

## S1 Appendix

### Recursion equations

In each generation we census the genotype frequencies in male and female gametes/gametophytes (hereafter, gametes) between meiosis (and any meiotic drive) and gametic competition. At this stage we denote the frequencies of X- and Y-bearing gametes from males and females  $x_i^\circ$  and  $y_i^\circ$ . The superscript  $\circ \in \{\delta, \varphi\}$  specifies the sex of the diploid that the gamete came from. The subscript  $i \in \{1, 2, 3, 4\}$  specifies the genotype at the selected locus **A** and at the novel sex-determining locus **M**, where  $1 = AM$ ,  $2 = aM$ ,  $3 = Am$ , and  $4 = am$ . The gamete frequencies from each sex sum to one,  $\sum_i x_i^\circ + y_i^\circ = 1$ .

Competition then occurs among gametes of the same sex (e.g., among eggs and among sperm separately) according to the genotype at the **A** locus ( $w_1^\circ = w_3^\circ = w_A^\circ$ ,  $w_2^\circ = w_4^\circ = w_a^\circ$ , see Table 1). The genotype frequencies after gametic competition are  $x_i^{\circ,s} = w_i^\circ x_i^\circ / \bar{w}_H^\circ$  and  $y_i^{\circ,s} = w_i^\circ y_i^\circ / \bar{w}_H^\circ$ , where  $\bar{w}_H^\circ = \sum_i w_i^\circ x_i^\circ + w_i^\circ y_i^\circ$  is the mean fitness of male ( $\circ = \delta$ ) or female ( $\circ = \varphi$ ) gametes.

Random mating then occurs between gametes to produce diploid zygotes. The frequencies of XX zygotes are then denoted as  $xx_{ij}$ , XY zygotes as  $xy_{ij}$ , and YY zygotes as  $yy_{ij}$ , where **A** and **M** locus genotypes are given by  $i, j \in \{1, 2, 3, 4\}$ , as above. In XY zygotes, the haplotype inherited from an X-bearing gamete is given by  $i$  and the haplotype from a Y-bearing gamete is given by  $j$ . In XX and YY zygotes, individuals with diploid genotype  $ij$  are equivalent to those with diploid genotype  $ji$ ; for simplicity, we use  $xx_{ij}$  and  $yy_{ij}$  with  $i \neq j$  to denote the average of these frequencies,  $xx_{ij} = (x_i^{\varphi,s} x_j^{\delta,s} + x_j^{\varphi,s} x_i^{\delta,s})/2$  and  $yy_{ij} = (y_i^{\varphi,s} y_j^{\delta,s} + y_j^{\varphi,s} y_i^{\delta,s})/2$ .

Denoting the **M** locus genotype by  $b \in \{MM, Mm, mm\}$  and the **X** locus genotype by  $c \in \{XX, XY, YY\}$ , zygotes develop as females with probability  $k_{bc}$ . Therefore, the frequencies of XX females are given by  $xx_{ij}^\varphi = k_{bc} xx_{ij}$ , XY females are given by  $xy_{ij}^\varphi = k_{bc} xy_{ij}$ , and YY females are given by  $yy_{ij}^\varphi = k_{bc} yy_{ij}$ . Similarly, XX male frequencies are  $xx_{ij}^\delta = (1 - k_{bc}) xx_{ij}$ , XY male frequencies are  $xy_{ij}^\delta = (1 - k_{bc}) xy_{ij}$ , and YY males frequencies are  $yy_{ij}^\delta = (1 - k_{bc}) yy_{ij}$ . This notation allows both the ancestral and novel sex-determining regions to determine zygotic sex according to an XY system, a ZW system, or an environmental sex-determining system. In addition, we can consider any epistatic dominance relationship between the two sex-determining loci. Here, we assume that the ancestral sex-determining system (**X** locus) is XY ( $k_{MMXX} = 1$  and  $k_{MMXY} = k_{MMYY} = 0$ ) or ZW ( $k_{MMZZ} = 0$  and  $k_{MMZW} = k_{MMWW} = 1$ ) and epistatically recessive to a dominant novel sex-determining locus, **M** ( $k_{Mmc} = k_{mmc} = k$ ).

Selection among diploids then occurs according to the diploid genotype at the **A** locus,  $l \in \{AA, Aa, aa\}$ , for an individual of type  $ij$  (see Table 1). The diploid frequencies after selection in sex  $\circ$  are given by  $xx_{ij}^{\circ,s} = w_l^\circ xx_{ij} / \bar{w}_D^\circ$ ,  $xy_{ij}^{\circ,s} = w_l^\circ xy_{ij} / \bar{w}_D^\circ$ , and  $yy_{ij}^{\circ,s} = w_l^\circ yy_{ij} / \bar{w}_D^\circ$ , where  $\bar{w}_D^\circ = \sum_{i=1}^4 \sum_{j=1}^4 w_l^\circ xx_{ij} + w_l^\circ xy_{ij} + w_l^\circ yy_{ij}$  is the mean fitness of diploids of sex  $\circ$ .

