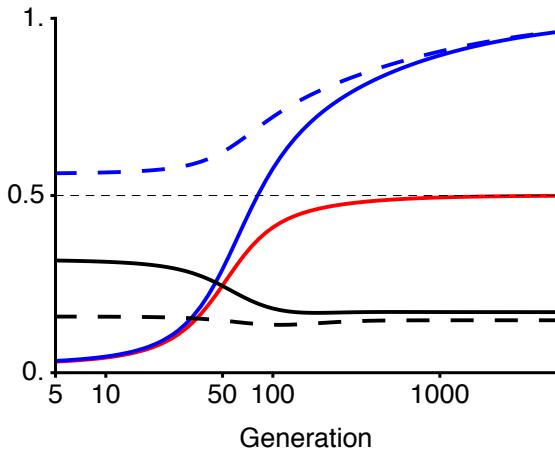
Plot A frequency in males

```
Amalvec = {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0};
Table[
  {Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Amalvec / 2},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Amal = ListLogLinearPlot[%, Joined → True,
  PlotRange → {{startplot, endtime}, {0, 1.1}},
  PlotStyle → {Black, Thick, Dashing[Large]}, loglinearplotINVoptions]
```



Plot all

```
plotA = Show[Wfem, Yfem, Ymal, Afem, Amal,
  LogLinearPlot[0.5, {x, 5, endtime}, PlotStyle -> {Black, Dashed}]]
```



Panel B - overdominance in both sexes, no haploid selection (parameters from fig S.2C green)

Parameters

```
tryFAA = 0.6;
tryFAa = 1;
tryFaa = 0.6;
tryMAA = 0.7;
tryMAa = 1;
tryMaa = 0.5;
trywAf = 1;
trywaf = 1;
trywAm = 1;
trywam = 1;
tryaf = 1/2;
tryam = 1/2;
tryr = 0.005;
tryR = 1/2;

param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, trywAf, trywaf, trywAm,
  trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};
```

Simulation

```
sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
  tryMAa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[2]]
{0.720702, 0.82555, 0.0461837, 0.5}
```

```

run[0] =
  generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMaa, tryMaa,
    trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[2]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]]

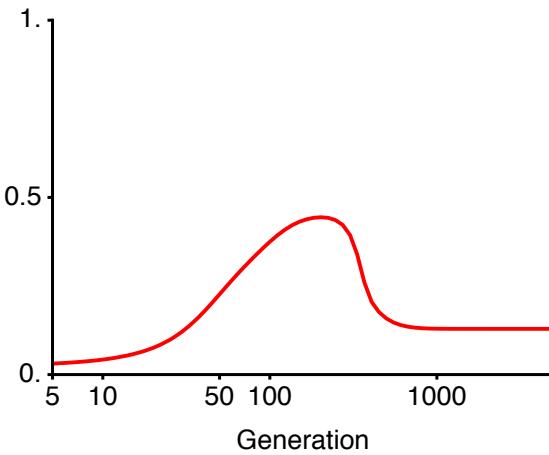
```

Plot neo-W frequency in females

```

Wfemvec = {0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0};
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Wfemvec},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Wfem = ListLogLinearPlot[%, Joined → True, PlotRange → {{startplot, endtime}, {0, 1}},
  PlotStyle → {Red, Thick}, loglinearplotINVoptions]

```

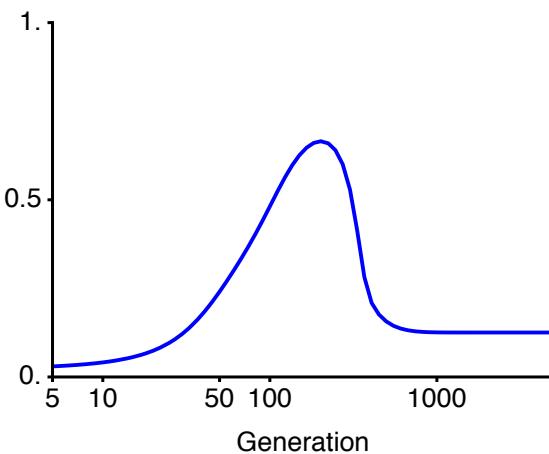


Plot Y frequency in females

```

Yfemvec = {0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0};
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Yfemvec},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Yfem = ListLogLinearPlot[%, Joined → True, PlotRange → {{startplot, endtime}, {0, 1}},
  PlotStyle → {Blue, Thick}, loglinearplotINVoptions]

```

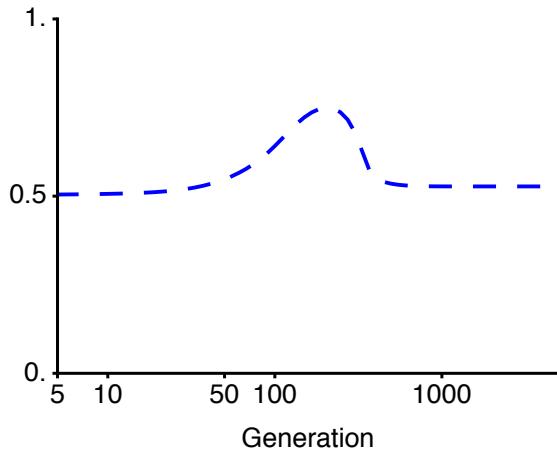


Plot Y frequency in males

```

Ymalvec = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1};
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Ymalvec},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Ymal = ListLogLinearPlot[%, Joined → True, PlotRange → {{startplot, endtime}, {0, 1}},
  PlotStyle → {Blue, Thick, Dashing[Large]}, loglinearplotINVoptions]

```

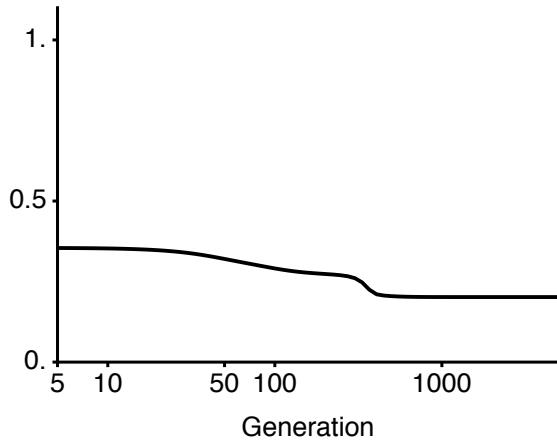


Plot A frequency in females

```

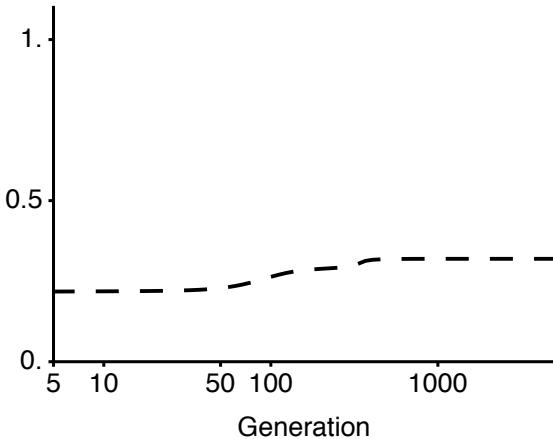
Afemvec = {1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0};
Table[
  {Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Afemvec / 2},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Afem = ListLogLinearPlot[%, Joined → True,
  PlotRange → {{startplot, endtime}, {0, 1.1}},
  PlotStyle → {Black, Thick}, loglinearplotINVoptions]

```



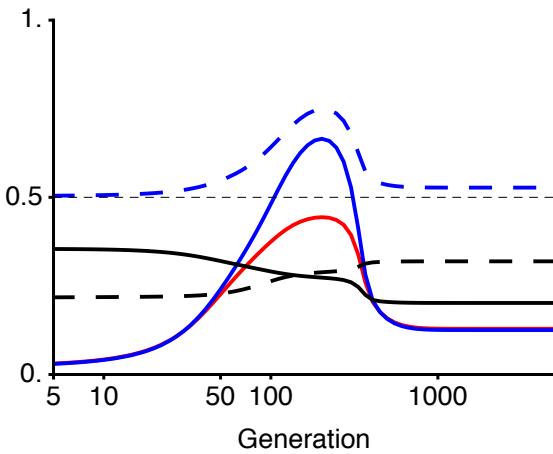
Plot A frequency in males

```
Amalvec = {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0};
Table[
  {Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Amalvec / 2},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Amal = ListLogLinearPlot[%, Joined → True,
  PlotRange → {{startplot, endtime}, {0, 1.1}},
  PlotStyle → {Black, Thick, Dashing[Large]}, loglinearplotINVoptions]
```