Finally, these diploids undergo meiosis to produce the next generation of gametes. Recombination and sex-specific meiotic drive occur during meiosis. Here, we allow any relative locations for the **X**, **A**, and **M** loci by using three parameters to describe the recombination rates between them.  $R$  is the recombination rate between the **A** and **M** loci,  $\rho$  is the recombination rate between the **M** and **X** loci, and  $r$  is the recombination rate between the **A** and **X** loci (Fig 1). S1 Table shows replacements that can be made for each possible ordering of the loci assuming that there is no cross-over interference. During meiosis in sex  $\circ$ , meiotic drive occurs such that,

in *Aa* heterozygotes, a fraction  $\alpha^\circ$  of gametes produced carry the *A* allele and  $(1 - \alpha^\circ)$  carry the *a* allele.

Among gametes from sex  $\circ$ , the frequencies of haplotypes (before gametic competition) in the next generation are given by

$$\begin{aligned} x_1^{\circ'} = & xx_{11}^{\circ,s} + xx_{13}^{\circ,s}/2 + (xx_{12}^{\circ,s} + xx_{14}^{\circ,s})\alpha^\circ \\ & - R(xx_{14}^{\circ,s} - xx_{23}^{\circ,s})\alpha^\circ \\ & + (xy_{11}^{\circ,s} + xy_{13}^{\circ,s})/2 + (xy_{12}^{\circ,s} + xy_{14}^{\circ,s})\alpha^\circ \\ & - r(xy_{12}^{\circ,s} - xy_{21}^{\circ,s})\alpha^\circ - \rho(xy_{13}^{\circ,s} - xy_{31}^{\circ,s})/2 \\ & + [-(R+r+\rho)xy_{14}^{\circ,s} + (R+\rho-r)xy_{41}^{\circ,s} \\ & + (R+r-\rho)xy_{23}^{\circ,s} + (R+\rho-r)xy_{32}^{\circ,s}]\alpha^\circ/2 \end{aligned} \quad (S1.1a)$$

$$\begin{aligned} x_2^{\circ'} = & xx_{22}^{\circ,s} + xx_{24}^{\circ,s}/2 + (xx_{12}^{\circ,s} + xx_{23}^{\circ,s})\alpha^\circ \\ & - R(xx_{23}^{\circ,s} - xx_{14}^{\circ,s})\alpha^\circ \\ & (xy_{22}^{\circ,s} + xy_{24}^{\circ,s})/2 + (xy_{21}^{\circ,s} + xy_{23}^{\circ,s})(1 - \alpha^\circ) \\ & - r(xy_{21}^{\circ,s} - xy_{12}^{\circ,s})(1 - \alpha^\circ) - \rho(xy_{24}^{\circ,s} - xy_{42}^{\circ,s})/2 \\ & + [-(R+r+\rho)xy_{23}^{\circ,s} + (R+\rho-r)xy_{32}^{\circ,s} \\ & + (R+r-\rho)xy_{14}^{\circ,s} + (R+\rho-r)xy_{41}^{\circ,s}](1 - \alpha^\circ)/2 \end{aligned} \quad (S1.1b)$$

$$\begin{aligned} x_3^{\circ'} = & xx_{33}^{\circ,s} + xx_{13}^{\circ,s}/2 + (xx_{23}^{\circ,s} + xx_{34}^{\circ,s})\alpha^\circ \\ & - R(xx_{23}^{\circ,s} - xx_{14}^{\circ,s})\alpha^\circ \\ & (xy_{33}^{\circ,s} + xy_{31}^{\circ,s})/2 + (xy_{32}^{\circ,s} + xy_{34}^{\circ,s})\alpha^\circ \\ & - r(xy_{34}^{\circ,s} - xy_{43}^{\circ,s})\alpha^\circ - \rho(xy_{31}^{\circ,s} - xy_{13}^{\circ,s})/2 \\ & + [-(R+r+\rho)xy_{32}^{\circ,s} + (R+\rho-r)xy_{23}^{\circ,s} \\ & + (R+r-\rho)xy_{41}^{\circ,s} + (R+\rho-r)xy_{14}^{\circ,s}]\alpha^\circ/2 \end{aligned} \quad (S1.1c)$$

$$\begin{aligned} x_4^{\circ'} = & xx_{44}^{\circ,s} + xx_{34}^{\circ,s}/2 + (xx_{14}^{\circ,s} + xx_{24}^{\circ,s})\alpha^\circ \\ & - R(xx_{14}^{\circ,s} - xx_{23}^{\circ,s})\alpha^\circ \\ & (xy_{44}^{\circ,s} + xy_{42}^{\circ,s})/2 + (xy_{41}^{\circ,s} + xy_{43}^{\circ,s})(1 - \alpha^\circ) \\ & - r(xy_{43}^{\circ,s} - xy_{34}^{\circ,s})(1 - \alpha^\circ) - \rho(xy_{42}^{\circ,s} - xy_{24}^{\circ,s})/2 \\ & + [-(R+r+\rho)xy_{41}^{\circ,s} + (R+\rho-r)xy_{14}^{\circ,s} \\ & + (R+r-\rho)xy_{32}^{\circ,s} + (R+\rho-r)xy_{23}^{\circ,s}](1 - \alpha^\circ)/2 \end{aligned} \quad (S1.1d)$$