Plot all

```
plotB = Show[Wfem, Yfem, Ymal, Afem, Amal,
  LogLinearPlot[0.5, {x, 5, endtime}, PlotStyle → {Black, Dashed}]]
```



Panel C - sex-antagonism with male drive (parameters from fig S.3C green)

Parameters

```

tryFAA = 1.05;
tryFAa = 1;
tryFaa = 0.85;
tryMAA = 0.85;
tryMAa = 1;
tryMaa = 1.05;
trywAf = 1;
trywaf = 1;
trywAm = 1;
trywam = 1;
tryaf = 1 / 2;
tryam = 1 / 2 - 8 / 100;
tryr = 0.005;
tryR = 1 / 2;

param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};

```

Simulation

```

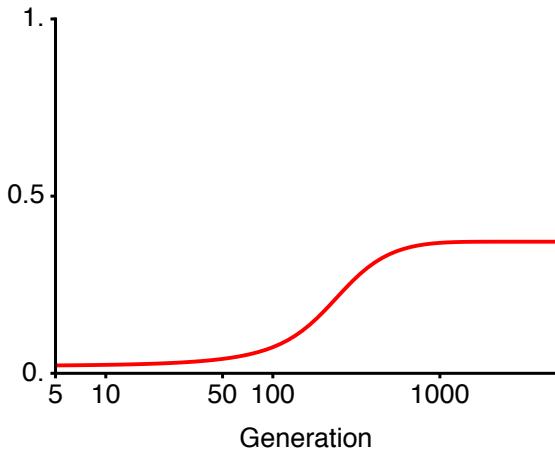
sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]]
{0.418811, 0.363484, 0.00693408, 0.531719}

run[0] =
  generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa,
trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]]

```

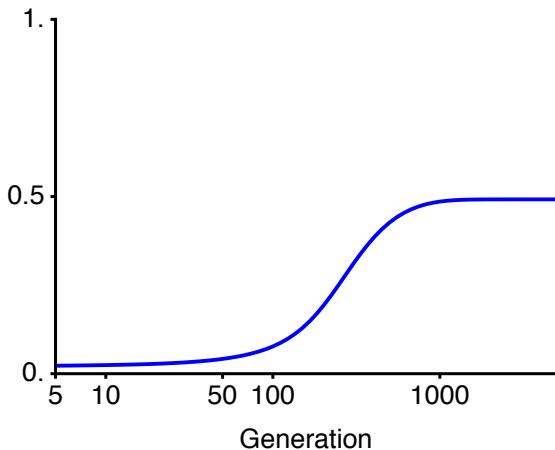
Plot neo-W frequency in females

```
Wfemvec = {0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0};
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Wfemvec},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Wfem = ListLogLinearPlot[% , Joined → True, PlotRange → {{startplot, endtime}, {0, 1}},
  PlotStyle → {Red, Thick}, loglinearplotINVoptions]
```



Plot Y frequency in females

```
Yfemvec = {0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0};
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Yfemvec},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Yfem = ListLogLinearPlot[% , Joined → True, PlotRange → {{startplot, endtime}, {0, 1}},
  PlotStyle → {Blue, Thick}, loglinearplotINVoptions]
```

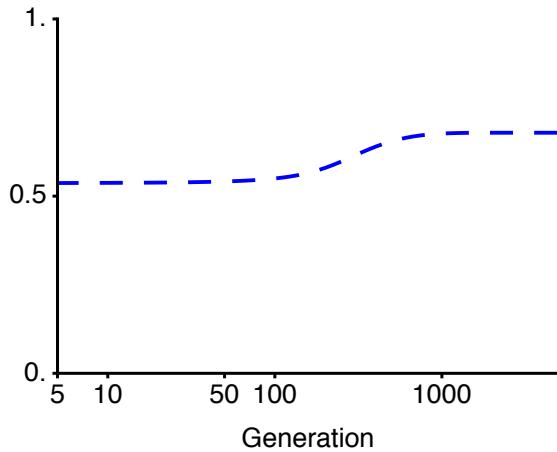


Plot Y frequency in males

```

Ymalvec = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1};
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Ymalvec},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Ymal = ListLogLinearPlot[% , Joined → True, PlotRange → {{startplot, endtime}, {0, 1}},
  PlotStyle → {Blue, Thick, Dashing[Large]}, loglinearplotINVoptions]

```

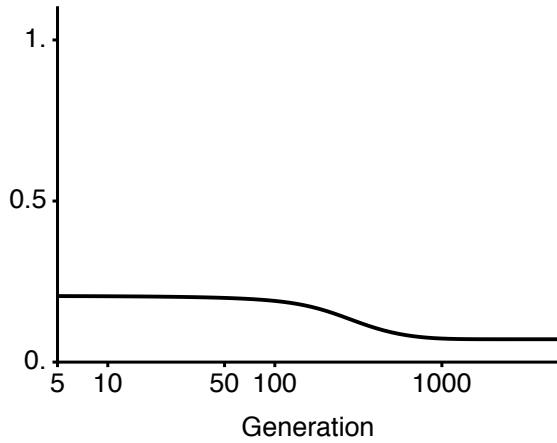


Plot A frequency in females

```

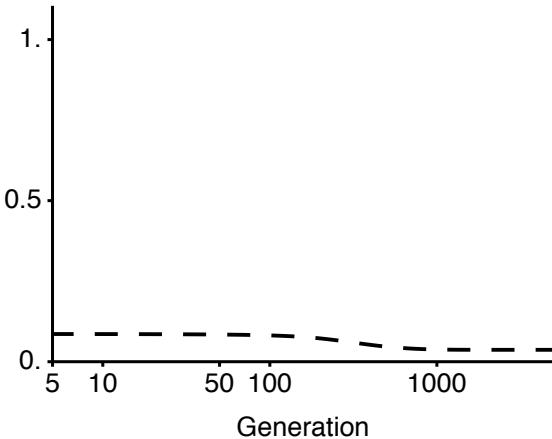
Afemvec = {1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0};
Table[
  {Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Afemvec / 2},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Afem = ListLogLinearPlot[% , Joined → True,
  PlotRange → {{startplot, endtime}, {0, 1.1}},
  PlotStyle → {Black, Thick}, loglinearplotINVoptions]

```



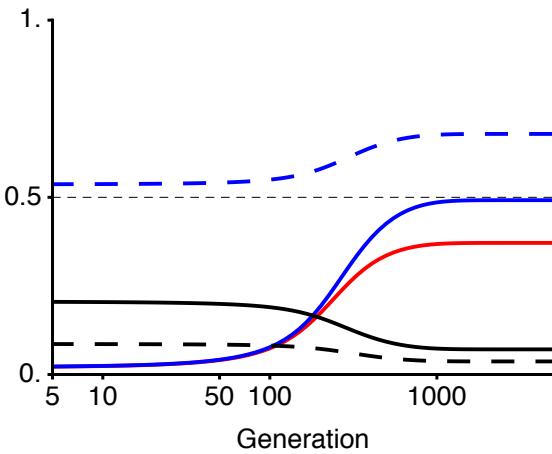
Plot A frequency in males

```
Amalvec = {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0};
Table[
  {Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Amalvec / 2},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Amal = ListLogLinearPlot[%, Joined → True,
  PlotRange → {{startplot, endtime}, {0, 1.1}},
  PlotStyle → {Black, Thick, Dashing[Large]}, loglinearplotINVoptions]
```



Plot all

```
plotC = Show[Wfem, Yfem, Ymal, Afem, Amal,
  LogLinearPlot[0.5, {x, 5, endtime}, PlotStyle → {Black, Dashed}]]
```



Panel D - ploidy-antagonism with pollen competition (parameters from fig S.8F green)

Parameters

```

tryFAA = 1 + (1 / 10);
tryFAa = 1 + (3 / 4) (1 / 10);
tryFaa = 1;
tryMAA = 1 + (1 / 10);
tryMAa = 1 + (3 / 4) (1 / 10);
tryMaa = 1;
trywAf = 1;
trywaf = 1;
trywAm = 1 - 1 / 10;
trywam = 1;
tryaf = 1 / 2;
tryam = 1 / 2;
tryr = 1 / 200;
tryR = 1 / 2;

param = {tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa, trywAf, trywaf, trywAm,
         trywam, tryaf, tryam, tryr, tryR, tryr (1 - tryR) + tryR (1 - tryr), tryk};