$$\begin{aligned} y_1^{\circ'} = & yy_{11}^{\circ,s} + yy_{13}^{\circ,s}/2 + (yy_{12}^{\circ,s} + yy_{14}^{\circ,s})\alpha^\circ \\ & - R(yy_{14}^{\circ,s} - yy_{23}^{\circ,s})\alpha^\circ \\ & (xy_{11}^{\circ,s} + xy_{31}^{\circ,s})/2 + (xy_{21}^{\circ,s} + xy_{41}^{\circ,s})\alpha^\circ \\ & - r(xy_{21}^{\circ,s} - xy_{12}^{\circ,s})\alpha^\circ - \rho(xy_{31}^{\circ,s} - xy_{13}^{\circ,s})/2 \\ & + [-(R+r+\rho)xy_{41}^{\circ,s} + (R+\rho-r)xy_{14}^{\circ,s} \\ & + (R+r-\rho)xy_{32}^{\circ,s} + (R+\rho-r)xy_{23}^{\circ,s}]\alpha^\circ/2 \end{aligned} \quad (S1.1e)$$

$$\begin{aligned}
 y_2^{\circ'} = & yy_{22}^{\circ,s} + yy_{24}^{\circ,s}/2 + (yy_{12}^{\circ,s} + yy_{23}^{\circ,s})\alpha^{\circ} \\
 & - R(yy_{23}^{\circ,s} - yy_{14}^{\circ,s})\alpha^{\circ} \\
 & (xy_{22}^{\circ,s} + xy_{42}^{\circ,s})/2 + (xy_{12}^{\circ,s} + xy_{32}^{\circ,s})(1 - \alpha^{\circ}) \\
 & - r(xy_{12}^{\circ,s} - xy_{21}^{\circ,s})(1 - \alpha^{\circ}) - \rho(xy_{42}^{\circ,s} - xy_{24}^{\circ,s})/2 \\
 & + [-(R + r + \rho)xy_{32}^{\circ,s} + (R + \rho - r)xy_{23}^{\circ,s} \\
 & + (R + r - \rho)xy_{41}^{\circ,s} + (R + \rho - r)xy_{14}^{\circ,s}](1 - \alpha^{\circ})/2
 \end{aligned} \tag{S1.1f}$$

$$\begin{aligned}
 y_3^{\circ'} = & yy_{33}^{\circ,s} + yy_{13}^{\circ,s}/2 + (yy_{23}^{\circ,s} + yy_{34}^{\circ,s})\alpha^{\circ} \\
 & - R(yy_{23}^{\circ,s} - yy_{14}^{\circ,s})\alpha^{\circ} \\
 & (xy_{33}^{\circ,s} + xy_{13}^{\circ,s})/2 + (xy_{23}^{\circ,s} + xy_{43}^{\circ,s})\alpha^{\circ} \\
 & - r(xy_{43}^{\circ,s} - xy_{34}^{\circ,s})\alpha^{\circ} - \rho(xy_{13}^{\circ,s} - xy_{31}^{\circ,s})/2 \\
 & + [-(R + r + \rho)xy_{23}^{\circ,s} + (R + \rho - r)xy_{32}^{\circ,s} \\
 & + (R + r - \rho)xy_{14}^{\circ,s} + (R + \rho - r)xy_{41}^{\circ,s}]\alpha^{\circ}/2
 \end{aligned} \tag{S1.1g}$$

$$\begin{aligned}
 y_4^{\circ'} = & yy_{44}^{\circ,s} + yy_{34}^{\circ,s}/2 + (yy_{14}^{\circ,s} + yy_{24}^{\circ,s})\alpha^{\circ} \\
 & - R(yy_{14}^{\circ,s} - yy_{23}^{\circ,s})\alpha^{\circ} \\
 & (xy_{44}^{\circ,s} + xy_{24}^{\circ,s})/2 + (xy_{14}^{\circ,s} + xy_{34}^{\circ,s})(1 - \alpha^{\circ}) \\
 & - r(xy_{34}^{\circ,s} - xy_{43}^{\circ,s})(1 - \alpha^{\circ}) - \rho(xy_{24}^{\circ,s} - xy_{42}^{\circ,s})/2 \\
 & + [-(R + r + \rho)xy_{14}^{\circ,s} + (R + \rho - r)xy_{41}^{\circ,s} \\
 & + (R + r - \rho)xy_{23}^{\circ,s} + (R + \rho - r)xy_{32}^{\circ,s}](1 - \alpha^{\circ})/2.
 \end{aligned} \tag{S1.1h}$$

The full system is therefore described by 16 recurrence equations (three diallelic loci in two sexes,  $2^3 \times 2 = 16$ ). However, not all diploid types are produced under certain sex-determining systems. For example, with the  $M$  allele fixed and an ancestral XY sex-determining system, there are XX females and XY males ( $x_3^{\circ} = x_4^{\circ} = y_3^{\circ} = y_4^{\circ} = y_i^{\circ} = 0, \forall i$ ). In this case, the system only involves six recursion equations, which we assume below to calculate the equilibria.