```

Simulation

```

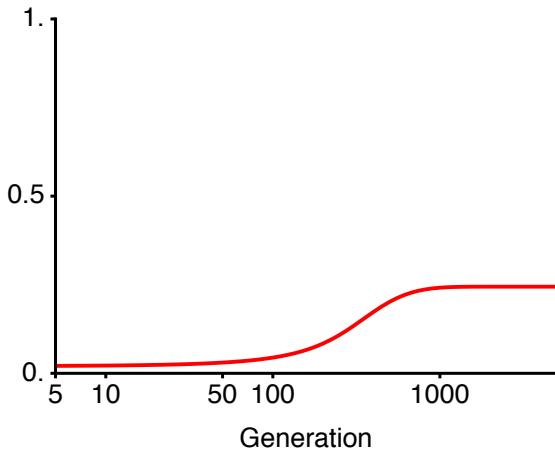
sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa,
        tryMaa, trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]]
{0.819467, 0.82566, 0.0576155, 0.5}

run[0] =
  generation[param, startgen[sieveXY[tryFAA, tryFAa, tryFaa, tryMAA, tryMAa, tryMaa,
        trywAf, trywaf, trywAm, trywam, tryaf, tryam, tryr][[1]], trypm]];
For[time = 1, time < endtime, time++, run[time] = generation[param, run[time - 1]]]

```

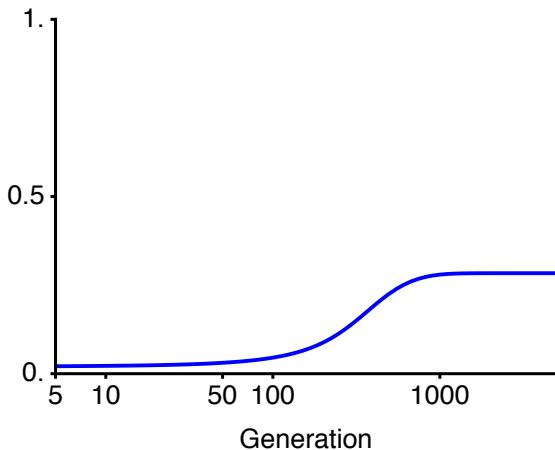
Plot neo-W frequency in females

```
Wfemvec = {0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0};
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Wfemvec},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Wfem = ListLogLinearPlot[% , Joined → True, PlotRange → {{startplot, endtime}, {0, 1}},
  PlotStyle → {Red, Thick}, loglinearplotINVoptions]
```



Plot Y frequency in females

```
Yfemvec = {0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0};
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Yfemvec},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Yfem = ListLogLinearPlot[% , Joined → True, PlotRange → {{startplot, endtime}, {0, 1}},
  PlotStyle → {Blue, Thick}, loglinearplotINVoptions]
```

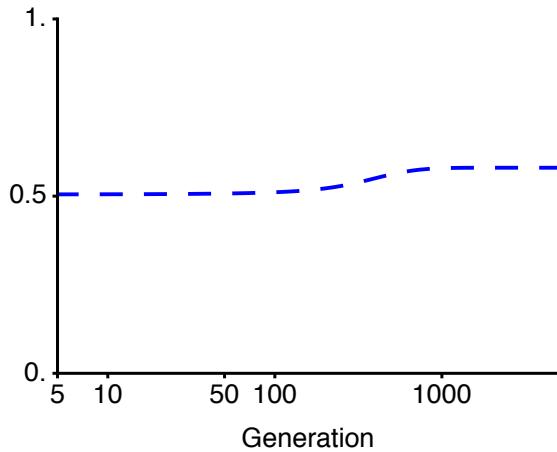


Plot Y frequency in males

```

Ymalvec = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1};
Table[{Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Ymalvec},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Ymal = ListLogLinearPlot[% , Joined → True, PlotRange → {{startplot, endtime}, {0, 1}},
  PlotStyle → {Blue, Thick, Dashing[Large]}, loglinearplotINVoptions]

```

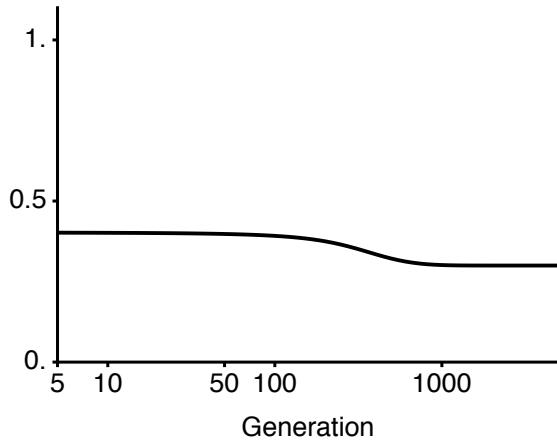


Plot A frequency in females

```

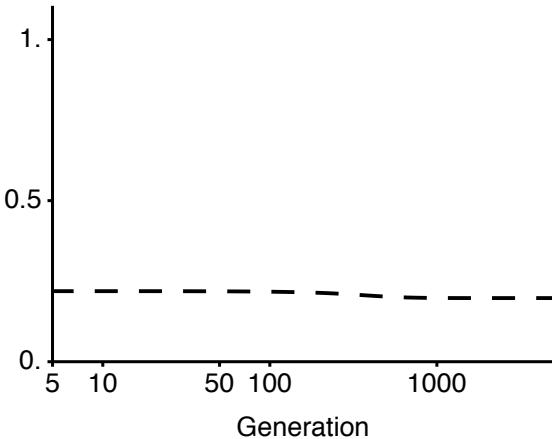
Afemvec = {1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0};
Table[
  {Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Afemvec / 2},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Afem = ListLogLinearPlot[% , Joined → True,
  PlotRange → {{startplot, endtime}, {0, 1.1}},
  PlotStyle → {Black, Thick}, loglinearplotINVoptions]

```



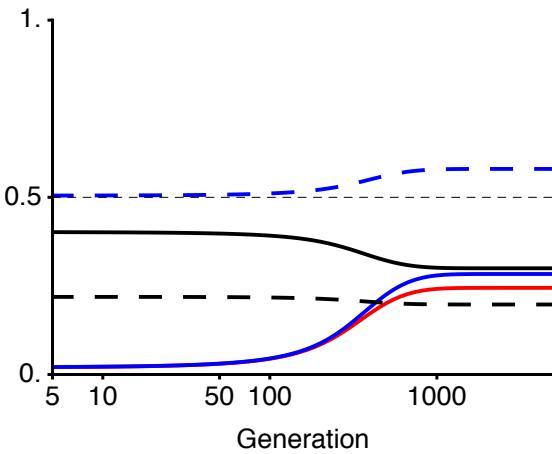
Plot A frequency in males

```
Amalvec = {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0};
Table[
  {Round[Exp[exptime]], generation[param, run[Round[Exp[exptime]]]].Amalvec / 2},
  {exptime, Log[startplot], Log[endtime], 0.1}];
Amal = ListLogLinearPlot[%, Joined → True,
  PlotRange → {{startplot, endtime}, {0, 1.1}},
  PlotStyle → {Black, Thick, Dashing[Large]}, loglinearplotINVoptions]
```



Plot all

```
plotD = Show[Wfem, Yfem, Ymal, Afem, Amal,
  LogLinearPlot[0.5, {x, 5, endtime}, PlotStyle → {Black, Dashed}]]
```



All panels

```

GraphicsRow[
{
Show[
plotA,
Epilog -> {
Rotate[Text[Style["Frequency", 16], Scaled@{-0.15, 0.5}], π/2],
Text[Style["A", 16, Bold], Scaled@{0.05, 0.95}],
Text[Style["ploidy-antagonism", 16], Scaled@{0.5, 0.95}]
},
PlotRangeClipping -> False
],
Show[plotB,
Epilog -> {
Text[Style["B", 16, Bold], Scaled@{0.05, 0.95}],
Text[Style["overdominance", 16], Scaled@{0.5, 0.95}]
}
],
Show[plotC,
Epilog -> {
Text[Style["C", 16, Bold], Scaled@{0.05, 0.95}],
Text[Style["sex-antagonism", 16], Scaled@{0.5, 0.95}]
}
](*,
Show[plotD,
Epilog -> {
Text[Style["D", 16, Bold], Scaled@{0.05, 0.95}]
}
]*)
},
Spacings -> -50
]

Export[plotdir <> "Freq_plot_combined_PloidAntag.eps",
Magnify[%(*//rasterTrick*), 7.5/10]];

```

